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METEOROLOGICAL OFFICE

Scientific Paper No. 14

Variation of the difference between
two earth temperatures

by P. B. SARSON, M.A.

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Contents

Summary	1
Theoretical treatment	1
Practical applications	4

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SUMMARY

In the study and analysis of earth temperatures it is often necessary to compare data from one station with those from another or with data from a different depth at the same station. Despite the natural smoothing of the data by the slow diffusion of heat from above or below, earth temperatures often show greater differences between neighbouring stations than between stations a hundred miles or more apart. These differences arise from variations in diffusivity: the diffusivity depends not only on soil type but also on drainage. Nevertheless, by forming simple systematic differences it is possible to compare data from such neighbouring stations in some detail, and even to make estimates of missing data or extend the period of means generally within $\pm 0.1^\circ\text{F}$ accuracy. The method depends on the natural smoothing by heat diffusion, such errors as do arise being solely attributable to the variation with time of the coefficient of thermal diffusivity caused by large variations in sunshine or rainfall.

THEORETICAL TREATMENT

We can assume that the surface temperature may be expressed by the equation:

$$\theta = a_o + \sum b_i \sin\left(\frac{2\pi t}{T_i} + \alpha_i\right) \quad . . . (1)$$

where the periods of oscillation T_i and the phases α_i are unspecified. The coefficients a_o , b_i and the phases α_i are all constant. If we can assume the diffusivity κ ($=k/\rho c$ where k is the thermal conductivity, ρ the density and c the specific heat) is constant with depth, then, from the normal laws of conduction, the earth temperature at depth z is

$$\theta_z = a_o + \sum b_i e^{-2\pi z/\lambda_i} \sin\left(\frac{2\pi t}{T_i} + \alpha_i - \frac{2\pi z}{\lambda_i}\right) + \delta_1 \quad . . . (2)$$

where $\lambda_i = 2\sqrt{(\pi \kappa T_i)}$, and δ_1 is a small random error introduced to correct for variations in diffusivity with time. At a neighbouring station we may assume the surface temperature is

$$\theta' = a'_o + \sum b'_i \sin\left(\frac{2\pi t}{T_i} + \alpha'_i\right).$$

The earth temperature at this station, at depth z' , is

$$\theta'_{z'} = a'_o + \sum b'_i e^{-2\pi z'/\lambda'_i} \sin\left(\frac{2\pi t}{T_i} + \alpha'_i - \frac{2\pi z'}{\lambda'_i}\right) + \delta_2$$

where δ_2 is a small random error similar to δ_1 . The difference between the earth temperatures at the two stations is

$$\begin{aligned} \theta_z - \theta'_{z'} &= a_o - a'_o + \sum \left\{ b_i e^{-2\pi z/\lambda_i} \sin\left(\frac{2\pi t}{T_i} + \alpha_i - \frac{2\pi z}{\lambda_i}\right) - b'_i e^{-2\pi z'/\lambda'_i} \sin\left(\frac{2\pi t}{T_i} + \alpha'_i - \frac{2\pi z'}{\lambda'_i}\right) \right\} + \delta_1 - \delta_2 \\ &= a_o - a'_o + \sum A_i b_i e^{-2\pi z/\lambda_i} \sin\left(\frac{2\pi t}{T_i} - \frac{2\pi z}{\lambda_i} + \alpha_i + \varepsilon_i\right) + \delta_1 - \delta_2, \quad . . . (3) \end{aligned}$$

where

$$A_i^2 = 1 - 2(b'_i/b_i)e^{\zeta_i} \cos(\alpha_i - \alpha'_i - \zeta_i) + (b'_i/b_i)^2 e^{2\zeta_i}, \quad \dots (3a)$$

$$\tan \varepsilon_i = \frac{b'_i \sin(\alpha_i - \alpha'_i - \zeta_i)}{b_i e^{\zeta_i} - b'_i \cos(\alpha_i - \alpha'_i - \zeta_i)} \quad \dots (3b)$$

and

$$\zeta_i = 2\pi \left(\frac{z}{\lambda_i} - \frac{z'}{\lambda'_i} \right). \quad \dots (3c)$$

Since $a_o, a'_o, b_i, b'_i, \alpha_i, \alpha'_i$ are all constants, being the harmonic coefficients and phase angles of the expressions of the surface temperature at the two stations, the difference between two earth temperatures is the sum of three constituent parts:

- (i) a constant, $a_o - a'_o$, being the difference between the annual mean surface temperatures at the two stations,
- (ii) a small random error, $\delta_1 - \delta_2$, depending on the variations of thermal diffusivity with time,
- (iii) a summation of the original harmonics of the earth temperature at one of the stations each multiplied by a factor A_i depending only on values of ζ_i (that is, on the difference $z/\lambda_i - z'/\lambda'_i$), and with a phase change ε_i , depending only on values of ζ_i .

