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DEPRESSIONS AS VORTICES

By A. H. R. GOLDIE, D.Sc., F.R.S.E.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

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DEPRESSIONS AS VORTICES

§ 1—INTRODUCTION

In a recently published memoir (1)* the writer, from a detailed kinematical study of four depressions and from partial studies of other depressions, reached the following amongst other conclusions :—

1. In a certain region outside the central area, the average surface air motion, notably in the cold air, approximates to a simple (vr=constant) vortex.

2. In the polar part of the central area, on the other hand, there is sometimes, but not always, an approximation of the average speed to direct proportionality to distance from centre (v/r=constant). In the cases examined this was found not to be genuine " solid rotation " ; there was considerable incurvature, and the dynamical conditions for a disc of air spinning like a solid and having a general translational motion were not satisfied.

3. There was evidence that the central polar area, characterized by the approximate v/r=constant type of motion, increases in diameter with the life of the depression.

4. The above were average relationships between speed and distance from isobaric centre. The more detailed analysis of the conditions throughout the central area revealed a cell-like structure with convergence of air, both the cold and the warm, below the two-kilometre level, towards a length of the main front running through the centre of the depression—and thence upward motion. The analogy with a feature of Aitken's model cyclones was indicated, and also the fact that the convergence here in turn explained the vortical nature of the air movement in the outer regions of the depression.

The depressions which were principally studied and to which further reference will be made in the present paper were those of October 1–3, 1929, October 7–9, 1930, October 17–18, 1930, and March 18–19, 1933.

The objects of the present paper are to broaden the basis of that discussion, to show the intimate relation between the depth of a depression and the intensity of its outer vortex, to discover the extent of the relation of that depth to the intensity of the circulation in the upper part of the troposphere, to consider quantitatively whether the connexion between surface and upper air could suffice for the transfer (from above) of the energy found in surface regions of a depression, and to explore the average air movement in the upper part of the troposphere over depressions.

* The numbers in brackets refer to the bibliography on p. 18.

§ 2—A DEPRESSION OF SPECIAL INTENSITY (OCTOBER 26–28, 1936)

Whilst the previous memoir was in the press the opportunity arose of studying another depression which makes a valuable addition to the series, namely one with the characteristics almost of a tornado. Between October 26 and 28, 1936, a deepening depression (which seems to have originated as a secondary to another deep depression at the time when the latter was lying northward of Faroe) travelled rapidly from the west, its centre passing between Shetland and the mainland of Scotland and then moving across Norway and finally north-eastward over the northern part of the Baltic. At first it moved practically on the course of the depression of October 1–3, 1929, but it finished almost on the same course as the depression of October 7–9, 1930. Its rate of progression—18 m./sec. or 40 m.p.h.—was even slightly higher than that of the last-mentioned depression, and the isobars were distinctly rounder in shape than those of either of the depressions just mentioned. The violence attained was also greater—maximum hourly wind speeds of 56, 67 and 67 m.p.h. at Stornoway, Tiree and Bell Rock and maximum gusts of 84, 104 and 94 m.p.h.

The results of analysing this depression after the methods followed in the earlier memoir are shown in Figs. 1, 2 and 3. It was found (see Fig. 1) (a) that the speed

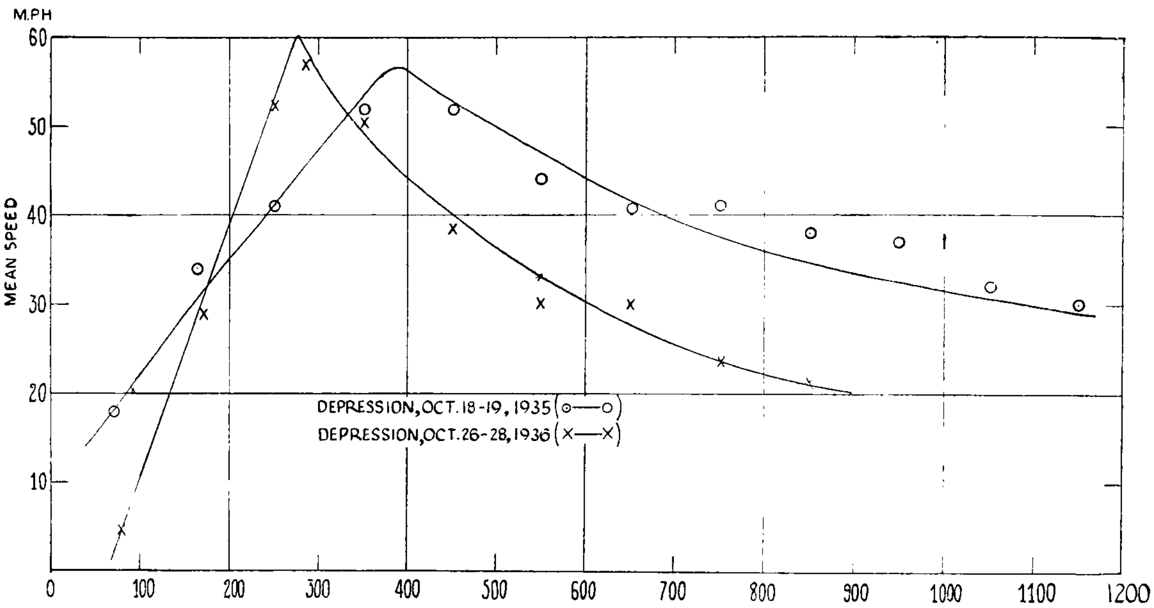

FIG. 1.—MEAN WIND SPEEDS IN THE DEPRESSIONS OF OCTOBER 18-19, 1935, AND OCTOBER 26-28, 1936.

distribution of a simple vortex was maintained up to a closer approach to the isobaric centre than in any of the large primary depressions previously studied ; and (b) that the region between the ring of maximum speed and the centre had certain pretty definite " solid rotational " characteristics. The second conclusion was reached in the following manner. On the diagram of actual winds (Fig. 2, i.e., the one corresponding to Fig. 8 of *Geophysical Memoirs* No. 72) a kinematic centre, 0′, could be determined by inspection ; and on the diagram of wind relative to the centre (Fig. 3, i.e., the one corresponding to Fig. 9 (a) of *Geophysical Memoirs* No. 72) the tornado centre, 0, could also be determined by inspection. The distance 00′ was thus measured as being 230 Km. and by theoretical calculation the distance 0′ 0″ was found to be 80 Km. The position thus given for 0″, the isobaric centre, corresponded closely to that determined empirically by the use of the autographic pressure records.

The rate of rotation of the central area as computed from the distance 00′ and the known rate of progression came out as once in about 23 hours. An estimate of the time of a complete rotation from Fig. 3 gives 22 hours which is in fair agreement. The radius of the part in rotation was about 200 to 250 Km. Thus this depression,

