Met O 11 Technical Note No 80

AN INVESTIGATION INTO BAROCLINIC INSTABILITY IN THE 10-LEVEL MODEL

by

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1. INTRODUCTION

This report describes the results of an experiment performed using the 'stripped' form of the 10-level model octagon. For a full description of the stripped octagon, see James (1976). The experiment was set up with the intention of comparing the growth rate and phase speed of an unstable baroclinic disturbance in the 10-level model with the results from a similar integration by the Reading University spectral model.

Spectral coefficients for the initialisation of baroclinic wavenumber 6 were sent by Reading University on magnetic tape and were subsequently converted for use in the 10-level model. A 6-day integration was carried out and results were written to disk every 16 timesteps (4 hours) resulting in a series of 37 datasets which were analysed to calculate the growth rate and phase speed of wavenumber 6.

2. GROWTH RATE AND PHASE SPEED CALCULATION

Simmons and Hoskins (1976) describe a method of monitoring the growth rate and phase speed of a wave in the spectral model and this is modified slightly to be used with the 10-level model.

We denote the spectral expansion of the amplitude of the zonal wavenumber m component of the height field at level l and time T "time-units" by

$$h_l^{(m)}(\mu, T) = \sum_{n=|m|}^{|m|+J} h_{nl}^m(T)\, P_n^m(\mu) \quad ,$$

where J is the meridional truncation and μ is sin (latitude). Now define two amplitude-weighted measures as follows:

$$\dot{g}(h) = \frac{1}{W} \sum_{l=1}^{10} \sum_{n=|m|}^{|m|+J} \left\{ |h_{nl}^m(T)| \times \left| \frac{|h_{nl}^m(T)|}{|h_{nl}^m(T-1)|} - \frac{|h_{nl}^m(T-1)|}{|h_{nl}^m(T-2)|} \right| \right\} \quad \ldots (1)$$

$$g(h) = \frac{1}{W} \sum_{l=1}^{10} \sum_{n=|m|}^{|m|+J} \left\{ |h_{nl}^m(T)| \times \frac{|h_{nl}^m(T)|}{|h_{nl}^m(T-1)|} \right\} \quad \ldots (2)$$

where

$$W = \sum_{l=1}^{10} \sum_{n=|m|}^{|m|+J} |h_{nl}^{m}(T)|$$

The function $\dot{g}(h)$ is a measure of how close the wave is to perfect exponential growth. When $\dot{g}(h)$ is actually zero the wave is growing exponentially, and the growth rate of the wave is taken to be

$$N \ln(g(h)) \quad (\text{day})^{-1}$$

where N is the number of "time-units" in 24 hours.

Simmons and Hoskins also describe a similar method used to calculate the phase speed of a wave. However, using this method computationally was found to introduce difficulties when following wave crests from one time to the next. Calculation of the phase speed was, therefore, performed manually, but in a similar fashion to the calculation of growth rate, that is by summing over all meridional wavenumbers and pressure levels.

3. RESULTS

Figures 1 and 2 show the 500 mb height fields and 550 mb vertical velocity fields for days 0, 3, 5 and 6. The wave is seen to grow and progress steadily throughout the 6-day period with increasing vertical motion. The characteristic tilting of the trough and ridge axes is also in evidence: to the north of the jet maximum, the tilt is NW - SE, while to the south the tilt is NE - SW - a structure consistent with the convergence of westerly momentum in the neighbourhood of the jet.

Detailed plots of the function $\dot{g}(h)$, together with the growth rate and phase speed are shown in figure 3. We see from the graph that after 160 timesteps (nearly two days) the value of $\dot{g}(h)$ has fallen to less than 10% of its initial value, and so we assume that exponential growth is occurring (to a close approximation) from this point until the end of the integration. However, when we look at the graph of growth rate above, we see a distinct tailing-off beginning after about 288 timesteps (3 days), suggesting that Simmons' and Hoskins' criterion for exponential growth based on the size of $\dot{g}(h)$ may not be wholly appropriate. Between 160 and 288 timesteps the value of the growth rate remains fairly steady at about 0.64 $(\text{day})^{-1}$.

Looking at the graph of phase speed we see that after the initial 'settling-down' period, the wave progresses steadily at about 8.5 degrees per day until the very end of the integration when breakdown is occurring.

It should be remembered that the values of growth rate and phase speed are obtained by summing over all ten levels of the model. For an assessment of growth from level to level we examine the change of the total amplitude of wavenumber 6 with time for 200, 500 and 900 mb (figure 4). The graph shows that the period of uniform exponential growth varies from level to level, as does the rate of growth. At 900 mb, uniform exponential growth occurs approximately between 32 and 256 timesteps, during which period the growth rate is measured directly from the graph as 0.599 $(\text{day})^{-1}$ (see table 1).

