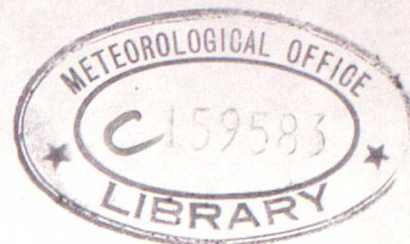**METEOROLOGICAL OFFICE**

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A Summary of Observed Properties of Atmospheric Ice Crystals

By

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A SUMMARY OF OBSERVED PROPERTIES OF ATMOSPHERIC ICE CRYSTALS

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1. INTRODUCTION

Precipitation of the ice phase in the atmosphere occurs in a wide variety of physical forms. The growth of single ice crystals is far from simple, there being many crystal habits for crystal growth. The particular habit in which a crystal grows is dependent upon the ambient temperature and also the water vapour supply throughout the period of its growth.

The purpose of this note is to document some of the parameters important in determining the growth and fall of ice crystals of several of the most common habits. The data are not new but are compiled from some of the principal sources and put in forms convenient for numerical evaluation and comparison. This is done as an aid to numerical modelling and observational study in that both the bulk properties of ice precipitation (other than hail and aggregates) and the sensitivity of a bulk model to assumed crystal properties can be estimated.

Various dimensional relations are included, as well as terminal velocities and parameters occurring in the theory of diffusional growth in terms of both crystal size and mass. The diffusional mass growth rate of an ice crystal is factorised into three terms, namely the growth coefficient, crystal capacitance and the ventilation factor, each of which are discussed in turn. The range of crystal sizes considered is from 10 to 1000 μm in length or diameter. The following parameters are derived: thickness, volume, bulk density, mass, capacitance, terminal velocity and ventilation for each of the principal crystal types over the pertinent size range, as outlined schematically in figure 1.

2. DERIVATION OF ICE CRYSTAL MASS

The size of simple ice crystals is characterised by a scale length. For plates and dendrites this length is equal to the crystal diameter, while in the case of columnar crystals and bullets it is taken to be the length of the crystal, see figure 2.

A number of observational studies have provided dimensional relationships linking the thickness of plates and dendrites to their maximum diameter and also the width of columnar crystals and bullets to their length. These have been tabulated by Auer and Veal (1970), Heymsfield (1972) and more recently by Pruppacher and Klett (1978). These expressions, together with the expression for the area of the basal face of the crystal, were used to derive the crystal volume.

The bulk density of ice crystals is also dependent on crystal habit and crystal size, tending to decrease with increasing size generally. Pruppacher and Klett (1978) have tabulated relationships between the maximum crystal dimension and the bulk density based on the data of

Heymsfield (1972). These expressions were applied to calculate the ice crystal bulk densities. However, for small crystal sizes all the expressions for bulk density of the crystals tend to that of bulk ice (0.9 gcm^{-3}) except for the expression given by Pruppacher and Klett for bullet crystals. Reference to Heymsfield's (1972) original data in which he assigned bullets to three categories dependent upon cloud type provided a more consistent expression for the bulk density of bullet crystals observed in cirrostratus clouds and this was employed in the calculations.

Table 1 summarises the dimensional relationships for:-

1. Crystal dimension
2. Area of basal face
3. Volume
4. Bulk density

    where the scale length of the crystal is denoted by diameter d for plate-like crystals (plates and dendrites) length l for columnar crystals (bullets and columns)

The crystal mass is now simply the product of the volume and bulk density of the crystal for a particular scale length. Note that these values are only valid over the range of crystal scale length from 0.01 mm to 1 mm. Figure 3 shows the ice crystal mass as a function of maximum crystal dimension for the range of crystal size over which the expressions in table 1 are valid.

Figure 4 indicates the dependence of density on crystal mass for the principal ice crystal types in the mass range 10^{-9} to 10^{-5}g. The bulk density for solid plates, solid columns, warm hollow columns and bullets remains close to that of bulk ice throughout the range of mass considered here. However, the bulk density of cold hollow columns (formed at higher vapour pressure) decreases steadily over the mass range considered (from 0.9 gcm^{-3} down to 0.68 gcm^{-3}) whilst the bulk density of dendritic crystals tends to that of solid plates for crystals smaller than 10^{-6}g but decreases markedly as the crystal mass increases above this value (to 0.63 gcm^{-3} at 10^{-5}g). For large dendrites riming also increases density significantly (eg Locatelli and Hobbs, 1974).

3. CALCULATION OF ICE CRYSTAL GROWTH RATES

The diffusional mass growth rate of ice crystals by sublimation is given by the equation (Pruppacher and Klett 1978, equation 13.71)

$$\frac{dm}{dt} = \frac{4\pi C F_{v,L*} S_{v,i}}{\dfrac{RT_\infty}{e_{sat,i}(T_\infty) D'_v M_w} + \dfrac{L_s}{k'_a T_\infty}\left(\dfrac{L_s M_w}{RT_\infty} - 1\right)} = 4\pi C F_{v,L*} GRO$$

This expression actually represents only a limited aspect of the growth process, as it does not take into account the factors leading to the growth habit of a crystal. The rate-determining quantities in this model are seen as the rates of transport of water molecules and heat into the vicinity of the crystal. The mode of growth is embodied partly in the capacitance but

also in the aspect ratio subsequently used to convert mass growth into a length change, which is usually specified empirically. A more complete theory should account for the different rates of growth along different crystal axes and their different temperature dependences. Since crystal habit is a function of temperature and vapour pressure (see Figure 5a) the kinetics of uptake of molecules and distribution of latent heating at different crystal sites must ultimately be described. An extended version of the diffusion theory with different growth rates has been put forward, and growth rates along the distinct planes have been measured at a range of temperatures. The factors leading to the temperature/vapour pressure dependences are complex, however, and not completely understood. Pruppacher and Klett (section 13.3.3) and Mason (1971, Ch 5) give some insight into the crystallographic factors in this question, while more recent work (eg Parungo and Weickmann, 1983) has extended the scope of discussion to the role of the additional quasi-liquid surface layer, which appears to introduce additional complications at temperatures above ≒10C.

The simplified growth expression given above may conveniently be represented as a product of three terms:

a. The growth coefficient dependent only upon the physical conditions of the crystal's environment ie. pressure, temperature and the supply of water vapour given by the water vapour mixing ratio.

b. The capacitance of the crystal, which is solely a function of the crystal geometry.

c. The ventilation factor which is dependent on the crystal geometry, its fall velocity and the ambient air density.

