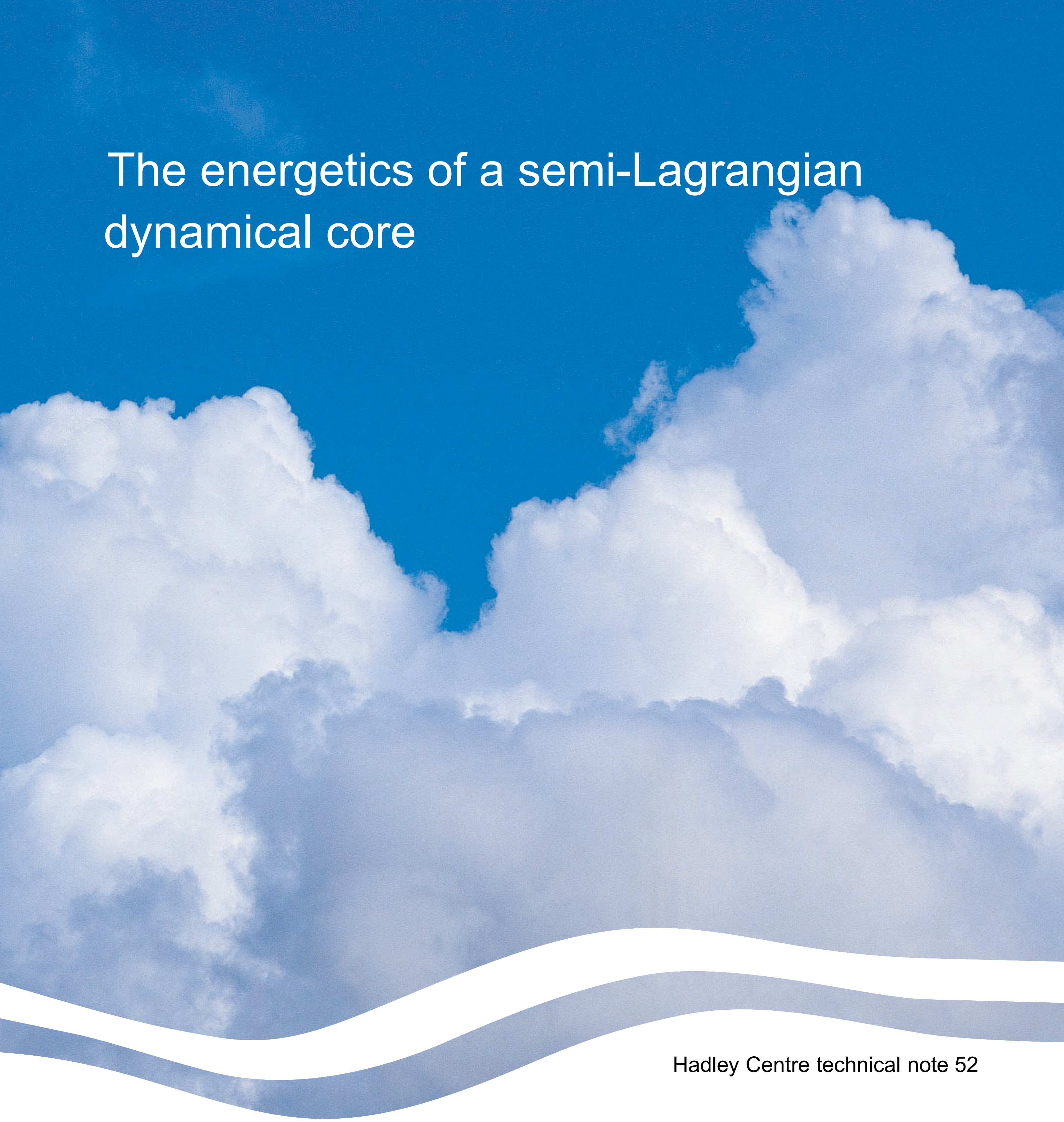
The energetics of a semi-Lagrangian dynamical core



Hadley Centre technical note 52

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23 September 2004

Met Office

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Abstract

The dependence of the energetics of the new Met Office semi-Lagrangian dynamical core on resolution, departure point calculation and diffusion are investigated and compared with results from the old Eulerian model at resolutions typically used for climate modelling. At a typical climate model resolution the semi-Lagrangian core energetics differ from the Eulerian. The most significant difference is lower transient eddy kinetic energy in the semi-Lagrangian core. Increasing horizontal resolution increases the transient eddy kinetic energy. Vertical resolution and time resolution have only a small impact on the energetics. The energetics of the dynamical core show a strong sensitivity to the departure point calculation; higher order schemes increase the transient eddy kinetic energy of the dynamical core.

1. Introduction

The energetics of a full climate model are very important in determining how well it will simulate mid-latitude variability and storms. While the treatment of moisture and physical parameterisations are important in the simulation of mid-latitude storms, much can be learnt about the underlying energetics by looking at a dry dynamical core version of the full model. Previous studies comparing Semi-lagrangian and Eulerian climate models and dynamical cores, (Chen et al (1997), Williamson et al (1998), Willamson & Olson (1994)) have shown that the semi-lagrangian dynamics tends to have less eddy kinetic energy than equivalent resolution Eulerian dynamics at resolutions typically used for climate modelling (e.g. T42 or 2.x2.5 degrees). This study looks at the energetics of the new Met Office Semi-Lagrangian dynamical core (briefly described in section 2), to be used in the new Hadley Centre model HadGEM1, and compares it with the old Eulerian core used in the Hadley Centre atmospheric climate model HadAM3 (Pope et al 2000). The aim is to determine whether the new model has less eddy kinetic energy at lower resolutions and whether anything can be done in the choice of the semi-lagrangian dynamics to try to improve the energetics at lower resolution.

