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The 27–Day Recurrence Interval in Magnetic Disturbance

An Examination made with the aid of hourly Character Figures

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LIST OF ILLUSTRATIONS

THE 27-DAY RECURRENCE INTERVAL IN MAGNETIC DISTURBANCE

AN EXAMINATION MADE WITH THE AID OF HOURLY CHARACTER FIGURES

Since 1904, when Maunder first pointed out the existence of a quasi-periodicity in magnetic storms, considerable attention has been directed to this aspect of terrestrial magnetic theory in the hope of defining accurately the length of the time interval between successive outbreaks. By examining the dates of occurrence of 276 storms as recorded at Greenwich Observatory during 22 years, Maunder[1] himself estimated the "period" to be approximately 27¼ days. By statistically more precise methods Chree[2] arrived at an almost identical time for the mean length of the interval between disturbances at Kew Observatory.

These estimates corresponded closely with that synodic rotation period of the sun which is most frequently deduced from sunspot movements in both solar hemispheres and, according to Carrington's estimates, is to be associated with solar latitudes 15°–20°. Other investigators have since obtained different values of the interval. Except, however, for the weight given by Schuster[3] to 13½ days after an analysis of Greenwich statistics by periodogram methods, all the supposed intervals were comprised within the limits 25½ to 30½ days.

The synodic rotation period of the sun's more superficial layers varies by at least a day in passing from heliographic latitudes 30° to 10°, which are the approximate mean latitudes of spots at the beginning and end of the solar cycle. It is also known that the longer lived spots tend to progress more slowly than those other spots in the same latitude which reappear a smaller number of times. They have also different periods of rotation between successive appearances. All sunspots, moreover, have proper motions. Hence, when it was recognized that magnetic disturbance on the earth was in some way associated with sunspot phenomena, the mean times of recurrence of disturbance were naturally expected to exhibit variations.

Another factor was introduced when Schmidt[4] and, later, Angenheister[5] claimed that an interval of 30 days or multiples of that number which seemed to them to separate great disturbances was to be explained by deeply seated commotions in the sun's atmosphere. The increase in rotation period with depth of the solar layers necessary for the acceptance of this hypothesis seemed to be substantiated by the spectroscopic observations of Adams and St. John at Mount Wilson Observatory.

With these and other obviously influential factors (as, for example, the distance and orientation of the earth with respect to the sun) in mind, it will be seen that a somewhat detailed method of dealing with the problem is necessary. Hitherto, the ways by which recurrence intervals have been estimated have permitted only approximations to fractions of a day, and in many cases even whole days have been the safest acceptable value. The methods used by Chree[6] seemed capable of further extension provided measures of disturbance for time units smaller than the day were forthcoming.

To provide the latter, character figures 0, 1 and 2 were assigned to each hour of the three years 1921, '22 and '23 on the basis of the declination and horizontal force traces from the Kew magnetograph. 1917 was, subsequently, similarly treated.

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[1] E. W. Maunder. *Monthly Notices, R.A.S.* Vol. LXV, 1904–5, pp. 2–34; 538–59.
[2] C. Chree. *Phil. Trans.* A. Vol. 212, 1912, pp. 75–116, and Vol. 213, 1913, pp. 245–77.
[3] A. Schuster. *Monthly Notices, R.A.S.* Vol. LXV, 1904–5, pp. 186–97.
[4] A. Schmidt. *Met. Zeitschrift*, 1909, pp. 509–11.
[5] S. Angenheister. *Terr. Mag.* and Atmos. Electricity, 1922, pp. 64–69.
[6] C. Chree. *Proc. R. Soc.* A. 1914. Vol. 90, pp. 583–600, etc.

(54) Wt. 16616—779/1083. 625. 3/28. T.S. 66.