Table 1

P (mb)	Growth Rate $(\text{day})^{-1}$	Phase Speed (degrees/day)	Period of Exponential Growth (timesteps)
200	0.570	8.5	288–480
500	0.656	8.5	32–480
900	0.599	8.5	32–256
Reading University –	0.522	9.6	

Similarly, at 500 mb the period starts after 32 timesteps and only tails off slightly towards the end of the integration, and the growth rate is 0.656 $(\text{day})^{-1}$; whereas at 200 mb the wave does not start to grow until about 112 timesteps and only really becomes uniform after 288 timesteps when the growth rate is measured to be 0.570 $(\text{day})^{-1}$. The values of growth rate plotted in figure 3 are in some sense an average of the different growth rates over the ten levels, and the graph therefore obscures some of the detailed characteristics of the growth of the wave which are shown up in figure 4.

It is noticeable from figure 4 that at 900 mb the wave gradually stops growing and begins to decrease in amplitude during the last day of integration, and that at 200 mb the wave does not begin to grow at all until the second day. It would appear that the growth of the wave is slightly 'out of phase' from level to level, with the growth at lower levels occurring earlier than at upper levels. It is not known whether this is due to a real physical effect, to a peculiarity of the 10-level model or to the initial data.

Figures 5 and 6 show phase/time plots for two of the wave components (6,6) and (6,10) at 200, 500 and 900 mb. Apart from the initial unsteadiness in the 200 mb (6,6) wave and the acceleration of both waves at 900 mb towards the end of the period, the waves at all three levels are remarkably uniform in their phase speeds. Measured directly from the graphs, this works out to be approximately 8.5 degrees/day which agrees with the value derived from figure 3. For the (6,6) component there is a phase lagbetween 900 mb and 200 mb of a little over $\pi/2$ with most of this occurring below 500 mb, while for the (6,10) component the phase lag is about 0.3π between 900 mb and 500 mb with very little phase lag above 500 mb. As the wave at 900 mb ceases to grow, the phase speeds of its components increase thereby making the phase lag with height greater. It is not easy to explain this feature.

The values of growth rate and phase speed obtained by Reading University were 0.522 $(\text{day})^{-1}$ and 9.56 degrees/day respectively. The value for the growth rate is about 20% lower than in the 10-level model, whereas the value of the phase speed is about 1 degree/day more than in the 10-level model.

4. CONCLUSION

The experiment has shown that it is very difficult to specify one particular value of 'growth rate' for an unstable baroclinic wavenumber 6 disturbance in the 10-level model due to the fact that both the period of exponential growth and the rate of growth vary from one level to the next. However, a fairly definite figure of 8.5 degrees/day can be given to the phase speed which is slightly less than that obtained by the Reading University team. Similarly, their value of 0.522 (day)-1 for the growth rate is less than the values obtained at 200, 500 and 900 mb in the 10-level model.