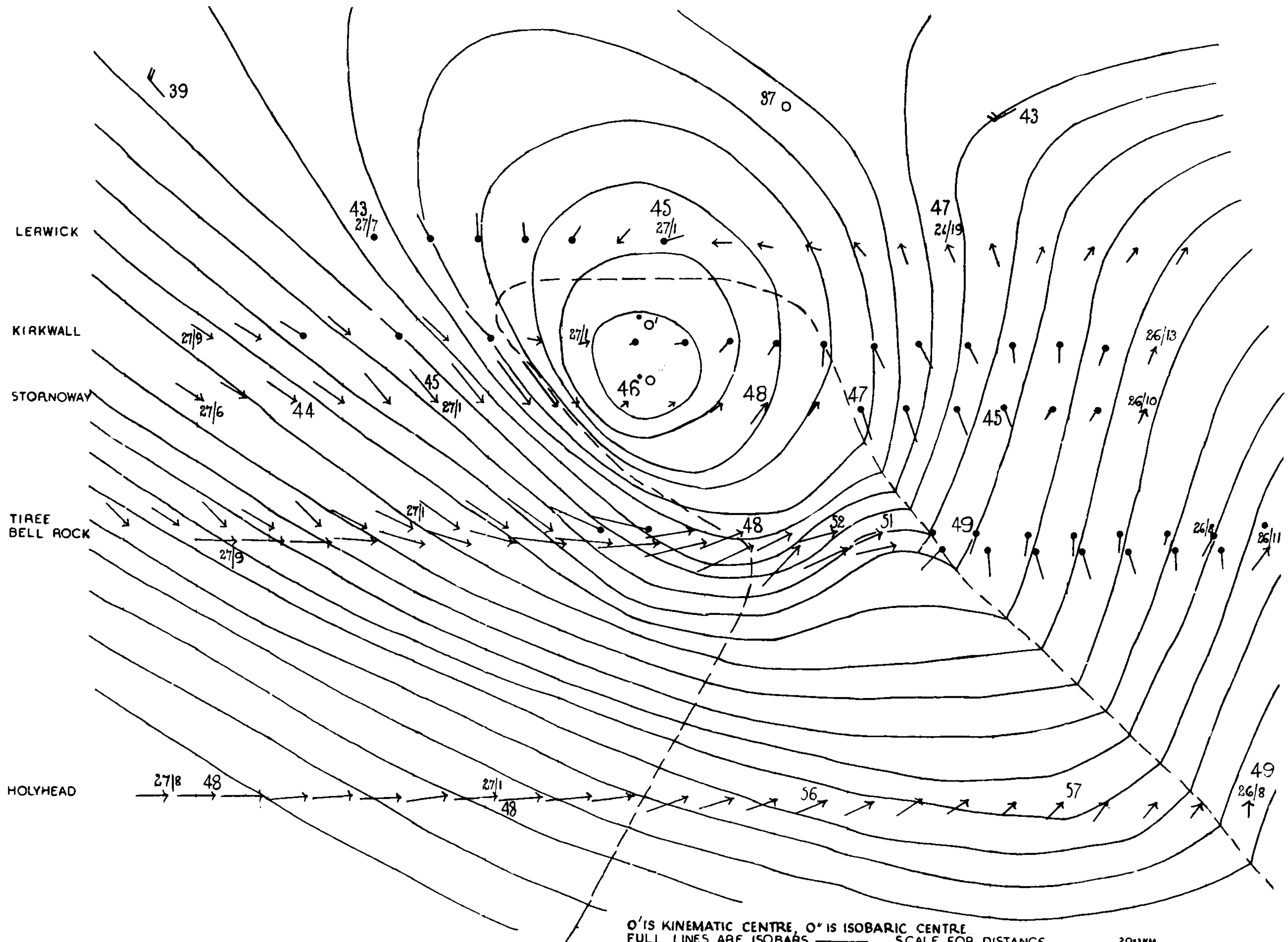

FIG 2—CROSS SECTIONS OF DEPRESSION, OCTOBER 26-27, 1936

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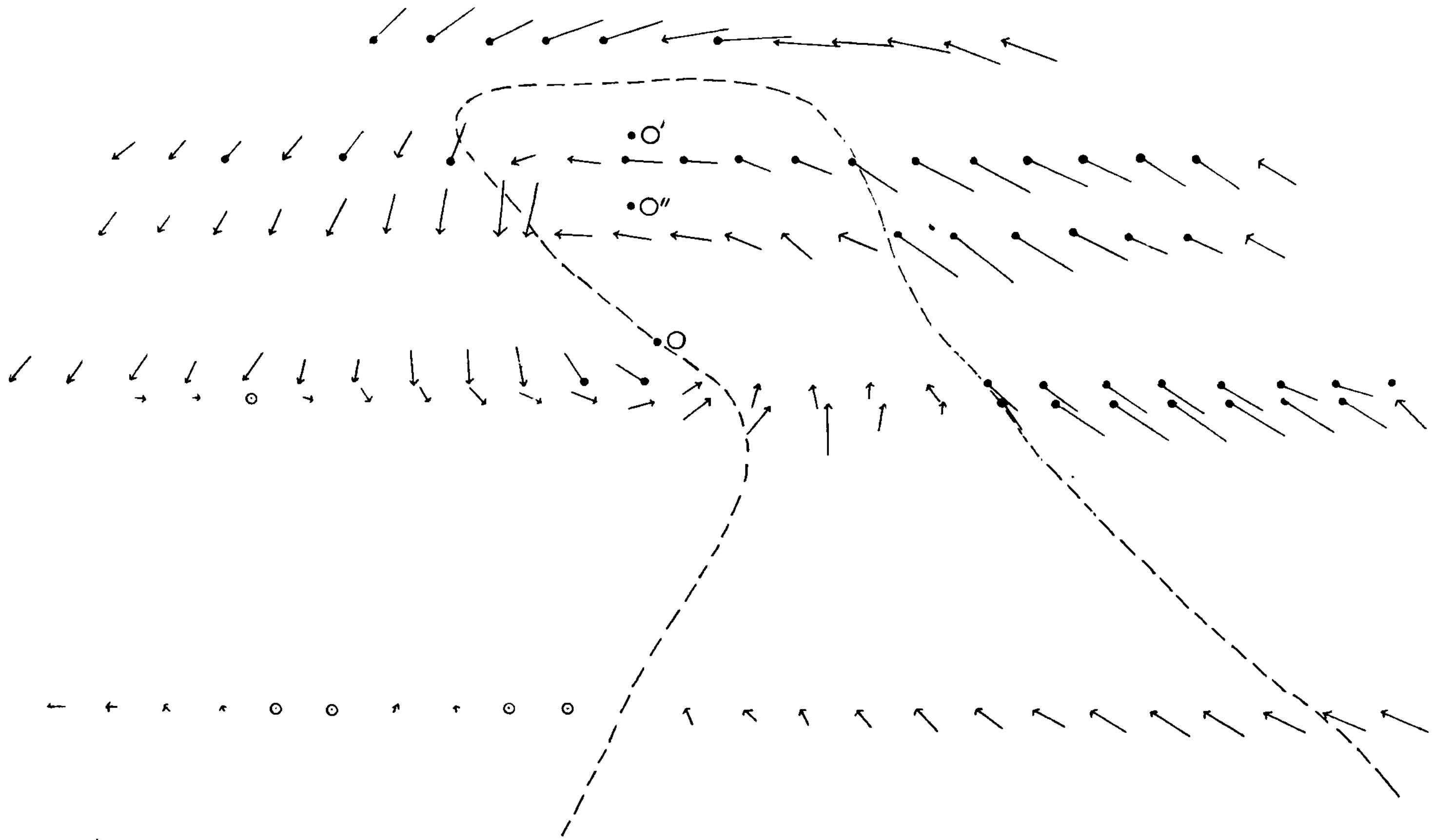


O′ IS KINEMATIC CENTRE, O″ IS ISOBARIC CENTRE, O IS TORNADO CENTRE.
FULL LINES ARE ISOBARS, ——— SCALE FOR DISTANCE, ——200 KM—— BROKEN LINES INDICATE DISCONTINUITIES OF SURFACE AIR MOVEMENT– – – – – – LENGTH OF ARROW IS PROPORTIONATE TO ANEMOMETER SPEED, THUS ——60 MPH——. ARROWS WITH FLECHES ARE ESTIMATES ON BEAUFORT SCALE, THUS ⤡——. TIMES ARE INDICATED THUS 26/11 = 11h ON THE 26TH.
• INDICATES RAIN FALLING. NUMBERS REPRESENT TEMPERATURE IN °F ——THUS 56.

FIG 3 —SURFACE MOVEMENT OF AIR RELATIVE TO ISOBARIC CENTRE, OCTOBER 26-27, 1936 more nearly than others previously studied, approximated to the dynamical conditions for " solid rotation." The approximation to " solid rotation " was, however, somewhat spoilt by the presence of the "fronts", and it cannot be represented as really good, at least at surface level.

Whatever may be said of the central region it is to be noted that the winds in the outer regions of this and the other depressions, when considered in relation to distance from the isobaric centre, give quite good representations of simple vortices, and thus the speed developed in each air mass (e.g., old cold air in front or new cold air in rear) in the outer regions can be very well explained on the simple vortex basis as the result of convergence towards an inner area around the isobaric centre of the depression.

The behaviour of the air within the central area of a depression is complex, and it is pretty obvious that it must, and does, vary during the life of the depression. It is certain that in the early stages there is in the lower layers strong horizontal convergence, balanced or more than balanced by considerable upward movement. It is also pretty certain—from considerations of weather as well as of continuity—that in the final stage when the depression is filling up there is little or no upward movement. As regards the intermediate stages, it is also the case that the general effect of turbulence would be to bring the movement towards symmetry and a minimum of relative motion between the various parts, and, in fact, that the closer the approximation towards solid rotation the less would be the subsequent rate of dissipation of energy by turbulence and thus the greater the expectation of life for the depression.

Two further depressions, December 27–28, 1930, and October 18–19, 1935, were also examined in some detail. The mean speeds in the latter are shown in Fig. 1, but need not be separately discussed.

