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VERTICAL PROFILES OF MEAN WIND IN THE SURFACE LAYERS OF THE ATMOSPHERE

BY

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VERTICAL PROFILES OF MEAN WIND IN THE SURFACE LAYERS OF THE ATMOSPHERE

This memoir was originally written in 1948 *and has since been modified, in detail only, in the light of later work.*

SUMMARY

A set of sensitive recording cup anemometers was installed in 1941 over an open grassland site at Porton, Wiltshire, adjacent to equipment for recording vertical temperature gradient. The wind-profile observations obtained there up to 1945, together with some data for other surfaces including the sea, have been analysed with particular attention to the effects of roughness of the surface and of thermal stratification.

The well known logarithmic relationship of Prandtl is shown to be a very adequate representation of the wind profiles under neutral conditions. For other conditions of stability the form of the wind profiles and the gustiness of the wind is found to be satisfactorily related to the Richardson number except under very stable conditions. The effect of thermal stratification on the wind profile is small near the surface but increases markedly with height, in keeping with the fact that the Richardson number is very close to zero at the surface and increases numerically almost linearly with height. A generalized wind-profile relationship is proposed which enables the eddy viscosity to be evaluated, and this is compared with some previous formulations. Observations bearing on the critical value of the Richardson number for the onset of turbulence are discussed in relation to the laboratory results of Reichardt.

INTRODUCTION

Considerable research has been made in the last 30 or 40 years on vertical wind profiles in the atmosphere near the ground. While much useful information has been obtained by these studies, the data collected have frequently been subject to rather considerable limitations, the three principal being as follows :—

(i) Many of the earlier investigations on wind profiles were not accompanied by simultaneous observations of the vertical temperature distribution which greatly influences the rate of vertical interchange of momentum by turbulence.

(ii) The anemometers available for many of these investigations were unsuitable for the measurement of mean wind velocities with good precision, and facilities for frequent calibration in a wind tunnel were generally lacking.

(iii) In most cases the appreciable influence of irregularities in the roughness of the ground surface for considerable distances around the anemometer installation was not sufficiently appreciated.

In the work reported in the present paper a set of recording cup anemometers of high sensitivity was employed to investigate the wind profile in the lowest 10 m. of the atmosphere. These were a development of the Sheppard cup anemometer[1]* which was specially designed to achieve high accuracy in the measurement of vertical wind-speed gradients. The anemometers were frequently calibrated in a wind tunnel designed to give good accuracy down to very low air speeds. Simultaneous records of the vertical temperature distribution in the atmospheric layer

* The index numbers refer to the bibliography on p. 63.

investigated were obtained by using Johnson's temperature-gradient recording system[2]. Considerable attention was given to obtaining observations which were as far as possible characteristic of an area of uniform surface roughness.

In addition to the speed-and temperature-profile observations made in the years 1941–45 over short- and long-grass surfaces, an extensive series of observations of the gustiness of the wind was made using bi-directional vanes† to give further information on interchange processes in the atmosphere near the ground. Some velocity-profile observations over a very level desert surface and others over the sea are also given, the former having been kindly put at the author's disposal by K. L. Calder and the latter by Sir Nelson Johnson.

PART I—INSTRUMENTS AND SITE

§ 1—INSTRUMENTS

Anemometer system.—The anemometers and recording system have already been described[4], together with particulars of the method of calibration. The anemometers were re-calibrated at intervals of one to two months, and in all cases the differences between successive calibrations were small; in most cases they were hardly measurable. The recorder gave a photographic record at quarter-hourly intervals of the electric counters actuated by the anemometers, so that the mean wind speeds over 15-min. periods constitute the basic material for the wind-speed profiles. The photo-electric type of anemometer (used over the long-grass surface) gave about 250 counts/15 min. with 1 m./sec. wind speed, while the micro-contact type used in the remainder of the investigation gave half as many at the same speed. The counting rate was therefore, in all cases, much greater than with the ordinary forms of electric-contact cup anemometers, and errors caused by the recording system, ignoring fractions of a revolution of the contact spindle, were negligible.

Cup anemometers over-estimate a fluctuating wind to an extent depending on the mass per unit projected area of the cups[5]. This effect was reduced as far as possible by using spun aluminium cups giving a weight per unit area of 0·20 gm./cm.^{2}, the moment of inertia of the complete rotating system being about 800 gm. cm.^{2} (i.e. only a little greater than for the Sheppard cup anemometer). The over-estimation error of cup anemometers of this type has been investigated experimentally[6] for sinusoidal fluctuations, and from this it appears that the over-estimation of the wind speed by the cup anemometers was in general less than 3 or 4 per cent. except for very low wind speeds. Best's measurements of the gustiness of the wind at various heights[7] show that the lateral gustiness of the wind does not vary markedly with height so the over-estimation error will also vary only slowly with height. The form of the speed profile is not, therefore, affected to any appreciable extent by the over-estimation, but the gradients are all slightly too great, although not to a significant extent taking into account other uncertainties in the observations.

In comparing the wind-speed profile with the speed profile measured in wind tunnels, it must be remembered that the cup anemometer gives the mean wind speed in the horizontal plane and not the mean wind speed in the mean wind direction as measured by a pitot tube in a wind tunnel. The cup anemometer therefore gives the mean value of

$$u_c = \sqrt{\{(\bar{u} + u')^2 + v'^2\}}$$

over a period of time where u_c is the speed recorded by a perfectly responsive cup anemometer $\bar{u}$ is the true mean speed in the mean wind direction, u' and v' are the horizontal eddy components in the mean wind direction and at right angles to the mean wind direction respectively. If the eddy velocities are only a fraction of the mean wind velocity, as is generally the case, and $u' \simeq v'$, this gives

† An improved type of the instrument described by Taylor[3].

$$\bar{u}_c = \bar{u}\left\{1 + \overline{\left(\frac{u'}{\bar{u}}\right)^2}\right\} .$$

In practice the cup anemometers do not respond completely to the eddy velocities as is shown by the following data obtained in wind-tunnel experiments :

Fluctuation period (sec.)	1·5	2·95	6·2
Mean speed $\bar{u}$ (m./sec.)	4·7	4·6	4·45
True amplitude as percentage of $\bar{u}$ ..	36	45	50
Amplitude indicated by cup anemometer as percentage of true amplitude	45	70	90

The indicated speed $\bar{u}_c$ will not therefore, in general, be so much greater than $\bar{u}$ as shown by the above equation, but it will serve as giving an upper limit to the difference. Some experiments made with a responsive spring-controlled pressure-plate anemometer over grassland under overcast conditions ($\bar{u} \doteqdot 4$ m./sec.) gave the mean square ratio of u' to $\bar{u}$ as 0·03 to 0·04 at 2 m. above the surface. This value will be relatively constant with wind speed but subject to variation with change in temperature gradient. In general it seems that over-estimation of $\bar{u}$ by using cup anemometers under the conditions of the present work may amount altogether to some 4–6 per cent., perhaps somewhat more in very light or turbulent winds. No correction has, however, been applied to the indicated wind speeds in the following work.

Temperature gradient apparatus.—The temperature differences 17·1–1·2 m. and 7·1–1·2 m. referred to in the following work were recorded using the installation described by Johnson[2]. The method used in the measurement of the 5–1 m. and 1–0·2 m. temperature differences over the short-grass area was the same, except that the element housings were of the type described by Best[7], and the sensitivity of the bridge circuit was increased so that 1° F. corresponded to 9·4 mm. on the records given by the Cambridge thread recorder. Frequent zero checks were made under overcast conditions (i.e. approximately dry adiabatic lapse rate) using Assmann psychrometers, and special care was taken with all junctions in the wiring. The convention as to the sign of temperature gradient used in the following work is inversions positive, lapses negative. In studying the vertical stability of a moist atmospheric layer, the vertical distribution of water vapour, as well as of temperature, should strictly be taken into account. The condition for stability is that the lapse rate of virtual temperature shall not exceed the dry adiabatic lapse rate[8]. In practice, since vertical density gradients due to humidity variation in the lowest 10 m. of the atmosphere are generally so small compared with those due to temperature variation, it is sufficiently accurate to use the dry-air criterion, which is that the atmospheric layer under consideration is in stable, unstable or neutral equilibrium according as the vertical gradient of potential temperature is positive, negative or zero.

§ 2—SITE

The observations fall into four series as follows :—

Series	Period	Surface	Nature of observations
I	June, July 1941	Long grass	Temperature at 1·2, 7·1 and 17·1 m. Wind speed at 1, 1·98, 5·22 and 13·27 m.
II	June–September 1943	Mown grass	Temperature at 1·2, 7·1 and 17·1 m. Wind speed at 1, 1·98, 5·22 and 13·27 m.
III	April, May 1944	Mown grass	Temperature at 1·2, 7·1 and 17·1 m. Wind speed at 0·5, 1, 2, 4 and 8 m.
IV	June–December 1944	Mown grass	Temperature at 0·2, 1 and 5 m. Wind speed at 0·5, 1, 2, 4 and 8 m.

The site of the anemometer installation at Porton, Salisbury Plain, was about 180 m. (200 yd.) to the west of the temperature-gradient tower described by Johnson[2], the terrain around being undulating downland. A map of the surrounding country is given in Fig. 1. The exposure to the north is obstructed by many buildings only a quarter of a mile distant. Records obtained with wind directions between NW. and NE. have accordingly not been utilized.

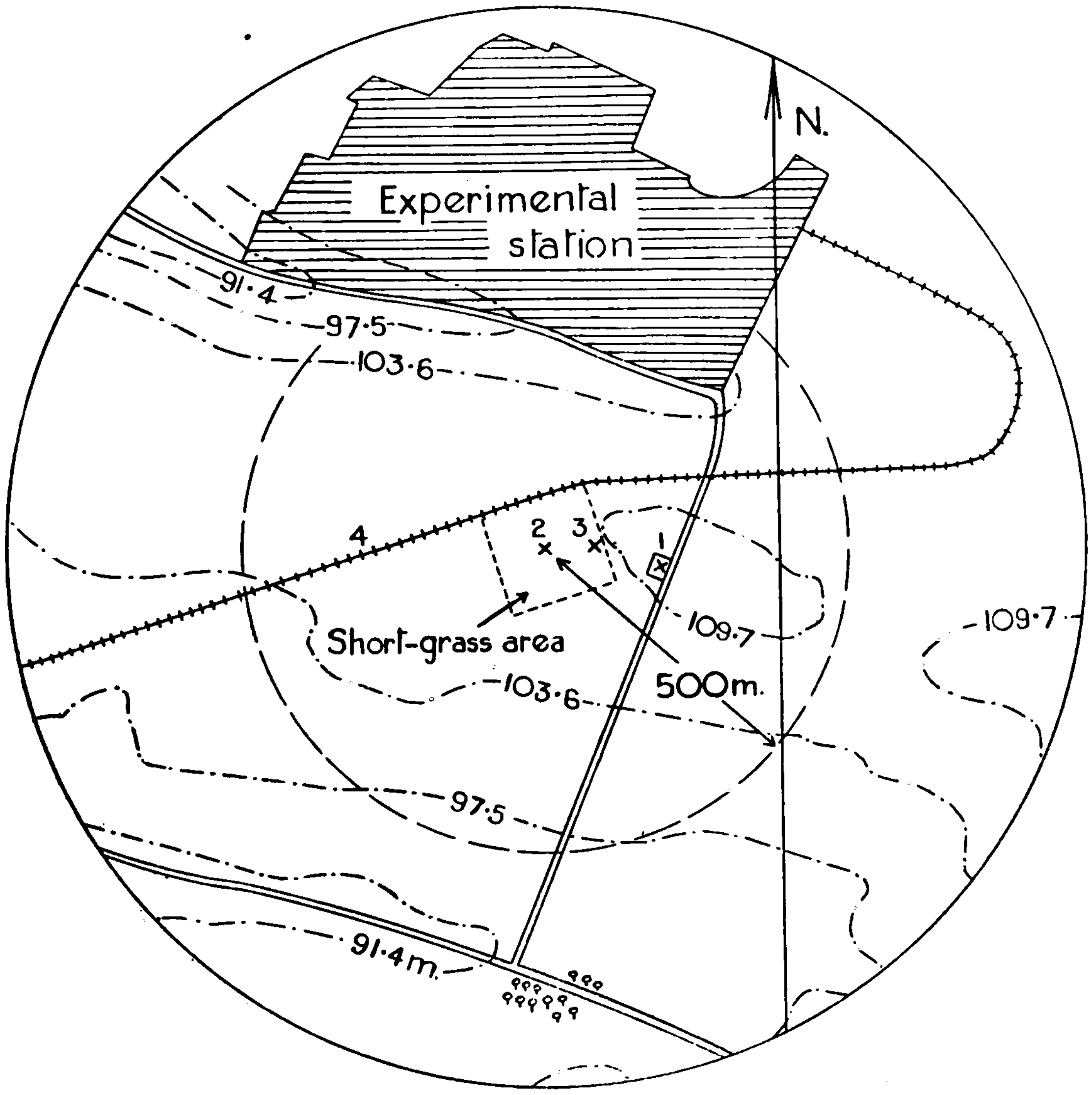


FIG. 1—SITE OF OBSERVATIONS
1. Temperature-gradient tower
2, 3. Anemometer sites
4. Light railway

For the observations of Series I and II the anemometers were erected on the tops of wooden poles $4\frac{1}{2}$ m. (5 yd.) apart, one pole for each anemometer. During Series I observations, the grass was long in the sense that the flower stalks extended to some 60–70 cm. with the leafy undergrowth about 30 cm. high. Grass of this description extended over an area of approximately 200 m. round the anemometers; beyond this, to the north of the light railway shown on the map, was a rather thin crop of oats 50–60 cm. high. In other directions the virgin downland comprised rough grass 10–15 cm. in height. During the recording periods winds were generally from a westerly point, and hence their immediate travel for approximately 500 m. to the anemometers was over a surface of relatively uniform roughness.

The interval between Series I and II was necessary for the preparation of the short-grass area, 180 m. (200 yd.) square, centred on the anemometer installation. Repeated cutting resulted in a good level uniform area of short grass nearly equal to a sports field in smoothness.

Series II observations over the prepared short-grass area showed clearly that 90 m. fetch of the wind over short grass was not nearly sufficient to produce a turbulent boundary layer truly characteristic of the short-grass surface up to a height of 13 m. In fact when smoke was generated at the upwind edge of the short-grass area (sky overcast, wind speed 5–7 m./sec.) it was observed that only occasional wisps of smoke rose to a height greater than about 6·5 m. at the position of the 13-m. mast. From this it seemed probable that even the 5-m. anemometer was somewhat affected by the rough grass beyond the prepared area. That this was so, was confirmed by the simultaneous measurement of wind profiles at several different distances from the upwind edge. This work showed that the lower air layers accelerated considerably over the short grass with the production of a speed profile of the type shown in Fig. 26. It was found that using nearly the full width of the prepared area (170 m. fetch) was only sufficient to give a speed profile characteristic of the short grass up to a height of about 5 m. In view of this, only the ratios of the 2-m. to 1-m. wind speeds in the Series II observations have been utilized in the present work.

To obtain a profile characteristic of the short grass to as great a height as possible, the anemometers were moved for the Series III and IV observations to near the eastern edge of the prepared area, to give a fetch for westerly winds of 170 m. over short grass.

Anemometers at 1, 2, and 4 m. were mounted on arms of galvanized pipe extending out from a mast of $1\frac{1}{2}$-in. water pipe steadied by guy wires. The aspirated platinum-resistance thermometer elements at 0·2, 1, and 5 m. were similarly mounted on arms from the same pole*, the whole being so arranged that no mutual interference resulted with westerly winds, which were, of course, the only ones which gave useful results at this anemometer site. The anemometer at 0·5 m. was mounted about 3 m. away from the mast, and the 8-m. anemometer was mounted on the mast of the Dines pressure-tube anemograph which stood on the short-grass area some 25 m. to the south of the anemometer installation. The grass in the vicinity of the installation was cut by hand to be as similar as possible to the rest of the short-grass area. It was expected that the 8-m. anemometer would give observations indicating the limited size of the short-grass area, and there was evidence of this in the results as will be seen later. It was also found, from an analysis of the results under overcast conditions with moderate or strong winds, that only the results for wind directions between 230° and 300° true could be relied upon to give speeds up to 4 m. unaffected by the surrounding downland, and only the observations from these directions were used in the following work. Although winds from these directions were reasonably frequent during the recording periods, it was found that occasions of strong instability with these directions were rather infrequent, settled fair weather being more usually associated with easterly winds.

The anemometers were not at any time fenced round in the manner of the usual meteorological enclosure, and consequently the lower anemometers were accidentally damaged on two occasions. This, however, was considered to be a lesser evil than surrounding the anemometers with a fence which not only acts as a direct obstruction to the wind but makes it difficult to maintain a uniform surface.

PART II—RESULTS FOR CONDITIONS OF NEUTRAL STABILITY

§ 3—WIND PROFILES OVER A SHORT-GRASS SURFACE

The data for occasions of approximately neutral stability (i.e. lapse rate close to the dry adiabatic value) were selected from two periods of Series III, one of 9 days and the other of 13 days, during which the wind was often in the required direction to give a fetch of 170 m. over the prepared short-grass area before reaching the anemometer installation. It was soon found in the study of wind profiles that changes in roughness of the ground surface have a considerable effect. In these comparatively short periods, however, the change in roughness due to the

* Thermometer elements at the same heights as the anemometers would have been more convenient in the analysis of the results but could not be arranged on the mast without mutual interference.

growth of the grass was not very marked. The results are presented in Table I, the speeds at the various heights being given as speeds relative to that at 1 m. (e.g. R_4 is the ratio of the wind speed at 4 m. to the wind speed at 1 m.). There were insufficient observations with winds lighter than 2 m./sec. at 2 m. for this range to be included. It will be noted that the higher the wind speed the greater is the range of temperature gradient for which results have been included. This is due to the fact (dealt with later) that the effects of thermal stratification diminish rapidly with increasing wind speed. That the error incurred by allowing this latitude is very small may be seen from the observations in the 6–8 m./sec. wind-speed range, which have been split into two groups for the second period, one for zero temperature gradient or small lapse rate and the other for moderate lapse rate (17·1–1·2 m. temperature difference from −1·7° to −2·5° F.).

TABLE I—WIND PROFILES OVER SHORT GRASS

Range of wind speed at 2 m.	No. of observa-tions	Range of 17·1–1·2 m. temperature difference	Mean 17·1–1·2 m. potential temperature difference	Mean wind speed at 2 m.	$R_{0\cdot5}$	R_1	R_2	R_4	R_8
m./sec.		°F.	°F.	m./sec.					
Period: May 26, 1944–June 3, 1944							Average grass length: 1·7 cm.		
2–4	19	0 to −0·7	−0·20	3·25	0·8725	1·000	1·117	1·242	1·349
4–6	46	0 to −1·5	−0·37	5·19	0·8750	1·000	1·112	1·2245	1·325
6–8	23	0 to −2·2	−1·33	6·87	0·8855	1·000	1·1115	1·2245	1·304
			Weighted mean	5·21	0·877	1·000	1·113	1·229	1·326
			Value calculated from logarithmic law†		0·8795	0·997	1·113	1·230	1·347
			Percentage difference (observed−calculated)		−0·25	0·3	0·05	−0·1	−1·5

Range of wind speed at 2 m.	No. of observa-tions	Range of 17·1–1·2 m. temperature difference	Mean of 17·1–1·2 m. potential temperature difference	Mean wind speed at 2m.	$R_{0\cdot5}$	R_1	R_2	R_4	R_8
m./sec.		°F.	°F.	m./sec.					
Period: June 4–16, 1944							Average grass length: 2·3 cm.		
2–4	27	0 to −0·7	+0·06	3·61	0·861	1·000	1·126	1·262	1·374
					20/4‡	..	12/2·3	26/5	58/11
4–6	172	0 to −1·5	−0·17	4·92	0·867	1·000	1·122	1·246	1·353
					17/1·5	..	16/1·2	33/2·5	52/4
6–8	42	0 to −1·7	−0·88	6·81	0·877	1·000	1·123	1·248	1·349
					18·5/3	..	14/2	31/5	52/8
6–8	42	−1·7 to −2·5	−1·86	7·31	0·884	1·000	1·121	1·246	1·341
					15·5/2·5	..	17/2·5	30/5	41/6
>8	14	0 to −3	−2·25	8·23	0·878	1·000	1·129	1·259	1·351
			Weighted mean	5·55	0·8710	1·000	1·1226	1·2483	1·3526
			Value calculated from logarithmic law†		0·8723	0·9977	1·1232	1·2487	1·3741
			Percentage difference (observed−calculated)		−0·15	0·25	−0·05	−0·05	−1·6

† The logarithmic laws fitted by least squares to the observations for 0·5 to 4 m. were :—

1st period $u/u_1 = 0\cdot3883 \log_{10} (z/0\cdot272)$

2nd period $u/u_1 = 0\cdot4167 \log_{10} (z/0\cdot404)$

u being the wind speed at height z cm. and u_1 that at height 1 m.

‡ The figures in the last five columns to the left of the solidus are the standard deviation of the observations multiplied by 1,000 while the figures to the right are the standard error of the mean value also multiplied by 1,000.

It is apparent that over the range of wind speed 2–8 m./sec. at 2 m. the profiles of relative speed are practically independent of speed under adiabatic conditions. Therefore the gradient of mean speed at any given height over the short grass is directly proportional to the mean wind speed.

The weighted mean speed ratios from Table I are shown in Fig. 2 plotted against the logarithm of the height above the surface. Straight lines have been fitted by the method of least squares to the points from 0·5 to 4 m. and the calculated values of the speed ratios are given in Table I. The 8-m. speeds were not used as there was reason to believe that the 170 m. fetch of the wind over the short grass was not quite sufficient to give profiles wholly characteristic of this surface up to a height as great as 8 m. The discrepancies between the observed values and those appropriate to a logarithmic variation of wind with height are less than 0·3 per cent. from 0·5 to 4 m., but increase to about 1·5 per cent. at 8 m., probably as a result of a residual effect of the rougher downland beyond the short-grass area. Change in roughness of the ground causes the slope of the line relating wind speed to the logarithm of the height to change, showing that the wind-speed gradient is increased in the same proportion at each height by an increase in surface roughness.

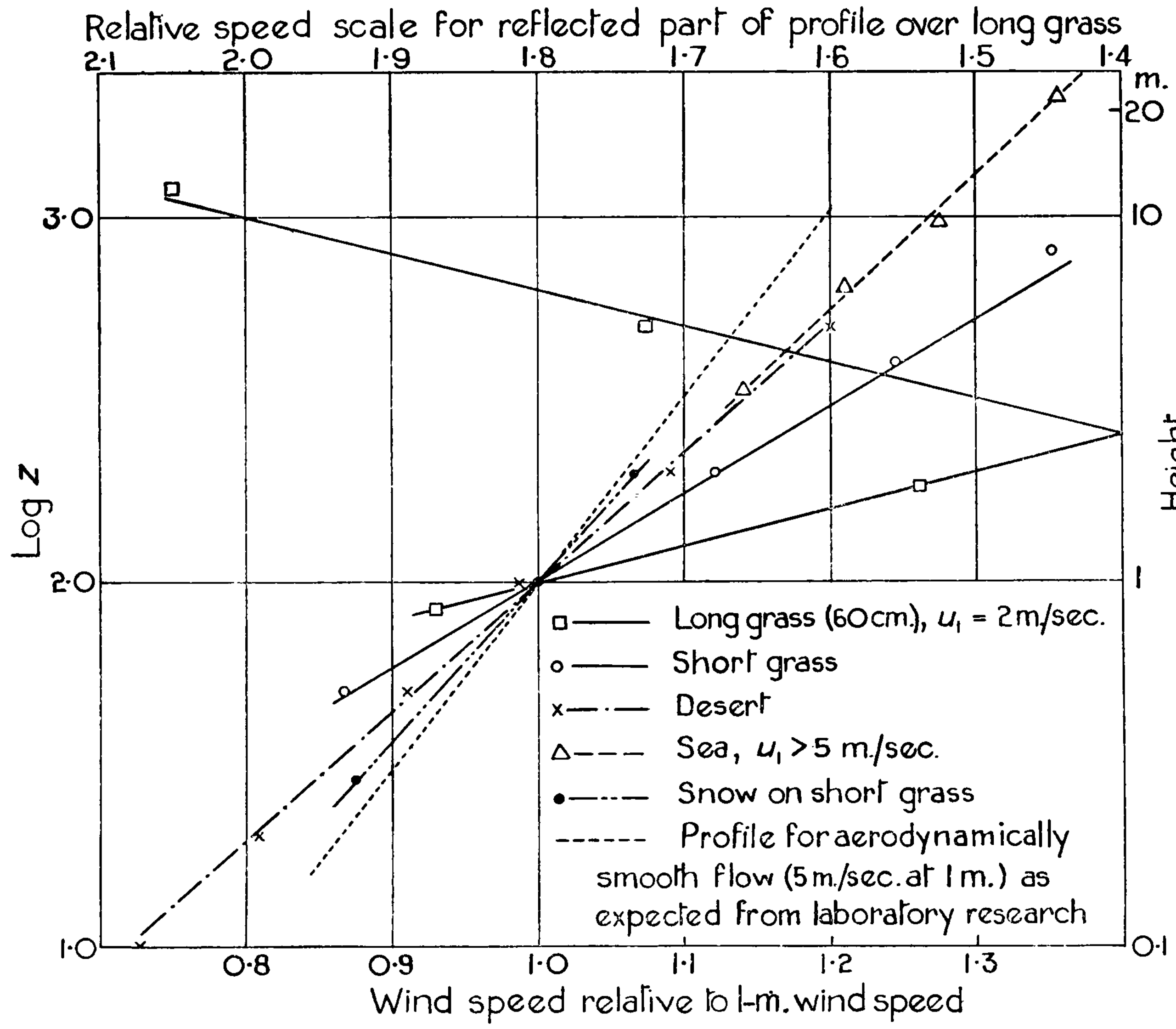


FIG. 2—WIND PROFILES OVER VARIOUS SURFACES

Profiles obtained under approximately neutral conditions of stability over other surfaces have been included in Fig. 2. The observations over long grass and snow will be discussed in due course, while the details of the observations for desert and sea surfaces are given in the Appendix. All these observations are in agreement with earlier work, for example that of Best[7], Sverdrup[9], Paeschke[10] and others who agree in finding that, under neutral conditions of stability, a linear relationship between speed and the logarithm of the height satisfactorily fits the observations.

§ 4—LOGARITHMIC WIND PROFILE

The flow conditions in turbulent boundary layers both in pipes and channels and over plane surfaces have been extensively investigated during the last twenty years, and certain laws of wide applicability have been established. Major contributions on the theoretical side have been made by von Kármán and Prandtl[11], while notable experimental investigations are those of Nikuradse[12], Dryden[13] and Schlichting[14].

This work has shown that the fundamental relationship for turbulent flow over a plane surface, in the absence of thermal stratification, is that relating the shear to the shear stress, namely

$$\frac{du}{dz} = \frac{u_*}{kz} \qquad \ldots\ldots (1)$$

where u = the mean speed at a height z above the surface

$u_* = \sqrt{(\tau/\rho)}$, so called " friction or shearing-stress velocity "

τ = the shearing stress at height z

ρ = fluid density

k = von Kármán's constant = $0 \cdot 40$.

As k has hitherto been found to be a universal constant of turbulent isothermal boundary-layer flow, this relationship shows that the speed gradient at any height z above the surface is solely determined (in the case of a fluid of constant density) by the shear stress and the distance z. It has accordingly been found that equation (1) applies to flow over both rough and smooth surfaces. The only essential difference between the two cases is in the way the surface drag is generated. The fundamental features of the flow pattern are not, however, affected by the manner in which the drag is produced except in the immediate vicinity of the surface.

On integration, equation (1) gives a logarithmic law for the layer in which the shearing stress is effectively constant ($\tau \doteq \tau_0$, the value at the surface) :—

$$\frac{u}{u_*} = \frac{1}{k} \log_e \left(\frac{z}{c}\right) \qquad \ldots\ldots (2)$$

where c is the integration constant.

So, for flow over both smooth and rough surfaces, a logarithmic law is observed to hold in the layer of constant shearing stress†. Flow over aerodynamically smooth surfaces will be considered first. In this case projections on the surface are small enough to be totally submerged in the thin surface film in which laminar flow exists. As the drag exerted by the boundary surface must be transmitted to the fluid through this laminar sublayer, it follows that the drag coefficient is a function of the Reynolds number only. Experiments by Nikuradse[12] with smooth pipes and by Dryden[13] with smooth flat plates have shown that the appropriate form of equation (2) is

$$\frac{u}{u_*} = \frac{1}{k} \log_e \frac{9 \cdot 05\, u_* z}{\nu} \qquad \ldots\ldots (3)$$

where ν = kinematical viscosity of the fluid. The logarithmic term is of the nature of a Reynolds number while u/u_* (the dimensionless speed ratio) is simply related to the drag coefficient C_D

† Montgomery[15] has shown clearly why it happens that the logarithmic law is also, in the case of pipe flow, a good approximation nearly to the pipe axis.

$$C_D = \frac{\tau_0}{\frac{1}{2}\rho u^2}$$
$$= 2\frac{{u_*}^2}{u^2}.$$

With rough surfaces, provided that the fluid speed is sufficiently high, the drag is due to the impact of fluid on the bluff surfaces of roughness elements, and hence varies as the square of the fluid speed in a similar manner to the drag of a bluff body. The resistance coefficient is not, therefore, in this régime (fully rough flow) a function of the Reynolds number but is solely dependent on the form and density of distribution of the roughness elements on the surface. The form of equation (2) which applies to fully rough flow has been shown by Nikuradse[12] and Schlichting[14] to be

$$\frac{u}{u_*} = \frac{1}{k}\log\left(\frac{z}{z_0}\right) \quad \ldots\ldots \quad (4)$$

where z_0, the roughness parameter of the surface, has the dimensions of a length and is constant for any given surface.

A flow régime transitional between aerodynamically smooth flow and fully rough flow has been shown to exist, and as a result of the experiments with sanded surfaces Nikuradse[16] found that whether the flow was fully rough, transitional or aerodynamically smooth depended on certain criteria, which Schlichting[14] suggested may be used in the case of other rough surfaces, provided the roughness parameter (z_0) for the fully rough régime is known. Nikuradse's criteria may then be re-stated† in terms of z_0:

$$\left.\begin{array}{lr} \text{fully rough flow} \quad \ldots\ \ldots\ \ldots & \dfrac{u_* z_0}{\nu} > 2\cdot 5 \\[2ex] \text{transitional flow} \quad \ldots\ \ldots\ \ 2\cdot 5 > & \dfrac{u_* z_0}{\nu} > 0\cdot 13 \\[2ex] \text{aerodynamically smooth flow} \ \ldots\ \ 0\cdot 13 > & \dfrac{u_* z_0}{\nu}. \end{array}\right\} \quad \ldots\ldots \quad (5)$$

While it seems probable that these criteria should be approximately valid for surfaces roughened with protuberances of compact shape, it is doubtful if they apply when the roughness elements are spiky, like grass blades. In such cases it would be expected that the upper limiting values of $u_* z_0/\nu$ would be less than those given above.