REFERENCES

James, I.N. (1976) Met O 11 Tech. Note No. 60

Simmons, A.J., and Hoskins, B.J. (1976) J. Atmos. Sci 33 pp. 1454-1477

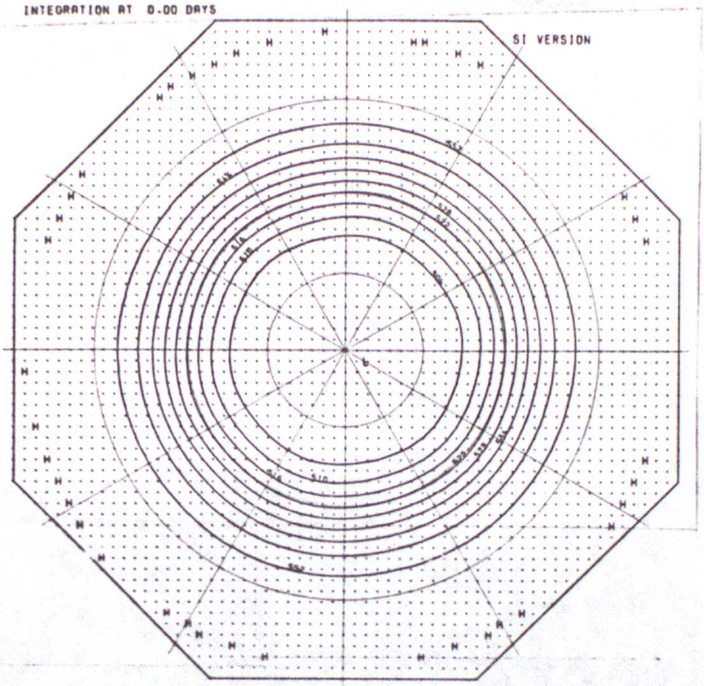
FIGURE 1

DAY 0

500 MB CONTOURS

500 MB CONTOURS   CONTOUR LINES AT 6 DECAMETER INTERVALS

INTEGRATION AT 0.00 DAYS



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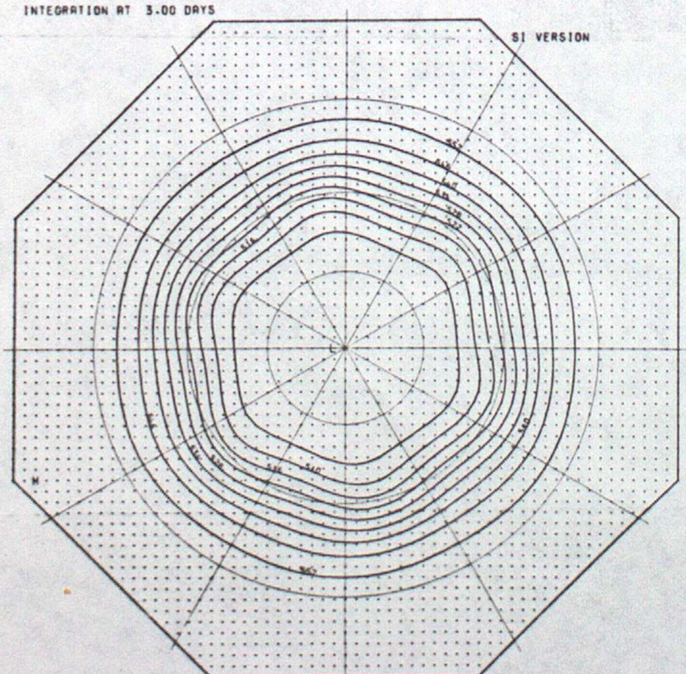
DAY 3