The thermal diffusivity of the soil, κ , varies from 0.0025 to 0.010 cm²/sec, so that values of λ_i (in centimetres) vary from $12\sqrt{(6\pi T_i)}$ to $24\sqrt{(6\pi T_i)}$ where T_i is measured in days. The principal oscillations of temperature have periods of one day or one year; therefore corresponding to the diurnal oscillation λ_d varies from about 50 to 100, and corresponding to the annual oscillation λ_y varies from about 1,000 to 2,000, the suffixes d and y referring to diurnal and yearly values. Values of ζ_i depend on the depths to which the particular earth temperatures refer. In comparing earth temperatures at different levels at the same station, $\lambda'_i = \lambda_i$, $a'_o = a_o$, $b'_i = b_i$, $\alpha'_i = \alpha_i$, and therefore

$$A_i^2 = 1 - 2e^{\zeta_i} \cos \zeta_i + e^{2\zeta_i} = 2e^{\zeta_i} (\cosh \zeta_i - \cos \zeta_i),$$

$$\tan \varepsilon_i = \sin \zeta_i / (\cos \zeta_i - e^{-\zeta_i})$$

and

$$\zeta_i = 2\pi(z - z')/\lambda_i.$$

For a diurnal oscillation, the maximum values of ζ_d for a comparison between temperatures at 4 in. and 8 in., and between 4 in. and 1 ft are about 1.2 and 2.4, respectively. For an annual oscillation, the maximum values of ζ_y for a comparison between temperatures at 4 in. and 8 in., between 4 in. and 1 ft, between 1 ft and 2 ft and between 1 ft and 4 ft are about 0.06, 0.12, 0.18 and 0.55, respectively.

In comparing earth temperatures at the same level at different stations, $z = z'$. The approximate maximum values of ζ_i are then: $\zeta_d = 2\pi z/100$ for a diurnal oscillation, and $\zeta_y = 2\pi z/2,000$ for an annual oscillation. The maximum values of ζ_d at 4 in., 8 in. and 1 ft are about 0.6, 1.2 and 1.8, respectively; the maximum values of ζ_y at 4 in., 8 in., 1 ft, 2 ft and 4 ft are about 0.03, 0.06, 0.09, 0.2 and 0.4, respectively.

Strictly, the amplitude of the diurnal temperature oscillation is not constant; average values have an annual oscillation about a mean value; thus

$$b_d = B_o + b_o \sin\left(\frac{2\pi t}{T_y} + \alpha_b\right) \quad . . . (4)$$

where B_o and b_o are constants. The contribution of the diurnal oscillation to the surface temperature is then

$$b_d \sin\left(\frac{2\pi t}{T_d} + \alpha_d\right) = B_o \sin\left(\frac{2\pi t}{T_d} + \alpha_d\right) + \frac{1}{2}b_o \left\{ \cos\left(\frac{2\pi t}{T_d + \tau_b} + \alpha_d - \alpha_b\right) - \cos\left(\frac{2\pi t}{T_d - \tau_b} + \alpha_d - \alpha_b\right) \right\}. \quad (5)$$

where τ_b/T_b is approximately $1/365$. The values of ζ_t corresponding to $T_d \pm \tau_b$ differ from ζ_d by $\zeta_d/730$. These differences are quite negligible, and we may therefore use the same values ζ_d , A_d , ε_d for all three parts of the diurnal oscillation in equation (5). The contribution of the diurnal oscillation to the difference between two earth temperatures is then

$$\begin{aligned} & A_d B_o e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right) + A_d b_o e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t}{T_y} + \alpha_b\right) \sin\left(\frac{2\pi t}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right) \\ & = A_d b_d e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right). \quad . . . (6) \end{aligned}$$

For earth temperatures, particularly average values at neighbouring stations, the two significant oscillations are the diurnal and annual variations, and it is sufficient to consider only the first harmonics of these. The earth temperature difference is then

$$\begin{aligned} \theta_z - \theta'_z &= A_d b_d e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right) + A_y b_y e^{-2\pi z/\lambda_y} \sin\left(\frac{2\pi t}{T_y} - \frac{2\pi z}{\lambda_y} + \alpha_y + \varepsilon_y\right) \\ &\quad + a_o - a'_o + \delta_1 - \delta_2 \\ &= A_d \theta_d + A_y \theta_y + a_o - a'_o + \delta_1 - \delta_2, \end{aligned}$$

where θ_d is difference of the earth temperature at depth z from the daily mean at a time $\varepsilon_d T_d/2\pi$ later than t , and θ_y is the difference of the daily mean earth temperature from the annual mean at a time $\varepsilon_y T_y/2\pi$ later than t . These time lags, depending only on ε_d and ε_y , are constant providing there are no changes in the régime of earth temperature at either station.

