

AIR MINISTRY.

METEOROLOGICAL OFFICE.

GEOPHYSICAL MEMOIRS, No. 22.

(SECOND NUMBER OF VOLUME III.)

ABSOLUTE DAILY RANGE
OF
MAGNETIC DECLINATION

AT
KEW OBSERVATORY, RICHMOND, 1858 TO 1900.

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Published by the Authority of the Meteorological Committee.



LONDON :

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1923

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ABSOLUTE DAILY RANGE OF MAGNETIC DECLINATION AT KEW OBSERVATORY, RICHMOND, 1858 TO 1900.

ABSTRACT.

The absolute daily range—*i.e.*, the excess of the absolute maximum over the absolute minimum of the day—is found for all available days of the 42 years 1858 to 1873 and 1875 to 1900. The times of occurrence of the daily maximum and minimum are also determined. Mean values are found of the daily range for every month and year of the period and their relations to sunspot frequency are investigated. The year is subdivided into 73 five-day periods, and mean values of the range are found for each period, to assist in the study of the annual variation in the range. The distribution of the ranges in groups according to size is considered. The times of occurrence of the daily maximum and minimum are studied for different periods of years, with a view to investigating whether the phenomena show any secular change.

§1. INTRODUCTION.

EVEN in the case of Meteorology there are few experienced students of the subject who are prepared to admit that after an element has been observed at a station for a dozen years there is little to be gained by further observations. If this attitude is justified in meteorologists, it is doubly warranted in the case of magneticians. It is generally believed that if meteorological conditions are undergoing any secular change in Britain, that change is so slow as to be scarcely, if at all, appreciable in half a century. Assuming this, it follows that normals derived from, say, ten years would be as good as normals from 50 years, were not accidental departures from the mean in individual years so large that a short term of years is inadequate for their elimination. But in Terrestrial Magnetism we have an element highly dependent on sunspot frequency. Also the sunspot period is fully 11 years, and the differences between sunspot phenomena in successive 11-year periods are large. Thus the elimination of the “accidental” element would obviously call for a considerable multiple of 11 years. Further, we have what we may regard as the equivalent of a progressive change of by no means negligible amount in climate. For not only are the absolute values of the magnetic elements continually altering, but there is also a progressive change in the direction of the resultant magnetic force. In 1858, when the present magnetograph was started at Kew Observatory, H (horizontal force) was approximately 0.1750 C.G.S., and D (declination) was $21^{\circ} 55' W$. In 1900, the last year of the period now dealt with, H was 0.1843, and D was $16^{\circ} 53' W$. To produce a change of $1'$ in D , the necessary forces were 5.09γ in 1858, and 5.36γ in 1900, and the two forces being perpendicular to the magnetic meridians at the time were inclined to one another at an angle of 5° . If the force system to which the diurnal inequality is due had remained invariable from 1858 to 1900, a decrease of 5% would naturally have ensued in the diurnal range of D . But there is no *a priori* reason why the diurnal inequality force system should remain invariable, any more than the declination itself. A well-known theory associated with the names of Dr. Balfour Stewart and Sir Arthur Schuster would make the diurnal inequality forces vary with the vertical component of force, and apart from theory some dependence of the diurnal inequality on the earth's field is what we should naturally expect. We should not in any event anticipate a direct proportionality between the diurnal inequality forces and the local value of any magnetic element. For it is natural to suppose that the diurnal inequality is due to electrical currents at a very considerable height in the atmosphere, and a considerable area of a high current sheet would naturally exert a finite influence at any particular spot on the earth's surface.

§2. THE DATA AVAILABLE FOR THE EXAMINATION OF THE ABSOLUTE DAILY RANGE OF DECLINATION.

The present paper is confined to a discussion of the *absolute daily range* of D , *i.e.*, the difference between the two extreme (instantaneous) values recorded in the course of 24 hours commencing at Greenwich midnight, and to the times of occurrence of the extremes. Greenwich and Kew local times differ by only 1 min. 15 sec., so that for practical purposes the time used may be regarded as *local mean time*.

In the earlier years, up to January, 1868, westerly declination increased down the sheet, and the time scale was approximately 19·25 mm. to the hour. After 1868 westerly declination increased up the sheet, and the equivalent of one hour was 0·6 inch, or 15·25 mm. No change, however, is believed to have been made in the ordinate scale value, which has throughout been 1 mm. = 0'·87. Particulars of the relative positions of the magnet-mirror and photographic paper are given in the original description of the magnetograph by Balfour Stewart,* and they were in good agreement with measurements made in 1913—when the magnetograph was moved pending structural changes—which confirmed the accepted scale value.

The series of observations was not unbroken. The change in the time scale referred to above entailed a loss of trace in 1868 from January 8 to 15 inclusive. A much more serious break occurred in 1874. The *Report of the Kew Committee* for 1873 mentions that arrangements had been made to dismount the magnetographs "on the occasion of painting the basement." The *Report* for 1874 announces that "the magnetograph instruments were dismounted in January, 1874, for the purpose of thorough examination and readjustment. . . . The scale values were accordingly redetermined, and the instruments handed over to Mr. Adie for examination and repair. They were returned and remounted in May, but have not been set in continuous action as yet, inasmuch as it is intended that the automatic records should be suspended for the entire year. . . . The cost of these operations has been £77 10s."

The old *Minutes* of the Kew Committee show that H.M. Office of Works had decided to repaint the basement, and that General Sabine had suggested that as this entailed dismounting the magnetographs, the opportunity should be taken to give them an overhaul. It was also on General Sabine's initiative that regular recording was not resumed until 1875. The maker's charge is compatible with the execution of material structural alterations, but as the *Minutes* state that the maker's account "was far in excess of his estimate," and as there is no mention of structural change either in the *Minutes* or the annual *Report*, there presumably was none. The interruption is to be regretted, especially as the mean monthly and annual values of the daily range in Table I show an exceptionally large fall in amplitude between 1873 and 1875, which may cause some uneasiness. It can, however, I think, be satisfactorily accounted for. The earlier months of 1873 were exceptionally disturbed. The mean daily range for January was 2'·6 larger than that of any previous January. But a marked decline in disturbance set in as the year advanced. After midsummer the 1873 ranges are markedly less than the corresponding ranges in 1872. December 1873 was a really quiet month with a smaller mean range than any previous December. There were five complete days' record in January, 1874, before the magnetograph was dismounted, and the mean range from these was only 11'·28. The phenomena are thus all compatible with the setting in of a large decline *before* the magnetographs were dismounted.

Though the magnetograph, officially regarded, was out of action during 1874, records were actually obtained on a considerable number of days. These did not appear numerous enough to make it desirable to include 1874 with the other years,

* *Report of the British Association*, 1859, p. 200.

but the following mean monthly values of daily range from that year may be recorded: 12'·25 for May from 11 days, 12'·28 for June from 15 days, 15'·46 for July from 17 days, and 9'·68 for December from 9 days. As a precaution against an unrecorded change of scale value in 1874 a comparison was made of diurnal inequality ranges for Kew with corresponding ranges given by W. Ellis* for Greenwich for years before and after 1874. The accordance appeared equally satisfactory for the earlier and later years.

The most serious gap in the data apart from 1874 occurred in the end of August and beginning of September, 1877. Traces had been obtained, but their appearance suggested that a frictional effect arising from a hair, whose attachment to the magnet was eventually noticed, had existed for some time before discovery. Satisfactory corrections for the reduction in range arising from the friction could not be made, so the doubtful data were discarded. A gap of 4 or 5 days, including a large magnetic storm, occurred in September, 1881. A record showed that the curves had been lent to a distinguished physicist, but apparently not returned. The Astronomer Royal was kind enough, however, to supply a tracing of the corresponding Greenwich curves, so that the gap was satisfactorily filled. In 1891 and subsequent years any gaps could be filled by means of the Falmouth curves, which are now kept at Kew Observatory.

The curves were measured with a glass scale showing minutes of arc directly. The ordinates were usually read to 0'·1, and times were estimated to the nearest minute. The accuracy attainable was really less than this. Accuracy in time to one minute is difficult to secure even under the most favourable conditions. In the case of ordinary maxima and minima differences of several minutes may occur between the estimates made at different times. Some of the older traces had faded so much that accuracy in the ordinate to even 0'·5 was doubtful; in some few cases in fact the trace was quite invisible. In some days registration had been interrupted, and there was reason to fear that the maximum or minimum for the day had been lost. This was especially true of the days, usually in January, of scale value determinations. The method used was that of Broun, which requires deflections to be made on the *D* as well as on the *H* and *V* (horizontal and vertical force) traces. The procedure for a time was to put a piece of ordinary white paper on the drum, and mark on it in pencil the positions of the dots of light. Deflections being made at several distances, the scale value operations lasted quite a considerable time. The time selected was usually near midday, so there was appreciable risk of losing the *D* maximum (extreme westerly position).

Ordinary days of incomplete trace, when the maximum or minimum was believed to have been lost, were omitted from the monthly means, as their retention would have unduly lowered the mean. Days, however, of large disturbance when the trace was lost through the limits of registration being exceeded were included. Whilst the range shown on the sheet is in such a case unquestionably too small, it is usually much in excess of the average, so the monthly mean value would suffer more from the exclusion of the day than from its inclusion. The time when the trace was off the sheet was usually quite short, and there was very seldom much doubt as to the hour to which the missing maximum or minimum should be assigned. The cases in which the limits of registration were exceeded were not numerous, and in most of them the appearance of the trace did not suggest that the true daily range was much in excess of that recorded. The effect on the mean monthly range is believed in most cases to be quite insignificant. The largest magnetic storm which occurred during the period was completely recorded, so far as *D* is concerned. Changes which were made in the position of the light kept the trace on the sheet, though the range recorded exceeded the equivalent of the full width of the paper. A magnetic storm, it should be noted, often has its two extreme values on successive days, and the range during the storm may considerably exceed that on any one day.

* *Phil. Trans. R. Soc.*, Vol. 171 (1880), p. 541.

§3. THE ANNUAL VARIATION OF THE DAILY RANGE OF DECLINATION.

The daily ranges measured exceeded 15,000. To have chronicled them individually would have entailed excessive printing. Table I is confined to mean monthly and annual values. The largest and least of the monthly and annual means are in heavy type. The years of sunspot maximum and minimum are also distinguished by heavy type, 1860 being the first year of maximum and 1867 of minimum. The year 1870 having the largest mean range had the largest sunspot frequency of the 19th century; 1872 was the only other year in which the daily mean range exceeded 20'. On the other hand, the mean range fell short of 10' in only four years, viz., 1900 and the three successive years 1877, 1878 and 1879. The lowest sunspot frequency during the period considered occurred in 1878, but that year had a larger range than 1900.

The largest mean monthly value was 28'·17 in April, 1871, and the least 4'·73 in December, 1900, the latter being the only mean monthly value less than 5'. While the range of April, 1871, is pre-eminent, it is really less remarkable than that for November 1882. The excess of the former over the next largest April range is only 2'·01, and ten Aprils in all had ranges exceeding 20'; whereas the November, 1882, range exceeded the next largest November range by no less than 8'·04. June showed the narrowest limits of mean range, no range exceeding 20' on the one hand or falling short of 10'·5 on the other. December is the only other month in which no mean range exceeding 20' is encountered; while March, April and July are the only other months in which the mean range did not fall below 10'.

There is considerable irregularity in the run of even the mean annual ranges. Some years, however, show rather a marked rise or fall in the great majority of the monthly values as compared with the previous year. The most conspicuous examples of a rise as compared with the previous year are afforded by 1869, 1870, 1880, 1891 and 1892. The most conspicuous falls as compared with the previous year are afforded by 1867, 1873 (after January), 1877, 1897 and 1900. The big fall between 1873 and 1875 has been already referred to.

In some months of the year there is normally a considerable progressive change in the amplitude of the daily range. Table II represents an attempt to find the times of the normal year at which the largest and least ranges occur. It was realised that even 42 years would not suffice to fix the dates to a day. The range on a highly disturbed day may be ten times that of the average day of the month, so that the "accidental" element even in a 42-year mean may be very sensible. Accordingly the year was divided into 5-day groups, and mean daily ranges were got out for these. In this case the means, being each based on 210 days, are much less affected by accident. Occasionally, of course, a day or two was missing. The first column in Table II gives the date of the central day of each 5-day group. In leap years February 29 was included with March 1 in the group referred to February 27. The second column of Table II gives the mean daily range from the forty-two 5-day groups which centred at the date specified in the first column. The greatest and the least of the 42 values appear in the third and fourth columns respectively. The data in the fifth column are smoothed, representing means from three successive entries in the second column. They thus represent means from 15-day groups centering at the date given in the first column. The figures in the sixth and seventh columns are of the same nature exactly as those in the fifth column, except that they are derived from selected groups of years. The one group, termed for brevity sunspot maximum years, includes the following 14 years of large sunspot frequency 1859, 1860, 1861, 1869, 1870, 1871, 1872, 1882, 1883, 1884, 1892, 1893, 1894 and 1895. The other group, described as that of sunspot minimum years, includes the 12 years 1866, 1867, 1876, 1877, 1878, 1879, 1887, 1888, 1889, 1890, 1899 and 1900.

TABLE I.—MEAN MONTHLY AND ANNUAL VALUES OF ABSOLUTE DAILY RANGE.