Firstly the question of sensitivity to resolution is examined. The dimensions of a model, which can be varied, are;

(i) Horizontal resolution i.e. separation of the grid-points in a grid-point model or spherical truncation in a spectral model,
(ii) Vertical resolution i.e. separation of the vertical levels in height or pressure.
(iii) Time resolution i.e. length of time over which the model is integrated each timestep.

Ideally the dynamical behaviour of the model should not be significantly altered by varying any of the above but in practice at the resolutions most climate models operate at, the dynamical behaviour of the model has often not fully converged (Boer & Denis 1997).

Sensitivity of the dynamical core of a model to horizontal resolution has been looked previously by Boer and Denis (1997) using a spectral model with Eulerian advection and Pope & Stratton (2002) using HadAM3 the atmospheric component of HadCM3 (a grid point model with Eulerian advection). Results from Pope & Stratton

(2002) suggest that the dependence of the energetics of the dynamical core on horizontal resolution closely follow the dependence seen in the full model. As the model resolution increases the transient eddy kinetic energy of the model increases. At higher resolutions the model is better able to resolve smaller scale features eg storms with more extreme values of winds leading to higher transient eddy kinetic energy.

The sensitivity of a dynamical core to vertical resolution has been studied for Eulerian and semi-lagrangian dynamics by Williamson et al (1998) who looked at the impact on the tropical temperature profile around the tropopause. They found convergence of the temperature profile in the semi-lagrangian model when they increase the vertical resolution from 18 to 36 levels but do not comment on the impact on the model energetics. Boer and Denis (1997) compare 18 and 31 levels but find that any differences are of similar order of magnitude to differences introduced by interpolation from model to pressure levels.

The other dimension of a model is time. The sensitivity of a model to time step resolution is often neglected. A recent study by Williamson and Olson (2003) of time step dependence in an aqua-planet has shown that model physical parametrizations are often strongly dependent on time step. Their study was prompted by their need to understand the differences in behaviour between an Eulerian and a semi-lagrangian aqua-planet both running with the same physical parametrizations but with different time steps. The time step of a semi-lagrangian model is not restricted by Courant number considerations as is an Eulerian model and is therefore usually chosen to be longer as this reduces the computational cost. For this reason it is important to establish whether the basic dynamics of the model show any timestep sensitivity. Section 3 describes the resolution tests and their results.

Certain choices can be made when settings up a semi-lagrangian scheme. The sensitivity of the eddy kinetic of these choices explored here are:

(1) Choice of the interpolation scheme for the departure point calculation.
(2) The choice of the weights in the off-centring of the semi-implicit two-time level scheme.
(3) The impact of diffusion on the dynamics.

Semi-lagrangian schemes are known to be dissipative due to the interpolation required to calculate the value of the field at the departure point. Higher order interpolation schemes are expected to be less diffusive as they use information from more surrounding grid-points and are therefore more likely to retain highs and lows in a field. Work by Semazzi & Dekker (1994) suggest that low order interpolation schemes like linear and quadratic are not accurate enough but cubic or above is suitable for use in a numerical weather prediction model. Section 4 gives results from varying the interpolation scheme.

The more off-centred the semi-implicit time calculation the more likely the model fields are to be damped. Some off-centring is desirable in a two time level semi-implicit scheme to overcome problems of spurious orographic resonance (Rivest et al 1994). Results from looking at the off-centring are given in section 5.

The addition of diffusion to a model will tend to reduce the eddy kinetic energy. In a full model increments from physical parametrizations often introduce grid scale noise which can be controlled by adding some diffusion. Therefore it is useful to know the impact of diffusion on the energetics of the dynamical core. The impact of diffusion on the model is investigated in section 6.

2. Description of model used.

The dynamics of the new Met Office model is described in great detail in Staniforth et al (2003); a brief description of the main points is provided here. The model is non-hydrostatic using the full primitive equations. The advection scheme is semi-Lagrangian with an off-centred two-time level semi-implicit time discretisation. The dynamics uses an Eulerian treatment of the continuity equation and conserves the dry mass of the model. The model is a grid point model with variables held on the Arakawa C grid (Arakawa & Lamb 1977) in the horizontal and the Charney-Phillips grid (Charney & Phillips 1953) in the vertical (i.e. winds, density and pressure are held on different levels to potential temperature, vertical velocity and moisture). The vertical coordinate is hybrid height with levels near the bottom boundary following the surface terrain and those at the top of the model being flat. In the case of the dynamical core, as the surface is flat, all points on a model level have the same height.

The old Eulerian dynamics used in HadAM3 (Pope et al 2000) is a grid-point, hydrostatic primitive-equation model with a split-explicit advection scheme. The vertical coordinate is hybrid pressure with no staggering of winds and temperature in the vertical and using the Arakawa B-grid in the horizontal.

Both models were run using the Held Suarez (1994) dynamical core forcing for a period of 3 years 3 months. Means formed from the last 3 years of the integrations were used for the comparisons.

3. Sensitivity to resolution

3.1 Horizontal resolution

The sensitivity of the new semi-Lagrangian version of the Met Office model has been examined using the same set of horizontal resolutions as in Pope & Stratton (2002,) i.e. N48 (2.5° latitude x 3.75° longitude), N72 (1.66° x 2.5°), N96 (1.25° x 1.875°) and N144 (0.833° x 1.25°), with the addition of N24 (5° x 7.5°) and N216 (0.766° x 0.833°), (N216 is the resolution currently used in the Met Office operational NWP model). Figure 1 shows the globally integrated components of the energy cycle (Ulbrich and Speth 1991) and Figure 2 the mean vertical profiles of the transient eddy kinetic energy component for the different resolutions. Considering only quasi-cubic interpolation at this stage (see section 5 for further details) it is clear that the new model shows a very strong sensitivity to horizontal resolution particularly at the lower resolutions. Figure 2 shows that the increase in eddy kinetic energy occurs throughout the atmosphere though is largest around 250 hPa, the level of the westerly jet cores. From about N96 upward the model starts to show convergence.

