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Brief notes on the dispersion of pollutants in the
atmosphere.

by

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Brief notes on the dispersion of pollutants in the atmosphere

Introduction

The Earth's atmosphere responds very positively to the different amounts of heat that are received per unit surface area in equatorial regions and in polar regions. On a planet (unlike the Earth) that didn't rotate about its own axis the excess heat would be transferred from the ground to the air in low latitudes, the heated air would rise, cold air would flow south from polar regions to take its place and the warm air aloft would recirculate towards the poles, sink, give up some of its heat to the ground, where it would be largely lost by long-wave radiation to outer space during the long nights.

On Earth, the rotation of the planet alters this simple circulation very considerably; instead of mainly northerly wind near the surface in the northern hemisphere, the prevailing wind is much more variable with latitude so that at around 50° - 60° N a slight predominance of west to south-westerly winds are experienced, and the transfer of heat northwards takes place in a rather complex way by the succession of depressions and anticyclones in the lower atmosphere and by so-called jet-streams at greater height. The remarkable difference between the winds on a rotating planet and on a non-rotating planet is that the effect of the rotation is to force the wind to flow not from hot to cold (or from high pressure to low pressure) but at right angles to this direction. In the northern hemisphere winds, at heights away from the drag of the surface, flow nearly along the lines of equal pressure (the isobars) with low pressure to the left and high pressure to the right. The strength of the wind depends directly on the steepness of the pressure gradient. Mathematically we call this wind the geostrophic wind G and it is proportional to the pressure gradient ∇P

$$G \propto \nabla P$$

The geostrophic wind can be easily read off a weather chart using a transparent scale which relates wind speed to the spacing of the isobars on the chart.

In the lowest part of the atmosphere (say below 1 km) the wind is not geostrophic but is affected by the drag of the underlying surface. The ground is a sink of momentum (density \times velocity) and if it were not replenished from above, the air just above the surface would come to a standstill. However, this drag, in causing a braking action to the airflow, results in an overturning, or turbulent breakdown, of the flow causing air from different levels to be mixed and momentum to be transferred from higher levels to replace that which has been lost to the ground. The speed of the wind now varies