A.   The growth coefficient GRO

This is dependent upon the diffusion of water vapour through the environmental air towards the crystal, and the thermal conduction of environmental air responsible for removal of the latent heat of sublimation released at the crystal surface. While temperature and pressure are important factors in determining the growth rate, atmospheric variations tend to be dominated by the supersaturation of water vapour in the environmental air with respect to ice.

$$GRO = \cfrac{S_{v,i}}{\cfrac{R T_\infty}{e_{sat,i}(T_\infty) D'_v M_w} + \cfrac{L_s}{K'_a T_\infty}\left(\cfrac{L_s M_w}{R T_\infty} - 1\right)}$$

R = universal gas constant = 8314 $JKmol^{-1}K^{-1}$
M_W = molecular weight of water = 18.0 $KgKmol^{-1}$
T_∞ = temperature of environmental air
$e_{sat,i}(T_\infty)$ = saturation vapour pressure over a plane ice surface.
L_s = specific latent heat of sublimation of ice = 2.839 JKg^{-1}.
$S_{v,i}$ = supersaturation of water vapour in the environmental air with respect to ice.

3

D_v'> ~ D_v = diffusivity of water vapour in air. (Pruppacher and Klett (1978), equation 13.3).

$$D_v (p,T) = 2.11 \times 10^{-5} \left(\frac{T}{T_o}\right)^{1.94} \left(\frac{p_o}{p}\right)$$

T_o = 273.15K
p_o = 1013.25 mb
$= 101325$ Nm^{-2}
D_v in $m^2 sec^{-1}$

K_a' ~ K_a = Thermal conductivity of dry air. For typical atmospheric conditions the thermal conductivity of environmental air is approximately equal to that of dry air. (Pruppacher and Klett 1978 equation 13.16).

$K_a = (2.382 + 0.00703\, T_c) \times 10^{-2}$ $Wm^{-1}K^{-1}$; T_c = temperature in degrees centigrade.
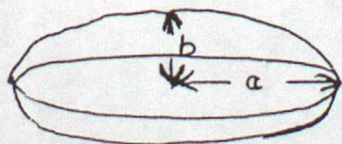
Strictly the diffusivity and conductivity should include correction terms for the molecular nature of processes at small diameter, but these are of no consequence above micron sizes (Pruppacher and Klett ch.13.1.1).

A Fortran program for calculating the growth coefficient now exists, consisting of two parts. The subroutine "GROWTH" generates a table of values of the specific growth function at increments of 100 mb and 1°C over the range 200 → 1100 mb and 0 → -50°C. The function "GRO" applies linear interpolation to this table of values and returns the appropriate growth function value for pressure and temperature within the limits of the tabulated values. The water vapour mixing ratio must also be supplied with the pressure and temperature as arguments in order to compute the supersaturation with respect to ice of the environmental air, and hence the growth rate. In figure 5b the growth coefficient for ice crystals growing in an environment at water saturation is plotted as a function of temperature at two levels 1000 mb and 500 mb. Note that the maximum growth coefficient on both curves occurs close to -15°C where growth becomes dendritic near water saturation.

B. Ice crystal capacitance C

The capacitance of the crystal is solely a function of the crystal geometry. The geometries of the different crystal types considered here are divided into three classes.

(i)   Hexagonal plates and dendrites are approximated by an oblate spheroid, the length of the semi-major a and semi-minor b axes being equal to the radius and half-thickness of the crystal respectively.

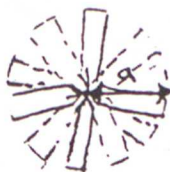$$C = \frac{ae}{\sin^{-1}e}, \quad e = \left(1 - \frac{b^2}{a^2}\right)^{1/2}$$

(ii)   Columns and bullets are approximated by a prolate spheroid, the length of the semi-major b and semi-minor a axes being equal to half the

width and length of the crystal respectively

$$C = \frac{A}{\ln[(a+A)/b]} \qquad , \ A = (a^2 - b^2)^{1/2}$$

(iii) The bullet rosette is approximated by a sphere, the radius of the sphere being equal to one half of the scale length of the bullet rosette.

$$C = a$$

Figure 6 indicates the dependence of capacitance on the maximum dimension (ie. scale length) of the crystal. Figure 7 shows the dependence of capacitance on the crystal mass for different crystal types. While the dependence of capacitance on mass is much the same for bullets and columns, the capacitance of plates, dendrites and bullet rosettes increases significantly more rapidly than for the columnar crystals, essentially because of the larger diameter/mass ratio.

C. Ventilation of ice crystals

Ventilation is an important factor which enhances the diffusion mass growth rate of ice crystals falling in the atmosphere. The amount of ventilation is determined by the portion of particle area exposed directly to the oncoming flow. It is necessary here to note that hexagonal ice plates and columns fall through the atmosphere with their maximum dimension orientated perpendicular to the direction of fall. Hall and Pruppacher (1976) have derived from experimental data expressions for the ventilation coefficient $F_{V,L*}$. The important parameters which control the amount of ventilation are the terminal velocity V and characteristic-length L^*.

$$L^* = \frac{\text{Total surface area of crystal}}{\text{Perimeter of crystal area projected in the direction of the flow.}}$$

$$F_{V,L*} = 1 + 0.14X^2 \qquad X < 1$$
$$F_{V,L*} = 0.86 + 0.28X \qquad X \geq 1$$

Where $X = N_{sc}^{1/3} \ N_{Re,L*,v}^{1/2}$

Schmidt Number $N_{sc} = \frac{\nu}{D} = \frac{\eta}{\rho_{air}D}$

Reynolds Number $N_{Re,L*,v} = \frac{vL^*}{\nu} = \frac{\rho_{air} \ v \ L^*}{\eta}$

Values of the parameter X at the surface and 400 mb level were almost identical:-
$$X = 219.2 \ (V_{surf} \ L^*)^{1/2} \quad \text{at surface}$$
$$X = 205.9 \ (V_{surf} \ L^*)^{1/2} \quad \text{at 400 mb}$$

this indicates that the ventilation is only weakly dependent on height.
Ventilation factors at the surface are tabulated in table 2 and plotted as
a function of both the size and mass of the crystal in figures 8 and 9
respectively. The ventilation factor is seen to be unimportant for
crystals smaller than 0.1 mm and 10^{-7}g. For larger crystals the
ventilation factor is most important for crystals with higher terminal
velocities.

Approximations have been made to the crystal geometries in deriving values
for L^*. Hexagonal plates and dendrites have been approximated by a
circular disc, bullets and columns by cylinders and the bullet rosette by a
sphere. The recent work of Schemenauer and List (1985) however should also
be consulted. They have carried out measurements using similar models of
actual crystal shapes, and not surprisingly found significant complications
arising from flow around crystals with detailed, eg branched, structure.

