MET.O.14

METEOROLOGICAL OFFICE
BOUNDARY LAYER RESEARCH BRANCH
TURBULENCE & DIFFUSION NOTE

T.D.N. No. 42

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by

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November 1973

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Some theoretical considerations of vertical dispersion of pollution in convergent flow.

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1. Statement of the problem in relation to modelling of urban air pollution.

It is now widely accepted that over a modern urban area the flow field is modified, as a consequence of increased roughness and 'heat island' effects, with a tendency for an inflow and a corresponding up-flow to be superimposed on an otherwise quasi-uniform horizontal wind field when this is weak. Thus (see McCormick 1970), 'a light indraft of as much as 1-2 m sec^{-1} ' may occur towards a city centre. Also work on urban temperature fields has shown the existence of an elevated 'heat plume' downwind of a city in otherwise stable conditions at night. Again, Gronskei (1972) has reported several case studies of the wind field in winter time in the Oslo region, in which marked convergence of the wind field was observed.

Modelling of air pollution from multiple sources is currently attempted with various degrees of elaboration in the representation of the dispersion, but (in practice) usually with the assumption of quasi-uniform-constant wind. At one extreme there is the full numerical summation of the individual (plume) distributions from designated source points - at the other the simplified, partly algebraic, summation advocated and developed into simple form by Gifford and Hanna (1971). Discussion continues on the relative merits of these 'models' as they stand, but the additional problem of adequately allowing for a non-uniform wind field has also been recognised. Attempts to overcome this difficulty follow various lines :

(a) use of 'sequential puff' representations of the travel and dispersion (e.g. see discussion in Stern 1970).

(b) detailed numerical solution of the K-type diffusion equation (Lamb and Neiburger 1971).

(c) finite-difference solution of the continuity equation in a grid system, specifically allowing for divergence in the wind field and consequent finite mean vertical velocities (Gronskei 1972).

In disregarding mean vertical motions Lamb and Neiburger assume that upward motion accompanying convergence must result in a reduction of concentration, hence that neglect of the effect will give a conservative (high) estimate of the concentration. Also, Gronskei regards his calculations as demonstrating the beneficial effects of the organised vertical lifting of the air over Oslo in keeping concentrations low. On the other hand in some quarters there seem to have been misgivings, qualitatively at least, that the reduction of wind speed in a city would lead to adverse effects on pollution.

If one considers a marked volume of air which is supposed to contain pollution, it follows from mass continuity that the volume can never change purely as a result of divergence in the wind field. Thus the concentration of pollution for unit volume cannot change purely as a result of wind divergence. Strictly this implies incompressible flow but even in compressible flow the same constraint applies to mixing ratio. In other words there must be a mixing process leading to an effective change of the volume occupied by a given amount of pollutant. It is accordingly not immediately obvious from any simple considerations that there should be any effect of convergence on concentration. The following mathematical analysis has been undertaken with the object of obtaining some further insight into this rather obscure point, and specifically to examine whether the complication of convergence may be satisfactorily incorporated in the algebraic integrations of the effect of an area source.

2. Two-dimensional equations

For simplicity we will confine our analysis to the two-dimensional case of advection in the x -direction and vertical transfer in the z -direction, for which in the usual notation the steady state equation of conservation is

$$\bar{u}\,\frac{\partial \chi}{\partial x} + \bar{w}\,\frac{\partial \chi}{\partial z} \simeq -\frac{\partial}{\partial z}\left(\overline{w'\chi'}\right) \simeq \frac{\partial}{\partial z}\left(K\,\frac{\partial \chi}{\partial z}\right), \qquad (1)$$

the eddy flux term in the x -direction being as usual neglected.

Analytic solutions for $\bar{u}$ other than constant are not immediately available, and in any case we do not have at present any information on the magnitude of the effect of a divergence on the properties of K .

An alternative approach is to write the expression for the vertical spread of the cloud. In the horizontally uniform case this procedure is followed in Lagrangian similarity analysis, the mean vertical displacement $\bar{Z}$ of particles being expected to follow the relation (in neutral conditions)

$$\frac{d\bar{Z}}{dt} = b\,u_* \qquad (2)$$

and the mean horizontal travel

$$\frac{d\bar{X}}{dt} = \bar{u}(c\bar{Z}) \qquad (3)$$

so

$$\frac{d\bar{Z}}{d\bar{X}} = \frac{b\,u_*}{\bar{u}(c\bar{Z})} \qquad (4)$$

As a crude representation of the foregoing, but otherwise neglecting the variation of wind speed with height, and removing the restriction to neutral flow, we may write speculatively

$$\frac{dh}{dt} = a_1 f(u)\, g(h) \tag{5}$$

where h is now the effective upper limit to the plume of particles from a ground-level source. The $f(u)$ term represents the effect of mean wind speed (for convenience the bar is omitted from here onwards) on the level of turbulence at a given height and the $g(h)$ term the variation of the effect of turbulence with height (which latter will implicitly be dependent on stability).