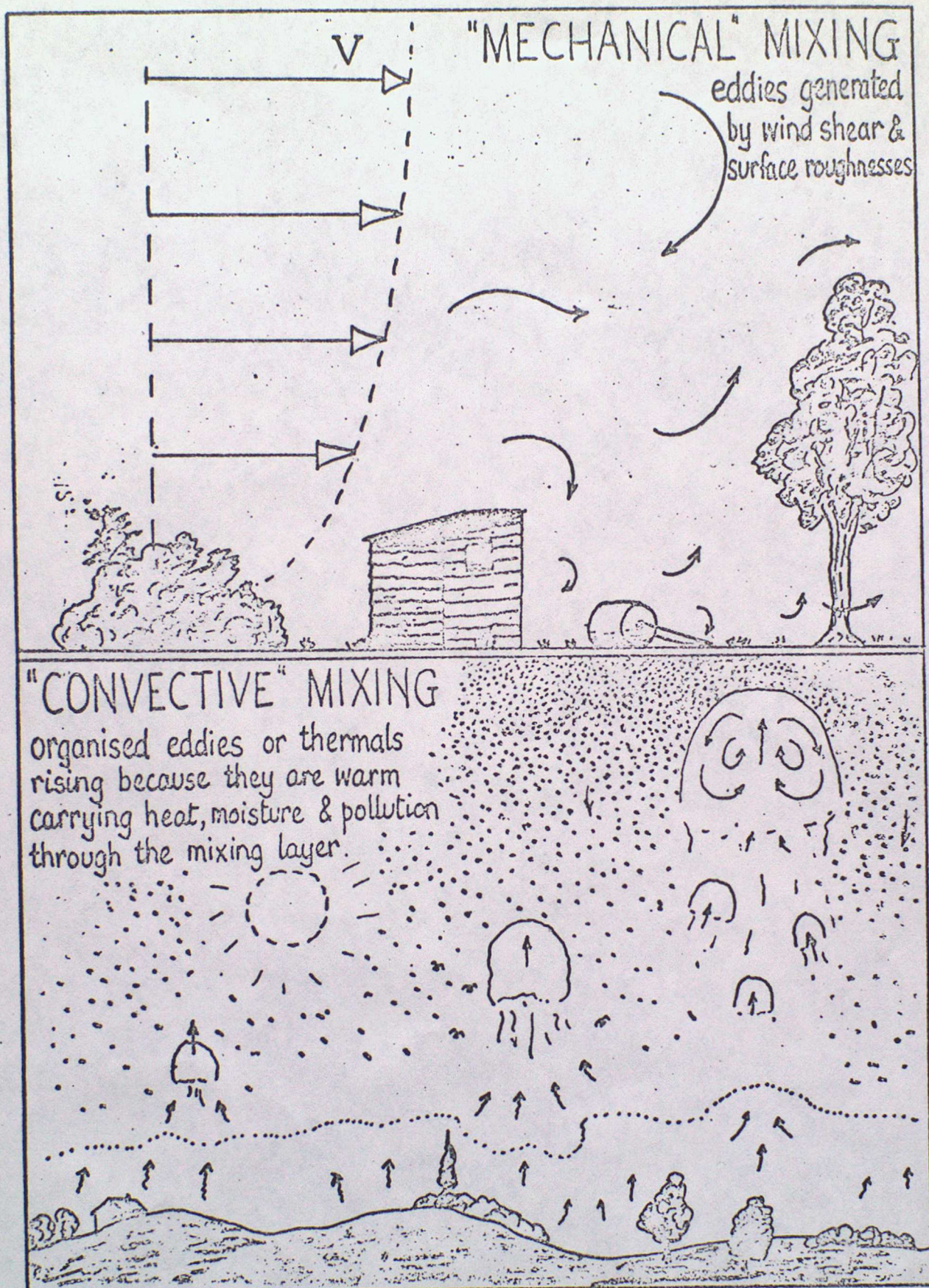
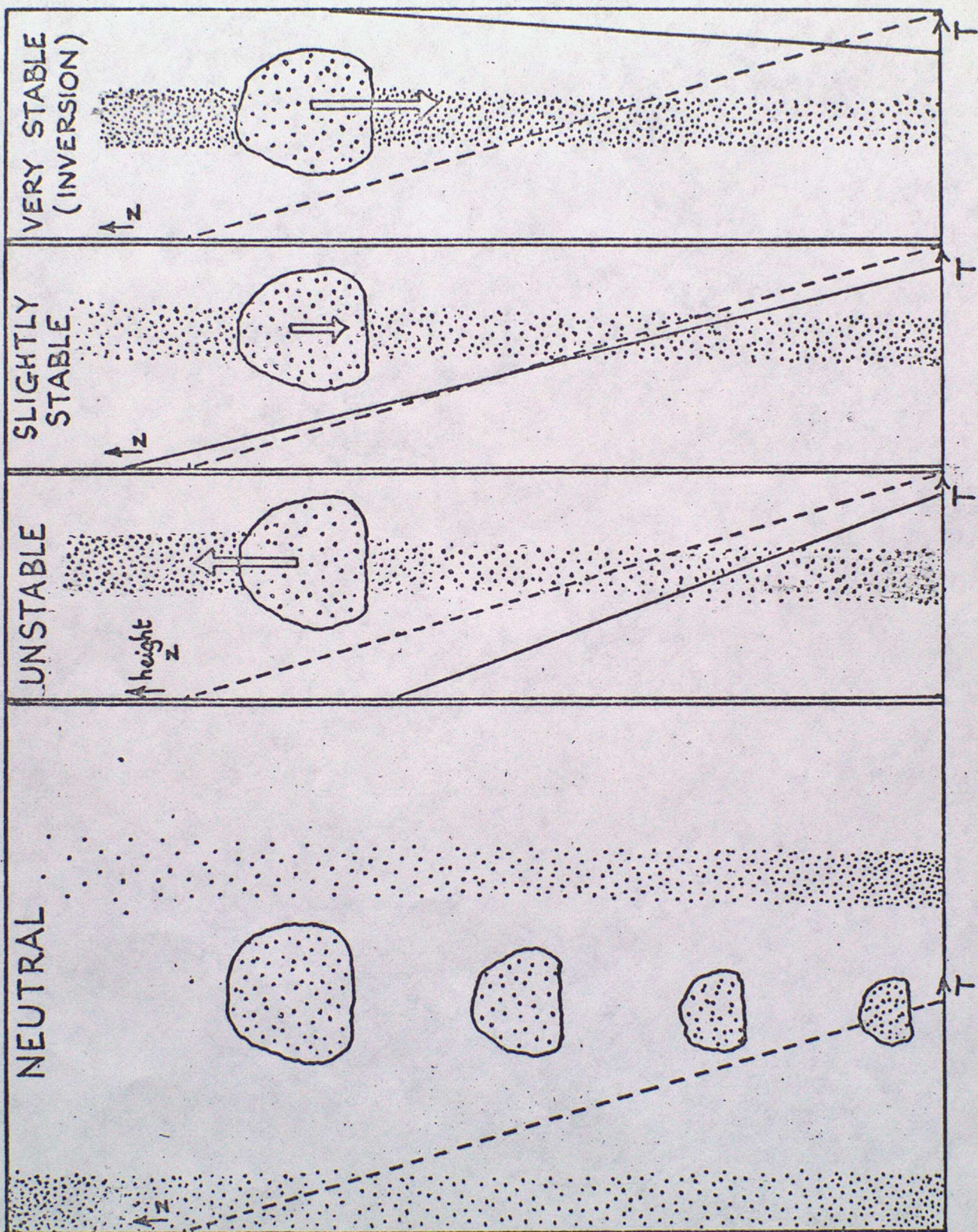


FIGURE 1.



UNSTABLE

COLD AIR ALOFT

WARM, SUNNY
LIGHT WINDS

LOW POLLUTION FROM ALL SOURCES

NEUTRAL

CLOUDY
FRESH WINDS

FAIRLY LOW POLLUTION: BUT CONTRIBUTIONS FROM
ALL SOURCES

STABLE

WARM AIR ALOFT.

LIGHT WINDS.
RADIATIONAL COOLING
AT GROUND

HIGH POLLUTION: BUT FROM LOW-LEVEL SOURCES ONLY

continuously through this layer from zero at the ground to the geostrophic wind speed at the top. The layer is called the atmospheric boundary layer or the mixing layer.

Another additional source of mixing in the boundary layer comes from the transfer of heat from the ground when warmed by the sun to the overlying air. This is illustrated in Figure 1. On warm summer days, and especially when the wind is light, this source of mixing predominates. The magnitude of the vertical velocities induced under these conditions turns out to be proportional to the third power of the flux of heat from the ground.

What makes the mathematical description of these mixing processes so difficult is the vast and continuous range of scales of turbulent "eddies" present. On the largest scale we have, as already seen, the global circulation from equator to poles, the depressions and anticyclones (several thousand kilometres across), and the smaller "weather" features like warm and cold fronts, rain belts etc. Then we have the so-called meso-scale features like sea-breezes, out and in-flows connected with thunderstorms, and gravity-induced flows near mountains. On a still smaller scale we have organised circulations within the boundary layer itself which occasionally are apparent to the eye when they result in extensive parallel rows of cumulus clouds (cloud-streets) above the ascending parts of the circulating rolls. Finally, we have the turbulent eddies induced by surface drag and by surface heating as already mentioned. The scales of these motions range from hundreds of metres down to fractions of a millimetre.