Absolute Daily Range.														
Year.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Sun-spot No.
1858	'	'	'	'	'	'	'	'	'	'	'	'	'	54.8
59	12.57	17.07	19.94	20.06	15.09	16.50	16.52	15.04	19.58	21.04	13.53	14.96	16.83	93.8
60	11.19	17.17	19.90	23.85	16.04	17.77	17.25	22.85	26.10	22.65	14.70	16.30	18.81	95.7
61	13.37	15.61	23.05	21.26	17.50	19.80	21.17	26.94	20.25	16.38	12.41	11.74	18.29	77.2
62	18.85	16.77	19.71	19.71	15.93	16.49	15.77	19.36	17.76	16.93	13.32	16.46	17.26	59.1
63	14.50	14.39	14.20	17.52	15.45	15.21	19.02	21.31	18.50	24.81	19.19	18.02	17.68	44.0
64	20.90	19.32	18.46	18.93	16.88	15.34	16.36	17.94	19.97	18.65	16.64	15.55	17.91	47.0
65	12.41	13.16	18.30	18.67	16.53	17.18	16.44	18.56	18.23	19.42	19.16	16.73	17.07	30.5
66	18.99	22.01	17.17	16.66	15.81	16.41	14.57	21.66	18.85	22.78	19.05	10.13	17.84	16.3
67	13.49	21.52	18.86	17.17	14.13	12.23	13.36	15.07	16.88	20.54	17.20	10.89	15.94	7.3
68	10.69	13.76	15.93	14.41	14.00	14.40	13.22	14.94	20.60	17.94	12.16	7.95	14.17	37.3
69	9.78	13.50	17.03	21.89	14.85	14.57	16.18	19.40	19.74	20.87	12.85	7.54	15.68	73.9
70	10.63	19.89	19.95	26.16	19.77	17.85	17.73	18.68	23.44	17.75	14.35	11.83	18.17	139.1
71	17.06	21.24	21.75	23.15	20.90	19.75	18.97	22.27	27.31	22.13	17.68	14.64	20.57	111.2
72	14.04	20.71	21.80	28.17	18.77	19.21	18.80	22.78	17.76	19.61	19.72	13.18	19.55	101.7
73	16.14	19.91	21.07	22.29	17.40	18.31	21.56	23.16	20.47	25.57	19.31	17.19	20.20	66.3
74	23.53	19.41	20.73	20.33	16.76	18.33	17.26	18.41	17.84	16.17	13.28	6.85	17.41	17.1
75	8.60	11.33	14.64	14.38	13.17	12.83	12.16	12.29	12.55	11.30	8.88	6.95	11.59	11.3
76	9.24	12.11	12.88	10.99	9.92	11.82	11.97	12.34	11.69	11.49	9.29	7.81	10.96	12.3
77	9.01	8.20	10.77	11.19	12.57	10.93	10.37	9.40	7.85	9.34	8.91	5.74	9.52	3.4
78	6.65	8.78	10.08	11.58	10.39	13.08	10.14	10.66	10.88	8.05	6.48	7.32	9.51	6.0
79	6.75	7.06	10.96	10.67	10.43	10.95	10.73	12.01	11.18	9.47	7.13	7.35	9.56	32.3
80	6.73	7.28	12.05	12.63	12.31	11.98	12.61	18.14	13.87	14.28	12.14	10.15	12.01	54.3
81	12.09	12.06	14.72	13.38	12.54	14.94	13.97	13.72	17.56	13.47	14.29	13.13	13.82	59.7
82	12.28	15.13	13.94	21.57	17.15	14.90	13.18	15.93	14.91	17.63	27.76	12.81	16.43	63.7
83	11.58	18.96	17.45	17.80	13.21	14.31	17.38	13.89	16.99	16.00	13.22	9.92	15.06	63.5
84	9.88	13.22	16.86	17.54	14.26	14.95	14.92	13.97	15.18	14.09	13.16	10.98	14.08	52.2
85	11.33	11.94	14.12	14.14	17.45	15.66	14.51	15.56	16.60	13.48	10.62	8.51	13.66	25.4
86	13.07	12.01	18.67	16.76	15.80	14.43	15.26	14.73	13.73	15.70	14.20	13.15	14.79	13.1
87	11.78	15.27	13.31	15.32	13.50	12.14	12.90	14.32	15.75	11.61	10.65	10.61	13.10	6.8
88	12.05	11.36	12.64	13.72	13.51	12.69	12.41	12.98	13.01	12.73	10.61	8.75	12.20	6.3
89	7.31	9.46	12.45	11.62	11.16	11.30	11.58	12.28	11.82	12.10	12.43	8.35	10.99	7.1
90	8.33	9.25	11.44	11.11	10.49	10.79	10.96	11.55	13.01	13.18	10.79	7.09	10.67	35.6
91	7.98	10.92	15.44	16.37	17.03	12.64	13.35	13.86	17.00	16.54	13.54	10.30	13.75	73.0
92	12.77	20.75	24.02	17.98	20.33	17.31	20.69	17.99	16.32	17.56	12.93	13.70	17.70	84.9
93	13.71	15.19	16.89	17.10	16.01	16.41	16.51	17.55	17.16	15.69	15.14	10.08	15.62	78.0
94	12.29	19.50	17.65	17.47	16.61	15.34	18.35	17.72	19.01	16.71	16.56	10.85	16.50	64.0
95	12.04	16.59	17.27	17.31	16.65	17.05	15.94	13.57	14.82	17.81	16.96	10.94	15.58	41.8
96	16.55	16.96	17.48	16.02	15.67	12.78	13.64	15.04	15.16	13.17	10.70	10.92	14.51	26.2
97	9.06	11.24	14.48	16.03	13.15	12.05	12.17	12.69	11.63	11.50	9.66	11.99	12.14	26.7
98	9.94	10.14	15.79	12.87	13.07	12.28	12.13	13.27	15.31	13.69	9.69	9.38	12.30	12.1
1899	9.85	11.99	13.58	12.75	14.40	12.92	11.29	12.02	11.64	10.21	7.62	7.84	11.34	9.5
1900	10.23	8.07	11.18	10.00	10.54	10.53	10.41	11.15	9.26	8.67	5.23	4.73	9.17	

It is clear from the second column of Table II that the amplitude of the range has a marked double oscillation in the course of the year, the principal minimum occurring in December, and the principal maximum occurring towards the end of March or beginning of April. During the latter part of April and the early part of May there is a considerable decline. The value would seem to be nearly stationary for a couple of months near midsummer. A decided rise is visible during August and September, a secondary maximum appearing late in September or early in October. There follows a decline, slow at first but accelerated during November.

"Accidental" fluctuations in the figures in the second column of Table II catch the eye, and even in the fifth column there is not a perfectly smooth progression. Judging by the run of the figures, the true normal date of the principal maximum lies between March 29 and April 13, the dates suggested by the second and fifth columns respectively. Thus it would seem to fall a little after the spring equinox. Both the second and fifth columns suggest for the secondary maximum a date close to the autumnal equinox. The principal minimum according to these columns falls

TABLE II.—ANNUAL VARIATION IN DAILY RANGE.

Days centering at	Mean of 42 years.	Larg-est.	Least.	Smoothed Means.			Days centering at	Mean of 42 years.	Larg-est.	Least.	Smoothed Means.		
				42 years.	S. Max.	S. Min.					42 years.	S. Max.	S. Min.
Jan. 3	11.06	28.00	4.00	10.69	11.81	8.14	July 2	15.37	36.36	9.42	14.92	17.70	12.13
" 8	11.74	31.06	4.40	11.50	12.24	9.00	" 2	14.82	34.88	9.64	15.10	18.32	11.89
" 13	11.69	25.56	3.86	11.92	12.34	9.77	" 12	15.10	26.26	9.42	15.09	18.45	11.78
" 18	12.32	25.94	4.50	12.39	13.04	10.62	" 17	15.34	32.00	9.20	14.99	18.30	11.44
" 23	13.17	40.74	6.20	12.63	14.24	10.17	" 22	14.53	27.92	8.16	14.71	17.38	11.33
" 28	12.40	32.08	5.76	13.09	15.88	9.82	" 27	14.26	21.40	9.04	14.75	16.81	11.64
Feb. 2	13.71	34.30	5.04	13.32	16.33	9.72	Aug. 1	15.45	26.96	8.70	15.27	17.83	11.85
" 7	13.85	26.80	5.10	14.30	17.33	11.09	" 6	16.10	35.16	9.34	16.00	19.12	12.13
" 12	15.35	38.90	6.54	14.19	16.39	11.92	" 11	16.45	48.82	5.56	16.03	19.19	11.81
" 17	13.37	33.86	5.34	14.80	17.80	12.41	" 16	15.54	26.86	9.90	15.99	18.99	12.03
" 22	15.67	31.92	7.08	14.69	18.36	11.55	" 21	15.99	32.64	9.74	15.92	18.78	12.33
" 27	15.02	34.34	6.54	15.05	18.91	11.12	" 26	16.23	31.94	10.00	16.34	19.33	12.94
Mar. 4	14.46	25.88	6.92	15.11	18.29	11.55	" 31	16.81	58.20	10.06	16.35	19.69	12.65
" 9	15.86	29.20	9.32	15.79	18.15	12.35	Sept. 5	16.02	36.42	9.52	16.55	18.97	12.96
" 14	17.04	36.38	8.74	16.52	18.69	13.56	" 10	16.82	27.18	9.38	16.54	19.48	12.57
" 19	16.65	28.42	8.68	16.65	19.18	13.38	" 15	16.79	27.08	8.14	16.61	18.43	13.29
" 24	16.27	28.22	9.12	16.97	20.36	13.36	" 20	16.23	30.12	7.92	16.68	19.37	13.67
" 29	18.00	35.92	10.28	17.25	21.33	13.50	" 25	17.02	62.18	8.78	16.11	18.65	13.27
April 3	17.48	34.34	9.32	17.52	21.51	13.47	" 30	15.08	29.64	8.84	16.31	19.06	13.46
" 8	17.07	32.14	8.22	17.41	21.23	13.50	Oct. 5	16.84	45.80	7.12	16.15	18.18	12.85
" 13	17.68	40.22	9.08	17.56	21.80	13.10	" 10	16.54	33.86	7.86	16.62	19.24	12.65
" 18	17.93	42.24	8.98	17.11	21.61	12.45	" 15	16.48	54.14	7.70	16.09	18.55	11.98
" 23	15.73	34.02	8.96	16.31	20.42	11.53	" 20	15.25	29.22	7.62	15.80	18.84	11.84
" 28	15.28	31.66	8.54	15.41	18.32	11.86	" 25	15.68	36.84	6.10	15.12	17.02	11.78
May 3	15.23	26.34	8.72	14.99	17.40	11.66	" 30	14.43	29.34	5.38	14.92	17.25	11.28
" 8	14.45	25.44	6.42	15.15	17.11	12.41	Nov. 4	14.66	28.12	6.20	14.49	17.05	10.70
" 13	15.76	37.70	8.78	15.12	17.79	11.44	" 9	14.38	28.68	5.28	14.21	17.22	10.15
" 18	15.15	34.84	7.46	15.04	17.50	11.71	" 14	13.58	32.70	4.72	14.17	17.75	9.81
" 23	14.22	21.08	9.60	14.76	17.20	11.72	" 19	14.55	87.70	4.14	13.33	16.73	9.18
" 28	14.91	22.24	8.38	14.65	16.76	12.45	" 24	11.87	19.60	4.50	12.45	14.88	9.44
June 2	14.83	24.40	9.30	14.70	17.06	12.38	" 29	10.94	18.65	4.12	11.31	12.85	8.64
" 7	14.36	23.56	9.32	14.51	17.06	12.04	Dec. 4	11.12	24.54	3.62	11.25	13.02	8.30
" 12	14.33	24.14	8.98	14.32	17.10	11.35	" 9	11.68	24.12	3.32	11.54	14.01	7.44
" 17	14.27	27.72	8.25	14.47	16.97	11.66	" 14	11.81	28.12	3.78	11.51	14.34	7.71
" 22	14.82	25.88	7.30	14.55	17.01	11.79	" 19	11.03	29.18	3.50	11.08	13.20	7.89
" 27	14.57	26.50	9.52	14.92	17.45	12.14	" 24	10.39	24.78	3.48	10.23	11.83	8.14
							" 29	9.28	15.38	3.84	10.24	11.46	7.98

a few days after the winter solstice. The rate of change near the secondary minimum in summer is so slight that its time of occurrence is somewhat doubtful. It would seem, however, to precede the summer solstice.

The figures in the third column of Table II are very irregular. Disturbances of the very first magnitude may occur at any season of the year, and a single storm of that class has a profound effect. The outstanding example was the great storm of November 1882, the mean range from the five days November 17 to 21 being 87'.7. This one storm was responsible for the conspicuous interruption in the decline of

the range during November shown in the second column. The high values in the second column answering to the dates August 31 and September 25 similarly arise from exceptional disturbances in individual years. Some of the low values in the second column, notably that for September 30, arise from an accidental (?) scarcity of large disturbances in the 5-day group. The figures in the fourth column of Table II are comparatively regular. They emphasise the fact that December is a month of less range than January. The minimum in December arises partly from the comparative scarcity of disturbances as compared with the equinoxes, and partly from the small range of the regular diurnal variation characteristic of winter. Towards midsummer, as will be seen more clearly presently, there is a marked scarcity of disturbances, but the amplitude of the regular diurnal variation is then large. Thus the absence of any clear indication of a secondary minimum in the fourth column is not surprising. The entry 5'·56 in that column for August 11 was derived from 1878—the year of lowest sunspot frequency from 1824 to 1900—It was quite abnormal, the next lowest value for that 5-day group being 9'·06 (in 1890).

The figures from the groups of years of sunspot maximum and minimum in the sixth and seventh columns agree with those in the fifth column in showing a double oscillation with a principal minimum in December. But, while the sunspot maximum years agree with the 42 years in making the spring maximum decidedly the larger, the sunspot minimum years make the autumn maximum equally prominent. In the sunspot maximum group of years the time of the autumn maximum is very indefinite, there being persistently high values from early in August to early in October. In the sunspot minimum years, on the contrary, the time of the autumn maximum close to the equinox is clearly shown. The difference between the size of the ranges from the sunspot maximum and sunspot minimum years is conspicuous. In fact no single one of the seventy-three 5-day groups fails to show a much larger range in the former group of years than in the latter.

§4. COMPARISON BETWEEN THE MEAN DAILY RANGE AND THE SUNSPOT FREQUENCY —WOLF'S FORMULA.

With the results in Table II before us, it will be clear that sunspot frequency must be taken into account when discussing the significance of declination data. Accordingly Tables III and IV should be considered together. Table III gives mean values of the daily range for the 12 months and the year derived from four successive periods of years, from the whole 42 years, and from the groups of sunspot maximum and minimum years already specified. In view of the progressive change in the force equivalent of 1', the force equivalents of the mean ranges for the year are given in the last column for the four periods. Table IV shows the sunspot frequencies which correspond with the ranges in Table III. They are calculated from the monthly and yearly values given by Wolfer.