For the logarithmic laws, equations (3) and (4), to be valid in the surface layers of the atmosphere it is necessary to show that, in the atmospheric boundary layer, the drag force is related to the wind gradient as indicated in equation (1) with a value of von Kármán's constant, k, equal to that found by laboratory experiments, $0\cdot 40$. The measurement of the drag is not nearly as simple in the atmosphere as it is with fluid flow in pipes and channels, and accordingly the data on this point are rather meagre. Rossby and Montgomery[17] have shown, however, that wind gradients measured over open grassland give values of τ_0 (on the assumption that $k = 0\cdot 4$) which are in agreement with those deduced by Taylor[18] from an analysis of data of pilot-balloon ascents over Salisbury Plain. Sheppard[19] has also demonstrated that k is very close to $0\cdot 4$ under conditions of slight lapse by measurement of the surface drag of the wind over an extensive concrete surface.

Calder[20] has shown that the vertical diffusion of matter in the lower atmosphere under adiabatic conditions is very accurately predicted by a treatment which derives eddy diffusivities

† This re-statement is equivalent to Schlichting's although not quite in the same form.

from the logarithmic wind profile on the assumption of $k = 0{\cdot}40$. The diffusion data, which were obtained from a very large number of carefully controlled experiments conducted over downland, provide a very sensitive test of the correctness of this formulation. It therefore appears that there is now little doubt that $k = 0{\cdot}40$ is equally applicable to atmospheric boundary-layer flow as to the smaller-scale boundary layers investigated in laboratory studies*.

§ 5—WIND PROFILES OVER SMOOTH SURFACES

Wind profiles over smooth surfaces may also be compared with laboratory results as a further test of the general validity of the laws of boundary-layer flow. Advantage was taken of an occasion when a thin layer (about 4 cm.) of snow covered the 180 m. square short-grass area already mentioned. The wind was light during the period of snowfall, so the layer was very smooth and even, much more so than is normal with natural snow-covered surfaces. The wind speeds were measured at three heights near the lee side of the area using newly calibrated Sheppard anemometers, and as a further precaution the anemometers were systematically interchanged between each 5-min. run to minimize any residual errors in the calibrations. The mean results from observations over a period of an hour, during which very steady conditions obtained and the temperature gradient was very close to the dry adiabatic lapse rate, are given in Table II.

TABLE II—WIND PROFILE OVER VERY SMOOTH SNOW; NEUTRAL STABILITY

	Height (cm.) 28·5	96	196
	metres per second		
Mean wind speed	4·243	4·820	5·176
Calculated speed from logarithmic law ..	..	4·824	..

The closeness with which the observations fit a logarithmic law of the form of equation (2) is shown by comparing the observed speed at 96 cm. (4·820 m./sec.) with that calculated from the logarithmic law (4·824 m./sec.) using the other two wind speeds to determine the constants. The wind ratio, R_2, for use in Table III below was found by interpolation to be 1·068.

Some observations of R_2 have also been made over mud flats. Observations for over an hour were made under adiabatic conditions using similar anemometers and the same technique as for the snow surface. The site was near Foulness, Essex, the off-shore wind having a fetch of 4,500 m. over level mud and shallow pools of water before reaching the site. In this case the mean value of R_2 was 1·060.

Comparison may now be made between the observed values of R_2 and those calculated from the logarithmic law for smooth flow, equation (3), taking $k = 0{\cdot}40$ and $\nu = 0{\cdot}14$ C.G. S.units.

TABLE III—WIND RATIOS OVER SMOOTH SURFACES

	Mean speed at 2 m.	Wind ratio (R_2) observed	calculated*
	m./sec.		
Very smooth snow on short grass ..	5·18	1·068	1·059
Mud flats	5·10	1·060	1·059
Short grass (from Table I)..	5·19	1·112	1·059

* Calculated from equation (3).

* Since this was written, Pasquill[21] has measured the surface drag of the wind over grassland and found $k = 0{\cdot}37$, a value not differing significantly from 0·40 in view of the experimental error.

It will be seen that for both the mud flats and the snow surface the wind shear is very close to that predicted by the laboratory laws for aerodynamically smooth flow, although it appears that the snow surface, as might be expected, is slightly rougher than the wet mud surface. The considerably greater shear with even short grass is clearly displayed. The application of the laboratory laws of fluid flow to the atmosphere near the ground appears to be justified by these results over smooth surfaces.

§ 6—WIND PROFILES OVER A LONG-GRASS SURFACE

From the Series I observations over grass 60–70 cm. long, the values were extracted of the wind speeds recorded under neutral conditions of stability with wind directions between 240° and 300°. This was the sector of best exposure, these winds having a fetch of about 500 m. over ground of relatively uniform roughness. The mean values are given in Table IV.

TABLE IV—WIND RATIOS OVER LONG GRASS—ALL SPEEDS

Mean wind speed at 2 m. : 3·91 m./sec. Number of observations : 138.
Mean potential temperature difference, 17·1 — 1·2 m. : — 0·10°F.

	Height (cm.)			
	100	198	522	1327
Observed ratio	1·00	1·302	1·712	2·035
Calculated ratio* ..	1·00	1·305	1·695	2·052

* From $u/u_1 = 0 \cdot 854 \log_{10} \{(z - 25)/5 \cdot 07\}$.

These results do not indicate a linear relationship between u and $\log z$ when z is taken to be the height above the soil surface, but the last line in Table IV shows that if the heights are reckoned from a datum level 25 cm. above the soil surface then the logarithmic law gives a good representation of the data. It appears, therefore, that in this case the effective or " aerodynamic " surface is about 25 cm. above the soil surface. In such cases therefore the appropriate form of the logarithmic law for aerodynamically rough flow is

$$\frac{u}{u_*} = \frac{1}{k} \log_e \left(\frac{z - d}{z_0}\right), \qquad \ldots . \quad (6)$$

where z is the height measured from the soil surface and d is the zero-plane displacement which may be considered as due to the layer of air trapped among the dense vegetation.

Zero-plane displacement.—Paeschke[10] also found that a positive zero-plane displacement was necessary, and he concluded that it was generally equal to the measured height of the vegetation. While this may be approximately true when the vegetation is very dense, it is not supported by the results given above for long grass.

Sverdrup[9, 22, 23], following Rossby and Montgomery[17, 24], takes d to be a negative quantity numerically equal to the roughness parameter z_0, on the reasoning that the mixing length should have a finite value adjacent to a rough surface. The Rossby-Montgomery form of the logarithmic law accordingly gives, on extrapolation to the surface, $u = 0$ at $z = 0$. An examination of the detailed profiles obtained by Paeschke (two of which are reproduced in Fig. 3) shows that this formulation is not valid. A rough surface is essentially a three-dimensional assemblage of bluff bodies, and the mean effective level at which the drag is generated must depend on the vertical distribution of active surface and also on the closeness of packing of the roughness elements. The effective datum level for the turbulent flow must therefore be at some height between the base and the top of the roughness elements which can only be determined from the wind profiles.

For relatively smooth surfaces, such as snow, short grass, etc. the zero-plane correction is small enough to be negligible in most cases, and for these surfaces it appears best to retain the original Prandtl form of the logarithmic law, as given by equation (4).

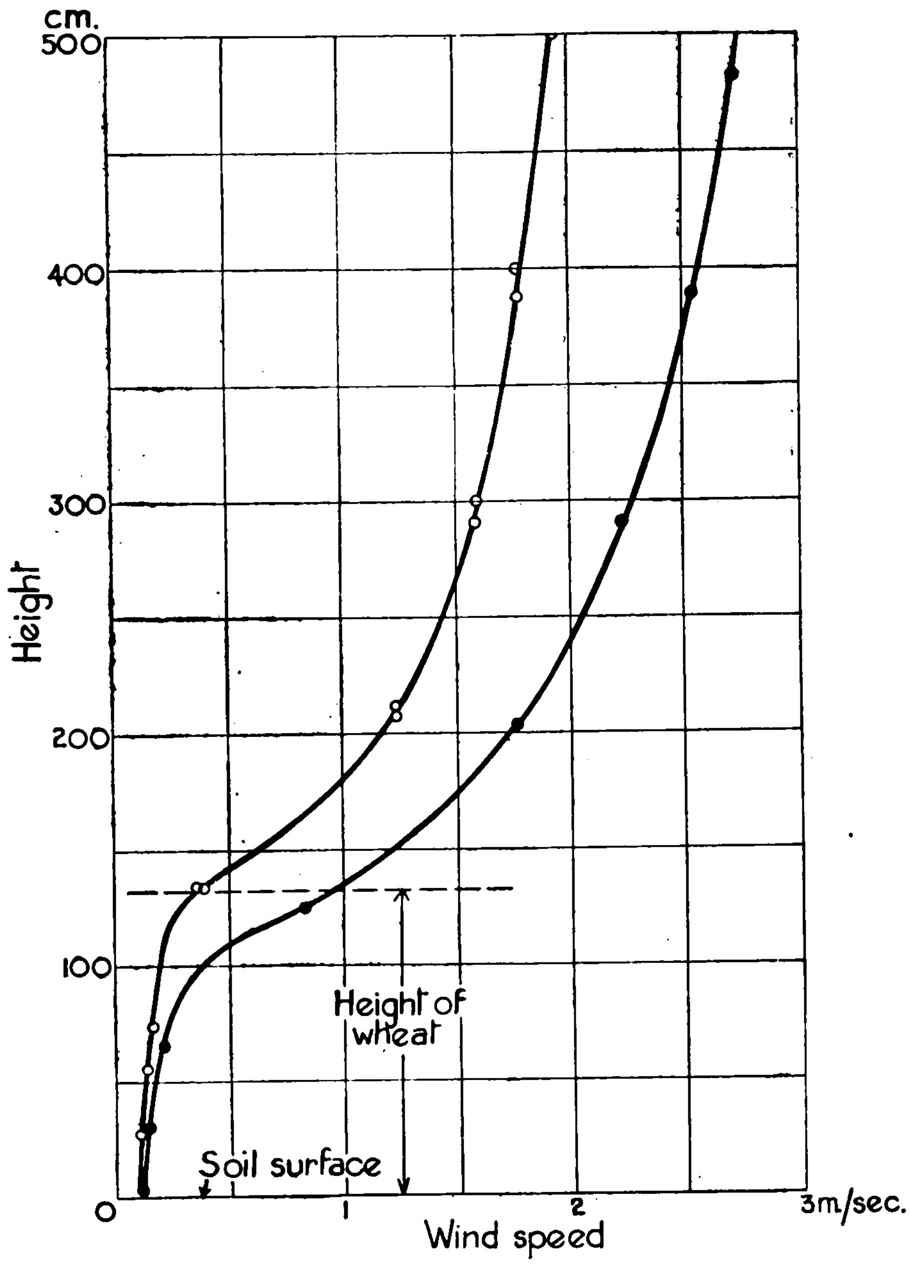


FIG. 3—WIND PROFILES OVER A CORNFIELD
(after Paeschke[10])

Variation of roughness of long grass with wind speed.—The data of Table IV indicate a roughness parameter of 5·1 cm., but when separated in groups according to the wind speed it is found that, in contrast to the short-grass results, the roughness parameter is not constant but decreases with increasing wind speed as will be seen from Table V. A constant zero-point displacement d of 25 cm. has been applied in calculating roughness parameters, as there is no evidence of any significant change in this quantity with wind speed—a change of only a few centimetres would be obscured by experimental and casual errors. The relation between u and log $(z - 25)$ is plotted in Fig. 4.

The considerable decrease in roughness parameter with increasing wind speed displayed in Table V is attributed to the bending of the long-grass blades in the wind to produce a smoother surface. Grass blades, at right angles to the flow in light winds, bend as the wind increases until they become more nearly parallel to the flow over a considerable part of their length, with a consequent decrease in resistance. The change in z_0 from 9 to 3·7 cm. corresponds to a decrease of the drag coefficient (referred to the speed at 2 m.) by nearly 40 per cent.

TABLE V—WIND PROFILE OVER LONG GRASS SHOWING THE EFFECT OF WIND SPEED

Wind speed at 2 m. Range	Wind speed at 2 m. Mean	17·1–1·2 m. temperature difference Range	17·1–1·2 m. temperature difference Mean	No. of obs.	Wind ratios* $R_{1\cdot98}$	Wind ratios* $R_{5\cdot22}$	Wind ratios* $R_{13\cdot27}$	Roughness parameter
m./sec.	m./sec.	°F.	°F.					cm.
1–2	1·48	−0·1 to −0·5	−0·30	20	1·363	1·878	2·360	9·0
2–3	2·48	0 to −0·7	−0·42	30	1·368	1·850	2·221	8·2
3–4	3·43	0 to −1·0	−0·49	49	1·327	1·761	2·083	6·1
4–6	4·76	0 to −1·5	−0·46	90	1·292	1·677	1·973	4·4
>6	6·22	+0·5 to −1·5	+0·16	5	1·279	1·647	1·937	3·7

*R_z = ratio of wind at z m. to that at 1 m., both heights measured from the soil surface.

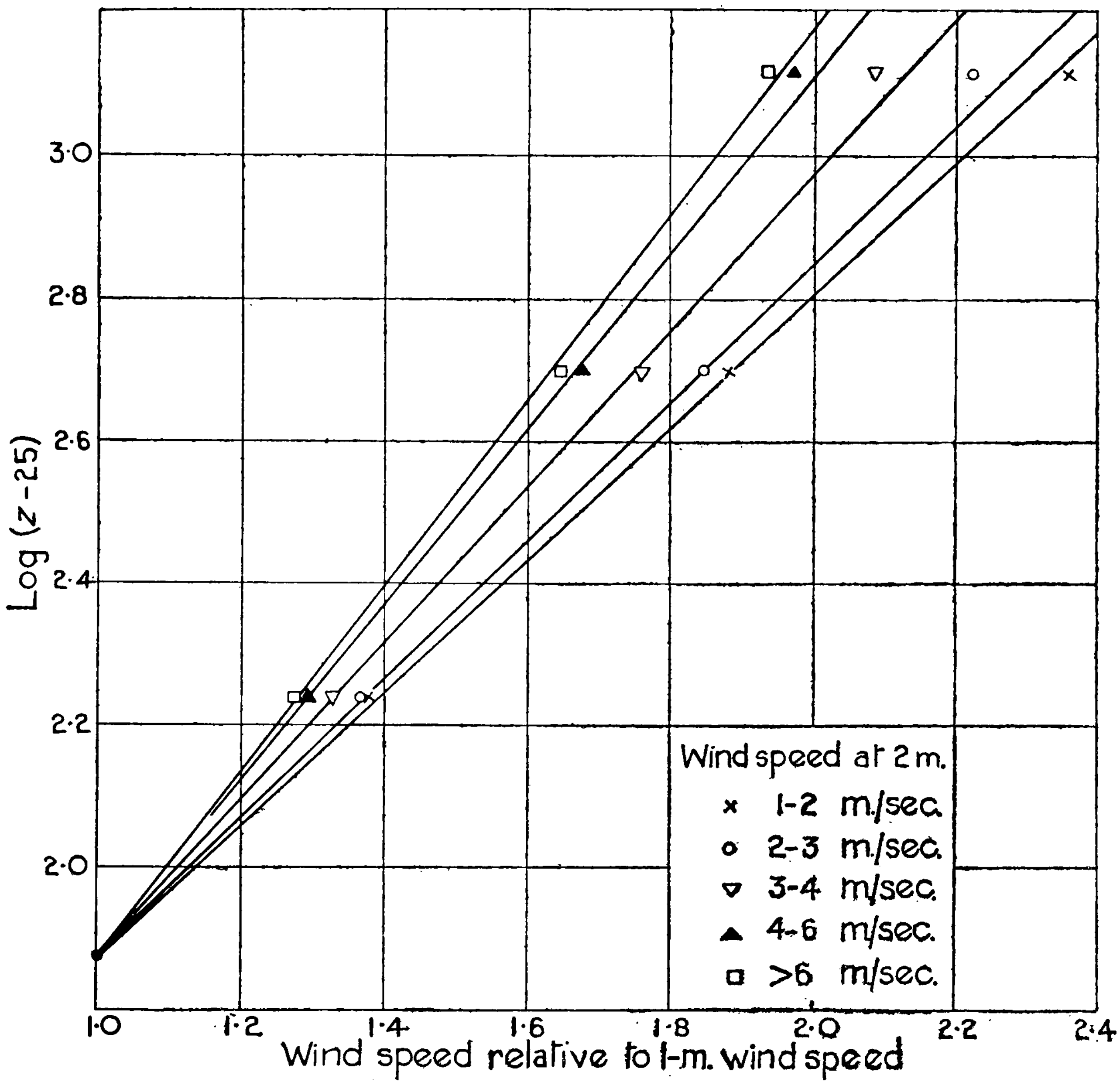


FIG. 4—WIND PROFILES OVER LONG GRASS
z is measured in centimetres

This effect was also displayed, although to a much smaller degree, by the results of measurements over the mown area when the grass was 4–5 cm. long. The R_2 wind ratios for this surface (neutral conditions) and for shorter grass (1·7–2·5 cm.) are shown in Fig. 5 against wind speed, the values being taken from Table VIII (see p. 26). There is a quite definite variation in the ratio for the former surface with wind speed corresponding to a decrease in roughness parameter from about 2·4 cm. with a wind of 2 m./sec. to 1·7 cm. with a wind of 6–8 m./sec. at 2 m. This is equivalent to a decrease of the resistance coefficient (2-m. reference speed) of about 14 per cent. For the shorter grass, however, there is no significant variation, the roughness parameter remaining constant at 0·6 cm.

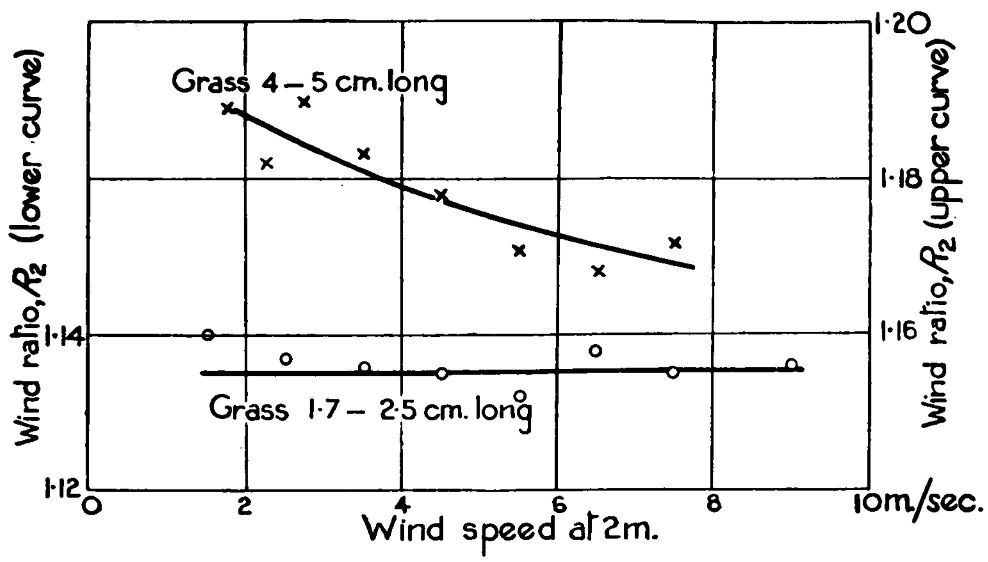


FIG. 5—RELATION BETWEEN R_2 AND WIND SPEED AT 2 M.

It is probable that many surfaces covered in vegetation vary in roughness depending on the wind speed, although the effect may generally be less than for long grass which is very readily deformable.

§ 7—CHANGE OF ROUGHNESS OF A MOWN-GRASS SURFACE ARISING FROM GROWTH

Fig. 6 shows the values of R_2, recorded over the mown-grass area for all occasions of adiabatic lapse, plotted against time in days, the occasions when the grass was cut being indicated. The rapid increase in roughness with growth of the grass is very marked. In the latter half of May, when the growth of grass is most rapid, the roughness parameter increased from 0·3 to 1·2 cm. in 10 days, which corresponds to an increase in the drag coefficient at 2m. from 0·0076 to 0·0123.

During this period of three months the length of the grass was measured at intervals in the following manner. A flat piece of cardboard about 15 cm. square was dropped at random on

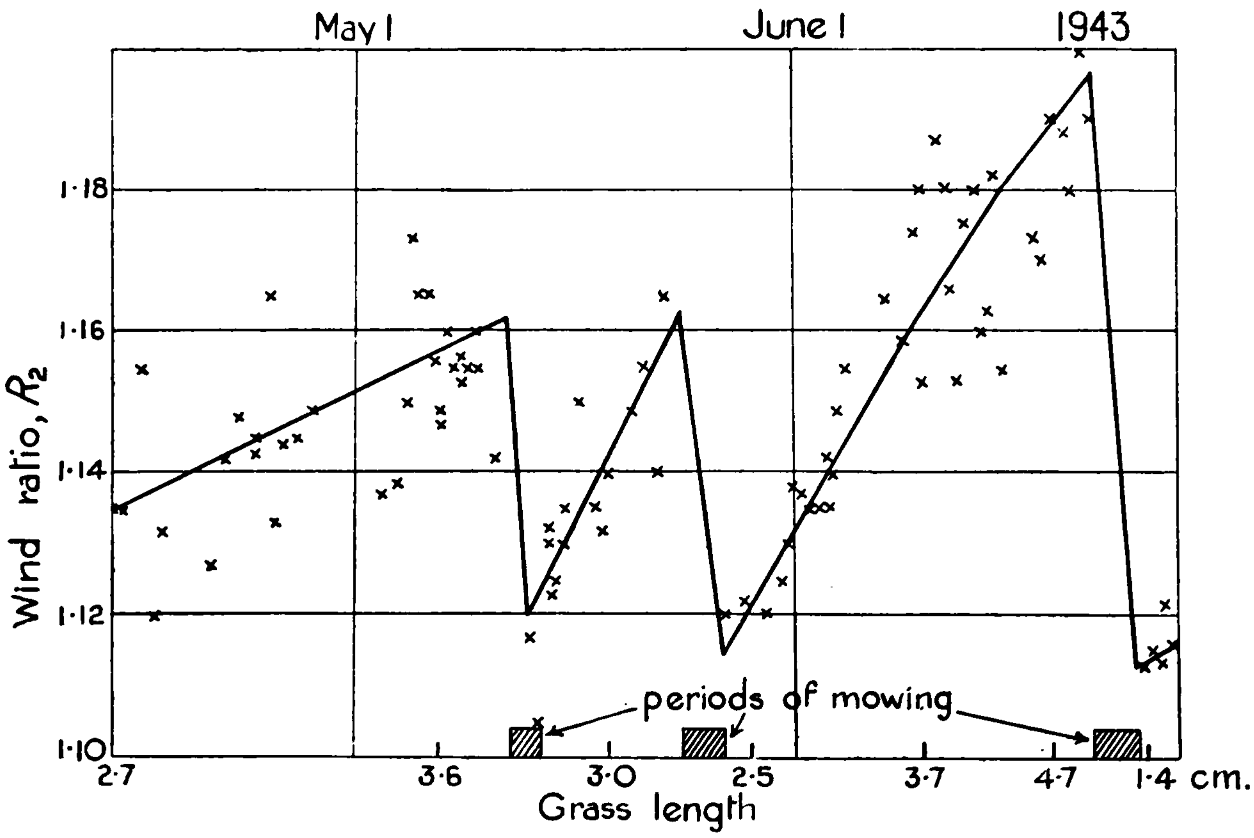


FIG. 6—EFFECT OF GROWTH OF GRASS ON R_2

to the grass and the heights of the four corners from the ground surface were measured. This was repeated 30 times moving about 3 m. each time, starting near the anemometers and proceeding in the direction of the prevailing SW. wind. The mean of the 120 measurements so obtained was taken as the grass length of the area, the method being found to give reproducible results. Fig. 7 shows the variation of the roughness parameter of the grass surface with the length of grass as measured by the above method.

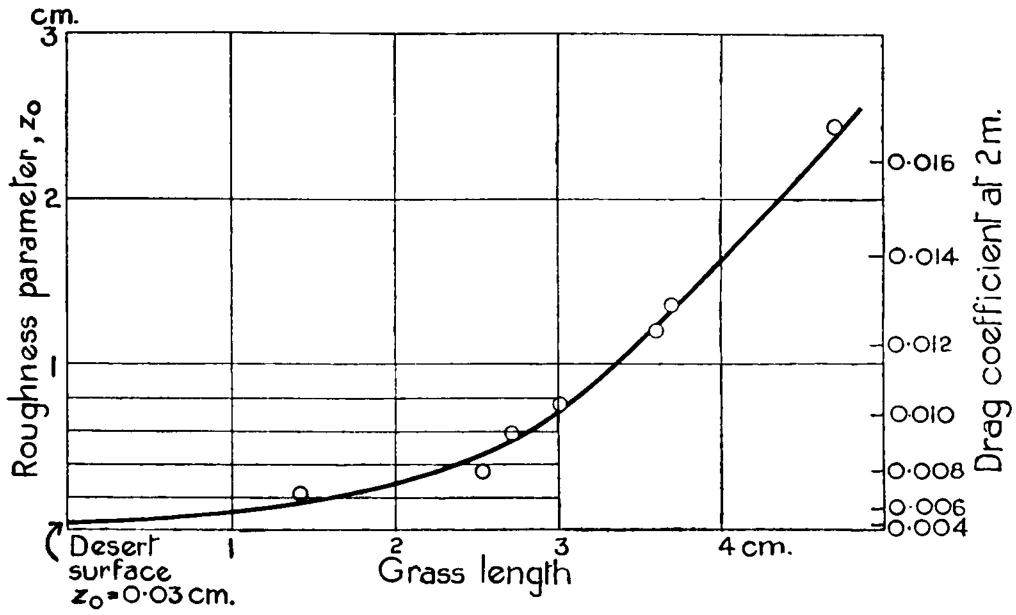


FIG. 7—ROUGHNESS PARAMETER RELATED TO GRASS LENGTH

§ 8—ROUGHNESS PARAMETER OF VARIOUS NATURAL SURFACES

Values of the roughness parameter for the various surfaces mentioned in the foregoing are collected in Table VI together with some values for the natural downland surface at Porton (several different points on the Porton experimental ground). The values of R_2 and the drag coefficients at 2 m. are also given.

TABLE VI—ROUGHNESS PARAMETERS OF VARIOUS SURFACES

Type of surface	R_2	z_0	Drag coefficient at 2 m.
		cm.	
Smooth mud flats	1·06	0·001	0·0021
Smooth snow on short grass	1·07	0·005	0·0024
Sea*	..	0·02	0·0038
Desert near Karachi†	1·085	0·03	0·0041
Snow surface, Canada‡	1·10	0·10	0·0055
Mown grass surface			
grass length 1·5 cm.	1·11	0·2	0·0067
grass length 3·0 cm.	1·14	0·7	0·010
grass length 4·5 cm. u_2 = 2 m./sec.	1·185	2·4	0·017
grass length 4·5 cm. u_2 = 6–8 m./sec.	1·15	1·7	0·014
Long grass, 60–70 cm. high u_2 = 1·5 m./sec.	1·275	9·0	0·033
Long grass, 60–70 cm. high u_2 = 3·5 m./sec.	1·25	6·1	0·026
Long grass, 60–70 cm. high u_2 = 6·2 m./sec.	1·21	3·7	0·021
Downland at Porton winter	1·15–1·18	c. 1–2	0·011–0·015
Downland at Porton summer	1·18–1·22	c. 2–4	0·015–0·021

* Observations made by Sir Nelson Johnson (see Appendix).
† Observations made by K. L. Calder (see Appendix).
‡ Observations made by C. H. B. Priestley.

For purposes of comparison Fig. 8 shows on the left the values of Table VI indicated against a vertical scale of roughness parameter, and on the right values found by other workers. There is, in general, satisfactory agreement between the values for land surfaces. No very exact agreement can be expected when account is taken of the difficulty of describing natural surfaces which vary greatly in texture. As already explained the difference in the case of the snow surface is due to the snow on short grass being much smoother than the great majority of natural snow surfaces.

The roughness parameter for a sand surface, given in Fig. 8, is from wind-tunnel measurements by Bagnold[27]. A level layer of sand (grain diameter 0·25 mm.) was subjected for a short time to the action of an air current sufficiently strong to cause saltation of the grains, thereby producing a normal surface of wind-blown sand ; that is one not only pitted with tiny bombardment craters a few grain diameters in size but undulating in the usual flat transverse ripples. This surface was " fixed " by spraying with water mist and the profile measurements made. The roughness parameter for this sand surface and for the desert surface of sun-baked sandy alluvium are very similar, as would be expected.

