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The relation between Pasquill Stability P and Kazanski-Monin Stability μ.

by F. B. Smith

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Summary

In 1961, Pasquill introduced his well-known stability classification for the rate at which ground-level continuous-source plumes diffuse upwards as a function of downwind distance. He deliberately expressed his stability classes in terms of two readily assessable parameters: a broad assessment of the intensity of the incoming solar radiation and the magnitude of the wind velocity at 10 metres.

It was obvious both then and now that could the sensible heat flux be estimated without recourse to complex measurements it would be a much better parameter to use than the incoming solar radiation.

This paper argues that for a given surface roughness z_o, the diffusion process in ideal conditions is determined entirely by the friction velocity u_* and the sensible heat flux H, and the argument is seen to be entirely consistent with Pasquill's concept. It goes on to demonstrate that Pasquill's stability P (as defined by Smith (1972) in his numerical representation of Pasquill's classes) can be expressed in terms of H and u_* in the non-dimensional combination

$$\mu = -\frac{g k^2 H}{\rho c_p T f u_*^2} \simeq -0.038 \frac{H}{u_*^2}$$

defined by Kazanski and Monin (1960).

An approximate means of allowing for different z_o in the development of the plume out to one or two kilometres downwind is discussed.

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The dispersion of a passive gas or an ensemble of 'particles' released continuously from a source near the ground is controlled by the wind and turbulence fields. Vertical turbulence has a relatively short time-scale (order of minutes or tens of minutes) and comes into quasi-equilibrium with the wind and temperature fields and the surface sources of energy. The wind and temperature fields on the other hand have much longer time-scales (order of hours) and they respond rather slowly to varying terrain, changes in the synoptic pattern, and the surface fluxes of heat and momentum. These changes are almost always present. The question is: how important are they to the structure of turbulence in the boundary layer? Turbulence measurements obtained during field experiments are admittedly somewhat biased in this respect; they are normally taken in relatively stationary meteorological situations at sites which are remarkable for their spatially uniform surface characteristics. However measurements made at non-ideal sites like Cardington do not appear to show dramatic departure from those collected over very homogeneous countryside, like that in the Minnesota and Kansas experiments, for example.

Generally then, it appears that provided the spatial and temporal inhomogeneities are not too great, the structure of the vertical turbulence, and hence the diffusion rate, should be principally governed by the surface sources of turbulent energy.

Although it does not affect the generality of the argument, we will restrict further thoughts on this matter to diffusion conditions which can be usefully described in terms of an eddy diffusivity K(z).

The two dimensional diffusion equation for a continuous passive plume emanating from a fixed source is

$$u \frac{\partial c}{\partial x} = \frac{\partial}{\partial z}\left[K \frac{\partial c}{\partial z}\right] \qquad (1)$$

where $K(z)$ is the eddy diffusivity. Pasquill (1974) describes the limitations to be recognised in the application of (1), especially in non-neutral conditions and for elevated sources. He quotes two possible forms for $K(z)$ in his equations (6.3) and (6.4). An exactly equivalent form is:

$$K(z) = a\,\sigma_w \lambda_m \qquad (2)$$

where σ_w is the root-mean-square turbulent vertical velocity and λ_m is the spectral length-scale (related to the physical dimension of the eddies of greatest energy density). Now σ_w, λ_m and the wind speed $u(z)$ appearing in (1) all depend on the basic internal parameters that describe the structure of the atmospheric boundary layer. Of these, the two paramount parameters are the surface fluxes of heat and momentum which are the sources of turbulence and hence determine the dispersive character of the layer. The surface heat flux H is normally positive (i.e. upwards) during the day except near sunrise or sunset, or if relatively warm air is flowing over cold ground, and is negative (downwards) at night. The flux of momentum is identical to the shearing stress τ that the atmosphere exerts on the underlying surface. We find it convenient to replace τ (which has rather complex dimensions) by the friction velocity $u_* = \sqrt{\frac{\tau}{\rho}}$ (ρ = air density), which has, as its name implies, the dimensions of a velocity. Usually u_* is about 0.02 to 0.06 of the geostrophic wind G, depending on how rough the surface is, and how large H is.