500 MB CONTOURS

500 MB CONTOURS   CONTOUR LINES AT 6 DECAMETER INTERVALS

INTEGRATION AT 3.00 DAYS

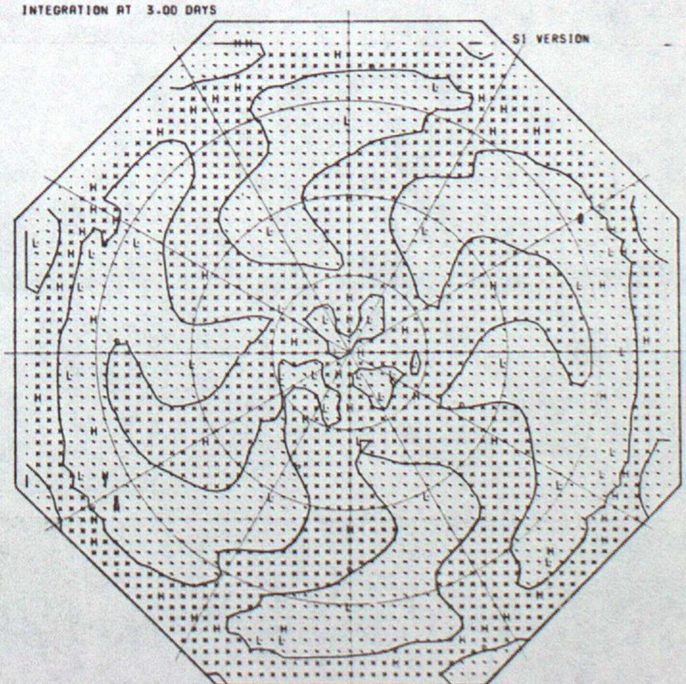


R0659U01

550 MB VERTICAL VELOCITY

550MB VERT VEL   ISOPLETH INTERVAL = 10 MB/HOUR

INTEGRATION AT 3.00 DAYS



R0659U01

. INDICATES 0 TO -5MB/HR   x INDICATES 0 TO +5MB/HR

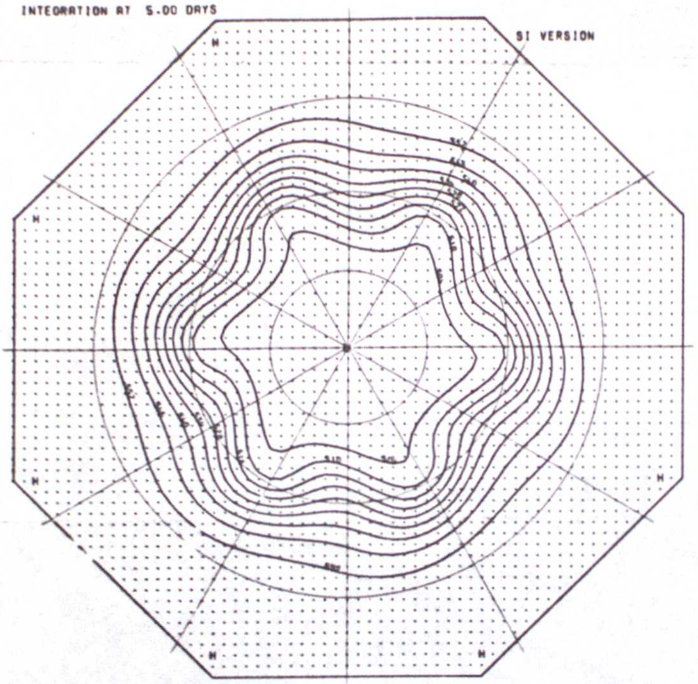
FIGURE 2

DAY 5

500 MB CONTOURS

500 MB CONTOURS CONTOUR LINES AT 6 DECAMETER INTERVALS

INTEGRATION AT 5.00 DAYS

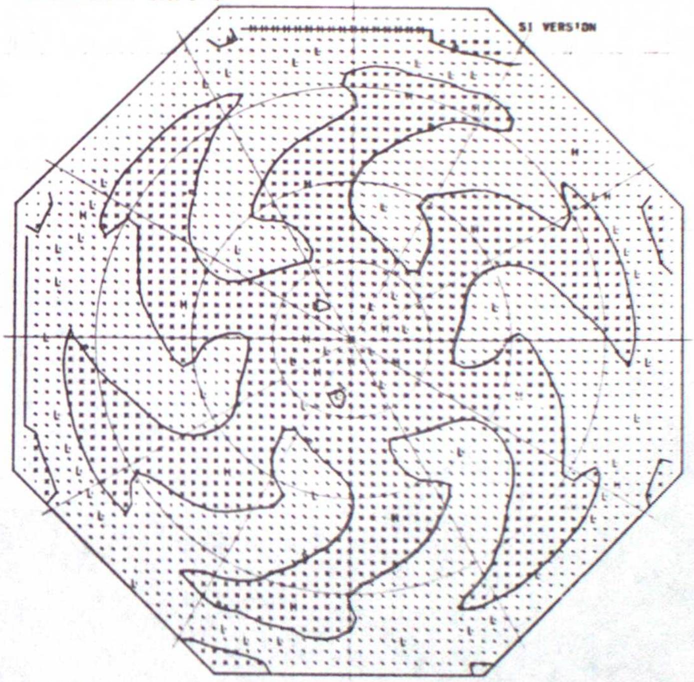


R0659U01

550 MB VERTICAL VELOCITY

550MB VERT VEL ISOPLETH INTERVAL = 10 MB/HOUR

INTEGRATION AT 5.00 DAYS



R0659U01

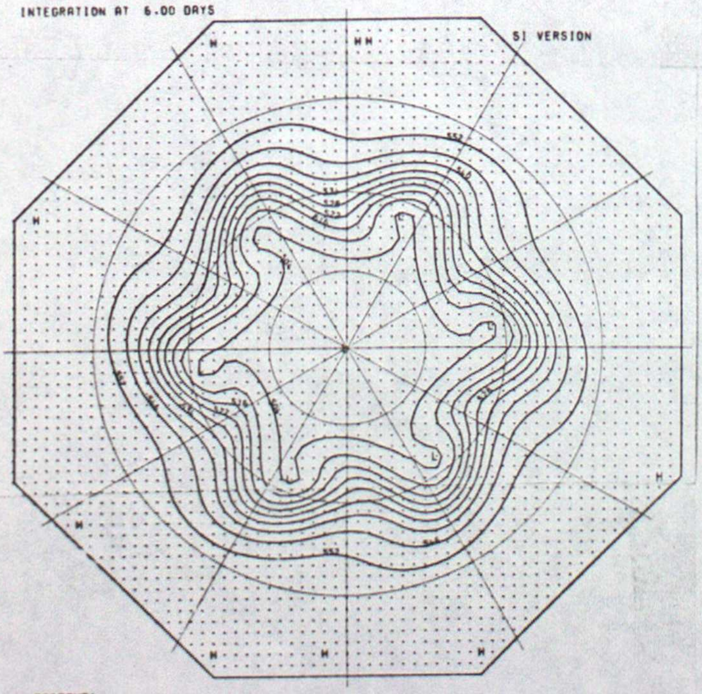
. INDICATES 0 TO -5MB/HR X INDICATES 0 TO +5MB/HR