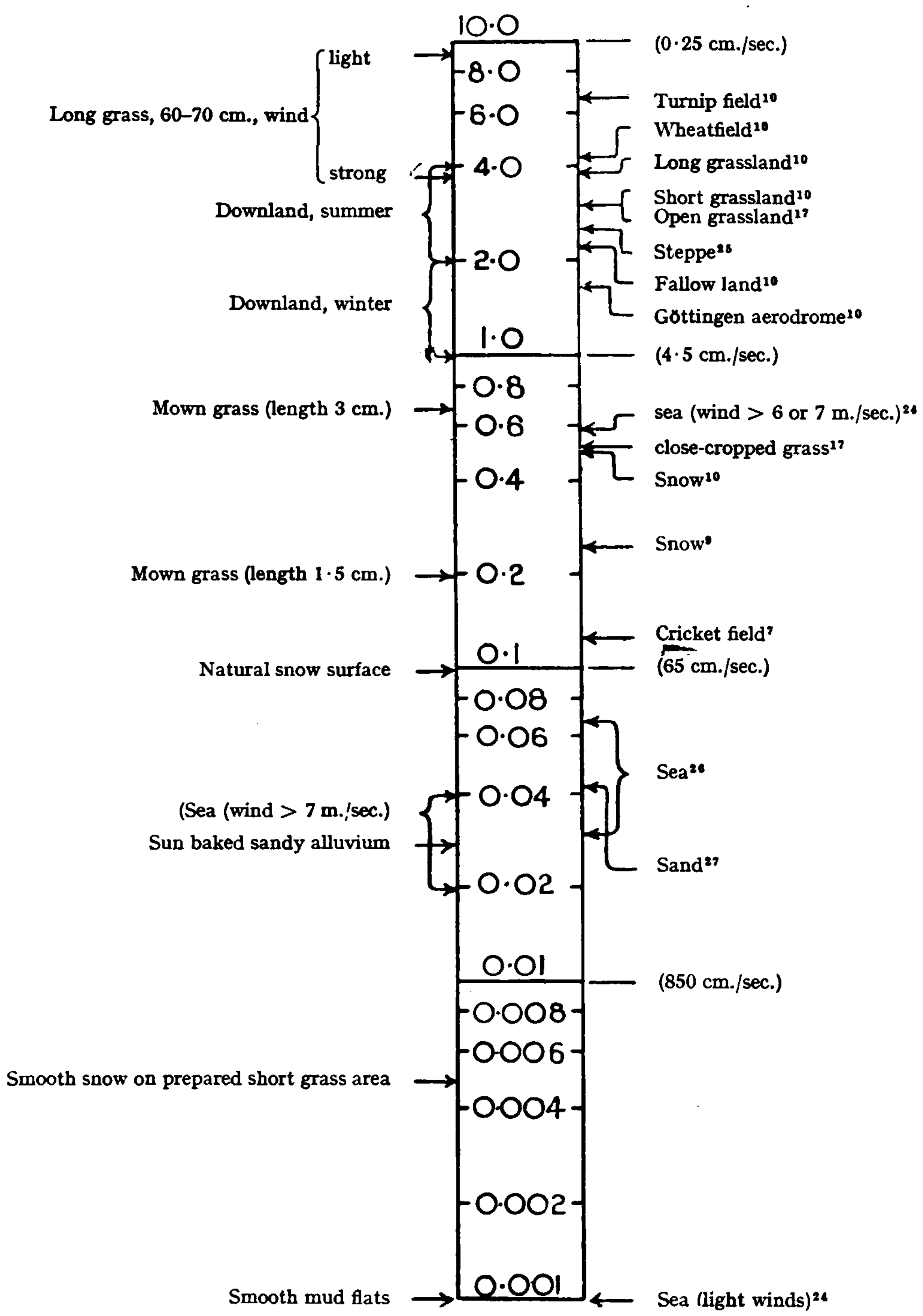


FIG. 8—ROUGHNESS PARAMETER (CM.) OF VARIOUS SURFACES
The speeds on the right-hand side are the minimum speeds at 1 m. for which fully rough flow would be expected

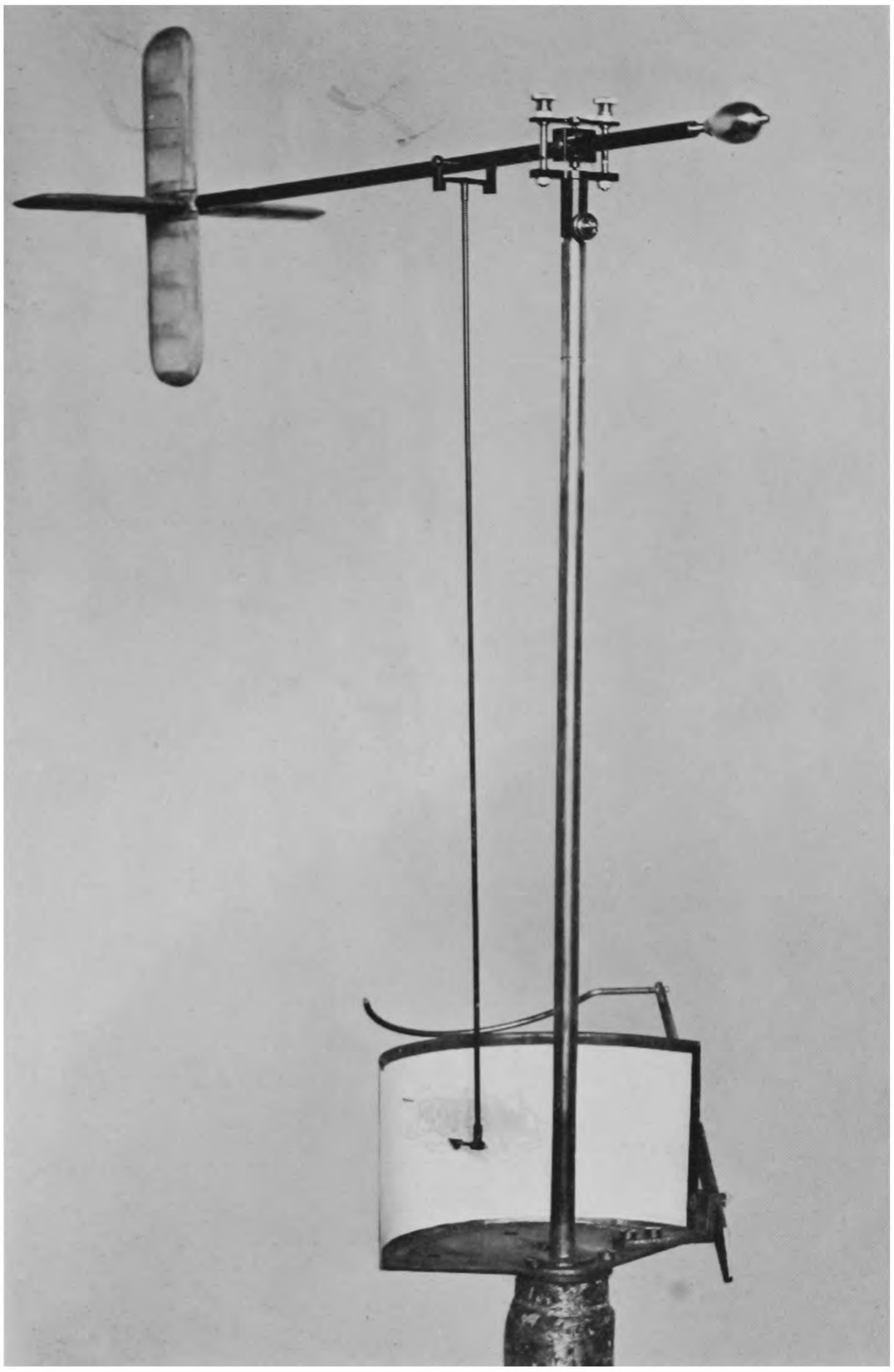


FIG. 9—SHEPPARD-PATTERN BI-DIRECTIONAL WIND VANE

The roughness parameter of the sea surface is rather controversial owing to the paucity of data, which moreover are not consistent. Rossby and Montgomery[24] found that at wind speeds less than about 6–7 m./sec. at 15 m. the sea surface is aerodynamically smooth, but that with stronger winds the surface is rough with a roughness parameter of about 0·6 cm., which is independent of wind speed. Model[26], however, on the basis of profile measurements by Bruch[28] concludes that the roughness parameter is from 0·03 to 0·06 cm. (depending on whether or not a zero-point displacement should be applied), but agrees in finding the roughness independent of wind speed for moderate and strong winds. The roughness parameter of the sea for winds greater than 7·5 m./sec. at 15 m. given by Sir Nelson Johnson's measurements is about 0·02–0·04 cm., which agrees more closely with Model's result than with that of Rossby and Montgomery. The observations were few in number, but wind gustiness values obtained at the same time also support the value given by the wind profiles. The sea observations are discussed more fully in the Appendix. Fig. 8 also indicates the minimum speeds (at 1 m.) for which fully rough flow would be expected on the basis of the criteria in expression (5), p. 13. This suggests that transitional or smooth flow is only likely to occur over surfaces with a vegetation cover at wind speeds of 0·5 m./sec. or less. With smoother surfaces, however, such as very level snow, mud flats, ice-sheets and water surfaces (at low wind speeds) transitional or aerodynamically smooth flow is likely to occur within the normal meteorological range of wind speeds.

§ 9—VERTICAL EDDY VELOCITIES UNDER CONDITIONS OF NEUTRAL STABILITY

On Prandtl's mixing-length hypothesis the eddy velocities at any given distance above a boundary surface should be proportional to the friction velocity as both are assumed to be proportional to $l du/dz$, where l is the mixing length[52]. From this reasoning, it is to be expected that the ratio of mean eddy velocity to the mean wind speed will be independent of wind speed under neutral conditions of stability. A knowledge of eddy velocities and the vertical wind gradient would therefore be illuminating in the study of vertical interchange processes. A considerable body of data has been obtained using bi-directional vanes over various types of terrain, and mean values for adiabatic conditions are presented below.

The bi-directional vane is admittedly a far from perfect instrument for the measurement of eddy velocities ; in the first place it is only possible to obtain from the records a kind of " mean extreme " value of the eddy velocities, and secondly, because of inertia, the vane tends to respond unduly to eddies of about 1-sec. period. Fortunately, however, the aerodynamic damping is considerable so the distortion due to this failing is not very great. The chief merit of the bi-directional vane is its simplicity, which has permitted considerable data to be collected under conditions which would often have precluded the use of more refined apparatus. The pattern of vane used is shown in Fig. 9 and is an improvement, due to Sheppard, of the Taylor vane. The pivot-tail distance is 25 cm. and the moment of inertia 8,600 C.G.S. units. Jewelled bearings are used in the universal joint to secure a low bearing friction.

The method of analysing the traces given by the bi-directional vane is that described by Best[7] (p. 48), and in all cases the duration of the record was three minutes. The smallest ovals were drawn round the records consistent with not excluding more than 9 loops in each case. The drawing of inner ovals round the denser parts of the traces was abandoned as experience has shown the uncertainty of this process. From the semi-diameters of the ovals were obtained the corresponding angular excursions of the vane (in radians) from the mean wind direction. These values are termed the gustiness values—g_y the lateral and g_z the vertical gustiness. As an approximation these gustiness values are equal to the ratio which the " mean extreme " eddy velocity in the appropriate direction bears to the mean wind velocity, i.e.

$$g_z \simeq \frac{W'}{u}$$

where W' is the " mean extreme " vertical eddy component.

Variation of gustiness with wind speed.—The values of gustiness measured under neutral conditions at a height of 2 m. over grass about 3 cm. long have been grouped according to the 2-m. wind speed, and the mean values of these groups are shown in Fig. 10 plotted against wind speed.

The lateral gustiness values, g_y, are rather scattered, and there is no significant correlation with wind speed. The coefficient of variation of lateral gustiness values is nearly 50 per cent. greater than that of the vertical values, owing to the mean wind direction in the horizontal plane being rather ill defined, on account of the action of large-scale turbulence and "wind swings". There were occasions when a definite wind shift during the 3-min. period resulted in a dumb-bell shaped record instead of the usual oval. These records were rejected, but there is, of course, no clear-cut distinction between wind shifts and the normal eddy motion. Particularly over irregular terrain, the mean wind direction in the horizontal plane is more a convenient mathematical idealization than a physical reality.

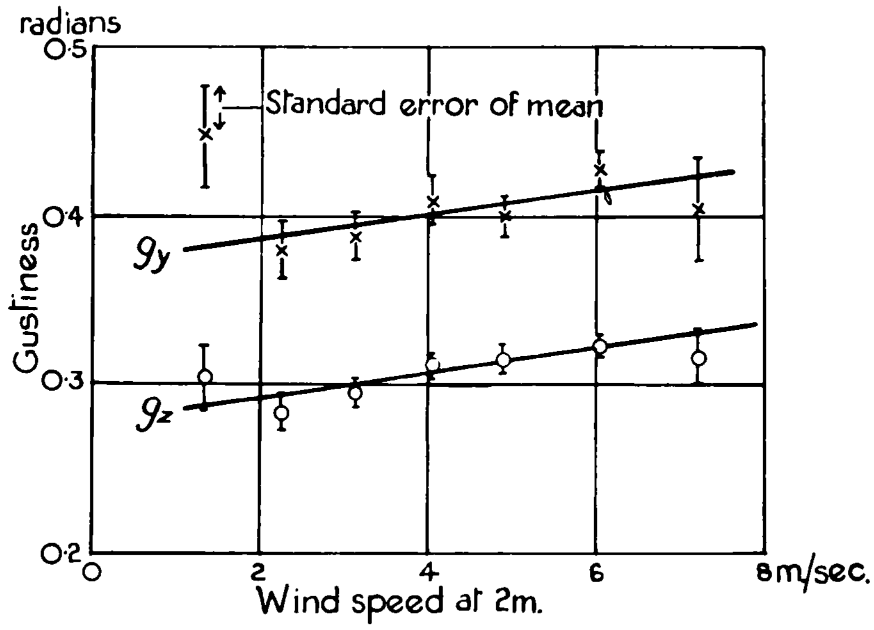


FIG. 10—VARIATION OF GUSTINESS WITH WIND SPEED

The vertical gustiness values show a tendency to increase with wind speed; the correlation coefficient between g_z and u is significant but small ($+ 0{\cdot}27$), the number of observations being 144, each the mean of two records taken in a 15-min. period. The regression line given in Fig. 10 indicates about a 10 per cent. increase in g_z for a four-fold increase of wind speed. When account is taken of the fact that the response characteristics of the bi-directional vane vary with wind speed, being rather poor in light winds, it is probable that the vertical gustiness may be taken to be approximately independent of wind speed, in conformity with Prandtl's theory and with the constancy of wind ratios over short grass under neutral conditions of atmospheric stability.

Variation of vertical gustiness with roughness of surface.—Considerable numbers of simultaneous observations of gustiness by bi-directional vane and of R_2 are available for a variety of surfaces, and the mean values for neutral conditions of stability are given in Table VII together with the roughness parameter (z_0) and u_*/u_2 calculated from the logarithmic velocity law. In all cases the gustiness was measured at 2 m. above the surface and the standard method of analysis adopted. The vanes used were all Sheppard pattern and of standardized dimensions, so that differences due to instrumental technique should be quite small.

The mean values of g_z from Table VII are plotted in Fig. 11 against u_*/u_2, and in those cases where it could be calculated the standard error of the mean value has been indicated by ellipses round the points. Within the limits of experimental error the vertical gustiness is directly

TABLE VII—VARIATION OF GUSTINESS WITH TYPE OF SURFACE: NEUTRAL STABILITY

	No. of obs.	u_2	R_2	z_0	u_*/u_2	g_z at 2 m.	$\frac{g_y}{g_z}$
		m./sec.		cm.			
Downland	108	3–7	1·188	2·0	0·086	0·359	1·52
Short grass	278	2–9	1·140	0·7	0·071	0·308	1·35
Sparse grassland ..	12	..	1·14	0·7	0·071	0·29	1·48
Natural snow surface*	83	..	1·100	0·1	0·052	0·205	1·37
Desert near Karachi† ..	53	3–7	1·085	0·03	0·045	0·190	1·33

* Observations obtained by Priestley. † Observations obtained by Calder.

proportional to u_*/u_2 giving the relationship

$$\frac{W'_2}{u_*} = 4 \cdot 15 \qquad \dots\dots (7)$$

where W'_2 is the " mean extreme " vertical eddy velocity at 2 m.

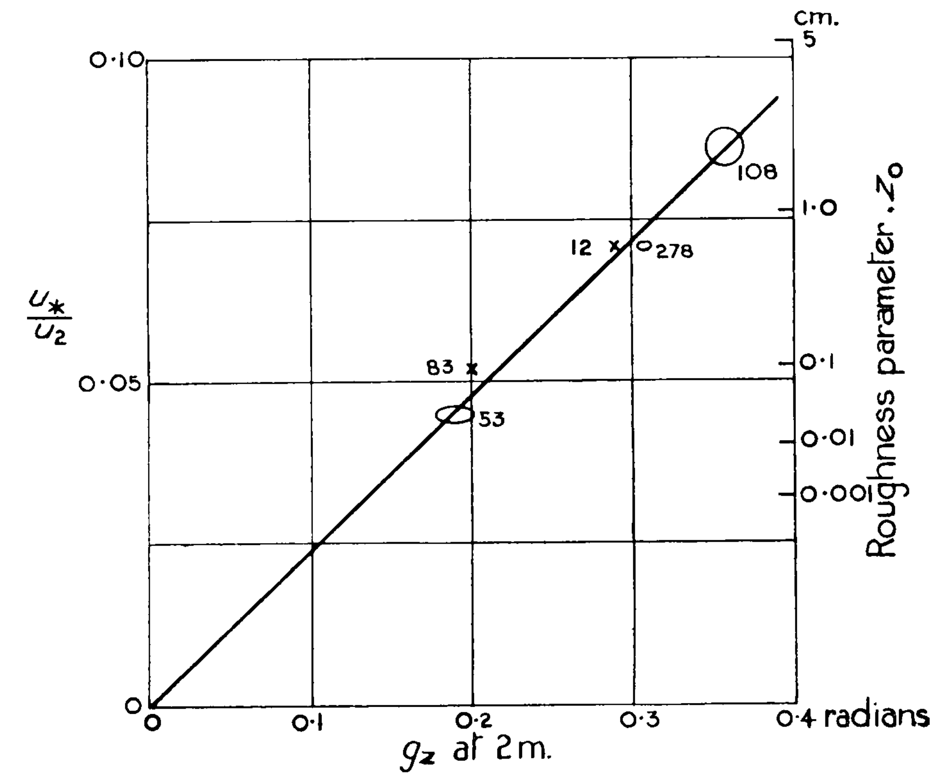


FIG. 11—VERTICAL EDDY VELOCITIES RELATED TO THE FRICTION VELOCITY

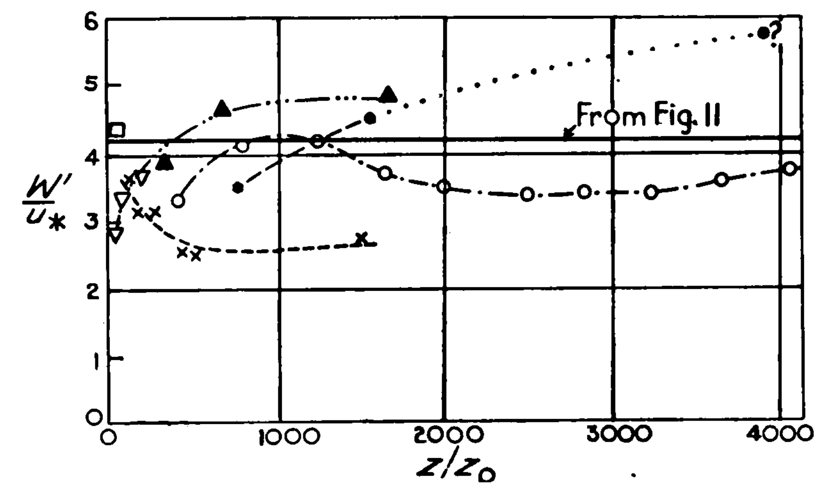
The number of observations is given at the side of each plotted point. The standard error of the mean value where it could be calculated has been indicated by ellipses round the points.

Variation of vertical gustiness with height.—On the mixing-length hypothesis the mean vertical eddy velocity should be invariable with height in the layer in which the shearing stress is constant. In Fig. 12 the results of three investigations using bi-directional vanes are given in the form of a non-dimensional plotting of the ratio of the " mean extreme " vertical eddy velocity, W', to the friction velocity, u_*, against the ratio of the height above the surface to the roughness parameter, z/z_0. Taking the vertical dimensions of the larger eddies effective in momentum transfer in a layer of air adjacent to the ground to be of similar magnitude to the depth of the layer, it follows that the vane records should not be relied upon at heights above the surface much less than 10 times the pivot-tail length of the vane. This dimension was about 20 cm. in the vanes used so that observations at 1-m. height may be somewhat in error ; at lower heights than 1 m. the records are unlikely to have much significance for the present purpose and accordingly have been omitted.

The vanes used in these three sets of experiments were of different construction, and likely therefore to have somewhat different response characteristics. The responsiveness of a vane depends on the ratio of the aerodynamic restoring couple to the moment of inertia. Taking the ratio for the Sheppard vane (Fig. 9) as unity, the relative values are

Vane used by Scrase[29] ..	0·25
Vane used by Best[7] ..	0·65
Sheppard vane	1·0

Considering now the observations shown in Fig. 12, it is seen that the values given by Scrase[29] are considerably lower than the others, which may well be owing to the relatively poor response characteristics of his vane, which also had greater bearing friction than the later types. There is, however, some uncertainty in the u_* values for Scrase's experiments as vertical wind gradients were not measured at the same time. The value of u_* was calculated from a value of z_0 of 1·2 cm. derived from wind velocities at 13·4 m. and 3·0 m. measured by vane air-meters over the same site under neutral conditions†. This value of z_0 appears to be a little low for downland, but a greater value, by leading to a higher friction velocity, would increase the discrepancy between Scrase's result and the rest.



Key	Author	No. of obs. at each height	z_0 cm.
×----×	Scrase[29]	2 to 10	1·2
●—–●	Best[7]	circa 50	0·13
▲—···▲	Simpson—shortgrass ..	31	0·3
▽—···▽	Simpson—long grass ..	15	3·1
○—·—○	Fage and Townend[30] ..	..	$1·4 \times 10^{-4}$
□	Fage[31]	..	0·020

FIG. 12—VARIATION OF VERTICAL EDDY VELOCITY WITH DISTANCE FROM SURFACE

The observations by Best[7] (Table 34) and those by J. H. Simpson (unpublished) both show an increase in gustiness with height, but Best's 5-m. value is probably unduly high, as the cricket field over which he made his measurements is not of sufficient size for 5-m. measurements to be entirely characteristic of the short-grass surface. The travel of the wind over short grass was nearly twice as great—165 m. (180 yd.) against 91 m. (100 yd.)—in Simpson's experiments, which should also be rather more accurate as he used three similar instruments (at 1, 2 and 5 m.) simultaneously.

The nature of the variation of W' with height is seen to be rather uncertain. Scrase's results indicate, mainly, a decrease with height; the observations over various surfaces give a constant value over a considerable range of z/z_0, while the observations of Best and Simpson show W' to increase with height.

Fig. 12 also gives the results of Fage and Townend[30] who measured extreme eddy velocities by the ultra-microscopic method in a smooth pipe of square cross-section. The extreme eddy velocities given by this method should be comparable, within reasonable limits, with the values given by the bi-directional vane. In the pipe experiments the value of friction velocity at the

† The measurements are referred to by Scrase[29] in the footnote to p. 9. The relation $u \propto z^{0·13}$ quoted by Scrase was based on these observations, together with a greater number, using records from pressure-tube anemographs. These latter observations have been discarded as insufficiently accurate for determining the roughness parameter.

wall is given by the pressure drop along the pipe, and u_* at each distance from the wall is found from the linear decrease of shear stress from the wall value to zero at the pipe centre. The range of distances from the pipe wall covered by the values given in Fig. 12 is $0\cdot05s$ to $0\cdot5s$, s being equal to a half-side of cross-section of the pipe. Nearer the centre of the pipe conditions cannot be expected to be similar to those over a plane surface. The value of z_0 for the smooth surface† is $\nu/9\cdot05u_*$ which, under the conditions of the experiments, was $1\cdot4 \times 10^{-4}$ cm.

Similar measurements by Fage[31] in a rough pipe gave the following values :—

z/s ..	0·2	0·25	0·3	0·35	0·4
z/z_0 ..	12	15	18	21	24
W'/u_* ..	5·25	4·0	4·7	4·3	4·5

$\underbrace{4\cdot0 \quad 4\cdot7 \quad 4\cdot3 \quad 4\cdot5}_{\text{Mean } 4\cdot35}$

The mean value, 4·35, is shown in Fig. 12, and it will be seen that the pipe experiments as a whole indicate a ratio of W' to u_* constant within about $\pm$ 15 per cent. over the range z/z_0 from 15 to 4,000, the mean value being 3·7 as compared with 4·15 given by the observations of gustiness at a height of 2 m. over various surfaces, the range of z/z_0 in that case being from 100 to 7,000. Taking into account the great difference in scale between the pipe experiments and the atmospheric observations, the concordance between the results is rather striking.

PART III—EFFECTS OF THERMAL STRATIFICATION

§ 10—WIND PROFILES OVER A SHORT-GRASS SURFACE

The wind-speed measurements obtained over short grass (Series II) will be considered first. In this case the wind had a fetch of 90 m. over the short grass before reaching the anemometer installation (anemometers at heights of 1·00, 1·98, 5·22 and 13·27 m.). As already mentioned in Part II, evidence was obtained that the wind profiles with this fetch were not characteristic of the short-grass surface up to 13 m., and that, even at 5 m., there was probably a residual effect of the surrounding longer-grass surface. In the analysis of these results, therefore, most attention was paid to the 1·98 : 1 m. wind ratio which approximates so closely to the 2:1 m. ratio, R_2, that the difference can be neglected.

§ 11—DEPENDENCE OF R_2 ON TEMPERATURE GRADIENT AND WIND SPEED

To eliminate as far as possible the effect of varying surface roughness, the periods during which the value of R_2 under neutral conditions was constant within $\pm0\cdot01$ were selected from graphs such as Fig. 6. The observations obtained during these periods were grouped for wind speed at 2 m. and temperature difference between heights of 17·1 and 1·2 m. Table VIII gives the mean values of R_2 for each group and the numbers of observations.

The values of Table VIII for grass length 2·5–3·2 cm., which contains the greatest numbers of observations, are shown plotted against the 17·1–1·2 m. temperature difference in Fig. 13. The points for the ranges of wind speed between 4 and 7 m./sec. have, however, been omitted for the sake of clarity. For wind speeds above 5 m./sec. at 2 m. the variation of the wind ratio with temperature gradient is seen to be slight. The variation progressively increases as the wind speed decreases, as was also found by Best[7]. This shows, as would be expected, that at high wind speeds the buoyancy forces evoked by the temperature gradient become almost negligible as compared with the dynamical forces due to friction.

† Compare equations (3) and (4).

TABLE VIII—MEAN VALUES OF R_2 RELATED TO TEMPERATURE GRADIENT AND WIND SPEED AT 2 M.

Grass length: 2·5–3·2 cm. Neutral value of R_2: 1·125–1·145

Wind speed at 2m.	Temperature difference, 17·1–1·2 m. (°F.) −3·5 to −3·0	−3·0 to −2·5	−2·5 to −2·0	−2·0 to −1·5	−1·5 to −1·0	−1·0 to −0·5	−0·5 to 0·0	0·0 to 0·5	0·5 to 1·0	1·0 to 1·5	1·5 to 2·0	2·0 to 2·5	2·5 to 3·0	3·0 to 3·5	3·5 to 4·0	4·0 to 4·5	4·5 to 5·0	5·0 to 6·0	6·0 to 7·0	7·0 to 8·0	No. of obs.
m./sec. 10–12	..	..	..	1·120 1/..	1·125 5/..	1·135 3/..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	9
8–10	..	1·125 2/..	1·120 3/..	1·135 4/..	1·130 6/..	1·138 8/..	1·125 1/..	..	..	..	..	..	..	..	..	..	..	..	..	..	24
7–8	..	..	1·132 3/..	1·129 6/..	1·131 8/..	1·134 5/..	1·134 7/..	..	..	..	..	..	..	..	..	..	..	..	..	..	29
6–7	1·120 2/..	1·120 12/5	1·127 16/12	1·132 12/9	1·132 15/8	1·138 17/11	1·137 11/7	1·138 6/..	..	..	..	..	..	..	..	..	..	..	..	..	91
5–6	1·115 1/..	1·113 19/11	1·123 26/12	1·125 22/9	1·126 16/9	1·123 7/..	1·138 19/8	1·136 26/12	1·135 2/..	..	..	..	..	..	..	..	..	..	..	..	138
4–5	..	1·116 8/..	1·118 27/10	1·126 23/11	1·125 19/14	1·135 17/8	1·143 27/10	1·132 21/16	1·137 9/..	1·150 1/..	..	..	..	..	..	..	..	..	..	..	152
3–4	..	1·112 15/17	1·113 28/11	1·121 15/14	1·132 7/..	1·132 11/8	1·139 21/15	1·135 22/15	1·146 19/15	1·151 15/10	1·157 3/..	..	..	..	..	..	..	..	..	..	156
2–3	..	1·112 3/..	1·109 16/12	1·108 3/..	1·127 5/..	1·125 2/..	1·146 3/..	1·152 3/..	1·148 8/..	1·181 12/16	1·185 13/26	1·183 10/33	1·214 7/..	1·290 6/..	1·260 4/..	..	1·320 3/..	..	..	..	98
1–2	..	..	1·092 2/..	1·092 12/12	1·117 6/..	1·152 5/..	1·137 2/..	1·135 1/..	1·200 1/..	1·215 7/..	1·248 10/54	1·289 30/81	1·318 26/78	1·319 26/80	1·308 26/88	1·362 10/103	1·321 8/..	1·220 9/..	1·130 1/..	1·240 3/..	185
0·5–1	..	..	..	..	..	1·140 1/..	1·130 1/..	1·180 1/..	..	1·259 5/..	1·180 10/57	1·280 5/..	1·305 10/130	1·330 10/150	1·200 1/..	1·280 3/..	..	..	..	..	47
No. of observations	3	59	121	98	87	76	92	80	39	40	36	45	43	42	31	13	11	9	1	3	929

The number to the left of the solidus is the number of observations meaned; where the number is 10 or more the standard deviation multiplied by 1,000 is shown to the right of the solidus.

TABLE VIII—*continued*

Grass length : 1·7–2·5 cm.

Neutral value of R_2 : 1·114–1·126

Wind speed at 2 m.	Temperature difference, 17·1–1·2 m. (°F.) −3·5 to −3·0	−3·0 to −2·5	−2·5 to −2·0	−2·0 to −1·5	−1·5 to −1·0	−1·0 to −0·5	−0·5 to 0·0	0·0 to 0·5	0·5 to 1·0	1·0 to 1·5	1·5 to 2·0	2·0 to 2·5	2·5 to 3·0	3·0 to 3·5	3·5 to 4·0	4·0 to 4·5	4·5 to 5·0	No. of obs.
m./sec. 6–7	..	1·121 4	1·113 3	..	1·120 2	1·115 1	..	..	..	..	..	..	..	..	..	..	..	10
5–6	1·115 1	1·113 5	1·119 8	1·122 4	1·122 9	1·120 1	..	..	..	..	..	..	..	..	..	..	..	28
4–5	..	1·102 4	1·113 14	1·121 6	1·111 4	1·120 4	1·125 1	1·140 1	1·125 2	..	..	..	..	..	..	..	..	36
3–4	..	1·097 16	1·103 6	1·104 13	1·117 2	1·119 7	1·127 5	1·134 12	1·139 16	1·145 6	1·152 2	1·140 1	..	..	..	..	..	86
2½–3	..	1·100 1	..	1·105 5	1·110 1	1·095 1	1·112 4	1·141 6	1·146 5	1·155 2	1·170 6	1·155 1	..	..	..	..	..	32
2–2½	..	..	1·080 1	1·110 2	1·121 4	1·108 3	1·135 1	1·150 1	1·163 6	1·182 3	1·190 2	1·186 7	1·233 5	..	..	..	..	35
1½–2	..	..	..	1·092 8	1·101 3	1·113 2	1·123 2	1·143 1	1·182 5	1·176 3	1·213 2	1·253 3	1·220 2	1·315 3	1·320 2	1·135 3	..	39
1–1½	..	..	1·060 1	1·071 4	1·106 4	1·102 3	1·127 2	..	1·171 5	1·202 5	1·209 6	1·230 1	1·222 4	1·302 10	1·283 8	1·410 2	1·235 1	56
No. of observations	1	30	33	42	29	22	15	21	39	19	18	13	11	13	10	5	1	322

The numbers below the mean values denote number of observations.

TABLE VIII—*continued*

Grass length: 4·0–4·7 cm. Neutral value of R_2: 1·170–1·190

Wind speed at 2 m.	Temperature difference, 17·1–1·2 m. (°F.) −4·0 to −3·5	−3·5 to −3·0	−3·0 to −2·5	−2·5 to −2·0	−2·0 to −1·5	−1·5 to −1·0	−1·0 to −0·5	−0·5 to 0·0	0·0 to 0·5	0·5 to 1·0	1·0 to 1·5	1·5 to 2·0	2·0 to 2·5	2·5 to 3·0	3·0 to 3·5	3·5 to 4·0	4·0 to 4·5	4·5 to 5·0	No. of obs.
m./sec. 7–8	1·137 8	1·159 6	1·158 6	1·169 4	1·160 6	1·167 2	1·167 3	1·175 1	..	..	..	..	..	..	..	..	..	..	37
6–7	1·154 7	1·152 6	1·152 11	1·167 7	1·152 10	1·162 12	1·170 7	1·182 10	1·175 1	..	..	..	..	..	..	..	..	..	71
5–6	1·146 4	1·151 9	1·149 13	1·152 10	1·155 14	1·151 5	1·165 11	1·170 30	1·185 2	..	..	..	..	..	..	..	..	..	98
4–5	1·155 4	1·140 3	1·157 14	1·159 15	1·158 12	1·167 14	1·175 16	1·167 22	1·193 6	1·200 1	..	..	..	..	..	..	..	..	107
3–4	1·156 6	1·159 7	1·155 4	1·152 2	1·167 9	1·169 9	1·174 9	1·185 34	1·194 18	1·201 8	1·185 1	..	..	..		..	..	..	107
2½–3	1·143 3	1·162 2	1·150 2	1·155 3	1·162 4	1·155 1	1·179 5	1·193 14	1·202 8	1·226 5	1·192 2	1·202 4	..	..	..	..	..	..	53
2–2½	..	..	..	1·150 1	..	..	1·170 1	1·189 9	1·194 11	1·216 5	1·249 5	1·230 5	..	1·265 3	..	..	..	..	40
1½–2	..	..	..	..	..	1·155 1	1·150 1	1·191 7	1·219 5	1·243 9	1·277 3	1·303 4	1·348 8	1·384 3	1·350 1	1·490 1	..	1·450 4	47
1–1½	..	..	..	..	..	..	..	1·237 2	1·202 2	1·355 2	1·250 1	1·440 2	1·412 4	1·400 2	1·500 2	1·427 11	1·330 6	1·460 1	35
No. of observations	32	33	50	43	55	44	53	129	53	30	12	15	12	8	3	12	6	5	595

The numbers below the mean values denote number of observations.