TABLE III.—MEAN MONTHLY AND ANNUAL VALUES OF DAILY RANGES FOR
DIFFERENT GROUPS OF YEARS.

Period.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Year Force.
1858–1867 ..	14·70	17·08	18·55	18·82	15·74	16·13	16·37	19·37	19·67	20·11	15·74	13·87	17·18	88·0
1868–1878 ..	12·47	15·51	17·07	19·01	15·45	15·67	15·51	16·94	16·95	16·23	13·08	9·90	15·32	79·5
1879–1889 ..	10·44	12·16	14·29	15·01	13·76	13·48	13·59	14·32	14·60	13·69	13·29	10·34	13·25	69·5
1890–1900 ..	11·16	13·69	15·93	15·00	14·90	13·65	14·13	14·22	14·57	14·07	11·71	9·80	13·57	72·0
1858–1900 ..	12·12	14·53	16·40	16·87	14·93	14·68	14·85	16·12	16·36	15·92	13·41	10·93	14·76	
Sunspot maximum	13·27	17·90	19·38	20·81	17·18	17·10	17·73	19·05	19·11	18·32	16·23	12·89	17·41	
Sunspot minimum	9·62	11·40	12·84	12·54	12·09	11·98	11·61	12·39	12·80	12·11	9·88	7·87	11·43	

TABLE IV—MEAN MONTHLY AND ANNUAL VALUES OF SUNSPOT FREQUENCY
FOR DIFFERENT GROUPS OF YEARS.

Period.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
1858-1867	51.6	53.5	59.7	47.6	50.5	56.2	54.8	56.6	51.4	52.4	49.0	47.4	52.6
1868-1878	46.5	59.4	65.3	62.6	65.1	56.7	52.9	56.9	53.7	54.2	60.3	54.6	57.4
1879-1889	32.3	37.4	35.8	40.6	33.2	37.5	38.0	33.7	34.7	33.5	32.3	28.8	34.8
1890-1900	39.8	42.7	35.5	41.6	43.2	45.4	44.8	45.4	45.1	41.8	35.0	40.2	41.7
1858-1900	42.2	47.8	48.5	47.8	47.5	48.6	47.3	47.7	45.9	45.1	43.6	42.4	46.2
Sunspot maximum ..	72.9	84.0	82.8	92.2	91.7	88.4	84.9	88.4	80.4	82.5	81.5	81.0	84.2
Sunspot minimum ..	11.0	10.1	11.3	7.9	9.0	8.9	9.4	8.7	9.3	9.2	8.3	8.1	9.3

The two last of the four groups of years each include 11 years, and answer very approximately to a complete sunspot cycle. The second group 1868-1878 also makes a very fair representation of a complete cycle. The year 1874 is missing, but its sunspot frequency 44.7 does not differ very much from the mean. The first group 1858-1867 is less satisfactory, and it is a distinct pity that no data are available for 1857. Still the period includes a well marked sunspot maximum in 1859-60 and the subsequent minimum. The mean monthly sunspot frequencies, even for the whole 42 years, show a very sensible variation. They are decidedly higher for the 7 months February to August than for the others, the values for January, December and November being the lowest. Thus we cannot claim that any one of the sets of mean monthly ranges in Table III is fairly representative of the ideal year of uniform sunspot frequency.

The double annual variation in the range of *D* is clearly visible in every group of years in Table III, the principal minimum appearing in each case in December. The secondary minimum appears in June in the two later 11-year periods, the 42 years and the sunspot maximum years; but the sunspot minimum years place it in July, and the two earliest groups of years in May. The first group of years shows the principal maximum of range in October. In every other case the spring maximum is the larger, though in the sunspot minimum years the autumn maximum is only slightly inferior. The March value exceeds that for April in the sunspot minimum group, and in the latest group of years; in every other case the spring maximum occurs in April.

As the last column of Table IV shows, the mean sunspot frequency was considerably higher for the first two periods of years than for the two latest. It was lowest of all for the third period. Thus we should expect differences between the figures for the four periods in the last two columns of Table III. These differences, in the case of the last three periods, are in the direction the sunspot figures would lead us to expect. But the excess of the figures for the first period over those for the second remains unexplained.

In view of the variability in the monthly values of sunspot frequency in Table IV, an attempt was made to arrive at daily ranges answering to a common sunspot frequency. For this purpose use was made of the two groups of sunspot maximum and sunspot minimum years already mentioned. Taking, for illustration, the month of January, we have the following data:—

	Mean daily range.	Mean sunspot frequency.
Sunspot maximum years ..	13.27	72.9
„ minimum years ..	9.62	11.0
Excess of sunspot maximum ..	3.65	61.9

Assuming after Wolf a linear relation between the range R and the sunspot frequency S , we have $R = a + bS$, where a and b are constants.

The above figures give us $3.65 = b \times 61.9$,
whence $b = 0.0590$.

Employing this value of b , we have

$$9.62 = a + 0.0590 \times 11.0,$$

and so $a = 8.97$.

Table V gives the values obtained in this way for a , b and $100b/a$. Obviously a is the range answering to zero sunspot frequency, and $100b$ is the increase in the range answering to a sunspot frequency of 100. The sunspot frequency has only once been 0.0 in the year of absolute minimum, and it is usually somewhat less than 100 in the year of absolute maximum, still $100b/a$ is a very convenient measure of the relative importance of the sunspot influence.

It is by no means unlikely that the declination range in any particular month may be influenced by the sunspot frequency in previous months. Also accidental features would naturally not be wholly eliminated from the monthly means derived from the groups of years of sunspot maximum and minimum; thus it would not be safe to assume that the annual variation shown by the values of b and b/a in Table V is absolutely normal. The annual progression of the figures is less smooth than one would expect *a priori*, and one would certainly not anticipate so large a difference between the values of b/a for January and December

TABLE V.—WOLF'S FORMULA: CONSTANTS FROM GROUPS OF SUNSPOT MAXIMUM AND SUNSPOT MINIMUM YEARS.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Arithmetic Means from		
														Wintr.	Eqnx.	Smmr.
a	8.97	10.51	11.81	11.77	11.54	11.41	10.85	11.66	11.97	11.33	9.16	7.31	10.69	—	—	—
b	.0590	.0880	.0915	.0981	.0615	.0644	.0811	.0836	.0887	.0847	.0867	.0689	.0798	.0757	.0907	.0727
$100b/a$	0.658	0.837	0.775	0.834	0.533	0.564	0.747	0.717	0.741	0.748	0.947	0.943	0.746	0.846	0.775	0.640

Accidental features would naturally have less influence on the seasonal and yearly values. According to the seasonal figures, the sunspot influence absolutely considered is greatest at the equinox, but relatively considered it is greatest in winter. This is in harmony with the conclusions reached for the range of the regular diurnal inequality* for the period 1890 to 1900. The values of b/a now found are larger than those obtained in the case of the diurnal inequality. The values now obtained for b and b/a in the case of the year are, however, somewhat smaller than those obtained previously for the absolute ranges† of the period 1890 to 1900.

The values of a in Table V, being the ranges answering to zero sunspot frequency, may be regarded theoretically as comparable ranges. But zero sunspot frequency is a very exceptional state, so that even if accidental influences were wholly eliminated, the relation between the monthly values of a could not claim to be representative of normal conditions. The results in Table VI have a better claim to be representative. They are derived from Table III by applying a correction based on the difference between the mean sunspot frequency of the particular month and 46.2, the mean sunspot frequency for all the months of the 42 years. The values assigned to b in the calculations for the several months were derived from Table V. The differences between the monthly means for ordinary years in Table IV

* *Phil. Trans. R. Soc.*, A 208, p. 245.

† *Ibid.*, p. 246.

and 46.2 are mostly fairly small, so that an error in the value accepted for b would prejudice the result less than in the case of the range given by the value of a in Table V. Comparing Table VI with Table III we see that the general effect of the sunspot allowances in Table VI has been to reduce the difference between the spring and autumn maxima. The period 1890 to 1900 shows, however, an exception to this rule. The results from the whole 42 years are only slightly modified. They give, as in Table III, a principal maximum in April and a secondary maximum in September, a principal minimum in December and a secondary minimum in June.

TABLE VI.—DECLINATION RANGES CALCULATED FOR A UNIFORM SUNSPOT FREQUENCY OF 46.2.

Period.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Year Force.
	'	'	'	'	'	'	'	'	'	'	'	'	'	?
1858-67 ..	14.38	16.44	17.31	18.68	15.48	15.49	15.67	18.50	19.21	19.58	15.50	13.79	16.67	85.5
1868-78 ..	12.45	14.35	15.32	17.40	14.29	14.99	14.97	16.05	16.28	15.55	11.86	9.32	14.43	75.0
1879-89 ..	11.26	12.93	15.24	15.56	14.56	14.04	14.26	15.36	15.62	14.77	14.50	11.54	14.16	74.5
1890-1900 ..	11.54	14.00	16.91	15.45	15.08	13.70	14.24	14.29	14.67	14.44	12.68	10.21	13.93	74.0
1858-1900 ..	12.36	14.39	16.19	16.71	14.85	14.53	14.76	16.00	16.39	16.01	13.64	11.20	14.76	

From the figures in the last column of Table VI, which represent the force equivalents of the ranges in the preceding column, it would appear that after allowance is made for the differences in the values of H and of the sunspot frequency, the mean ranges from the three last periods of years are in very close agreement. The excess in the ranges during the earliest period remains unaccounted for.

§5. CLASSIFICATION OF YEARS ACCORDING TO AMPLITUDE OF RANGE.

Table VII arranges the 42 years according to the size of the mean range, following the descending order in the first four columns, which deal with the 21 largest ranges, and the ascending order in the last four columns, which include the 21 smallest ranges. It also gives for comparison the corresponding Wolfer sunspot frequencies, and the mean latitudes of sunspots as measured at Greenwich. Latitudes of a satisfactory character were not available for 1862 and 1867.

There are a good many exceptions to the association of large ranges with large frequencies, and low ranges with low frequencies. One of the most outstanding cases is supplied by 1893, a year of absolute sunspot maximum, frequency 84.9. Its range only slightly exceeds that for 1895, a year with a frequency of 64.0, and is fully 2' below the range for 1892, which has a frequency of 73.0. On the other hand, 1863 and 1865, with frequencies of only 44.0 and 30.5 respectively, have ranges about 0'.6 in excess of that for 1861, a year with a frequency of 77.2; 1888 and to a lesser degree 1889, are examples of very small spot frequencies unaccompanied by correspondingly small ranges. The spot latitude seems to throw no light on these exceptions. 1888 had very nearly the same spot latitude as 1900 and 1878, but its range exceeds that for 1900 by 3'. The range for 1867 appears very high for a year of sunspot minimum, still it is 1'.5 less than the range of any other year prior to 1875.

A large mean annual daily range may arise either from a general tendency in the regular diurnal variation to be large, or from the presence of an unusual amount of disturbance. The former tendency is generally if not always visible in years of many sunspots. Disturbance, however, though generally if not always small in years of few sunspots, is not always conspicuous near sunspot maximum. It was the absence of disturbance in 1893 and its prominence in 1892 which led to the inverted positions of these years in Table VII.

TABLE VII.—YEARS IN ORDER OF AMPLITUDE OF RANGE.

Years of Large Range.				Years of Small Range.			
Year.	Range.	Sunspot.		Year.	Range.	Sunspot.	
		Frequency.	Mean Latitude.			Frequency.	Mean Latitude.
1870	20.57	139.1	18	1900	9.17	9.5	7.7
1872	20.20	101.7	14.5	1878	9.51	3.4	7.6
1871	19.55	111.2	16	1877	9.52	12.3	9.6
1859	18.81	93.8	17.5	1879	9.56	6.0	22.0
1860	18.29	95.7	17.5	1890	10.67	7.1	22.0
1869	18.17	73.9	21.5	1876	10.96	11.3	11.2
1863	17.91	44.0	11	1889	10.99	6.3	11.6
1865	17.84	30.5	10	1899	11.34	12.1	9.5
1892	17.70	73.0	18.4	1875	11.59	17.1	11.2
1862	17.68	59.1	—	1880	12.01	32.3	19.6
1873	17.41	66.3	12	1897	12.14	26.2	8.0
1861	17.26	77.2	14.5	1888	12.20	6.8	7.4
1864	17.07	47.0	11	1898	12.30	26.7	10.5
1858	16.83	54.8	20.5	1887	13.10	13.1	8.4
1894	16.50	78.0	14.2	1885	13.66	52.2	11.8
1882	16.43	59.7	17.8	1891	13.75	35.6	20.3
1866	15.94	16.3	9	1881	13.82	54.3	18.3
1868	15.68	37.3	23	1884	14.08	63.5	11.3
1893	15.62	84.9	14.5	1867	14.17	7.3	—
1895	15.58	64.0	13.5	1896	14.51	41.8	14.3
1883	15.06	63.7	13.1	1886	14.79	25.4	10.4

Table VIII juxtaposes the ranges from all years with sunspot frequencies less than 10, and gives the ranges calculated for zero frequency when use is made of the value of b ascribed to the year in Table V. The sunspot frequencies being all very low, any uncertainty arising from the value of b should be small. We should infer that even near sunspot minimum some sensible cause which is still unknown is operative. The decline in range from 1878 to 1879, and from 1888 to 1890, suggests that something may depend on how long an epoch of low sunspot frequency has prevailed. It is a little significant that the period of low sunspot frequency near the minimum in 1867 was unusually short.

TABLE VIII.—MEAN RANGES FROM YEARS OF LEAST SUNSPOT FREQUENCY.

Year.	1867.	1878.	1879.	1888.	1889.	1890.	1900.
Sunspot Frequency	7.3	3.4	6.0	6.8	6.3	7.1	9.5
Observed Range	14.17	9.51	9.56	12.20	10.99	10.67	9.17
Range for Zero Frequency ..	13.59	9.24	9.08	11.66	10.49	10.10	8.41

Table IX gives the largest and smallest of the mean ranges observed for each of the 12 months and the year, with the corresponding sunspot frequencies. In seven months the highest mean range occurs with a sunspot frequency exceeding 100, but the frequency accompanying the highest mean range in February, March and December is only about average. The smallest value of the mean range for each of the 12 months is associated with a frequency much below average; at the same time the frequency accompanying the smallest mean ranges in April, June and September exceeds 10. Of the lowest monthly mean ranges 1878 and 1900 each supply 4, and all not supplied by 1900 come from the period 1876 to 1879.