We now assume that the process of convergence in the mean flow does not affect the functions $f(u)$ and $g(h)$ and therefore write, for the convergent wind case

$$\frac{dh}{dt} = u\frac{dh}{dx} = a_1 f(u)\, g(h) + w(h) \tag{6}$$

with

$$w(h) = -\frac{du}{dx} . h . \tag{7}$$

Together these equations form

$$\frac{d(uh)}{dx} = a_1 f(u)\, g(h) . \tag{8}$$

We are thereby allowing the convergence possibly to affect the rate of vertical mixing through its affect on the wind speed and its (non-mixing) effect on the resultant height of the plume of particles.

To analyse the effect on concentration of material from a continuous source, we take the case of a crosswind line source of effectively infinite extent and rate of emission Q per unit length. Assuming similarity in the vertical profiles in terms of z/h (i.e. at any distance x, $\chi(z)/\chi(0)$ is a universal function of z/h) it follows, from rearrangement and applying conservation, that

$$\chi(z,x) = \frac{Q}{u(x)\,h(x)} \cdot f_1(z/h) \tag{9}$$

where f_1, is another universal function with some constant value j at $z = 0$, and therefore

$$\chi(0,x) = j\,\frac{Q}{u(x)\,h(x)} . \tag{10}$$

The simplest case for solution of (8) is with $f(u) = g(h) = 1$ i.e. basically the rate of turbulent spread is not dependent on either u or h itself, and as they are the only factors which convergence is assumed to affect we would expect the solution to indicate no effect of convergence on concentration. Equation (8) then gives

$$uh = a_1 x \tag{11}$$

and so

$$\chi(0,x) = j\,\frac{Q}{a_1 x} \tag{12}$$

which is indeed exactly the same as if there had been no convergence to start with and we had written (6) without the ω-term.

To investigate the more likely case that neither $f(u)$ nor $g(h)$ is unity it is necessary to introduce simple forms for these functions.

3. Solutions for linear convergence

Consider
$$f(u) = a_2 u \tag{13}$$
and
$$g(h) = a_3 h^{\alpha} \quad , \text{ where } \alpha < 1 , \tag{14}$$
then Eq. (8) reduces to
$$\frac{d(uh)}{dx} = a\, u\, h^{\alpha}, \tag{15}$$

which when integrated gives
$$(uh)^{1-\alpha} = a(1-\alpha)\int_0^x u(x')^{1-\alpha}\, dx' \tag{16}$$

(where a is a constant with dimensions $[L]^{-\alpha}$).

For the simple case of linear convergence,

i.e.
$$u(x) = u_0\,(1 - bx) \tag{17}$$
where b is a constant (to be distinguished from that in Eq. (2)) with dimensions $[L]^{-1}$, equation (16) yields
$$uh = u_0\left\{\frac{a(1-\alpha)}{b(2-\alpha)}\cdot\left[1-(1-bx)^{2-\alpha}\right]\right\}^{1/(1-\alpha)}. \tag{18}$$

Therefore
$$h(x) = \frac{\left\{\frac{a(1-\alpha)}{b(2-\alpha)}\cdot\left[1-(1-bx)^{2-\alpha}\right]\right\}^{1/(1-\alpha)}}{(1-bx)} \tag{19}$$

and
$$\chi(0,x) = \frac{jQ}{u_0}\left\{\frac{a(1-\alpha)}{b(2-\alpha)}\cdot\left[1-(1-bx)^{2-\alpha}\right]\right\}^{1/(\alpha-1)}. \tag{20}$$

Equations (19) and (20) then, are general solutions for the height to the top of the plume and the surface concentration, respectively, for the case of a linearly-converging horizontal wind field.

Since one of our main aims is to relate the convergent case to the non-convergent case it is convenient at this stage to introduce the general solutions to equation (15) when there is no convergence of the uniform wind u_o i.e. $b = 0$. In such circumstances, Eq. (15) reduces to

$$\frac{dh}{dx} = a h^{\alpha} \tag{21}$$

which gives

$$h_*(x) = \left[a (1-\alpha) x \right]^{1/(1-\alpha)} \tag{22}$$

for the height of the plume in non-convergent conditions and

$$\chi_*(0,x) = j \frac{Q}{u_o} \left[a (1-\alpha) x \right]^{1/(\alpha-1)} \tag{23}$$

for the corresponding surface concentration at x . These solutions can of course be directly obtained from Eqns. (19) and (20) by taking the limit as $b \longrightarrow 0$.