Even when earth temperatures are measured more than once a day it is sometimes convenient to compare values at a fixed hour, t_1 , say. Then, from equation (6),

$$\begin{aligned} A_d \theta_d &= A_d B_o e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t_1}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right) + A_d b_o e^{-2\pi z/\lambda_d} \sin\left(\frac{2\pi t}{T_y} + \alpha_b\right) \sin\left(\frac{2\pi t_1}{T_d} - \frac{2\pi z}{\lambda_d} + \alpha_d + \varepsilon_d\right) \\ &= B + B_d \theta'_y, \end{aligned}$$

where B and B_d are constants and θ'_y is the difference of the daily mean earth temperature from the annual mean at a time $(\alpha_b - \alpha_y + 2\pi z/\lambda_y)T_y/2\pi$ later than t .

Let us now suppose that the earth temperature on any day in any year at depth z is

$$\theta_z = a_o + \theta_m + \phi$$

where $a_o + \theta_m$ is the long-term average or normal temperature at any time of year, and ϕ is the departure of the daily mean from the average on that date in any year. It is clearly possible to find a time difference such that

$$B_d\theta'_y + A_y\theta_y = A(\theta_m + \phi),$$

where the quantities on each side of this equation refer to temperatures at different times of the year, the difference between the times being constant. Thus

$$\theta_z - \theta'_z = A\theta_m + B + a_o - a'_o + A\phi + \delta_1 - \delta_2. \quad \dots (7)$$

Of the terms on the right-hand side, $A\theta_m$ changes only from day to day and not from year to year; A increases with $(z/\lambda_i - z'/\lambda'_i)$ and is of the order of a tenth or less; B is a proportion of the mean diurnal amplitude and vanishes if the diurnal variation may be neglected; $a_o - a'_o$ is the difference in the annual mean temperature at the two stations and is almost constant; $A\phi$ is a second order variable since ϕ is the departure of the earth temperature from the long-term average; it is comparable in size with the random variables δ_1 and δ_2 except in extreme years; δ_1 and δ_2 are random corrections for the variation of diffusivity with time; over a short period, except after heavy rain or sudden thaws, changes in δ_1 and δ_2 are small; over longer periods or in comparisons between years or periods of years the mean values of δ_1 and δ_2 become very small except in years with exceptional soil conditions. Combining $A\phi$, δ_1 and δ_2 together we have the general equation for the difference between two earth temperatures:

$$\theta_z - \theta'_z = A\theta_m + B + a_o - a'_o + \delta \quad \dots (8)$$

where δ is the sum total of all the random fluctuations. Changes in régime at either station will be exhibited as changes in A and B , and conversely changes in A and B point to changes in régime at either station. B is negligible if the earth temperature has no diurnal variation at either station.

PRACTICAL APPLICATIONS

Estimation of missing daily values

Providing there are no sudden falls of heavy rain to cause sudden changes in diffusivity or rapid drainage through sandy soils, we can assume that all the terms on the right-hand side of equation (7) change slowly. Therefore, during a short period for which observations are missing at a station, a good estimate of the earth temperatures can be made by comparison with the earth temperatures at a neighbouring station, for the difference between the temperatures at the two stations can be estimated by interpolation from the temperature differences at the beginning and end of the period. This method is less successful for earth temperatures at the shallow depths when allowance must be made for even small falls of rain or long hours of hot sunshine, either of which will modify the diffusivity considerably near the surface.

The method was tested by choosing six months at random in the period 1936–50 using three groups of three stations:

- { Burnley, height: 458 ft, clay soil
- { Harrogate, height: 478 ft, 9–12 in. loam over yellow clay
- { Bradford, height: 439 ft, light soil (made up with ashes and clinker)
- { Cambridge Botanical Gardens, height: 41 ft, sandy loam over gravel and chalk
- { Clacton-on-Sea, height: 53 ft, 18 in. earth over gravel
- { Lowestoft, height: 82 ft, light sandy soil over chalk
- { Cambridge Botanical Gardens, height: 41 ft, sandy loam over gravel and chalk
- { Greenwich Observatory, height: 149 ft, hard gravel
- { Kensington Palace, height: 80 ft, light loam over gravel

The middle ten days (11th–20th) of each month were assumed to be missing at each station in turn, and the observations estimated by interpolation from the five-day mean earth-temperature difference before (6th–10th) and after (21st–25th) each period of “missing” observations. The results of the test are summarized in Table I.