The significance of the " characters " was the same as that adopted in the international scheme for allocating figures to whole days. An hour was described as an " 0 " hour when the magnetic condition during the hour was " quiet," with little or no departure from the mean daily curve for the season and year ; a " 1 " was allotted to each hour of moderate or small disturbance, while for an hour to merit a " 2 " it required to show unusual departure, either in amount of oscillation or range or both, from the normal quiet run for the epoch concerned. By assigning one or other of these three figures to each hour throughout the years under investigation, first on the evidence of the declination traces alone, and then independently using the horizontal force traces as the basis of judgment, and combining the two to form a final estimate, each hour in the four years was furnished with a character figure 0, 1 or 2 descriptive of the magnetic state at Kew.

It should be explained here that, with a special object in view, the original intention was to examine the three consecutive years 1921, '22 and '23 alone. Preliminary consideration of certain features of the results, however, suggested that similar figures for a year about sunspot maximum in the last cycle might provide useful comparative data, and since 1917 was a year of conspicuously high number (103·9 on the Wolf-Wolfer scale) assignment of character figures to each hour of that year was made in the same way as for the other three years.

Then assuming that every occurrence of a " 2 " indicated an enhanced state of disturbance which might be followed after some interval by another period of increased magnetic activity, every hour of character 2 was listed and the sequence of numbers formed by the 120 consecutive hourly characters included within the interval 25 days 13h. to 30 days 12 h. after the selected "2" hour were set down in horizontal array. Since the total number of 2's assigned to hours in the four years was 2,497, this number of sequences each of 120 constituents have gone to form the basis of the first set of results given here. As will be described later, a separate analysis of 1922 necessitated a 50 per cent extension of this part of the investigation.

The arrays were then grouped according to the months of the original 2's and 48 sequences of monthly totals for the 120 vertical columns formed in each of the two ways :—

1. Regarding the 2's as numerical quantities ; *i.e.*, calling n_1 the number of occurrences of 1 in any column and n_2 the number of 2's, the total for the column was taken as $2n_2+n_1$. This gave a measure of the " total " character for the hour concerned.
2. Considering the frequency of 2's alone ; *i.e.*, using simply n_2.

From these individual monthly totals, final sequences were formed, representing the mean annual run of disturbance from hour to hour over the five complete days ($25\frac{1}{2}$ to $30\frac{1}{2}$) subsequent to the initially disturbed " 2 " hour. There was thus available at this stage, two sets each of four (one for each year) final annual sequences, each comprised of 120 totals.

Since such an extended series of figures, even when restricted to the final totals, would be too cumbersome for complete reproduction, those for the total (i.e., the $2n_2 + n_1$) characters have been represented graphically in Fig. 1 on a small time scale, and both sets have been further telescoped into the forms shown in Tables I and II. These latter have been derived by grouping together every three consecutive hourly totals, taking for the first group the three hours 13h. to 15h. on the 25th day, for the second group, 16h. to 19h. on the same day, and so on. The 120 single hourly sums were thus reduced to 40 3-hourly totals. Each of these was then expressed as a percentage of the mean of the entire 40 ; so that the resulting figures in the tables represent the state of affairs for each $\frac{1}{8}$-day from $25\frac{1}{2}$ to $30\frac{1}{2}$ days after the selected disturbed hours, expressed as percentages of the mean character for that (5-day) interval. This has been effected for the total characters with 2's treated as numerical quantities (Table I), and for 2's alone (Table II), the latter showing the simple frequency distribution of hours of large disturbance throughout the five days.

Fig. 1. FINAL ANNUAL SEQUENCES OF TOTAL ($2n_2+n_1$) CHARACTER FIGURES OVER THE INTERVAL 25½ TO 30½ DAYS AFTER DISTURBED HOURS.