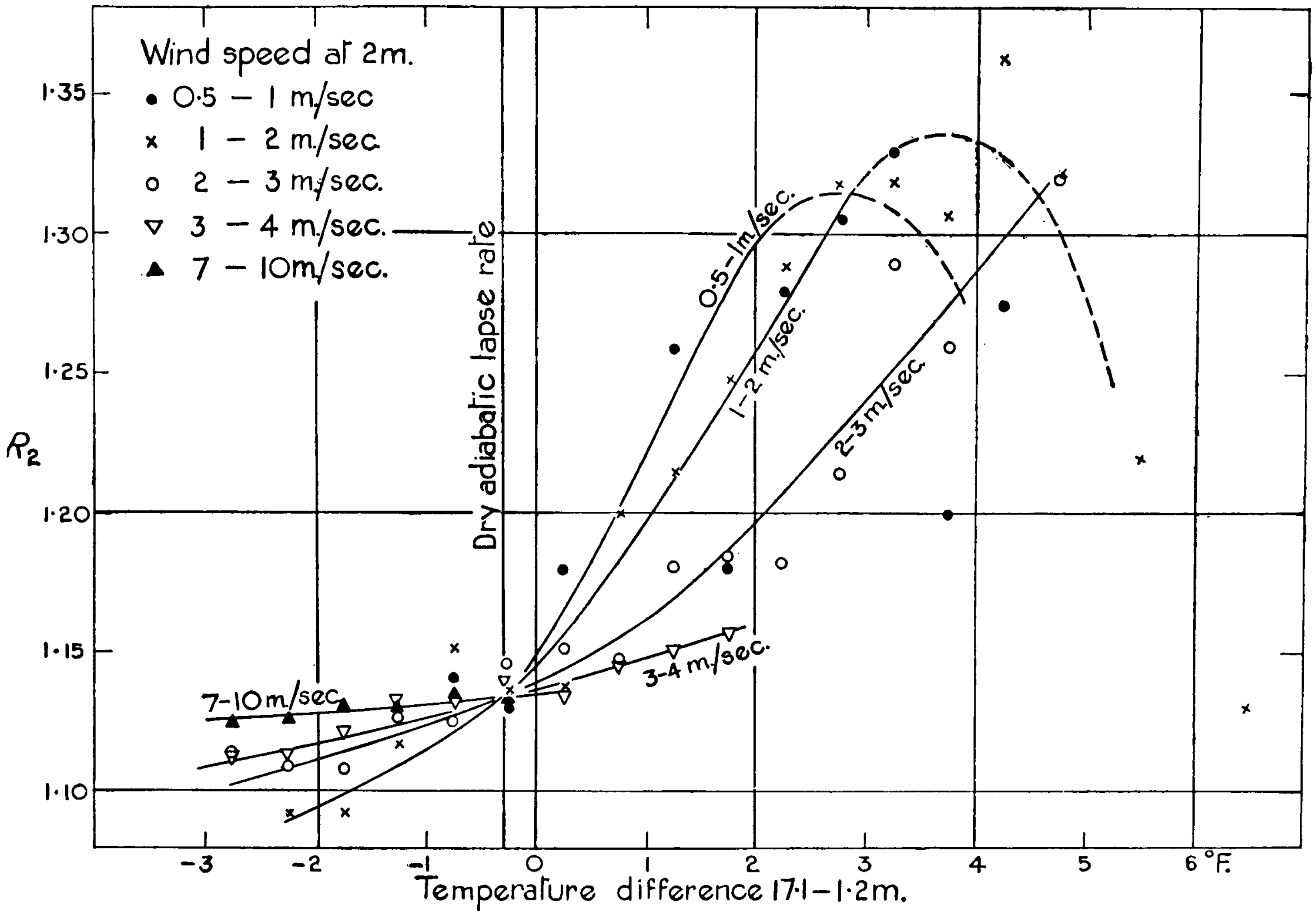


FIG. 13—DEPENDENCE OF WIND RATIO ON TEMPERATURE GRADIENT AND WIND SPEED OVER SHORT-GRASS SURFACE

Grass length 2·5–3·2 cm.

For conditions of large temperature inversion and wind speed, at 2 m., less than 2 m./sec. the results show a sudden decrease in the wind ratio from a high value ($> 1\cdot3$) to values not much greater in some cases than the adiabatic value. This occurs with wind speeds at 2 m. of 1–2 m./sec. when the inversion (17·1–1·2 m.) exceeds about 4° F.; for winds from 0·5 to 1 m./sec. the fall commences sooner, i.e. at + 2° or + 3° F. Turbulence under such conditions is very small or even nil, and a probable explanation of the above features is that shallow air currents of katabatic origin then become of importance. This explanation receives some support from the fact that a number of records during inversions approaching + 10° F. show "inverted" wind profiles, i.e. the greatest wind speed at 1 m. and the least at 13 m. No great reliance can be placed on these observations, as in these very light winds it is possible that some of the anemometer cup rotors were stationary for part of the time. The regularity of some of these inverted profiles suggests, however, that such was not always the case. Best also obtained some inverted wind profiles under conditions of strong inversion, the anemometers having been in continuous rotation.

The rather large effects of varying surface roughness, together with the effect of wind speed and possibly of katabatic currents, have in much previous work considerably obscured the relationship between wind ratios and the temperature gradient. In the present results random fluctuations are still quite appreciable as will be seen from the correlation coefficients in Table IX between R_2 and the 17·1–1·2 m. temperature difference for the data of Table VIII, grass length 2·5–3·2 cm.

Fig. 13 shows that the regression is not linear for the two lower wind ranges, but the departure is not great and should not greatly affect the values of the correlation coefficient. These values show that 50–70 per cent. of the variance of R_2 is accounted for by its dependence on temperature

TABLE IX—CORRELATION COEFFICIENTS BETWEEN R_2 AND THE TEMPERATURE DIFFERENCE, 17·1–1·2 m.

Wind speed at 2 m.	Range of temperature difference 17·1–1·2 m.	No. of obs.	Correlation coefficient
m./sec.	°F		
1 to 2	−2·5 to +4·5	165	+0·76
2 to 3	−3 to +5	98	+0·85
3 to 4	−3 to +2	156	+0·70

gradient, the wind speed and surface roughness being approximately constant. Taking into account the incomplete elimination of the effects of surface roughness and wind speed and the scatter due to experimental error, it seems probable that the wind gradient measured over a sufficiently large uniform surface is determined by the surface roughness, temperature gradient and wind speed, and that the effect of other variables is comparatively slight.

In Table VIII, grass length 2·5–3·2 cm., the standard deviations of the observations in each group from the mean are given. The standard deviations are reasonably constant at about 0·01–0·015 for lapse conditions, but increase rapidly for conditions of moderate or strong inversion. As the wind ratio is independent of wind speed at neutral stability, the standard deviation was calculated for the 168 values in the range of temperature difference 0° to − 1° F. and found to be 0·012. The frequency distribution of the values was symmetrical and very close to the normal form. Part of the variance is due to the range of surface roughness embraced in the analysis ; an approximate elimination of this gives a standard deviation of R_2, due to experimental and casual error, of about 0·010.

Errors in the temperature-gradient measurements as well as errors in the wind-speed measurements will, of course, contribute to the scatter of the points in Fig. 13. Because of the nature of the curves of R_2 plotted against temperature gradient, the magnitude of the errors caused by errors in temperature-gradient measurements will be at a minimum for lapses and increase for inversions.

The increase in the standard deviation of the values of R_2, from about 0·012 for neutral conditions to 0·06–0·08 under conditions of moderate inversion, is considered to be more connected with increased random errors in the measurement of the inversion temperature gradients than with errors in the wind-speed measurements themselves. This seems probable from a consideration of the sources of error (apart from instrumental error) which, under inversion conditions, may arise from :—

(i) fluctuating nature of inversion conditions
(ii) non-uniform nature of the inversions spatially
(iii) katabatic effects when turbulence has decreased to a low level.

Fluctuation of inversion conditions.—Steady conditions of moderate or large inversion are rare ; the normal behaviour is a more or less periodic building up, followed by a break-down which is often fairly rapid. The period of these fluctuations is frequently of the order of half to one hour (Johnson[2], p. 30 and Fig. 18). Any difference in phase between the temperature-gradient variations and the velocity-gradient changes would lead to errors in the analysis.

Non-uniformity of inversions spatially.—Owing to the low rate of turbulent mixing under inversion conditions large horizontal gradients of temperature occur. Best[32] showed that " During the hours of inversion . . . temperature differences up to 1·5° F. may persist between points at the same height 50 feet apart for periods up to 30 minutes ". The observations of temperature gradient in the work dealt with above were made at a point 180 m. away from the anemometer installation. This, coupled with the fact that the surface in the vicinity of the temperature-gradient tower was natural downland and not short grass as for the anemometers, would frequently give rise to rather large differences between the recorded temperature gradient and that actually influencing the wind-speed profiles over the short-grass area.

Katabatic effects.—When the turbulence has fallen to a low value and horizontal temperature differences of the order of those observed by Best are present, it is probable that katabatic drifts set in even over relatively level ground, and this tendency is, no doubt, the more pronounced in the presence of variations in the radiative and thermal properties of the soil from point to point.

§ 12—DEPENDENCE OF WIND RATIOS ON THE RICHARDSON NUMBER

As already mentioned, the decreasing dependence of the wind ratio upon the temperature gradient with increasing wind speed suggests that the ratio of buoyancy forces to frictional forces is the controlling factor in thermally stratified flow, in a similar manner to that in which the ratio of the inertial forces to the viscous forces is the important parameter (Reynolds number) in the absence of a density stratification. Now this ratio of buoyancy to frictional forces is expressed by the non-dimensional Richardson number,

$$\frac{g}{\theta}\,\frac{\dfrac{d\theta}{dz}}{\left(\dfrac{du}{dz}\right)^2},$$

where g is the acceleration due to gravity, $d\theta/dz$, the vertical gradient of potential temperature, and θ, potential temperature on the absolute scale. In dealing with the lowest few metres of the atmosphere θ may be taken equal to the mean air temperature of the layer. Strictly, the denominator of the Richardson number should contain the square of the vector wind shear, but in the lowest few metres of the atmosphere the variation of mean wind direction with height is so small as to be negligible.

Considering now the air layer between two constant levels, 1 m. and 2 m. for example, it would accordingly be expected that

$$R_2 = \frac{u_2}{u_1} = f\left\{\frac{\theta_2 - \theta_1}{(u_2 - u_1)^2}\right\} \qquad \ldots\ldots (8)$$

taking T and g to be constant and f denoting functional dependence. This gives

$$R_2 = f\left\{\frac{\theta_2 - \theta_1}{u_1^2\,(R_2 - 1)^2}\right\}$$

$$= f\left\{\frac{\theta_2 - \theta_1}{u_2^2}\left(\frac{R_2}{R_2 - 1}\right)^2\right\},$$

which can be put in the form

$$R_2 = f_1\left(\frac{\theta_2 - \theta_1}{u_1^2}\right)$$

$$= f_2\left(\frac{\theta_2 - \theta_1}{u_2^2}\right). \qquad \ldots\ldots (9)$$

Therefore if R_2 over a surface of constant roughness proves to be uniquely related to $(\theta_2 - \theta_1)/u_2^2$, then it follows that it is also uniquely related to the Richardson number in the 2–1 m. layer.

For the observations discussed above, the 2–1 m. potential-temperature differences are not available, but the difference $\theta_{17.1} - \theta_{1.2}$ for a given wind speed would be expected to be uniquely related to $\theta_2 - \theta_1$, so the values of $(\theta_{17.1} - \theta_{1.2})/u_2^2$ have been calculated for each group in Table VIII, grass length 2·5–3·2 cm., taking the temperature gradient and wind speed for each group to be the mid points of the respective ranges, except for the range 1–2 m./sec. in which case the true mean values were calculated as they sometimes differed appreciably from the mid points. Fig. 14 shows the relation of R_2 to $(\theta_{17.1} - \theta_{1.2})/u_2^2$. The wind ratio, for a surface of given roughness, is seen to be closely related, over a considerable range, to the stability parameter and hence to the Richardson number. The relationship is probably not exact for two reasons. First, long-wave radiation as well as eddy heat transfer must influence the temperature profile, but the effect is probably small, to judge from Elsasser's work[33], except when the turbulence has fallen to low intensity[34]. The second reason is that vertical interchanges of heat and

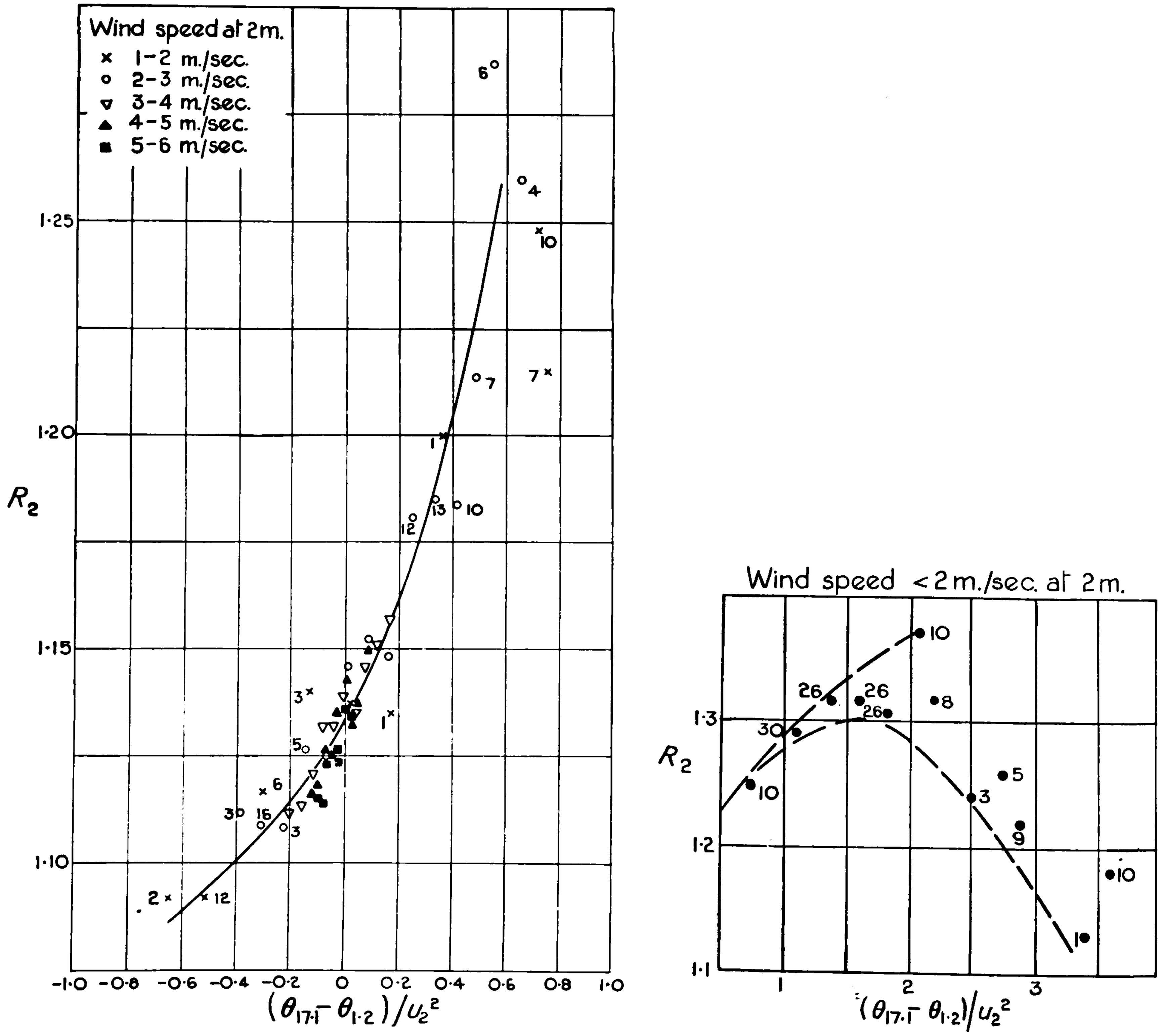


FIG. 14—R_2 AS A FUNCTION OF $(\theta_{17.1} - \theta_{1.2})/u_2^2$
The figures beside each point are the number of observations represented by that point

momentum are influenced by departures from horizontal uniformity, and, owing to buoyancy forces, the effects are different for the two entities as has been shown by Priestley and Swinbank[35]. For a given surface, however, it is likely that this influence is also roughly a function of the Richardson number as horizontal temperature differences are almost certainly strongly correlated with the vertical temperature gradient.

The remaining data of Table VIII display the same features, and all show a large scatter when $(\theta_{17.1} - \theta_{1.2})/u_2^2$ exceeds about $+ 0{\cdot}5$. This is probably owing to the effects already discussed in § 11 together with enhanced importance of radiative heat transfer and large horizontal temperature differences as found by Best under these conditions[32].

Further evidence for the dependence of the form of the wind profiles upon the Richardson number will be presented later. At this point it is convenient to examine the variation of the gustiness of the wind with temperature gradient and wind speed to see if this also displays a dependence on the Richardson number.

§ 13—VARIATION OF LATERAL AND VERTICAL GUSTINESS WITH TEMPERATURE GRADIENT AND WIND VELOCITY OVER SHORT GRASS

Observations of the lateral and vertical gustiness of the wind over short grass were made using a bi-directional vane (see § 9) once or twice daily over a period of two years. Each observation was the mean of two records taken in a period of 15 min., and the wind speed at 2 m. was also measured. The observations were grouped for wind speed and 17·1–1·2-m. temperature difference, and the means and standard deviations are given in Table X.

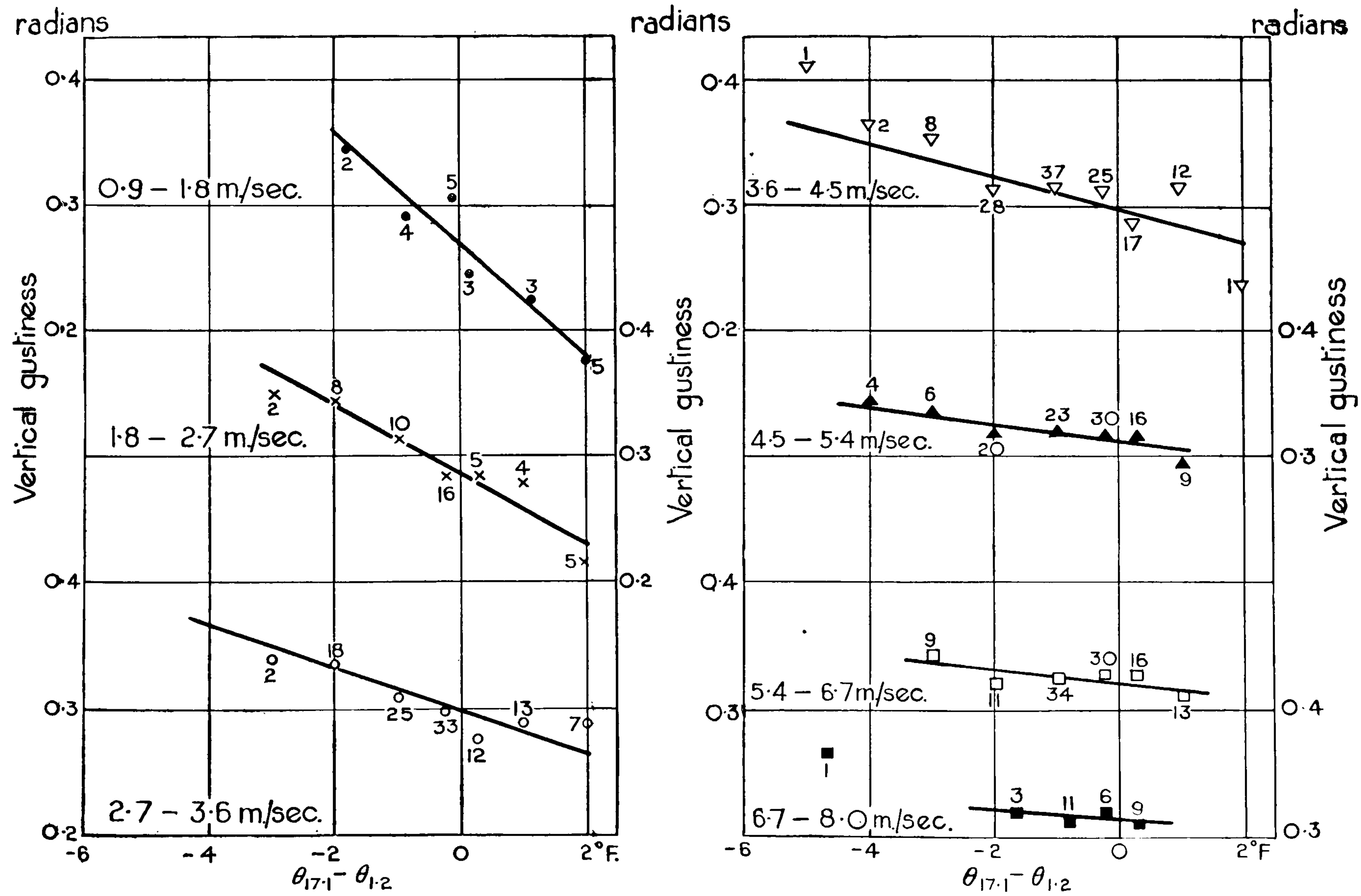


FIG. 15—VARIATION OF VERTICAL GUSTINESS WITH WIND SPEED AND VERTICAL AND VERTICAL TEMPERATURE GRADIENT OVER SHORT-GRASS SURFACE

TABLE X—VALUES OF LATERAL AND VERTICAL GUSTINESS AT A HEIGHT 2 M. OVER A SHORT-GRASS SURFACE

Period of observation : November 1, 1940–October 10, 1942

u_2		Temperature difference, 17·1–1·2 m. (°F.) −5·5 to −4·6	−4·5 to −3·6	−3·5 to −2·6	−2·5 to −1·6	−1·5 to −0·6	−0·5 to −0·1	0 to +0·4	+0·5 to +1·4	+1·5 to +2·4
m./sec.										
	g_y	..	..	..	0·395	0·423	0·449	0·405	0·278	0·271
	g_z	..	..	..	0·335	0·287	0·303	0·245	0·213	0·176
0·9–1·8	u_2 (m./sec.)	..	..	..	1·56	1·58	1·21	1·41	1·13	1·31
	$\theta_{17\cdot1} - \theta_{1\cdot2}$(°F.)	..	..	..	−1·80	−0·85	−0·14	+0·13	+1·13	+1·96
	n	..	..	..	2	4	5	3	3	5
	g_y	..	..	0·482	0·435	0·383	0·379	0·385	0·384	0·312
	g_z	..	..	0·350	0·344	0·315	0·284	0·284	0·281	0·219
1·8–2·7	σ_1	..	..	..	..	50	72	..	..	..
	σ_2	..	..	..	..	52	43	..	..	..
	n	..	..	2	8	10	16	5	4	5
	g_y	..	0·495	0·457	0·451	0·421	0·388	0·371	0·372	0·373
	g_z	..	0·410	0·340	0·334	0·309	0·296	0·274	0·289	0·230
2·7–3·6	σ_1	..	..	..	63	61	75	52	47	..
	σ_2	..	..	..	37	42	40	34	34	..
	n	..	1	2	18	25	33	12	13	17
	g_y	0·580	0·505	0·486	0·438	0·412	0·409	0·355	0·392	0·340
	g_z	0·410	0·365	0·353	0·312	0·313	0·311	0·285	0·313	0·235
3·6–4·5	σ_1	..	..	..	57	63	68	65	60	..
	σ_2	..	..	..	45	51	30	53	36	..
	n	1	2	8	28	37	25	17	12	1
	g_y	..	0·536	0·458	0·427	0·410	0·400	0·398	0·373	..
	g_z	..	0·344	0·335	0·318	0·319	0·315	0·315	0·291	..
4·5–5·4	σ_1	..	..	..	57	78	63	78	69	..
	σ_2	..	..	..	35	43	44	45	56	..
	n	..	4	6	20	23	30	16	9	..
	g_y	..	..	0·446	0·427	0·427	0·429	0·418	0·400	..
	g_z	..	..	0·340	0·318	0·321	0·323	0·322	0·307	..
5·4–6·7	σ_1	..	..	50	62	64	54	61	59	..
	σ_2	..	..	24	25	33	34	25	47	..
	n	..	..	9	11	34	30	16	13	..
	g_y	0·570	..	..	0·443	0·421	0·403	0·432	..	..
	g_z	0·370	..	..	0·317	0·310	0·317	0·307	..	..
6·7–8·0	σ_1	..	..	..	..	71	..	53	..	..
	σ_2	..	..	..	..	39	..	41	..	..
	n	1	..	..	3	11	6	9	..	..

σ_1 and σ_2 are the standard deviations (multiplied by 1,000) of the values of lateral and vertical gustiness respectively from the mean of the group when there are 9 or more observations in the group.

n is the number of observations in each group, each being the mean of two 3-min. bi-directional vane records taken in a period of 10 min.

The values of vertical gustiness are shown plotted against the 17·1 m.–1·2 m. temperature difference in Fig. 15, the numbers of observations meaned being indicated against the points. The values, few in number, for inversion temperature differences greater than + 2° F. have been omitted. Although the regressions of gustiness upon temperature difference are probably not linear, the departures from linearity are not noticeable over the range of variables in Fig. 15. Straight lines have accordingly been fitted by the method of least squares. The mean values in the 0·9–1·8 m./sec. range have been corrected to 1·35 m./sec. by graphical interpolation. In Fig. 16 the logarithms of the slope, m, of the straight lines in Fig. 15 are plotted against the logarithm of the wind speed at 2 m. The value for the highest velocity range has not been

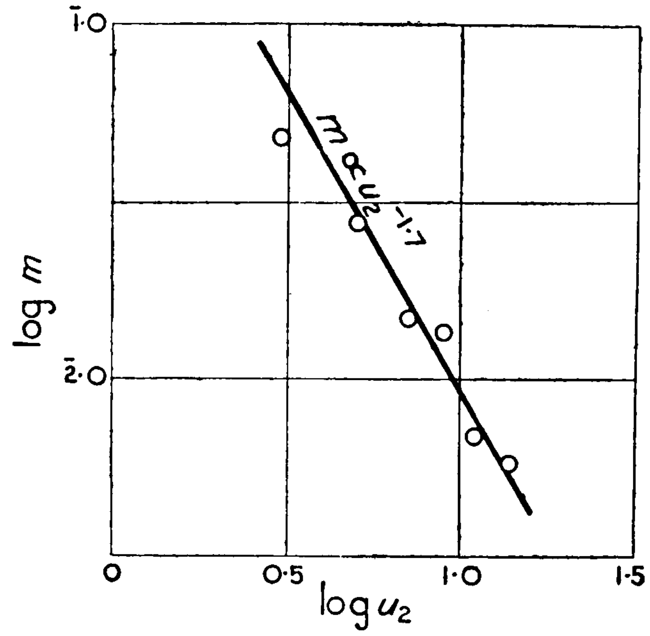


FIG. 16—SLOPE OF VERTICAL GUSTINESS ; TEMPERATURE GRADIENT REGRESSION LINES RELATED TO WIND VELOCITY

included as it is obvious from Fig. 15 that the slope is uncertain owing to the small range of temperature difference and the small number of observations. The straight line fitted by least squares to the weighted values of Fig. 16 gives

$$\frac{dg_z}{d\,(\theta_{17\cdot1} - \theta_{1\cdot2})} \propto u_2^{-1\cdot7}, \qquad \ldots . \text{(10)}$$

while neglecting the point corresponding to the lowest wind velocities (for which the bi-directional vane is least satisfactory) gives $-$ 1·85 for the index in the above relationship. A value of the index of $-$ 2 would be consistent with the vertical gustiness being uniquely related to the Richardson number. Taking into account the random error of the observations and the imperfections of the bi-directional vane technique, it is considered that the observed values of the index in expression (10) are not significantly different from $-$ 2, and that the gustiness data support the velocity gradient observations in indicating that the vertical interchange of momentum over a surface of given roughness is a function of wind speed and the Richardson number to a reasonable degree of approximation.

The lateral gustiness values give similar indications to those from the vertical gustiness values, but the greater scatter of the lateral values renders the evidence less conclusive.

§ 14—WIND AND TEMPERATURE PROFILES OVER SHORT GRASS

In view of the unsatisfactory features, already referred to (§ 11), which attended the correlation of wind-structure measurements at one site with temperature-gradient measurements at another about 180 m. away, another temperature-gradient installation was set up on the anemometer mast (see Part I for the instrumental details). This apparatus measured the temperature differences between 5 and 1 m. and 1 and 0·2 m. Westerly winds had a fetch of 170 m. over short grass before reaching the installation.