All these eddies interact upon each other to a greater or lesser degree and this is fortunate otherwise once formed it would be very difficult to get rid of them. One eddy's circulation will tend to stretch and distort the circulation of another eddy within its range and in so doing it tends to increase the velocity gradients. In mathematical terms the interaction of two eddies causes energy to flow partially to larger scales but mainly to smaller scales as each eddy gets thinner and thinner as it is distorted by its neighbours. Ultimately the gradients are sufficiently strong that the natural stickiness or viscosity of the air can dissipate the turbulent energy into heat. Normally viscosity is unimportant to eddies larger than a few tens of centimetres across but is all important to eddies down in the millimetre range.

This process of distortion and ultimate dissipation means that eddies have a finite life in the boundary layer. The size of the eddies, before distortion really gets underway, depends mainly on height; in the first 20 to 100 metres the diameter is typically about equal to the height but above that the size increases more slowly and above typically 300 m becomes

virtually constant equal to about one quarter of the boundary layer depth (which might be 600-1200 metres). The life-time of the eddy depends on its size or diameter d ; very roughly

$$\text{life-time} \approx \frac{6d}{G}$$

where G is the geostrophic wind speed.

Thus if $G = 5 \text{ ms}^{-1}$ and $z = 10$ metres, then $d \approx 10 \text{ m}$ and the life-time is about 12 seconds. Strictly this time is not the time to total disappearance but a much shorter time when distortion has become considerable (it has a definite mathematical definition which is inappropriate to quote here).

This complex interacting field of eddies is important in the transfer of momentum to the surface to overcome drag, in transferring heat and moisture from the surface into the atmosphere, and last but not least in dispersing any pollutant emitted into the air.

We have seen that the wind is an important source of turbulent energy and that during sunny weather so is the heat flux from the surface into the air. At night, particularly when the ground is cooling rapidly because of radiational losses of heat, the lower layers of air also cool and temperature often then actually increases with height (a so-called "inversion"). This makes the air stable to vertical motions and turbulence often dies out completely. Deprived of its replenishing supply of momentum the wind speed in the first 10 metres or so then becomes very light indeed. The idea of stability is pictorially demonstrated in Figure 2.

Two properties of the atmosphere relevant to the dispersion of pollutants are affected by changes in "stability": one is the wind speed especially in the lower layers, the second is the strength or intensity of vertical mixing and dispersion.

2. Plumes

The concern shown in the Introduction about the qualitative nature of the airflow in the boundary layer is purely a reflection of the importance that these motions have in transporting and dispersing plumes of pollutants leaving chimney stacks or other sources. Ultimately the question must be "given an array of sources what is the best estimate we can make of the concentrations of the various pollutants at some specified receptor point given any required information about the terrain and the current meteorology?" We would also need to know certain details about the pollutants themselves; how much is being emitted per unit time, are they chemically reactive, are they buoyant, heavy or passive, or are they toxic or explosive, is any heat being released through the same chimney, and so on.

Clearly there are lots of difficult questions here and we cannot hope to cover more than a few. In fact only two types of pollutants will be discussed (i) passive, non reactive, conservative, steady continuous emissions (ii) as in (i) except the pollutant is hot when it leaves the source.

Observing a visible plume, of the first kind, being emitted from a fairly tall stack one sees that normally

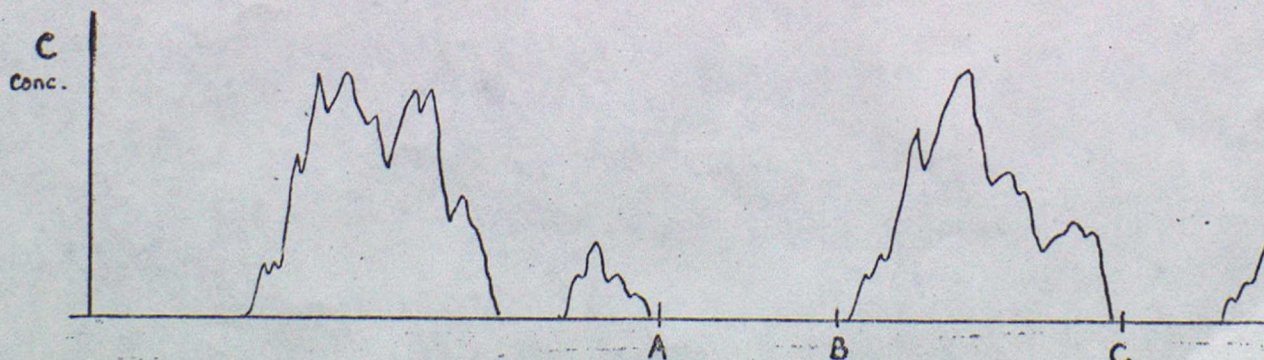
- (a) the plume is carried away downwind.
- (b) the plume meanders both in the vertical and acrosswind.
- (c) the plume grows in size with distance (it looks rather like a narrow distorted cone with its apex at the stack exit) and ultimately impinges on the underlying ground.

Sometimes the plume breaks up completely into discrete puffs which eventually may join up again as the individual puffs grows larger and larger. This break-up is more commonly observed with plumes of the second kind.