The combination of Eqns. (22) and (23) with (19) and (20) gives

$$h(x) = \frac{h_*(x)}{1-bx} \left[\frac{1-(1-bx)^{2-\alpha}}{(2-\alpha)\, bx} \right]^{1/(1-\alpha)} \tag{24}$$

and

$$\chi(0,x) = \chi_*(0,x) \left[\frac{(2-\alpha)\, bx}{1-(1-bx)^{2-\alpha}} \right]^{1/(1-\alpha)} . \tag{25}$$

Therefore we can write

$$\frac{h}{h_*} \equiv \frac{h}{h_*}(\sigma;\alpha) = \sigma^{-1}\left[\frac{1-\sigma^{2-\alpha}}{(2-\alpha)(1-\sigma)}\right]^{1/(1-\alpha)} \tag{26}$$

and

$$\frac{\chi}{\chi_*} \equiv \frac{\chi}{\chi_*}(0,\sigma;\alpha) = \left[\frac{(2-\alpha)(1-\sigma)}{1-\sigma^{2-\alpha}}\right]^{1/(1-\alpha)} \tag{27}$$

where

$$\sigma = 1 - bx = \frac{u(x)}{u_o} \tag{28}$$

is the fractional reduction of the horizontal wind due to the linear convergence.

4. Practical interpretation of the solutions for the infinite cross-wind source.

It follows immediately from Eqn. (27) that

$$\chi = \chi_* \iff \sigma^{2-\alpha} - (2-\alpha)\sigma + (1-\alpha) = 0 . \tag{29}$$

The only solution to Eqn. (29) for $\alpha < 1$, is the trivial solution, $\sigma = 1$, which holds for all α. Therefore, when $b \neq 0$, $\chi = \chi_*$ only at $x = 0$ and trivially $\chi \equiv \chi_*$ for all values of x only if there is no convergence i.e. $b = 0$.

Further, the solutions are such that

$$\frac{\chi}{\chi_*}(0,\sigma;\alpha) > 1 \qquad \text{for all } \alpha < 1,\ 0 \leq \sigma < 1 . \tag{30}$$

Therefore our solutions to Eqn. (15) imply that the surface concentration at any distance x downwind from an infinite cross-wind line source in a linearly-convergent flow is always greater than the corresponding concentration with no convergence ; provided of course that the rate of turbulent spread is a function of u. The range of α of interest can be estimated. In the non-convergent wind situation we expect $h \propto t^p$ where $1/2 \leq p \leq 1$ ($p = 1/2$ implies the parabolic growth for h of the classical Fickian diffusion case and $p = 1$ implies a linear development). Therefore $dh/dt \propto h^{\frac{p-1}{p}}$ and so $\alpha \simeq \frac{p-1}{p}$ is assumed to be most probably in the range $-1 \leq \alpha \leq 0$. Fig. 1 illustrates χ/χ_* as a function of σ for several values of α in the range of interest and we note that this ratio is insensitive to the value of α when the convergence is not too large. For example, for σ as low as 0.5, χ/χ_* ranges only from 1.31 to 1.33 as α varies from -1 to 0. Therefore, if the advecting horizontal wind is reduced to half the value it had at the line source then the surface concentration will be about 33% greater than in the non-convergent case with the same u_o.

An interesting feature of Figure 1 is that

$$\frac{\chi}{\chi_*}(0,\sigma;0) > \frac{\chi}{\chi_*}(0,\sigma;\alpha) > \frac{\chi}{\chi_*}(0,\sigma;-1) \tag{31}$$

for $0 > \alpha > -1$ and $0 \leq \sigma < 1$. Thus for $\sigma < 1$ the lowest values for χ/χ_* are for $\alpha = -1$ which is the parabolic growth case for h in non-convergent conditions and the highest values are when $\alpha = 0$ which corresponds to a linear growth in non-convergent conditions. The result that the concentration is affected relatively more for $\alpha = 0$ than for $\alpha = -1$ is perhaps surprising but it should be remembered that these are ratios and not absolute values.

Indeed specific examples indicate that the differences in the concentrations between convergent and non-convergent cases are such that

$$\left[\chi(0,x;-1)-\chi_*(0,x;-1)\right] > \left[\chi(0,x;0)-\chi_*(0,x;0)\right] \tag{32}$$

Therefore the absolute values of the concentrations react in the expected manner to the inclusion of convergence even although in terms of percentage changes the greatest effects are experienced where the concentrations are least.

5. Extension of the Model to an Area Source

In this section we consider a uniform area source with a uniform emission rate q per unit horizontal area and whose upwind edge is at the origin $x=0$. The area source is of infinite extent in the cross-wind direction and is considered as a distribution in the x-direction of elemental cross-wind strips of width δx and emission rate $q\,\delta x$ per unit cross-wind length such that $Q(X) = \int_0^X q\,dx = qX$. See Figure 2.

The surface concentration $\chi(0,X)$ at X is determined as the integrated effect of the contributions $\delta\chi(0,X;x)$ from the elemental strips at $0 \leqslant x \leqslant X$, as determined using the model of the previous sections.