Given u_* and H, then $K(z)$ is almost determined, but not quite. For highly elevated sources the height z_i of the top of the boundary layer can be important. Often z_i can be roughly estimated in terms of u_* and H but there are occasions when this is not so. Excluding these cases : $K = K(u_*, H, z)$. The wind speed is also determined by the same parameters, except that the surface roughness z_o (related to the physical dimensions of the surface elements - grass blades, sand particles etc) is also relevant and causes a factor constant with height to be added to u.

The concentration profile for a ground-level source is therefore determined by

(i) the source strength Q

(ii) u_* and H

(iii) surface roughness z_o

For a given surface roughness and heat flux there is a direct relation between u_* and the wind speed measured at some standard height, say 10 metres. The root-mean-square height of the pollutant "particles", σ_z, is in itself independent of Q; thus $\sigma_z = \sigma_z(u_{10}, H, z_o)$.

Pasquill (1961) when he first developed the classification scheme which allowed

σ_z to be estimated in terms of observable parameters u_{10} and the intensity of incoming solar radiation, was calling upon more intuitive principles, but in so far as H is determined largely by the incoming radiation, we can see that his intuition was correct. The form of the relationship for σ_z was determined empirically from actual observations of σ_z at relatively short range and it remains a largely empirical relationship even to this day. However in 1972, Smith, using tentative profiles for the eddy diffusivity K(z) in unstable, neutral and stable conditions (see Figure 1), solved the diffusion equation numerically and Pasquill's classification scheme was broadly confirmed.

Pasquill's scheme categorised different dispersive conditions into 4 classes in unstable conditions, calling them A (the most unstable), B, C and D (near neutral). These have been later called Pasquill Stability classes (P). This scheme and the corresponding σ_z curves are displayed in Figure 2. Smith's (1972) modified scheme in addition allows for

(1) the effect of surface roughness z_o

(2) a more continuous variation of stability. Figure 3 is a graphical rationalisation of Pasquill's table used by Smith.

In recent times much more data have become available from large-scale field experiments on the turbulence structure in the boundary later which has enabled proper parameterisation of the K-profiles to be made. New solutions of the diffusion equation are being obtained but await full verification.

From a theoretical standpoint the more fundamental relation

$$\sigma_z = \sigma_z(u_*, H, z_o) \qquad (3)$$

is incomplete. The question must be faced: what is the form of this dependance? One would anticipate that, for a given z_o , the other two parameters should be combined in a way that expresses "stability". The only such combination which is independent of height is in terms of the Kazanski-Monin stability μ :

$$\mu = \frac{k u_*}{fL} \qquad (4)$$

where L is the Monin-Obukhov length scale = $-\frac{\rho c_p T u_*^3}{g k H}$

f is the coriolis parameter = 1.12×10^{-4} at latitude 51°N,

k is von Karman's constant ≈ 0.4

Thus $\mu \propto H/u_*^2$ and at our latitude $\mu = -0.038 \frac{H}{u_*^2}$. It would be reasonable to postulate that:

$$\sigma_z = \sigma_z(\mu, z_o) \qquad (5)$$

If we view Figure 3 in this light we see that the μ- dependance is roughly, if not exactly confirmed. The dashed line on the Figure represents $|\mu| = 25$ and corresponds reasonably well with a constant value of P = 2.5. Some of the differences may well be due to the inevitably approximate nature of the graphical interpolation of Pasquill's original table.

Making a more thorough analysis of the relationship between P and μ over all likely values of H and u_{10} (or u_*) shows that a good relationship is given by :

$$P = \frac{3.6}{1 + X(\mu_*)} \qquad (6)$$

where $X(\mu_*) = 0.53\mu_* + 4.9\mu_*^2 - 2\mu_*^3$

and $\mu_* = 0.01|\mu| = 3.8 \times 10^{-4} \frac{H}{u_*^2}$

where H is in Wm^{-2} and u_* is in ms^{-1}.