To investigate the dependence of wind gradient upon the Richardson number, several periods were selected of a few days each, during which the surface roughness was tolerably constant and favourable wind directions* prevailed. The mean Richardson numbers, $R_i(4/0\cdot5)$, for the layer $0\cdot5$ to 4 m. were calculated from the average potential-temperature gradients and wind gradients between these levels. The 4–$0\cdot5$ m. temperature differences were obtained graphically from the 5–1 m. and 1–$0\cdot2$ m. differences. The ratios of the wind speed at 4 m. to the wind speed at $0\cdot5$ m., $R_{4/0\cdot5}$ for two of the selected periods are shown in Figs. 17 and 18. From

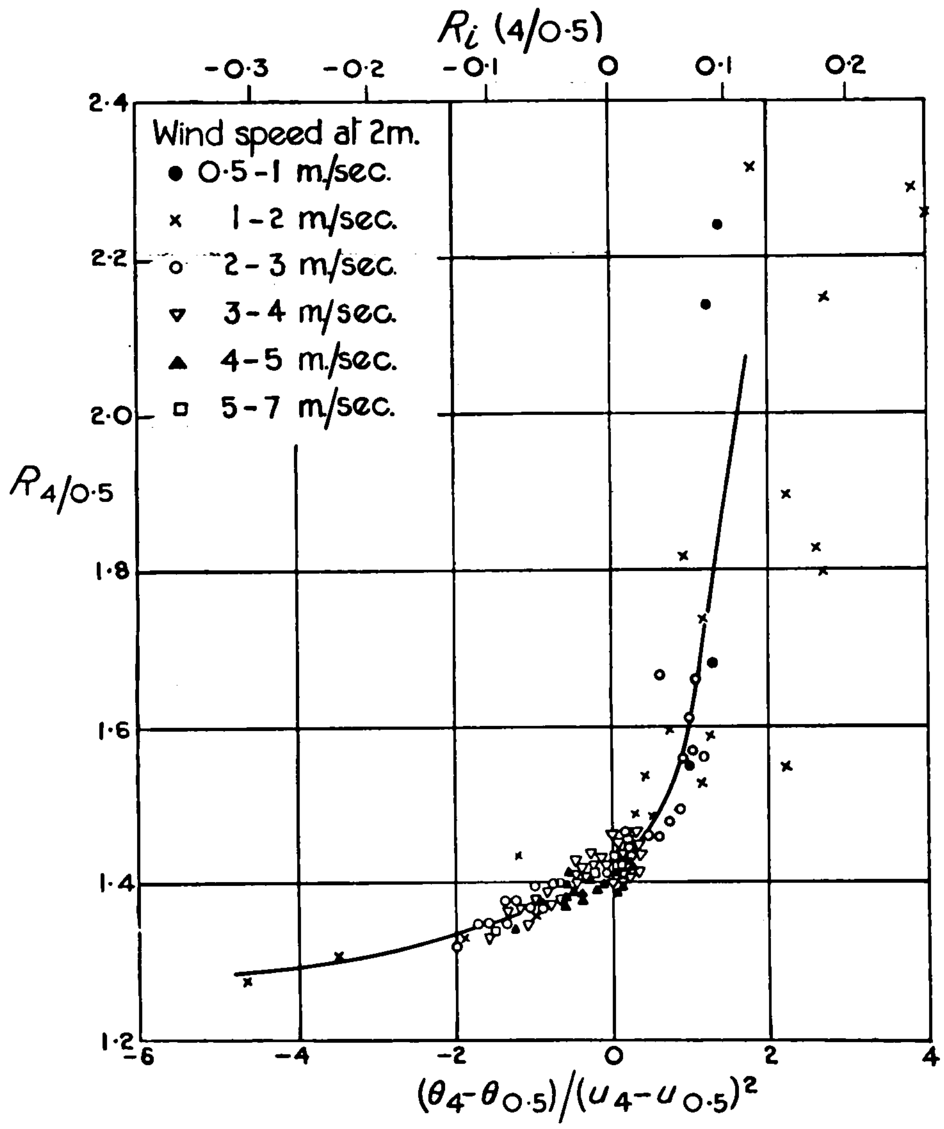


FIG. 17—RELATION BETWEEN $R_{4/0\cdot5}$ AND R_i (4/0·5) OVER SHORT-GRASS SURFACE, JULY 13–16, 1944

Fig. 17 it is apparent that conditions are very erratic when the stability is more than slight; measurement of the temperature and wind gradients at the same spot has not resulted in any substantial decrease in the scatter of values in conditions of marked stability. In Fig. 18 the values for inversion conditions exceeding $R_i(4/0\cdot5) = 0\cdot1$ have been omitted (the dispersion is similar to that in Fig. 17) to enable the remainder to be shown on a more open scale. It is apparent that, within the limits of experimental error, the previous finding, that wind gradient over a surface of constant roughness is closely related to the Richardson number, is supported by these observations covering a range from strong instability to slight or moderate stability. When the stability is great, it appears that conditions are more complex, but further consideration of this state will be deferred to later sections.

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* Only observations for directions between 230° and 300° true (inclusive) were used.

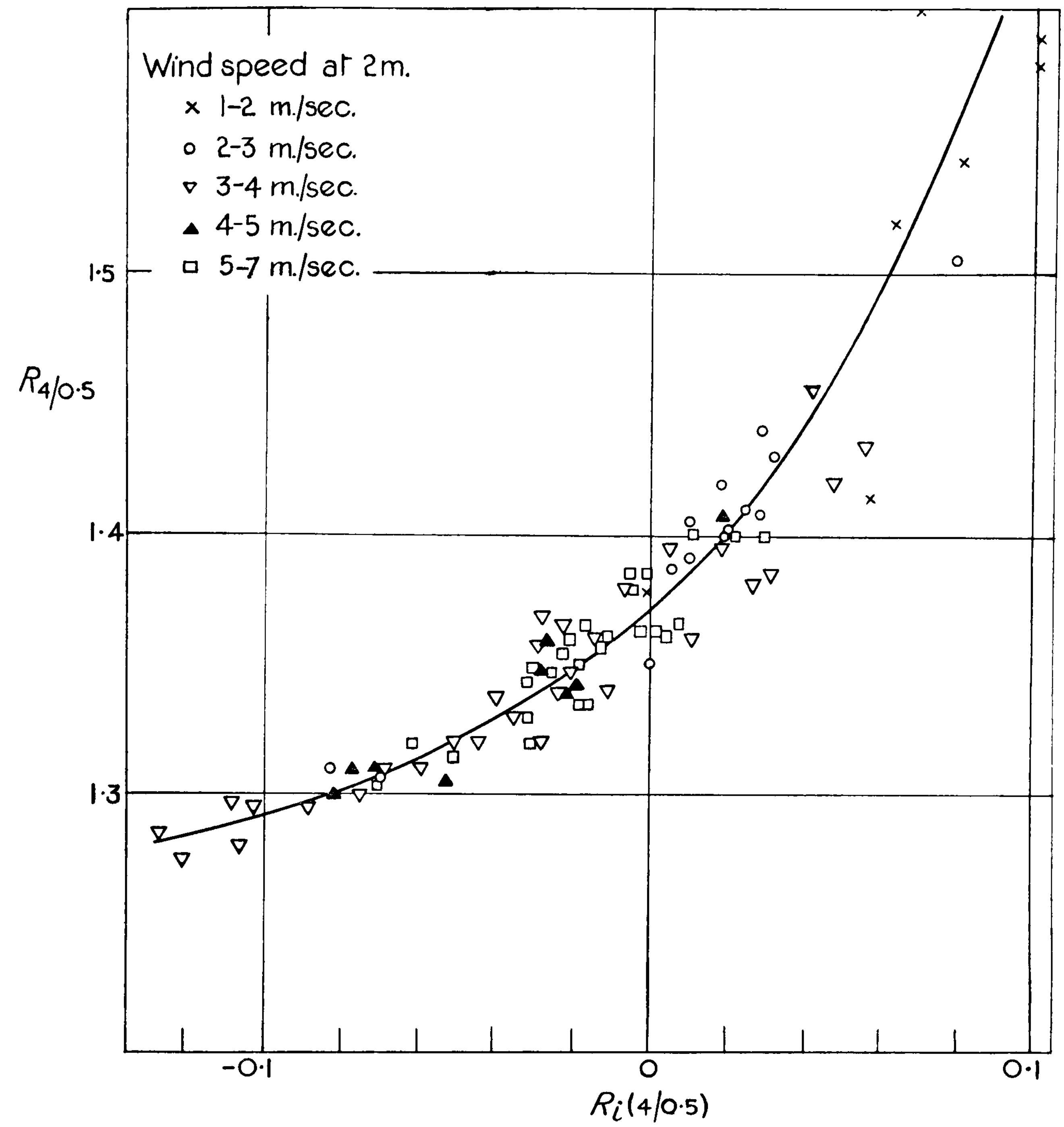


FIG. 18—RELATION BETWEEN $R_{4/0\cdot5}$ AND $R_i(4/0\cdot5)$ OVER SHORT-GRASS SURFACE, JULY 27–28, 1944

§ 15—FORM OF THE WIND PROFILES UNDER VARIOUS CONDITIONS OF ATMOSPHERIC STABILITY

Having established that the Richardson number provides a working measure of atmospheric stability in the layers of air near the ground*, it becomes possible to examine the effect of stability and instability on the form of the vertical profiles of wind speed. This was done by collecting the results for two reasonably small ranges of surface roughness from the data of Series IV. These two sets of results are called Series A and Series B. The two series involve different calibrations of the anemometers, and comparison of the two series should give some measure of the reproducibility of results. The observations in each series were grouped for stability using the parameter $(\theta_5 - \theta_{0\cdot2})/u_1^2$, where $\theta_5 - \theta_{0\cdot2}$ is measured in degrees Fahrenheit and u_1 in metres per second. The mean values of these groups are presented in Table XI and Fig. 19.

The striking feature of Fig. 19 is the fact that the curves of $R_{4/0\cdot5}$ plotted against height are convex to the velocity axis under unstable conditions, while under stable conditions the

* Since this memoir was written Batchelor[53] has shown from an analysis of the conditions for dynamical similarity in perfect gas atmospheres under non-adiabatic conditions that this conclusion is justified for the lowest few metres of the atmosphere.

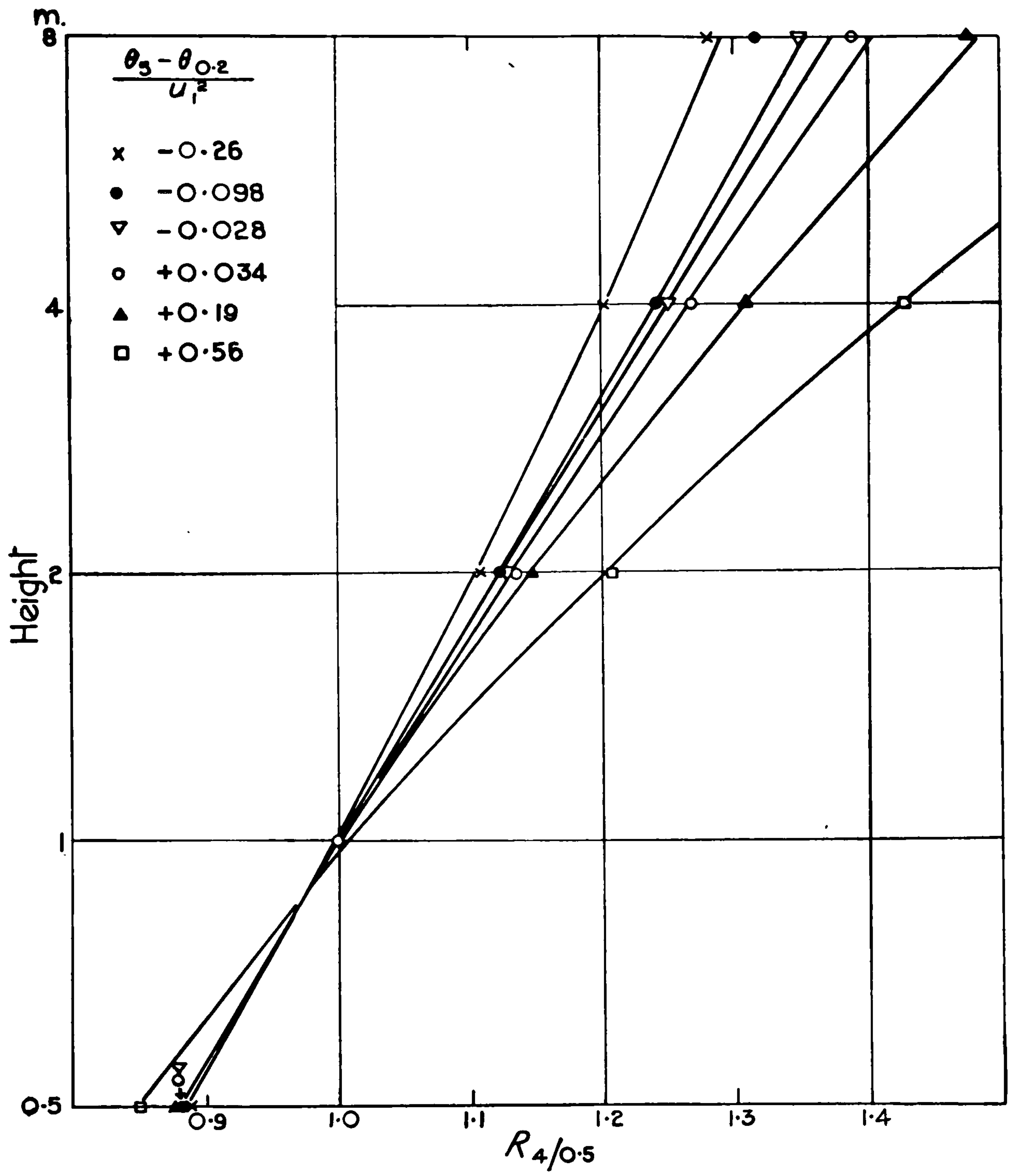


FIG. 19—EFFECT OF STABILITY AND INSTABILITY ON THE WIND PROFILES OVER GRASS

curvature is of the reverse sign. The curvature becomes, in each case, more pronounced the greater the departure of the conditions from neutral stability. The form of the profiles suggests that relationships of the type

$$\frac{du}{dz} = az^{-\beta} \qquad \dots. (11)$$

might well represent the data adequately. To test this possibility, the following procedure was adopted. Integration of equation (11) gives

$$u = \frac{a}{1-\beta} z^{1-\beta} + b\,, \qquad \dots. (12)$$

and considering the wind velocities u_a, u_b, u_d for three heights z_1, z_2, z_3 it follows that

$$\frac{u_d - u_b}{u_b - u_a} = \frac{z_3^{1-\beta} - z_2^{1-\beta}}{z_2^{1-\beta} - z_1^{1-\beta}}. \qquad \dots. (13)$$

For three given heights, the value of the right-hand side of equation (13) may readily be evaluated

TABLE XI—WIND SPEED PROFILES OVER SHORT GRASS

	$\theta_5-\theta_{0\cdot 2}/u^2]$ Range	Mean	No. of obs.	$\overline{u_1}$	$\theta_5-\theta_{0\cdot 2}$	$T_{17\cdot 1}-T_{1\cdot 2}$	Speed ratios $R_{0\cdot 5}$	R_1	R_2	R_4	R_8	β	$\frac{a}{u_1}$	$\frac{b}{u_1}$
Series A				m./sec	°F.	°F.								
1	<−0·2	−0·26	36	2·64	−1·83	−2·28	0·886	1·0 (0·999) −0·1	1·107 (1·104) −0·3	1·201	1·280 (1·291) +0·8	1·10	0·156	2·560
2	−0·19 to −0·05	−0·098	26	3·64	−1·27	−1·82	0·880	1·0 (1·002) 0·2	1·125 (1·122) −0·3	1·241	1·317 (1·358) 3·1	1·02	0·175	9·74
3	−0·04 to −0·001	−0·028	28	3·24	−0·34	−0·85	0·880	1·0 (1·003) 0·4	1·129 (1·127) −0·2	1·251	1·350 (1·376) 1·9	0·99	0·177	−1·72
4	0 to +0·09	+0·034	63	2·61	+0·21	−0·31	0·881	1·0 (1·004) 0·4	1·135 (1·133) −0·2	1·268	1·389 (1·408) 1·4	0·94	0·182	−2·025
5	+0·10 to +0·29	+0·19	10	1·70	+0·50	−0·06	0·877	1·0 (1·004) 0·4	1·148 (1·148) 0	1·309	1·475 (1·490) 1·0	0·83	0·195	−1·178
6	+0·30 to +1·5	+0·56	11	1·35	+0·96	+0·54	0·849	1·0 (1·012) 1·2	1·207 (1·203) −0·4	1·428	1·722 (1·693) −1·7	0·765	0·254	−0·068
7	+1·5	+12·2	10	0·64	+3·74	+1·59	..	1·0	1·355	1·68	2·14	..	..	..
Series B														
1	<−0·2	−0·78	6	1·14	−0·80	−1·45	0·906	1·0 (0·995) −0·5	1·073 (1·074) +0·1	1·141	.. (1·200) ..	1·20	0·1195	1·593
2	−0·19 to −0·15	−0·18	8	3·32	−1·98	−2·15	0·899	1·0 (1·000) 0·0	1·097 (1·094) −0·3	1·180	1·248 (1·259) 0·9	1·11	0·140	2·272
3	−0·14 to −0·10	−0·125	19	3·21	−1·34	−1·54	0·900	1·0 (1·003) 0·3	1·105 (1.100) −0·5	1·190	1·266 (1·276) 0·8	1·09	0·143	2·593
4	−0·09 to −0·05	−0·073	29	4·03	−1·19	−1·56	0·892	1·0 (1·001) 0·1	1·109 (1·105) −0·4	1·204	1·277 (1·299) 1·7	1·07	0·154	3·201
5	−0·04 to −0·001	−0·030	75	4·17	−0·56	−1·12	0·888	1·0 (1·002) 0·2	1·118 (1·114) −0.4	1·224	1·317 (1·328) 0·8	1·025	0·163	7·510
6	0 to +0·09	+0·045	120	4·34	+0·66	−0·14	0·891	1·0 (1·005) 0·5	1·124 (1·119) −0·5	1·235	1·331 (1·351) 1·5	0·99	0·1705	−16·050
7	+0·10 to +0·19	+0·151	56	2·80	+1·19	+0·43	0·883	1·0 (1·003) 0·2	1·129 (1·128) −0·1	1·260	1·406 (1·399) −0·5	0·93	0·1765	−1·522
8	+0·20 to +0·29	+0·23	13	2·59	+1·59	+0·80	0·880	1·0 (1·004) 0·4	1·142 (1·138) −0·4	1·283	1·431 (1·439) 0·6	0·89	0·1865	−0·690
9	+0·30 to +0·49	+0·41	28	2·06	+1·74	+0·87	0·872	1·0 (1·005) 0·5	1·155 (1·157) 0·2	1·331	1·544 (1·531) −0·8	0·81	0·2065	−0·082
10	+0·50 to +0·99	+0·68	25	1·83	+2·37	+1·58	0·845	1·0 (1·005) 0·5	1·191 (1·199) 0·7	1·429	1·696 (1·704) 0·5	0·75	0·258	−0·032
11	+1·00 to +1·49	+1·31	15	1·54	+2·98	+1·75	0·819	1·0 (1·001) 0·1	1·216 (1·227) 0·9	1·475	1·733 (1·780) 2·7	0·75	0·286	−0·144
12	>+1·50	+4·1	51	1·02	+3·54	+1·73	0·801	1·0 (1·012) 1·2	1·276 (1·271) −0·5	1·589	1·922 (1·979) 2·9	0·70	0·339	−0·117

The figures in brackets are the calculated speed ratios from equation (12) using the constants β, a/u_1 and b/u_1 given in the last column which have been calculated from $R_{0\cdot 5}$ and R_4. Below these calculated speed ratios are the percentage differences between calculated and observed values.

as a function of β. This graph may then be used to solve equation (13) for β when the left-hand side has been computed from the wind observations. To find the value of β appropriate to the wind speeds at 0·5, 1, 2 and 4 m. (the speed at 8 m. is considered to be somewhat influenced by the terrain beyond the short grass) the speeds at 1 and 2 m. were meaned, and the mean considered to be the speed at 1·414 m. (i.e. the geometric mean of the speeds at 1 and 2 m.). This assumes that the profile between 1 and 2 m. is logarithmic, an assumption which leads to values of β very slightly closer to unity than the true values, although the error is negligible in practice.

The values of β calculated as above are given in the last column of Table XI, together with the values of a/u_1 and b/u_1, a and b being the constants in equation (12) ; b/u_1 is non-dimensional, but a/u_1 has the dimensions (metre) $^{\beta-1}$. Underneath the observed values of the wind ratios in these tables are the calculated values corresponding to the values of the constants given in the last column, a/u_1 and b/u_1 having been evaluated from β and $R_{0\cdot5}$ and R_4. The percentage differences between the calculated and observed values are given below the calculated values. Inspection of these shows that equation (12) satisfactorily represents the speed profiles up to 4 m. over a considerable range of stability conditions ; only under conditions of great stability are the departures more than 0·5 per cent. The agreement at 8 m. is somewhat variable, being a little closer in Series B than in Series A. This may partly be due to the results of Series A being for the summer, a time when the difference in roughness between the short-grass area and the surrounding downland was greater than in the autumn when the results of Series B were obtained. In any case, the discrepancies at 8 m. are no greater than is to be expected considering the limited fetch (170 m.) of the wind over short grass.

The relationship between β and $(\theta_5 - \theta_{0\cdot2})/u_1^2$ is shown in Fig. 20. The values from the two sets of data both give similar trends. The fact that Series A give somewhat lower values of β than Series B is very probably owing to small systematic errors in anemometer calibrations, because β, involving as it does the second derivative of the velocity profile, is consequently very sensitive to errors. The course of the curve in Fig. 20 is somewhat doubtful for the greater stabilities due to the fluctuating nature of the profiles under these conditions.

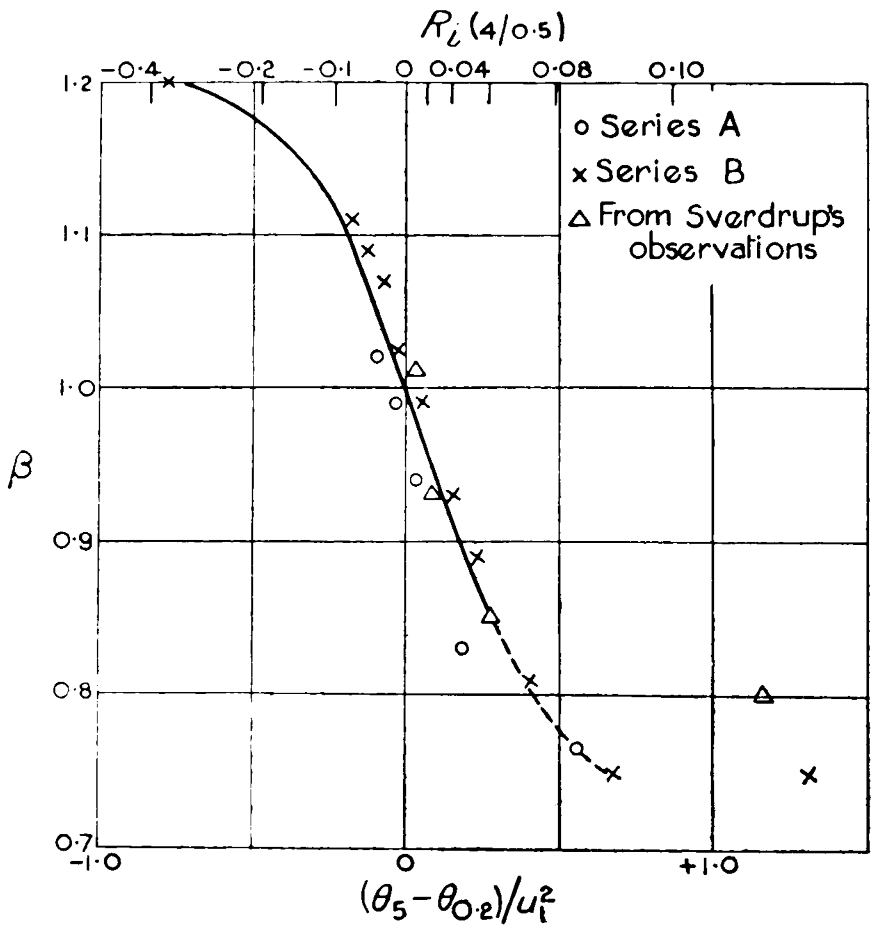


FIG. 20—VARIATION OF β WITH STABILITY

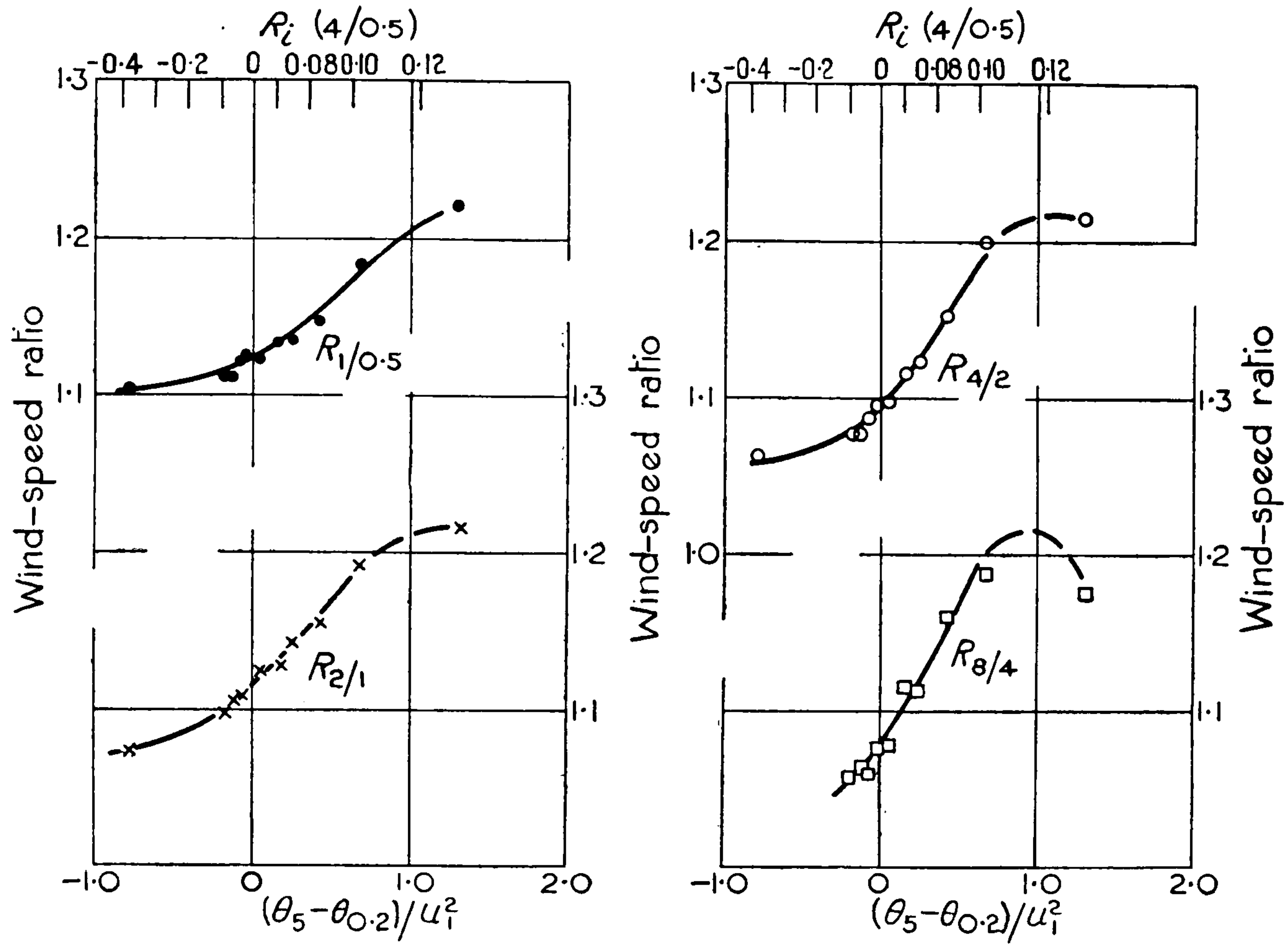


FIG. 21—DEPENDENCE OF WIND RATIOS ON STABILITY OVER A SHORT-GRASS SURFACE

The profiles show that the effects of stability or instability become more marked with increasing height. This is very clearly shown by the graphs of wind ratios plotted against $(\theta_5 - \theta_{0\cdot 2})/u_1^2$ in Fig. 21 and by the following figures :—

Wind ratio	$R_{1/0\cdot 5}$	$R_{2/1}$	$R_{4/2}$	$R_{8/4}$
Percentage variation of the wind ratio over the range $(\theta_5 - \theta_{0\cdot 2})/u_1^2 = -0\cdot 2$ to $(\theta_5 - \theta_{0\cdot 2})/u_1^2 = 0\cdot 3$	2·6	4·2	5·5	7·2

It appears from these results that the form of the wind profile in the lowest few centimetres should be almost completely uninfluenced by considerable changes in temperature gradient. This matter is discussed further in § 17.

§ 16—VERTICAL TEMPERATURE PROFILES

The temperature measurements over the prepared grass area were made at three heights (5, 1 and 0·2 m.). These heights are too few for any very detailed study of the vertical temperature profile. Best[7] (p. 18, Fig. 8) has, however, given some temperature profiles, over the height range 0·025 to 17·1 m. for clear June days, which show characteristics similar to those displayed by the wind-speed profiles discussed above. That is to say, the graphs of potential temperature against the logarithm of the height are not quite linear, and the departures from linearity are such that in lapse conditions $d\theta/dz$ decreases (numerically) with height more rapidly than is consistent with a logarithmic relationship, while in inversion conditions the reverse is true, i.e. $d\theta/dz$ decreases less rapidly than if it were directly proportional to the height. These data are shown in Fig. 22, potential temperature (2·5-cm. potential temperature taken as zero) being plotted against log z. The curves in the figure are calculated from the relationship

$$\frac{d\theta}{dz} = az^{-\delta} \qquad \ldots . \ (14)$$

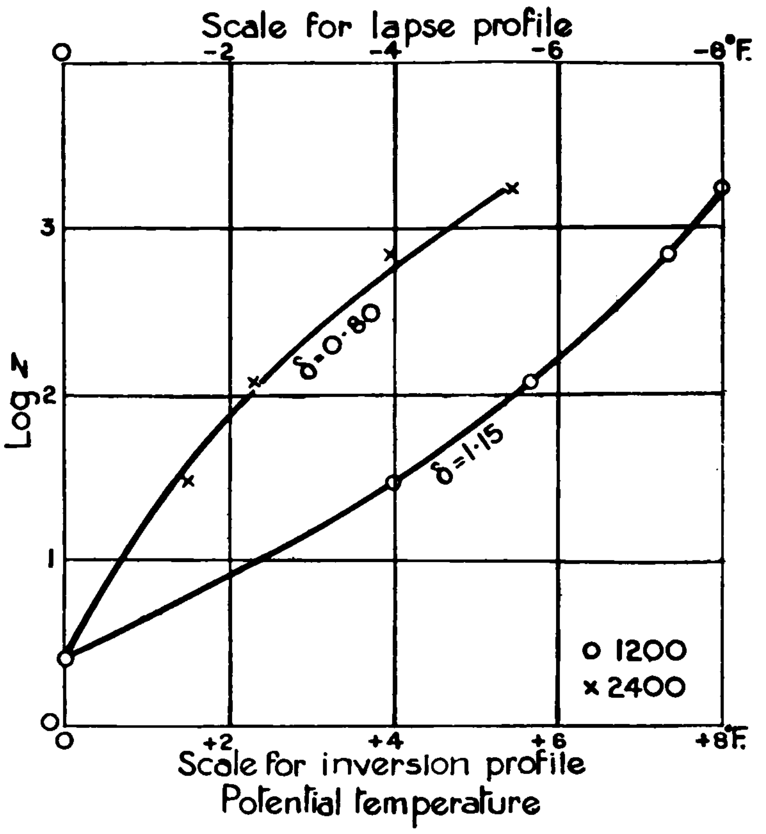


FIG. 22—VERTICAL TEMPERATURE PROFILES FOR CLEAR DAYS IN JUNE ACCORDING TO BEST[7]

analogous to that found suitable for the wind-speed profiles. It is seen that this relationship gives a very close fit to the midday potential-temperature profile ($\delta = 1\cdot15$) and tolerably good agreement with the midnight profile ($\delta = 0\cdot80$). This indication from Best's work that equation (14) gives a good representation of the temperature profiles was taken to justify the calculation of values of δ from the observations of temperature at three heights in the present experiments.

The ratio (R_θ) of the potential-temperature difference between 5 and 1 m. to that between 1 and 0·2 m. was evaluated for occasions of suitable wind direction (230–300° true) and 5–0·2 m. temperature differences numerically greater than 1° F. Occasions of smaller temperature difference than 1° F. were not analysed because experimental error then became too great for reliable ratios to be obtained (1° F. = 9·4 mm. on the autographic records). Under conditions of lapse the dots on the record of the Cambridge thread recorder are considerably scattered owing to the variability of the air temperature and the responsiveness of the thermometer elements. Johnson[2] has given particulars of the speed of response of elements of the pattern used and sample records under various conditions. To reduce the error of the mean temperature differences for each 15-min. period obtained from these records, the value for each dot was read off and the mean taken. The occasions of inversion conditions analysed were restricted to those when the trace of the adjacent pressure-tube anemograph (orifice height 13·3 m.) still showed slight turbulence to be present. The mean values of the ratio R_θ are given in Table XII, together with the standard deviation of the values from the mean of each group of 5 or more values. The values of the stability parameter $(\theta_5 - \theta_2)/u_1^2$ are also given in the table.

