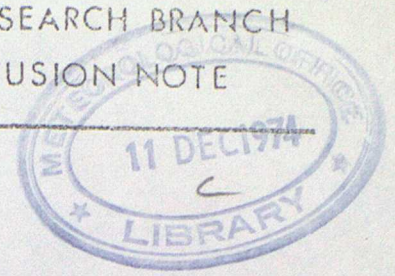


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COMPARISONS AND VERIFICATION OF MODELS FOR THE
EVOLUTION OF THE CONVECTIVE BOUNDARY LAYER.

by

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Summary

Three models for the evolution of the convectively-unstable boundary layer are compared. Two of the models are highly complex numerical models, while the third is a very simple analytical model.

All three models have been tested using the 0600 temperature profile from Day 33 of the Wangara Data as the initial state.

The height and potential temperature of the mixing layer from each of the models are compared between models, and with the observed values.

Introduction

A realistic model of the diurnal evolution of the convective boundary layer is required for the estimation of the dispersion of pollutants in the lower atmosphere. There are many models which produce a realistic time-dependent mixing layer, ranging from simple analytical models, to fully 3-dimensional numerical models. A comparison is made in this work between the simple model proposed by Carson (1973) for the dry, inversion-capped, convective layer, and two numerical models, one a 3-dimensional model (Deardorff, 1974), and also a second-order closure model (Wyngaard, AFGL internal report). All three models were used to simulate the growth of the convective layer on Day 33 of the Wangara Data (Clarke et al, 1971)

The simple thermal model

Neglecting any large scale vertical motions, the simple thermal model of Carson gives

$$h^2(t) = h^2(t_{0,i}) + \frac{2(1+2A)}{\gamma_i} \int_{t_{0,i}}^t \frac{H(0,\tau)}{\rho c_p} d\tau ,$$

where h is the height of the mixing layer, and $H(0,t)$ is the upward surface heat flux at time t . The subscripts refer to the initial temperature profile, which is assumed to be made up of a number of sections, each section having a constant lapse rate. The lapse rate γ_i applies to the section $h_{0,i} \leq z < h_{0,i+1}$ as illustrated in fig. 1, and $t_{0,i}$ is the time at which the height of the mixing layer reaches $h_{0,i}$.

A is an empirical constant relating the heat flux at the top of the mixing layer to that at the bottom, and is taken to be 0.2.

The model was integrated using $H(0,t) = H_{\max} \sin \Omega t$, to give

$$h^2(t) = h^2(t_{0,i}) + \frac{2(1+2A)}{\gamma_i} \frac{(\cos \Omega t_{0,i} - \cos \Omega t)}{\Omega \rho c_p} ; t_{0,i} \leq t \leq t_{0,i+1} . \quad \Omega = \frac{\pi}{T} ,$$
where T is the length of the day. T was taken to be 11 hours, with H_{\max} obtaining at 1300 local time. A value of $0.216 \text{ ms}^{-1} \text{ } ^\circ\text{C}$ was used for $\frac{H_{\max}}{\rho c_p}$; this is the figure obtained by Wyngaard by fitting a sine wave to the surface heat flux results from Deardorff's 3-dimensional model.

During this work it was realised that there is an inconsistency in the simple thermal model. The imposed conditions on the temperature imply that the temperature of the mixing layer, θ_e , must change discontinuously as the top of the mixing layer passes through the discontinuities in the lapse rate, γ . The jump in θ_e can be of the order of 2°C at the large discontinuities in γ .

This arises from the equation for the rate of change of the temperature discontinuity, $\Delta\theta$, at the top of the mixing layer. The equation for the heat flux at the top of the layer gives

$$\frac{d}{dt}(\gamma h) = - \frac{\gamma H(h,t)}{\rho c_p \Delta\theta} , \quad (1)$$

and the equation for the rate of change of temperature in the layer gives

$$\frac{d}{dt}(\gamma h) - \frac{d}{dt}(\Delta\theta) = \frac{H(0,t) - H(h,t)}{\rho c_p h} . \quad (2)$$