Figure 3 shows the energy cycle for old Eulerian model at N48, N96 and N144 when run with a similar set of 38 levels in the vertical to the new semi-Lagrangian model and with diffusion settings as used in Pope and Stratton (2002). Looking at just the curve labelled Eulerian L38 (the other curves will be explained in section 6), the results are very different from Fig 1; N48 has more transient eddy kinetic energy and more transient available potential energy, and there is less difference between N48 and N96 though this will be explained in section 6. The rate of conversion of transient available potential energy to transient eddy kinetic energy is higher in the Eulerian case at all resolutions. At N48 the Eulerian conversion rate CK is higher between transient eddy kinetic energy and zonal kinetic energy.

These results confirm the findings of Chen et al (1997), Williamson et al (1998) and Williamson and Olson (1994) that a semi-Lagrangian model will have less transient eddy kinetic energy than an Eulerian model at a typical climate resolution eg. N48.

3.2 Vertical resolution

In this study the standard resolution of 38 levels (N48L38) in the vertical was reduced to 19 (N48L19) and increased to 60 (N48L60, to see if changing the vertical resolution had any significant impact on the results. The reduction and increase in model levels was done throughout the troposphere while keeping the top of the model roughly constant. Figure 4 shows the results in terms of the global mean profile of the transient eddy kinetic energy. Altering the vertical resolution has little impact on the energetics of the model when compared to the impact of horizontal resolution. As might be expected increasing vertical resolution tends to increase the transient eddy kinetic energy.

3.3 Time resolution

In this study the N48L38 model was run with the following timesteps; 5, 10, 30 and 60 minutes using quasi-cubic and cubic interpolation (see later). Table A gives the global transient eddy kinetic energy and the transient available potential energy, i.e. the components showing the largest changes in figure 1. The results show the semi-Lagrangian dynamics of the model have only a small (~3-5%) sensitivity to timestep. As timestep is reduced the results from the quasi-cubic start to approach those from the cubic. The semi-Lagrangian calculations are expected to be more accurate with shorter time step so this result is reassuring though the difference in behaviour between quasi-cubic and cubic interpolation is interesting and will be discussed in section 4.

Table 1 Global transient eddy kinetic and available potential energy.

Timestep (minutes)	5	10	30	60
EKE quasi-cubic	4.86	4.98	5.17	5.11
APE quasi-cubic	3.55	3.72	3.84	3.82
EKE Cubic	4.70	4.66	4.43	4.23
APE Cubic	3.50	3.52	3.43	3.36

4. Sensitivity to interpolation used for departure point calculation

The Met Office semi-Lagrangian dynamics has three different interpolations schemes coded and available for use, these are;

1. ECMWF quasi-cubic, a blend of linear and cubic interpolations (Ritchie et al 1995) using 32 gridpoints in 3D.
2. Cubic Lagrangian using 64 gridpoints in 3D.
3. Quintic Lagrangian using 216 grid points in 3D

The quasi-cubic interpolation is less computational expensive than cubic interpolation but is also less accurate. The interpolation options 1 and 2 were assessed at a number of different horizontal resolutions. Quintic interpolation, because of its cost, was only assessed at N48.

Fig 1 shows results from using the different interpolation schemes assessed in terms of the global energy cycle with figure 5 showing the vertical transient eddy kinetic energy profiles for the different interpolation schemes at N48. At N48 the scheme with the most eddy kinetic energy i.e. least diffusive is the quintic interpolation, next is the quasi-cubic and then with least energy the cubic. We would expect the most accurate scheme i.e. the quintic to be the least diffusive and the least accurate quasi-cubic to be the most diffusive. N Woods (private communication) has investigated this anomaly and looking in 1D suggests the inaccuracy of the quasi-cubic scheme can in some situations lead to an overestimate of highs and lows accounting for the less diffusive result. Results from section 3.3 suggest that as the time step is increased the differences between the quasi-cubic and cubic energetics become larger, the inaccuracies in the quasi-cubic scheme leading to higher transient eddy kinetic energy whereas the time step increases in the cubic scheme the inaccuracies lead as might be expected to lower transient energy kinetic energy.

It is worrying that the choice of the interpolation scheme used to calculate the value of the fields at the departure point could have such a large influence on the energetics of the model. The influence of this choice does not seem to alter much with resolution (fig 1); it is still as important at N144 as at N48.

5. Semi-Lagrangian weights

The semi-Lagrangian scheme used is a semi-implicit off-centred two-time-level scheme (Rivest et al 1994) i.e. the value of a field F at any gridpoint at time level n+1 is given by

$$F^{n+1} - F_d^n = \Delta t\left[w\Psi^{n+1} + \left(1-w\right)\Psi_d^n \right] \qquad (1)$$

Where Ψ is the source term, d is at the departure point and w is a weight ($0.5 < w \le 1$). The scheme has highest accuracy and least damping for w close to 0.5 with least accuracy and most damping with w close to 1. The particular implementation of the scheme in the Met Office model involves different weights for the horizontal and vertical parts of the momentum equation. Referring to these as w1 and w2 respectively; the default settings are w1 =0.6 and w2=1.0. The default values were chosen on the basis of tests using the full atmospheric model and may not be essential to ensure stability in the dynamical core.

Two sensitivity integrations were done, the first altering w2 to 0.7 (advice suggested against reducing this much further) and the second increasing w1 to 1.0. Figure 6 shows profiles of the global average transient eddy kinetic energy for all three integrations. As expected reducing the weight w2 to 0.7 increases the energy slightly but the change is very small. Increasing w1 to 1.0 has a much larger impact, reducing the eddy kinetic energy by the order of 13%. The results imply that the weights controlling the off-centring of the horizontal part are more important in influencing the model’s eddy kinetic energy than those controlling the vertical. This is perhaps expected as vertical velocity is small relative to horizontal velocity and contributes only a small amount to the total kinetic energy.