From Eqn. (10) the contributions to the surface concentration at X from the elemental strip source at x is

$$\delta\chi(0,X;x) = jq\frac{\delta x}{u(X)\,h(X;x)} \tag{33}$$

where from Eqn. (16)

$$\begin{aligned} u(X)\,h(X;x) &= \left[a(1-\alpha)\int_x^X u(x')^{1-\alpha}\,dx'\right]^{1/(1-\alpha)} \\ &= u_0\left\{\frac{a(1-\alpha)}{b(2-\alpha)}\left[(1-bx)^{2-\alpha}-(1-bX)^{2-\alpha}\right]\right\}^{1/(1-\alpha)} \end{aligned} \tag{34}$$

Therefore, integrating Eqn. (33)

$$\chi(0,X) = jq\int_0^X \frac{dx}{u(X)\,h(X;x)}$$

$$= \frac{jq}{u_0}\left[\frac{a(1-\alpha)}{b(2-\alpha)}\right]^{1/(\alpha-1)} \int_0^X \left[(1-bx)^{2-\alpha} - (1-bX)^{2-\alpha}\right]^{1/(\alpha-1)} dx. \qquad (35)$$

If we now put $$\sigma = 1-bx = \frac{u(x)}{u_0}, \qquad (36)$$

$$\Sigma = 1-bX = \frac{u(X)}{u_0}, \qquad (37)$$

and $$r = \frac{\sigma}{\Sigma} = \frac{u(x)}{u(X)}, \qquad (38)$$

i.e. $$dx = -\frac{\Sigma}{b}\,dr, \qquad (39)$$

then Eqn. (35) becomes

$$\chi(0,X) \equiv \chi(0,\Sigma;\alpha) = \frac{jq}{u_0}\left[\frac{a(1-\alpha)}{2-\alpha}\right]^{1/(\alpha-1)} \frac{\Sigma^{1/(\alpha-1)}}{b^{\alpha/(\alpha-1)}} \int_1^{\Sigma^{-1}} \left(r^{2-\alpha}-1\right)^{1/(\alpha-1)} dr. \qquad (40)$$

The integral in Eqn. (40) generally cannot be expressed in terms of elementary functions and must be evaluated numerically. The integration can be expressed in terms of elementary functions only for $\alpha = 0$ in the range of α of interest to us, however then the definite integral becomes infinite. We shall return to the case $\alpha = 0$ below.

The corresponding solution, $\chi_*(0,X)$ for the surface concentration at X due to the area source when convergence is absent and the wind speed is the same u_0 is, from Eqn. (35),

$$\chi_*(0,X) = \frac{jq}{u_0}\int_0^X \frac{dx}{h_*(X;x)} \qquad (41)$$

$$= \frac{jq}{u_0}\left[a(1-\alpha)\right]^{1/(\alpha-1)} \int_0^X (X-x)^{1/(\alpha-1)}\,dx$$

where h_* has the form of Eqn. (22) after integration of Eqn. (21).

The integral in Eqn. (41) is finite only if $\alpha < 0$, when

$$\chi_*(0,X) = \frac{jq}{\mu_0}\left[a(1-\alpha)\right]^{1/(\alpha-1)} \cdot \frac{(\alpha-1)}{\alpha} \cdot X^{\alpha/(\alpha-1)} \tag{42}$$

Therefore for $\alpha < 0$, the ratio of the surface concentration in the convergent case to that in the non-convergent case with the same μ_0 can be written

$$\frac{\chi}{\chi_*} \equiv \frac{\chi}{\chi_*}(0,R;\alpha) = \left[\frac{\alpha(2-\alpha)}{\alpha-1}\right]^{1/(1-\alpha)} \cdot \frac{R}{(R-1)^{\alpha/(\alpha-1)}} \cdot \mathrm{I}(R;\alpha) \tag{43}$$

where
$$\mathrm{I}(R;\alpha) = \int_1^R (r^{2-\alpha}-1)^{1/(\alpha-1)}\, dr \tag{44}$$

and
$$R = \Sigma^{-1} = \frac{\mu_0}{\mu(X)} . \tag{45}$$

The function $\mathrm{I}(R;\alpha)$, $\alpha<0$, must generally be evaluated numerically. However as $R\to 1$ i.e. either $X\to 0$ or alternatively the convergence is small, then we may write $r=1+\varepsilon$ and $R=1+\delta$ where $\varepsilon, \delta \ll 1$ and the integrand in Eqn. (44) can be expanded in powers of ε and the integral expressed as

$$\mathrm{I}(R;\alpha) \equiv \mathrm{I}_1(\delta;\alpha) = \left[\frac{(\alpha-1)(2-\alpha)}{\alpha}\right]^{1/(\alpha-1)} \delta^{\alpha/(\alpha-1)}\left[1 - \frac{\alpha\delta}{2(2\alpha-1)} + \frac{\alpha(\alpha+6)}{24(3\alpha-2)}\delta^2 + O(\delta^3)\right] \tag{46}$$

and so

$$\frac{\chi}{\chi_*}(0,R;\alpha) = (1+\delta)\left[1 - \frac{\alpha\delta}{2(2\alpha-1)} + \frac{\alpha(\alpha+6)}{24(3\alpha-2)}\delta^2 + O(\delta^3)\right]$$

$$= R\left[1 - \frac{\alpha(R-1)}{2(2\alpha-1)} + \frac{\alpha(\alpha+6)}{24(3\alpha-2)}(R-1)^2 + O\left[(R-1)^3\right]\right] . \tag{47}$$

Therefore for small values of δ , we would expect χ/χ_* to behave as described by Eqn. (47). (Note that $\lim_{\delta \to 0} \chi/\chi_* \to 1$, as of course it should.)