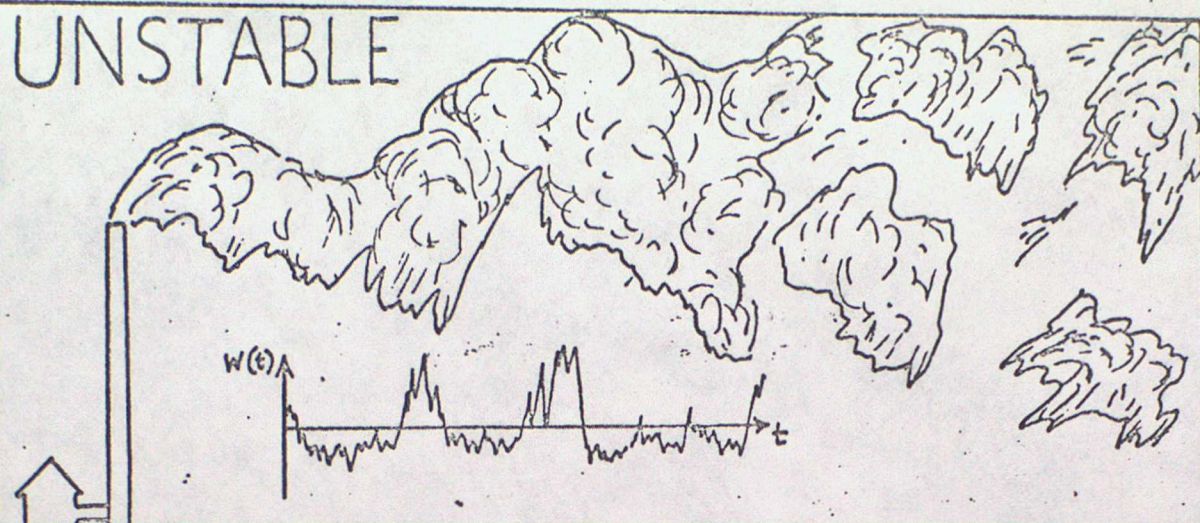
An observer at a fixed point may

- (i) never be affected by the plume until a significant change in the meteorology occurs.
- (ii) be almost always in the plume if he is directly downwind of the source.
- (iii) be affected by the plume in a very intermittent way.

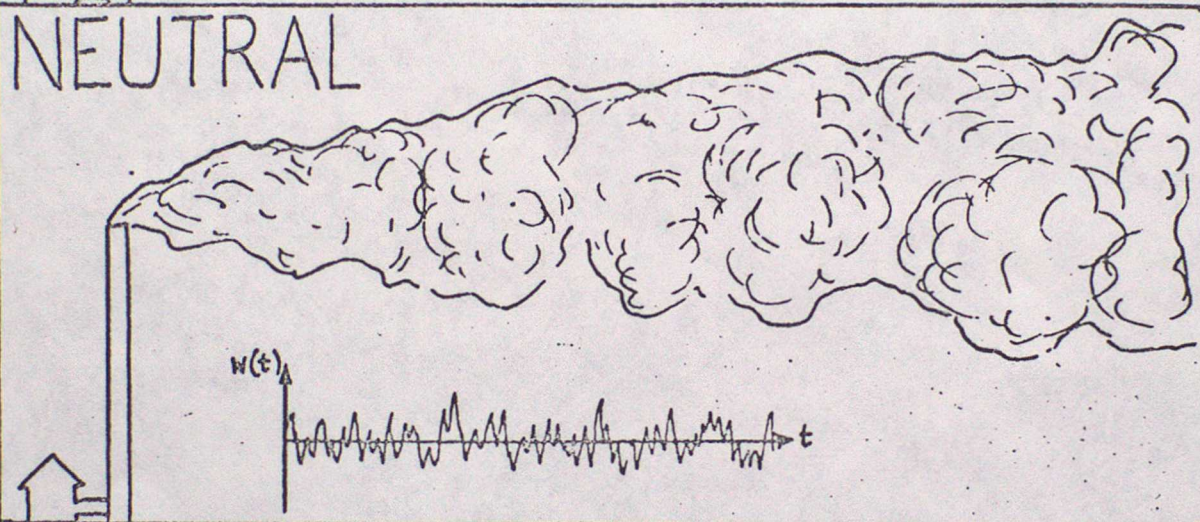
In the last situation the record of concentration with time may look like this:



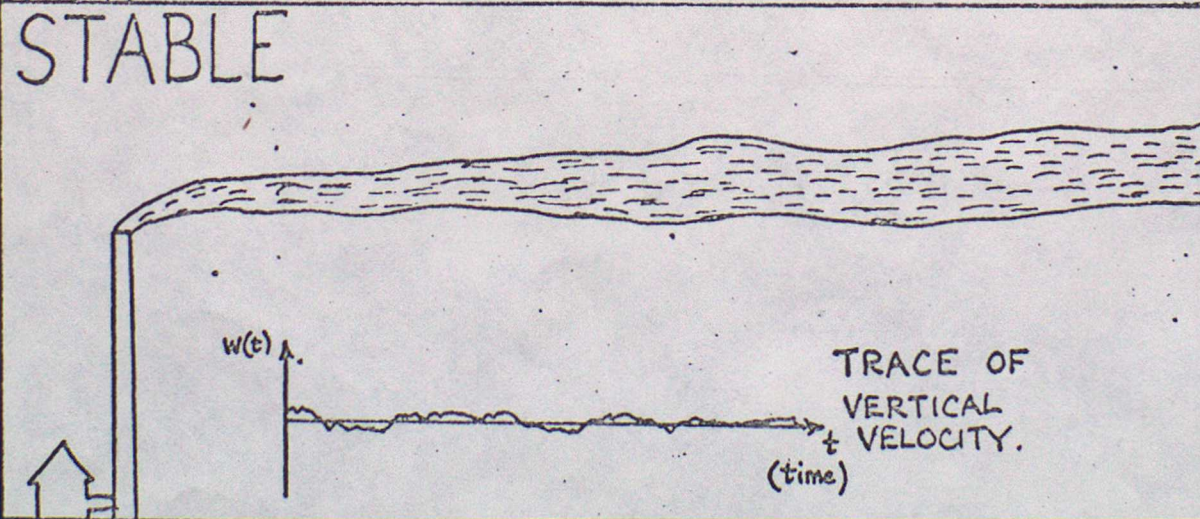
UNSTABLE



NEUTRAL



STABLE



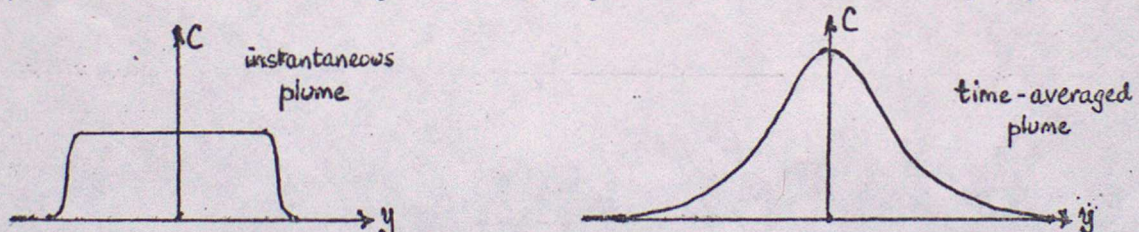
To find the effect of the pollutant on the receptor, the concentration may be estimated by analysing a continuous sample of air over a certain time interval. Clearly a very different picture would be obtained if that time interval were AB or BC, valid though each may be for its own interval. To get a representative average over a much longer time period, the interval should be at least AC if not very much longer.

The time over which we measure the concentration is called the sampling time and it is clear that the answer we get depends in a very important way on this time,

At any moment and at a given distance downwind x from the source the plume will have a certain cross-wind dimension d . Measured some time later the width may be d' , and again later d'' and so on. Typically there may be something like a four-fold variation in the various values of d .

Due to the meandering of the plume, which comes about because of the larger eddies, the centre of the plume is constantly moving in the y - z plane (y - acrosswind, z - vertical). The concentration and the plume width averaged over many tens of minutes will reflect both the instantaneous width distribution d and the extent of the plume meandering. The time average width D is always as large as or larger than d .

Usually the concentration is nearly constant within the instantaneous plume, but varies in a nearly-Gaussian way within the time-averaged plume



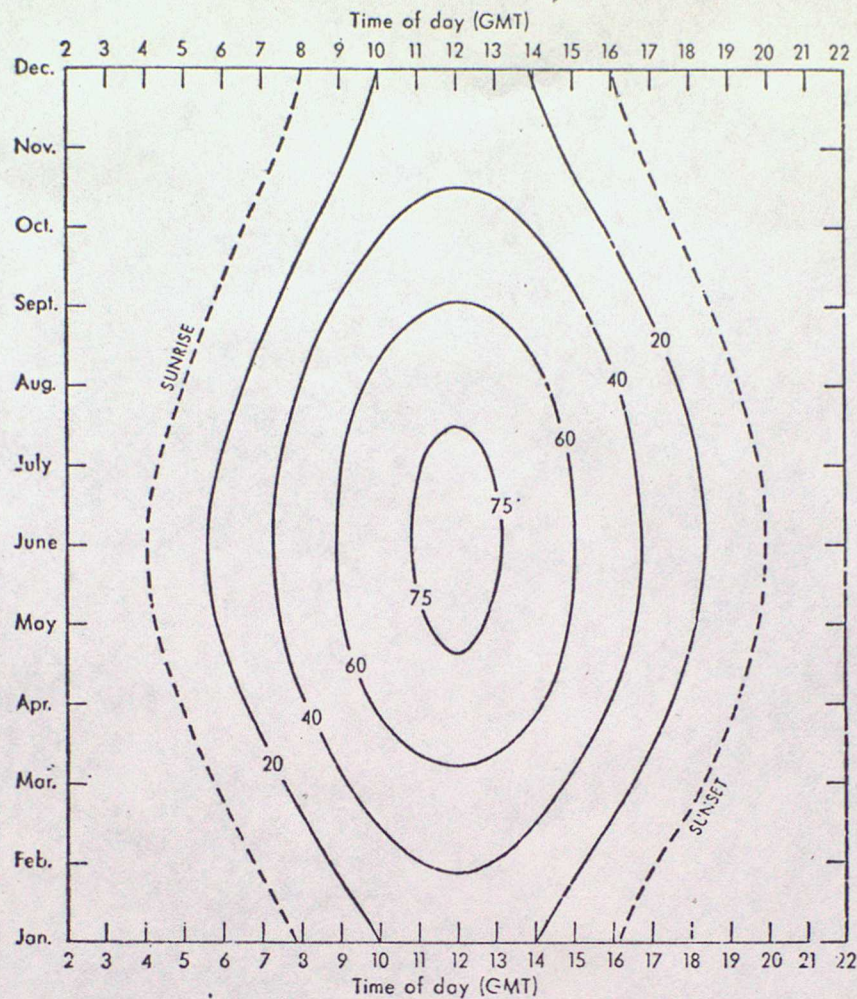
Mathematically we could describe the width in terms of the "total" width, from where concentration C first becomes non-zero on the left of the plume to where it becomes just zero on the right. This might be adequate for the instantaneous plume, but because of the long "tails" to the distribution is unsatisfactory for the time-averaged plume.