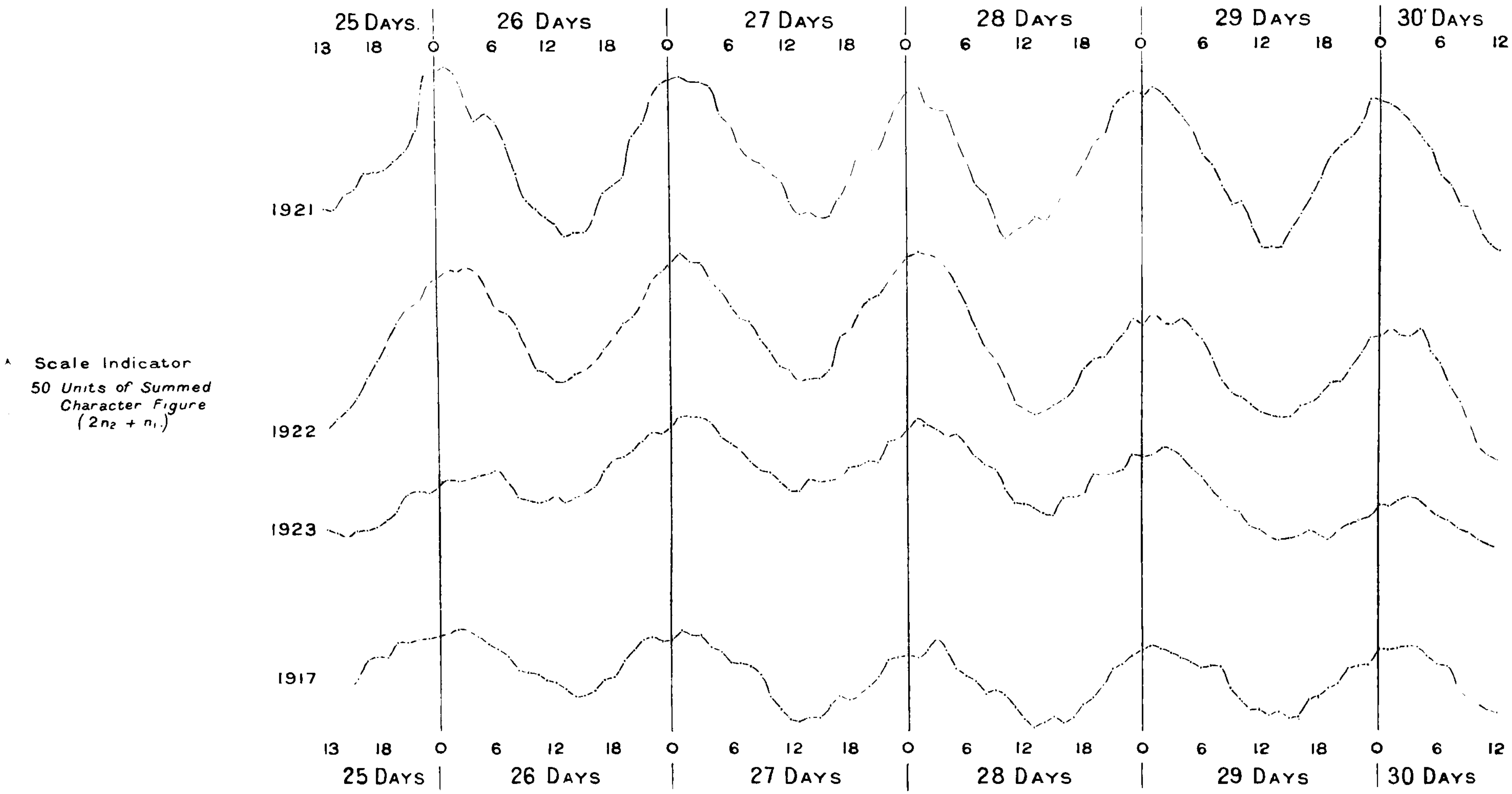


Fig. II. 4-HOURLY CONTRIBUTIONS TO THE TOTAL VARIATION OF DISTURBANCE DEDUCED FROM ALL SELECTED 2's IN 1922.