Table 1 compares values of P read off Smith's curves and calculated using equation (6).
The root-mean-square difference is $\sigma_P = 0.22$.

H	u_{10}	u_*	μ	$P_{equ(6)}$	P_{curves}
250	8	0.73	17.83	2.91	3.00
	6	0.566	29.65	2.34	2.40
	5	0.488	39.89	1.93	1.85
	4	0.407	57.35	1.42	1.37
	3	0.335	84.65	0.96	0.92
	2	0.248	154.50	0.59	0.60
200	8	0.725	14.46	3.07	3.15
	6	0.560	24.23	2.59	2.58
	5	0.483	32.58	2.22	2.10
	4	0.402	47.03	1.69	1.58
	3	0.328	70.64	1.16	1.24
	2	0.245	126.6	0.66	0.92
150	8	0.717	11.09	3.22	3.25
	6	0.553	18.64	2.86	2.80
	5	0.476	25.16	2.55	2.36
	4	0.395	36.53	2.06	1.90
	3	0.320	55.66	1.46	1.56
	2	0.239	99.79	0.81	1.30
100	8	0.710	7.54	3.37	3.40
	6	0.543	12.89	3.14	3.08
	5	0.465	17.57	2.92	2.62
	4	0.384	25.77	2.52	2.24
	3	0.309	39.80	1.93	2.00
	2	0.233	70.00	1.17	1.80
50	8	0.695	3.93	3.50	3.50
	6	0.528	6.82	3.40	3.30
	5	0.448	9.47	3.30	3.00
	4	0.369	13.95	3.09	2.75
	3	0.294	21.98	2.70	2.58
	2	0.221	38.90	1.97	2.45

TABLE 1 $\quad z_o = 0.1$ metres.

Table 2 shows the bounds of the Pasquill stability categories:

Pasquill Class	Stability P		Kazanski-Monin Stability			
	P_{min}	P_{max}	μ_{max}	μ_{min}	μ'_{max}	μ'_{min}
A	0	1	∞	87.5	∞	$(7/3)^2$
B	1	2	87.5	38	$(7/3)^2$	$(7/3)$
C	2	3	38	16	$(7/3)$	1
D	3	3.6	16	0	1	0

TABLE 2 $\quad \mu' = \frac{7}{3} \frac{H}{10^3 u_*^2}$

One should note the regular "geometric" progression in the bounds of μ' (and hence also of μ).

A revised set of curves yielding P consistent with equation (6) in terms of u_{10} and H is given in Figure 4, and these are seen to be very similar to the original curves in Figure 3.

Pasquill's original table is not very specific about the magnitude of the sensible heat flux and talks instead in terms of strong, moderate or slight incoming solar radiation. Smith(1972) in his graphical representation of this table made the following assumptions:

(1) strong insolation implies $R_n > 600 \text{ Wm}^{-2}$
moderate insolation implies $300 < R_n < 600 \text{ Wm}^{-2}$
slight insolation implies $0 < R_n < 300 \text{ Wm}^{-2}$

(2) the sensible heat flux H is broadly related to R_n by the simple formula

$$H = 0.4 (R_n - 100) \qquad (7)$$

in typical countryside conditions.

Generally category D occurs much more frequently than category C, and C more frequently than B, and B more than A. Consequently the average heat flux within an insolation range is likely to lie between the lowest value and the mid-point arithmetic mean.

Thus for strong insolation the average H probably lies between 200 and 250 Wm^{-2}
for moderate ... ... 100 and 150
for slight ... ... 0 and 50

Using Figure 4, we may construct a Table comparing Pasquill's original class estimates derived from actual σ_z -data, and those derived from equation (6), using the above likely ranges for H.