Total Area	$\pi d\left(\dfrac{d}{2}+h\right)$	$\pi d\left(\dfrac{d}{2}+\ell\right)$	$\pi d\left(\dfrac{d}{2}+\ell\right)$	πd^2
Perimeter of area directed in flow direction	πd	$2(\ell+d)$	πd	πd
L^*	$\dfrac{d}{2}+h \approx \dfrac{d}{2}$	$\dfrac{\pi d\left(\frac{d}{2}+\ell\right)}{2(\ell+d)} \approx \dfrac{\pi d\ell}{2(\ell+d)} \approx \dfrac{\pi}{2}.d$	ℓ	d

4. ICE CRYSTAL TERMINAL VELOCITIES

For a detailed discussion of ice crystal terminal velocities the reader is
referred to Heymsfield (1972). The terminal velocities of ice crystals of
up to 1mm in length or diameter were computed from the formulae presented
by Heymsfield for bullets, columns and plates, which are reproduced in
table 3a. The data for dendrites were obtained by reference to Kajikawa
(1972) reproduced here as figure 10. From this diagram scaling factors
were derived and applied to the existing data for plates.

Heymsfield (1972) suggests a pressure dependent relation for the terminal
velocity $U_t \sim (p_o/P)^{0.4}$ which implies that the terminal velocities at 400
(700) mb are 1.6 (1.2) times greater than the 1000 mb values quoted here.
The terminal velocities computed by the above procedure at 1000 mb are
tabulated in table 3b and are plotted as a function of crystal size in
figure 11 and crystal mass in figure 12.

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5. LARGER CRYSTALS AND AGGREGATES

Numerous studies have been carried out of larger snowflakes, particularly at ground level. The values for the dimensional relations, densities and terminal velocities of such aggregates are extremely variable and are not considered in detail here. However, no review or textbook discussion of this range is known to us. Some supplementary references are however provided for completeness.

REFERENCES

Auer, A.H., and Veal, D.L.   The dimension of ice crystals in natural
    1970   clouds. J. Atmos. Sci. Vol. 27 p.919.

Hall, W.D., and Pruppacher, H.R. The survival of ice particles falling from
    1976   cirrus clouds in subsaturated air.
                                    J. Atmos. Sci. Vol. 33 p.1995.

Heymsfield, A.J.   Ice crystal terminal velocities.
    1972   J. Atmos. Sci. Vol. 29 p.1348.

Kajikawa, M.   Measurement of falling velocity of
    1972   individual snow crystals. J. Meteor. Soc.
                                    Japan Vol. 50 p.577.

Pruppacher, H.R. and Klett, J.D. Microphysics of clouds and precipitation.
    1978   D. Reidel Publishing Company ISBN
                                    90-277-0515-1.

Large crystals and aggregates:

Hobbs, P.V., Chang, S., and   The dimensions and aggregation of ice
 Locatelli, J.D. 1974.   crystals in natural clouds. J. Geophys.
                                    Res. Vol. 79 p2199.

Locatelli, J.D. and Hobbs, P.V. Fall speeds and masses of solid
    1974   precipitation particles. J. Geophys. Res.
                                    Vol 79 p.2185.

Magono, C. and Nakamura, T.   Aerodynamic studies of falling snowflakes.
    1965   J. Meteor. Soc. Japan Vol. 43 p.139.

TABLE 1

	HEXAGONAL PLATE Pla	DENDRITE Ple	SOLID BULLET Clc	HOLLOW BULLET Cld	SOLID COLUMN Cle	HOLLOW COLUMN Clf
DIMENSIONAL RELATIONSHIP (mm)	$h=0.0473d^{0.474}$	$h=0.0379d^{0.377}$	$d=0.250\ell^{0.786}$ $\ell \leq 0.3mm$	$d=0.185\ell^{0.532}$ $\ell \geq 0.3mm$	$d=0.308\ell^{0.927}$	$d=0.309\ell^{0.930}$
AREA OF BASAL FACE (mm²)	$a=0.65d^2$	$a=0.65d^2$	$a=0.0406\ell^{1.571}$	$a=0.0223\ell^{1.064}$	$a=0.0615\ell^{1.854}$	$a=0.0621\ell^{1.860}$
VOLUME (mm³)	$v=0.0308d^{2.474}$	$v=0.0246d^{2.377}$	$v=0.0406\ell^{2.571}$	$v=0.0233\ell^{2.064}$	$v=-0.0615\ell^{2.854}$	$v=0.0621\ell^{2.860}$
BULK DENSITY (gcm⁻³)	$\rho=0.9$	$\rho=0.588d^{0.377}$ $d \geq 0.323mm$	$\rho=0.81\ell^{-0.054}$ $\ell \geq 0.150mm$	$\rho=0.81\ell^{-0.054}$ $\ell \geq 0.150mm$	$\rho=0.9$	COLD COLUMN $\rho=0.65\ell^{-0.0915}$ $\ell \geq 0.028mm$ WARM COLUMN $\rho=0.848\ell^{-0.014}$ $\ell \geq 0.014mm$

TABLE 2. Characteristic Lengths L^* and Ventilation Factors F_{V,L^*}

CRYSTAL TYPE	PLATES	DENDRITES	SOLID BULLET	HOLLOW BULLET	SOLID COLUMN	HOLLOW COLUMN	BULLET ROSETTE
Characteristic length L^* (metres) d or ℓ in millimetres	$5 \times 10^{-4}d$	$5 \times 10^{-4}d$	$10^{-3} \times ℓ$	$10^{-3} \times ℓ$	$4.84 \times 10^{-4}ℓ^{0.927}$	$4.85 \times 10^{-4}ℓ^{0.930}$	$10^{-3} \times d$

CRYSTAL TYPE / CRYSTAL SCALE LENGTH (mm)	PLATES	DENDRITES	SOLID BULLET	HOLLOW COLD COLUMN	HOLLOW WARM COLUMN	BULLET ROSETTE
0.1	1.03	1.01	1.10	1.06	1.06	1.10
0.2	1.11	1.06	1.33	1.20	1.22	1.33
0.3	1.23	1.14	1.55	1.34	1.38	1.55
0.4	1.35	1.22	1.75	1.48	1.52	1.75
0.5	1.46	1.29	1.94	1.60	1.66	1.94
0.6	1.57	1.35	2.12	1.71	1.78	2.12
0.7	1.67	1.40	2.28	1.80	1.88	2.28
0.8	1.76	1.45	2.42	1.88	1.97	2.42
0.9	1.85	1.49	2.54	1.95	2.05	2.54
1.0	1.94	1.52	2.64	2.00	2.11	2.64

Table 3a

CRYSTAL TYPE	SPECIFIC EQUATIONS FOR TERMINAL VELOCITY AT 1000mb U_{1000} (cms^{-1}) in terms of L(=1000L*)
BULLET	$-1.2 + 169.65L$ (mm) $- 84.5L^2$(mm)
COLD COLUMN (T≤−22°C) WARM COLUMN (−3.5≥T≥−9.5°C)	$-0.528 + 157.33L$ (mm) $- 85.27L^2$(mm) $-1.073 + 180.45L$ (mm) $- 93.41L^2$(mm)
PLATE	$-0.55 + 90.69L$ (mm) $-= 23.44L^2$(mm) $-$ $5.26L^3$(mm)

Table 3b

SCALE LENGTH OR DIAMETER OF CRYSTAL (mm)	TERMINAL VELOCITIES AT 1000mb in ms^{-1}				
	HEXAGONAL PLATES	DENDRITES	BULLETS	COLD COLUMNS	WARM COLUMNS
0.1	0.083	0.041	0.149	0.144	0.160
0.2	0.166	0.091	0.294	0.275	0.313
0.3	0.244	0.138	0.421	0.390	0.447
0.4	0.317	0.174	0.531	0.488	0.562
0.5	0.383	0.199	0.625	0.568	0.658
0.6	0.443	0.214	0.702	0.632	0.736
0.7	0.496	0.220	0.762	0.678	0.795
0.8	0.543	0.231	0.804	0.708	0.835
0.9	0.583	0.233	0.830	0.720	0.857
1.0	0.615	0.234	0.840	0.715	0.860

FIGURE 1.