TABLE IX.—YEARS OF LARGEST AND LEAST MEAN RANGE, AND ASSOCIATED SUNSPOT FREQUENCIES.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Year of largest Range ..	1873	1865	1892	1871	1870	1860	1872	1860	1870	1872	1882	1862	1870
Sunspot Frequency ..	86.7	39.3	49.9	162.4	176.0	108.6	105.5	100.3	136.0	103.5	84.4	40.9	139.1
Range	23.53	22.01	24.02	28.17	20.90	19.80	21.56	26.94	27.31	25.57	27.76	18.02	20.57
Year of least Range ..	1878	1879	1878	1900	1876	1900	1878	1877	1877	1878	1900	1900	1900
Sunspot Frequency ..	3.3	0.6	7.8	16.0	5.1	12.1	0.1	6.3	16.4	1.1	4.5	0.3	9.5
Range	6.65	7.06	10.08	10.00	9.92	10.53	10.14	9.40	7.85	8.05	5.23	4.73	9.17

Table X is a parallel table to IX. It gives particulars of the monthly and yearly ranges associated with the absolutely highest and lowest sunspot frequencies. Only three of the months of largest range are months of largest spot frequency, and only four of the months of least range are months of lowest spot frequency. The ranges for these seven months are distinguished by heavy type.

TABLE X.—YEARS OF LARGEST AND LEAST SUNSPOT FREQUENCY AND ASSOCIATED MEAN RANGES.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
Year of largest Frequency	1884	1871	1870	1871	1870	1870	1870	1870	1870	1870	1870	1870	1870
Frequency	91.5	125.3	159.4	162.4	176.0	135.6	132.4	153.8	136.0	146.4	147.5	130.0	139.1
Range	9.88	20.71	21.75	28.17	20.90	19.75	18.97	22.27	27.31	22.13	17.68	14.64	20.57
Year of least Frequency	1867	1879	1879	1878	1879	1890	1878	1878	1875	1878	1886	1900	1878
Frequency	0.0	0.6	0.0	0.1	2.4	1.3	0.1	0.0	2.4	1.1	0.3	0.3	3.4
Range	10.69	7.06	10.96	11.58	10.43	10.79	10.14	10.66	12.55	8.05	14.20	4.73	9.51

§6. VALUES OF CONSTANTS IN WOLF'S FORMULA AND CORRELATION COEFFICIENTS.

The method of calculating the values of a and b given in Table V affords no measure of the degree of correlation between the D ranges and sunspot frequency. Also the secular change apparent in the value of the range renders the ordinary method of investigating correlations inapplicable to the 42 years as a whole. Accordingly ordinary correlation methods with a "regression equation" $y = bx$, where x and y represent departure from the mean values of frequency and range respectively, were applied to the four periods 1858–67, 1868–78, 1879–89 and 1890–1900. The value of b having been found from the equation

$$b = \Sigma xy \div \Sigma x^2$$

the corresponding value of a —the range for zero frequency—was obtained by subtracting $b\bar{S}$ from the mean range for the period, \bar{S} being the mean value of S .

The correlation coefficient r was found from the usual equation

$$r = \Sigma xy \div (\Sigma x^2 \Sigma y^2)^{\frac{1}{2}}$$

The calculations were restricted to the year, and to January and December representatives of winter, March and September representatives of equinox, and June and July representatives of summer.

The resulting values of a , b , $100 b/a$ and r appear in Table XI. The values ascribed to the seasons—unlike the values ascribed to the year—are simply arithmetic means of the corresponding values from the two months taken to represent the season. The data in the last four columns are arithmetic means of the results obtained for the four periods of years. Values of b/a which are arithmetic means naturally differ from the values that would be obtained from the arithmetic means of b and of a . There are conspicuous differences between the results from the different periods of years. The values of a from the first period are in every instance considerably the largest, while the values of b from that period are invariably less than those from the second and fourth periods. The third period has in two out of the six months a slightly lower value for b than the first period, but its values for b/a are always the larger. The values of b and b/a from the third period are all inferior to the corresponding values from the second and fourth periods. The second and fourth periods give very similar values of a , b and b/a for the year. In the case of the individual months the higher values of b and b/a are found thrice in the second and thrice in the fourth period.

TABLE XI.—VALUES OF CONSTANTS IN WOLF'S FORMULA AND OF CORRELATION COEFFICIENTS.

	1858-1867				1868-1878				1879-1889				1890-1900				Means 4 periods.			
	u	b	$b/a \times 100$	r	a	b	$b/a \times 100$	r	u	b	$b/a \times 100$	r	a	b	$b/a \times 100$	r	a	b	$b/a \times 100$	r
January ..	13.75	0.0185	0.134	0.13	6.60	1.262	1.912	0.85	9.51	0.288	0.303	0.33	9.19	0.496	0.539	0.54	9.76	0.558	0.722	0.46
March ..	14.26	0.0551	0.386	0.71	12.28	0.0733	0.597	0.86	12.13	0.0604	0.498	0.73	11.91	0.1132	0.950	0.72	12.65	0.0755	0.608	0.75
June ..	13.65	0.0441	0.323	0.75	12.27	0.0600	0.489	0.89	11.62	0.0497	0.427	0.83	10.79	0.0630	0.584	0.88	12.08	0.0542	0.456	0.84
July ..	13.16	0.0585	0.445	0.88	11.44	0.0769	0.672	0.91	11.58	0.0528	0.456	0.79	10.36	0.0842	0.813	0.87	11.63	0.0681	0.597	0.86
September ..	17.89	0.0347	0.194	0.51	10.82	1.1141	1.054	0.89	12.88	0.0497	0.386	0.59	10.21	0.0968	0.948	0.81	12.95	0.0738	0.646	0.70
December ..	11.84	0.0428	0.362	0.40	6.26	0.0666	1.063	0.80	9.14	0.0417	0.457	0.43	7.55	0.0560	0.741	0.72	8.70	0.0518	0.656	0.59
Mean—																				
6 months ..	—	0.0423	0.307	0.56	—	0.0862	0.964	0.87	—	0.0455	0.408	0.61	—	0.0771	0.762	0.76	—	—	—	—
Winter ..	12.80	0.0307	0.248	0.27	6.43	0.0964	1.487	0.83	9.33	0.0353	0.380	0.38	8.37	0.0528	0.640	0.63	9.23	0.0538	0.689	0.53
Equinox ..	16.07	0.0449	0.290	0.61	11.55	0.0937	0.825	0.87	12.50	0.0550	0.442	0.66	11.06	0.1050	0.949	0.76	12.80	0.0747	0.627	0.72
Summer ..	13.40	0.0513	0.384	0.82	11.85	0.0685	0.580	0.90	11.60	0.0513	0.441	0.81	10.58	0.0736	0.699	0.88	11.82	0.0612	0.527	0.85
Year ..	15.37	0.0343	0.223	0.78	10.23	0.0884	0.864	0.95	11.11	0.0605	0.545	0.76	9.96	0.0867	0.870	0.94	11.67	0.0675	0.626	0.86

In every case the value obtained for r is highest in the second period, the value from the fourth period coming next in four months out of the six. In five months the first period supplies the lowest value of r . It follows that the correlation between D range and sunspot frequency is decidedly closer during the second and fourth periods than during the first and third, and it is decidedly less close during the first period than during the third. The value of r is always highest—*i.e.*, the correlation is closest—for the two summer months, and in most cases it is decidedly higher for the equinoctial than the winter months. In the second period, however, the difference between the different months and seasons is not large. In all cases the value of r for the year is considerably higher than the mean of the values for the six months. In the second and fourth periods the approach of r to unity in the case of the year is very close.

As regards the mean values in the final columns of Table XI, the value of a is invariably greater and the value of b invariably less than the corresponding quantity in Table V. The two tables agree in making b highest for equinox, but b/a highest for winter.

The sunspot cycle is exceptionally prominent in the second period of Table XI, and somewhat poorly represented in the third period, which may account for the difference between them. There is no obvious explanation of the poor correlation in the first period.

§7. FREQUENCY OF RANGES OF VARIOUS AMPLITUDES—SECULAR AND ANNUAL VARIATION.

Table XII gives the distribution of the ranges according to amplitude in the different years. The first column gives the date—years of sunspot maximum and minimum being in heavy type—the second the number of days to which no range could be assigned. The third column gives the number of days in which the range did not exceed 5', the fourth the number of days in which the range was greater than 5' but did not exceed 10', and so on. The number of days with ranges greater than 40' is small, and for ranges from 40' to 90' the step was 10'. Of ranges exceeding 90' there were only seven, and of these two exceeded 2°. Owing to loss of trace through the limits of registration being exceeded, the distribution of ranges exceeding 40' may not be absolutely correct. The small number of ranges not exceeding 5' in years near sunspot maximum, and their large number in years near sunspot

TABLE XII.—NUMBER OF RANGES OF SPECIFIED SIZE.

Year.	Un- cer- tain.	0' to 5'	5' to 10'	10' to 15'	15' to 20'	20' to 25'	25' to 30'	30' to 35'	35' to 40'	40' to 50'	50' to 60'	60' to 70'	70' to 80'	80' to 90'	90' to 120'	>120'
1858	3	3	53	141	75	40	27	7	7	7	1	—	1	—	—	—
1859	—	0	48	95	111	51	23	17	6	8	1	1	1	1	2	—
1860	1	6	44	94	123	46	21	10	5	6	7	3	—	—	—	—
1861	3	0	58	111	107	37	22	11	3	8	3	1	—	1	—	—
1862	—	3	54	123	88	36	23	16	11	6	2	1	1	1	—	—
1863	—	4	31	114	106	56	23	15	11	5	—	—	—	—	—	—
1864	—	4	59	122	80	36	36	13	9	6	—	1	—	—	—	—
1865	—	7	55	135	63	38	25	13	13	10	5	1	—	—	—	—
1866	4	2	84	125	68	38	18	13	5	6	2	—	—	—	—	—
1867	2	10	98	141	56	25	18	9	3	3	—	—	—	—	—	—
1868	14	14	68	120	69	37	19	12	6	5	—	2	—	—	—	—
1869	1	7	45	102	107	42	32	14	7	2	3	—	—	2	1	—
1870	1	0	27	67	124	83	26	11	11	7	3	1	1	2	1	—
1871	2	0	27	77	132	65	32	12	6	5	4	2	1	—	—	—
1872	5	0	18	90	121	64	30	11	10	10	2	4	—	1	—	—
1873	11	7	35	97	108	62	29	7	3	4	2	—	—	—	—	—
1875	3	29	104	172	31	19	5	1	0	—	1	—	—	—	—	—
1876	5	30	132	158	25	6	4	4	1	—	1	—	—	—	—	—
1877	32	39	165	102	12	9	3	2	1	—	—	—	—	—	—	—
1878	10	47	173	114	15	1	3	1	0	—	1	—	—	—	—	—
1879	4	30	185	123	16	6	1	0	0	—	—	—	—	—	—	—
1880	9	15	123	150	44	12	6	4	2	—	1	—	—	—	—	—
1881	9	5	62	180	81	17	6	2	0	1	—	1	1	—	—	—
1882	1	3	58	160	85	28	8	8	3	4	2	—	1	1	1	2
1883	1	9	65	161	71	28	15	4	5	3	3	—	—	—	—	—
1884	—	2	77	165	84	22	7	6	1	2	—	—	—	—	—	—
1885	—	19	72	161	70	27	8	4	2	1	1	—	—	—	—	—
1886	4	6	67	160	72	31	12	3	5	3	—	1	1	—	—	—
1887	—	13	94	163	56	25	10	1	3	—	—	—	—	—	—	—
1888	—	17	128	141	51	16	7	2	2	2	—	—	—	—	—	—
1889	2	34	140	138	34	5	9	2	1	—	—	—	—	—	—	—
1890	—	12	161	155	25	10	1	1	0	—	—	—	—	—	—	—
1891	—	16	77	157	69	29	5	5	5	2	—	—	—	—	—	—
1892	—	2	42	132	108	35	16	14	2	7	2	—	6	—	—	—
1893	—	5	47	124	132	37	11	6	2	1	—	—	—	—	—	—
1894	—	2	53	159	88	27	13	4	6	6	5	1	—	1	—	—
1895	—	6	57	136	99	32	18	14	3	—	—	—	—	—	—	—
1896	—	14	71	157	61	34	15	5	4	1	—	—	—	—	—	—
1897	—	19	116	155	45	21	5	1	1	1	—	—	—	—	—	—
1898	—	21	122	149	40	22	6	2	0	—	1	1	—	1	—	—
1899	—	35	123	147	39	10	6	2	2	1	—	—	—	—	—	—
1900	—	56	176	117	8	4	2	2	0	—	—	—	—	—	—	—
Totals	127	553	3,494	5,590	2,999	1,269	606	291	167	136	55	21	14	11	5	2

minimum, at once catch the eye. In this respect, however, the sunspot minimum in 1867 is much less conspicuous than the subsequent minima. Subsequent to 1873 at least two ranges not exceeding 5' appeared in every year, but previous to that date there were five years with no single example of so small a range. The absence of large ranges near sunspot minimum is worthy of notice. 1877 and 1878 between them supplied only five ranges exceeding 30', while 1879 had no single range as large as this. 1890 and 1900 had no range exceeding 35', and 1889 had only one such range. Ranges exceeding 40' numbered 14 in 1859, 16 in 1860, 13 in 1861, 11 in 1862, 16 in 1865, 15 in 1870, 12 in 1871, 17 in 1872, 11 in 1882, 15 in 1892 and 13 in 1894, no other year possessing as many as 10 such ranges. All these years except 1865—the large mean range of which has been already commented on—had large sunspot frequencies. But 1893, though a year of sunspot maximum, had only one range exceeding 40', and 1884, with a spot frequency in excess of that for 1882, had only two ranges exceeding 40', and none greater than 50'.