§ 3—GENERAL FEATURES OF SIX DEPRESSIONS

The broad features of the five October depressions and the December depression, which have now been examined, are shown together—in somewhat generalised form—in Fig. 4. A feature common to all is the existence of the outer vortex. Beyond a radius of some 400–500 Km. from the centre the outer vortices did not differ much from one another in intensity, the vr products being roughly of the same order. Five of the six had in common the feature that the radius just mentioned was the inner limit of their outer vortices—at least at the time when the depressions

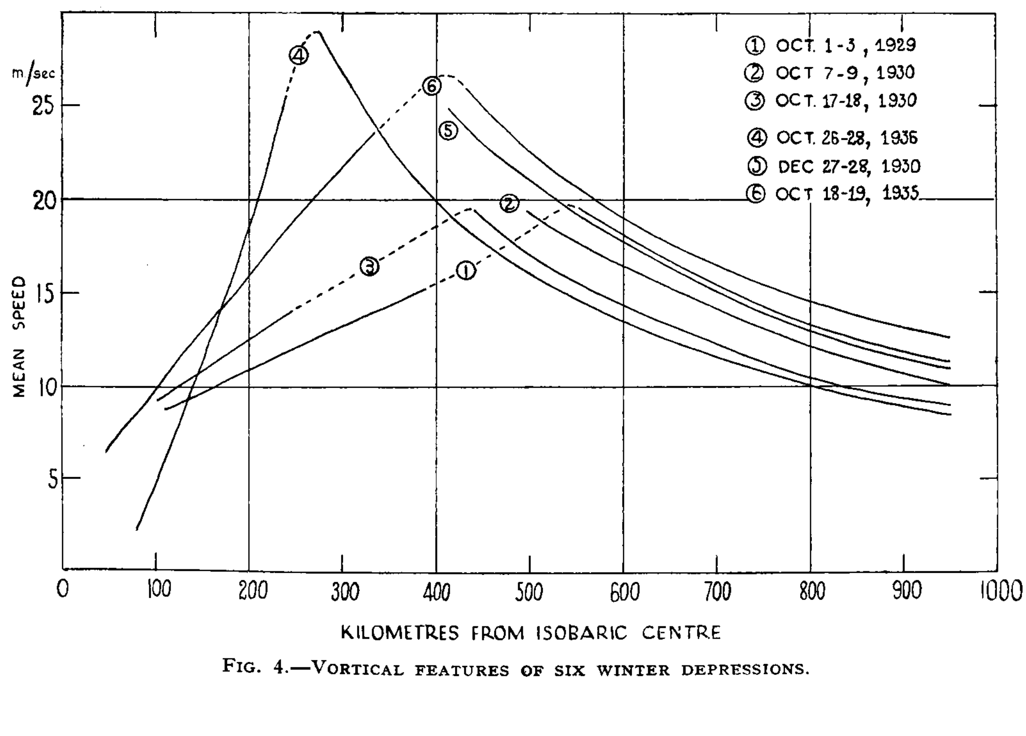


FIG. 4.—VORTICAL FEATURES OF SIX WINTER DEPRESSIONS.

crossed the British Isles. The highest speeds existing in three of these depressions were practically identical—hourly mean speeds of 19 or 20 m./sec. The lowest pressures near the centre of these three depressions were respectively 970, 975 developing to 965, and 980 developing to 970 mb. But in the depression of October 26–28, 1936, the vortex, though—or possibly because—its vr product was less than in any of the others, was maintained at least up to within 280 Km. of the isobaric centre, with a corresponding maximum hourly mean wind speed of about 29 m./sec. The pressure near the centre of this depression, namely 961 mb., was also lower than the minimum pressures in the three other depressions, at least at the time when they crossed the British Isles. It would obviously be of great practical value if some reason for this superdevelopment could be discovered. The two remaining depressions showed an intermediate degree of development in regard to speed, but had even lower pressures at their centres.

The first of the depressions (October 1–3, 1929) at the time of study was a dying depression, had become nearly stationary, and in its central area had v proportional to r with, however, still some convergence and evidences of cellular structure.

The second (October 7–9, 1930) was a very active depression, with no definite evidence of v proportional to r in the central area, but instead the cellular structure in strongly marked degree with every evidence of convergence and upward movement. It deepened by at least 10 mb. in 24 hours and travelled a long way; the wind arrows on a subsequent synoptic chart, namely at 7 h. on the 9th when the centre was over Sweden, afford some evidence that a v proportional to r condition may have been established by that time.

The third depression (October 17–18, 1930) was of secondary type, and it was possible to study its life history to some extent (*Geophysical Memoirs* No. 72, § 8). In its case there was almost from the beginning an approximation to v proportional to r in the inner region; also the inner region grew and the speed in the zone of maximum speed rose for a time. After that the depression passed to northward of Scotland and—so far as could be determined from the synoptic charts—filled up.

In the case of the fourth depression—the tornado-like storm of October 26–28, 1936—it is not possible to say with certainty how deep it was when the centre was on the Atlantic. So far as can be determined, however, it arrived almost fully developed, attained its greatest depth when crossing the British Isles, and afterwards filled up pretty rapidly, showing greatly diminished violence when over Sweden and the Baltic. Its greater violence was associated with the feature that the vortex was developed up to some 280 Km. from the centre, as against 450 to 550 Km. in the cases previously examined.

The fifth depression (December 27–28, 1930) had attained its maximum depth and greatest intensity by the time it reached our Islands. It was already beginning to fill up and was travelling very slowly.

In the sixth case (October 18–19, 1935) the depression was approximately at its maximum intensity; it afterwards slowly filled up with gradually diminishing rate of travel, but travelled a long way.

§ 4—RELATION BETWEEN DEPTH OF A DEPRESSION AND INTENSITY OF OUTER VORTEX

The examination of depressions by the comprehensive methods applied to those discussed above is laborious, and in only relatively few cases are sufficient data available. The obvious next step, assuming the outer regions to conform approximately to vortical structure, would be to determine the intensity of the vortex in as many cases as possible and to see what relation, if any, the vortex showed to other features of the depression.

In the previous memoir (pp. 8 and 15) reference was made to the fact that in two October depressions of about the same depth, as measured by the lowest pressures on the synoptic charts, but otherwise not conspicuously similar, the outer vortices were found to be of closely similar intensity. It was pointed out that this result was in accord with a mathematical deduction of Brunt, who, in amplifying Rayleigh's theoretical discussion on rotating fluid, noted that the intensity of the vortex added by removal of fluid from the region of the axis should, other things being equal, be proportional to the amount of fluid removed. Granting that in the atmosphere, owing to the diversity of initial conditions, the lowest pressure attained in a depression can be regarded as only very roughly an indication of the " amount of fluid removed " in the initial formation of the depression, it seemed worth determining the vortical intensities in a sufficient number of cases to examine this question. Accordingly the details were worked out for twenty cases, the additional cases being selected from different years and seasons, but being analysed only to the extent necessary to get an estimate of the vortical intensity of the outer region and the minimum pressure at the centre. The results are set out in Table I and Fig. 5. Actually the agreement with theory is remarkably good in eighteen of the twenty cases. The two remaining cases, which diverge from the others, were somewhat

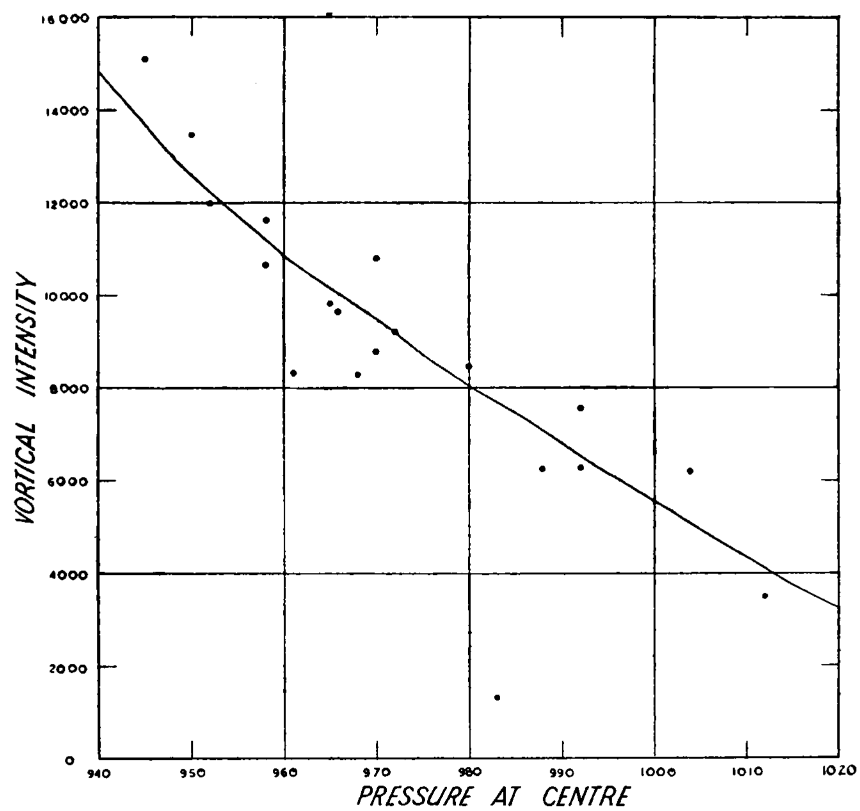


FIG. 5.—VORTICAL INTENSITY (OF 20 DEPRESSIONS) IN RELATION TO PRESSURE AT CENTRE.