Fig. 23 shows the dependence of R_θ on $(\theta_5 - \theta_{0\cdot2})/u_1^2$. The points for the various ranges of wind speed at 2 m. all lie close to one curve over the whole range of unstable conditions, and for stable conditions up to $(\theta_5 - \theta_{0\cdot2})/u_1^2 \simeq 0\cdot5$. This indicates that over a considerable range of stabilities, for a given surface, the forms both of the temperature profile and the wind profile are, to a first approximation, related uniquely to the value of the Richardson number in the layer of air under consideration.

The values of δ in equation (14) have been calculated from the values of R_θ given by the curve of Fig. 23, and the dependence of both δ and β on $\theta_5 - \theta_{0\cdot2}/u_1^2$ is shown in Fig. 24. Both

TABLE XII—RATIO OF THE 5 M. : 1 M. POTENTIAL-TEMPERATURE DIFFERENCE TO THE 1 M. : 0·2 M. POTENTIAL-TEMPERATURE DIFFERENCE

Observations over short grass

	$u_2 = 1$–2 m./sec.		$u_2 = 2$–3 m./sec.		$u_2 = 3$–4 m./sec.		$u_2 = 4$–5 m./sec.		$u_2 = 5$–7 m./sec.		$u_2 = 7$–12 m./sec.	
$\theta_5-\theta_{0\cdot2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$	R_θ	$\frac{\theta_5-\theta_{0\cdot2}}{u_1^2}$
°F. −2·65	..	..	..	..	0·58 1/..	−0·26	0·65 5/0·07	−0·16	0·85 8/0·11	−0·09	..	..
−2·10	..	..	..	..	0·64 3/..	−0·21	0·66 10/0·11	−0·13	0·80 9/0·11	−0·07	..	..
−1·65	..	..	0·56 6/0·19	−0·33	0·65 10/0·12	−0·17	0·75 4/..	−0·10	0·79 13/0·11	−0·05	0·89 1/..	−0·03
−1·35	0·47 2/..	−0·44	0·55 5/0·16	−0·27	0·78 6/0·12	−0·13	0·83 8/0·12	−0·08	0·79 18/0·11	−0·05	0·70 2/..	−0·025
−1·05	0·51 5/0·28	−0·56	0·66 8/0·16	−0·21	0·79 6/0·19	−0·11	0·76 8/0·12	−0·07	0·75 14/0·13	−0·03	0·83 3/..	−0·02
+1·05	1·62 6/0·45	+0·50	1·21 10/0·26	+0·23	1·19 12/0·27	+0·11	1·06 9/0·55	+0·06	1·04 9/0·37	+0·04	0·92 23/0·15	+0·015
+1·35	2·02 1/..	+0·56	1·33 13/0·33	+0·29	1·25 10/0·39	+0·13	1·03 7/0·23	+0·08	0·89 8/0·16	+0·05	0·89 9/0·07	+0·02
+1·65	1·47 5/0·27	+0·98	1·41 10/0·31	+0·36	1·09 8/0·22	+0·18	1·00 11/0·22	+0·10	0·88 17/0·08	+0·06	..	..
+1·95	1·52 5/0·22	+1·24	1·48 12/0·31	+0·42	1·05 9/0·22	+0·21	0·94 4/..	+0·12	0·94 2/..	+0·07	..	..
+2·35	1·20 5/0·36	+1·5	1·46 10/0·32	+0·51	1·29 13/0·37	+0·25	..	..	..	..	..	..
+3·05	1·14 2/..	+2·1	1·40 9/0·34	+0·73	1·41 1/..	+0·33	..	..	..	..	..	..
+3·75	..	..	1·60 7/0·32	+0·90	..	..	..	..	..	..	..	..
+4·50	..	..	1·46 5/0·16	+1·05	..	..	..	..	..	..	..	..

The number to the left of the solidus is the number of observations in the group while the number to the right is the standard deviation of the values from the mean value of the ratio.

these indices tend to be greater than unity for lapse conditions and less than unity in inversions. It will, however, be noted that the value of δ for conditions close to neutral stability is about 1·1, whereas the value of β is close to unity (logarithmic profile). Under these conditions thermal influences on the atmospheric eddy motion should be negligible. It is probable that the departure of the observed values of δ from unity under these conditions is mainly due to experimental error. Systematic errors of less than 0·1° F. in the temperature differences would account for the discrepancy, and the equipment used was not capable of maintaining an appreciably greater accuracy.

From Fig. 24 it appears that δ varies more markedly than β with changing stability, but the difference between the two indices, under stable conditions, is too small to have certain significance in view of the rather large effect of experimental errors. This finding of approximate similarity between the vertical distributions of wind and potential temperature, under stable conditions, is in harmony with Sverdrup's results from observations over snow-fields[9]. The greater value of δ than β under lapse conditions is consistent with a more rapid increase of the eddy conductivity with height than of the eddy viscosity, a result explicable in terms of the theoretical considerations of Priestley and Swinbank[35] when taken in conjunction with the observed increase in the effects of buoyancy with height.

Using the equation for potential temperature corresponding to equation (13) $\theta_4 - \theta_{0\cdot5}$ could be calculated from the values of δ (Fig. 24) and $\theta_5 - \theta_{0\cdot2}$. In this way, the value of the Richardson number, $R_i(4/0\cdot5)$, for the 4 to 0·5-m. layer corresponding to any value of $(\theta_5 - \theta_{0\cdot2})/u_1^2$ could be calculated. Scales of $R_i(4/0\cdot5)$ have accordingly been inserted in Figs. 16, 19, 20, 22 and 23.

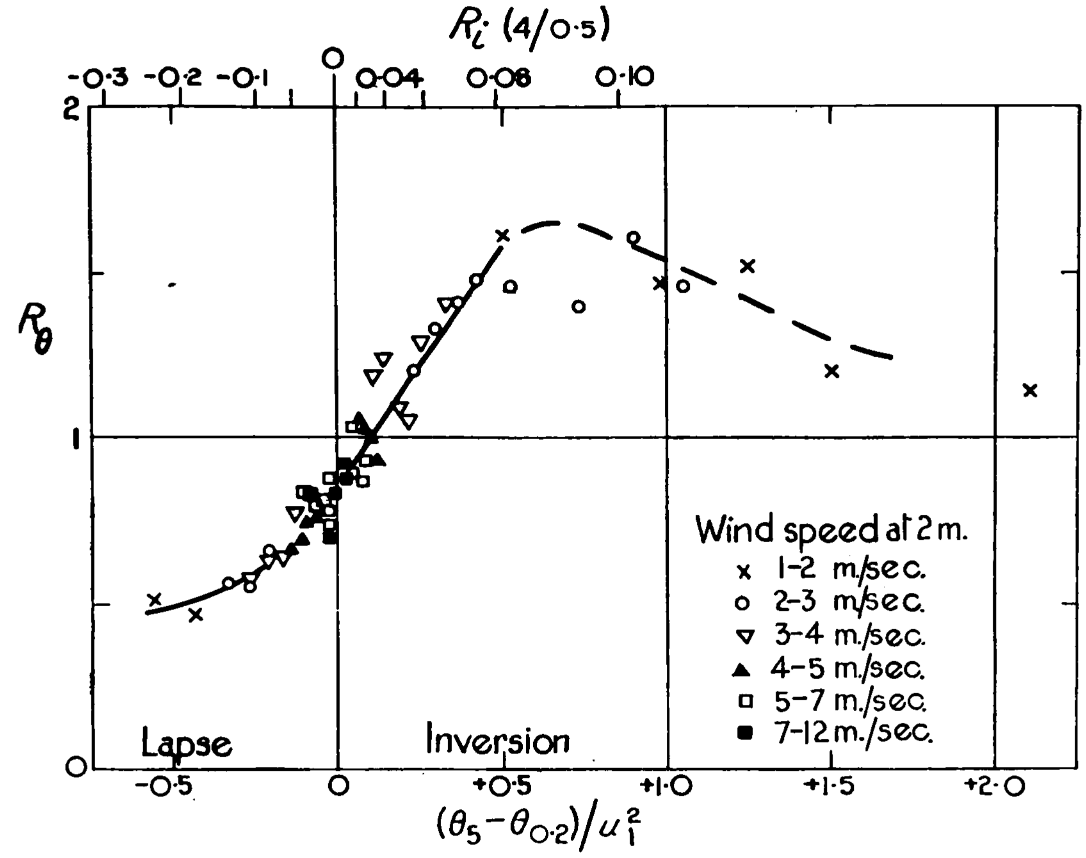


FIG. 23—POTENTIAL-TEMPERATURE-DIFFERENCE RATIO RELATED TO STABILITY

Figs. 17 and 21 show that the steady increase in the wind-speed ratios with increasing stability only holds up to $R_i(4/0\cdot5) \simeq 0\cdot1$. After this, the values generally decrease again and become very erratic, an effect probably due to the turbulence becoming very small. The ratio, R_θ, of the potential-temperature difference between 5 and 1 m. to that between 1 and 0·2 m. displays

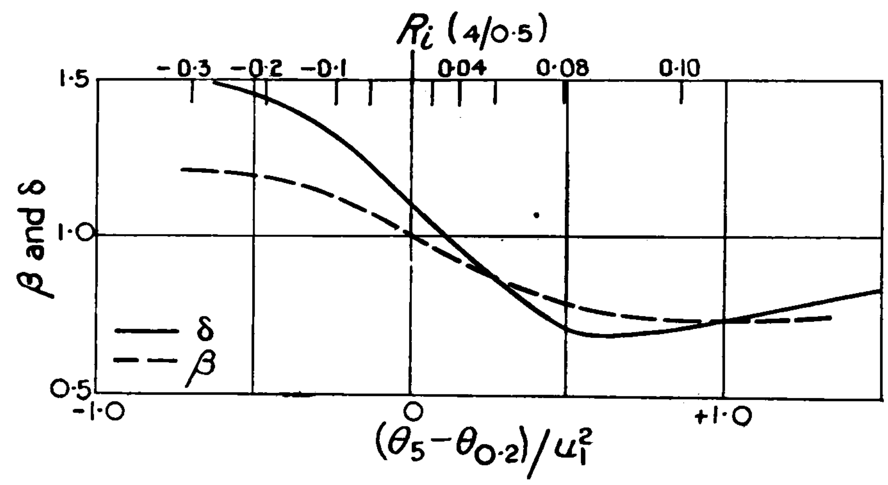


FIG. 24—COMPARISON OF VALUES OF δ AND β

an exactly similar behaviour (Fig. 23) which reinforces the conclusion that some marked change in conditions must begin to occur when the stability exceeds a certain value. The critical value of the Richardson number for the suppression of turbulence will be discussed in § 23 after the variation of stability with height has been considered.

§ 17—VARIATION OF STABILITY WITH HEIGHT

The Richardson number, being

$$R_i = \frac{g}{T}\frac{d\theta/dz}{(du/dz)^2} \qquad \ldots . (15)$$

it follows from equations (11) and (14) that

$$R_i \propto z^{2\beta-\delta} . \qquad \ldots . (16)$$

From Fig. 24 it is seen that $2\beta - \delta$ has the following approximate values for various conditions :—

conditions of marked instability	0·9
conditions near neutral equilibrium ..	0·9
conditions of marked stability	0·85

It is apparent, therefore, that the Richardson number increases almost in proportion with height in both stable and unstable conditions. It follows that the effect of a vertical temperature gradient decreases as the surface is approached. This accords with the evidence given by the wind-speed profiles, which, as already seen (Fig. 21) show a marked decrease in the influence of thermal stratification as the surface is approached. Flow conditions very near the surface should therefore approximate closely to those existing under neutral conditions.

§ 18—GENERALIZED WIND-PROFILE RELATIONSHIP

As the wind profile close to the surface is expected always to approximate closely to the logarithmic law found under neutral conditions, the boundary condition $u = 0$ at $z = z_0$ which is fulfilled by the logarithmic law should also be valid for conditions other than neutral. Applying this condition and integrating equation (11) gives

$$u = \frac{a{z_0}^{1-\beta}}{1-\beta}\left\{\left(\frac{z}{z_0}\right)^{1-\beta} - 1\right\} \qquad \ldots . (17)$$

which, on expansion, becomes

$$u = a{z_0}^{1-\beta}\left[\log_e\left(\frac{z}{z_0}\right) + \frac{(1-\beta)}{2\,!}\left\{\log_e\left(\frac{z}{z_0}\right)\right\}^2 + \ldots\right].$$

It follows that for small heights, i.e. $(z/z_0) \rightarrow 1$ and values of β not greatly different from unity, the profile will tend to the logarithmic form, i.e.

$$u = \frac{u_*}{k}\log_e\left(\frac{z}{z_0}\right) \qquad \ldots . (18)$$

provided that

$$a = \frac{u_*}{k{z_0}^{1-\beta}},$$

which substituted in equation (17) gives the generalized profile relationship

$$u = \frac{u_*}{k\,(1-\beta)}\left\{\left(\frac{z}{z_0}\right)^{1-\beta} - 1\right\}. \qquad \ldots . (19)$$

When $\beta \rightarrow 1$ this transforms into the logarithmic law.

Equation (19) used to calculate values of z_0 from the data of Table XI indicates a large variation of z_0 with stability from about twice the neutral value, under strong instability, to nearly zero at a quite moderate degree of stability, $R_i\,(4/0{\cdot}5) = +\,0{\cdot}05$. That this is an unacceptable result may be seen as follows. Under unstable conditions, because the ratio of wind speed at 1 m. to that at 0·5 m. is almost constant, an increasing z_0 with increasing instability would entail an increase in wind ratios near the surface, at 20 and 10 cm. for example. The observations of Best[7], which extended to within 2·5 cm. of the surface, give no evidence of variation of the low-level wind ratios in a sense inverse to that at higher levels. It seems very probable, therefore, that the variation of z_0 found by applying equation (19) directly to the wind observations between 0·5 and 4 m. is a consequence of equation (11) being only an approximate relationship. A z_0 constant with stability would be consistent with a small variation of β with height, β being closer to unity near the surface than at higher levels. The evidence for a variation of β with height is examined in § 19.

The finding that flow conditions near the surface are but little affected by large variations in stability would make it seem reasonable to assume, for a surface with rigid roughness elements, that the roughness parameter is also independent of stability. Under very stable conditions a decrease of roughness parameter might perhaps be expected as a result of the small wind gradient in the air layer among the grass stems (see Fig. 3). A non-turbulent layer adjacent to the soil might then form and partly submerge the roughness elements so decreasing the roughness parameter.

The assumption of a constant z_0 in equation (19) equal to the neutral value permits β and hence u_* to be found from the winds at two heights, since

$$\frac{u_2}{u_1} = \frac{(z_2/z_0)^{1-\beta} - 1}{(z_1/z_0)^{1-\beta} - 1}, \qquad \ldots\ldots (20)$$

which may be solved graphically for β if z_0 is known from a measurement of the wind ratio under neutral conditions. The values of β so obtained from R_4 over short grass are generally somewhat closer to unity than those giving the best fit to the whole wind profile between 0·5 and 4 m. This is shown in Table XIII where the values of β calculated by the two methods are distinguished by writing those obtained using equation (20) as β'.

TABLE XIII—VALUES OF VARIOUS PARAMETERS FOR SERIES B OBSERVATIONS OVER SHORT GRASS AND THE RESULTING DIFFERENCES BETWEEN CALCULATED AND OBSERVED SPEEDS

	Group	$R_i(4/0{\cdot}5)$	β	β'	u_*/u_1	Percentage difference between calculated and observed speeds at 0·5 m.	2 m.	8 m.	K/u_1† at 1 m.	4 m.
										cm.
Increasing instability	1	−0·38	1·20	1·13	0·096	+1·5	+0·1	..	8·4	40
	2	−0·09	1·11	1·065	0·081	+1·5	−0·5	+1·3	4·8	21
	3	−0·06	1·09	1·05	0·077	+0·5	−0·7	+1·1	4·2	18
	4	−0·03	1·07	1·03	0·073	0	−0·6	+2·0	3·5	14·5
	5	−0·01	1·025	1·01	0·069	+0·3	−0·5	+1·3	3·1	12·5
Increasing stability	6	+0·015	0·99	0·995	0·066	−0·1	−0·6	+1·7	2·6	10·2 (8·7)
	7	+0·055	0·93	0·97	0·061	−0·9	0	−0·8	2·05	7·9 (7·4)
	8	+0·065	0·89	0·945	0·056	−1·6	−0·3	+0·1	1·65	6·1 (6·0)
	9	+0·09	0·81	0·90	0·049	−2·4	+0·5	−1·9	1·1	3·7
	10	+0·10	0·75	0·82	0·037	−2·7	+0·9	−0·6	0·5	1·6 (3·6)
	11	+0·135	0·75	0·79	0·033	−1·1	+0·4	+2·1	0·4	1·1
	12	+0·25	0·70	0·72	0·026	−2·5	−0·8	+3·0	0·2	0·5

β is calculated from the speed profiles between 0·5 m. and 4 m. by equation (13); β', u_*/u_1 and the calculated velocities are obtained from R_4 with $z_0 = 0{\cdot}25$ cm. by equations (19) and (20).

† The bracketed values of K_4/u_1 are those deduced by Sverdrup[9] from observations over a snow surface of similar roughness to the short grass.

§ 19—VARIATION OF β WITH HEIGHT

Unstable conditions.—As β involves the second derivative of the wind profile it is difficult to obtain reliable indications of its change with height over the rather limited range $0 \cdot 5$ to 4 m. The values of β have, however, been calculated from the speeds at heights 4, 2 and 1 m. and again from those at 2, 1 and $0 \cdot 5$ m. The mean values, using equation (13), for all the conditions of marked lapse, $(\theta_5 - \theta_{0 \cdot 2})/u_1^2 < -0 \cdot 1$, for the results given in Table XI were :—

For heights 4, 2 and 1 m. $\beta = 1 \cdot 21$
For heights 2, 1 and $0 \cdot 5$ m. $\beta = 1 \cdot 07$

There is a statistically significant indication of an increase in β with height, but remembering that systematic errors in the wind speeds at the various heights of about $0 \cdot 2$ per cent. are sufficient to produce errors in the values of β of the order of 5 per cent., it is obvious that little can be said as to the possible magnitude of the increase.

Some part of the increase of β with height could be a result of advected large-scale eddy motion produced by the rougher surround to the short-grass area. Such eddy motion would augment vertical interchange at greater heights more than very close to the ground, and so cause β to increase with height more rapidly than might be characteristic of a very large level expanse of short grass.

Stable conditions.—A similar calculation of values of β, using equation (13), to that given above for unstable conditions, gave the following values for stabilities in the range $0 \cdot 1 < (\theta_5 - \theta_{0 \cdot 2})/u_1^2 < 1$.

For heights 4, 2 and 1 m. $\beta = 0 \cdot 88$
For heights 2, 1 and $0 \cdot 5$ m. $\beta = 0 \cdot 77$

The difference between these values is less significant than the corresponding difference under unstable conditions, because of the larger scatter of values. Systematic errors are also likely to be rather larger than under unstable conditions owing to the small wind speeds generally found under stable conditions. The change in β with height indicated above, if real, is not of the sign required to explain the anomaly referred to in § 18. The effects of advected large-scale eddy motion should not be very appreciable under stable conditions, as large-scale vertical motion is rapidly damped by a stable stratification. Under stable conditions, however, the rate of development in depth of the boundary layer with increasing distance from the upwind edge of the short-grass area is less than under adiabatic or unstable conditions. It is possible that the speeds at 4 m. under stable conditions may be somewhat affected by an insufficiently deep boundary layer characteristic of the short-grass surface.

§ 20—EDDY VISCOSITY

Assuming approximate constancy of the horizontal turbulent shearing stress τ in the lowest 30 m. or so of the atmosphere[36], the vertical component of the eddy viscosity K can be obtained from the relationship

$$\tau = \rho K \frac{du}{dz} \qquad \ldots . (21)$$

using equation (19), which gives

$$\frac{du}{dz} = \frac{u_*}{kz_0 \, (z/z_0)^{\beta'}} \qquad \ldots . (22)$$

leading to

$$K = ku^* \, z_0 \, (z/z_0)^{\beta'} \, , \qquad \ldots . (23)$$

which indicates the eddy viscosity to be proportional to the height raised to a power β' greater than unity under unstable conditions and less than unity when stability prevails. The values of K/u_1 for heights of 1 m. and 4 m. are given in Table XIII. It will be seen that there is moderately good agreement between these values and Sverdrup's values from observations over a snow surface of similar roughness to the short grass, except under conditions of strong stability. Sverdrup's observations are discussed more fully in § 22.

On the assumption that the eddy viscosity is equal to the eddy interchange coefficient for matter, values of K deduced from wind profiles by means of equations (19), (20) and (23) have been applied in the study of the vertical diffusion of gas[37] with moderately encouraging results, and Pasquill has also tested the method in a study of the evaporation of moisture from a meadow[38]. He found good agreement between calculated and observed rates of evaporation (measured gravimetrically) under unstable conditions ; under stable conditions the theory gave appreciably smaller rates of transfer than were measured.

§ 21—WIND PROFILES OVER A LONG-GRASS SURFACE

The observations over grass about 60 cm. high (Series I) have not been analysed in so much detail as those over the short-grass surface, because the variation in surface roughness of the long-grass surface, due to deflexion of the grass blades by the wind, introduces such formidable complexities into the study of the effect of stability on the wind profiles that a detailed analysis of the comparatively small body of data available would not be profitable. The observations are valuable, however, as the roughness of the ground surface was much more uniform over distances up to 1,000 m. than was the case for the observations over the short-grass area 180 m. square. All the observations for which the wind at 13·3 m. ranged between 2·5 and 5·5 m./sec. have simply been grouped according to 17·1–1·2 m. temperature difference (adiabatic lapse rate — 0·29° F.), and the mean wind-speed ratios (wind speed at 13·3 m. equal to unity) are given in Table XIV.

TABLE XIV—WIND PROFILES OVER LONG GRASS

Height of grass about 60 cm.

$\theta_{17\cdot1} - \theta_{1\cdot2}$ Mean	Temperature difference 17·1–1·2 m. Range	Length of record	$\bar{u}_{13\cdot3}$	Speed ratios $R_{1/13\cdot3}$	$R_{1\cdot98/13\cdot3}$	$R_{5\cdot22/13\cdot3}$	β
°F.	F°.	hr.	m./sec.				
−3·90	<−3	3	4·21	0·515	0·683	0·879	1·20
−2·35	−3 to −2·1	6	4·91	0·481	0·654	0·879	1·205
−0·95	−2 to −1·1	7	4·17	0·475	0·635	0·847	1·08
−0·15	−1 to 0	10	4·08	0·443	0·605	0·832	1·04
+0·75	0 to +0·9	6	3·10	0·385	0·538	0·768	0·89
+1·52	+1 to +1·9	3	3·11	0·321	0·494	0·752	0·905
+2·83	+2 to +2·9	5	2·90	0·275	0·437	0·707	0·81
+4·20	>+3	3	3·41	0·252	0·402	0·678	0·74

The values of β have been calculated from equation (13).

The profiles of Table XIV are plotted in Fig. 25 against $\log_{10}(z - 25)$ the zero-plane correction of 25 cm. found for the profiles under adiabatic conditions being assumed to be approximately constant over the range of stability encountered. The profiles so plotted exhibit the same change in the sign of the curvature (as indicated by the values of β) with change from stable to unstable conditions as was shown by the short-grass profiles. It is very noticeable that the gradient of relative wind speed near the ground remains nearly constant despite a considerable variation in stability conditions. This again shows that thermal effects on the wind profiles become very small in close proximity to the surface.

The variation of the Richardson number with height over the long grass could not be investigated with much accuracy as temperature-gradient records were only available for the two height intervals 17·1–1·2 m. and 7·1–1·2 m. The differences in temperature between 7·1 and 17·1 m. are rather small, so that the accuracy of the information given on the variation of temperature gradient with height is not good. To minimize the effects of scatter as much as possible the mean value of the ratio of potential-temperature difference under strong lapse

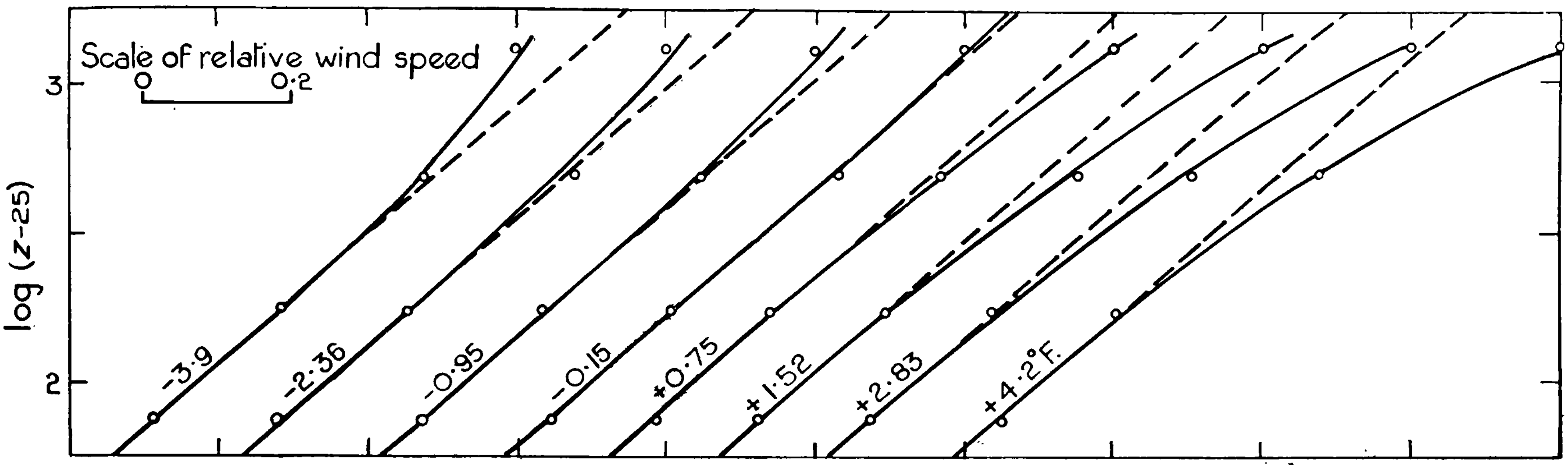


FIG. 25—WIND PROFILES OVER LONG GRASS

The potential-temperature difference, $\theta_{17} - \theta_{1\cdot2}$, for each profile is shown along the profile. Starting from the profile with $\theta_{17} - \theta_{1\cdot2} = -3\cdot9°$ F. the profiles are displaced to the right in steps of 0·2. z is measured in centimetres.

conditions $(\theta_{17\cdot1} - \theta_{1\cdot2})/(\theta_{7\cdot1} - \theta_{1\cdot2})$ was evaluated for each of 20 years for which records were available. This was done by taking the mean of the values recorded between 1100 and 1400 G.M.T. (inclusive) on each clear day in May, June and July, clear days for this purpose being those giving a continuous sunshine trace from 0800 to 1500 G.M.T. Occasions when the wind was from the obstructed northerly quarter were not included. The annual means show considerable variation from year to year (total range 1·05–1·55) owing to variation in the state of surface and distribution of obstructions, differences in mean wind speed and experimental error. The means for the two 10-yr. periods, together with the corresponding values of δ in $d\theta/dz \propto z^{-\delta}$ are* :—

	1923–33	1934–43
Mean ratio	1·275	1·350
Standard deviation of annual means ..	0·15	0·19
Mean value of δ	1·43	1·26
	1923–43	
20-yr. mean value of δ	1·35	
20-yr. mean potential-temperature difference, 17·1–1·2 m.	−2·3° F.	

It is probable from the above values that, for strong lapse conditions during the period of the wind observations, δ was between about 1·2 and 1·5, while β from the wind profiles was approximately 1·2. The Richardson number under conditions of moderate to strong instability is therefore indicated to be proportional to the height raised to some power between about 0·9 and 1·2, a result substantially in agreement with that from the observations over short-grass surfaces.

* No zero-point correction was applied in calculating δ as the temperature-gradient tower was surrounded by a small area of short grass.

§ 22—COMPARISON WITH THE RESULTS OF OTHER INVESTIGATIONS

Best's investigation.—Best[7] made very detailed measurements of the wind profile over a sports field under varying conditions of stability. He found under neutral conditions that the profile from 2·5 cm. up to 5 m. was well represented by a logarithmic law after making a zero-plane correction of 1 cm. His values of the constants in the logarithmic law (Best[7], p. 37) give a roughness parameter of 0·13 cm. for the closely cropped grass, a value in good agreement with the values obtained in the present work.

From Best's results under stable and unstable conditions (his Table XVIII) the values of β have been calculated for various heights using equation (13) and a zero-plane correction of 1 cm. The wind speeds at 5 m. have not been used as it was considered that the sports field was not of sufficient size to give homogeneous profiles up to this height. The 2·5-cm. speeds have also not been used, as they were measured with a hot-wire anemometer, all the other measurements being made with vane air-meters. In any case uncertainty as to the exact zero level becomes very important at such small heights. The results are given in Table XV.

TABLE XV—VALUES OF β CALCULATED FROM BEST'S OBSERVATIONS

Wind speed at 1 m., 1·5–4 m./sec.

$T_{1\cdot 1} - T_{0\cdot 1}$ (1)	Values of β calculated from $u_{0\cdot 05}$, $u_{0\cdot 25}$, and u_1 (2)	$u_{0\cdot 1}$, $u_{0\cdot 5}$, and u_2 (3)	$u_{0\cdot 05}$, $u_{0\cdot 10}$, and $u_{0\cdot 25}$ (4)	Mean of columns (2) and (3) (5)
°F.				
−3	1·12	1·10	1·24	1·11
0	1·09	1·04	1·095	1·065
+1	1·01	0·99	0·83	1·00

The mean values of β for the height range 5–200 cm. display the same trend as that found in the present work, but the value $\beta = 1 \cdot 0$ occurs under stable conditions instead of at neutral stability. This difference is considered to be due to the fact that Best worked on the lee side of the cricket pitch, the rest of the sports field being somewhat rougher. This would give rise (as has been found by experiments on the transition from rough to smooth surfaces) to a speed profile somewhat as indicated, in an exaggerated form, in Fig. 26. When the irregularity in the resulting composite profile is ignored, the effect of the increased speed at the lower levels, due to the lower drag of the smoother surface, gives rise to an anomalously low-wind gradient in the layer AB as compared with either above or below. The values of β are correspondingly affected, but the change with varying stability is probably reliable qualitatively. The values of β from the speeds at 5, 10 and 25 cm., which are probably less affected by the roughness discontinuity, also give a similar variation with stability.