Specify scale length L equal to the crystal diameter for plates & dendrites length for bullets & columns.

Terminal Velocity

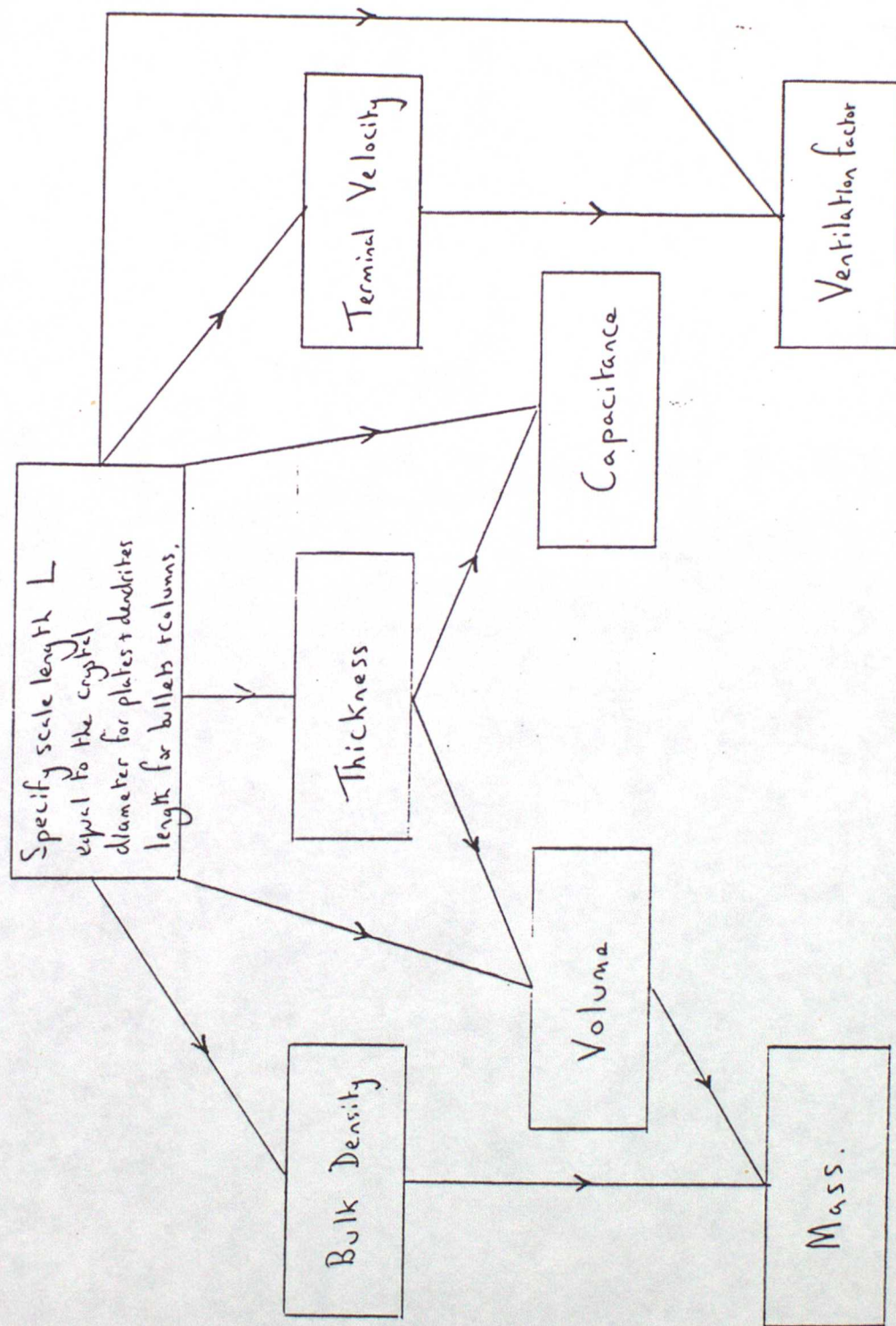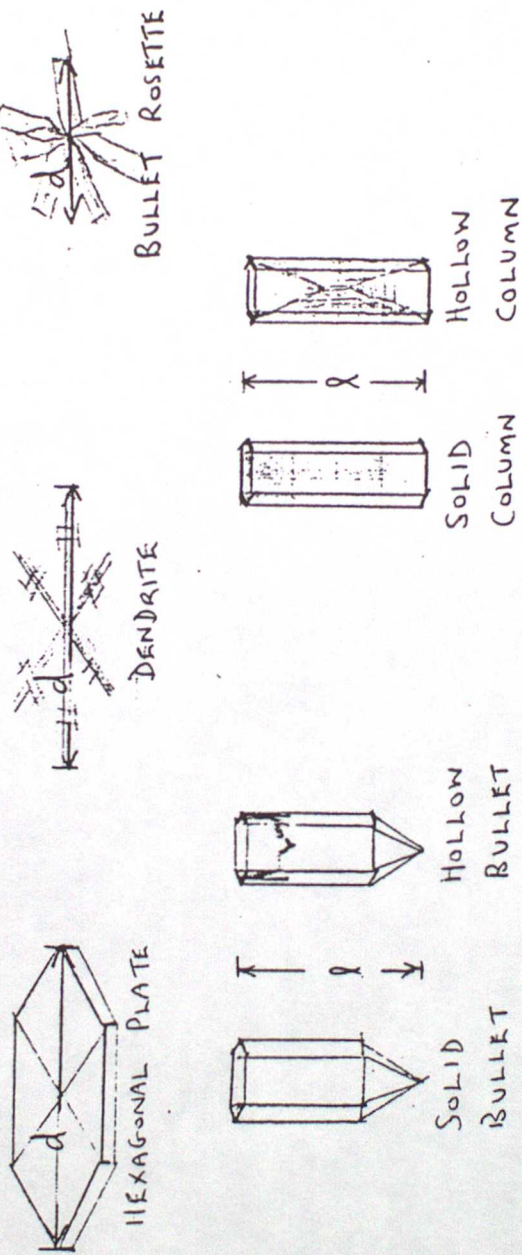Ventilation factor

Capacitance

Thickness

Volume

Bulk Density

Mass.