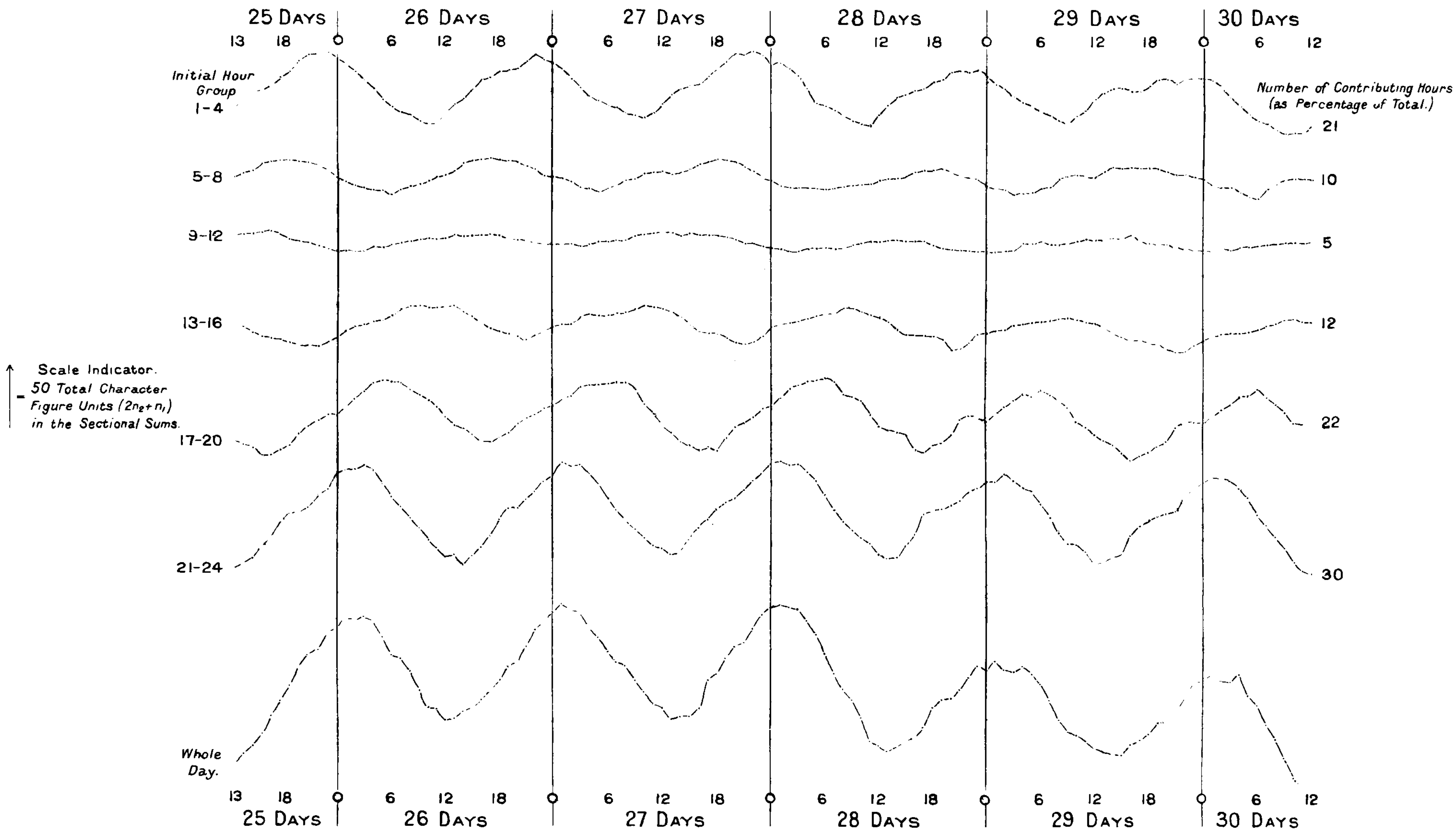