complex ones, with several centres showing on the synoptic charts. They occurred in February and March respectively, months when complex synoptic systems, very often in polar air, tend to predominate It is clear that with the great diversity in initial pressure conditions which characterizes polar air, and with vortices very probably superposed on successive vortices, consistent relationships between vortical development and pressure at the nearest centre can not be expected. However, even including these exceptional cases, the correlation co-efficient between vortical intensity and lowest pressure in the twenty cases taken together comes out at $-0 \cdot 88$ with a standard error of $0 \cdot 05$; and if the two exceptional cases are excluded the correlation is $-0 \cdot 93$, with a standard error of $0 \cdot 03$.

The association of the intensity of the outer vortex in an intimate degree with the depth of the depression is thus established. The question next arises as to what pre-existing conditions might determine either depth of depression or intensity of vortex.

TABLE I—ROTATIONAL FEATURES OF 20 DEPRESSIONS

No.	Date	Speed of Centre	Pressure at Centre	Maximum Wind Speed (hourly Mean) (v)	Distance from Centre (r)	Vortical Intensity	Maximum $\frac{v}{r}$
		m./sec.	mb.	m./sec.	Km.		
1	1929 Oct. 1– 3	5	970	19·6	550	10,800	·036
2	1930 Oct. 7– 9	16	965	19·5	500	9,800	·039
3	1930 Oct. 17–18	18	970	19·5	450	8,800	·043
4	1936 Oct. 26–28	18	961	29·0	280	8,190	·100
5	1930 Dec. 27–28	5	958	25·0	420	10,700	·060
6	1935 Oct. 18–19	15	952	26·6	450	12,000	·059
7	1933 Mar. 18–19	7	983	18·0	70	1,310	·026
8	1936 Apr. 21–22	24	992	18	360	6,300	·050
9	1936 June 1– 3	4	1004	19	320	6,100	·059
10	1936 July 18–19	6	992	20	380	7,600	·053
11	1936 July 23–25	4	980	18	470	8,460	·038
12	1936 Aug. 2– 3	9	988	13	480	6,240	·027
13	1936 Aug. 5– 6	9	1012	13	270	3,510	·048
14	1936 Jan. 5– 6	7	968	26	320	8,320	·081
15	1936 Jan. 8–10	22	958	27	430	11,610	·063
16	1935 Jan. 9–12	18	972	20	460	9,200	·043
17	1935 Jan. 25–26	14	950	27	500	13,500	·054
18	1935 Feb. 2– 3	26	945	24	650	15,100	·037
19	1935 Feb. 16–17	24	965	23	700	16,100	·033
20	1935 Apr. 10–11	8	966	24	400	9,600	·060
	Mean Values	13	973	21	420	9,170	·050

§ 5—RELATION BETWEEN DEPTH OF A DEPRESSION AND THE PRESSURE GRADIENT IN THE UPPER TROPOSPHERE

In *Geophysical Memoirs* No. 72 it was concluded from qualitative considerations that a dual causation of depressions was indicated, in which the energy was drawn mainly from the high speed of the warm air above the main polar front. The quantitative aspect of this point is open to examination, though somewhat indirectly, on a statistical basis.

Other things being equal, the speed of the warm air at high levels in the region above a principal front depends on the difference of temperature between the warm and the cold air masses, this difference being the chief element in leading to the increase with height of the pressure gradient for westerly winds. Upper air temperatures obtained almost daily from two or three stations in the British Isles and from a number of stations on the continent are published in the Upper Air Section of the *Daily Weather Report*. From these ascents it is very often possible to determine the temperatures, at various isobaric levels up to the 500 mb. surface or beyond, of the warm and cold air masses entering into particular depressions. Taking in this way the mean temperature difference on the 700, 600 and 500 mb. surfaces as a measure of the general temperature difference between the air masses, some 38 cases were got out where this temperature difference could be obtained, and the lowest pressure approximately in the associated depression was also noted from the synoptic charts of the International Section of the *Daily Weather Report*. The correlation between temperature difference and lowest pressure is represented by a co-efficient of $-0 \cdot 76$ with a standard error of $0 \cdot 07$; the results are shown in a dot diagram in Fig. 6. Thus there is a fairly close correlation between depth of a depression and temperature difference between the constituent air masses; that is in effect between depth and the increase of pressure gradient in the upper levels of the troposphere.

The correlation may be taken to mean that when a depression is formed at least half the controlling cause of the depth of the depression is the air speed in the upper levels of the troposphere, but equally it could be interpreted as meaning that the depth is determined by the potential energy associated with the juxtaposition of the cold and warm air masses. One objection to the second interpretation is that the greatest

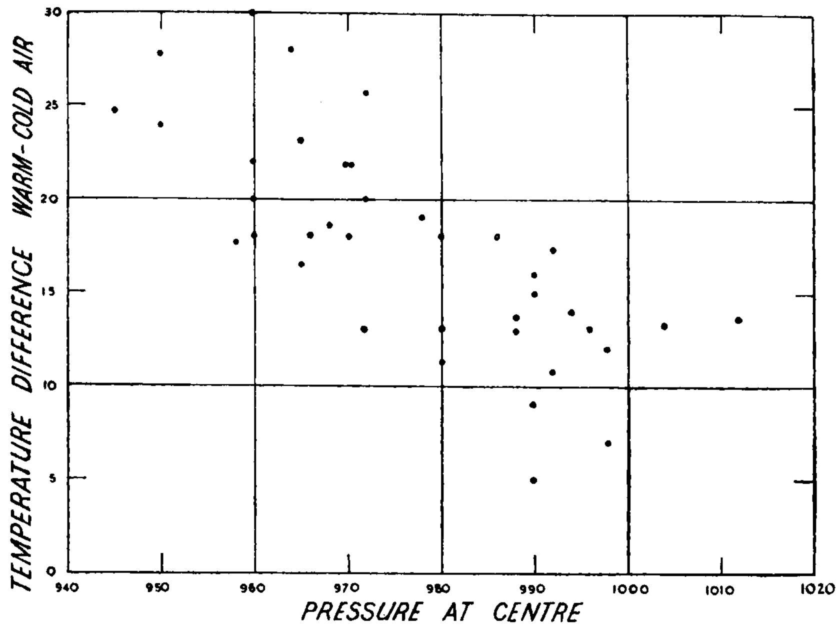


FIG. 6.—PRESSURE AT CENTRE (OF 38 DEPRESSIONS) IN RELATION TO TEMPERATURE DIFFERENCE BETWEEN WARM AND COLD AIR.

depth and the maximum kinetic energy of the depression in surface levels are certainly attained before most of the potential energy is made available by a re-arrangement of masses in which the cold comes mainly under the warm instead of lying mainly alongside the warm. In the final occluding stage, when decay has started, there is an approximation to such a process, and in the previous memoir it was, in fact, shown that even part of the already developed kinetic energy of the surface layers of the outer vortex is given back at this stage (at the time when the kinetic energy of the system as a whole is diminishing) in helping to push up into higher levels a relatively cool inner core ; it was also shown in the previous memoir how part of the kinetic energy of a depression may be expended in the formation of an anticyclone, i.e., in effect transformed into potential energy. Another and more serious objection is that the circulation in the higher levels of the troposphere, even when a depression is in existence, is much more vigorous than in surface levels. This is demonstrated later in this paper by the diagrams in §8 and §9 ; yet it is particularly the surface levels that should benefit from any transformation of potential to kinetic energy.