DAY 6

500 MB CONTOURS

500 MB CONTOURS CONTOUR LINES AT 6 DECAMETER INTERVALS

INTEGRATION AT 6.00 DAYS



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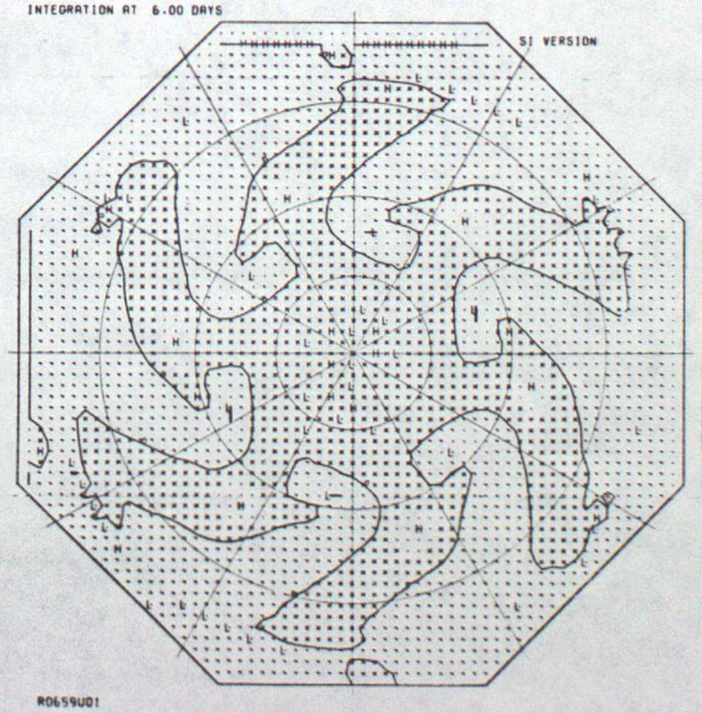
550 MB VERTICAL VELOCITY