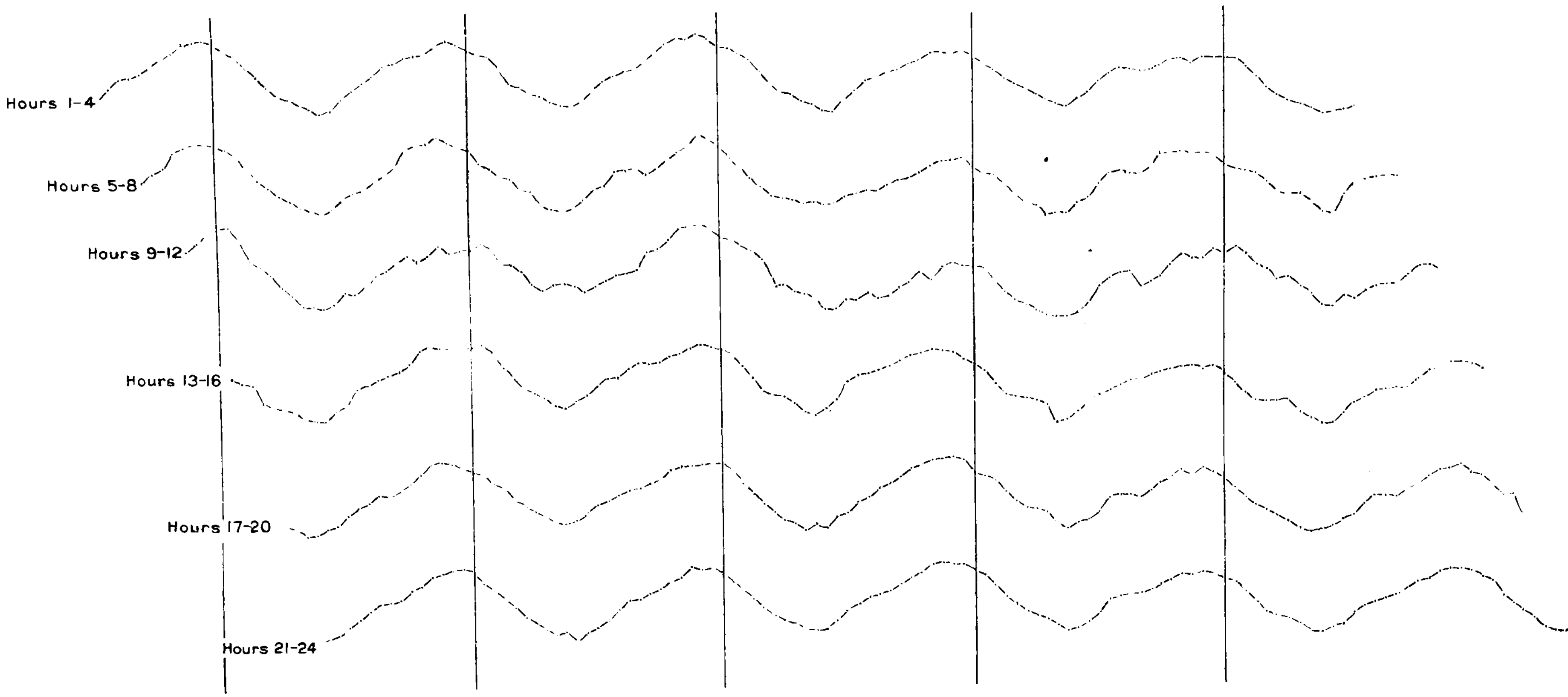