TABLE I. *Comparison of estimates of individual earth temperatures*

	Maximum error in estimate						Error in monthly mean formed by combining estimates from two comparisons					
	Daily values		10-day mean		Monthly mean							
	1 ft	4 ft	1 ft	4 ft	1 ft	4 ft	1 ft	4 ft	1 ft	4 ft	1 ft	4 ft
	<i>degrees Fahrenheit</i>						<i>degrees Fahrenheit</i>					
							Burnley		Harrogate		Bradford	
Mar. 1940	2.8	0.8	0.6	0.3	0.2	0.1	0.2	0.1	0.1	0	0.1	0.1
Apr. 1946	1.0	0.4	0.3	0.1	0.1	0	0.1	0	0.1	0	0	0
Aug. 1943	1.5	0.4	0.7	0.2	0.2	0.1	0.1	0	0.1	0.1	0	0
							Cambridge		Clacton		Lowestoft	
June 1936	2.4	1.7	0.8	0.4	0.3	0.1	0.2	0.1	0.2	0.1	0.3	0.1
Nov. 1941	2.0	0.5	0.6	0.2	0.2	0.1	0.1	0	0.1	0	0.2	0
							Cambridge		Greenwich		Kensington	
May 1943	1.5	1.0	0.5	0.3	0.2	0.1	0.1	0	0.1	0.1	0	0

A value zero given in this table means that the estimate was within 0.05°F of the true value of the earth temperature mean. The large error of 2.8°F in an estimate of 1 ft earth temperature in March 1940 was caused by a day or two's delay in the arrival of the thaw* at Burnley as compared with its arrival at Bradford and Harrogate on the other side of the Pennines.

The irregular results in June 1936 are more typical of a thundery month in summer with rainfall occurring sometimes at one station and not at the comparison station. It is of interest to examine the individual differences and therefore some of them are plotted in Figure 1. The greater randomness at the 1 ft level is at once apparent. The largest change (3.6°F) in earth temperature difference between one day and the next occurred between the 2nd and 3rd (Cambridge–Lowestoft); this sudden change did not appear in the difference, Cambridge–Clacton. The reason for the sudden change was that 10.4 mm of rain were recorded at Cambridge and 8.4 mm at Clacton, but none at all at Lowestoft. The 4 ft difference, Cambridge–Lowestoft, shows a rogue difference on the 14th. This is thought to be a 1°F error made in reading the thermometer at Lowestoft.

The broken lines in each diagram of Figure 1 represent the linear interpolations used to form the estimates of “missing” observations summarized in Table I. No improvement is to

* The winter of 1939–40 was exceptionally cold.