6. Diffusion

Adding diffusion to a field acts to reduce local high and low values and will therefore tend to reduce the eddy kinetic energy of the model by reducing the deviations of the u and v velocity components about their mean values. The horizontal diffusion operator used in the model is a ∇^2 style operator, which can be applied several times and operating on a field Q is given by

$$D(Q) = \frac{1}{r^2}\left[\frac{\partial}{\partial\lambda}\left(\frac{k_\lambda}{\cos^2\phi}\frac{\partial Q}{\partial\lambda}\right) + \frac{\partial}{\partial\phi}\left(k_\phi\frac{\partial Q}{\partial\phi}\right)\right]$$

Where is ϕ latitude, λ longitude, r distance from the centre of the earth and k_λ and k_ϕ are diffusion coefficients.

Fig 7 shows the results of applying different orders of diffusion with an e-folding time of 4 hours (i.e. a two grid-length wave of amplitude 1 is damped to an amplitude of 1/e in 4 hours). The higher the order of the diffusion operator the more scale selective the diffusion is. The ∇^2 operator damps the large scale waves in the model, significantly reducing the eddy kinetic energy, whereas the $(\nabla^2)^3$ operator has little impact. In fig 7 the $(\nabla^2)^3$ operator actually increases eddy kinetic energy in the run but this increase is very small and assumed to be due to the sampling period not being long enough.

In the case of the Eulerian model diffusion is essential to stabilise the model. The curve labelled just Eulerian in Figure 3 refers to a set of runs using different amounts of diffusion at different resolutions. This is the approach taken by Pope and Stratton (2002) when setting up the full model integrations at different horizontal resolution. N48 used a $(\nabla^2)^3$ operator whereas N96 and N144 used a $(\nabla^2)^2$ operator. This choice accounts for the smaller difference in energetics between N48 and N96. The other curves on the plot show the energy for the cases where the same e-folding time of 3 hours and the same order of operator are used at all resolutions. These additional curves resemble more closely the behaviour of the semi-Lagrangian model (Figure 1).

These results suggest that selective application of different order diffusion to the semi-Lagrangian model could be used to produce a flatter curve for, say, the transient eddy kinetic energy against horizontal resolution, but the net impact would be to reduce the already generally lower transient eddy kinetic energy of the semi-Lagrangian model.

Conclusions/ Summary

Ideally a model dynamics should converge with increasing resolution and the resolution used should be close to convergence. Results from the Met Office semi-lagrangian model show it converges with increasing resolution in time and space but at the highest resolution tested (i.e. N216) the solution is still dependent on the departure point calculation. The model dynamics shows far less dependence on vertical resolution and time resolution than on horizontal resolution.

The dependence of results on the order of the interpolation scheme used for the departure point calculation is of some concern but it does offer a means of increasing transient eddy kinetic energy. Altering the off-centring weights from their current settings offers little improvement in the energetics. Diffusion as expected tends to damp the model. In the case of a full climate model where diffusion may be necessary the choice of a higher order operator will have least impact on the larger scale waves and energetics of the model.

At the resolution usually used in the past for climate modelling, i.e. N48 the dynamics has not converged and is still some way from convergence. The eddy kinetic energy of the Semi-Lagrangian model is less than that of the old Eulerian model. Results from the full model using Eulerian dynamics (Pope and Stratton 2002) suggest the model has too little transient eddy kinetic energy at N48 when compared with ERA (ECMWF reanalyses). This implies that the eddy kinetic energy of the Semi-Lagrangian core needs to be increased. Choosing a higher order departure point calculation (i.e. quintic interpolation) would help but would also increase the CPU cost. A cheaper but less accurate option is to use the quasi-cubic interpolation.

At N96, a resolution under consideration for future climate integrations, the situation is improved. The semi-Lagrangian dynamics is closer to convergence and the eddy kinetic energy is similar to the old Eulerian dynamics. The solution is still dependent on the interpolation scheme used in the departure point calculation.

The results of this study suggest that, for climate modelling, changing from Eulerian to Semi-Lagrangian dynamics may be beneficial for higher resolutions i.e. of order N96 or above but may not benefit resolutions of order N48. A major benefit of switching to semi-Lagrangian dynamics at resolutions of N96 or above is the reduction in computational costs. A semi-Lagrangian model can run with a longer time step, whereas an Eulerian model must have an increasingly shorter time step with increasing horizontal resolution to maintain stability.

Acknowledgements

United Kingdom Government Meteorological Research Program. G Martin for her helpful comments on earlier versions of this paper. T Davies for help in settings up the dynamical core tests and advice on the dynamics.

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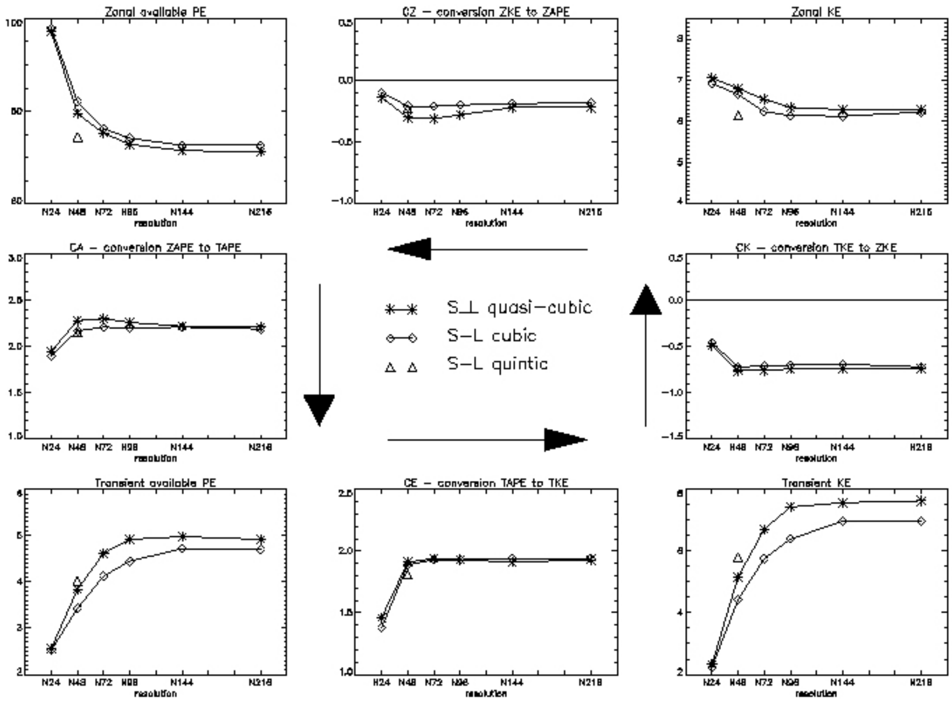
Figure Captions