It is interesting to note that although Eqn. (47) has not been derived for $\alpha = 0$ it does hold for all $\alpha < 0$ and indeed it implies

$$\lim_{\alpha \to 0} \frac{\chi}{\chi_*} = R = \frac{u_0}{u(X)} \tag{48}$$

for small values of δ . In fact, although $\chi(0,X)$ and $\chi_*(0,X)$ are infinite when $\alpha = 0$, which we recall corresponds to $h \propto t$ in the non-convergent wind situation, their ratio χ/χ_* is finite for all values of R and the integrals in Eqns. (40) and (41) can be expressed in terms of simple functions.

Thus

$$\frac{\chi}{\chi_*}(0,R;0) = 2R \frac{\int_1^R (r^2-1)^{-1}\,dr}{\int_1^R (r-1)^{-1}\,dr}$$

$$= R \lim_{\omega \to 0} \left\{ \frac{\left[\ln\left(\frac{r-1}{r+1}\right)\right]_{1+\omega}^{R}}{\left[\ln(r-1)\right]_{1+\omega}^{R}} \right\}$$

$$= R \lim_{\omega \to 0} \left[\frac{\ln(R-1) - \ln(R+1) + \ln(2+\omega) - \ln\omega}{\ln(R-1) - \ln\omega} \right]$$

$$= R = \frac{u_0}{u(X)} . \tag{49}$$

Therefore when $\alpha = 0$ we have the simple result that χ/χ_* is the inverse of the fractional reduction of the wind at X from its value u_0 at the leading edge of the area source. For example, if the convergence is strong enough, or X is far enough down wind from the leading edge such that the wind has dropped to half of its original value then $\chi = 2\chi_*$

i.e. the inclusion of a linearly convergent wind field in this case would double the expected concentration at X .

In general we would not expect the convergence to be strong enough to produce a value of $R \gg 2$ and most probably we would be concerned with values much closer to unity where Eqn. (47) could be used to determine the ratio χ/χ_* . Figure 3 gives the ratio χ/χ_* as a function of $R \leqslant 1.3$ and α , as determined from Eqn. (47). The deviation of the curves for $\alpha < 0$ from the line $\chi/\chi_* = R$ for $\alpha = 0$ increases with increasing R and decreasing α ; however even at $R = 1.3$ the lower limit to the ratio is 1.24 for $\alpha = -1$. Therefore, in practical terms, the ratio χ/χ_* is effectively independent of α for typically experienced degrees of convergence, and a good first approximation is $\chi/\chi_* = u_o/u(X)$.

Figure 4 shows χ/χ_* as a function of the probable extreme values of α, viz. 0 and -1, for a much larger range of R . The curve for $\alpha = -1$ was evaluated from Eqns. (43) - (46) as

$$\frac{\chi}{\chi_*}(0,R;-1) = \frac{\sqrt{3}\,R}{2\,(R-1)^{1/2}}\left[\int_{1+\delta}^{R}\frac{dr}{(r^3-1)^{1/2}} + I_1(\delta;-1)\right] \qquad (50)$$

where $I_1(\delta;-1)$ was evaluated for $\delta = 0.1$ and the integral term was integrated graphically from 1.1 to R . We note that even for our example of very marked convergence when $u(X) = \frac{1}{2}u_o$ i.e. $R = 2$, the ratio χ/χ_* is as high as 1.74 for $\alpha = -1$, our considered lowest value.

6. Summary and concluding remarks

As a starting point for discussion of the effects of convergence we have assumed, (a) that the rate of vertical spread is a function only of wind speed and height, (b) that this functional form is unchanged by (the degree of) convergence, (c) that the vertical profile of concentration exhibits similarity when height is scaled with respect to the height of cloud.

With the rate of spread function linear in u and a simple power law in height ($\propto h^{\alpha}$), and with a linear convergence of wind, it appears that ground level concentration from a crosswind line source, at ground level, of effectively infinite extent acrosswind, is higher than would occur with a non-convergent wind having the same speed at the source.

Algebraic integration for an area source with uniform rate of emission per unit area results in a corresponding increase in concentration, which at any position on the area is by a factor roughly equal to that by which the wind speed is reduced from the speed at the upwind edge. This is consistent with the point that the accummulated concentration at any position must be dominated by the sources immediately upwind (unless vertical spread has become slowly varying with distance at distances similar to the downwind length of area source involved). The results suggest the working rule that the correct accummulated concentration on an area source would be calculated by assuming the wind speed at the sampling position to apply over the whole source area upwind.