Dividing (2) by (1), and using $H(h,t) = -A H(0,t)$ gives

$$\frac{d(\Delta\theta)}{d(\gamma h)} + \left(\frac{1+A}{A}\right) \frac{\Delta\theta}{\gamma h} - 1 = 0 . \quad (3)$$

The general solution of this equation is $\Delta\theta = C(\gamma h)^{-\frac{1+A}{A}} + \frac{A\gamma h}{1+2A}$

where C is an arbitrary constant. Strictly, equation (3) is only valid, for $\gamma = \gamma_i$, in the section $h_{0,i} \leq h < h_{0,i+1}$. Hence if continuity of $\Delta\theta$ is required, C must be chosen in each section to match $\Delta\theta$ across the boundary. For the first

section i.e. $0 = h_{0,1} \leq h < h_{0,2}$, the initial condition is $\Delta\theta = 0$ when $h = 0$ as stated by Carson. Hence the solution of (3) is

$$\Delta\theta = C_i (\gamma_i h)^{-\frac{1+A}{A}} + \frac{A \gamma_i h}{1+2A} \quad \text{for } h_{0,i} \leq h < h_{0,i+1}$$

where

$$C_1 = 0$$

$$C_{i+1} = C_i \left(\frac{\gamma_{i+1}}{\gamma_i} \right)^{\frac{1+A}{A}} + \frac{A h_{0,i+1}}{1+2A} (\gamma_i - \gamma_{i+1}).$$

It is still possible to obtain an analytical solution for the height of the mixing layer, the solution being

$$\left[\frac{1}{2} \frac{A}{1+2A} (\gamma_i z)^2 - A C_i (\gamma_i z)^{-\frac{1}{A}} \right]_{h_{0,i}}^h = \frac{A \gamma_i H_{max}}{\Omega \rho c_p} (\cos \Omega t_{0,i} - \cos \Omega t)$$

where

$$[f(z)]_{h_{0,i}}^h = f(h) - f(h_{0,i})$$

This model was also tested on Day 33 at Wangara.

Results

The resulting heights from all four models are shown in fig. 2.

First compare the simple model as proposed by Carson with the slightly more complex model which keeps $\Delta\theta$ continuous. The main differences between the two results are near the points where γ changes by very large jumps. However, the differences are small, and apart from the region where γ is very small, around $t = 3.5$ hours, the two solutions are very similar indeed.

Furthermore, the results of the simple model compare very favourably with the results from the numerical models. The values obtained for the height of the mixing layer from the simple model are within 50 - 100 m of the numerical model values for virtually the whole of the integration period.

Another parameter available for comparison is the potential temperature of the mixed layer, θ_c . Results from the models and observed temperatures are presented in table 1. The simple model referred to is the modified analytic model, since the original model has discontinuities in θ_c . As can be seen, all three models produce accurate temperatures in the mixed layer, and there is no evidence of the numerical models giving temperatures closer to the observations.

Conclusions

The simple model proposed by Carson seems to be completely adequate for predicting the diurnal evolution of the height of the convective boundary layer. The only information required for the model is the surface heat flux, and the initial temperature profile. The results obtained using this model agree with the more elaborate numerical models to a high degree. The differences involved are of the same order as the error in the estimation of the height of the mixing layer from observations.

The differences between the simple model, and the model with continuous $\Delta\theta$ are also as small as the observational errors. However, if the temperature of the mixing layer is required, then the simple model as proposed originally is not adequate. But if the modification to give a continuous $\Delta\theta$ is used, a very acceptable temperature evolution can be obtained. In the case studied here, this model gave results of at least the same accuracy as the numerical models.

Thus, it appears that for overall parameters, such as the height and temperature of the mixing layer, the simple analytical models predict the evolution as accurately as any of the more complicated models examined here.

References

Carson, D J (1973) "The development of a dry, inversion-capped, convectively unstable boundary layer".

Quart. J. R. Met. Soc. 99 pp 450-467

Clarke, R H et al (1971) "The Wangara Experiment : boundary layer data".