1. Global integrated values of various components of the energy cycle from the semi-Lagrangian integrations plotted against resolution. The arrows indicate the direction of the cycle if the values are positive. Key: Z, zonal; PE, potential energy; KE, kinetic energy; C, conversion; A, available; T, transient.
2. Vertical profiles of the global average transient eddy kinetic energy for different horizontal resolutions from integrations using quasi-cubic interpolation.
3. As figure 1 but for values from Eulerian integrations with varying amounts of diffusion.
4. Vertical profiles of global average transient eddy kinetic for integrations with different vertical resolution (all using quasi-cubic interpolation).
5. Vertical profiles of global average transient eddy kinetic for N48L38 integrations using different interpolations for the departure point calculation.
6. Vertical profiles of global average transient eddy kinetic for integrations with different weights for the off-centred semi-Lagrangian advection.
7. Vertical profiles of global average transient eddy kinetic for N48L38 integrations with different amounts of diffusion.

Fig 1 Global integrated values of



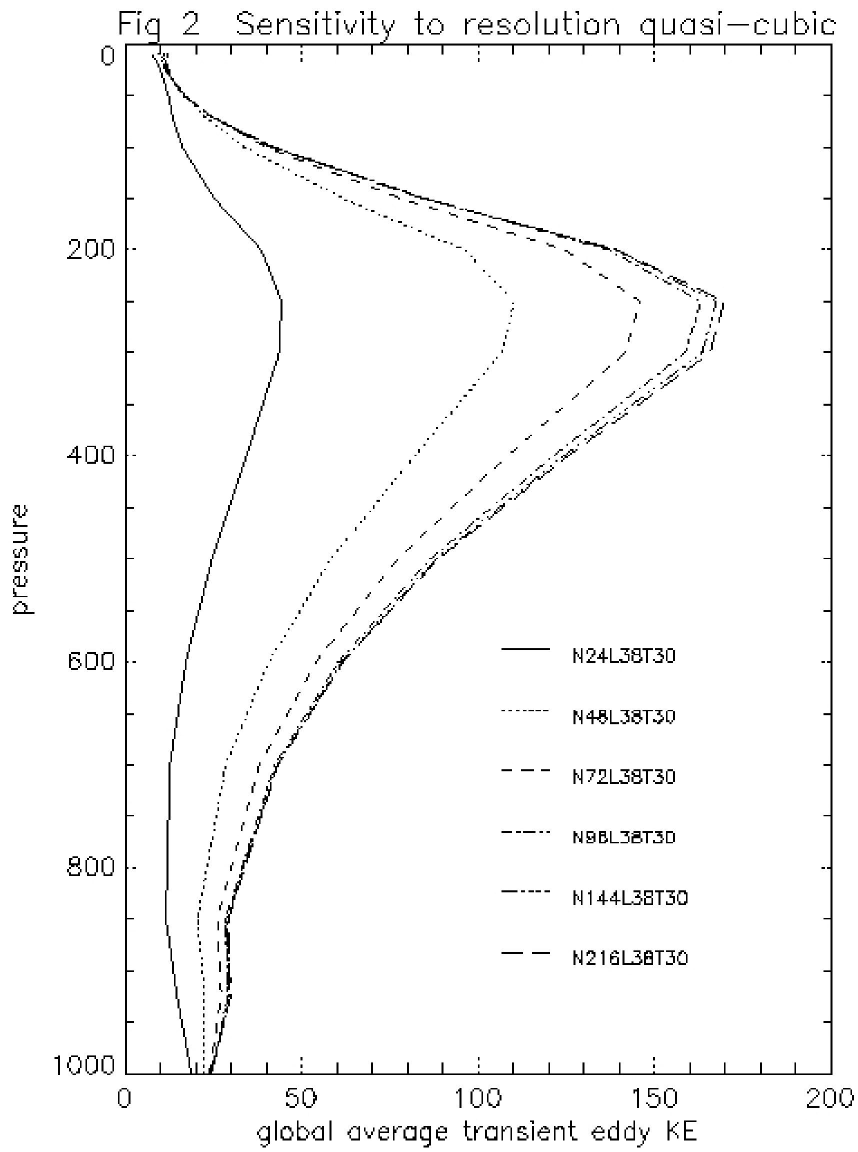


Fig 2 Sensitivity to resolution quasi-cubic
pressure
global average transient eddy KE
0
200
400
600
800
1000
0
50
100
150
200
N24L38T30
N48L38T30
N72L38T30
N96L38T30
N144L38T30
N216L38T30