The interpretation for the case when wind speed is reduced to zero by the convergence is not obvious. Also there are other questions, such as the effect of assuming no dependence of rate of spread on wind speed but retaining dependence on height, and the significance of an elevation of the sources, which need further consideration.

<u>References</u>

Gifford, F. and Hanna, S.R. 1971 Urban air pollution modelling, Proc. 2nd International Clean Air Congress, Ed. H.M. England and W.T. Beery, Academic Press.

Gronskei, K.E. 1972 A three-dimensional transport model for air pollution in an urban area with application to SO_2 concentration in Oslo, Proc. 3rd meeting of the expert panel on air pollution modelling, NATO Committee on the Challenges of Modern Society.

Lamb, R.G. and Neiburger, M. 1971 An interim version of a generalised urban air pollution model, Atmos. Environ., 5, 239-264.

McCormick, R.A. 1970 Meteorological aspects of air pollution in urban and industrial districts, WMO Technical Note No 106, Secretariat of the World Meteorological Organisation, Geneva, Switzerland.

Stern, A.C. 1970 Proceedings of Symposium on multiple-source urban diffusion models.

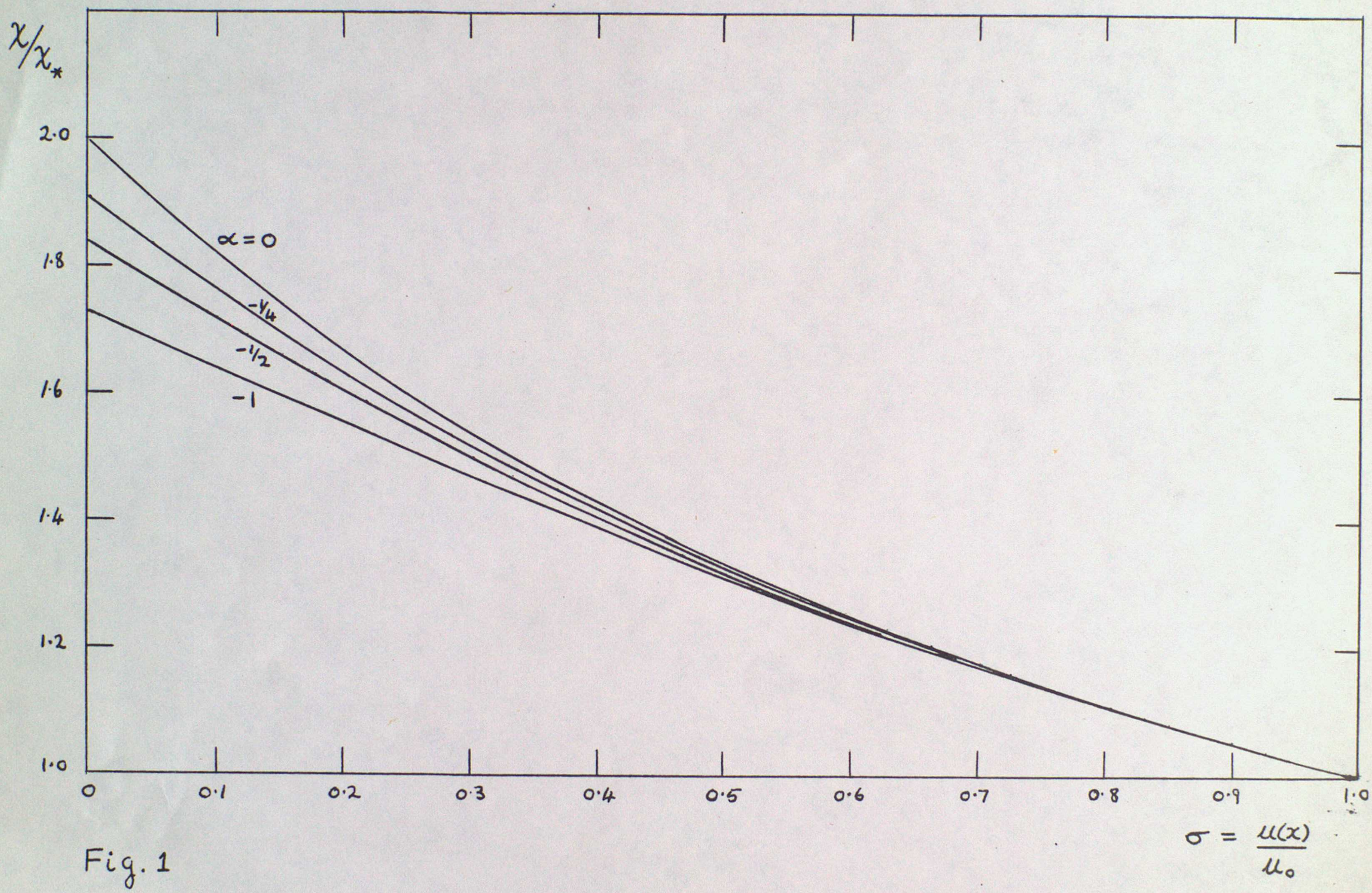
FIGURE LEGENDS

Figure 1 χ/χ_* , the ratio of the surface concentration at x , due to an infinite cross-wind line source at the origin, in a linearly-convergent wind to that in a uniform wind, as a function of $\sigma = u(x)/u_0$, the fractional reduction of the horizontal wind due to the convergence, for several values of the parameter α .