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TABLE 1

Evolution of mixing layer temperature from the models and observation

Time	Deardorff	Wyngaard	Simple Model	Observed
0800	-	-	0.3	-
0900	3.7	3.9	3.6	3.8
1000	7.5	6.6	6.9	-
1100	9.0	8.5	9.4	-
1200	9.9	9.7	9.7	9.7
1300	-	-	10.5	-
1400	11.6	11.5	11.2	-
1500	-	-	11.8	11.8
1600	12.7	12.6	12.3	-
1700	-	-	12.7	-
1800	-	-	12.8	12.7

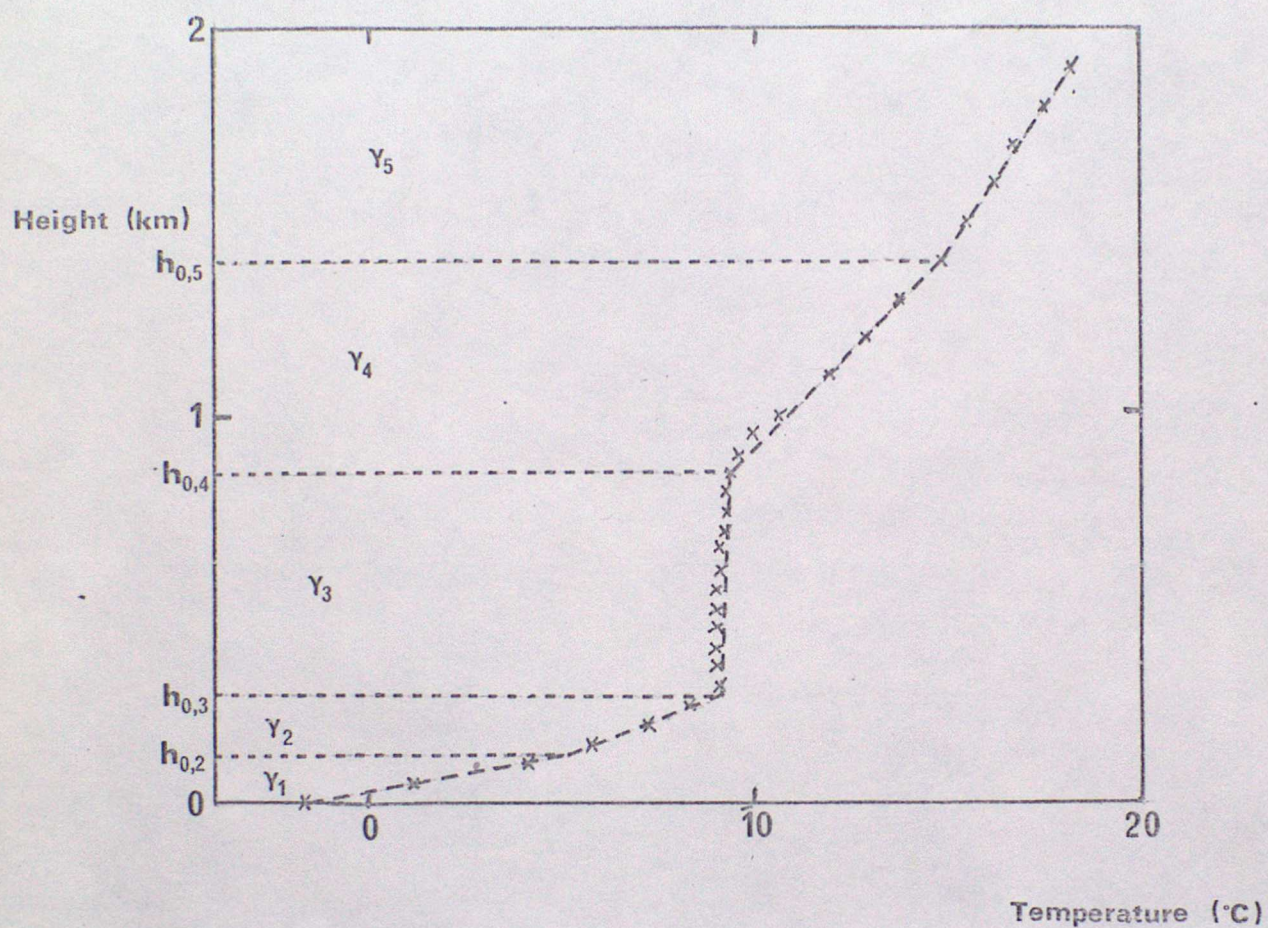


Fig. 1 Potential temperature profile at 0600

- x denotes the observed values
- is the piecewise-linear initial profile for the simple thermal model