Fig 3 Global integrated values of

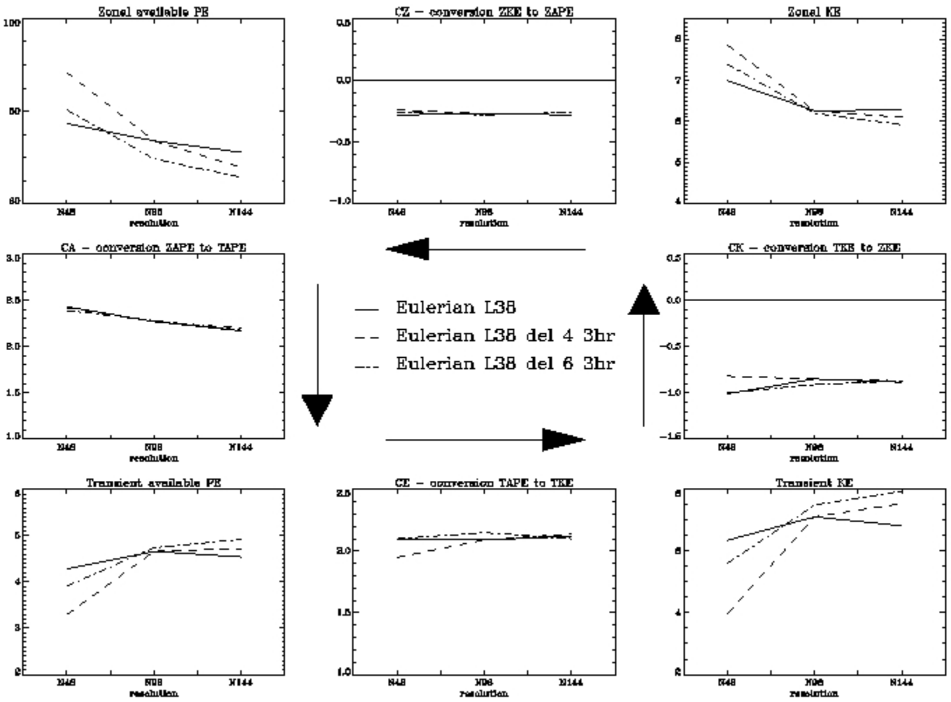
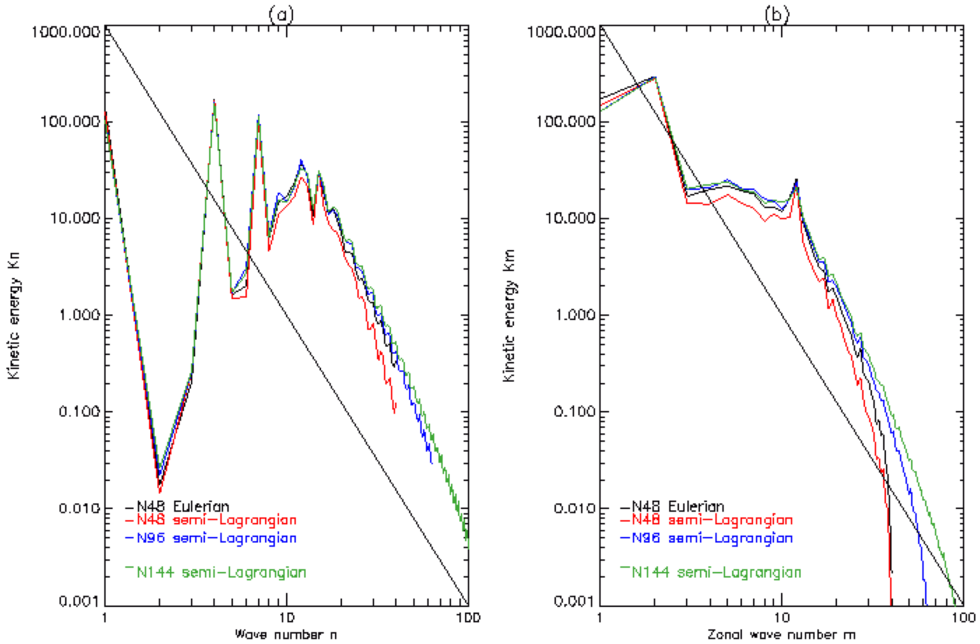