550MB VERT VEL ISOPLETH INTERVAL = 10 MB/HOUR

INTEGRATION AT 6.00 DAYS



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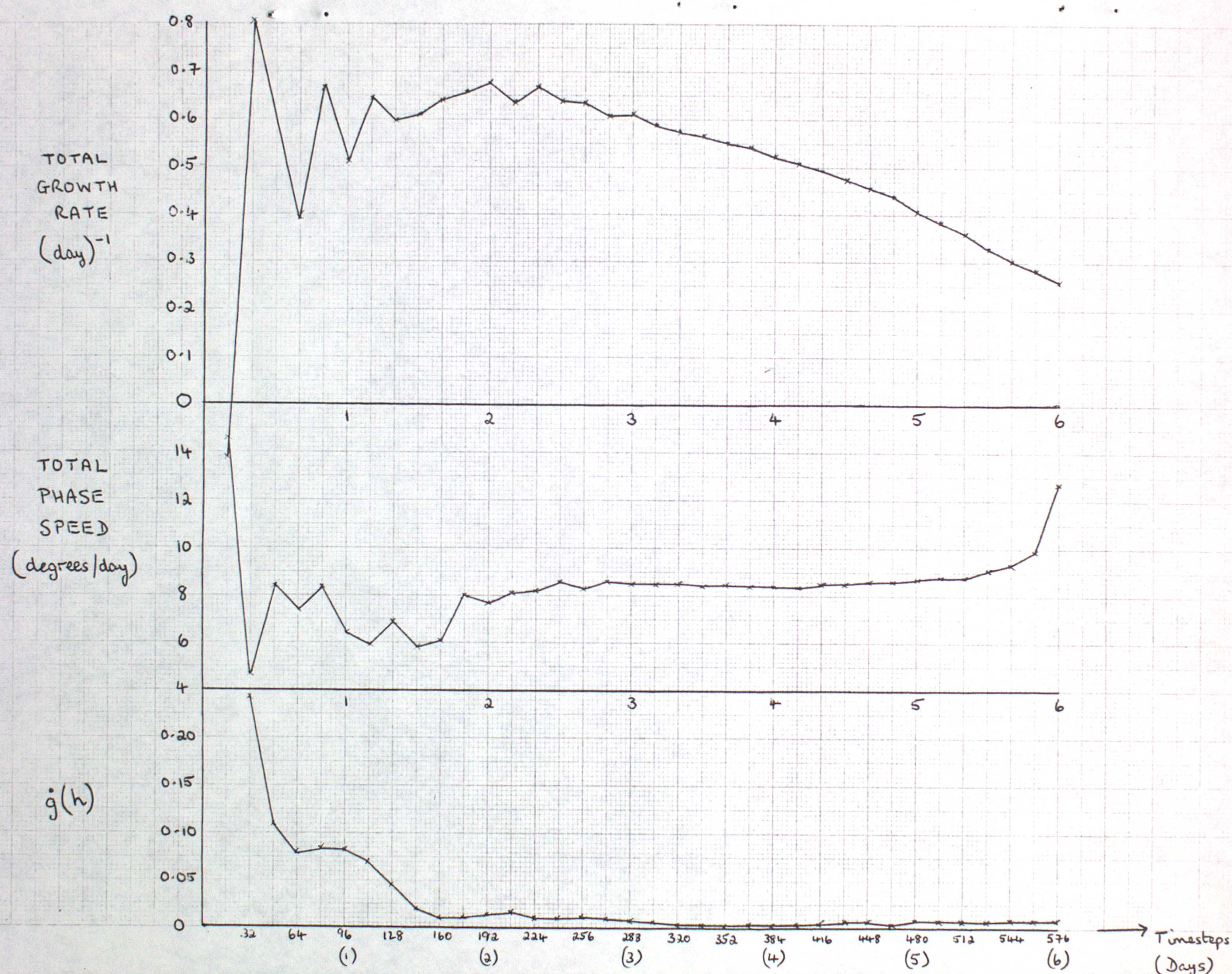
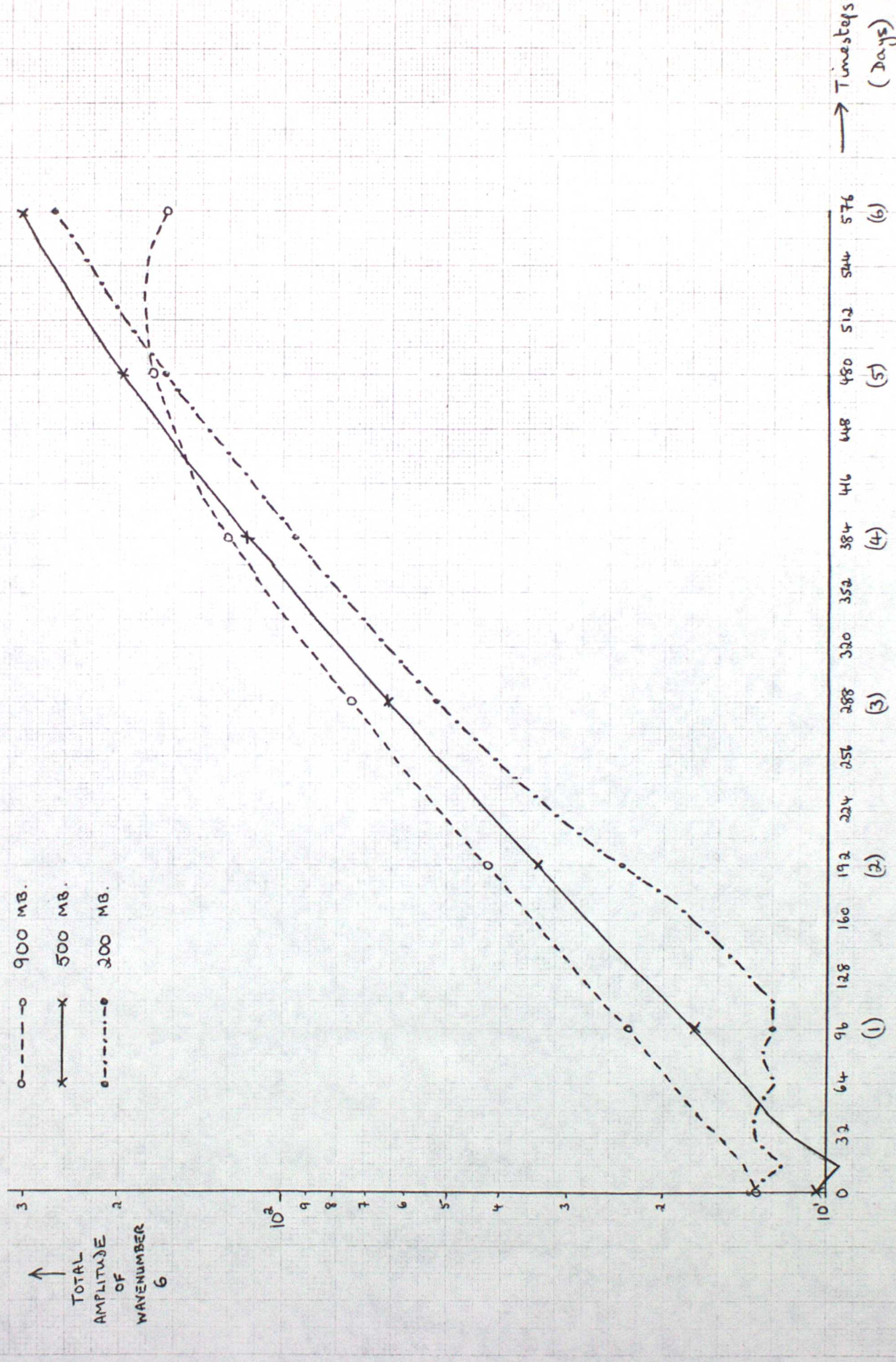
. INDICATES 0 TO -5MB/HR X INDICATES 0 TO +5MB/HR