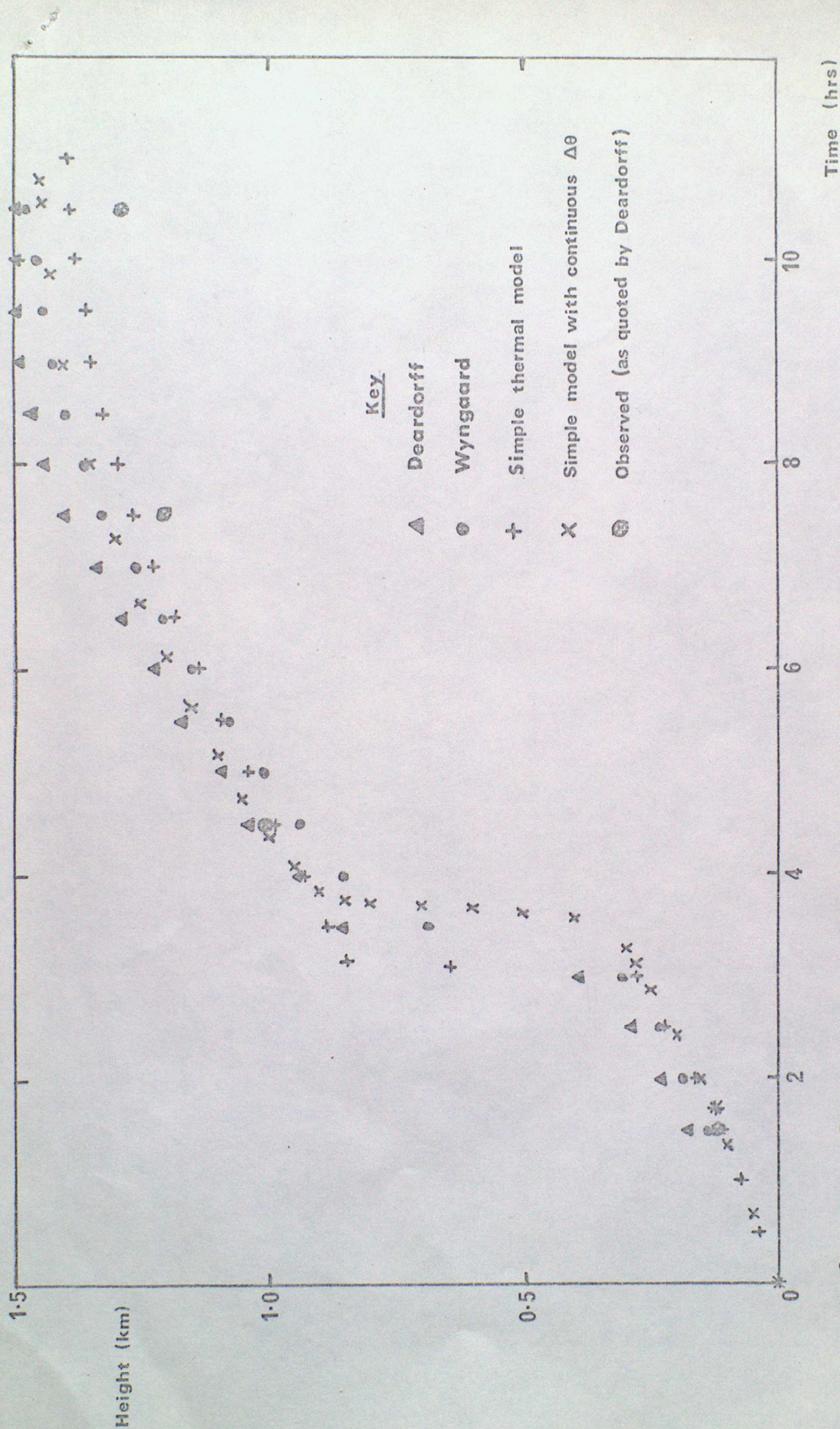


Fig. 2 Evolution of boundary layer height from the models and observation.