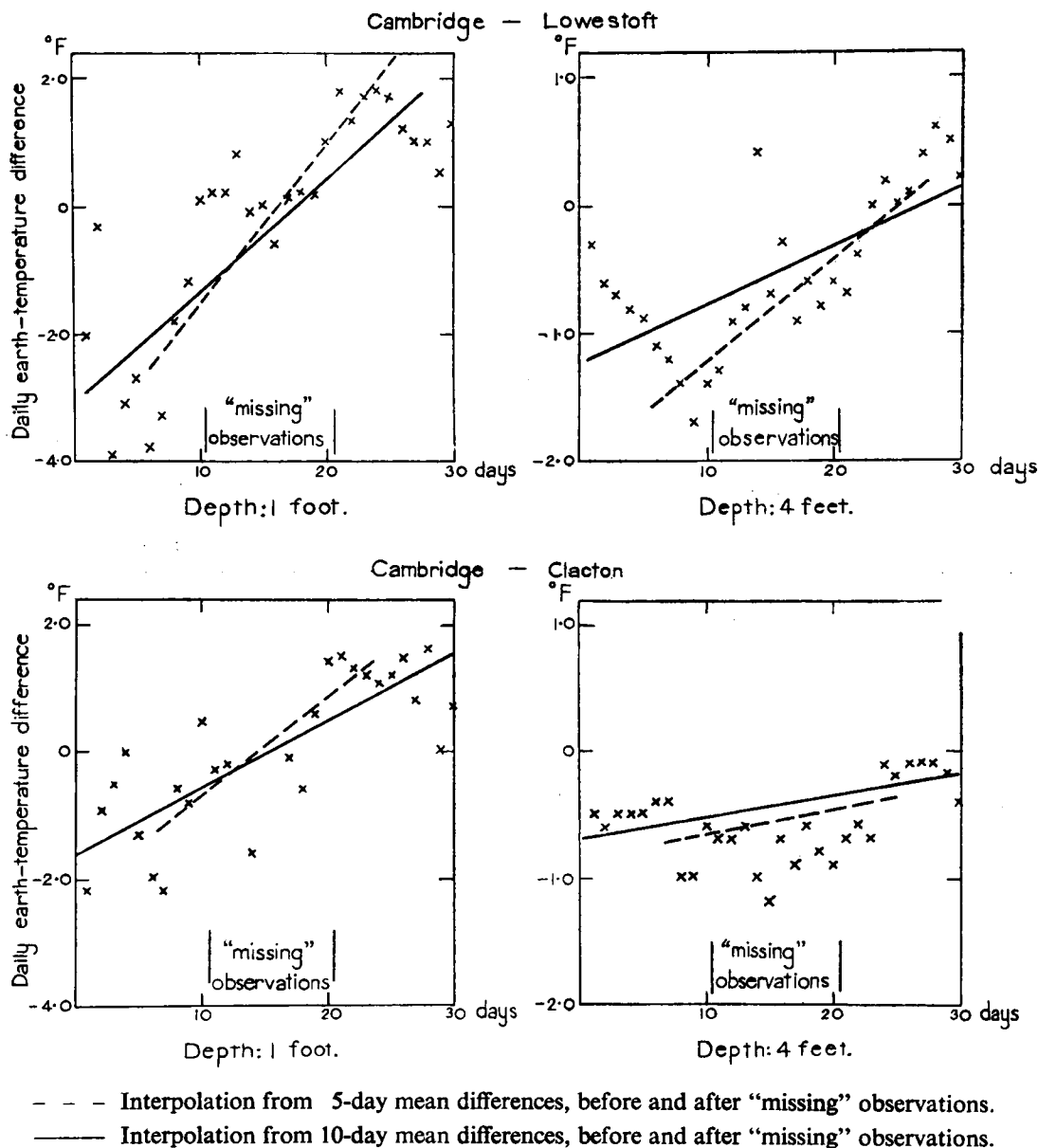


FIGURE 1. Daily earth-temperature difference, Cambridge Botanical Gardens, Lowestoft and Clacton-on-Sea, June 1936.

The vertical scale on the 4-ft diagrams is twice that in the 1-ft diagrams.

be obtained by interpolating from the means of 10-day periods before and after the period of missing data; such interpolations are represented by the full lines in the diagrams.

Estimation of missing monthly means

Mean values of δ_1 and δ_2 , being random variables, become quite small over a period of a month, and the mean value of $A\theta_m$ is approximately constant. Therefore, in equation (7),

only $A\phi$ can vary appreciably from year to year with extreme values of about $\pm 1^\circ\text{F}$ (range of ϕ is less than 20°F and A is about a tenth). The 10-year mean value of $A\phi$ is therefore probably accurate within $\pm 0.2^\circ\text{F}$ and the 15-year mean probably accurate within $\pm 0.1^\circ\text{F}$.^{*} An estimate based on ten years' observations of a missing monthly mean is therefore probably accurate within $\pm 1.2^\circ\text{F}$ providing the mean values of δ_1 and δ_2 over a period of a month are negligible. The poorest estimates will be for the coldest or warmest months. Inclusion of such an estimate in averages over ten years or more should give an average of accuracy within $\pm 0.12^\circ\text{F}$ which is comparable with the accuracy to which the means are calculated.

In comparing data from neighbouring stations of very different soil types to estimate a monthly mean, the monthly difference in earth temperature may sometimes appear to be very irregular. In such instances the mean values of δ_1 and δ_2 are not negligible over a period of a month and the estimate of a missing monthly mean may therefore have a larger error. It is possible to increase the accuracy of the estimate by regression analysis, taking into account both rainfall and earth temperature, but the improvement is only slight (in one case examined the standard error from the regression analysis was only 75 per cent of the standard error obtained by using the simple 10-year average temperature difference); it is more practicable to make a further estimate by comparison with a third station and to combine the estimates.

The method was tested at the one-foot level by comparing observations from Buxton (soil: 3 in. loam over limestone rock, height: 1,007 ft) with those from Sheffield (soil: clay, height: 428 ft) and Wakefield (soil: heavy loam over yellow clay, height: 115 ft); and tested at the four-foot level by comparing observations at Bath (soil: loam over gravel, height: 67 ft) with those from Marlborough (soil: loam over chalk, height: 424 ft) and Holton Heath (soil: sand over gravel, height: 64 ft). In each case a period of 25 years was divided into 5 lustra, and the observations from the 2nd and 4th lustra were assumed to be missing. Mean monthly earth-temperature differences were calculated from the 10-year period formed by the two lustra surrounding each "missing" lustrum and these mean differences used to estimate the "missing" monthly means.

These estimates were then compared with the true monthly mean temperatures. At the one-foot level 47 per cent of the estimates were within $\pm 0.5^\circ\text{F}$ of the true means, 81 per cent within $\pm 1.0^\circ\text{F}$ and 94 per cent within $\pm 1.5^\circ\text{F}$. At the four-foot level the corresponding figures were 58 per cent within $\pm 0.5^\circ\text{F}$, 87 per cent within $\pm 1.0^\circ\text{F}$ and 96 per cent within $\pm 1.5^\circ\text{F}$. All the large errors occurred in estimating the monthly means of unusually cold or warm months. In view of the exceptional soil conditions at Buxton this is hardly surprising.

Estimation of normals

At many stations earth-temperature observations are not available at a single site for the whole of a standard period and it is desirable, for purposes of comparison, to estimate the normal or average for a standard period. The same principles as outlined above may be used, but it is possible to estimate the normals without estimating the means for individual missing months. All possible means can be used in the estimation. The best estimate will clearly be that which contains estimates for the least possible period.