DIAGRAM INDICATING LOGICAL STEPS IN DERIVING THE PARAMETERS FOR A GIVEN SCALE LENGTH L.

Figure 2.

Principal crystal types.

Hexagonal Plate
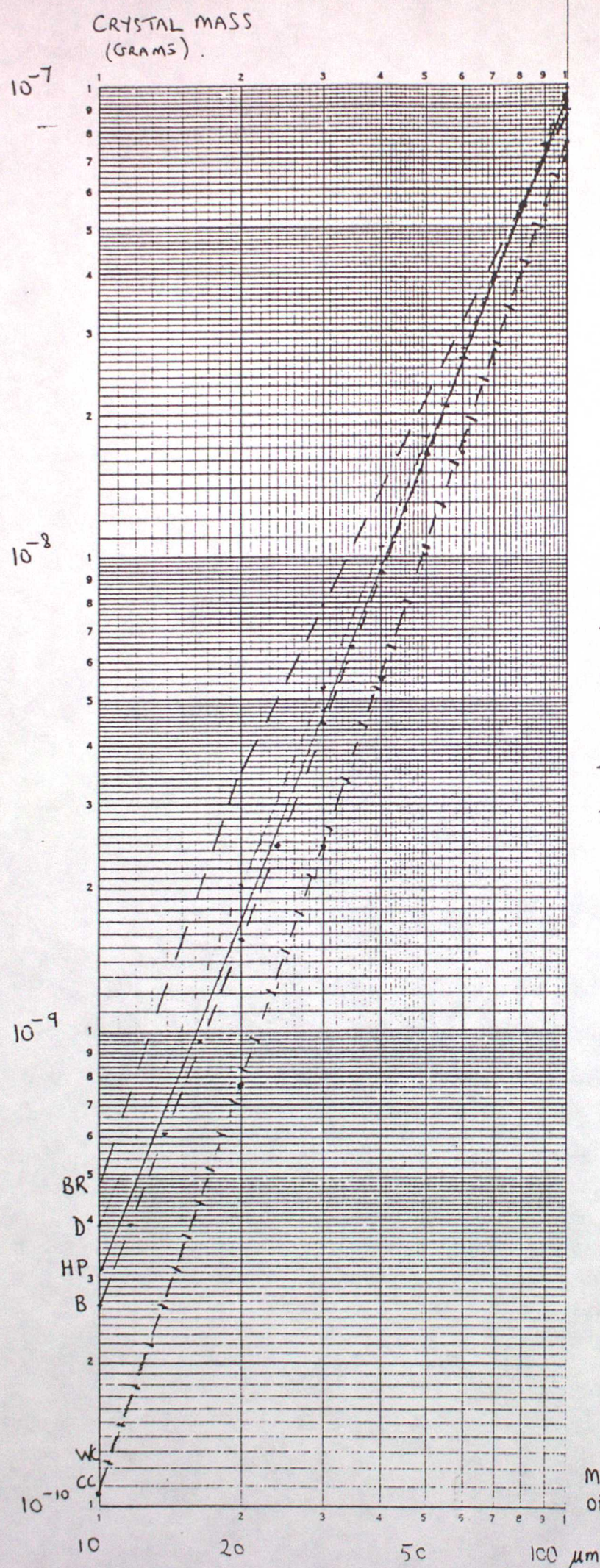
Solid Bullet

Hollow Bullet

Dendrite

Solid Column

Hollow Column

Bullet Rosette

FIGURE 3.

MASS OF ICE CRYSTALS

AS A FUNCTION OF

THEIR MAXIMUM DIMENSION.
(ie. LENGTH SCALE)

FOR DIFFERENT CRYSTAL TYPES.

————— HEXAGONAL PLATES HP

— — • BULLETS B

————— HOLLOW COLD COLUMNS CC

—I—I HOLLOW WARM COLUMNS WC

————— DENDRITES D

— — BULLET ROSETTE BR

CRYSTAL MASS (GRAMS).

CRYSTAL MASS (GRAMS)

MAXIMUM DIMENSION OF CRYSTAL

Maximum Dimension OF CRYSTAL

FIGURE 4. DENSITY OF ICE CRYSTALS AS A FUNCTION OF MASS FOR DIFFERENT CRYSTAL HABITS.

Hexagonal Plates HP
Bullets B
Hollow Warm Columns Wc
Hollow Cold Columns Cc
Dendrites D

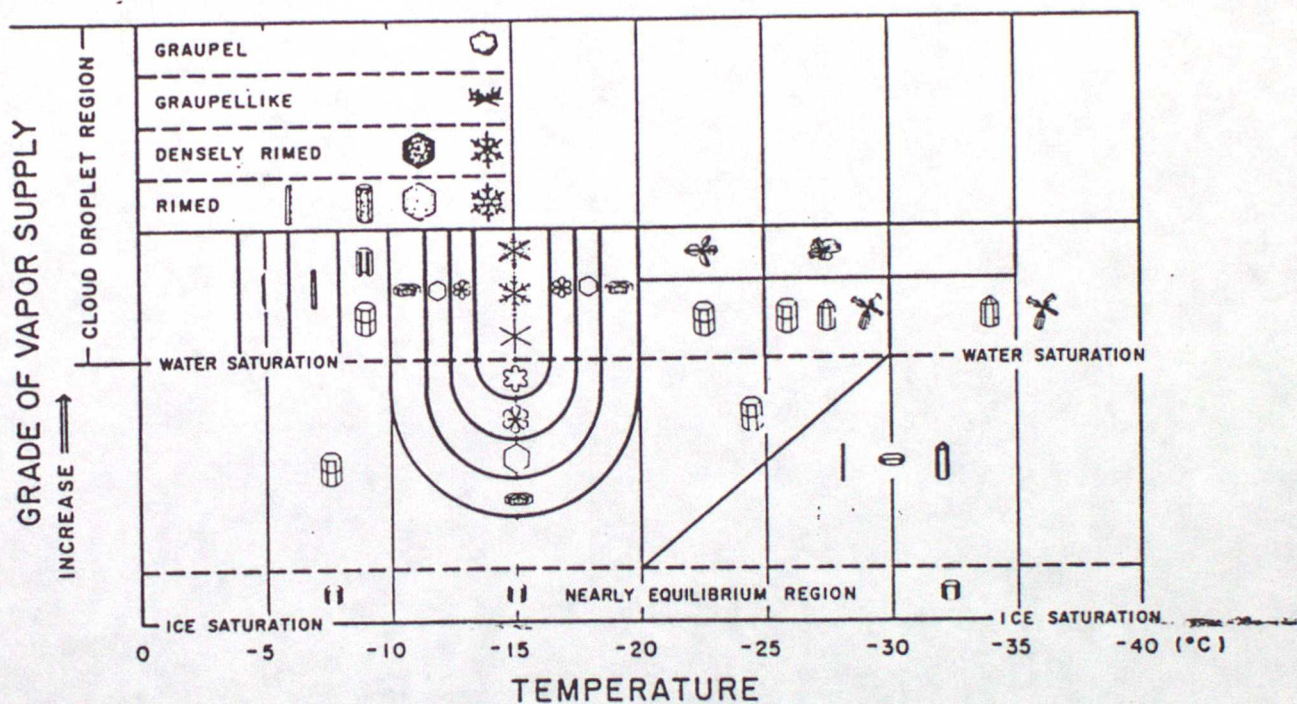ICE CRYSTAL DENSITY (g·cm-3)