Taking the 42 years as a whole, ranges from 10' to 15' were much the most numerous group, nearly 37 per cent. of the total number. Ranges between 5' and 10' were somewhat more numerous than ranges between 15' and 20'. In some individual years near sunspot minimum the group 5' to 10' was the most numerous, ranges exceeding 15' being much fewer than this one group. On the other hand, near sunspot maximum, some years including 1859, 1860, 1869, 1870, 1871, 1872, 1873 and 1893 had more ranges between 15' and 20' than between 10' and 15'.

To show the influence of time, the data in Table XII have been subdivided into the same four periods as before, and the results are given in Table XIII. To make the results strictly comparable, occurrences in the several groups are expressed as percentages of the total number of ranges. Data are added for years of many and few sunspots, employing the same two groups of years as before.

TABLE XIII.—PERCENTAGE OCCURRENCES OF RANGES OF SPECIFIED SIZE.

Period.	≥5'	≥10'	≥15'	≥20'	>20'	>30'	>40'	>50'	>60'
1858-1867	1.1	17.1	50.1	74.2	25.8	8.2	2.8	1.0	0.4
1868-1878	4.9	27.2	57.9	78.7	21.3	5.3	1.9	1.0	0.5
1879-1889	3.8	30.7	73.4	90.0	10.0	2.3	0.8	0.4	0.2
1890-1900	4.7	30.7	70.2	88.0	12.0	3.1	1.1	0.5	0.2
1858-1900	3.6	26.6	63.3	83.1	16.9	4.6	1.6	0.7	0.3
Spot Max. Years	0.8	13.9	46.7	75.9	24.1	7.0	2.8	1.5	0.8
Spot Min. Years	7.5	45.9	83.5	92.9	7.1	1.6	0.4	0.1	0.0

The column headed ≥5 in Table XIII is equivalent to the column 0' to 5' in Table XII. The column headed ≥10 in Table XIII includes the ranges in the two columns 0' to 5' and 5' to 10' in Table XII, and so on. As regards the last five columns in Table XIII, the column >60' includes all ranges exceeding 60', *i.e.*, all included in the last five columns of Table XII. The column >50 includes in addition to these all ranges from 50' to 60', and so on.

The differences between the periods 1879-1889 and 1890-1900 are comparatively small; but the later period, which had the higher sunspot frequency, has somewhat more large ranges. The second period, 1868-78, resembles the two later periods in the number of ranges not exceeding 10', but in all the groups for ranges exceeding 20' it has more than double the entries of the third period. The first period differs conspicuously from the third and fourth at all stages, having much smaller numbers in the first two categories. In its case ranges exceeding and ranges not exceeding 15' are almost equally numerous. Ranges exceeding 20' formed quite a quarter of the whole number during the first period, as against only 10 per cent. in the third period. The fewness of the numbers in the first two categories in the first period arises presumably from its containing only one year, 1867, of really low sunspot frequency. The difference in this respect between years of many and few sunspots is illustrated by the figures in the last two lines of Table XIII.

During the 42 years the ranges exceeding 50' numbered 108. Of these 74 occurred in the 14-year group of years of large spot frequency, as compared with only four in the 12-year group of years of small frequency. Of ranges exceeding 60', numbering 5 in all, not one occurred in the group of years of low sunspot frequency as against 39 in the group of large frequency.

Table XIV shows the occurrence of ranges of different sizes in the different months of the year, and Table XV gives these results as percentages after the scheme adopted in Table XIII. During the whole 42 years there was only one occurrence of a range not exceeding 5' from April to September; while such ranges represent one sixth of the total number in December. The average equinoctial month has a considerably larger percentage of ranges not exceeding 15' than the average summer month. What brings the mean equinoctial range above the mean summer range is the large excess in the number of ranges exceeding 20' during the former season. The average June range is much larger than the average December range, but the percentage of ranges exceeding 20' is actually less in June than in December.

TABLE XIV.—NUMBER OF RANGES OF SPECIFIED SIZE—ANNUAL VARIATION.

Month.	0' to 5'	5' to 10'	10' to 15'	15' to 20'	20' to 25'	25' to 30'	30' to 35'	35' to 40'	40' to 50'	50' to 60'	>60'	Total Days.
January ..	145	515	285	147	75	45	15	16	15	1	3	1,262
February ..	67	396	307	190	91	55	25	18	21	8	5	1,183
March ..	6	234	479	267	155	88	39	16	4	7	5	1,300
April ..	0	131	527	315	148	67	29	12	15	8	5	1,257
May ..	0	213	587	324	97	43	13	8	8	1	2	1,296
June ..	0	158	635	328	75	19	16	9	5	3	0	1,248
July ..	0	185	650	298	100	29	16	8	8	2	2	1,298
August ..	0	121	614	333	106	52	20	10	9	8	9	1,282
September	1	198	490	282	129	59	37	21	12	3	8	1,240
October ..	6	342	436	218	136	68	41	18	19	8	8	1,300
November	112	453	318	170	86	49	22	25	13	3	4	1,255
December..	216	548	262	127	71	32	18	6	7	3	2	1,292
Total ..	553	3,494	5,590	2,999	1,269	606	291	167	136	55	53	15,213

The tendency shown in each of the five last columns of Table XV to maxima towards the equinoxes, and minima at midsummer and midwinter, is presumably normal; but the want of smoothness in the figures, especially in the last three columns, suggests that a much longer period of years would be required to show the normal annual variation in the larger ranges. During the 42 years considered

TABLE XV.—PERCENTAGE OCCURRENCES OF RANGES—ANNUAL VARIATION.

Month.	>5'	>10'	>15'	>20'	>20'	>30'	>40'	>50'	>60'
January	11.5	52.3	74.9	86.5	13.5	4.0	1.5	0.3	0.2
February	5.7	39.1	65.1	81.2	18.8	6.5	2.9	1.1	0.4
March	0.5	18.5	55.3	75.8	24.2	5.5	1.2	0.9	0.4
April	0.0	10.4	52.3	77.4	22.6	5.5	2.2	1.0	0.4
May	0.0	16.4	61.7	86.7	13.3	2.5	0.9	0.2	0.2
June	0.0	12.7	63.5	89.8	10.2	2.6	0.6	0.2	0.0
July	0.0	14.3	64.3	87.3	12.7	2.8	0.9	0.3	0.2
August	0.0	9.4	57.3	83.3	16.7	4.4	2.0	1.3	0.7
September	0.1	16.1	55.6	78.3	21.7	6.5	1.9	0.9	0.6
October	0.5	26.8	60.3	77.1	22.9	7.2	2.7	1.2	0.6
November	8.9	45.0	70.4	83.9	16.1	5.3	1.6	0.6	0.3
December	16.7	59.1	79.4	89.2	10.8	2.8	0.9	0.4	0.2

a range exceeding 60' was never encountered in June, and only four times in all from May to July, the biggest range from these three months being only 84'.5. But how unjustifiable it would have been to conclude that times of outstanding disturbance will never be encountered in these months, is obvious when we note that May, 1921, supplied a range exceeding 133'. That single month supplied four daily ranges exceeding 40', as against a total of only 11 such ranges from the Mays of the 42 years now under review.

§8. FREQUENCY OF OCCURRENCE OF MAXIMA AND MINIMA OF DECLINATION
THROUGHOUT THE TWENTY-FOUR HOURS.

Table XVI gives the number of times when the maximum (westerly extreme) fell between midnight and 1 h., between 1 h. and 2 h., etc., for the several years,

TABLE XVI.—NUMBER OF OCCURRENCES OF *D* MAXIMA AT EACH HOUR.

Year.	Hour ending at																								Days used.
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h	
1858	2	4	3	2	4	4	3	2	0	0	6	23	106	133	53	4	4	1	1	1	0	1	2	2	361
1859	3	2	0	4	2	3	1	1	1	1	1	21	102	153	47	9	3	4	1	1	1	0	0	2	363
1860	0	3	1	1	1	0	1	2	0	1	2	20	113	162	37	8	1	2	2	1	3	1	0	3	365
1861	1	2	0	0	1	4	1	1	0	0	2	23	109	153	42	10	5	0	6	1	0	1	0	1	363
1862	1	5	5	2	6	0	4	5	0	1	4	27	108	136	42	4	4	1	2	2	0	1	2	2	364
1863	3	5	7	5	5	7	8	0	1	3	1	25	105	121	43	10	1	6	3	2	1	1	0	2	365
1864	7	5	6	4	4	3	4	7	2	3	5	30	94	113	46	14	2	10	4	0	1	0	1	1	366
1865	7	3	8	5	5	1	4	6	3	2	3	28	108	107	45	12	10	2	0	1	0	1	1	2	364
1866	7	7	6	5	2	2	5	2	2	0	3	26	105	114	41	16	6	3	1	1	1	1	1	3	360
1867	3	9	7	2	4	3	5	2	0	1	6	24	130	119	31	8	4	2	0	0	1	0	1	1	363
1868	1	4	2	1	2	1	3	1	3	1	4	15	106	135	50	13	5	2	0	0	1	0	0	0	350
1869	1	1	6	3	1	1	2	1	1	0	2	20	116	127	59	8	3	1	4	1	0	0	0	3	361
1870	2	3	1	0	0	4	3	0	0	0	2	16	110	142	45	11	5	9	0	3	3	1	0	2	362
1871	3	3	3	2	0	0	1	2	0	0	0	16	107	154	50	6	4	1	3	4	2	0	0	2	363
1872	4	0	1	0	4	2	5	1	1	0	2	13	97	134	62	12	5	5	1	2	3	1	2	2	359
1873	1	3	5	3	2	3	3	1	0	0	1	14	105	127	54	17	3	3	2	0	1	2	1	1	352
1875	1	5	2	2	4	0	1	0	1	0	3	36	136	113	38	10	0	5	1	0	2	1	0	0	361
1876	2	3	4	2	0	2	0	0	1	2	0	12	112	160	40	9	5	2	1	1	0	0	0	2	360
1877	1	3	4	2	2	1	0	1	2	0	5	20	141	134	29	7	1	3	3	0	1	0	0	0	360
1878	2	3	2	1	1	0	0	0	1	0	3	33	139	134	25	8	3	0	0	0	0	0	0	0	355
1879	0	0	1	0	1	0	0	0	0	1	0	15	143	163	23	8	3	2	2	0	0	0	0	0	362
1880	2	0	1	0	1	1	0	2	3	0	0	18	97	169	51	7	0	3	0	0	1	0	0	0	356
1881	2	0	3	3	0	0	0	0	1	1	2	16	98	169	40	4	3	4	0	1	0	0	0	0	347
1882	1	2	1	0	6	2	6	0	4	1	2	14	94	153	49	11	5	3	4	0	2	2	1	0	363
1883	2	0	1	3	3	0	0	2	0	0	2	12	93	166	57	10	1	2	4	2	0	1	1	2	364
1884	0	2	0	2	0	5	2	0	1	0	0	13	103	177	46	7	3	3	0	0	0	0	1	0	365
1885	3	0	2	2	1	3	1	0	2	0	3	14	100	164	47	13	3	4	2	1	0	0	0	0	365
1886	1	3	3	4	6	4	1	0	2	2	3	14	102	129	54	14	8	6	1	1	0	1	1	0	360
1887	1	3	3	1	0	2	0	2	2	2	2	15	120	154	40	9	2	2	1	0	0	0	0	1	362
1888	4	3	2	2	5	1	0	1	2	1	1	24	117	152	29	6	1	3	0	2	0	0	1	2	359
1889	1	5	3	1	4	2	4	0	1	0	1	26	130	152	26	2	1	1	1	2	0	0	0	0	363
1890	1	0	5	2	1	0	3	1	0	1	1	19	153	129	35	4	0	5	2	0	0	0	0	0	362
1891	1	2	1	2	1	0	2	2	0	2	0	17	111	148	55	7	9	1	1	0	0	0	0	0	362
1892	4	2	2	1	2	2	3	2	2	0	0	12	101	156	43	11	2	4	3	1	3	3	0	3	362
1893	2	2	0	3	1	2	4	0	0	0	0	11	85	176	58	8	5	3	2	0	0	1	0	0	363
1894	2	0	3	2	1	3	7	0	0	0	0	8	92	152	68	10	1	3	4	2	1	0	1	2	362
1895	0	0	1	2	1	5	2	1	1	1	4	13	120	152	41	12	1	4	2	0	0	0	0	0	363
1896	0	2	1	1	1	1	0	0	2	0	4	19	99	156	58	10	6	4	0	0	0	0	0	0	364
1897	1	3	0	1	2	3	0	1	2	1	0	15	122	144	42	15	3	3	3	0	0	0	0	1	362
1898	3	1	5	1	3	2	3	1	0	2	6	15	126	133	44	11	3	0	2	1	1	0	1	0	364
1899	2	3	4	2	2	1	1	0	1	2	2	21	130	135	47	2	3	1	1	0	0	1	1	1	363
1900	1	0	1	1	0	0	1	1	1	1	1	9	154	158	28	6	0	0	0	0	0	0	1	0	363

and Table XVII fulfils the same function for the minima. When the measurement placed the time exactly at an hour, the occurrence was usually assigned to the 60 minutes which began at that hour. A midnight value may be a maximum (or a minimum) both for the day ending and the day beginning. In such a case no one would dispute its claim to being a true maximum (or minimum). But when a midnight value figures as a maximum or a minimum, it is almost invariably for one only of the two consecutive days. It may be a true maximum or minimum for the curve in its immediate vicinity, but generally this is not the case, the curve sloping in the same direction on both sides of the midnight point. A maximum or minimum falling at midnight may signify a persistent rise or fall of the element extending over a number of hours. If the day had been regarded as commencing at noon, for instance, instead of midnight, the latter hour would in such a case have contributed no maximum or minimum. Thus the incidence of maxima and minima in hours adjacent to midnight may be sensibly affected by the accident of the time scheme we choose to adopt. *D* maxima near midnight are very rare, and the question of midnight occurrences hardly arises in their case. It is different, however, with minima as they not infrequently occur near midnight in the winter months. Attention is here merely drawn to the point. It would be of interest to see how the incidence of *D* minima would be affected if the day started at noon.