		INSOLATION								
		STRONG			MODERATE			SLIGHT		
10m. wind speed (ms^{-1})	H=	Old	New 250	200	Old	New 150	100	Old	New 50	0
2		A	A	A	A/B	A	A/B	B	A/B	D
2-3		A/B	A	A/B	B	A/B	B	C	B	D
3-5		B	B	B	B/C	B/C	C	C	C	D
5-6		C	C	C	C/D	C	C/D	D	D	D
> 6		C	C	C/D	D	D	D	D	D	D

TABLE 3 a comparison of old and new estimates of Pasquill stability classes.

As can be seen, very good agreement is achieved, supporting the postulated relationship between P and μ.

The above analysis has all been carried out for a surface roughness $z_o = 0.1$m typical of fairly open agricultural countryside. The question remains, can we predict the behaviour of σ_z for other z_o? Smith (1972) supplied a relationship by solving the diffusion equation repeatedly for different z_o. An alternative but rather approximate approach is achieved by noting that in neutral conditions where

$$u(z) = \frac{u_*}{k} \ln\left[\frac{z}{z_o}\right]$$

then $$u(z, z_{o1}) = u(z, z_{o2}) + \text{constant} \tag{8}$$

for a <u>specified</u> u_*. The constant is $$\tilde{u} = \frac{u_*}{k} \ln\left[\frac{z_{o2}}{z_{o1}}\right] \tag{9}$$

The same is also true in unstable conditions. If $z_o/|L|$ is small the constant is identical to (10), otherwise it is rather more complicated in form.

For specified H and u_*, the terms in the diffusion equation (1) are identical for two different z_o surfaces, except for the velocities $u(z)$ which differ by $\tilde{u}$. If $\tilde{u}$ is positive ($z_{o2} > z_{o1}$) then the plume is advected more quickly downstream and is shallower at a given x for $z_o = z_{o1}$ than for z_{o2}. By appealing to the close identity between the growth of an ensemble of serially released particles in terms of time of travel, as given by Lagrangian similarity theory and the growth of a continuous-source plume, with distance, given by the Eulerian diffusion equation (Smith, 1978) we can relate distance x to a mean time of travel t of those particles at x :

$$t = \frac{\bar{Z}_2}{ku_*} \tag{10}$$

where $\bar{Z}_2$ is the mean height of those particles over the surface z_{o2}. The addition of a constant $\tilde{u}$ to $u(z, z_{o2})$ increases x by $\tilde{u}t = \tilde{u}\frac{\bar{Z}_2}{ku_*}$. Consequently

$$\bar{Z}_1\left(x + \tilde{u}\frac{\bar{Z}_2}{ku_*}\right) = \bar{Z}_2(x) \tag{11}$$

The plume development over a surface z_{o1} can therefore be constructed from that over surface z_{o2}.

These relationships are strictly valid only in the lower part of the boundary layer below about 100 metres where Monin-Obukhov similarity theory applies. This approximate approach can only be applied then out to one or two kilometres downwind of relatively low level sources.

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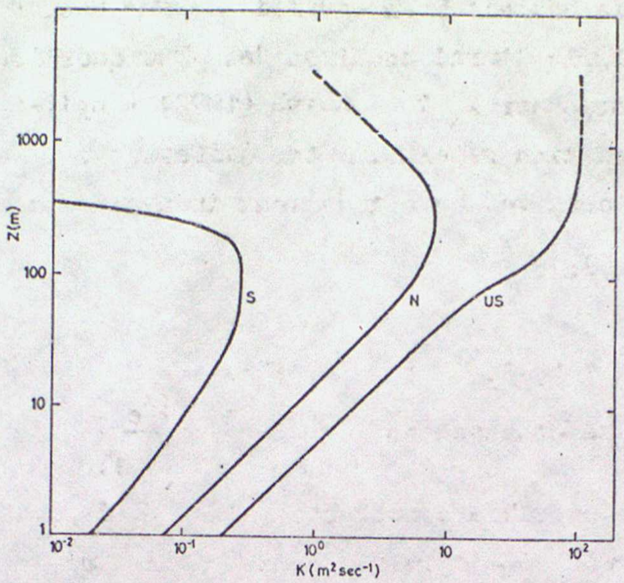

FIG. 1. Tentative profiles of K based on Eq. (6.4) and limited data for ϵ and λ_m. $z_0=3$ cm, geostrophic wind 4 m/sec, L −7, ∞ and +4 m. (From F. B. Smith 1973).