FIGURE 3



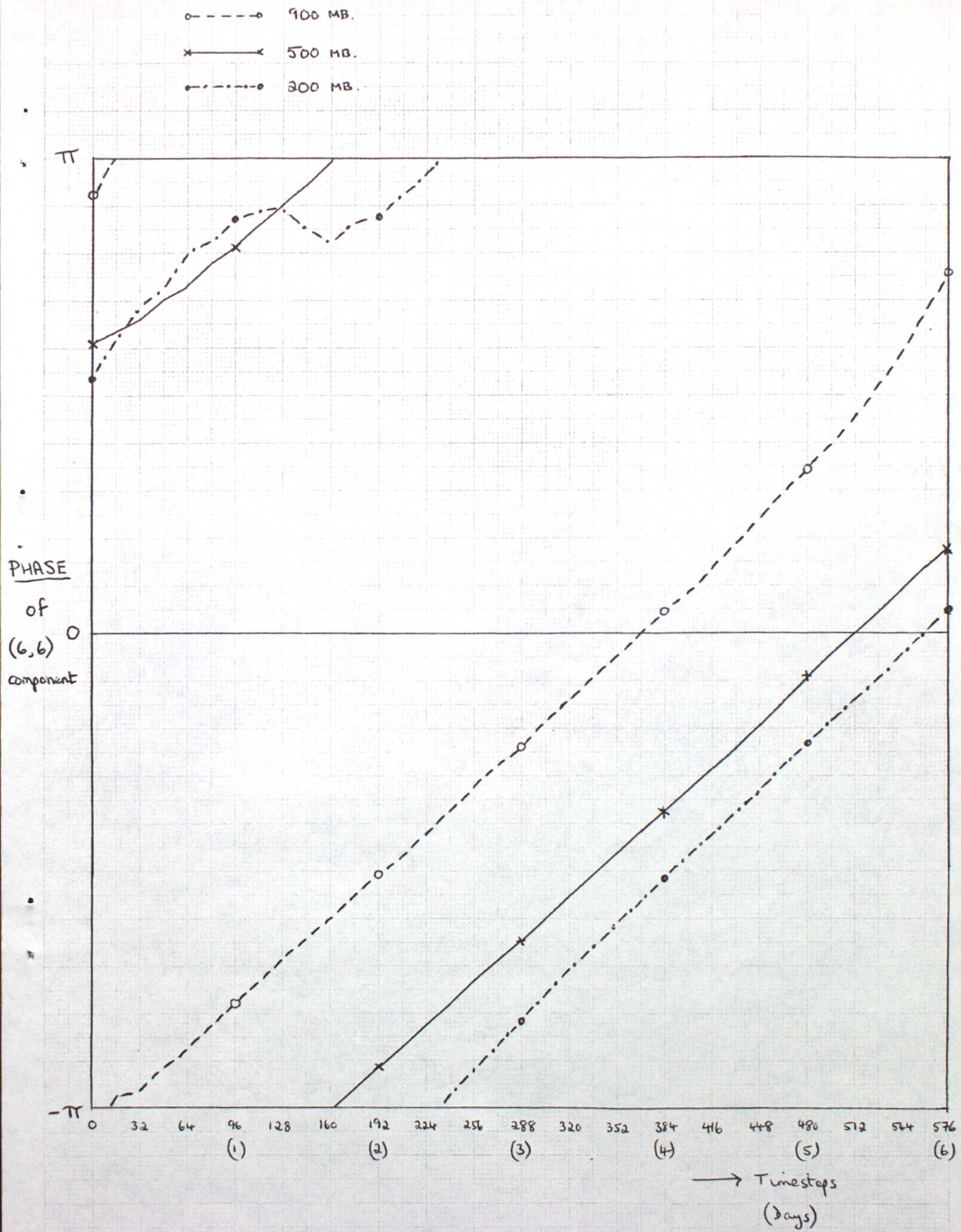
TOTAL GROWTH RATE (day)$^{-1}$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
6
TOTAL PHASE SPEED (degrees/day)
14
12
10
8
6
4
$\dot{g}(h)$
0.20
0.15
0.10
0.05
.32
64
96
128
160
192
224
256
288
320
352
384
416
448
480
512
544
576
(1)
(2)
(3)
(4)
(5)
(6)
Timesteps (Days)