Referring to Table XVI, it will be seen that the largest number of maxima occurs invariably either under 13h. (*i.e.*, between 12h. and 13h.) or under 14h., the latter hour supplying the larger number in 36 out of the 42 years. Hour 14-15 always comes third on the list, except in two years when hour 11-12 equals or surpasses it. No other hour of the 24, with the occasional exception of hour 15-16, gives more than 10 occurrences in any year.

The incidence of minima, as Table XVII shows at a glance, is much less concentrated. There is usually a substantial entry in each of the four hours ending at 6h., 7h., 8h. and 9h., as well as in all the night hours, from that ending at 19h. to that ending at 3h. Occurrences of minima are rare between 3h. and 5h., and very rare during the six hours ending at 16h. In the great majority of years the largest number of occurrences is found between 7h. and 8h., or between 8h. and 9h.; but in 1865 and in the three consecutive years 1867, 1868 and 1869 the highest entry occurs in one of the night hours.

The seasonal variation in the incidence of maxima and minima is shown in Tables XVIII and XIX. The entries represent percentages of the total occurrences in the 24 hours.

The seasonal variation in the occurrences of maxima is not very prominent. It will be seen, however, that as we pass from January to April there is a marked rise in the excess of entries under 14h. over the entries under 13h., indicating a progressively later occurrence of the maximum in the average day. In June and July the occurrences during the hour ending at 14h. are more than double those in the previous hour. In August there is a marked decline in the excess of the entry under 14h., and in the last four months of the year the largest entry occurs under 13h., November being the most conspicuous example of this. Thus the average hour of occurrence of the *D* maximum in summer is later than it is either in the earlier or the later months of the year. There is an apparent lack of symmetry between the figures from September to December as compared with those from April to January. This seems, however, mainly due to the fact that the magnetic phenomena follow solar, not mean, time. In February the sun is, on the average, about 14 minutes after the clock, whereas in November it is as much early. Thus we should expect the hour (G.M.T.) of maximum in November to be some 28 minutes earlier than in February. The occurrences of a maximum (or minimum) within an hour may be by no means uniformly distributed. For instance, the occurrences in the first 30 minutes may largely outnumber those in the second 30. Still we may expect in general to make a fair first approximation to the mean time of occurrence by regarding the occurrences during each hour as

concentrated at the half-hour. Also, in the case of the D maximum, we may as a first approximation confine our attention to the occurrences during the four hours ending at 15 h. Taking, for example, the February figures in Table XVIII, we have to find the C.G. of masses 5.93, 25.68, 40.17 and 14.91, whose time co-ordinates relative to 12 h. 0 m. are in hours -0.5 , $+0.5$, $+1.5$ and $+2.5$. For the time co-ordinate of the C.G. we have at once $+1.24$, *i.e.*, the estimated mean time of occurrence of the maximum in February is 13 h. 14 m. A similar calculation for November gives 12 h. 50 m. The difference, 24 minutes, is much what the equation of time would lead us to expect.

If we regard the concentration of the occurrences of D maximum near noon as measured by the sum of the percentages from the four consecutive hours giving

TABLE XVII.—NUMBER OF OCCURRENCES OF D MINIMA AT EACH HOUR.

Year.	Hour ending at																								Days used.
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h	
1858	34	9	14	3	6	8	31	41	52	7	0	0	1	0	0	0	2	6	13	21	24	27	34	30	363
1859	25	16	9	7	7	17	34	69	52	9	0	0	2	0	0	0	2	3	9	11	14	25	26	27	364
1860	24	11	16	11	6	13	40	59	56	14	1	0	0	0	0	0	0	2	10	13	19	19	23	28	365
1861	30	17	11	9	3	11	41	46	48	12	0	0	0	0	0	2	2	3	10	11	21	33	22	30	362
1862	39	17	4	11	7	11	27	49	36	3	0	0	0	0	0	1	4	3	20	19	21	33	27	32	364
1863	31	13	4	8	5	17	24	40	21	4	0	0	0	1	0	1	10	7	25	22	21	50	29	31	364
1864	29	9	9	4	11	8	14	42	28	5	0	0	0	0	0	0	3	11	26	25	36	37	32	34	363
1865	24	11	8	9	2	11	27	25	28	2	0	0	0	1	0	1	9	20	18	37	34	26	35	37	365
1866	19	12	12	9	2	13	25	39	22	1	0	0	0	0	0	3	3	21	26	34	24	33	35	28	361
1867	25	11	6	7	8	13	21	37	26	4	0	0	0	0	0	3	4	7	24	24	36	33	41	32	362
1868	33	25	16	9	4	17	18	25	32	1	0	0	0	0	1	0	5	4	14	19	27	37	29	38	354
1869	48	18	11	7	9	13	35	30	31	1	0	0	0	0	0	1	2	4	17	15	29	29	30	33	363
1870	30	13	25	9	9	11	34	57	47	7	1	0	1	0	0	0	1	3	6	13	13	21	33	29	363
1871	29	18	28	7	10	13	35	50	44	3	0	0	0	0	0	1	0	8	4	5	18	27	28	34	362
1872	27	17	22	7	4	13	36	48	45	4	0	0	1	0	0	1	2	7	5	14	24	17	28	39	361
1873	39	18	10	9	9	6	20	41	28	5	0	1	0	0	0	1	3	7	14	16	27	29	32	39	354
1875	20	15	11	5	9	15	26	57	49	6	0	0	0	0	0	1	1	4	10	15	34	29	30	25	362
1876	25	9	6	4	8	21	29	59	67	4	0	0	0	0	0	0	1	5	9	20	24	24	28	18	361
1877	25	5	5	3	6	8	26	83	62	4	0	0	0	0	0	1	3	4	13	19	19	23	26	26	361
1878	19	8	7	5	8	12	20	76	70	4	0	0	0	0	0	0	0	7	9	13	22	22	30	26	358
1879	22	7	3	10	7	16	27	89	62	5	0	0	0	0	0	0	0	1	6	7	16	30	24	29	361
1880	27	15	9	2	5	12	18	57	83	9	0	0	0	0	1	0	3	5	10	20	17	21	16	31	361
1881	13	12	4	9	2	14	44	60	55	6	1	0	0	0	0	1	1	9	10	18	22	29	15	22	347
1882	31	11	14	8	11	13	22	46	50	7	2	1	0	0	1	0	4	5	14	22	31	24	22	24	363
1883	29	5	14	9	8	14	30	62	66	9	0	0	0	0	0	0	3	3	16	15	18	20	27	16	364
1884	35	16	10	4	7	23	34	51	68	18	0	0	0	0	0	0	1	0	4	7	11	23	27	27	366
1885	39	15	8	9	11	21	30	46	47	18	0	0	0	0	0	0	0	5	8	11	20	23	16	37	364
1886	28	17	16	7	6	15	29	35	36	11	0	0	0	0	0	2	5	6	16	19	34	30	18	31	361
1887	38	16	12	7	4	15	28	55	32	3	0	0	0	0	0	1	3	5	16	21	22	30	29	25	362
1888	27	15	7	7	4	24	29	33	34	6	0	0	0	0	0	0	3	9	13	24	30	31	31	33	360
1889	28	7	7	14	8	27	30	44	34	6	0	0	0	0	0	1	0	7	9	18	36	28	27	34	365
1890	30	9	7	8	3	21	38	62	41	8	0	0	0	0	1	1	2	8	10	14	27	19	30	26	365
1891	34	14	5	9	4	10	27	54	40	7	0	0	0	0	0	0	2	9	19	27	21	29	25	26	362
1892	34	19	17	13	5	13	34	44	42	4	1	0	0	0	0	0	4	12	10	13	20	21	23	37	366
1893	33	23	8	10	3	8	35	66	59	13	0	0	0	0	0	1	0	3	10	9	17	20	15	29	362
1894	31	10	10	3	8	15	31	51	50	14	0	0	0	0	0	0	4	7	10	23	23	21	22	31	364
1895	30	14	5	6	5	14	37	56	32	1	0	0	0	0	1	2	3	12	16	23	27	30	28	22	364
1896	44	11	7	7	4	18	28	39	46	8	0	0	0	0	0	1	5	2	12	16	31	26	27	34	366
1897	23	16	17	4	8	22	34	45	33	8	1	0	0	0	0	0	3	3	7	19	29	38	26	28	364
1898	29	14	7	11	8	18	36	45	37	8	0	0	0	0	1	1	2	5	12	26	29	24	25	27	365
1899	23	7	10	6	3	17	30	44	50	12	0	0	0	0	1	0	5	4	14	23	30	31	33	22	365
1900	27	5	5	7	5	18	46	48	76	18	0	0	0	0	0	0	3	0	5	19	18	24	18	23	365

the largest sum (4 hours ending at 16h. 0m. in April, June and July, 4 hours ending at 15h. 0m. in the remaining nine months) we get the following results, going to 0·1:—

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
81·4	86·7	93·9	94·7	91·4	92·9	92·0	92·4	92·2	88·6	84·9	76·6

The concentration is thus decidedly least near midwinter, and is pretty nearly uniform from March to September.

TABLE XVIII.—PERCENTAGE OCCURRENCES OF MAXIMA AT EACH HOUR.

Month or Period.	Hour ending at																							
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
January ..	0·73	0·81	1·06	1·14	0·73	0·73	0·97	0·57	0·57	0·33	1·14	6·10	32·76	35·28	7·24	2·11	1·79	2·36	1·71	0·81	0·41	0·33	0·16	0·16
February ..	0·42	0·76	0·93	0·68	0·51	0·59	0·42	0·34	0·25	0·08	0·68	5·93	25·68	40·17	14·91	2·97	0·68	1·10	1·10	0·59	0·17	0·51	0·17	0·34
March ..	0·39	0·31	0·31	0·15	0·39	0·85	0·31	0·08	0·31	0·08	0·00	2·85	28·53	48·27	14·26	1·62	0·23	0·31	0·00	0·08	0·15	0·08	0·15	0·31
April ..	0·56	0·64	0·24	0·24	0·16	0·16	0·32	0·08	0·08	0·16	0·32	0·96	26·83	57·56	8·84	1·51	0·40	0·16	0·08	0·08	0·16	0·08	0·08	0·32
May ..	0·46	0·62	0·54	0·23	0·46	0·23	0·31	0·15	0·31	0·00	0·23	4·64	33·15	40·42	13·14	3·25	0·85	0·31	0·15	0·08	0·08	0·08	0·23	0·08
June ..	0·40	0·24	0·40	0·40	0·32	0·32	0·40	0·08	0·00	0·00	0·08	3·13	19·73	42·82	25·66	4·65	0·48	0·48	0·08	0·00	0·00	0·08	0·08	0·16
July ..	0·39	0·85	0·62	0·54	0·54	0·54	0·23	0·46	0·08	0·08	0·08	1·78	18·11	45·59	23·84	4·49	0·54	0·39	0·08	0·00	0·23	0·08	0·23	0·23
August ..	0·85	0·70	0·62	0·39	0·39	0·39	0·23	0·23	0·31	0·15	0·78	4·19	32·97	43·99	11·25	1·09	0·23	0·23	0·00	0·15	0·39	0·08	0·08	0·31
September ..	0·48	0·56	0·32	0·24	0·72	0·56	0·88	0·16	0·16	0·64	0·48	7·45	44·51	34·67	5·60	0·96	0·40	0·16	0·00	0·00	0·16	0·24	0·08	0·56
October ..	0·62	0·69	0·62	0·31	0·85	0·93	1·86	0·15	0·23	0·08	0·93	7·57	37·83	34·59	8·65	2·01	0·78	0·46	0·54	0·00	0·00	0·00	0·08	0·23
November ..	0·72	0·88	1·44	0·80	0·80	0·48	0·72	0·96	0·96	0·24	1·04	9·42	43·42	26·50	5·51	1·36	1·84	1·60	0·56	0·24	0·24	0·08	0·00	0·24
December ..	0·78	1·32	2·10	1·40	1·32	0·47	0·78	0·70	0·38	0·78	1·24	8·00	31·13	29·19	8·23	4·27	2·64	2·25	1·32	0·70	0·31	0·15	0·23	0·31
Year—																								
All years ..	0·57	0·70	0·76	0·54	0·61	0·53	0·62	0·34	0·30	0·22	0·58	5·15	31·23	39·93	12·26	2·52	0·90	0·81	0·46	0·22	0·19	0·14	0·13	0·28
S. Max. ..	0·49	0·43	0·39	0·45	0·45	0·65	0·73	0·26	0·22	0·08	0·37	4·17	28·40	42·46	13·86	2·62	0·87	0·87	0·71	0·35	0·35	0·22	0·12	0·43
S. Min. ..	0·58	0·90	0·97	0·48	0·51	0·32	0·44	0·23	0·30	0·55	5·63	36·33	39·33	9·10	1·96	0·67	0·55	0·28	0·14	0·07	0·05	0·12	0·23	
June—																								
S. Max. ..	0·24	0·24	0·00	0·72	0·00	0·00	0·48	0·00	0·00	0·00	0·00	3·10	17·66	47·98	23·87	4·77	0·00	0·72	0·00	0·00	0·00	0·00	0·00	0·24
S. Min. ..	0·28	0·56	0·56	0·00	0·00	0·28	0·28	0·00	0·00	0·00	0·00	3·38	21·70	43·40	23·67	5·07	0·28	0·00	0·00	0·00	0·00	0·00	0·28	0·28
September—																								
S. Max. ..	0·24	0·24	0·48	0·48	1·19	1·19	1·19	0·00	0·00	0·48	0·24	5·49	43·92	37·05	5·73	0·72	0·24	0·24	0·00	0·00	0·00	0·48	0·00	0·48
S. Min. ..	1·11	0·56	0·56	0·28	0·00	0·56	0·56	0·28	0·00	0·83	0·56	6·67	51·67	32·22	2·50	0·56	0·00	0·28	0·00	0·00	0·28	0·00	0·00	0·56
December—																								
S. Max. ..	0·23	0·70	0·23	1·17	0·93	0·23	0·70	0·47	0·70	0·23	0·47	8·63	25·87	35·43	10·02	4·43	3·03	1·40	1·86	1·63	0·70	0·23	0·23	0·47
S. Min. ..	1·62	1·89	3·23	1·35	1·62	0·00	0·54	0·27	0·00	0·81	1·89	7·28	41·23	24·25	4·58	4·85	1·62	1·89	0·54	0·27	0·00	0·00	0·27	0·00

The data in the nine lowest lines of Table XVIII are intended to elucidate the differences shown by years of many and few sunspots. Use was made of the same groups of years as before. In addition to results for the whole year, results are given for June, September and December, as representatives of summer, equinox and winter. If we take the figures in the 12h., 13h., 14h. and 15h. columns for the whole year, and calculate the mean time of occurrence in the same way as before, we obtain 13 h. 14·6 m. from the sunspot maximum years, 13 h. 10·2 m. from the 42 years, and 13 h. 4·6 m. from the sunspot minimum years. These regarded as absolute times are to some extent arbitrary, depending on the number of hours employed. If we take the *five* hours ending at 16 h., we get times a few minutes later, but we still get a 10 minutes' lag in the sunspot maximum as compared with the sunspot minimum years. Applying the same method to the data from the four hours ending at 15h. for June, September and December, we get the following results for the lag in sunspot maximum as compared with sunspot minimum years:—

June 3·1 minutes, September 8·3 minutes, December 14·9 minutes.