Best also made an extensive series of observations of the gustiness of the wind at a height of 2 m. using a bi-directional vane (his Table XXV). The average vertical gustiness values plotted against $(\theta_{1\cdot 1} - \theta_{0\cdot 1})/u_2^2$, are shown in Fig. 27. The mean velocity for each group of observations is not given, so the mid point of the range had to be assumed in each case. Best's observations show, within the limits of experimental error, that the gustiness depends upon the Richardson number as was found in the present work. The vertical gustiness appears to become vanishingly small at a value of $(\theta_{1\cdot 1} - \theta_{0\cdot 1})/u_2^2 \simeq +2$ corresponding to a Richardson number for the 1·1 to 0·1-m. layer of the order of 0·15, taking the difference in wind velocity between 110 and 10 cm. to be about 50 per cent. of the wind velocity at 2 m. (linear velocity profile*

* Best[7] (p. 39) did obtain an approximately linear velocity profile under conditions of strong stability. The value of $R_i(1 \cdot 1/0 \cdot 1)$ was also about $+0 \cdot 15$ on this occasion.

assumed in the absence of turbulence). This cannot be taken as a precise determination of the critical Richardson number for the suppression of turbulence, but it is probably sufficiently accurate to show that the critical value is appreciably less than unity in the atmospheric layer under discussion, a matter discussed further in § 23.

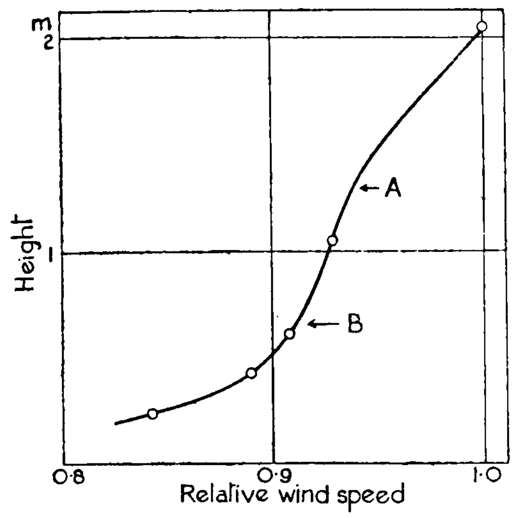


FIG. 26—WIND PROFILE OVER A CONCRETE AREA SURROUNDED BY DOWNLAND
Mean profile from 13 experiments at 90 m. from upwind edge of concrete

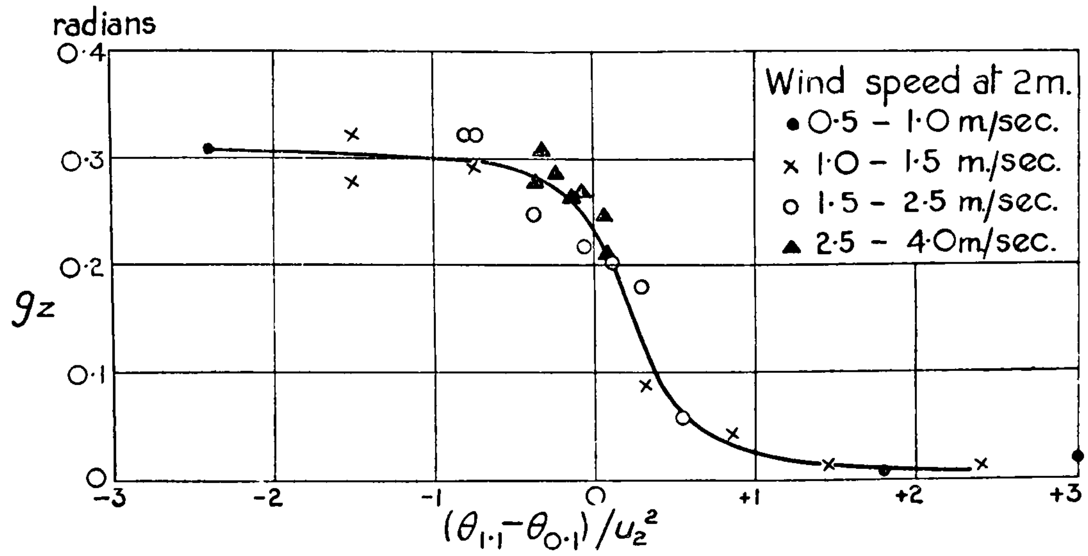


FIG. 27—RELATION BETWEEN BEST'S GUSTINESS VALUES AND STABILITY

Sverdrup's investigation.—Sverdrup[9] made an extensive series of observations of the wind and temperature profiles over a snow surface on Isachsens's plateau, Spitsbergen. The wind speeds were measured by small cup anemometers at 30 and 200 cm. above the surface and by an electric contact cup anemometer at 700 cm. Temperature and humidity were measured at 4, 100 and 500 cm. by Assmann psychrometers. The effective level (4 cm.) at which the lowest Assmann psychrometer measured the air temperature was ascertained by comparison with the results from a shielded thermo-couple at various small heights. Stable atmospheric conditions prevailed on a majority of occasions, and Sverdrup considered the wind speeds at 7 m. to be unreliable for many of the occasions of unstable conditions owing to the deposition of frost on the upper anemometer. He therefore only considered in detail the profiles under adiabatic and stable conditions.

The adiabatic wind profiles Sverdrup found to be well represented by a logarithmic law not differing materially from equation (4). He obtained the value 0·25 cm. for the roughness parameter of the snow surface, the same value as was found for the short-grass surface in the present work (Series B).

Average values of wind speed, potential temperature and vapour pressure from Sverdrup's Table 3 are reproduced in Table XVI, with the addition of values of β calculated from equation (13), together with the corresponding index δ_1 for the potential-temperature profiles. Values of the Richardson number for the 4–0·5 m. layer, R_i (4/0·5), are also given, to enable comparison to be made with the short-grass results.

The values of β from Sverdrup's observations given in Table XVI have been inserted in Fig. 20, and it will be seen that the variation of β with the Richardson number $R_i(4/0\cdot5)$ given by Sverdrup's observations is in very good agreement with that found in the present work. In both cases there is a suggestion that with increasing stability above a certain point ($R_i(4/0\cdot5) \simeq 0\cdot1$) some change in régime takes place. Best's gustiness data, already discussed, suggest that this effect may be owing to the turbulence becoming small in the neighbourhood of this value, in which case erratic values of β are perhaps to be expected as the profiles are then very susceptible to distortion by katabatic drifts.

TABLE XVI—SVERDRUP'S WIND-PROFILE OBSERVATIONS OVER SNOW IN SPITSBERGEN

Sverdrup's Group No. Mean wind speed†		VII $<2\cdot0$ m./sec.	VIII $2\cdot1$–$2\cdot95$ m./sec.	IX $3\cdot0$–$3\cdot95$ m./sec.	X >4 m./sec.	Mean
			metres per second			
Wind speed at levels of	700 cm.	2·07	3·74	4·97	6·63	
	200 cm.	1·55	2·96	4·06	5·60	
	30 cm.	0·98	2·03	2·84	4·00	
β		0·80	0·85	0·93	1·01	0·895
			degrees Centigrade			
Potential temperature at levels	500 cm.	2·68	2·22	1·67	1·48	
	100 cm.	2·02	1·65	1·33	1·25	
	4 cm.	1·12	1·00	0·87	0·84	
δ		0·84	0·76	0·835	0·95	0·845
			millimetres of mercury			
Vapour pressure at levels	500 cm.	5·42	5·19	4·99	4·96	
	100 cm.	5·23	5·06	4·92	4·89	
	4 cm.	4·92	4·89	4·82	4·81	
Richardson number, $R_i(4/0\cdot5)$		0·19	0·06	0·025	0·012	
No. of hours recording		74	77	84	50	

† Mean of 30- and 200-cm. wind speeds.

The values of δ are more irregular than the values of β, no doubt owing to the considerable experimental error in measuring small temperature differences. The trend, however, is for δ to approach unity as the stability decreases. The mean value of δ of 0·845 lies close to the mean value of β of 0·895. The values of the corresponding index for the vapour-pressure profiles are even more irregular than in the case of δ, a consequence of the very small differences in vapour pressure between the various levels. They are, however, all less than unity (0·77 to 0·92), and the mean value 0·84 agrees with the mean value of δ. There appears therefore to be a fair degree of similarity between the wind, potential-temperature and vapour-pressure profiles, in agreement with Sverdrup's conclusions.

Sverdrup[9] has given an analysis of the evaluation of the eddy viscosity from the speed and temperature profiles under stable conditions taking as starting point an equation due to Rossby and Montgomery[17], which in the notation of this paper is

$$\frac{du}{dz} = \frac{u_*}{kz}\sqrt{(1 + \sigma R_i(z))} \qquad \ldots\ldots (24)$$

where $R_i(z)$ is the Richardson number at height z and σ is a constant of proportionality which Sverdrup finds to be approximately 11. The justification for this relationship is not very clear, and will not be discussed here. Sverdrup was able to solve equation (24) by a method of successive approximations, assuming

$$\frac{d\theta}{dz} \propto (z + z_0)^{(1-n)/n} \text{ where } 3 < n < \infty,$$

and making a first estimate of u_* by assuming that the logarithmic wind profile can be applied from 30 cm. down to the surface. This is justified by the decrease of R_i as the surface is approached, the term under the surd in equation (24) then approaching unity.

Sverdrup deduces that equation (24) leads to the following relations with small and large stability :—

small stability .. $\frac{du}{dz} \propto z^{-1}$

large stability .. $\frac{du}{dz} \propto z^{-2/3}$

and says that the relationship shown in equation (11),

$$\frac{du}{dz} = az^{-\beta},$$

should be a suitable interpolation formula for intermediate stabilities, as is, in fact, found to be the case in the present work. This relationship gives such a close fit to the observations that it is used here to derive σ, in preference to the method adopted by Sverdrup. Re-arranging equation (24) and integrating between the levels z_1 and z_2 (at which heights the speeds are u_a and u_b and potential temperatures θ_a, θ_b) gives

$$\frac{k^2}{u_*^2}\int_{z_1}^{z_2} z^2\left(\frac{du}{dz}\right)^4 dz = \frac{\sigma g}{T}\int_{z_1}^{z_2}\frac{d\theta}{dz}dz + \int_{z_1}^{z_2}\left(\frac{du}{dz}\right)^2 dz\ . \qquad \ldots. (25)$$

Using equation (25) and the expression derived from it that

$$a = \frac{(1-\beta)\,(u_b - u_a)}{(z_2^{1-\beta} - z_1^{1-\beta})}$$

gives finally

$$\sigma = \frac{\theta\,(u_b - u_a)^2}{g\,(\theta_b - \theta_a)}\left\{\frac{(u_b - u_a)^2}{u_*^2}F\,(\beta, z_1, z_2) - G\,(\beta, z_1, z_2)\right\} \qquad \ldots. (26)$$

where $F\,(\beta, z_1, z_2) = k^2\left(\frac{1-\beta}{z_2^{1-\beta} - z_1^{1-\beta}}\right)^4\left(\frac{z_2^{3-4\beta} - z_1^{3-4\beta}}{3-4\beta}\right),$

and $G\,(\beta, z_1, z_2) = \left(\frac{1-\beta}{z_2^{1-\beta} - z_1^{1-\beta}}\right)^2\left(\frac{z_2^{1-2\beta} - z_1^{1-2\beta}}{1-2\beta}\right).$

This analysis has the advantage of avoiding assumptions as to the form of the temperature profile, only the difference in potential temperature between the levels z_1 and z_2 being required. The friction velocity u_* must, however, be evaluated before σ can be calculated, and this has been done by assuming, as Sverdrup also does, that flow conditions near the surface are but little affected by thermal influences, so that in the lowest 30 or 50 cm. the logarithmic law, equation (4), may be applied to evaluate u_*, a roughness parameter z_0 independent of stability being assumed. This first approximation to u_* gives, from equation (26), a first approximation to σ which enables a second approximation to u_* to be obtained in the manner indicated by Sverdrup. This gives a second approximation to σ which in the following instances was found to differ but little from the first.

Values of σ have been calculated by the above method from Sverdrup's data for comparison with those he obtained. The 4–0·5 m. wind-speed and potential-temperature differences were obtained by graphical interpolation, the observations being plotted against height on a logarithmic scale. The values of u_* were calculated from the 30-cm. wind speed using $z_0 = 0{\cdot}25$ cm. as found by Sverdrup. The comparison is as follows :

	Sverdrup's groups VII	VIII	IX	Mean
σ by equation (26)	12·6	10·4	12·8	11·9
σ found by Sverdrup	11·3	10·2	12·0	11·2

The differences between the results given by the two methods are insignificant, so the method given above has been used in the following. It should be noted that the values of σ are independent of the value of von Kármán's constant, k, as u_* in equation (26) involves k, and this cancels with k^2 in the F term*. It was also found that an uncertainty in β of say 5 per cent. only produced 2–4 per cent. uncertainty in σ under stable conditions†.

The 4–0·5-m. speed differences, the 4–0·5-m. potential-temperature differences (found by interpolation using the values of δ in Fig. 24) and the values of β over a short-grass surface (from Table XI) have been used to calculate σ. Errors in the values of δ would lead to only comparatively small errors in the estimated 4–0·5 m. potential-temperature differences. Constant values of z_0 of 0·23 and 0·40 cm. were employed. The values of σ so found are given in Table XVII, the values for the groups very close to neutral stability being omitted as not giving reliable results.

TABLE XVII—COMPARISON OF THE RICHARDSON NUMBER R_i(4-0·5) WITH THE STABILITY PARAMETER σ OF ROSSBY, MONTGOMERY AND SVERDRUP

Series A ($z_0 = 0 \cdot 40$ cm.)

Group	1	2	5	6
R_i(4/0·5) ..	−0·10	−0·03	+0·04	+0·08
σ	4·1	4·1	7·2	25

Series B ($z_0 = 0 \cdot 23$ cm.)

Group	1	2	3	4	7	8	9	10	11	12
R_i(4/0·5) ..	−0·38	−0·09	−0·06	−0·03	+0·055	+0·065	+0·09	+0·10	+0·135	+0·25
σ	2·0	5·0	6·3	6·6	5·4	7·0	10·5	26	29	26

Values of σ for unstable conditions have been included in Table XVII, because, although Sverdrup[9] at first only considered stable stratifications, in a later paper[22] he concluded from an analysis of Best's results[7] that equation (24) was valid also for unstable conditions. The values of σ are shown related to the Richardson number for the 4–0·5-m. layer in Fig. 28, which also includes the values given by Calder's desert data (see Appendix) analysed in an exactly similar manner to the short-grass results of Table XI. The desert observations are valuable as the roughness parameter of the surface, 0·025 cm., was ten times smaller than for the short-grass surface or Sverdrup's snow surface. Sverdrup's values are also shown in Fig. 28.

The agreement between the values of σ for the observations over short grass and those for the much smoother desert surface is reasonably satisfactory, but neither set of observations supports Sverdrup's conclusion that σ is a universal constant having a value of about 11·0. There is some indication that σ may be approximately constant over a restricted range of stabilities around neutral, but the results indicate a value of about 6 rather than 11. It should be noted, however, that the value of σ is very sensitive to the value assumed for the roughness parameter; had Sverdrup used a value $z_0 = 0 \cdot 30$ cm. instead of 0·25 cm. then he would have found $\sigma \simeq 7$ in good agreement with the present work. Taking into account that Sverdrup was not able to calibrate his anemometers very accurately and the climatic conditions to which they were exposed, it seems that the discrepancy noted above may not be significant.

* Sverdrup used $k = 0 \cdot 38$, whereas $k = 0 \cdot 40$ has been used in the present work.

† Under unstable conditions the results for the short-grass surface showed that a 5 per cent. β change produced a 5–8 per cent. change in σ.

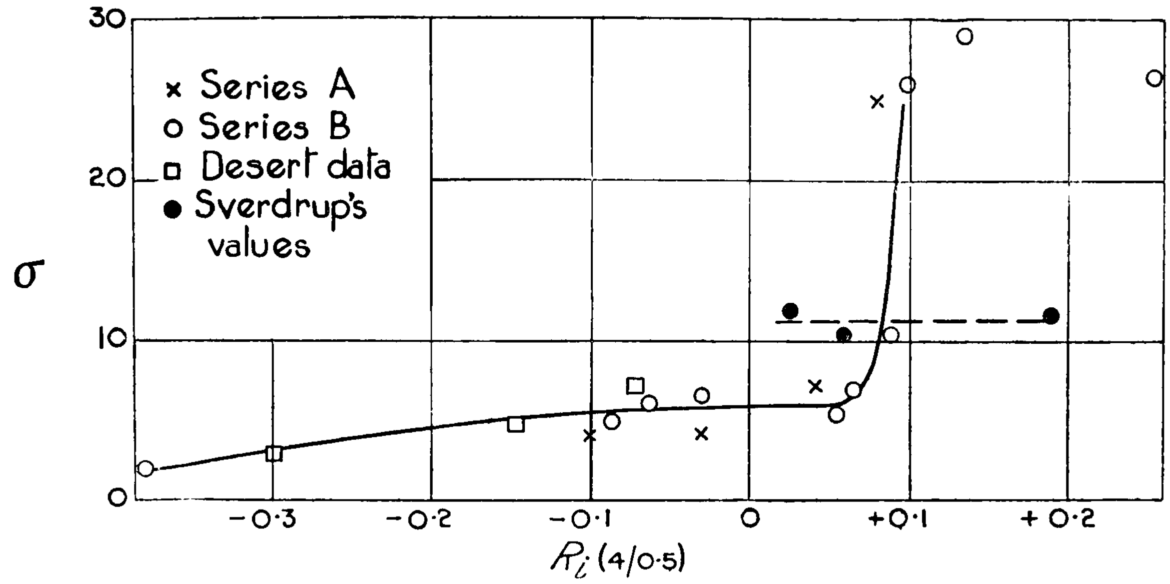


FIG. 28—ROSSBY-MONTGOMERY STABILITY CONSTANT σ

The reason for the decrease of σ with increasing instability is apparent from an examination of the Rossby-Montgomery relation, equation (24), which indicates imaginary values of the wind gradient if $\sigma R_i(z) < -1$. Now, both over the short grass and the desert, values of R_i (4/0·5) of $-0 \cdot 3$ have been observed with quite measurable speed gradients, and over the prairie by Tait (results unpublished) the following values have been recorded on different occasions of high sun and clear sky :—

Wind speed at 2 m. (m./sec.)	5·7	3·5	2·25	2·45
Wind speed difference 2 — 1 m. (m./sec.)	0·56	0·38	0·21	0·32
Temperature difference 2 — 1 m. (°F.)	−1·8	−1·8	−1·8	−2·3
Temperature difference 2 — 1 m. (°C.)	−1·0	−1·0	−1·0	−1·3
Richardson number (R_i(2/1))	−0·11	−0·23	−0·75	−0·42

Values of the Richardson number of around $-0 \cdot 5$ are therefore observed, on occasion, in the lowest few metres of the atmosphere, and for such values equation (24) necessarily indicates a value of σ of less than 2. As R_i is found to increase with height, fractional values of σ would probably be indicated by observations at, say, 10 m. under conditions of strong instability.

Sverdrup calculated the eddy viscosity at various heights over the snow surface, and from heat-balance considerations (radiation, ablation, albedo measurements, etc.) he concluded that

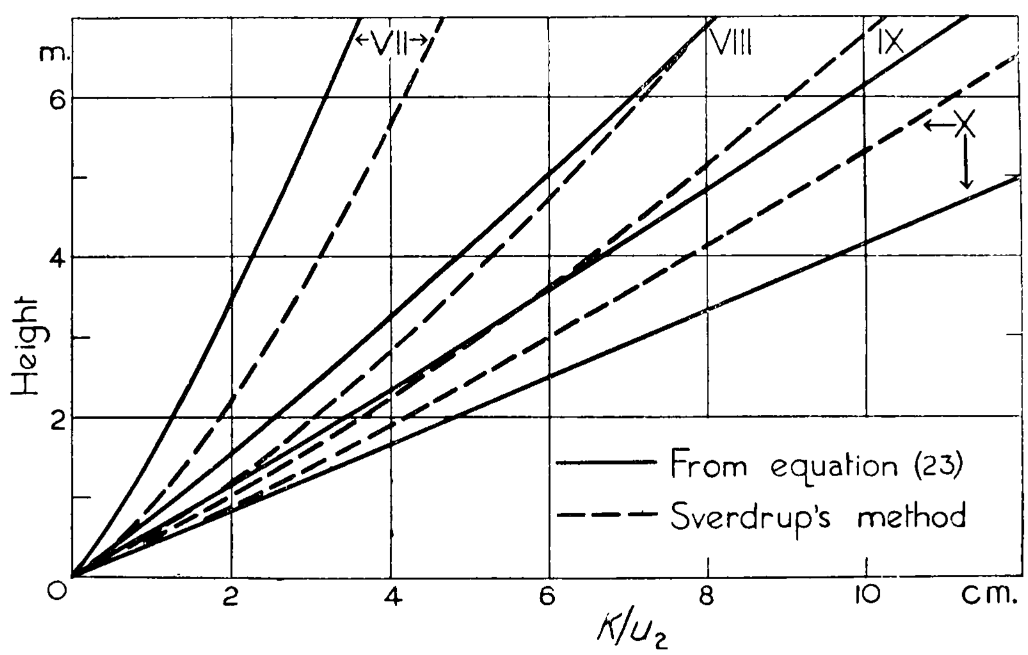


FIG. 29—EDDY VISCOSITIES OVER A SNOW SURFACE

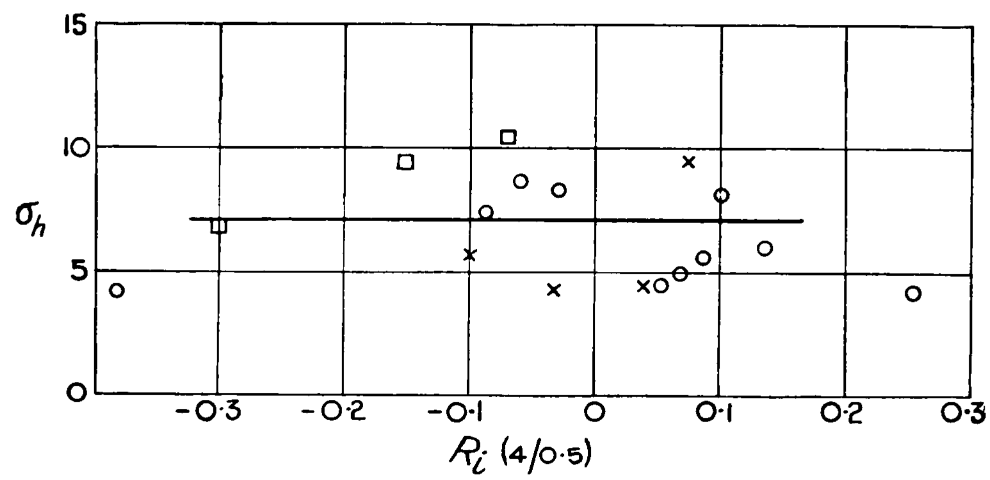
the eddy conductivity was equal to the eddy viscosity; the agreement was close, considering the uncertainties due to the considerable number of factors entering into the heat-balance equation. It seems, however, that the heat-balance work supported Sverdrup's estimated values of eddy viscosity. It is of interest, therefore, to compare the eddy viscosities calculated from Sverdrup's wind-speed measurements using equation (23) with those deduced by Sverdrup from the same data. $R_{7/0\cdot3}$ was used to calculate β', using $z_0 = 0\cdot25$ cm. as found by Sverdrup. The comparison of values of K/u_2 calculated by the two methods is given in Fig. 29, and it is evident that the differences are generally less than 10 per cent., except for the most stable conditions where the difference is 30 per cent. The fact that Sverdrup's heat-balance measurements agreed with his eddy viscosities in this case also is probably of little significance, as the errors in the estimates of heat transport by turbulence would naturally be greatest when the turbulence was small.

Holzman's investigation.—Holzman[39] has pointed out that the Rossby-Montgomery relationship, equation (24), gives no indication of any change of régime when the critical value of the Richardson number for the suppression of turbulence is reached. He suggested the relationship

$$\frac{du}{dz} = \frac{u_*}{kz}\left(1 - \sigma_h R_i(z)\right)^{1/2}, \qquad \ldots\ldots (27)$$

which, while almost identical with the Rossby-Montgomery equation for small values of R_i, would indicate a limiting value of positive stability at $1/\sigma$, i.e. about $R_i \doteqdot 0\cdot1$ if σ_h is of the order of 10.

An analogous relationship to equation (26) can be obtained from Holzman's equation, and values of σ_h so obtained from the observations over short grass and the desert surface are shown in Fig. 30, from which it is seen that, although the scatter is considerable, there is no systematic variation of σ_h with stability or surface roughness. The mean value, excluding the two points at the extremes of the stability range, is $7\cdot1$ with a standard error of $0\cdot6$.



× Short-grass surface, Series A, $z_0 = 0\cdot40$ cm.
○ Short-grass surface, Series B, $z_0 = 0\cdot27$ cm.
□ Desert near Karachi, $z_0 = 0\cdot025$ cm.

FIG. 30—HOLZMAN'S STABILITY CONSTANT σ_h

Thornthwaite and Kaser's investigation.—Thornthwaite and Kaser[40] have published wind profiles between $0\cdot15$ and $8\cdot5$ m. measured over a level field in Ohio. The profiles are of the logarithmic form at those times of day (1–2 hr. after sunrise and 1–2 hr. before sunset) when the stability conditions are approximately neutral. Under stable and unstable conditions the profiles of u against $\log z$ are curved, but in opposite senses in the two cases in the same way as observed in the present work.

Laikhtman's investigation.—While this account was being prepared, a translation of Laikhtman's paper[25] came to hand. He proposes the use of equation (20) with the difference that he uses the value of β found from the speeds at several heights which results in a roughness parameter varying with stability. He appears not to have appreciated that the diminishing effect of thermal forces near the surface warrants the use of a constant roughness parameter.

§ 23—CRITICAL VALUE OF THE RICHARDSON NUMBER FOR THE ONSET OF TURBULENCE

Sutton[52] has given, in Chapter 4 of his book, a review of the situation respecting the critical value of the Richardson number which marks the onset of turbulence in a boundary layer as the stability of the stratification is gradually decreased. Richardson[42] and Prandtl[43]* found the critical value to be unity. Later Taylor[44] and Goldstein[45], applying the method of small oscillations to examine the stability of plane laminar flow with a stable density stratification (linear velocity profiles being assumed for simplicity) found critical values of 0·25.

Schlichting[41] analysed the case of non-linear velocity profiles for comparison with results obtained by Reichardt in the Göttingen hot-cold wind tunnel since the velocity distribution in the tunnel was not linear. He was able to make the calculations for the case of a profile, linear from the boundary to a certain height then becoming part of a parabola connecting tangentially with the linear profile and with the line of constant velocity in the free stream. He found the critical Richardson number, $R_{i\,(\mathrm{crit.})}$, to be a function of the Reynolds number and the Froude number, but for large Reynolds number and small Froude number—the meteorologically significant case—the value of $R_{i\,(\mathrm{crit.})}$, according to Schlichting, is 0·041, and Reichardt's wind-tunnel results give good agreement with this theoretical prediction. It therefore appears that the critical Richardson number is very sensitive to change in form of the velocity profile.

The wind-speed observations of the present work indicate that the form of the profile becomes rather variable under conditions of marked stability, and the profiles in the absence of turbulence, though tending as shown by Best to be linear, are likely to be very subject to irregularity owing to katabatic drifts, except possibly over very level uniform sites. When a linear wind distribution occurs up to a considerable height, the conditions considered by Taylor might be closely approached ($R_{i\,(\mathrm{crit.})} = 0{\cdot}25$). At other times lower critical values, as found by Schlichting, may be appropriate. The best that can be said, in the absence of further detailed studies of the influence of wind-profile form, is that theoretical considerations indicate a critical Richardson number of the order of 0·1 for the surface layers of the atmosphere.

Before comparing the experimental evidence from wind-structure observations with the theoretical predictions, an examination of Reichardt's wind-tunnel profiles throws some light on what may be expected to occur in the atmosphere. Schlichting[41] (Figs. 10–14) gives a series of temperature and velocity profiles over the lower surface† for a range of air speeds covering the transition from stable laminar flow to turbulent flow, the transition being recognized by using a hot-wire anemometer, the output of which becomes fluctuating when turbulence sets in. The values of the Richardson number were obtained from photographic enlargements of these profiles, and are shown plotted against height in Fig. 31. At a free-stream speed of 87·5 cm./sec. the flow was laminar, and Fig. 31 shows that under these conditions R_i is almost constant with height at 0·05, the speed and temperature profiles being practically linear up to 4 cm. At 98·5 cm./sec. air speed the flow was at times laminar and at others turbulent, showing that conditions were close to the critical point. Fig. 31 shows that in the lowest 2 cm. the value of R_i fell below 0·04, and the occurrence of turbulence at times with this value accords with Schlichting's theoretical prediction. It should be noted, however, that the value of R_i exceeds 0·04 considerably at 4 cm. height and above. With speeds of 104–134 cm./sec. the flow was always turbulent, and Fig. 31

* Prandtl first obtained the value 0·5 but this was later shown by Taylor to be incorrect, the corrected value being 1·0.

† The lower surface of the wind tunnel was cooled by running water (10° C.) while the upper was heated by steam.

shows that only in the lowest centimetre or so of the flow was the Richardson number less than that occurring when the flow was laminar. It appears, therefore, that once stability near the surface has decreased sufficiently to permit turbulence to be generated, the turbulence is maintained in layers further removed from the surface even when the Richardson number in these layers is considerably greater than the critical value. It appears from this that the critical value of the Richardson number in boundary-layer flow should be determined from observations close to the surface.