Figure 2 Schematic representation of an area source as a distribution of elemental, crosswind, infinite strips of width δx and emission rate $q\,\delta x$ per unit cross-wind length. The surface concentration $\chi(0,X)$ at X is estimated by assuming a continuous source with uniform emission rate q per unit area between 0 and X .

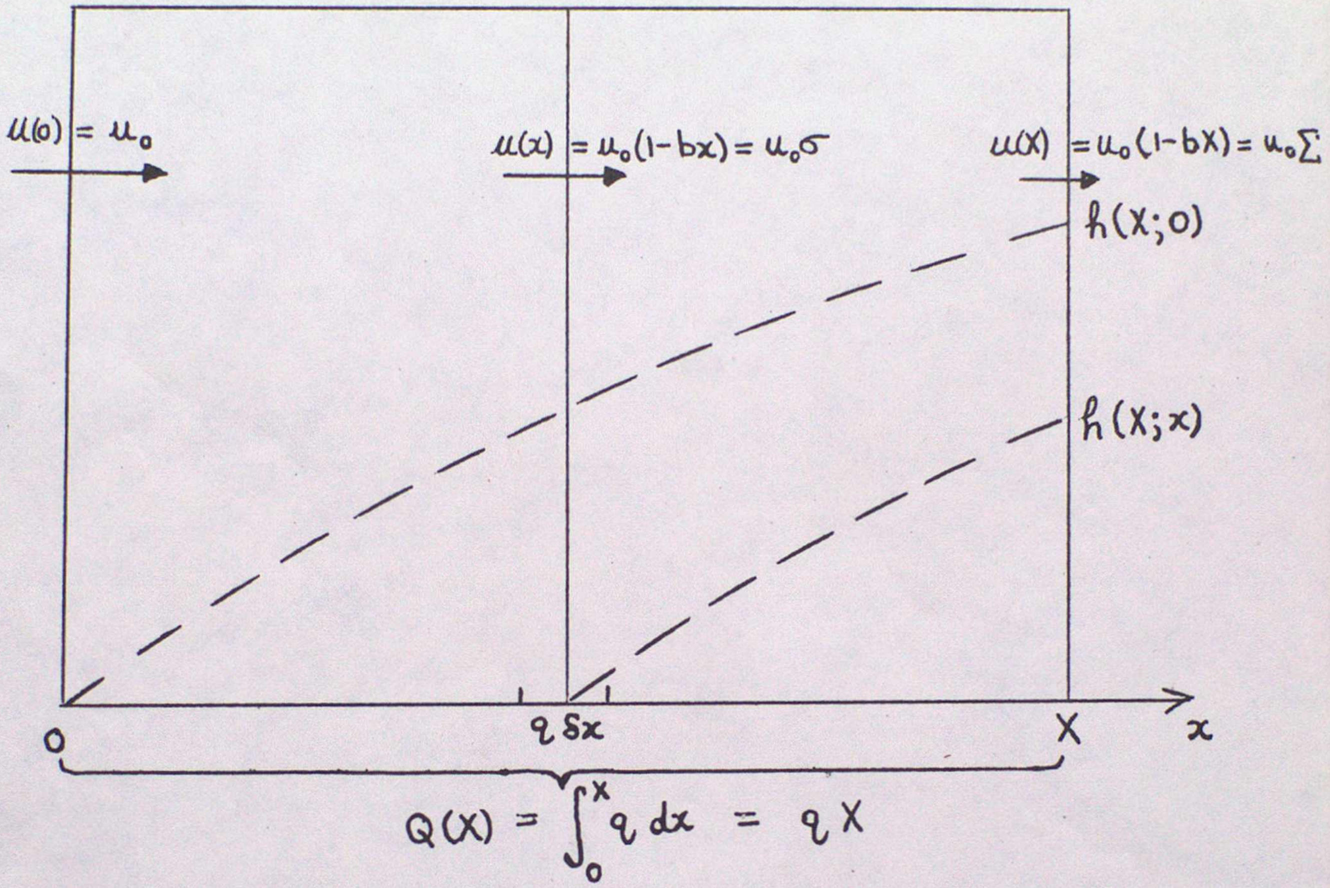
Figure 3 χ/χ_* , the ratio of the surface concentration at X , due to a uniform area source between 0 and X , in a linearly-convergent wind to that in a uniform wind, as a function of $R = u_0/u(X)$ and α . The curves for χ/χ_* are derived from Equation (47).