I have already described a tendency to a lag in sunspot maximum as compared with sunspot minimum years in the case of the 24-hour and 12-hour Fourier waves* of the regular diurnal inequality of declination. In that case, as in this, the lag was greatest in winter and least in summer.

The occurrences of *D* minima in Table XIX show much more seasonal variation. In the winter months, November, December, January and February, the principal maximum of frequency occurs in the night hours. There is a secondary maximum

* *Phil. Trans. R. Soc., A* 208, p. 223.

TABLE XIX.—PERCENTAGE OCCURRENCES OF MINIMA AT EACH HOUR.

Month or Period.	Hour ending at																							
	1 h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
January ..	12.28	4.83	3.01	1.98	1.58	0.87	0.23	2.14	7.84	2.69	0.00	0.00	0.08	0.08	0.16	0.32	1.66	1.43	3.80	6.81	10.06	12.92	13.08	12.12
February ..	12.63	3.56	3.47	2.03	1.78	1.27	0.51	1.19	9.58	5.76	0.00	0.00	0.00	0.00	0.00	0.08	1.10	2.63	4.75	6.70	9.41	10.33	11.78	11.44
March ..	8.85	4.08	2.62	2.85	1.31	0.62	0.85	5.77	23.92	3.62	0.08	0.00	0.08	0.00	0.00	0.00	0.46	1.38	5.77	6.15	7.69	6.69	8.69	8.54
April ..	6.21	3.58	3.34	2.87	1.19	0.88	3.18	19.59	27.87	1.35	0.00	0.00	0.00	0.08	0.00	0.00	0.24	0.88	2.87	3.66	4.62	5.57	6.77	5.25
May ..	3.94	3.25	3.09	1.62	2.40	6.11	18.39	30.37	11.20	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.15	1.08	1.86	4.17	2.87	3.25	5.80
June ..	3.70	2.57	2.57	0.96	2.33	15.04	24.44	29.10	10.45	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	0.56	1.61	1.37	2.09	2.65
July ..	3.87	2.55	2.24	2.31	2.24	14.77	23.90	24.53	9.28	0.77	0.08	0.00	0.00	0.00	0.00	0.15	0.00	0.93	1.16	1.93	2.17	2.71	4.41	4.41
August ..	4.72	3.18	3.25	2.48	2.56	4.57	19.60	26.72	6.51	0.46	0.00	0.00	0.00	0.00	0.08	0.00	0.15	0.62	1.55	4.11	4.49	5.04	4.65	5.27
September ..	7.77	4.41	2.88	2.88	2.16	1.68	6.17	18.75	10.82	0.32	0.00	0.08	0.08	0.00	0.00	0.16	0.48	2.56	4.97	6.33	7.77	6.33	4.97	8.49
October ..	7.85	3.23	3.08	1.31	1.00	0.38	0.62	5.16	20.78	2.77	0.15	0.00	0.15	0.00	0.00	0.38	1.23	4.54	5.62	7.85	7.54	9.71	7.85	8.78
November ..	13.29	3.66	2.63	1.83	1.19	1.04	0.64	1.04	9.47	2.31	0.16	0.08	0.00	0.00	0.24	0.88	2.23	2.95	4.78	7.17	9.47	13.22	11.23	10.51
December ..	12.32	4.49	2.25	1.63	0.93	1.40	0.23	0.93	3.18	2.94	0.08	0.08	0.00	0.00	0.08	0.39	1.16	3.10	5.50	7.67	11.54	14.41	11.77	13.94
Year—																								
All years ..	8.08	3.62	2.87	2.06	1.72	4.07	8.28	13.84	12.60	1.97	0.05	0.01	0.03	0.01	0.05	0.18	0.74	1.68	3.48	5.00	6.68	7.53	7.37	8.08
S. Max. ..	8.57	4.09	3.93	2.16	1.87	3.75	9.39	14.44	13.56	2.28	0.10	0.02	0.08	0.00	0.04	0.16	0.55	1.41	2.77	3.81	5.60	6.48	6.96	7.98
S. Min. ..	7.08	2.55	2.00	2.00	1.52	4.72	8.04	15.39	13.25	1.73	0.00	0.00	0.00	0.00	0.05	0.23	0.62	1.79	3.54	5.43	7.00	7.55	8.10	7.41
June—																								
S. Max. ..	2.63	2.63	2.63	0.48	2.86	12.65	27.92	34.37	7.64	0.72	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24	1.43	0.24	1.43	2.15
S. Min. ..	3.66	1.13	1.69	0.56	1.41	18.59	23.10	29.30	12.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.85	1.13	1.41	1.69	3.10
September—																								
S. Max. ..	6.70	6.46	3.11	3.11	2.63	1.44	7.18	21.05	11.72	0.24	0.00	0.00	0.24	0.00	0.00	0.00	0.24	1.91	3.35	5.26	6.22	4.55	6.46	8.13
S. Min. ..	8.64	3.90	1.67	3.90	1.67	2.23	5.01	21.45	11.98	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	3.62	5.01	5.85	8.08	5.01	3.62	8.08
December—																								
S. Max. ..	14.85	5.80	3.02	1.39	0.93	1.16	0.46	0.46	4.41	3.71	0.00	0.00	0.00	0.00	0.00	0.46	1.16	2.32	3.94	5.34	9.05	13.22	10.44	17.86
S. Min. ..	9.41	3.76	1.88	1.61	0.54	1.34	0.27	1.34	2.15	2.15	0.00	0.00	0.00	0.00	0.00	0.00	0.27	3.23	6.18	9.14	14.78	15.86	14.52	11.56

of frequency in the hour ending 9 h., but in December especially it is very inferior to the principal maximum. In both the equinoctial and the summer months the morning maximum of frequency is the principal one, the maximum at night becoming less and less prominent as we approach midsummer. The time of the morning maximum frequency is earlier in the summer than in the equinoctial months. The night maximum of frequency is too rounded to admit of a definite time being assigned to it in individual months. Also, it is at least open to doubt whether the fact that the day begins and ends at midnight is not partly responsible for throwing the maximum frequency there for the year as a whole. The rounded nature of the maxima of frequency in Table XIX is an obstacle to the adoption of the method followed in the case of Table XVIII for comparing sunspot maximum and minimum. A difference is, however, easily recognisable between these two groups of years as regards the night occurrences. In sunspot minimum years the night maximum of frequency appears pretty clearly in the hour ending at 23 h., while in the sunspot maximum years it appears in the hour ending at 1 h., indicating a lag of some two hours in the sunspot maximum years. The lag is responsible for the fact that the sunspot minimum percentage figure in the case of the year is the larger from 15 h. to 23 h. and the smaller from 24 h. to 5 h. This lag is particularly conspicuous in December. In September the run of the figures is too irregular, and in June the night maximum of frequency is too poorly developed, for a satisfactory comparison of the sunspot maximum and minimum years. In June, in the course of the whole 42 years, there was no single occurrence of a *D* minimum between 10 h. and 18 h.

Tables XX and XXI give the occurrences of *D* maxima and minima, expressed as percentages, for the year as a whole, treating separately the four periods 1858–1867, 1868–1878, 1869–1879 and 1890–1900. The differences in Table XX between the periods are small, and the irregularity in the run of the figures shows the necessity of allowing for accidental features. In all cases the largest percentages, in order of magnitude, occur in the hours ending at 14h., 13h., 15h., 12h. and 16h. respectively. The sum of the four largest percentages—coming from the columns headed 12h. to 15h.—comes to 84.35 for the first period, 88.64 for the second, 90.52 for the third and 90.48 for the fourth. Thus a slight increase would seem

to have taken place since the early years in the concentration of *D* maxima near noon. This may not, however, indicate a real secular change. A comparison of the times of occurrence of *D* maxima in the quieter and more disturbed days of some of the earlier years of the first period showed, as had in fact been expected, that on quiet days it was very exceptional for the maximum to fall outside the canonical hours (11h. 0m. to 15h. 0m.), but in highly disturbed days this was quite common. Thus the concentration of the *D* maxima in the four hours near noon is naturally greater the quieter the period. The third and fourth periods, especially the third, were quieter than the first two, and this may account for the greater concentration observed in their case.

TABLE XX.—PERCENTAGE OCCURRENCES OF MAXIMA AT EACH HOUR.

Period.	Hour ending at																								
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h	
1858-1867	..	0.94	1.24	1.18	0.82	0.94	0.74	0.99	0.77	0.25	0.33	0.91	6.80	29.72	36.08	11.75	2.61	1.10	0.85	0.55	0.28	0.22	0.19	0.22	0.52
1868-1878	..	0.50	0.78	0.84	0.45	0.45	0.39	0.50	0.20	0.28	0.08	0.61	5.44	32.63	37.96	12.61	2.82	0.95	0.87	0.42	0.30	0.36	0.14	0.08	0.34
1879-1889	..	0.43	0.45	0.50	0.45	0.68	0.50	0.35	0.18	0.45	0.20	0.40	4.57	30.19	44.10	11.66	2.30	0.76	0.83	0.38	0.23	0.08	0.10	0.13	0.13
1890-1900	..	0.43	0.38	0.58	0.45	0.38	0.48	0.65	0.23	0.23	0.25	0.43	3.98	32.41	41.08	13.01	2.41	0.83	0.70	0.50	0.10	0.13	0.13	0.10	0.18
1858-1900	..	0.57	0.70	0.76	0.54	0.61	0.53	0.62	0.34	0.30	0.22	0.58	5.15	31.23	39.93	12.26	2.52	0.90	0.81	0.46	0.22	0.19	0.14	0.13	0.28

To investigate the possibility of a progressive change in the time of the *D* maximum, the method already explained was applied to the entries in Table XX in the four columns 12h. to 15h. The following were the times calculated for the maximum of frequency:—

1858-1867.	1868-1878.	1879-1889.	1890-1900.
13 h. 7.7 m.	13 h. 9.1 m.	13 h. 11.6 m.	13 h. 11.9 m.

This suggests a slight progressive retardation in the time of the *D* maximum. Confirmation of this result appeared desirable in view of the differences in the times of maximum frequency for sunspot maximum and sunspot minimum years already described. To eliminate so far as possible any sunspot influence, a comparison was instituted between the seven earliest and the seven latest years of the sunspot maximum group and between the six earliest and the six latest of the sunspot minimum group. Two calculations were made, one using the data under four hours 12h. to 15h., the other using five hours. The results found for the retardation in the later group as compared with the earlier group of years were as follows:—

	Sunspot maximum years.	Sunspot minimum years.
Using 12 h. to 15 h. ..	+ 5 minutes.	+ 1 minute.
„ 12 h. to 16 h. ..	+ 5 minutes.	— 1 minute.

The time interval between the sub-groups compared was 23 years for the sunspot maximum group, and 18 years for the sunspot minimum. The same method of calculation was then applied to each month of the year and the year as a whole, comparing the 10 years 1861 to 1870 with the 10 years 1891 to 1900, so as to get a 30-year interval. The data employed were the entries under the four hours 12h., 13h., 14h. and 15h. The values, in minutes, obtained for the retardation were as follows:—

Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year
—4	+1	+11	+7	+3	+7	+7	+1	+8	+7	+11	+3	+5

The January results appearing anomalous, a comparison was instituted between January figures from 1861 to 1880 and from 1881 to 1900. This gave for a 20-year interval a retardation of + 1 minute.

There is unquestionably a considerable consensus of evidence in favour of a progressive retardation in the hour of the *D* maximum. Doubt was, however, felt in view of the more disturbed character of the earlier periods, whether the

phenomenon might not be a disturbance effect, and not a true secular change. In the absence of a workable definition of disturbance, it was not obvious how to attack the problem by way of the absolute daily range. Recourse was accordingly had to old data for the *D* diurnal inequality during 1890 to 1900 from different kinds of days. The following were the inequality figures* in the hours adjacent to noon:—

	11 h.	12 h.	13 h.	14 h.	15 h.
Quiet days	+1.71	+3.91	+4.84	+4.38	+3.14
Ordinary days	+2.20	+4.35	+5.26	+4.90	+3.70
Highly disturbed days ..	+3.83	+6.11	+7.04	+7.11	+5.78

Here the value under 12h., for example, may be regarded as the result for 60 minutes centring (not ending) at that hour. Thus the five figures in each case above really answer to a period having a 30-minutes' overlap on either side of the 4-hour period used in connection with the absolute range frequencies. If we apply the method we have used for the absolute ranges to the above inequality figures, we obtain for the hour of the *D* maximum:—

Quiet days.	Ordinary days.	Disturbed days.
13 h. 11.1 m.	13 h. 10.6 m.	13 h. 9.8 m.