^{*} BROOKS, C. E. P. and CARRUTHERS, N.; "Range" of values in a series of observations. *Met. Mag., London*, 76, 1947, p. 258.

Suppose the normal period is from the year $j=1$ to the year $j=N$, and that observations exist at station U for the period $j=m+1$ to $j=n$ where n is less than N . Then if observations also exist at station V for this same period the best estimate of the monthly mean difference between the earth temperatures at the two stations is

$$A\theta_m + B + a_o - a'_o + \delta = \frac{1}{n-m} \left[\sum_{m+1}^n u_j - \sum_{m+1}^n v_j \right]$$

where u_j, v_j are the monthly mean earth temperatures in year j at stations U and V . If u and v are the normals or estimated normals at U and V , then

$$\begin{aligned} u &= \frac{1}{N} \left[\frac{m}{n-m} \sum_{m+1}^n (u_j - v_j) + \sum_1^m v_j + \sum_{m+1}^n u_j + \frac{N-n}{n-m} \sum_{m+1}^n (u_j - v_j) + \sum_{n+1}^N v_j \right] \\ &= \frac{1}{n-m} \sum_{m+1}^n u_j + \frac{1}{N} \sum_1^N v_j - \frac{1}{n-m} \sum_{m+1}^n v_j \\ &= u_a + (v - v_a), \end{aligned} \quad . . . (9)$$

where u_a and v_a are the monthly averages at U and V for the common period of observations $j=m+1$ to $j=n$. Obviously, although for convenience the years $j=m+1$ to $j=n$ are assumed consecutive, this need not be the case and the formula (9) will still hold for a broken common period at both stations.

TABLE II. *Frequency of errors in estimated normals*

Period on which estimate is based	Error $\leq 0.1^\circ\text{F}$	Error $\leq 0.2^\circ\text{F}$	Error $\leq 0.3^\circ\text{F}$
<i>years</i>	<i>percentage frequency</i>		
Cambridge and Rothamsted (depth: 1 foot)			
10	48	83	96
15	61	96	100
20	83	100	100
Bath and Ross-on-Wye (depth: 4 feet)			
10	51	76	84
15	61	83	92
20	76	91	98
Bath and Cardiff (depth: 4 feet)			
10	29	52	63
15	34	65	81
20	51	78	94
Cardiff and Ross-on-Wye (depth: 4 feet)			
10	34	56	74
15	36	71	87
20	56	87	95

If, however, the common period of observations extends outside the standard period the best estimate cannot be reduced to such a simple formula as (9); the most practical formula is:

$$u = v + \frac{n}{N(n-m)} \sum_{m+1}^N (u_j - v_j) + \frac{m}{N(n-m)} \sum_{n+1}^n (u_j - v_j). \quad \dots (10)$$

In some parts of the United Kingdom no station within reasonable proximity has observations extending over the whole of a standard period. The normals at two stations which between them cover the whole standard period are given by:

$$\left. \begin{aligned} u &= \frac{1}{N} \left[\sum_{m+1}^N u_j + \sum_1^m v_j \right] + \frac{m}{N(n-m)} \sum_{m+1}^n (u_j - v_j) \\ v &= \frac{1}{N} \left[\sum_{n+1}^N u_j + \sum_1^n v_j \right] - \frac{N-n}{N(n-m)} \sum_{m+1}^n (u_j - v_j) \end{aligned} \right\} \quad \dots (11)$$

The method was tested by forming all possible 10-year, 15-year and 20-year averages from the six consecutive lustra forming the period 1921-50; and then, by comparison with another station, making an estimate of the monthly normal. Estimates of normals at a depth of one

TABLE III. *Relation between probable accuracy of estimated normal and range of earth-temperature difference over various periods*

		Probable accuracy of estimated normal	Range of earth-temperature difference during common period of observations				
			8 years	10 years	15 years	20 years	25 years
			<i>degrees Fahrenheit</i>				
Common period within standard period	{	±0.1	1.3	1.7	2.8	4.3	7
		±0.2	2.7	3.5	5.6	8	—
Common period extending 5 years outside standard period	{	±0.1	1.2	1.6	2.5	3.5	5
		±0.2	2.4	3.1	4.9	7	—
Neither station with observations throughout the standard period	<i>Period without obs.</i>						
	5 years	±0.1	2.8	3.5	4.9	6	7
		±0.2	5.6	7	—	—	—
	10 years	±0.1	2.0	2.5	3.4	4.3	—
		±0.2	4.0	5.0	6.9	—	—
	15 years	±0.1	1.6	2.0	2.8	—	—
		±0.2	3.3	4.1	5.6	—	—
	20 years	±0.1	1.4	1.7	—	—	—
±0.2		2.7	3.5	—	—	—	

foot were made using observations at Rothamsted (soil: clay with flints) and Cambridge (sandy); and at a depth of four feet using observations at Cardiff (clay), Ross-on-Wye (clay) and Bath (loam over gravel). The frequency of errors in the estimates of the monthly normals (180 estimates based on 10 or 20 years' observations, 240 estimates based on 15 years' observations) are given in Table II.