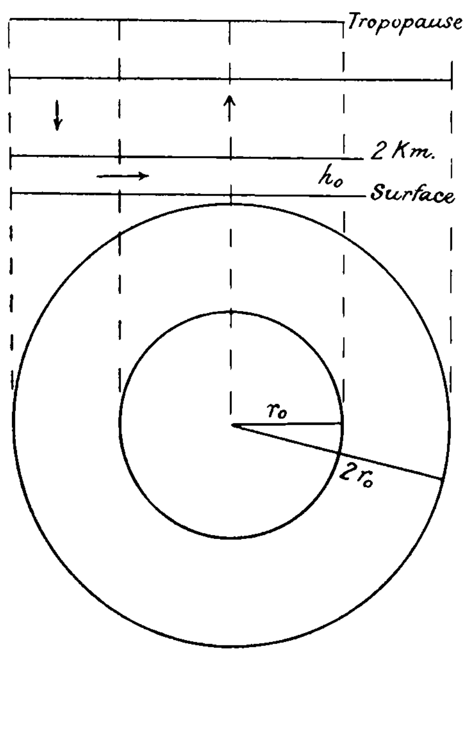
These apparently insurmountable objections to the second interpretation reinforce the claims of the alternative interpretation, i.e., that it is the increase in the speed of the air in the region above a principal front in the higher levels of the troposphere which controls the depth of the depression, or in effect that depressions occur essentially where there is high upper air speed and some degree of " shear " in the planetary circulation, and that the extent to which it is possible for surface barometric pressure to be reduced is controlled by—amongst other things—this upper air speed or the increase of the speed with height (i.e., the shear).

§ 6—THE TRANSFER OF ENERGY FROM UPPER TO SURFACE LEVELS

The other chief factor in the immediate initiation of depressions has (in the previous memoir) been inferred to be what might be called convectional connexion between the surface air and the upper levels where the horizontal speed is high. This point can be considered qualitatively on the following lines. The intensity of a depression may vary considerably from time to time, and the most obvious source of this variation lies not in any corresponding fluctuation of the upper air circulation, for that is unlikely, but in possible variations of the convectional connexion between the surface air and the upper air. Can it be said that the greater the convectional connexion the more active the depression ? The answer from general considerations seems to be yes. Other things being equal, the convection depends on the heat and moisture available to the surface layer. In winter, when the oceans are relatively warm, deep depressions originate almost invariably only over the oceans, and they decrease in intensity when they pass over large land areas or even inland waters. The greatest possible convectional disturbances in winter occur where cold air currents from a continental area blow out over the ocean, and these are precisely the places where depressions originate or deepen.

In summer in temperate latitudes there is no exactly corresponding condition. In the first place the high speed in the upper levels is less commonly present ; in the second place convectional disturbances over land are in general more localised and heat only, without water vapour, is available to cause them. The result shows itself in local thunderstorms and shallow ill-defined low pressure areas with no definite translational movement ; and the result is in agreement with the contention of Jeffreys (2), who has pointed out that any reduction of surface pressure from local heating alone would be of too small an order to account for the formation of the cyclones of temperate latitudes. The normal winter case, however, is different —i.e., there is a strong increase of wind speed with height. In that case the descending air in the surroundings of any region of upward movement, though of the same order of mass as that ascending in the centre, is bringing down from high levels a kinetic energy contribution far in excess of that which went up. Wherever this descending air is mixed with surface air which previously had little or no movement of its own, there will be increased speeds and a disturbance of the previously existing geostrophic balance. Given some initial motion relative to the centre there is, at once, as Aitken demonstrated in his experiments, the possibility of establishing a much greater reduction of pressure at the centre than in the case where relative calm exists initially all round ; because, where there is initially some motion, convergence towards a centre is hampered by the conservation of angular momentum.

Starting with known facts about rainfall, it can also be shown quantitatively that the convectional connexion between surface and the upper levels of the troposphere is of the right order to enable the required amount of kinetic energy to be transferred downwards. In *Geophysical Memoirs* No. 69, p. 14, it is stated that rates of rainfall up to 5 or 7 mm./hr. occur in warm fronts in cases where no orographic effect enters. From other information given in the same place it can be calculated that this corresponds to upward air movement at the rate of about 0·2 m./sec. Assume that in the initial state of a depression there is cell action in which within a radius r_0 (small initially) of the centre there is upward motion at the average rate of only half of this—i.e., at 0·1 m./sec. The corresponding downward motion from the upper levels is assumed to take place in an annular zone of inner radius r_0 and outer radius $2r_0$. Thus the average rate of downward motion of air of the same (unit) density as that which went up is $\frac{1}{3} \times 0\cdot 1$ m./sec. ; so that each hour the momentum of a layer of air of unit density and of thickness 120 metres is brought downwards from the upper levels of the troposphere. Therefore when the process is fairly started the lowest (say) 2 Km. of air in the outer annular zone loses the equivalent of



120 metres of air per hour by convergence towards the centre and gains 120 metres of air per hour from above, the air gained having a momentum corresponding to the speed in the highest levels of the troposphere. With a west-east air speed of 60 m.p.h. in these highest levels, the surface air in the outer annulus of the depression would gain west-east momentum at the rate of 3 or 4 m.p.h. per hour. Thus in half a day, so far as air speed in the lower levels is concerned, a small depression with a fair translational movement could be established. No account is taken here of the contribution to the energy of the whole system by the energy set free by condensation. The only assumption on this point is that initially, in a region around the centre, the surface air has become sufficiently humid and warm for continued ascent to take place to the upper levels of the troposphere.

Barometric pressure in the central area will fall only in so far as the air passing upward and thence carried away in upper levels exceeds the air converging below the 2 Km. level. If the lower air in the outer annulus has initially no moment of momentum relative to the centre, convergence will take place almost unrestrictedly, and there will be no appreciable fall of pressure in the inner area. But so soon as the outer zone acquires momentum the angular moment will be conserved during convergence, and there will then no longer be unrestricted convergence ; the inner area, on balance, will then lose air by reason of the ascent if the ascent continues. From this point the presence of water vapour in the ascending mass may well control further development. By way of giving an idea of the quantities involved it may be added that an unbalanced component of 0·04 m./sec. in the rate of ascent (i.e., not balanced by convergence at the surface) would lead in 48 hours to a fall of pressure of 30 mb. at the centre and 10 mb. at the periphery of an area 500 Km. in radius. This assumes the existence in the upper levels of the troposphere of a wind speed sufficiently in excess of surface wind speed to carry away all the rising air from the region of the ascent.

Another point which requires consideration is whether the flux of kinetic energy per hour from higher to lower levels as a result of the mechanism here portrayed, could suffice to maintain the depression when it reaches its full development and enters upon the final phase.

According to *Geophysical Memoirs* No. 72, footnote to p. 37, a typical October case at this stage would be one with an air speed of 20 m./sec. at the periphery of the inner core (500 Km. in radius), with convergence on the average at an angle of about 25° to the isobars round half the periphery, below the 2 Km. level. The equation of continuity—if no further development is by this time in progress—requires that at the 2 Km. level there should be a mean upward movement at the rate of 0·036 m./sec. The work required to lift the whole inner core at this rate, when the mean temperature has become 7° C. lower than the environment (roughly the average of W. H. Dines' results for old cyclones), is found to be approximately half the total inward flux of energy in the case specified. The question to be settled now is whether this inward flux of energy is of the same order as that brought down per hour to the outer annulus from higher levels. The inward flux per unit time is :

$$\frac{1}{2}\rho_0 v_0^2 \,.\, 2\pi r_0 \,.\, h_0 \,.\, v_0 \sin 25°$$

where ρ_0 is density, v_0 speed in the lowest 2 Km. and $h_0 = 2$ Km.

The downward flux of energy per hour when the rate of ascent in the inner core is 0·036 m./sec., is :

$$\frac{1}{2}\rho_0 V^2 \,.\, 3\pi r_0^2 \,.\, 40$$

where V is the speed in higher levels. If these two expressions are compared it is seen that V must be equal to or greater than $\sqrt{2}\, v_0$ i.e., 28 m./sec. or 63 m.p.h.

This would actually be a very moderate speed for the upper levels of the troposphere in October. At times when depressions form—of the intensity discussed —the speed is more likely to average 100 m.p.h. from 6 to 9 Km. (*Geophysical Memoirs* No. 72, p. 28).

A comment requires to be made also on the loss of kinetic energy suffered by the upper levels of the troposphere whilst a depression is being developed. To establish a depression of the type dealt with above, by energy transferred from the 6–9 Km. level of the troposphere, would involve a reduction in the mean speed everywhere above the depression to 93 m.p.h. if in the absence of the depression it would have been 100 m.p.h., or to 62 m.p.h. if in the absence of the depression it would have been 80 m.p.h. The maximum reduction should be in the centre area and especially to northward and north-eastward of the centre where the admixture of air ascended from the surface is greatest. (See §8, deduction 5.)