FIGURE 4



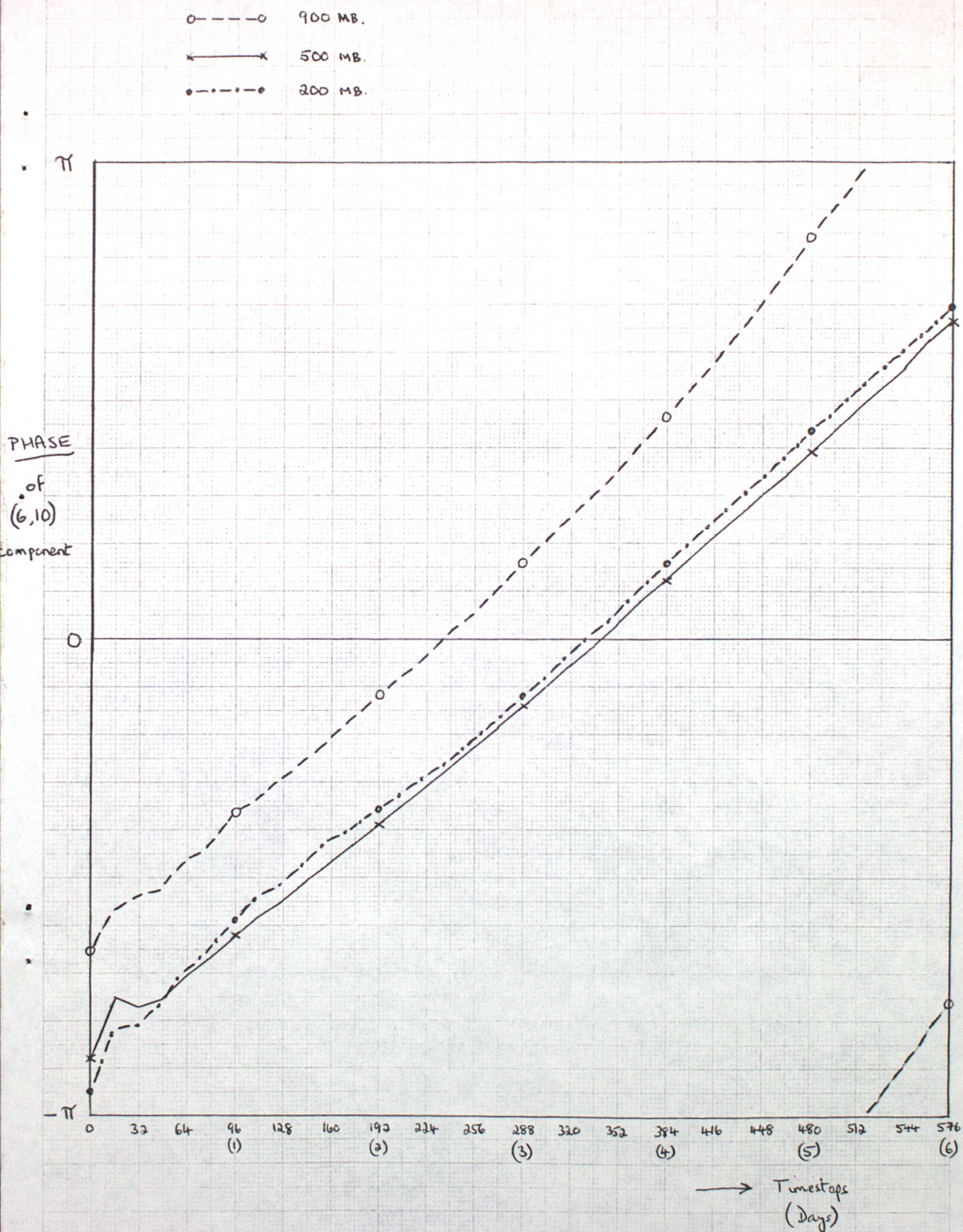
TOTAL AMPLITUDE OF WAVENUMBER 6
900 MB.
500 MB.
200 MB.
10'
2
3
4
5
6
7
8
9
10²
2
3
0
32
64
96
128
160
192
224
256
288
320
352
384
416
448
480
512
544
576
(1)
(2)
(3)
(4)
(5)
(6)
Timesteps
(Days)

FIGURE 5



900 MB.
500 MB.
200 MB.
π
0
−π
PHASE
of
(6,6)
component
0
32
64
96
128
160
192
224
256
288
320
352
384
416
448
480
512
544
576
(1)
(2)
(3)
(4)
(5)
(6)
→ Timesteps
(Days)

FIGURE 6



900 MB.
500 MB.
200 MB.
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PHASE
of
(6,10)
component
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32
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96
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128
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(2)
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256
288
(3)
320
352
384
(4)
416
448
480
(5)
512
544
576
(6)
Timesteps
(Days)