MASS of CRYSTAL (GRAMS)

Fig. 5a Temperature and humidity conditions for the growth of natural snow crystals of various types. (From Magono and Lee, 1966; by courtesy of *J. Fac. Sci.*, Hokkaido University.)
without
permission

FIGURE 5.b

GROWTH COEFFICIENT AS A FUNCTION OF TEMPERATURE

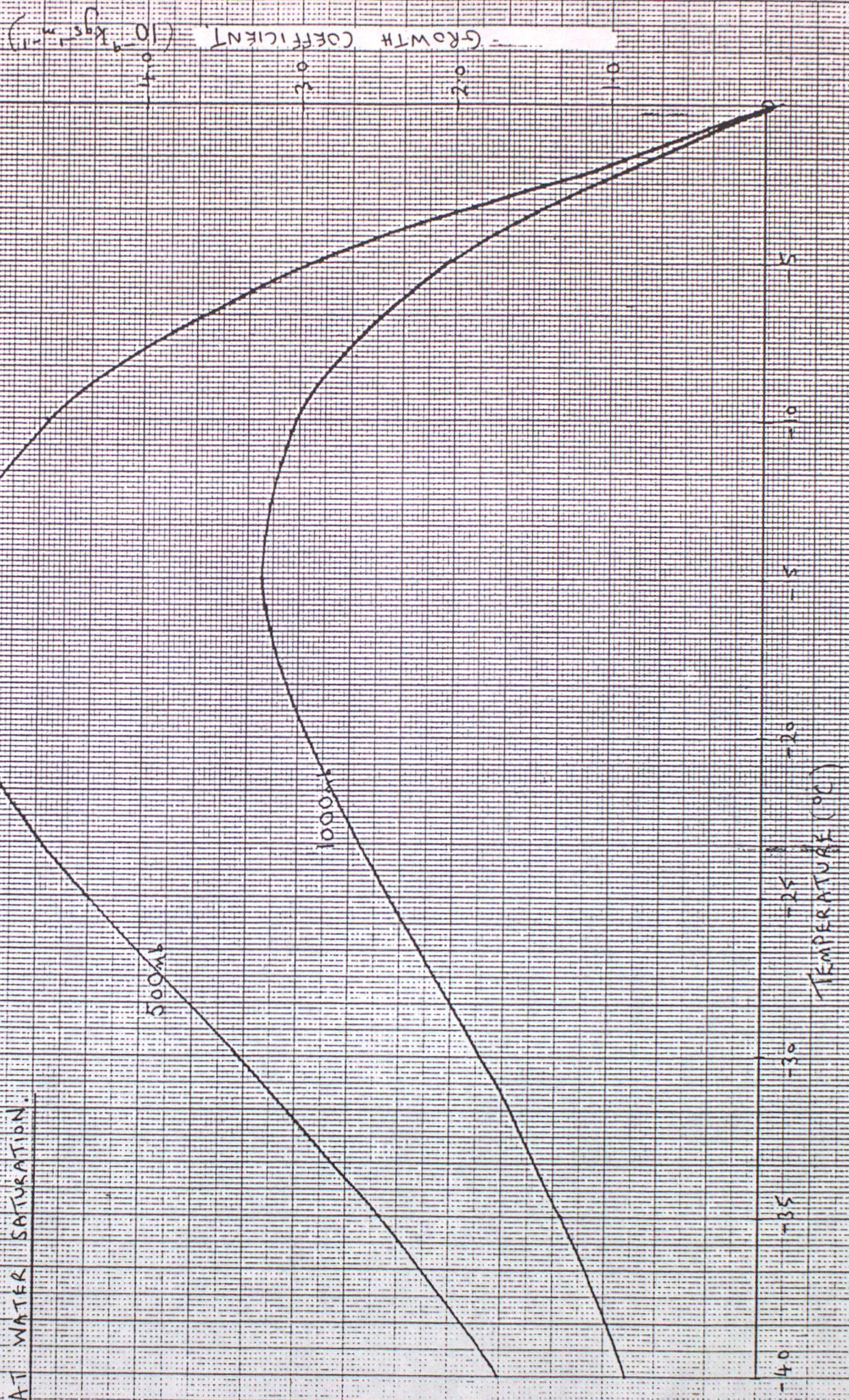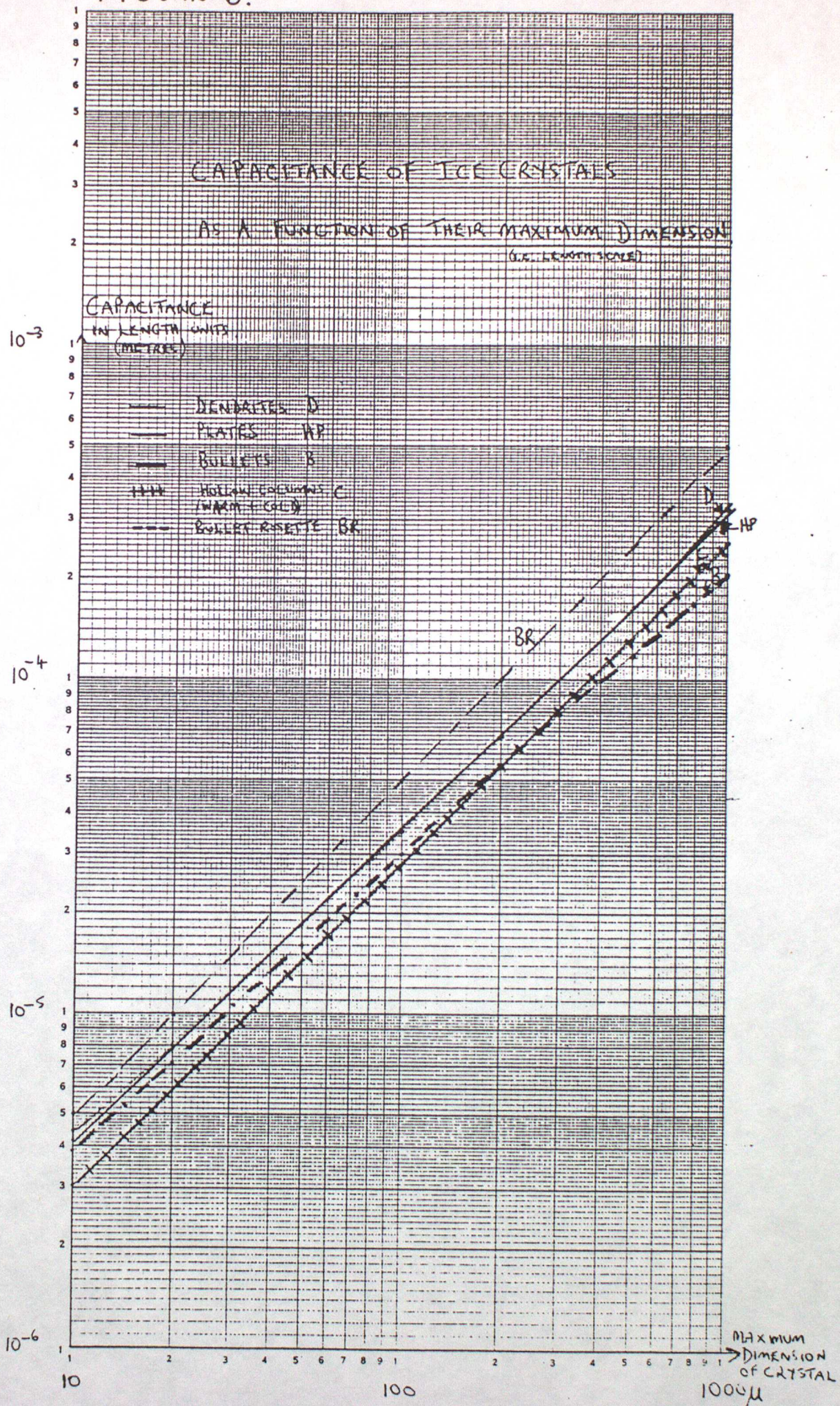FOR ICE CRYSTALS GROWING IN AN ENVIRONMENT