It is normal to define the width in terms of a root-mean-square width σ_p , defined mathematically as

$$\sigma_p^2 = \frac{\int_{-\infty}^{\infty} y^2 C(y) dy}{\int_{-\infty}^{\infty} C(y) dy}$$

For the nearly square distribution of the instantaneous plume where $C(y)$ is a constant $= C_0$

$$\sigma_p^2 = \frac{C_0 \int_{-d/2}^{d/2} y^2 dy}{C_0 \int_{-d/2}^{d/2} dy}$$



—INCOMING SOLAR RADIATION, R_0 , IN MILLIWATTS PER SQUARE CENTIMETRE FOR 0-1 OKTA OF CLOUD

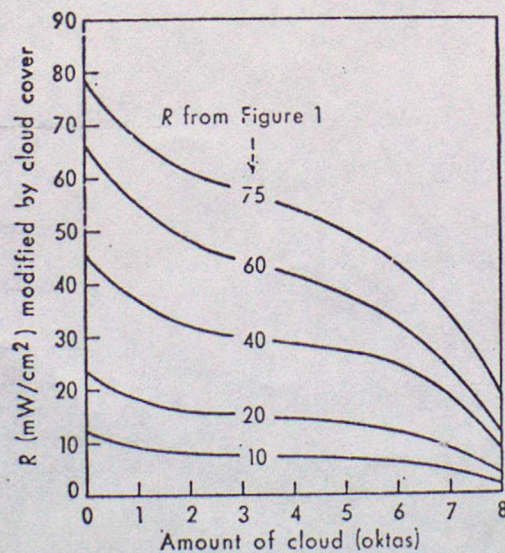


FIGURE 5 —INCOMING SOLAR RADIATION MODIFIED BY CLOUD COVER
As deduced from unmodified value R_0

$$i.e. \quad \sigma_p^2 = \left[\frac{y^3}{3} \right]_{-\frac{d}{2}}^{\frac{d}{2}} / \left[y \right]_{-\frac{d}{2}}^{\frac{d}{2}}$$

$$\sigma_p \approx 0.29 d$$

The variation of concentration with y in the time-averaged plume can be found very easily from mathematical tables giving the Gaussian or "normal" distribution.

Finally in this section it should be stated that although the concentration within the plumes may approximate to these mathematical distributions in a statistical sense quite well, in practice the pollutant is often very patchy within a plume, as can be seen by any observer. This is a very important property for pollutants either with an unpleasant odour or if they are explosive within certain concentration limits (like natural gas). Unfortunately the theory of this patchiness is almost non-existent.

3. The surface heat flux and stability

In the Introduction we discussed the concept that heat was an important source of turbulent energy during the day, when the heat came from the warming of the ground by the sun. In contrast night, radiational cooling of the ground, cools the lower layers of the air, causes air temperature to increase with height and damps down turbulence.

Situations in which heat is a positive source of turbulence during the day are called "convectively unstable" or just "unstable". Conversely when the temperature structure is a sink of turbulent energy, the situation is called stable; and if buoyancy forces are playing no role then the situation is neutral.

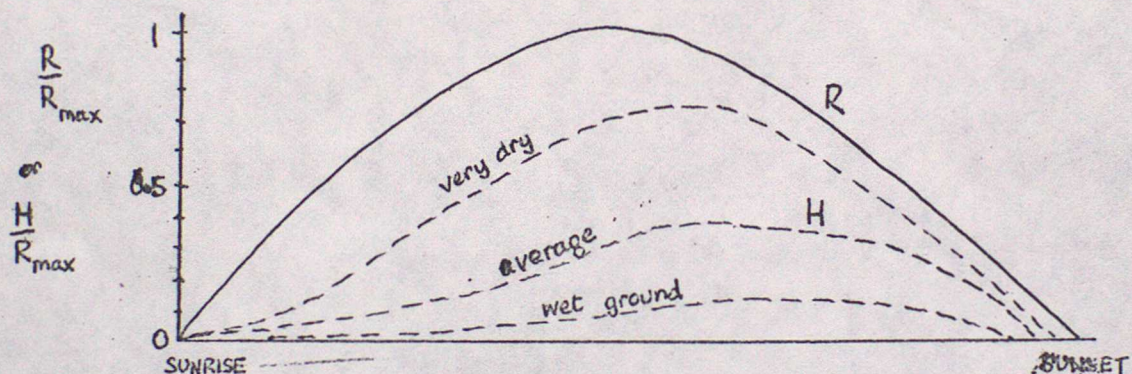
Often the atmosphere may be unstable in some layer (the boundary layer or mixing layer) up to some height h of the order of 400-2000 metres and stable above. In this case the stable layer provides a "lid" which will inhibit the pollutant escaping from the mixing layer to greater heights.

The more heat is rising from the ground, the more unstable the atmosphere is, and the faster is the dispersion of the plume (other things being equal). The estimation of heat flux is not easy. On a research basis, very fast response and delicate instruments are available but these are not suitable for general use. Instead a rather more empirical approach is preferred. The Meteorological Office is in the process of developing more accurate schemes based on detailed measurements made during 1976-77.

The problem can be subdivided as follows:

- (i) Estimate the net amount of radiation absorbed by the ground in clear sky conditions. This depends on the sun's elevation, the clarity of the atmosphere, the albedo of the ground.
- (ii) The modification of this amount by cloud. The fraction of sky covered, altitude and thickness will all be important factors.
- (iii) The plant cover. Heat received by leaves and branches will ultimately be given up to the atmosphere (except for a small fraction used in photosynthesis). Much of the heat getting to the soil or rock may simply warm the ground and only a fraction may go into the air.
- (iv) Latent heat of evaporation. According to the availability of moisture, much of the heat that flows into the air goes in latent form due to evaporation and contributes very little to generating turbulence unless the vapour condenses at some height to form cloud, thereby releasing the heat.
- (v) Whatever heat remains goes into the air as sensible heat, available for generating turbulence.