Thus all the quantities involved appear to be of about the right order, and there seems to be no reason why a depression once started should not continue in existence so long as there is adequate upper air speed in higher levels, and so long as no essential change, as for example occlusion or passing over a land surface, takes place in the surface levels.

§ 7—ROTATION AND OCCLUSION

For the 20 depressions examined the maximum wind speed divided by the distance from the centre was determined. This gives an approximate idea of the rate at which the inner core should rotate if and when a v/r = constant condition is reached. The mean maximum values obtained from the 20 depressions give a mean speed of 21 m./sec. at a mean distance of 420 Km. from the isobaric centres, or a rate of one complete rotation in 35 hours. The range of the individual cases is from about half to about double this mean rate of rotation.

The average speed of translation of these 20 depressions was 13 m./sec. Calculation on the basis of these mean values shows that in the average case the cold front (extending from the centre up to 420 Km. distance) would normally have a chance to rotate 180°—and this would generally involve occlusion up to the same distance from the centre—in about 45 hours from the time at which (apparent) " solid rotation " set in in the inner polar core. A more rapidly moving depression could take considerably longer to reach the completely occluded stage ; a stationary depression, if it could have the same structure, would reach occlusion in about eighteen hours.

Physical reasoning points to the same general result. If the depression draws its kinetic energy and its west-east momentum from the upper levels, then diminution of the convectional connexion between surface and the upper levels means both a decay of the depression and a falling off in the translational movement.

§ 8—AIR MOVEMENT IN THE UPPER LEVELS OF THE TROPOSPHERE

The whole results point to the importance of air movement in the upper levels. Various attempts, notably one by Douglas (3) have been made to explore this, using cloud observations. Douglas worked out diagrams showing the direction of movement of cirrus above depressions with open warm sectors and in the " dying " stage.

It has never, on existing data, been possible to explore this matter in an entirely satisfactory manner, but better and more ample data are now available in the nephoscope observations as printed in the Upper Air Supplement to the *Daily Weather Report*. Speeds of high cloud are there computed for an average height of 8 Km. (5 mi.) for cirro type cloud and 5 Km. (3 mi.) for alto type cloud. From these observations the writer has considered it worth the trouble to compile the diagrams, Figs. 7, 8 and 9, showing air movement above depressions in winter, early summer, and late summer. The method used has been as follows :—In each case where a depression existed whose centre and direction of motion were determinable, all the cirro cloud observations and most of the alto cloud observations within a radius of about 900 Km. from the isobaric centre 0″ were plotted on a large composite diagram, the arrows showing direction and speed of the cloud movement being plotted with reference to the direction of motion of the depression—i.e., in effect the result was the same as if cloud movements in each depression had been plotted on a preliminary diagram according to actual direction, and this diagram had then been rotated bodily to bring the direction of movement of the depression into the west-east direction before this diagram was added to the composite diagram. The periods covered were : for winter, October, 1930–March, 1931, October–December, 1935, and October, 1936–March, 1937 ; for early summer, April–June, 1935 and 1936 ; for late summer, July–September, 1935 and 1936. The first composite diagrams prepared in this way for winter, early summer, and late summer, contained respectively 170, 170, and 130 observations approximately. For simplicity the numbers of arrows were then reduced by collecting adjacent ones in groups mostly of four or five (but sometimes three or six), and replacing each group by one arrow indicating the average speed and direction. These average arrows are the ones actually shown on Figs. 7, 8 and 9. The diagrams have a scale of approximately 300 Km. to one inch.

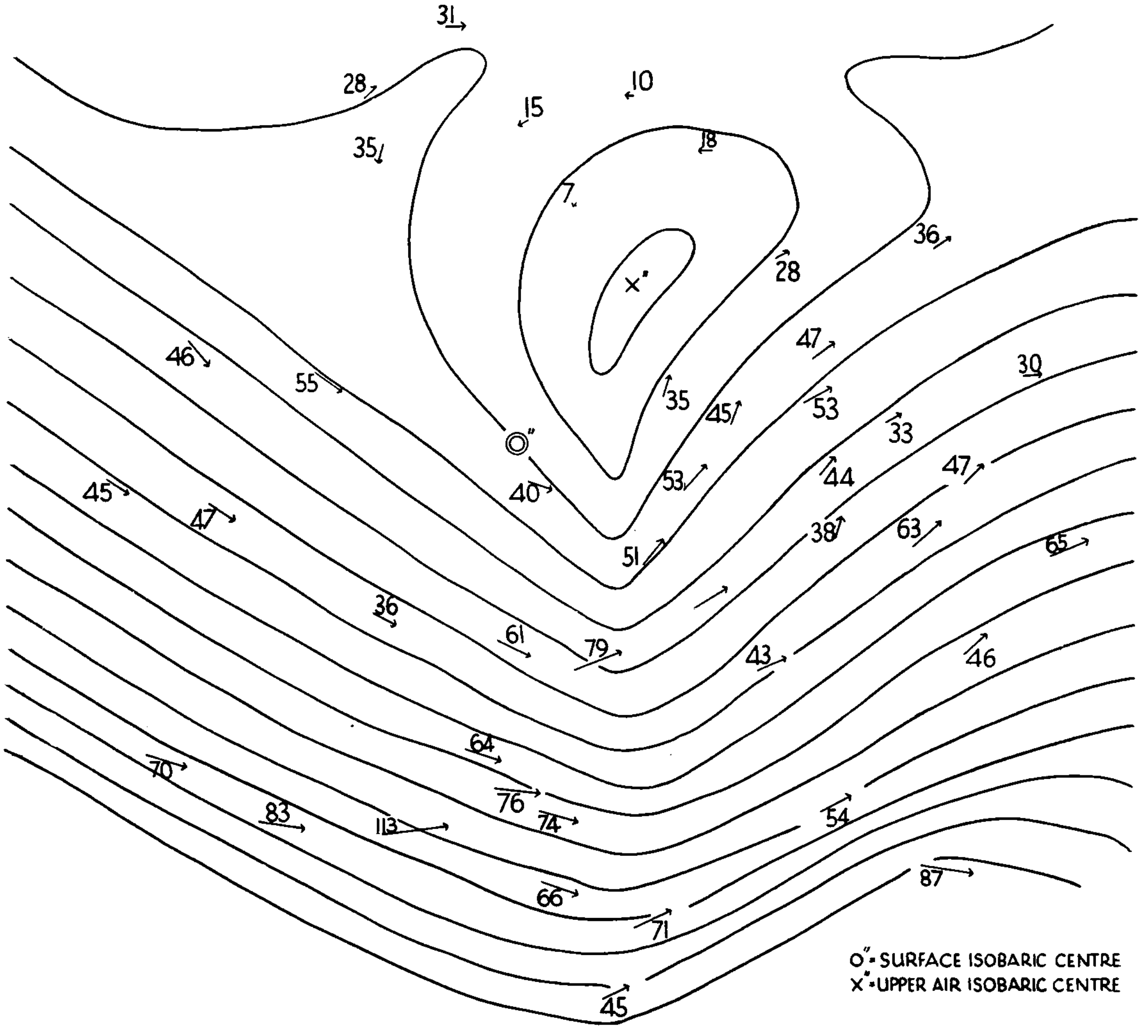


FIG. 7.—UPPER AIR MOVEMENT ABOVE DEPRESSIONS—WINTER (OCTOBER TO MARCH).

Next, isobars were drawn in to fit, so far as possible, the motion shown by the arrows, the distance apart of these isobars in the various parts of the diagrams being inversely proportioned to the speeds indicated by the arrows grouped into still larger areas. (The pressure step between the isobars was not computed). The upper air isobaric centre, thus determined, is marked as X".

Any number of observations are available for the southern halves of depressions, and there cannot be any doubt that the diagrams represent a satisfactory picture of air flow in the southern halves at an average height of about 7 Km.

The periods covered were chosen deliberately as being periods in which many depressions came unusually far south ; and in the winter diagram the results should be fairly satisfactory for the north side also ; but in the two summer diagrams there are too few observations on the north side, and a rather high proportion of these observations are of alto rather than cirro types. There is a double reason for this: near the centre, on the north side, alto cloud would in any case predominate; there are also very few observing stations in the northern part of the country, whilst very few depressions—especially in summer—pass on tracks sufficiently far south for any station to have an opportunity to observe cirrus in the northern half of the system. There is another difficulty—most of the depressions which do come far south are secondary to other depressions in the north, and there is the problem of deciding whether any particular cirrus observation some distance on the north side may not more properly belong to the northern low pressure system.