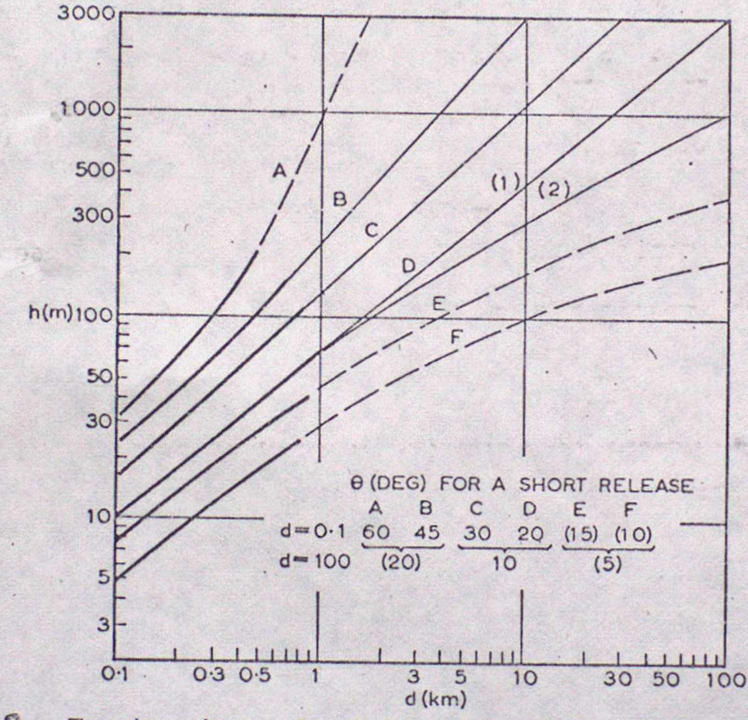

FIG. 2. Tentative estimates of vertical spread ($h\simeq 2{\cdot}15\sigma_z$) and angular lateral spread ($\theta\simeq 4{\cdot}3\sigma_y/x$) for a source in open country.

Key to stability categories

Surface wind speed (m/sec)	*Insolation*			*Night*	
	Strong	*Moderate*	*Slight*	*Thinly overcast or* ≥4/8 *low cloud*	≤3/8 *cloud*
<2	A	A–B	B	—	—
2–3	A–B	B	C	E	F
3–5	B	B–C	C	D	E
5–6	C	C–D	D	D	D
>6	C	D	D	D	D

(for A–B take average of values for A and B etc.)

Strong insolation corresponds to sunny midday in midsummer in England, slight insolation to similar conditions in midwinter. Night refers to the period from 1 hr before sunset to 1 hr after dawn. The neutral category D should also be used, regardless of wind speed, for overcast conditions during day or night, and for any sky conditions during the hour preceding or following night as defined above. The D(1) curve should be followed to the top of the dry-adiabatic layer; thereafter, in sub-adiabatic conditions, D(2) or a curve parallel to D(2) should be followed. (Pasquill 1961, from *The Meteorological Magazine*, February 1961, H.M.S.O. Crown Copyright Reserved)