This gives a small retardation in the quiet as compared with the disturbed days. The disturbed days employed here, however, were very highly disturbed—on the average the 19 most disturbed days of their year—so the difference between them and the quiet days was immensely greater than the difference between the average day of 1858–1867 (or 1861–1870) and the average day of 1890–1900 (or 1891–1900). Thus the natural inference is that the major part of the progressive retardation observed in the case of the absolute ranges does represent a true secular change. A retardation, it may be explained, is exactly what we should expect if the diurnal inequality were due to forces of invariable type. For the *D* maximum answers to the time when the radius of the vector diagram representing the diurnal variation forces in the horizontal plane is perpendicular to the magnetic meridian. Now the magnetic meridian was approaching the astronomical during the whole period considered, the total approach during the 42 years being approximately 5°. In the average year of the first period, 1858–1867, the radius of the vector diagram had about 3°·8 less to rotate to become perpendicular to the magnetic meridian than it had in the average year of the period 1890–1900. In the case of the diurnal variation during the quiet days of 1890–1900 the radius vector rotated through 67° between 11h. and 15h. At this mean rate the time required to rotate through 3°·8 would be roughly 14 minutes. Thus the retardation observed is really less than we should anticipate if we assumed that the diurnal inequality force system had remained invariable and that the difference in disturbance between the earlier and later periods had exerted no sensible influence.

In Table XXI, as in Table XX, the differences between the several periods are not large. The salient features are the same in all. There is a well marked double period. The forenoon maximum of frequency is always the principal one, and it is much less rounded than the night maximum. In the forenoon, the figure under 8h. is invariably the largest, those under 9h., 7h. and 6h. following in order of size, but these figures do not stand out from those under adjacent hours at all so prominently as in the corresponding case in Table XX. An attempt to use these figures to investigate a possible retardation led to contradictory results. The inference drawn was that owing to the large seasonal variation in the phenomena the material was too heterogeneous. The method was accordingly applied to individual months. The investigation was confined to the day maximum of frequency, because the night maximum appeared too rounded, and the four winter months were omitted, as the day maximum was too inconspicuous. The two

* *Phil. Trans. R. Soc., A* 202, pp. 360 and 361, and *A* 208, pp. 213 and 214.

TABLE XXI.—PERCENTAGE OF OCCURRENCES OF MINIMA AT EACH HOUR.

Period.	Hour ending at																							
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
1858-1867 ..	7.71	3.47	2.56	2.15	1.57	3.36	7.82	12.31	10.16	1.68	0.03	0.00	0.08	0.06	0.00	0.30	1.07	2.28	4.98	5.97	6.88	8.70	8.37	8.51
1868-1878 ..	8.20	4.06	3.92	1.81	2.11	3.58	7.75	14.61	13.20	1.08	0.03	0.03	0.06	0.00	0.03	0.17	0.50	1.47	2.81	4.14	6.58	7.17	8.17	8.53
1879-1889 ..	7.98	3.42	2.62	2.16	1.84	4.88	8.08	14.54	14.27	2.47	0.08	0.03	0.00	0.00	0.05	0.13	0.58	1.38	3.07	4.58	6.47	7.27	6.34	7.77
1890-1900 ..	8.43	3.54	2.45	2.10	1.40	4.34	9.38	13.82	12.63	2.52	0.05	0.00	0.00	0.00	0.10	0.15	0.82	1.62	3.12	5.29	6.79	7.07	6.79	7.61
1858-1900 ..	8.08	3.62	2.87	2.06	1.72	4.07	8.28	13.84	12.60	1.97	0.05	0.01	0.03	0.01	0.05	0.18	0.74	1.68	3.48	5.00	6.68	7.53	7.37	8.08

10-year periods 1861-1870 and 1891-1900 were compared, so as to get a 30-year interval. The values in minutes obtained for the retardation during the 30 years were as follows:—

March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Mean.
+7	+8	+2	-1	-4	+1	+11	+1	3.1

The mean is decidedly smaller than that obtained from the *D* maxima, and the probable error is rather large.

§9. THE EFFECT OF DISTURBANCE ON THE HOUR OF OCCURRENCE OF THE MINIMUM.

We have already referred to a disturbing factor arising from the difference between quiet and disturbed days. In the case of the *D* maxima it was not of primary importance; disturbance produced a "scatter" in the incidence of the maximum, but exerted no large directive influence in any one direction. In the case, however, of the *D* minima the difference between quiet and disturbed days is of the very first importance. If we compare *D* diurnal inequalities for quiet and less quiet days we find the night minimum rise in importance, as compared with the day minimum, as disturbance increases. It was accordingly decided to investigate the influence of disturbance on the incidence of *D* minima.

Use was made of the monthly five days of largest and five days of least range employed in a recent investigation* of the "27-day period." The present investigation was confined to the earliest and latest groups of years, 1858-1867 and 1890-1900. The results appear in Table XXII for the days of largest range, and in Table XXIII for the days of least range. The data are confined to the year and the seasons. In the case of the year, the percentage figures obtained for the two periods are given separately, as well as their arithmetic means. In the case of the seasons it appeared sufficient to give only the arithmetic means of the percentage figures from the two periods.

TABLE XXII.—PERCENTAGE OCCURRENCES OF MINIMA IN DAYS OF LARGEST RANGE.

Period.	Hour ending at																							
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
Year—																								
1858-1867 ..	10.50	6.00	3.50	2.17	0.67	1.33	3.67	2.33	1.83	0.50	0.17	0.00	0.17	0.00	0.00	0.83	1.67	4.33	6.00	12.50	10.17	12.00	10.83	8.83
1890-1900 ..	9.70	5.76	3.94	2.12	1.06	1.51	4.09	6.67	2.27	0.30	0.15	0.00	0.00	0.00	0.00	0.45	1.36	2.42	5.91	9.55	13.33	11.06	9.24	9.11
Mean ..	10.10	5.88	3.72	2.15	0.87	1.42	3.88	4.50	2.05	0.40	0.16	0.00	0.08	0.00	0.00	0.64	1.51	3.37	5.95	11.03	11.75	11.53	10.04	8.97
Winter Mean ..	9.57	5.50	2.32	1.20	0.00	0.00	0.00	0.23	0.00	0.23	0.23	0.00	0.00	0.00	0.00	1.18	3.32	4.84	8.59	12.91	16.66	15.18	10.00	8.04
Equinox „ ..	11.25	5.23	3.07	1.87	0.93	0.48	1.63	1.63	1.68	0.23	0.25	0.00	0.25	0.00	0.00	0.75	1.22	4.32	7.61	14.84	12.36	12.25	8.87	9.28
Summer „ ..	9.47	6.91	5.78	3.36	1.66	3.80	10.00	11.64	4.48	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.97	1.66	5.32	6.22	7.16	11.25	9.57

The results from the two periods agree excellently in their main features so attention may be concentrated on the mean results, which are more free from "accidental" features. In the year as a whole, in Table XXII, the largest forenoon percentage—viz., 4.50 under 8h.—is only about two-fifths of the percentage under 21h.; whereas in Table XXIII the largest night hour percentage—viz. 5.95 at 24h.,

* *Proc. R. Soc., A*, Vol. 101, p. 368.

TABLE XXIII.—PERCENTAGE OCCURRENCES OF MINIMA IN DAYS OF LEAST RANGE.

Period	Hour ending at																							
	1h	2h	3h	4h	5h	6h	7h	8h	9h	10h	11h	12h	13h	14h	15h	16h	17h	18h	19h	20h	21h	22h	23h	24h
Year—																								
1858-1867 ..	4.83	1.83	1.00	2.33	2.33	5.83	10.67	17.17	22.00	4.50	0.00	0.00	0.00	0.00	0.00	0.33	0.17	0.67	2.00	2.00	3.00	7.17	6.17	6.00
1890-1900 ..	6.97	1.21	0.61	2.58	1.06	6.21	12.12	18.33	23.94	6.21	0.15	0.00	0.00	0.00	0.30	0.00	0.45	0.30	1.06	1.36	3.03	4.70	3.48	5.91
Mean ..	.90	1.52	0.81	2.45	1.69	6.02	11.40	17.75	22.97	5.36	0.08	0.00	0.00	0.00	0.15	0.17	0.31	0.49	1.53	1.68	3.01	5.93	4.83	5.95
Winter Mean ..	.59	2.64	0.73	2.61	1.43	2.61	0.98	3.38	17.05	9.52	0.00	0.00	0.00	0.00	0.45	0.50	0.93	0.50	2.91	3.12	5.18	13.37	9.62	10.88
Equinox „ ..	.91	1.43	1.43	2.61	1.96	1.88	4.07	18.62	38.39	5.36	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.95	1.18	1.46	3.13	4.18	3.18	5.03
Summer „ ..	1.21	0.50	0.25	2.13	1.71	13.57	29.14	31.25	13.47	1.18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.48	0.72	0.25	1.68	1.96

—is little more than a quarter of the percentage figure under 9h. In the case of the five disturbed days the forenoon maximum of frequency absolutely disappears in winter, and is only slightly indicated in equinox. In the case of the five quiet days, on the other hand, the night maximum of frequency nearly disappears in summer, and in equinox it is but trifling as compared with the forenoon maximum. It is unnecessary to dwell on the difficulties which this enormous disturbance influence introduces into the comparison of different periods of years.

Tables XXIV and XXV aim at emphasising the more salient features of the variation in the incidence of *D* minima, by focusing attention on certain specified 4-hour intervals. These were selected as giving the largest sum of the percentage occurrences for the forenoon and the night respectively.

TABLE XXIV.—PERCENTAGE OCCURRENCES OF MINIMA.

Month.						Day 4-hour period.		Night 4-hour period.	
						Period.	Percentage.	Period.	Percentage.
January						7h-10h	12.90	22h- 1h	50.40
February						„	17.04	„	46.18
March						„	34.16	„	32.77
April						„	51.99	„	23.80
May						6h- 9h	66.07	21h-24h	16.09
June						„	79.03	22h- 1h	9.81
July						„	72.48	„	13.16
August						„	57.40	„	19.68
September						„	37.42	„	27.56
October						7h-10h	29.33	„	34.19
November						„	13.46	„	48.25
December						„	7.28	„	52.44
Years.									
1858-1867						6h- 9h	33.65	22h- 1h	33.29
1868-1878						„	39.14	„	32.07
1879-1889						„	41.77	„	29.36
1890-1900						„	40.17	„	29.90
1858-1900						„	38.79	„	31.06
S. Maximum						„	41.14	„	29.99
S. Minimum						„	41.40	„	30.14

Table XXIV employs all days. The four consecutive night hours giving the largest sum of percentage figures end at 1h. except in May, when they end at 24h. The four consecutive forenoon hours giving the largest sum of percentage figures end at 10h. from October to April ; for the remaining months and the year as a whole

they end at 9h. The monthly figures in Table XXIV for the morning group of hours show a very smooth progression from the minimum in December to the maximum in June, and from the maximum in June to the minimum in December. The fall to a minimum in June and the rise to a maximum in December is similarly prominent in the night group of hours. In the yearly results in Table XXIV there is an apparent progressive advance in the day figures, and a corresponding decline in the night figures, as we pass from the first to the third period of years, but this does not extend to the fourth period. Possibly the lesser amount of disturbance during the third period may account for its position as compared with that of the fourth. The figures from the sunspot maximum and minimum groups of years are remarkably alike.

Table XXV contains data analogous to those in Table XXIV, but it contrasts the days of largest and least range. The data are derived from the first and last periods of years only. In Table XXV the 4 hours giving the largest sum of percentage figures are not always the same for the two types of days. The contrast between the winter figures from disturbed days and the summer figures from quiet days is most striking.

TABLE XXV.—PERCENTAGE OCCURRENCES OF MINIMA IN DAYS OF LARGE AND SMALL RANGE.

Season.	5 Days of largest range.				5 Days of least range.			
	Day 4-hour period.		Night 4-hour period.		Day 4-hour period.		Night 4-hour period.	
	Period.	Per-centage.	Period.	Per-centage.	Period.	Per-centage.	Period.	Per-centage.
Year	6h- 9h	11·85	20h-23h	44·35	6h- 9h	58·14	22h-1h	22·61
Winter	8h-11h	0·69	20h-23h	54·75	7h-10h	30·93	22h-1h	45·46
Equinox	6h- 9h	5·42	20h-23h	48·32	7h-10h	66·44	22h-1h	17·30
Summer	6h- 9h	29·92	22h- 1h	37·45	6h- 9h	87·43	23h-2h	5·35

Considerable light on this extreme sensitiveness to disturbance in the time of occurrence of *D* minima may be derived from a study of the incidence of disturbance throughout the 24 hours. For some years past two-hourly values of declination at Kew Observatory have been issued weekly for the information of mining engineers, and an asterisk has been attached to all the two-hour intervals regarded as specially disturbed. The following shows the percentage incidence of such occurrences derived from the three-years 1919 to 1921 :—

0h-2h	2h-4h	4h-6h	6h-8h	8h-10h	10h-12h	12h-14h	14h-16h	16h-18h	18h-20h	20h-22h	22h-24h
12·0	11·1	6·8	4·4	2·4	1·5	2·8	5·9	10·5	15·1	14·6	12·9

Thus, so far as *D* is concerned, the day can be divided into two 12-hour periods, a quiet period, 4h-16h. supplying only 23·8 per cent. of the total disturbed hours, and a disturbed period, 16h-4h., supplying 76·2 per cent. of the disturbed hours. The former period includes the canonical hours of the *D* maximum and the *D* forenoon minimum, the latter the hours of the night minimum.

Disturbance seems to have a general tendency to increase the movements which the regular diurnal variation would inspire at the hour.

The exact way in which disturbance works is a matter of speculation, but not improbably it is largely a case of reducing the electrical resistance of the upper atmosphere, and so increasing the current associated with a given electromotive force. As disturbance, in Britain, is so much concentrated in the night hours, it

is natural that the D night phenomena should be those particularly affected. Apart from really large magnetic storms, disturbance of any size seldom lasts more than a few hours. If there is a considerable disturbance on one night from 18h. to 21h., and on another night from 21h. to 24h., the night minimum will naturally occur later on the second occasion. This helps to explain the very rounded character of the night maximum of frequency in the occurrences of D minima. Large disturbance is much less common near the canonical hours of occurrence of the D maximum, so it is only natural that the hour of maximum should appear less sensitive to disturbance.

The cost of the declination curve measurements made use of in the present investigation was defrayed out of grants received from the Government Grant Committee.