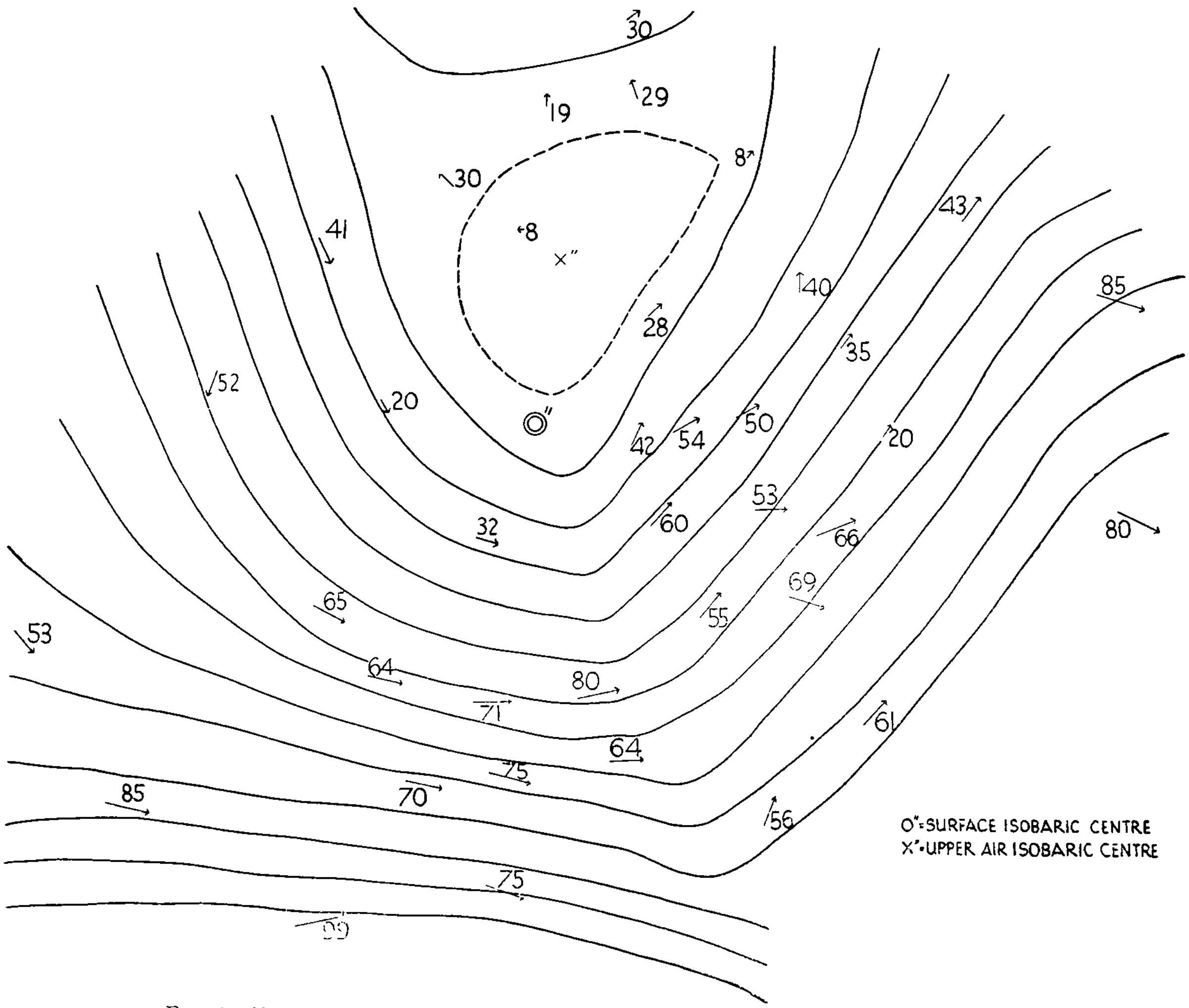


FIG. 8.—UPPER AIR MOVEMENT ABOVE DEPRESSIONS—EARLY SUMMER (APRIL TO JUNE).

Subject to these qualifications, the following deductions may be made :—

1. The three diagrams agree in indicating a trough of low pressure in the upper levels some distance ahead of the (surface centre of low pressure. The separation is more marked in winter, when this trough is about 150 Km. ahead of the surface centre and practically perpendicular to the line of advance. In summer the trough extends in a south-south-eastward direction.

2. The diagrams agree also in indicating only just a closed isobaric system in the upper levels, i.e., the depressions are almost "open" on their north sides, as if they were secondaries in a more general upper air circulation.

3. In winter the upper air centre is definitely about north-east by north of the surface centre and about 300 Km. away, that is, if the depression is moving from west. If, as often in winter the depression is moving from south-west by west, the

the point X″ lies due north of O″. In early summer and late summer—if the observations can be considered adequate—the centres lie a little east of north and a little west of north respectively and again some 300 Km. distant (see also deduction 6).

4. In all diagrams there is evidence of divergent movement in places on the eastern side.

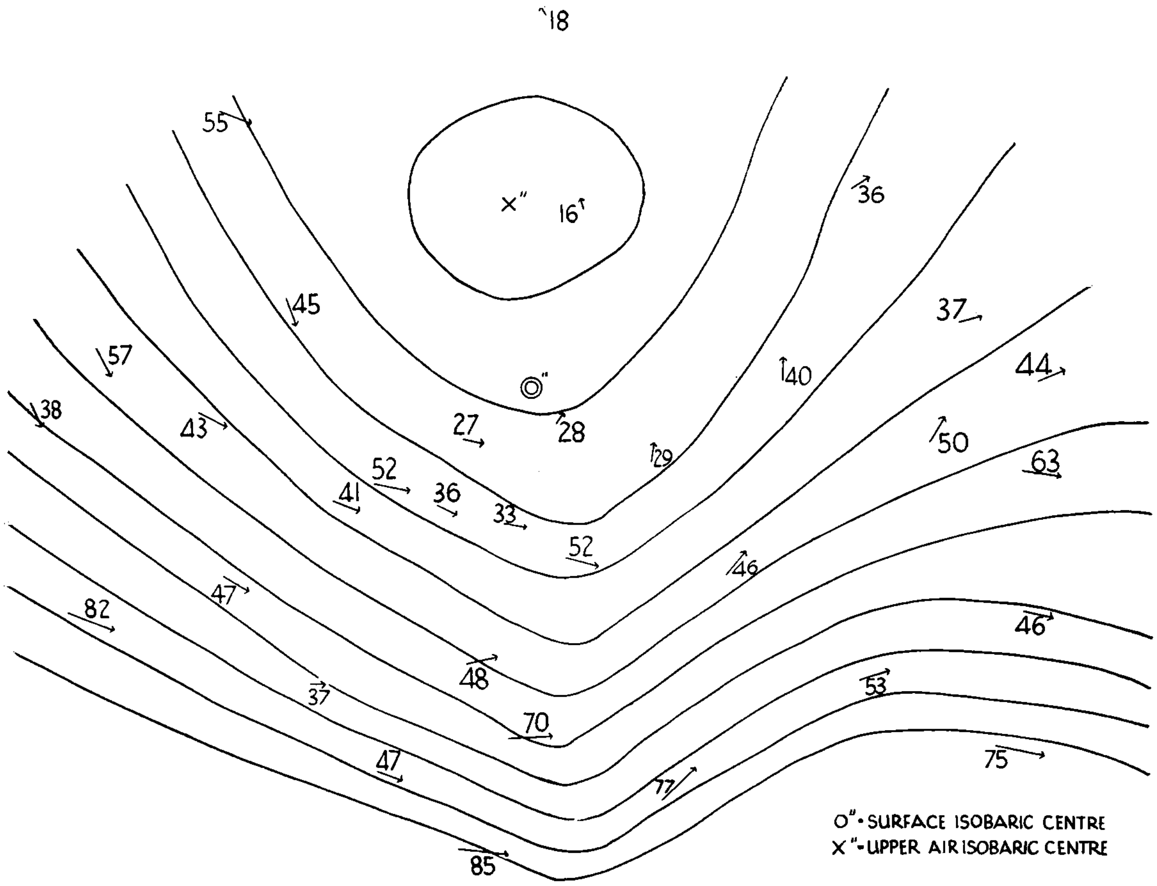


Fig. 9—Upper air movement above depressions—late summer (July to September).