Fig. III. PART-DAY CONTRIBUTIONS TO TOTAL DISTURBANCE VARIATION ARRANGED IN SAME PHASE AND REDUCED TO A PERCENTAGE BASIS.



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The general results of this part of the investigation are deducible from the tables and graphs in Fig. 1. From the latter especially it is immediately obvious that other influences which largely obscure the main issue have obtruded themselves; for instead of an expected well-defined maximum showing itself in each annual sequence, the character sums are seen to vary in surprisingly regular undulations of uniform wave length equal to 24 hours and with no one maximum in marked excess of the others. This regular variation of disturbance has been specially examined and the results of this examination have recently been published separately.[7]

TABLE I.—VARIATION OF TOTAL $(2n_2+n_1)$ CHARACTER FIGURE FROM $25\frac{1}{2}$ TO $30\frac{1}{2}$ DAYS AFTER HOURS OF CHARACTER 2.

3-HOURLY VALUES EXPRESSED AS PERCENTAGES OF THE MEAN FOR EACH SEQUENCE.

	25 days.				26 days.								27 days.							
3-Hourly Interval.	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
1917	94	105	114	117	121	116	106	98	91	97	113	117	121	111	104	84	79	88	95	108
1921	90	96	100	117	119	110	97	87	83	91	103	117	124	112	101	94	87	91	104	112
1922	83	92	105	112	117	112	103	93	93	99	107	116	119	112	104	96	92	99	109	117
1923	52	55	77	94	104	111	92	86	88	112	134	153	168	150	124	103	100	108	120	146

	28 days.								29 days.								30 days.			
3-Hourly Interval.	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12
1917	113	104	93	84	76	80	96	108	112	105	101	84	80	84	97	106	112	108	94	87
1821	114	106	90	82	86	95	107	114	114	105	92	84	80	91	102	110	110	100	89	82
1922	120	112	99	87	84	90	97	104	105	102	91	85	82	86	91	99	102	98	85	80
1923	157	145	119	84	71	86	108	119	128	114	83	60	44	49	51	68	81	73	54	51

TABLE II.—VARIATION OF OCCURRENCE FREQUENCY OF CHARACTER FIGURE 2 ALONE.

(3-HOURLY GROUPS AS PERCENTAGES.)

	25 days.				26 days.								27 days.							
3-Hourly Interval.	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24
1917	69	119	123	135	136	157	91	86	73	105	164	168	160	123	118	73	61	75	91	136
1921	86	96	104	122	140	124	106	90	81	85	108	129	138	129	117	100	90	94	114	113
1922	71	97	123	140	142	128	107	85	88	98	108	128	134	110	96	87	80	96	127	139
1923	28	44	67	123	128	118	84	103	115	141	195	221	269	203	151	90	59	108	136	159

	28 days.								29 days.								30 days.			
3-Hourly Interval.	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12	13 to 15	16 to 18	19 to 21	22 to 24	1 to 3	4 to 6	7 to 9	10 to 12
1917	173	136	95	59	48	59	102	132	159	118	75	25	23	41	86	116	134	73	48	34
1921	114	112	73	60	70	79	109	113	120	103	88	78	67	82	95	109	102	85	74	68
1922	140	118	92	66	57	70	91	100	107	104	78	68	67	75	91	104	105	92	69	60
1923	164	128	82	38	46	62	97	141	113	82	54	31	21	31	44	44	41	39	23	19

Without exception, the principal maxima in the curves of Fig. 1 are found in that part of the day in which the maximum of the normal daily variation of disturbance at Kew Observatory occurs. Hence, the search for the recurrence interval resolves itself into selecting that one of the five daily crests which seems in excess of its neighbours.

[7] *Geo. Mem.*, No. 32, 1926.

Now, in considering the time of this principal maximum, it is to be noted that since there is little difference between the crests, the selection of the greatest value (which presumably indicates the most frequent interval elapsing between the originally disturbed " 2 " hour and its recurrence) will depend on which total is accepted as the safest indication of the recurrence. Since a single hour is taken as the time unit for the starting point, individual hourly values might be considered to be the proper criteria for deciding the epochs of subsequent maxima. But scrutiny of the sequences of hourly totals clearly showed that the individual differences around the turning values are so small that an interchange of one or two " 1 " hours with " 2 " hours at critical points in the constituent arrays would frequently be sufficient to displace the " maximum " by at least a day. That is, such an insignificant change would suffice to convert a 27 into a 28 or 29-day recurrence interval for that year.

For this reason alone a 3-hourly or even quarter-day value may well be considered a more certain indication of a real disturbance recurrence. This suggestion is given force when it is remembered that the initially disturbed hours tend to occur in sequences rather than as isolated units. The times of incidence of the various maxima selected on the basis of hourly, 3-hourly and 6-hourly totals are shown in Table III, those for the grouped totals being given as centred at the middle of the group.

TABLE III.—TIMES OF INCIDENCE OF DISTURBANCE MAXIMA.

(DEDUCED FROM 3 MEASURES.)