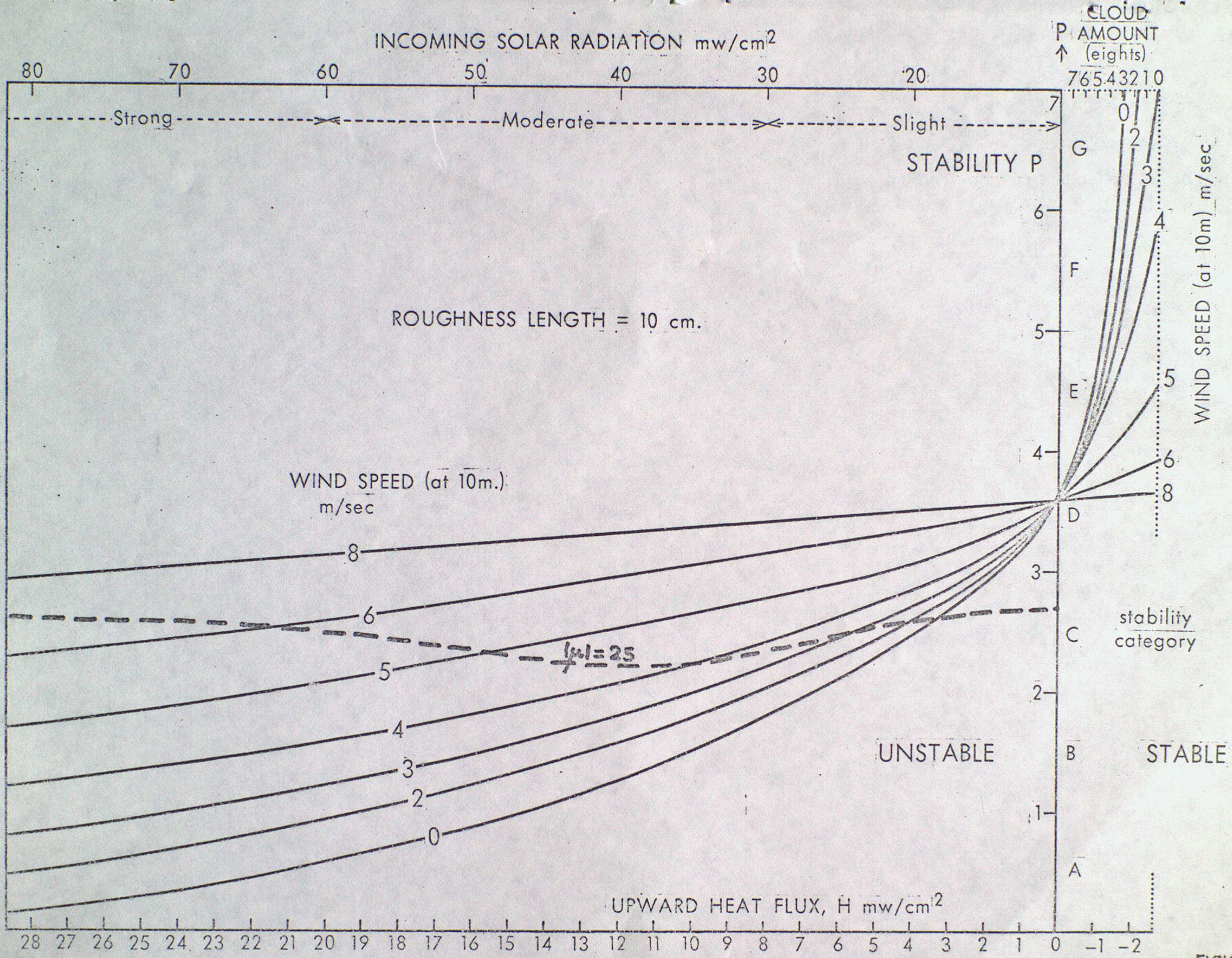
INCOMING SOLAR RADIATION mw/cm2
80
70
60
50
40
30
20
Strong
Moderate
Slight
STABILITY P
P
CLOUD AMOUNT (eights)
7 6 5 4 3 2 1 0
G
F
E
D
C
B
A
7
6
5
4
3
2
1
ROUGHNESS LENGTH = 10 cm.
WIND SPEED (at 10m.) m/sec
8
6
5
4
3
2
0
|μ| = 25
WIND SPEED (at 10m) m/sec
stability category
UNSTABLE
STABLE
UPWARD HEAT FLUX, H mw/cm2
28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0 −1 −2

FIGURE 3

FIGURE 4. The revised scheme for P based on lines of constant μ.