Figure 4 Kinetic energy spectrum of the rotational flow at 200hPa



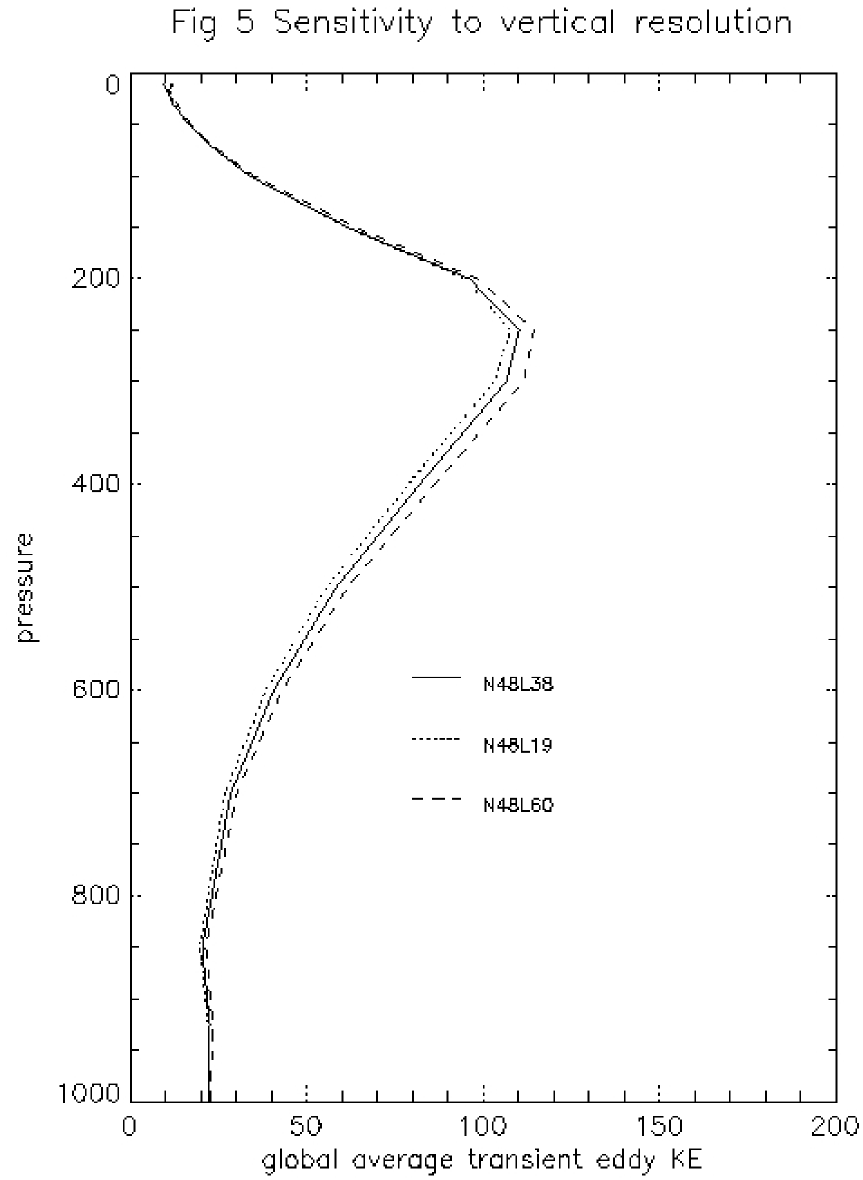


Fig 5 Sensitivity to vertical resolution
0
200
400
600
800
1000
pressure
0
50
100
150
200
global average transient eddy KE
N48L38
N48L19
N48L60

Fig 6 Sensitivity to interpolation

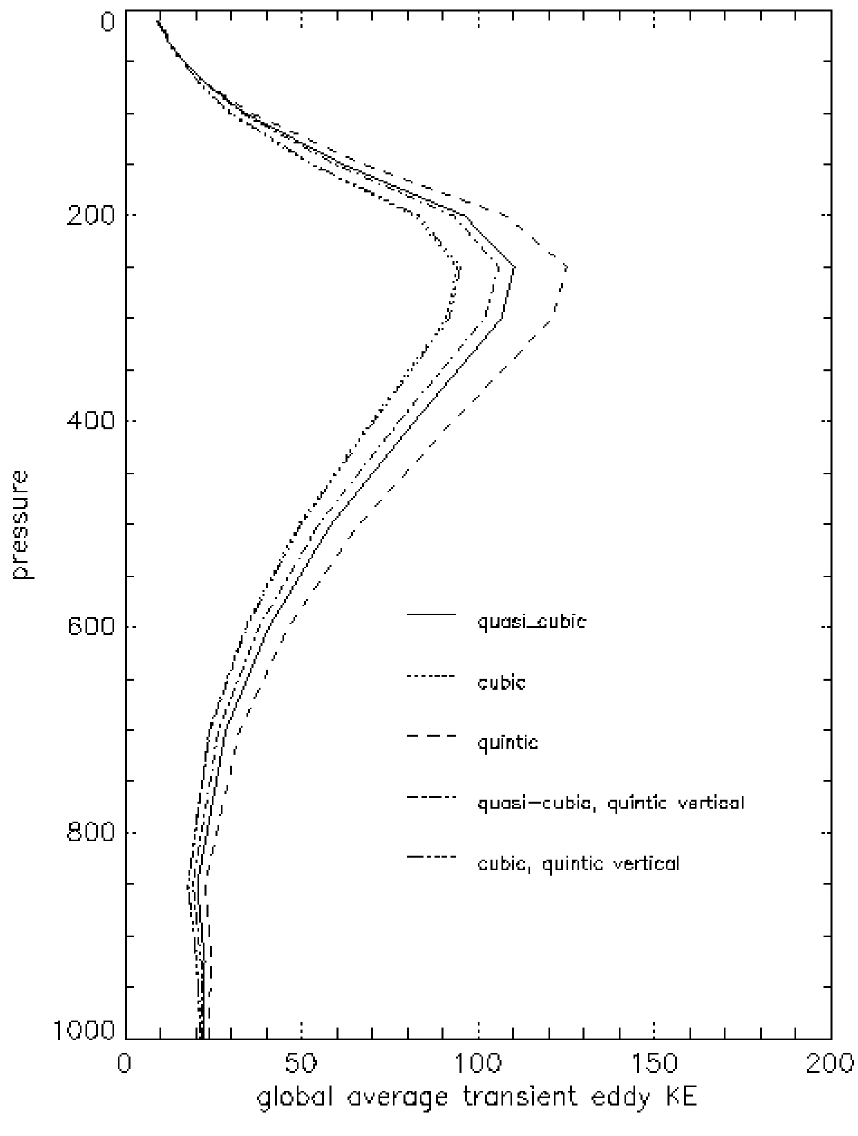
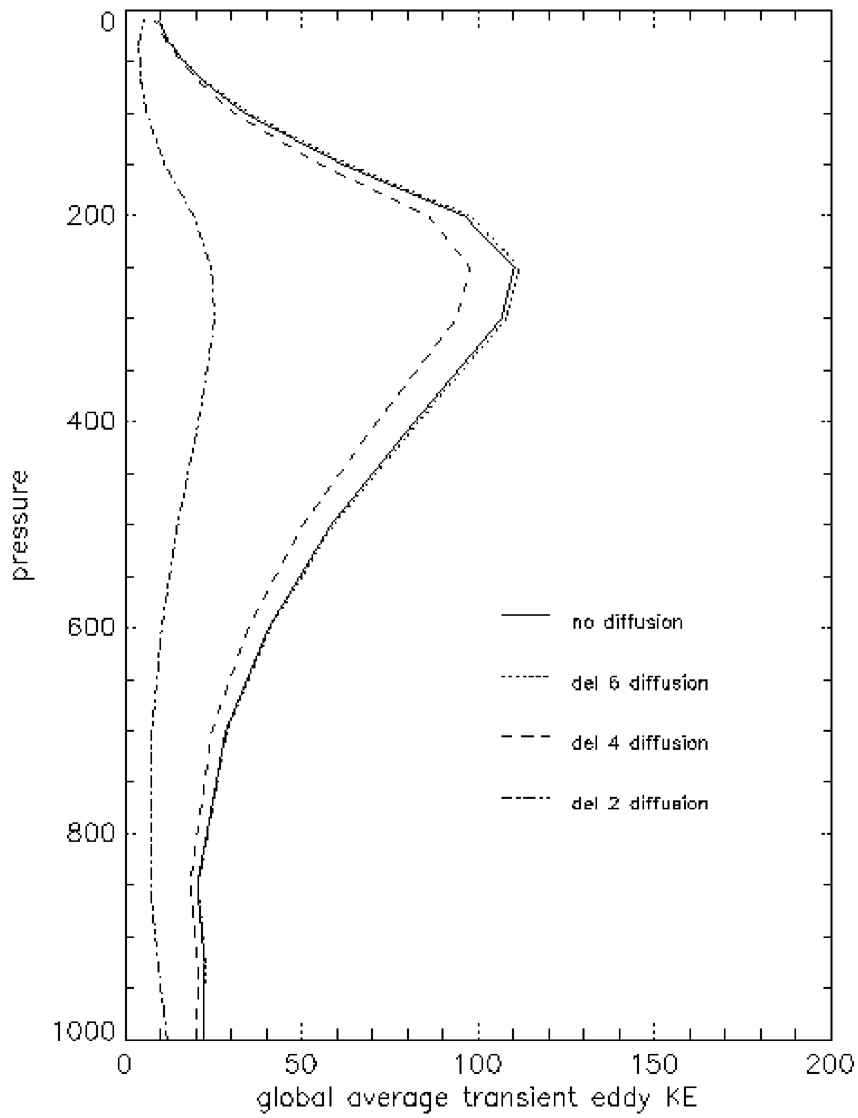