		Separate Hourly Totals. Day.	Hour.	3-Hourly Percentages. Day.	Hour.	Quarter-Day Totals. Day.	Hour.
Total ($2n_2+n_1$) characters	1917 ..	27	1	26 (=27)	$1\frac{1}{2}$	26 (=27)	0
	1921 ..	26	1	27	$1\frac{1}{2}$	27	0
	1922 ..	27 (=28)	1	28	$1\frac{1}{2}$	28	0
	1923 ..	27	1	27	$1\frac{1}{2}$	27	0
Occurrence frequency of 2's alone	1917 ..	26	21	28	$1\frac{1}{2}$	26	21
	1921 ..	26	2	26	$1\frac{1}{2}$	27	0
		=27	1				
	1922 ..	26	1	26	$1\frac{1}{2}$	26	0
	1923 ..	27	1	27	$1\frac{1}{2}$	27	0

Confining attention to the " total " ($2n_2+n_1$) character results, the main evidence of the graphs and table may be summarized as follows :—

1. The individual maxima invariably occur within two or three hours of an integral multiple (25, 26, 27, 28, 29 or 30) of 24 hours after the originally selected "2" hour.
2. In only one instance (that at 29 days 23h.) does a maximum precede the hour indicating the exact multiple of 24. All the other individual maxima, including the principal maxima, occur no later than two hours after the exact multiple.
3. The principal maxima are confined to the vicinity of the 26, 27 and 28 multiples of 24 hours after the initially selected hour.
4. For each one of the four years, the epoch of principal maximum (or one of the epochs if two equal maxima occur) may be within an hour of the 27th day from the initially selected hour on at least one of the three methods proposed for selecting the maximum.
5. The five individual maxima in each of the years 1922 and 1923 give the best indications of a rise from the 25th day to the chief maximum and subsequent subsidence to the 30th day. This would indicate that the recurrence interval was better defined in these two years than in the other years 1917 and 1921.

In summary, then, it appears that the most favoured epoch for recurrence of the disturbance maximum is approximately 27 days from the originally disturbed hours but at the same time, it is all but a matter of chance whether a particular disturbance reaches a higher state of development after this interval or at a time differing by a small multiple (1 or 2) of 24 hours from the complete 27 days.

When these conclusions are reviewed in terms of a regular diurnal variation of disturbance they are, at least in part, more readily understood. For it is then seen that though the epochs of the various turning points of the separate undulations differ somewhat among themselves and from the times of incidence of the turning values in the normal diurnal variation, they yet resemble the latter sufficiently to suggest a close association. The curves of Fig. 1 are indeed what would be expected to result from a combination of such variations with slightly differing phases and with the disturbance associated with the recurrence superimposed.

On this reasoning the cumulative disturbance specifically arising from a 27-day recurrence may contribute only a relatively small part to the final form of the curves. But it is this addition which is effective in throwing the balance of accumulated character figures from one maximum to a neighbouring maximum, and thus produces a 26, 27 or 28-day recurrence interval but not one involving a fraction of a day.

Viewed physically the interpretation would appear to be that, since the principal seat of the magnetic variations both regular and aperiodic is generally conceded to be in some conducting layer or layers of the earth's atmosphere, alterations in the conductivity conditions there due to the superimposed effect of an apparently insignificant sunspot or a chance concentration of two or more such diminutive sunspots may be responsible for converting what might have been a 27-day into a 26 or 28-day interval for the occasions concerned.

As indicated earlier the original hope in instituting this research was that the employment of hourly character figures would assist towards a minute analysis of such influences as drift of sunspots and seasonal variation of earth's distance and orientation with respect to the sun on the mean interval of recurrence of magnetic storms. The complications introduced by the diurnal variation have so far precluded the latter of these investigations and the data available do not give much scope for an examination of a possible shortening of the recurrence interval from 1917 to 1923 by reason of the equator-ward drift of spots during these years.

Further, although it was recognized that the " deeply seated " storms of Schmidt and Angenheister were presumed to be of a different nature from those which would be selected by a mere segregation of hours of character 2 in the above analysis, it was thought that if the examination were restricted to dealing with those more strongly disturbed hours, some slight differences in their behaviour compared with the previous results might be looked for. In particular, a tendency to retardation of maximum in the direction of the 30th midnight might be expected. The incidence of chief maximum for each year as indicated in the second part of Table II gives no reason for supposing that there is any essential difference in the length of the interval for the more highly disturbed hours as compared with that for the general $(2n_2 + n_1)$ disturbance.