AT WATER SATURATION.

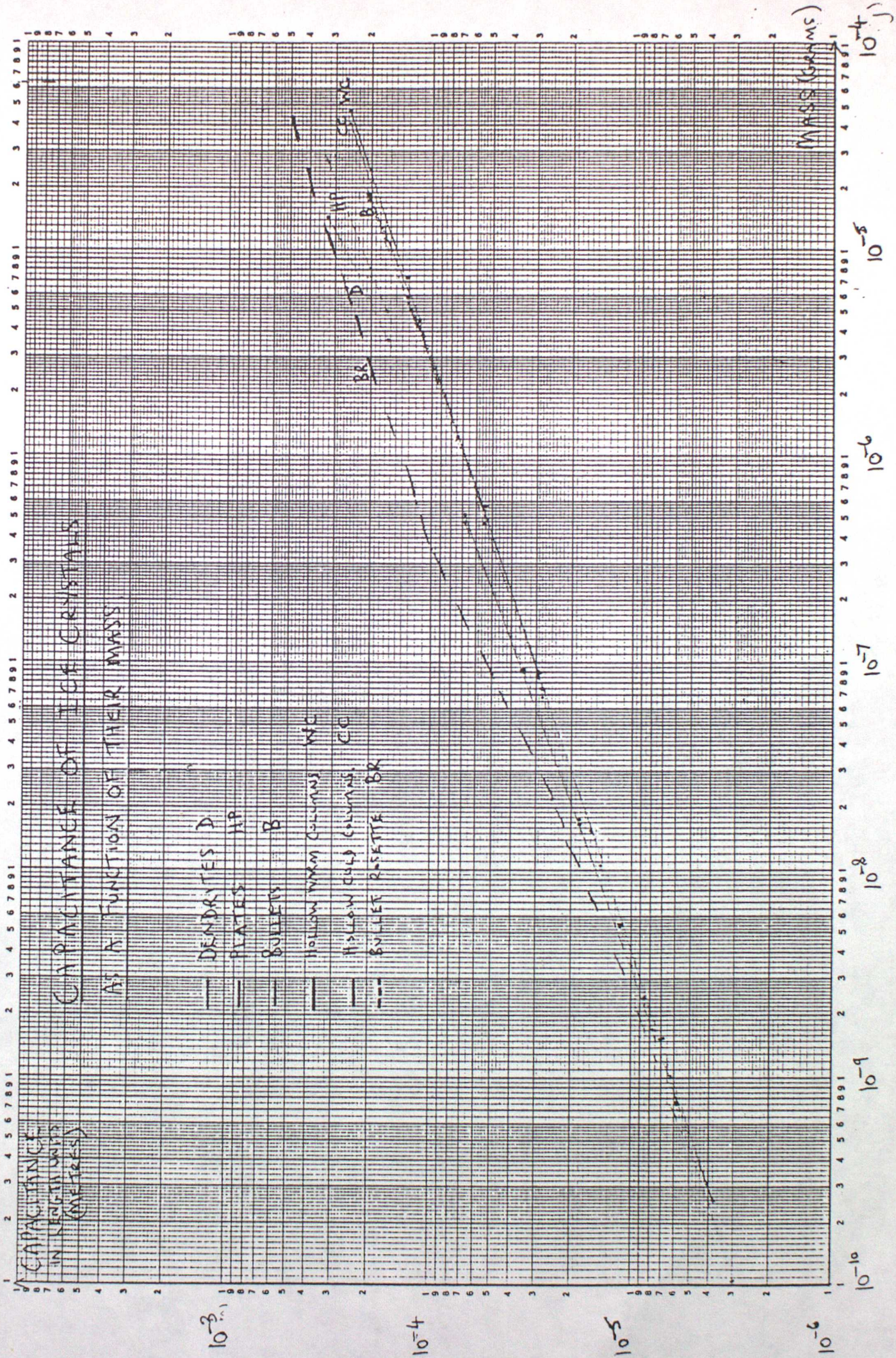GROWTH COEFFICIENT (10^{-7} kg s^{-1} m^{-1})

TEMPERATURE (°C)

FIGURE 6.

CAPACITANCE OF ICE CRYSTALS

AS A FUNCTION OF THEIR MAXIMUM DIMENSION
(i.e. LENGTH SCALE)

CAPACITANCE
IN LENGTH UNITS
(METRES)

DENDRITES D
PLATES HP
BULLETS B
HOLLOW COLUMNS C
(WARM & COLD)
BULLET ROSETTE BR

FIGURE 7.

CAPACITANCE OF ICE CRYSTALS
AS A FUNCTION OF THEIR MASS

FIGURE 8

VENTILATION COEFFICIENT $F_{v,L}^*$

AS A FUNCTION OF MAXIMUM DIMENSION (IE: LENGTH SCALE) FOR DIFFERENT ICE CRYSTAL TYPES.