Using the approximate relation between range and standard deviation found by Brooks and Carruthers (*loc. cit.*), a rapid assessment of the accuracy of such estimated normals can be obtained. It is convenient to be able to relate the probable error (that is, probable accuracy) directly to the range of earth-temperature difference, and Table III gives the maximum permissible ranges of earth-temperature difference, during various lengths of common period of observations at both stations, for probable accuracies of estimated normals of $\pm 0.1^\circ\text{F}$ and $\pm 0.2^\circ\text{F}$. These are the accuracies that will be achieved on 50 per cent of occasions. To achieve the same accuracies on 80 or 95 per cent of occasions the permitted ranges of monthly mean earth-temperature difference are about half or a third of those given in Table III.

Critical examination of data

Tables of the monthly mean difference between observations at two stations can be constructed rapidly. Examination of these monthly mean differences can, on occasion, show sudden changes between one period of years and another, as is shown by the extract in Table IV from the monthly mean difference between one-foot earth temperatures at Woburn and Rothamsted.

The difference of -1.9°F for July 1928 is distinctly queer and the daily observations at both stations then require close examination to see if there are any anomalous readings or arithmetical errors. But there also appears to be a permanent change of régime in October 1928, for then the temperature difference changes from being slightly greater in winter than in summer to being distinctly greater in summer than in winter. The change in régime is shown even more distinctly by the graph in Figure 2. To construct this graph the average difference in temperature 1941–50 was subtracted from all the other mean monthly differences, and the resulting second difference or monthly deviation from mean difference plotted against time. The mean difference 1941–50 is, in effect, an estimate of $A\theta_m + B + a_o - a'_o$ in equation

TABLE IV. *Monthly mean difference between one-foot earth temperatures at Woburn and Rothamsted*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
						<i>degrees Fahrenheit</i>						
1926	2.0	1.5	1.9	1.2	0.9	0.7	1.6	1.3	0.9	0.6	1.1	1.8
1927	1.5	1.0	1.7	1.9	0.2	0.4	1.4	0.6	0.8	0.6	1.7	1.2
1928	1.6	1.4	1.5	1.2	0.2	1.2	-1.9	0.2	0.5	0.7	0.4	-0.2
1929	-0.2	0.1	2.0	2.2	3.7	3.8	4.1	2.9	2.6	0.5	0.4	0.7
1930	1.3	0.9	1.4	2.0	2.4	4.1	3.0	1.7	1.1	0.3	0.0	0.3

(8) and the plotted deviations from this mean difference are monthly mean values of the random element δ . Thus, since before October 1928 the deviations show a non-random annual oscillation, the value A changed then. But A is a function of $(z/\lambda_a - z'/\lambda'_a)$ and $(z/\lambda_y - z'/\lambda'_y)$

only and can only change if z , z' , κ or κ' change. Therefore there was a maintained change in régime at either Woburn or Rothamsted occurring in the autumn of 1928. Subsequent investigation and correspondence showed that at Woburn in October 1928 the whole observing enclosure was reorganized and the earth thermometers dug up and placed in a different position.

Similar graphs can be constructed for any pair of earth-temperature records at approximately the same depth. They show immediately any significant changes of earth-temperature régime. Interesting examples are shown in Figures 3–5, showing the effects of changes of site or faulty observations.

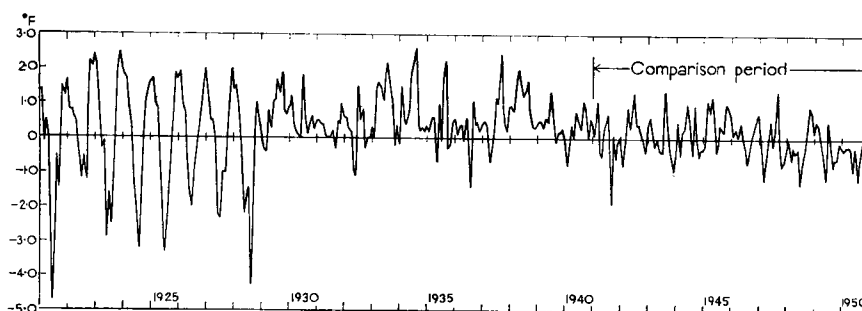


FIGURE 2. Deviations from the 1941–50 mean of earth-temperature difference, Woburn–Rothamsted at a depth of 1 ft.