Analysis of the Progressive Change of Phase of Disturbance Variation in the Final Sequences

Since the initially selected hours of disturbance symbolized by the character 2 were not restricted to the period centring at midnight, about which time the maximum of the diurnal variation normally occurs at Kew, and since the initial phases of the separate constituents of the final 5-day sequences of figures were determined by the time of the selected " 2," it was not immediately obvious how these final sequences formed such a regular progression with maxima approximately coincident with the exact day in each case. Stated otherwise, while it was evident that those selected 2's which occurred within an interval of $\frac{1}{4}$-day on either side of midnight would have associated with them just such a set of undulations as finally appeared, it was not clear how the contributions from the six hours centring at noon affected the final sequences.

In order to investigate this point the analysis of 1922 was repeated. Lists were drawn up of all those 2's which occurred at the same hour in the day and the sequence of 120 character figures covering the five days ($25\frac{1}{2}$ to $30\frac{1}{2}$) after the selected hours extracted afresh. In this way 24 blocks of sequences (one for each set of 2's with the same time of incidence) were formed and totals ($2n_2 + n_1$) derived as before. There thus resulted 24 sets of hourly totals representative of the run of disturbance over the five day interval starting $25\frac{1}{2}$ days subsequent to 2's taken from each hour of the day. The totals cannot be reproduced in full for evident reasons. The graphical representation in Fig. 2, however, illustrates the essential features of their variation. The first six curves in that figure have been obtained by plotting group totals formed by combining the hourly sequences in sets of four so that the resulting figures or their graphs really represent the course of magnetic disturbance subsequent to the " 2 " hours segregated in each $\frac{1}{6}$-day, 1h. to 4h., 5h. to 8h. ...... and 20h. to 24h.

An examination of the progressive differences shown by these curves explains how the final whole day curve exhibited maxima round the beginning of each exact day, for the contribution of each group to the sum-total necessarily depends on :

1. The number of hours of character 2 occurring in the particular group, *i.e.*, it depends ultimately on the normal frequency of 2's in different parts of the day as conditioned by the regular diurnal variation of disturbance, and
2. The phase of the middle of the four-hourly group with respect to the day.

Now the number of initial " 2 " hours included in the groups 1h. to 4h., and 20h. to 24h., comprise 51 per cent of the total number from every hour of the day and, if the group 17h. to 20h. be added, the percentage reaches 73. It is, therefore, an obvious result that the daily variation appropriate to the weighted mean of these hours will be the principal feature of the whole day sequence and will give rise to maxima occurring invariably within an hour or two of midnight.

The fact that there is no essential difference in type between these partial contributions from 4-hour groups is demonstrated graphically in Fig. III. To obtain the curves shown there the 120 constituents of the sequences from which the first six curves of Fig. II were derived were converted into percentages of the mean of their respective sequences. These percentages were then plotted with the initial entry of each sequence four hours in advance of its predecessor, thus keepng pace wi h the progressions of the groups throughout the day.

Keeping in mind that the smaller number of sequences (resulting from a diminished number of " 2 " hours) contnbuting to the central groups allows smaller irregularities to become more conspicuous, the diagram amply demonstrates that the six variations are identical in type and phase.

It also becomes evident now that little useful purpose can be served by treating these sequences of figures further. For while it might have been considered possible to obtain information of assistance in investigating some of the problems discussed above by comparison with the normal diurnal variation of disturbance for each respective year, it is now seen that such a comparison is precluded by the intrinsic heterogeneity of phase in the final "recurrence" variation. Whereas the diurnal variation discussed in *Geophysical Memoir* No. 32 is established by arranging daily distributions of disturbance in the same phase, the sequences obtamed in the above examination are entirely composite structures. They would require such a separate analysis as has been carried out for 1922 to permit any kind of comparison.

Fig. III does indicate, however, that the " purity " of the recurrence in any year or the extent to which the real recurrence maximum is well-defined in comparison with the neighbouring maxima will be as much dependent on the absence of disturbance hours in the middle of the day as on their concentration in the period immediately before and after midnight.

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