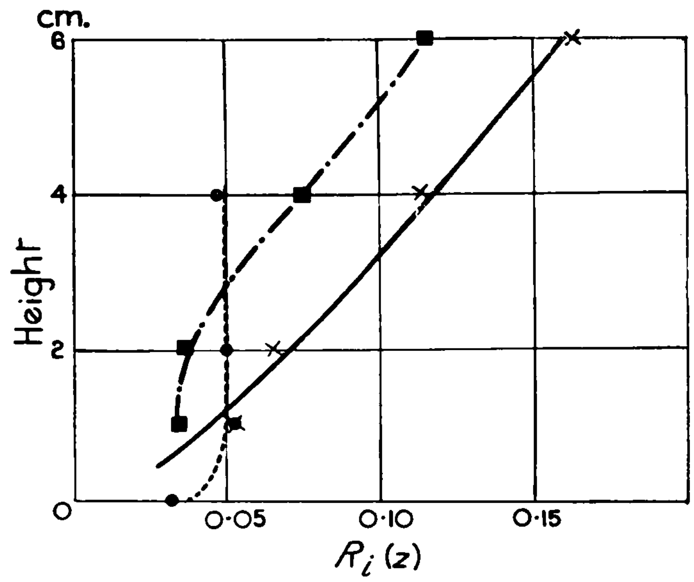


FIG. 31—VARIATION OF RICHARDSON NUMBER WITH HEIGHT

Reichardt's wind-tunnel results

· · · · · · 87·5 cm./sec. laminar flow

—·—·—·— 98·5 cm./sec. laminar and turbulent flow in turn

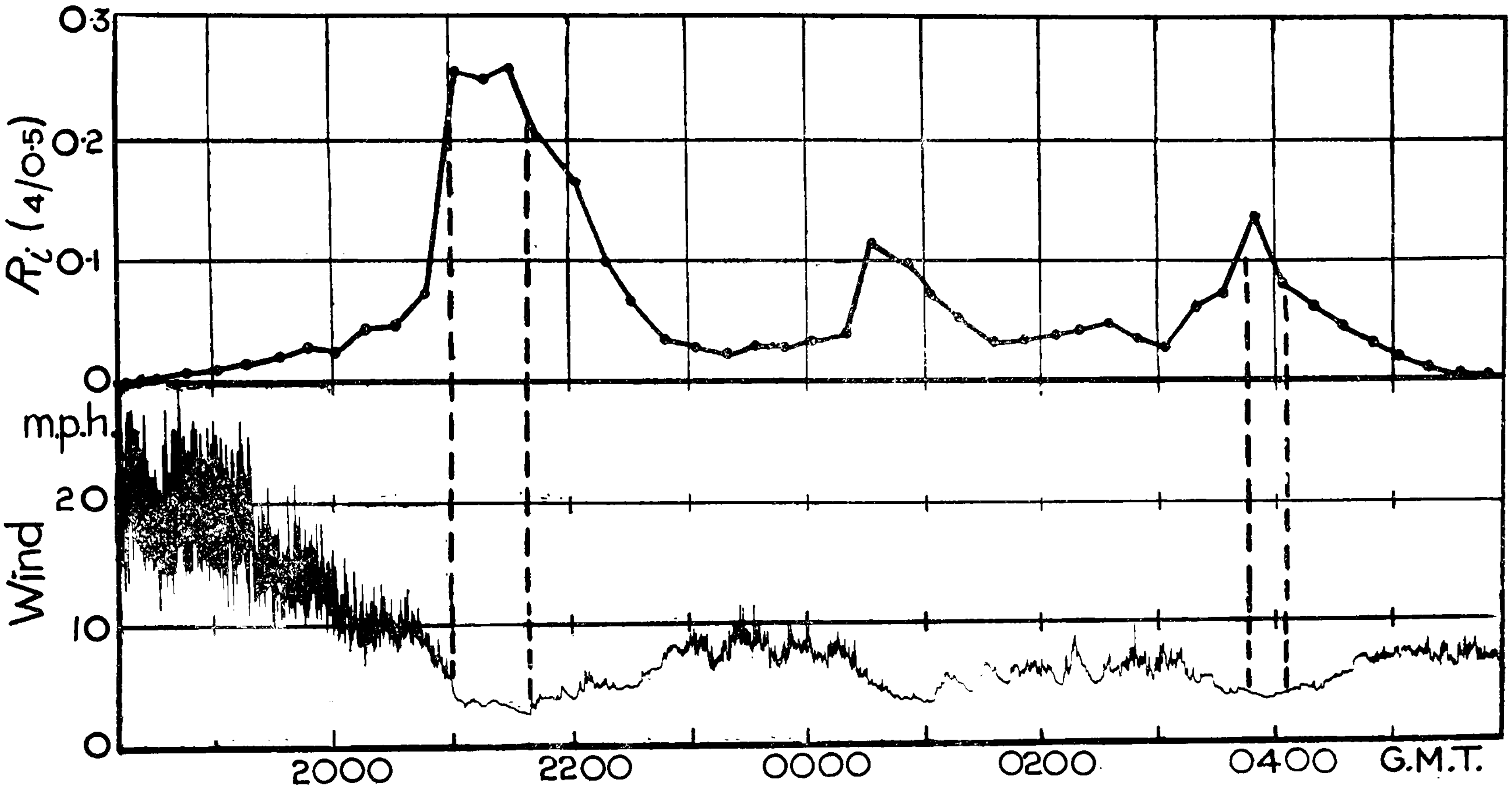
———— 104–134 cm./sec. turbulent flow, mean of three profiles

Comparing the atmospheric profiles with Reichardt's observations, it is noted that the increase of the Richardson number with height in the atmospheric boundary layer accords with the wind-tunnel observations when turbulence is present.

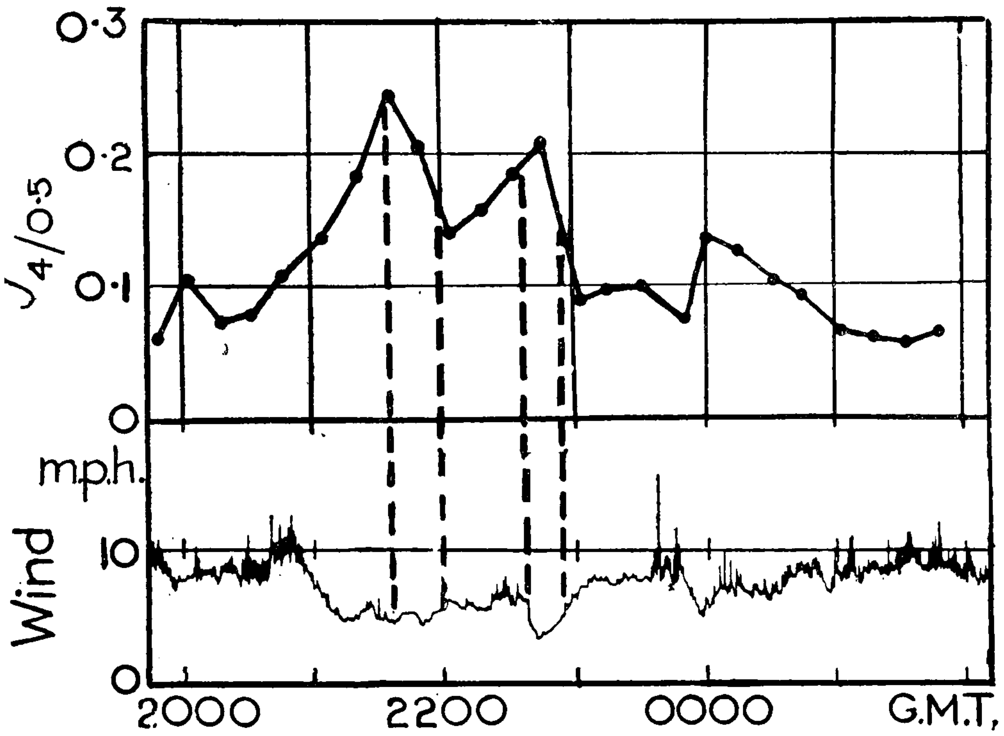
It is apparent from Reichardt's work that correlating the observations of the Richardson number, for a layer of the order of 10 m. deep, with the intensity of turbulence at a similar height, may lead to erroneous deductions being made as to the critical value of the Richardson number, as it seems that, at a height of several metres, R_i may considerably exceed the critical value while turbulence still persists owing to the instability of the flow at lower levels. This may be the reason for Richardson[46]* and other investigators finding the critical value of R_i to be in the neighbourhood of unity. Richardson measured temperature at 4·4 and 18·3 m. above the ground and wind velocities at 2 and 26 m., the turbulence at 26 m. being indicated by the breadth of trace given by a pressure-tube anemograph. Durst[47] also used observations at considerable heights (temperature at 1·2 and 43 m. (4 and 143 ft.) and velocities at 15 and 46 m. (50 and 150 ft.) but the upper anemometer was poorly sited for this work.

The records of the pressure-tube anemograph on the short-grass area (orifice height 13·3 m.) have been examined in conjunction with the values of the Richardson number for the air layer 4 to 0·5 m. given by the cup anemometer and temperature-gradient installation. It would have been preferable to have used a more sensitive measure of turbulence than that given by the character of the pressure-tube anemogram, but at least the results of this study should be comparable with the similar investigations of Richardson[46] and Durst[47]. In all, 14 suitable occasions of not too light wind in the requisite direction were available. Two typical examples

* Richardson remarks that a weakness of his observations was that they were made over a rather considerable range of height in a layer in which the speed and temperature gradients varied markedly with height.



Anemogram for July 27–28, and associated values of Richardson number, R_i (4/0·5)



Anemogram for November 3–4, 1944, and associated values of Richardson number

FIG. 32—SUPPRESSION OF TURBULENCE AT LARGE STABILITIES

are shown in Fig. 32, the times at which turbulence appeared to die out and to revive being indicated. In the examples shown, it appears that values of R_i (4/0·5) of from 0·1 to 0·2 are associated with the transition from turbulent to stream-line flow conditions and *vice versa*. The mean values given by all 22 occasions are as follows :—

	Turbulence died	Turbulence revived
Mean R_i(4/0·5)	0·16	0·14
Range of R_i(4/0·5) ..	0·09 to 0·29	0·09 to 0·23
Number of observations ..	22	15

This determination of the critical values is not very accurate because considerable changes of stability often occur from one quarter-hour period to the next, and frequently the value of $R_i(4/0 \cdot 5)$ rises or falls rapidly when the transition occurs. This behaviour in itself, however, gives an indication that a change of régime is taking place, and lends some support to the evidence given by the pressure-tube anemograms. The transition is indicated to occur at about $R_i\ (4/0 \cdot 5) = 0 \cdot 15$, which agrees with evidence considered in earlier sections that a change of régime occurs around this value and is consistent with Best's study of the variation of vertical wind gustiness with stability.

It is considered that the above results show that the critical value of the Richardson number for the 4 to 0·5 m. layer is considerably smaller than unity, and therefore smaller than the values obtained by Richardson and by Durst for surface air layers 20 to 40 m. deep. This finding accords with the expectation from Reichardt's work that greater heights of observation are likely to lead to higher critical values being deduced than indicated by observations close to the surface.

§ 24—DIURNAL VARIATION OF STABILITY IN FINE WEATHER

As the Richardson number in a layer of air near the ground is directly related to the potential temperature difference between the top and bottom of the layer divided by the square of the wind speed at some fixed height in the layer (see § 11), it was a simple matter to investigate the diurnal variation of stability in fine summer weather at Porton using the available records of potential temperature difference between 17·1 and 1·2 m. together with the wind speeds given by the pressure-tube anemograph (orifice height 13·3 m.). In all, 41 days in the months May to July (inclusive) during the period 1932–38 fulfilled the requirement of an almost uninterrupted sunshine trace and starshine trace. These occasions were divided into two groups according to whether the 13·3-m. wind averaged more or less than 3·5 m./sec. during the period 0700–1800 G.M.T. The mean diurnal variations of wind speed, potential temperature difference and $(\theta_{17 \cdot 1} - \theta_{1 \cdot 2})/u^2$ are shown in Fig. 33, different scales being used for positive and negative values of the latter quantity because the positive range is much greater than the negative.

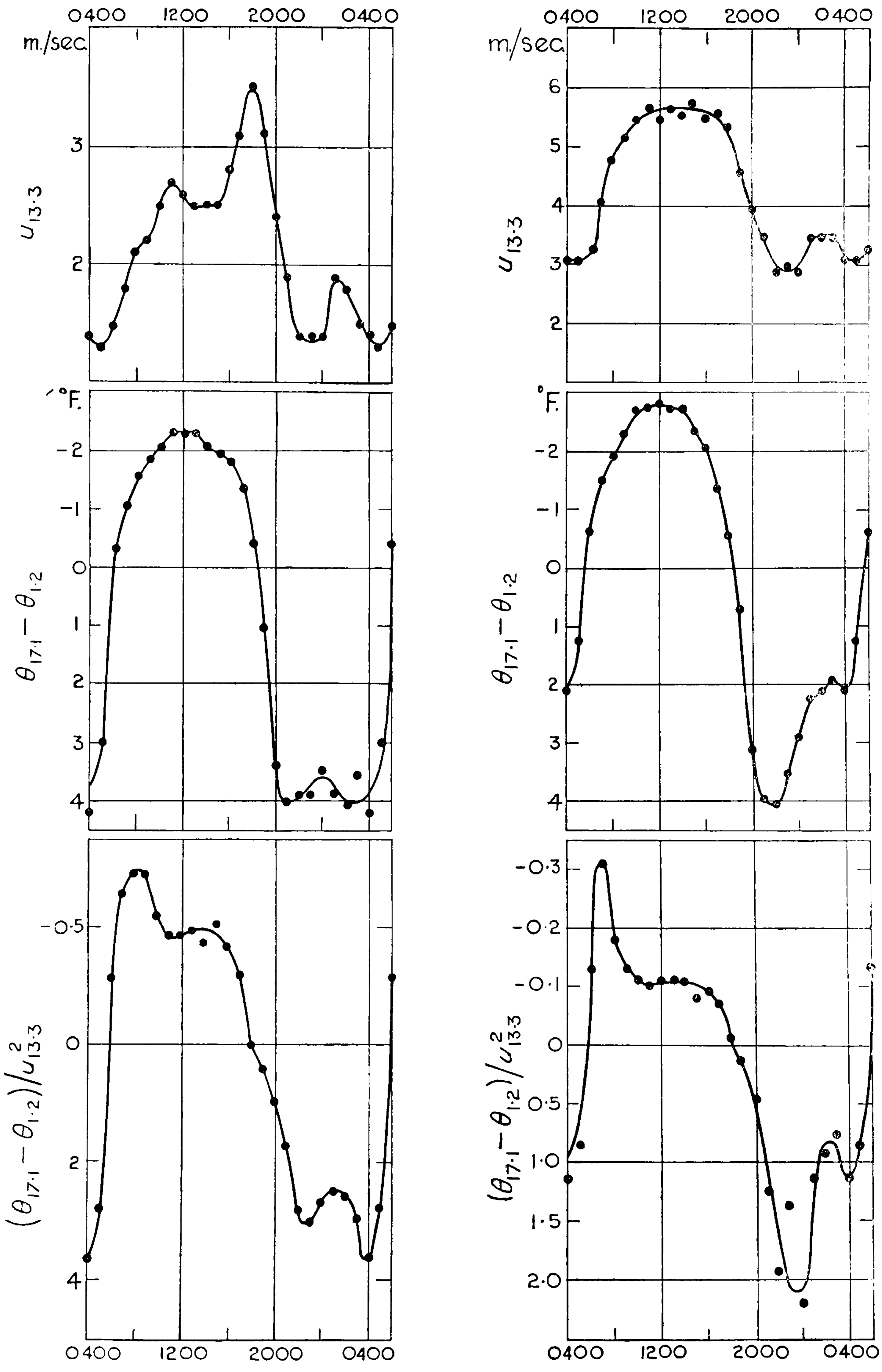
A noticeable feature of Fig. 32 is that, for both wind-speed ranges, the maximum instability in the surface layers occurs several hours before noon, owing to the difference in phase between the diurnal variations of wind speed and temperature gradient. The wind-speed variation lags behind the temperature-gradient change, with the result that in the morning light winds and moderate temperature gradients result in large instability, larger than in the afternoon when the wind speed is, on the average, greater. The phase difference between the wind and temperature-gradient variations also causes the rate of change of stability with time to be smaller around sunset than at the morning change-over from stable to unstable conditions.

It is interesting to note that at a height of 13 m. the wind speed has a secondary maximum at midnight or shortly after. At greater heights, this secondary maximum, as is well known[8], becomes increasingly pronounced, until finally at heights of the order of 300 m. the diurnal variation is the reverse of that near the ground.

§ 25—CONCLUSIONS

The main conclusions drawn from the observations of mean profiles of wind speed and temperature in the lowest 8 to 13 m. of the atmosphere can be briefly summarized as follows :—

(1) The wind profiles under neutral conditions of stability agree very well with the form of logarithmic law proposed by Prandtl[48]. The roughness parameters of various natural surfaces deduced from the logarithmic law are in good general agreement with those found by other



(*a*) Mean day-time wind < 3·5 m./sec. (*b*) Mean day-time wind > 3·5 m./sec.

FIG. 33—DIURNAL VARIATION OF WIND SPEED, TEMPERATURE GRADIENT AND STABILITY ON CLEAR SUMMER DAYS

workers. It appears that the flow in the surface layers of the atmosphere over ground covered with vegetation, or otherwise markedly roughened, is of the aerodynamically rough type. Transitional or aerodynamically smooth flow occurs over relatively smooth surfaces such as mud flats, very level snow, ice-sheets, etc.

(2) The influence of a temperature stratification on the wind profiles is found to be very small in close proximity to the surface, but to increase markedly with height with the result that the profiles depart from the logarithmic form in the following way :—

Stable stratification.—The wind gradient decreases with height more slowly than under neutral conditions, with the result that the graphs of u against $\log z$ are concave towards the u-axis.

Unstable stratification.—The wind gradient decreases more rapidly with height than under neutral conditions, so that the graphs of u against $\log z$ are convex towards the u-axis.

(3) The form of the wind profiles and the intensity of eddy motion over a surface of given roughness is found to be, to a good degree of approximation, dependent only on the value of the Richardson number at a standard height. The profiles are well represented by the relationship

$$\frac{du}{dz} = az^{-\beta},$$

β being a function of the Richardson number and having values greater than unity under unstable conditions and less than unity under stable conditions. There is, however, evidence (mainly indirect) that β is not quite constant with height, but may tend to unity close to the surface.

(4) The result that thermal effects on the wind profile are negligible very close to the surface enables a generalized speed-profile law to be obtained, namely

$$\frac{u}{u_*} = \frac{1}{k\,(1-\beta')}\left[\left(\frac{z}{z_0}\right)^{1-\beta'} - 1\right]$$

where z_0, the roughness parameter of the surface, is assumed to be constant over a considerable range of stability. This equation enables the eddy viscosities and diffusivities to be calculated from measurements of the wind speeds at two heights if the roughness parameter of the ground surface is known from measurements under neutral conditions. The analysis is only valid for the layer of effectively constant shearing stress, i.e. the lowest 30 to 50 m. of the atmosphere.

(5) The Rossby–Montgomery formulation[17] of the effect of stability on the wind gradient, i.e.

$$\frac{du}{dz} = \frac{u_*}{kz}\,(1 + \sigma R_i(z))^{1/2},$$

where $R_i(z)$ is the Richardson number at height z and σ is presumed to be a universal constant, is found not to agree with observations at large stability or instability. Holzman's proposed modification

$$\frac{du}{dz} = \frac{u_*}{kz}\,(1 - \sigma_h R_i(z))^{-1/2}$$

gives much better agreement over a wide range of stability and surface roughness, σ_h being found to be approximately 7.

(6) Measurements of eddy velocities with a modified form of Taylor bi-directional vane[7] indicate that turbulence becomes vanishingly small when the Richardson number for the layer 4 to 0·5 m. exceeds about 0·15. Pressure-tube anemograms also support this conclusion, and Reichardt's experiments on the effects of thermal stratification on air flow over a plane surface in a wind tunnel show the reason for the critical value of the Richardson number being smaller than has been found previously from observations at heights of the order of 10 to 40 m.

§ 26—ACKNOWLEDGEMENTS

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APPENDIX I—WIND PROFILES OVER A DESERT SURFACE AND OVER THE SEA

Desert surface.—A considerable number of observations of wind speeds at several heights above a desert surface were made in 1943 by K. L. Calder. The area was a large level expanse of uniform surface of sun-baked sandy alluvium at a site ten miles west of Karachi, free from vegetation and, for 300–400 yds. about the instrument position, completely flat.

The wind-speed profiles were measured at six heights simultaneously, using Sheppard cup anemometers which were systematically interchanged between each 3-min. run to reduce the effect of calibration errors. The results under neutral conditions of stability are given in Table XVIII.

TABLE XVIII—WIND-SPEED PROFILES OVER A DESERT SURFACE UNDER NEUTRAL STABILITY

Date	No. of obs.	Mean $\theta_{4\cdot25}-\theta_{0\cdot30}$	$\bar{u}_1$	Speed ratios $R_{0\cdot1}$	$R_{0\cdot2}$	$R_{0\cdot5}$	R_1	R_2	R_5
		°F.	m./sec.	*per cent.*					
July 27, 1943	5	0·0	4·60	73·1	82·3	93·1	100·0	110·0	121·9
August 5, 1943	8	−0·3	3·66	77·1	85·0	93·9	100·0	111·6	122·7
August 9, 1943	4	0·0	4·70	74·4	83·1	93·4	100·0	110·9	122·1
August 31, 1943	36	0·0	6·76	73·0	81·1	91·6	100·0	110·3	121·5
Weighted mean	..	−0·05	5·93	73·7	81·9	92·2	100·0	110·6	121·8
Calculated mean†	..	..	..	73·1	81·6	92·9	101·4	110·0	121·3
Percentage difference	..	..	..	+0·8	+0·3	−0·7	−1·4	+0·6	+0·4

† Calculated from the log law, $u = 28\cdot35 \log_{10} (z/0\cdot0264)$, where z is in centimetres.

Comparison of the observed mean speeds in Table XVIII with those given by a logarithmic law fitted by least squares shows that the agreement is good except at 1 m. where the difference amounts to 1·4 per cent. which is perhaps a little more than would have been expected on the grounds of experimental error. The roughness parameter is 0·026 cm., which indicates that this desert surface was very considerably smoother than short grassland.

The observations under unstable conditions* were grouped into three ranges of $(\theta_{4\cdot25} - \theta_{0\cdot30})/u_1^2$ and the weighted means are given in Table XIX.

TABLE XIX—WIND PROFILES OVER A DESERT SURFACE UNDER UNSTABLE CONDITIONS

Group	No. of obs.	Mean $\theta_{4\cdot25}-\theta_{0\cdot30}$	$\bar{u}_1$	Mean $\dfrac{\theta_{4\cdot25}-\theta_{0\cdot30}}{u_1^2}$	Velocity ratios $R_{0\cdot1}$	$R_{0\cdot2}$	$R_{0\cdot5}$	R_1	R_2	R_5
		°F.	m./sec.		*per cent*					
2	36	−3·35	7·72	−0·055	74·85	83·07	93·55	100·0	109·01	114·76
3	37	−5·0	7·43	−0·091	..	83·03	93·90	100·0	107·93	112·21
4	8	−5·2	5·64	−0·162	77·60	85·50	95·50	100·0	108·00	113·00

* The high wind speeds prevailing at Karachi at the season of the experiments precluded occasions of appreciable stability being observed.

The profiles exhibit the same tendency for the wind gradient to decrease more rapidly with height than $1/z$, as was observed over the grass surfaces at Porton. The values of β in $du/dz \propto z^{-\beta}$ calculated from the speeds at 20, 100 and 500 cm. are :—

	Group 1	2	3	4
β	0·89	1·085	1·20	1·065

The Group 1 (neutral) observations give a value of β somewhat less than unity while the value for Group 4 (only 8 observations) is somewhat low. Taking into account the small magnitude of the wind gradient over this very smooth surface, the agreement between the trend of β from these observations and that observed over grassland is tolerably good, remembering that each observation over the grass surfaces extended over a period 5 times longer than was the case for the observations over the desert.

Sea surface.—Observations of mean wind velocities at several heights over the sea were made in 1922 by Sir Nelson Johnson, considerable care being taken to ensure that the anemometers were exposed in such a way that they were not influenced by the presence of the ship (a destroyer). A Robinson cup anemometer with 3-in. cups on 5-in. arms was mounted at the top of the foremast at a height of 21·3 m. above water level. Readings at lower heights were obtained by means of a jury bowsprit which projected 20 ft. beyond the bows of the ship. At its extremity were carried a vane supporting an electrical fan anemometer 6·4 m. above the water and also a steel tube which projected downwards and terminated in another vane and anemometer, 3·35 m. above the water. All three anemometers were arranged to record on a tape-machine chronograph. On some occasions of high winds, the bowsprit arrangement was not used and a second Robinson anemometer was mounted on an extension to the jack-staff at 9·7 m. above the sea. Records were only taken when the ship was within 15° of being head to the wind; mostly the angle between the fore-and-aft line and the wind was much less. All the anemometers were carefully calibrated before and after the series of experiments. Observations of air temperature were made using Assmann psychrometers at heights of 0·9 and 5·8 m. above the water-level.

The results obtained with wind speeds greater than 7 m./sec. at 21 m. are given in Table XX*, the site of the observations being about four miles south of the Needles, Isle of Wight.

TABLE XX—WIND-PROFILE MEASUREMENTS OVER THE SEA

Date		$u_{21\cdot3}$	$\theta_{5\cdot8}-\theta_{0\cdot9}$	$R_{3\cdot35/21\cdot3}$	$R_{6\cdot4/21\cdot3}$	$R_{9\cdot7/21\cdot3}$	$R_{21\cdot3/21\cdot3}$	Wind direction
		m./sec.	°F.		*per cent.*			°true
June 1922	19th	9·15	−1·6	85·5	93·2	..	100	240
	20th	9·35	+0·9	83·3	92·4	..	100	250
	21st	8·8	−2·2	..	85·3	..	100	240
	21st	7·8	−0·8	81·6	88·5	..	100	310
	22nd	11·1	−1·0	..	..	91·9	100	235
	27th	12·3	−0·7	81·6	85·5	..	100	220
	29th	11·2	−1·6	..	..	95·2	100	250
	30th	9·95	+0·3	89·8	92·5	..	100	265
	30th	10·55	−0·9	..	..	94·7	100	240
Mean		10·0	−0·8	84·3	89·5	93·9	100	—

* Too few observations were obtained with lighter winds to be of much value taking into account that experimental error is likely to be greater with lighter winds.

The potential-temperature differences are relatively small, and taking into account the moderate to strong winds it may be taken that the mean values from the whole series are characteristic of conditions of approximately neutral stability. The mean values of relative wind speed are shown to a base of log z in Fig. 2, and it is seen that the observations fall close to a straight line which has been drawn paying least attention to the 9·7-m. point, for which the observations were few and not taken on the same occasions as the measurements at lower heights. The value of the roughness parameter is found to be 0·02 cm. If the first observation on June 30, 1922, is rejected as giving a noticeably smaller speed gradient than the rest, then the roughness parameter is found to be 0·04 cm.

A roughness parameter of 0·02 to 0·04 cm. agrees well with Model's analysis[26] of Bruch's extensive series of observations[28] over the Baltic. These observations were made at seven heights between 18 and 225 cm., although the speeds were not measured at all these heights simultaneously. Model concludes that these observations indicate a roughness parameter independent of wind speed of from 0·03 to 0·06 cm., the high value being given by a method of analysis similar to the above, while the lower value is found if a zero-point correction of 4 cm. is made, as suggested by Model, to allow for the presence of a laminar boundary layer. There appears to be little justification for supposing that a laminar boundary layer of this depth really exists. Model, together with other authors on this subject, ignores the fact that there must be an uncertainty of a few centimetres in the zero level of the wind profiles measured over sea surfaces using rafts or boats. Such an uncertainty leads to an appreciable uncertainty in the roughness parameter derived from profile measurements in the lowest two or three metres. A similar uncertainty in the zero level in the case of Sir Nelson Johnson's observations is, however, negligible, as the velocities were measured at considerably greater heights.

In contrast to the observations so far considered, which agree in indicating a roughness parameter of the sea surface in moderate to fresh winds of around 0·05 cm. is the result obtained by Rossby and Montgomery[24], who conclude that the roughness parameter of the sea surface is constant at about 0·6 cm. for 15-m. wind speeds of more than 6 or 7 m/.sec. At lower wind speeds they conclude that the flow over the sea is either transitional or of the smooth type, which may well be the case. They arrive at the value 0·6 cm. for moderate or strong winds by various methods, the principal being based on the angle between the surface and gradient winds and on the ratio between the corresponding speeds. They have not, however, discussed the possibly quite considerable influence of thermal stratification on the observations they utilized. Rossby and Montgomery also concluded that Wüst's observations[49] support this value, the data in this case being 13 observations at each of four heights between 20 and 600 cm. above the sea surface. Model[26], however, takes the apparently reasonable view that Wüst's early measurements with simple equipment should be given little weight in comparison with Bruch's observations, which were eight times more numerous and made with better facilities.

Some support for the smaller value of the roughness parameter of the sea is given by some measurements of the gustiness of the wind made by Sir Nelson Johnson in the trials referred to above. A Taylor bi-directional vane was used to compare the intensities of turbulence of the wind over sea and land. The " mean extreme " gustiness values of the wind (both horizontal and vertical) over the sea were found to be approximately one half as great as those measured over open downland using the same apparatus under the same conditions of approximately neutral stability and of height above the surface. This result was also confirmed by gustiness values evaluated from the tape-machine record of the fan-anemometer contacts.

The empirical relationship for neutral stability, between " mean extreme " vertical gustiness g_z as measured by the bi-directional vane and the roughness parameter z_0 shown in Fig. 10 can be used to estimate z_0 from the knowledge that the mean value of g_z at 2 m. over downland is about 0·36. The value of g_z over the sea at 2 m. would then be 0·18 corresponding to a roughness parameter of about 0·02 cm.—a value in reasonable agreement with that given by the velocity profiles.

The difference in roughness between open grassland and the sea indicated by Sir Nelson Johnson's observations corresponds with a 2 : 1 ratio of diffusivities under neutral conditions, the wind velocity being the same in each case. This 2 : 1 ratio of diffusivities has been found to accord well with the results of numerous experiments on the rate of diffusion of smoke over land and sea.

It might at first sight appear surprising that a roughness parameter as small as 0·02–0·05 cm. should be found for a water surface disturbed by waves several feet high. It must be remembered, however, that with fully developed wave motion characteristic of the wind at the time, the waves travel before the wind at a speed about 80 per cent. of the wind speed (Sverdrup[50], p. 143).

Westwater[51] has also published observations of wind veer with height over the sea indicating a low aerodynamic roughness of the sea as compared with the land, and he has pointed out that not only do the waves run with the wind but also that the surface layers of the water have a drift component in the direction of the wind (quite apart from wave motion)—an effect also contributory to the smallness of the resistance coefficient of the sea surface.

The small friction of the wind over the sea, and the correspondingly small wind shear, has the consequence that, in spite of the generally small vertical temperature gradients over the open sea, the stability variations in terms of the Richardson number are often of comparable magnitude to these occurring over land.

APPENDIX II—LIST OF SYMBOLS

C_D = drag coefficient
c = integration constant
d = zero-plane displacement
g_y = lateral gustiness of wind
g_z = vertical gustiness of wind
K = eddy viscosity
k = von Kármán's constant
l = mixing length
m = slope of vertical gustiness curve
R_i = Richardson number
$R_{i\,(\mathrm{crit.})}$ = critical Richardson number
$R_i\,(z)$ = Richardson number at height z
$R_i\,(4/0{\cdot}5)$ = mean Richardson number for the layer 0·5 to 4 m.
R_θ = ratio of potential temperature difference between 5 and 1 m. to that between 1 and 0·2 m.
$R_{0\cdot5}, R_2 \ldots$ = ratio of wind speed at 0·5 m., 2m. . . . to wind speed at 1 m.
$R_{4/0\cdot5}$ = ratio of wind speed at 4 m. to wind speed at 0·5 m.
T = Absolute temperature
u = horizontal wind speed at a distance z from the surface
$\bar{u}$ = true mean speed in the mean wind direction
u' = horizontal eddy component in the mean wind direction

u_a, u_b, u_d = horizontal wind speed at the 3 heights z_1, z_2 and z_3
u_c = speed recorded by a perfectly responsive cup anemometer
$\bar{u}_c$ = mean speed recorded by a perfectly responsive cup anemometer
$u_{0\cdot5}, u_1, u_2 \ldots$ = horizontal wind speed at 0·5 m., 1 m., 2 m. . . .
$u_* = \sqrt{(\tau/\rho)}$, so-called " friction or shearing-stress speed "
v' = horizontal eddy component at right angles to the mean wind direction
W' = " mean extreme " vertical eddy component
W_2' = " mean extreme " vertical eddy component at 2 m.
w' = vertical eddy component
z, z_1, z_2, z_3 = heights above surface
z_0 = roughness parameter of the surface
β = index of z in power-law variation of vertical wind gradient with height
β' = value of β obtained from a wind ratio, equation (20), and a constant known roughness parameter
δ = index of z in power-law variation of vertical temperature gradient with height
θ = potential temperature
θ_a, θ_b = potential temperature at heights z_1, z_2
$\theta_{0\cdot2}, \theta_1, \theta_{1\cdot2} \ldots$ = potential temperature at heights 0·2 m,. 1 m., 1·2 m. . . .
ν = kinematic viscosity
ρ = fluid density
σ = stability constant in the Rossby–Montgomery formula
σ_h = stability constant in Holzman's formula
σ_1, σ_2 = standard deviations (multiplied by 1,000) of the values of lateral and vertical gustiness respectively
τ = shearing stress at height z
τ_0 = shearing stress at the surface.

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