FIGURE 9.

VENTILATION COEFFICIENT F_{VL}*

AS A FUNCTION OF MASS

FOR DIFFERENT ICE CRYSTAL TYPES

Hexagonal Plates	HP	
Bullets, R₁ + Bullet Rosette	BR	
Hollow Cold Columns	CC	
Hollow Warm Columns	WC	
Dendrites	D	

FIGURE 10.

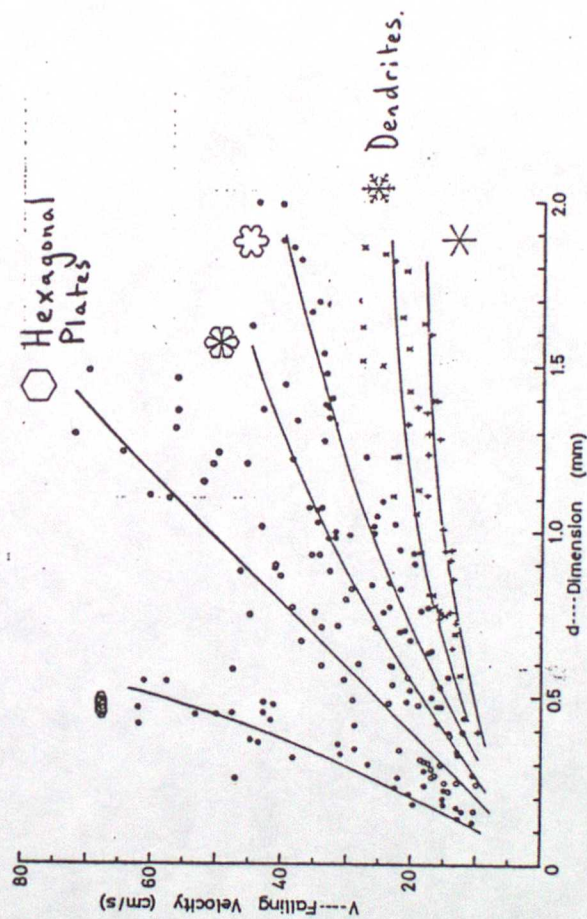TERMINAL FALL VELOCITY

OF PLANE TYPE CRYSTALS

AT −10°C AND 1000mb. (Kajikawa 1972).

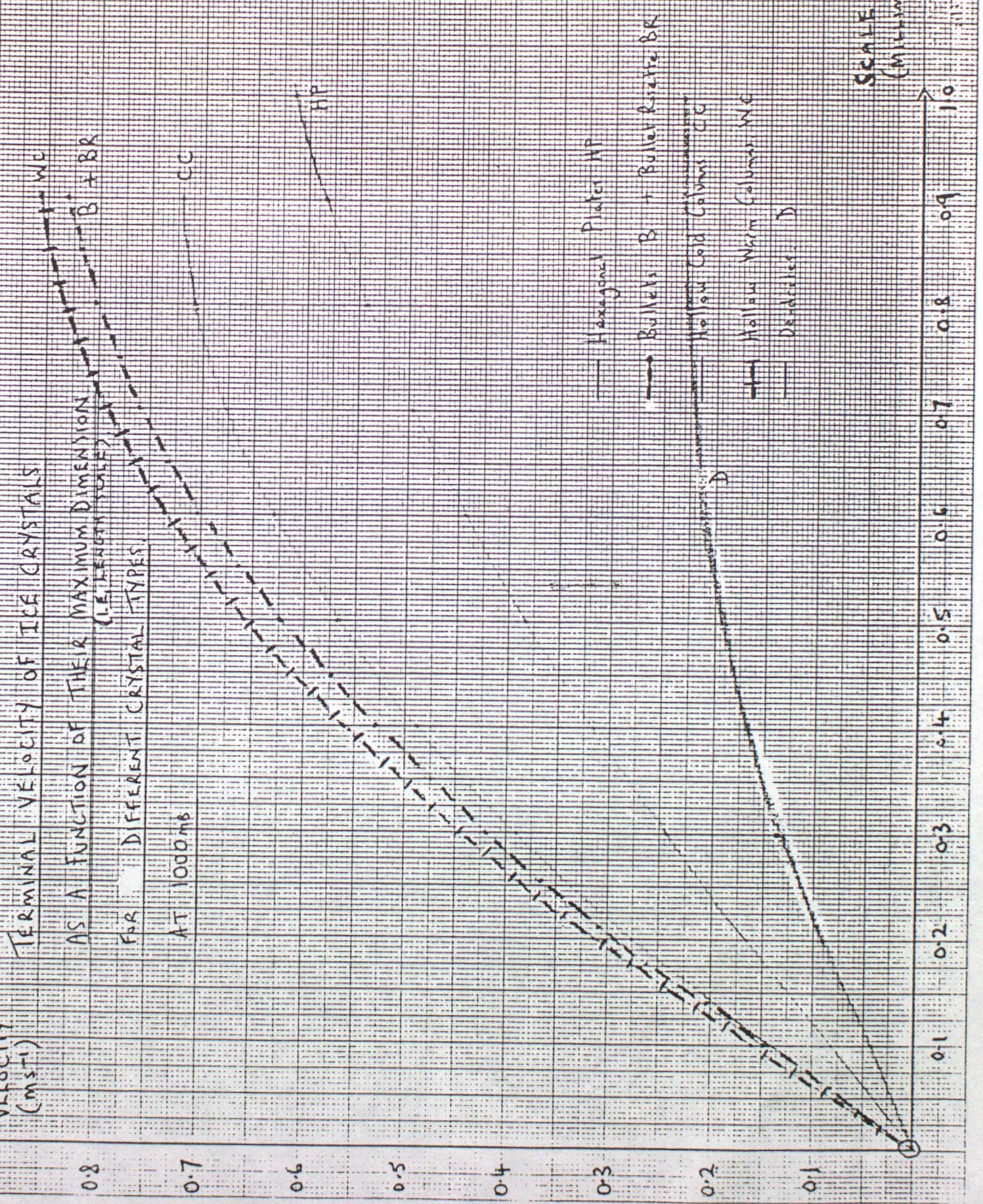Fig. 10. Falling velocity of plane type crystals.

FIGURE 11

TERMINAL VELOCITY OF ICE CRYSTALS

AS A FUNCTION OF THEIR MAXIMUM DIMENSION
(I.E. LENGTH SCALE)

FOR DIFFERENT CRYSTAL TYPES

AT 1000 mB

FIGURE 12.

TERMINAL VELOCITY OF ICE CRYSTALS.

AS A FUNCTION OF THEIR MASS.

FOR DIFFERENT CRYSTAL TYPES.

AT 0.00 mB.