Fig 7 Sensitivity to diffusion.



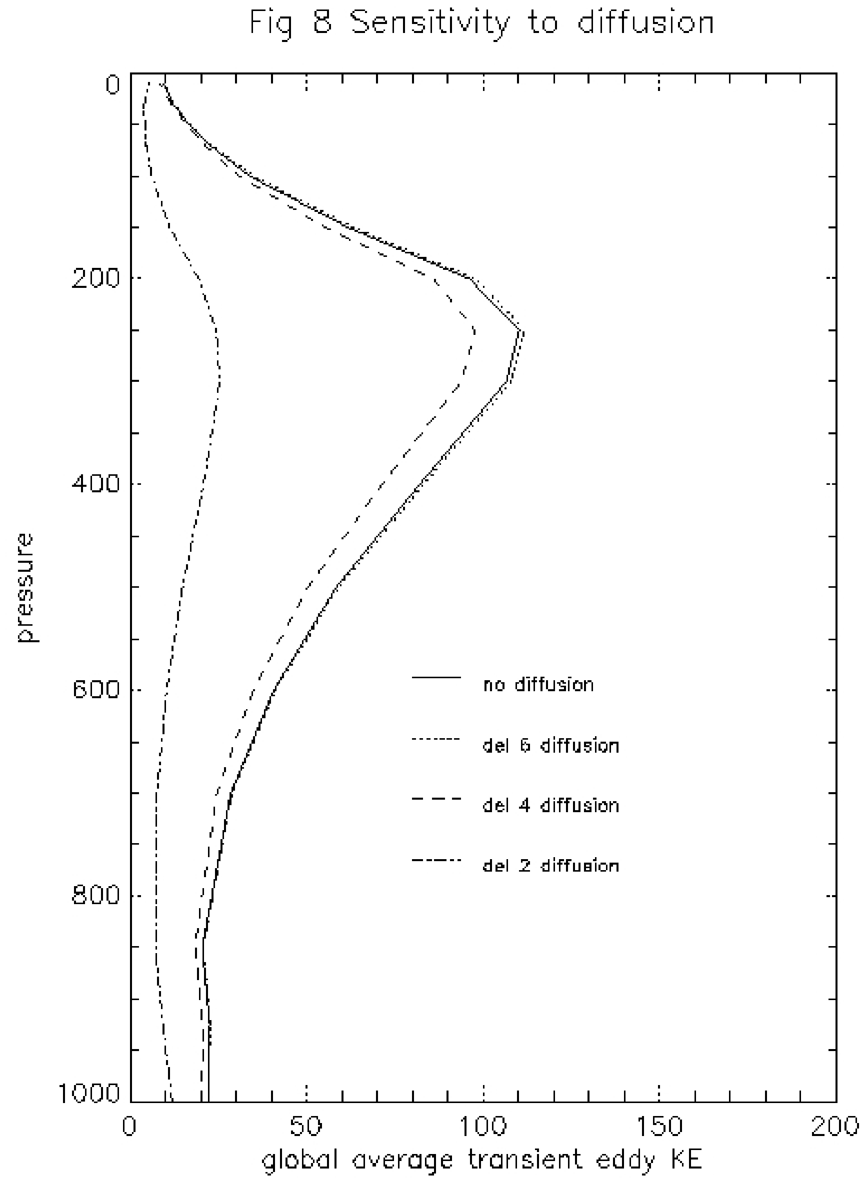


Fig 8 Sensitivity to diffusion
pressure
0
200
400
600
800
1000
0
50
100
150
200
global average transient eddy KE
no diffusion
del 6 diffusion
del 4 diffusion
del 2 diffusion