5. The upper air speeds indicated (see Table II) show that estimates made in the earlier memoir and referred to earlier in this paper are not unduly high. Even the average speed in a belt 500–800 Km. south of the surface centre comes out as being 78 m.p.h. in winter, 75 m.p.h. in early summer, and 71 m.p.h. in late summer; and these averages would be increased slightly if speeds near the trough line were excluded. The reduction in speed, and more notably in the average component of

TABLE II—MEAN CLOUD SPEEDS (M.P.H.) TAKEN IRRESPECTIVE OF DIRECTION

Zone south or north of (surface) centre	Winter	Early Summer	Late Summer
5–800 Km. S.	78	75	71
0–500 Km. S.	51	56	45
0–500 Km. N.	36	39	39
	Mean west-east drift of air.		
5–800 Km. S.	76	73	70
0–500 Km. S.	38	42	32
0–500 Km. N.	18	11	20

west-east drift of air in the zones from 0 to 500 Km. south and north of the (surface) centre is in accord with the idea that in these zones and particularly in the northward zone there is an important admixture of air, of negligible west-east momentum, ascended from surface levels (see §6).

The above points are mainly points of resemblance between the three diagrams. There are also certain interesting differences :

6. In the winter diagram the general flow of air is more nearly parallel to the course of the depression than in the summer diagrams. The speed of progression of the winter depressions was in general fairly high and the direction of motion clearly defined. The early summer diagram in particular has quite a number of arrows showing winds approximately perpendicular to the line of progression ; and on the north side of the centre there is tremendous divergence. The late summer diagram has some similar features. Some indefiniteness in many cases in the direction of motion of summer and especially early summer depressions may account in part for these features. Also, especially in early summer, the general current in the upper levels of the troposphere must contain a component directed from south or south-south-east towards north or north-north-west, representing the run-off of upper air from over the continent during the seasonal rise of land surface temperature. The general direction of movement of the winter and late summer depressions was from between west-south-west and south-west. Some of the early summer depressions moved from north or north-west or south or even south-east ; the average direction —if in such a case it can have any meaning—and the most frequent direction was from west or slightly north of west. A not infrequent form of behaviour of early summer depressions, however, is to extend a trough south-eastwards rather than to travel in the ordinary sense ; and the nephoscope observations indicate this also.

A comment should be made here on a comparison of Figs. 7, 8 and 9 with the diagrams given by Douglas, though the detail in his is not intended to be so great. Douglas also gives isobars for 8 Km., computed however from surface pressure and mean upper air temperature and referring mainly to winter cyclones, though intended to fit in with the observations of cloud direction. The north-west current some distance in front of the depression is commented upon and is evident in Douglas' as in the writer's diagrams. The trough to which reference is made in deduction 1 above does not appear in Douglas' diagrams, unless possibly in the one of the partially occluded cyclone and in this case it is relatively far in the rear. A definite centre is not marked on his pressure diagrams, but it would seem to lie north-west of the surface centre by at least 600 Km. in the case of the partially occluded cyclone, and to north-north-west and even farther away in the case of the younger cyclone. Probable explanations for these differences are that Douglas' diagrams seem to be based on cloud directional data only, and that the data relating to the north sides of low pressure systems may have included a rather large proportion of secondaries.

§ 9—UPPER AIR SPEED IN RELATION TO DISTANCE FROM CENTRE

Using the data of Figs. 7, 8 and 9, the mean speeds for the year, at distances 1–200, 2–300, 3–400 Km. and so on from the upper air isobaric centre, were worked out and these are plotted in Fig. 10 ; on the same diagram also there are shown the mean speeds in (surface) cold and warm air in the typical depression of October 1–3, 1929. In comparing the curves it has to be kept in mind that the surface and upper air centres are about 300 Km. distant from one another horizontally. The upper air curve shows that within 500 Km. of the centre there is a steady diminution of speed as the centre is approached, and a very close approximation to v/r=constant ; there is a slight local maximum some 500–600 Km. from the upper air centre, and this may be significant since the distance is about the same as that of the ring of maximum surface speed from the surface centre ; beyond this radius the observations indicate an asymptotic rise to a speed of 70 to 80 m.p.h.

The three curves give the picture to be expected provided (a) that in the absence of the depression the undisturbed upper troposphere speed were of the order of perhaps 80 m.p.h. or more, and the undisturbed speeds in the lower air—both warm and cold—were of the order of only 22 m.p.h. ; (b) that the enhanced speeds of the surface cold air and the diminished speed of the upper air, with the approximation to equality up to some 400–500 Km. from the surface and upper air centres respectively, are the result of mixing in regions nearer the centre, and (c) that the vortical rise in surface air speed in the region from 1,000 to 400 or 500 Km. from the surface centre is the result of forced convergence of the surface air, both cold and warm, towards the central area.

The diagram shows that about 600 Km. from the upper air centre (or 300 Km. from the surface isobaric centre) the speeds of the upper air and of the surface warm air are almost identical. This may be due in part to the fact that the upper curve

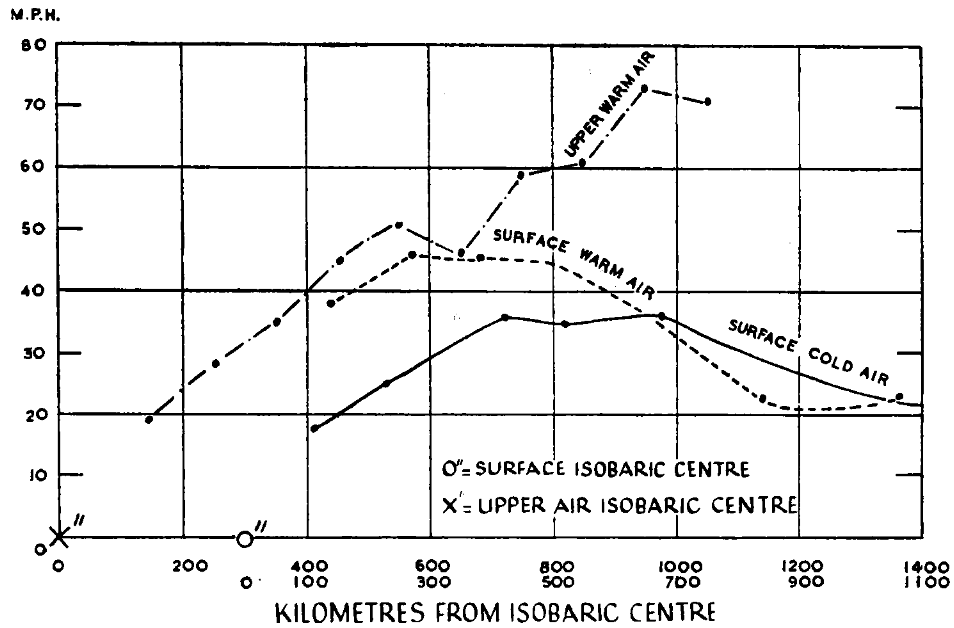


FIG. 10—UPPER AIR AND SURFACE SPEEDS IN RELATION TO UPPER AIR AND SURFACE (PRESSURE) CENTRES.

is an average taken over the year, whilst the lower curves are for a fairly deep October depression. But apart from that, it does not in any event mean that in the warm air at this place there is a uniform and solid current throughout the troposphere. In the previous memoir it was noted that the surface warm air here is converging at a considerable angle to the isobars, whilst even at the 2 Km. level the motion was found to be exactly along the isobars ; and according to Figs. 7, 8 and 9 of the present paper the motion at 7 Km. is in certain places divergent.

§ 10—CONCLUSIONS

It is shown that there is an intimate relation between the pressure at the centre of a depression and the intensity of its outer vortex. Twenty cases examined, after the elimination of two cases in which the initial circumstances were not altogether normal, give a correlation coefficient of $-0 \cdot 93$, with a standard error of $0 \cdot 03$, for this relationship.

A relationship, though of a less intimate nature, is found to exist between the pressure at the centre of a depression and the temperature difference between the constituent air masses, and this is interpreted as indicating a corresponding relationship between pressure at centre and increase of pressure gradient with height in the upper levels of the troposphere. The correlation coefficient (for thirty-eight depressions) in this case is $-0 \cdot 76$ with a standard error of $0 \cdot 07$.

It is shown that the amount of interaction between surface air and the upper levels of the troposphere, as determined from the upward and downward air movement required to account for the rates of rainfall observed in depressions, is of the right order to enable the amounts of kinetic energy which are required (to form and maintain the depressions) to be transferred from upper to surface levels.

From an analysis of some 470 nephoscope observations the structure of the upper troposphere over depressions is shown in relation to the surface structure ; and it is further confirmed that the speeds in the upper troposphere above depressions are of the order estimated by theoretical calculation in *Geophysical Memoirs* No. 72, and considered to be necessary to account both for the maintenance of depressions when formed and for their initiation as cell-like phenomena in regions having a more or less notable increase of wind speed with height.

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