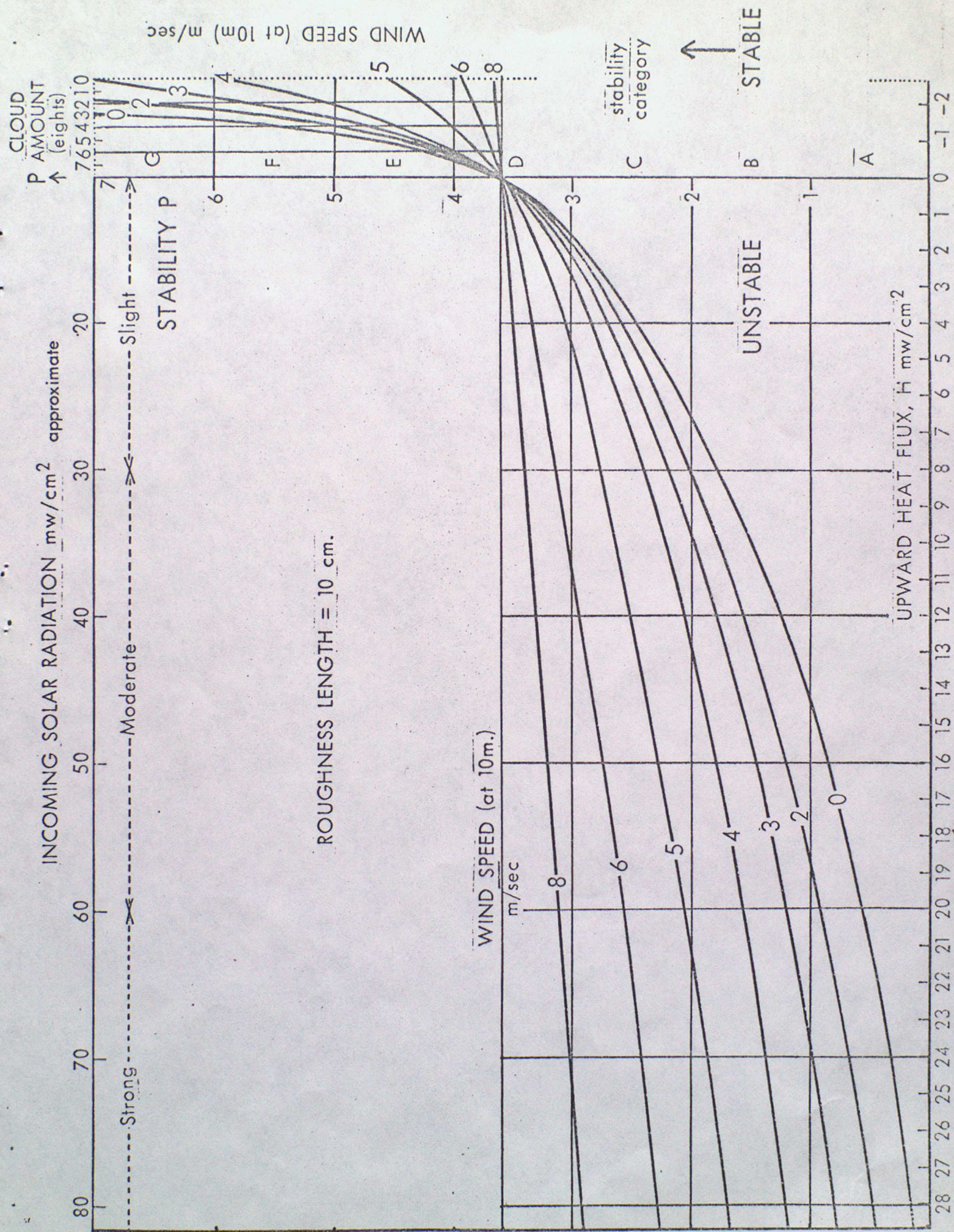
On a sunny day, the incoming radiation R and the sensible heat flux H follow a variation typified by the following diagram:



As a very rough guide we may take $H = 0.4(R-100)$ in units of watts per square metre. (Wm^{-2}), or $0.4(R-10)$ in milliwatts per square centimetre ($1 \text{ mw/cm}^2 \equiv 1 \text{ W/m}^2$).

Figure 5 shows how R may be approximately estimated as a function of month, time of day and cloud amount, so that at least a rough estimate of the surface sensible heat flux may be obtained. Hopefully a better scheme will be available soon.

Now heat flux is most effective when hot bubbles or plumes can form and carry heat and pollution upwards. If however the atmosphere is already turbulent due to the wind, these plumes may get broken up and dispersed. Moreover the stronger the wind, the longer in terms of distance downwind it takes for a hot plume to carry pollution upwards from a continuous stationary source to the top of the mixing layer. For both these reasons we see that with a heat source of energy and a mechanical (wind) source of energy, as the latter increases the effective rate of dispersion



through the vertical actually decreases. A rather curious result.

Pasquill in 1958 attempted to understand quantitatively the dual role of wind and heat flux on vertical dispersion. From carefully conducted experiments (supplemented in later years by mathematical solutions of the problem) he determined what combinations of the two yielded equal rates of growth of σ_p with distance. Classifying the latter in terms of stability categories A to F he arrived at a result which has subsequently been modified to give Figure 6.

A (or numerically $0 \leq P \leq 1$) is the most unstable class obtained on warm sunny days in summer; D is near neutral, and F is very stable and a night-time class. B, C and E are intermediate classes.

The interpretation of these classes in terms of dispersion requires an understanding of the role of the underlying surface and that is the topic of the next section.

4. Surface Roughness

Place a box on a wooden plank which is lifted at one end and the box will start to slide if the slope of the plank is large enough. The force of gravity acting on the box is large enough to overcome the frictional forces acting upslope between the box and the plank. If the plank is highly polished the box will slide with a relatively small slope but if it is rough then a larger slope will be required. The frictional drag between the two objects increases with increasing roughness. So it is with the frictional drag between the atmosphere and the ground. More energy is lost when the air blows over a city than over an ice field or a desert.