Figure 3 shows the comparison between Aberdeen Observatory and Craibstone at a depth of four feet. The site at Aberdeen changed three times during the period; only one of these changes made no significant difference to the earth-temperature régime. When, on 1 January 1930, the method of observation changed from lagging the thermometer with wood to lagging with wax the change in the record was no more noticeable than would be expected from the known change in thermometer depth from 124 cm to 122 cm.

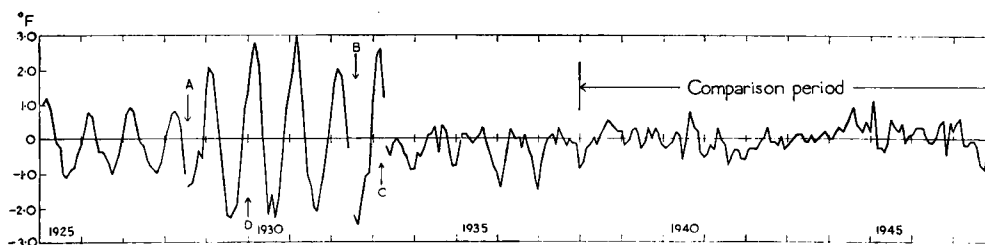


FIGURE 3. Deviations from the 1938–47 mean of earth-temperature difference, Aberdeen–Craibstone at a depth of 4 ft.

- A: Change of site from College Gardens to Ladymill, 1 July 1928
- B: Change of site from Ladymill to Athletic Ground, 8 June 1932
- C: Change of site from Athletic Ground to the Glebe, 31 March 1933
- D: Change of thermometer type, 1 January 1930

Figure 4 (Kensington Palace–Kew Observatory) shows the effect on the curve of deviations from mean difference of a period when the thermometer depth at Kensington Palace was, by accident, shallower than the nominal depth for about two years.

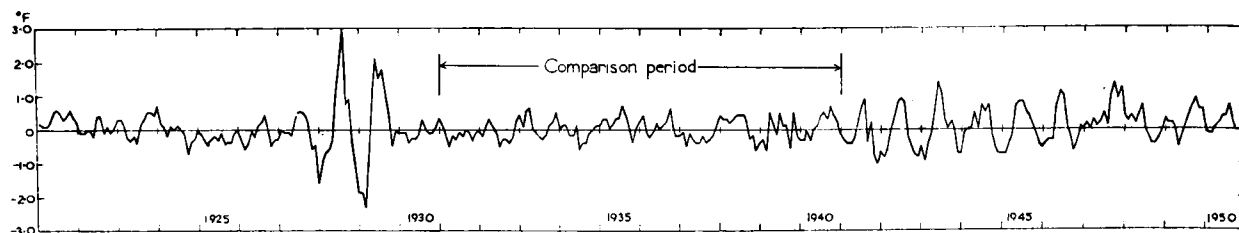


FIGURE 4. Deviations from the 1931-40 mean of earth-temperature difference, Kensington Palace-Kew Observatory at a depth of 4 ft. The depth of the 4-ft thermometer at Kensington Palace was incorrect from autumn 1927 to autumn 1929

Figure 5 (Kew Observatory-Worthing) shows the resulting curve from stations some distance apart in slightly different climatic regions (coastal and inland), in which a change of site made no significant change in the earth-temperature régime. The random variations seem to be somewhat larger than for stations with more exactly similar climates. It is possible, however, that there is a secular change in the climate which is more pronounced at one station than another. This would be hidden by comparison between stations closer together. This might also be the explanation of the suggestion of an annual oscillation from 1940 onwards. But this oscillation is mirrored very approximately in the curve of Figure 4 so that this oscillation, if it exists and is not simply random variation, is associated with the climatic régime at Kew Observatory rather than at Worthing.

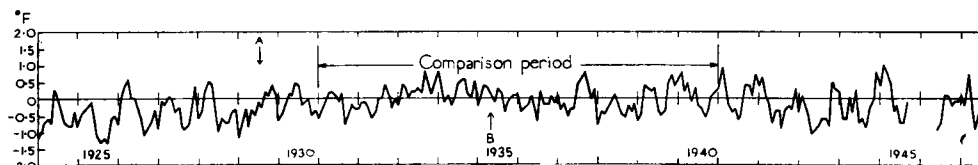


FIGURE 5. Deviations from the 1931-40 mean of earth-temperature difference, Kew Observatory-Worthing at a depth of 4 ft.

A: Change of site at Worthing from Victoria Park to Beach House Park, 10 June 1929
B: Change of site at Worthing within Beach House Park, 13 March 1935

Nevertheless the difference in climate between Kew Observatory and Worthing is still small enough for the annual oscillation indicative of significant change of earth-temperature régime not to be hidden by a greater range of randomness. With more widely differing climates the values of a'_i , b'_i and α'_i become so different from a_i , b_i and α_i that climatic differences outweigh all but the largest differences of observing régime at either station.

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