The frictional drag of the plank on the box may be easily assessed by the above simple experiment, comparing resolved forces along the slope.

It is obviously not so easy to measure the drag of the underlying surface on the atmosphere.

Firstly let us remind ourselves what we mean by drag. The total drag is obviously the total force between the ground and the atmosphere over a very large area. This isn't a very useful concept if we are solely concerned with what is going on over one large field, say. We usually mean by drag the force per unit area between ground and air. Force has dimensions of mass times acceleration or (density x volume) x acceleration.

Calling the drag (that is, the shearing stress) τ then the dimensions of τ/ρ (where ρ is the density of air) must be

$$\left[\frac{\tau}{\rho} \right] = \frac{1}{\rho} \frac{(\rho \times L^3) \times L T^{-2}}{L^2}$$

(where L = Length, T = time)

$$\text{ie } \left[\frac{\tau}{\rho} \right] = (L T^{-1})^2 = [\text{velocity}]^2$$

If we write this velocity u_* and call it the friction velocity, then u_* is simply another way of saying what the frictional drag of the surface is, and it's convenience is that it has the same simple dimensions as wind speed unlike τ which has rather more complicated dimensions.

One may speculate what other velocity u_* is related to. We have already discussed the relationship between the turbulent eddies and the drag of the ground in general terms. Perhaps u_* is related then to the typical velocities in the eddies. If we define a root-mean-square vertical velocity σ_w as

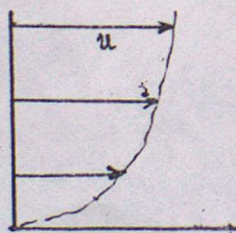
$$\sigma_w = \sqrt{\overline{w^2}}$$

where w is the instantaneous vertical velocity at a point, and the over-bar denotes a time average, then experiments show a nearly constant relation between u_* and σ_w .

$$\sigma_w = 1.3 u_*$$

at heights near the ground.

Now it is reasonable to relate σ_w to the local vertical gradient of the mean horizontal velocity \bar{u} since the eddies are formed by an overturning, tumbling action of the air as it flows over the ground



It follows that

$$u_* = (1.3)^{-1} \sigma_w = A \frac{du}{dz} \quad \text{where } A \text{ has to be determined.}$$

The friction velocity u_* , σ_w and \bar{u} are all velocities, and therefore to make the dimensions correct A must be proportional to a length to balance the dz in the velocity gradient. The most obvious length is height z itself.

We postulate then that

$$u_* = k z \frac{d\bar{u}}{dz}$$

where k is an absolute constant. Keeping u_* fixed we can integrate this equation to give the variation of \bar{u} with height:

$$u_* \ln z = k \bar{u} + \text{constant of integration.}$$

This can be rewritten
$$\bar{u} = \frac{u_*}{k} \ln \frac{z}{z_0}$$

where \ln is the natural logarithm and z_0 is a constant of integration which must have dimensions of length.

Experiments have amply verified that in near neutral stabilities this log-law holds very well indeed up to several tens of metres height, with $k = 0.4$.

What about its behaviour at small z ?

$$\ln 0 = -\infty$$

$$\ln 1 = 0$$

Therefore $\bar{u} = 0$ when $z = z_0$ and for $z < z_0$ the solution is obviously wrong, suggesting a reversed flow. In fact the log-law holds down to values of z several times larger than z_0 . But what is the physical significance of z_0 ? We would hope that z_0 is related to the size of the roughness elements on the ground - a measure of its smoothness. If wind speed recorders (called anemometers) measure the wind profile and \bar{u} is plotted on a graph against $\ln z$ then the curve should be a straight line which if extrapolated down to very small z should yield z_0 where the line implies $\bar{u} = 0$.

Many measurements have been made of z_0 over different surfaces

| | (cms) |
|-----------------|--------------------|
| Mud flats, ice | 1×10^{-3} |
| Smooth sea | 2×10^{-2} |
| Level desert | 3×10^{-2} |
| Lawn (1cm high) | 0.1 |
| Lawn (5cm high) | 1-2 |

| | |
|------------------------------------|--------|
| | (cms) |
| Large fields | 10 |
| Fully grown root crops | 10-14 |
| Small fields with hedges and trees | 25 |
| Forests | 100 |
| Towns | 50-400 |

Obviously z_0 depends on the type of surface but is not solely a measure of the height of the roughness elements but depends on the density of packing of the elements as well.

We see then that u_* and z_0 both describe to some degree the drag of the underlying surface:

z_0 is some measure of the physical dimensions of the roughness elements.

u_* is a description of the drag induced by a given wind over those elements.

In actual fact rather complex boundary layer theory can relate u_* to z_0 in terms of two other quantities: G , the geostrophic wind defined earlier, and f , the coriolis parameter = 1.12×10^{-4} secs $^{-1}$ at 51° N (related to latitude and the rotational period of the Earth). The theory gives u_*/G as a function of G/fz_0 and, to allow for the effects of stability, a parameter H/G^2 where H is the heat flux from the surface in Wm^{-2} and G is in ms^{-1} .

| $\log_{10} \frac{G}{fz_0}$ | H/G^2 | | | | | |
|----------------------------|---------|------|------|------|------|------|
| | 0 | 2 | 4 | 20 | 40 | 200 |
| 5 | .054 | .061 | .066 | .074 | .078 | .090 |
| 6 | .042 | .048 | .051 | .056 | .061 | .071 |
| 7 | .035 | .039 | .042 | .047 | .051 | .058 |
| 8 | .030 | .033 | .035 | .041 | .043 | .051 |
| 9 | .025 | .027 | .029 | .034 | .037 | .045 |

Table of $\frac{u_*}{G}$