Figure 4 χ/χ_* , the ratio of the surface concentration at X , due to a uniform area source between 0 and X , in a linearly-convergent wind to that in a uniform wind, as a function of $R = u_0/u(X)$ and the two 'limiting' values of α . The curve for $\alpha = -1$ is derived from Equation (50).



χ/χ_*
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\alpha = 0$
$-1/4$
$-1/2$
-1
$\sigma = \frac{u(x)}{u_0}$

Fig. 1



$u(0) = u_0$
$u(x) = u_0(1-bx) = u_0\sigma$
$u(X) = u_0(1-bX) = u_0\Sigma$
$h(X;0)$
$h(X;x)$
0
$q\,\delta x$
X
x
$Q(X) = \int_0^X q\,dx = qX$

Fig. 2.

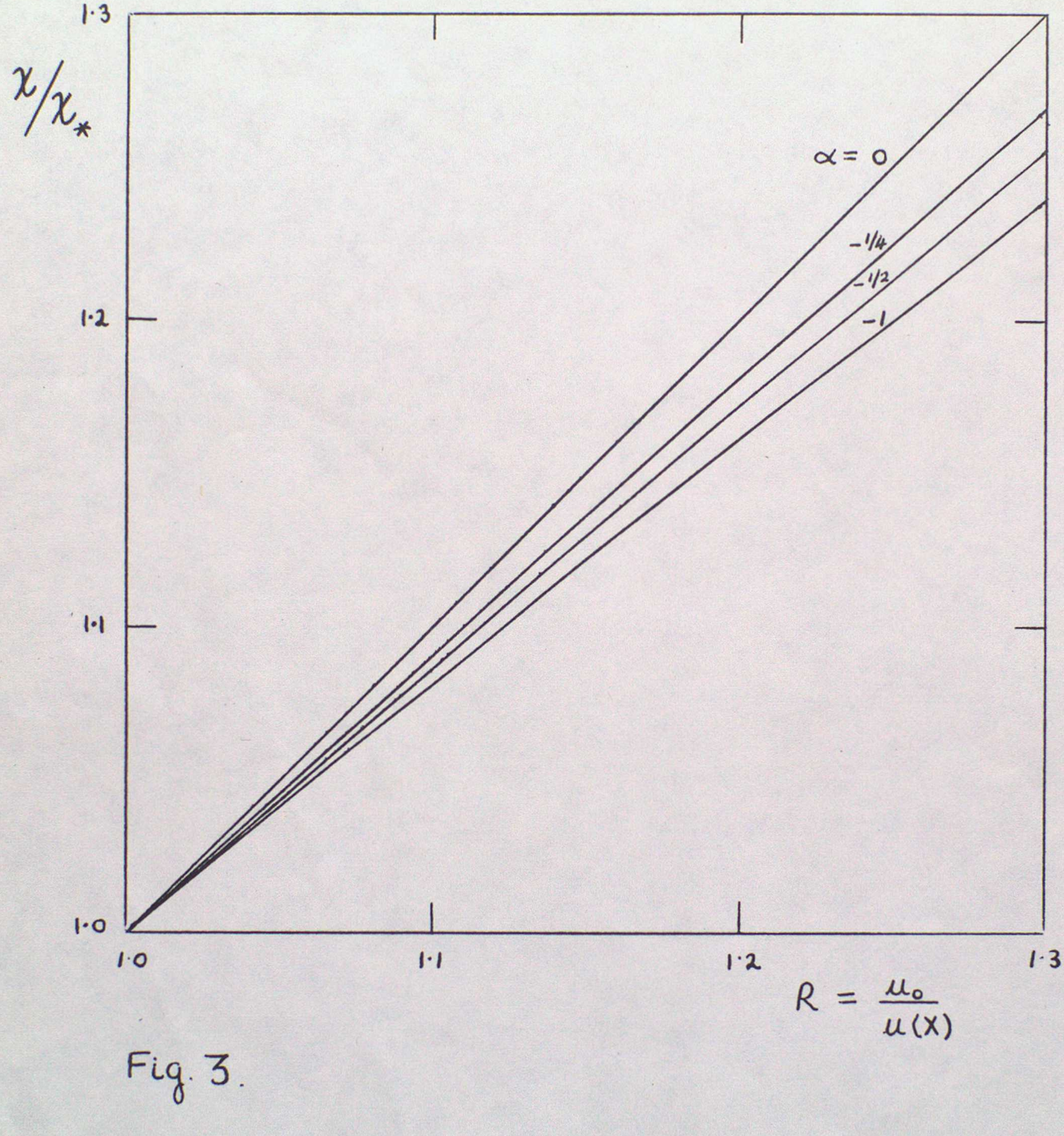


Fig. 3.

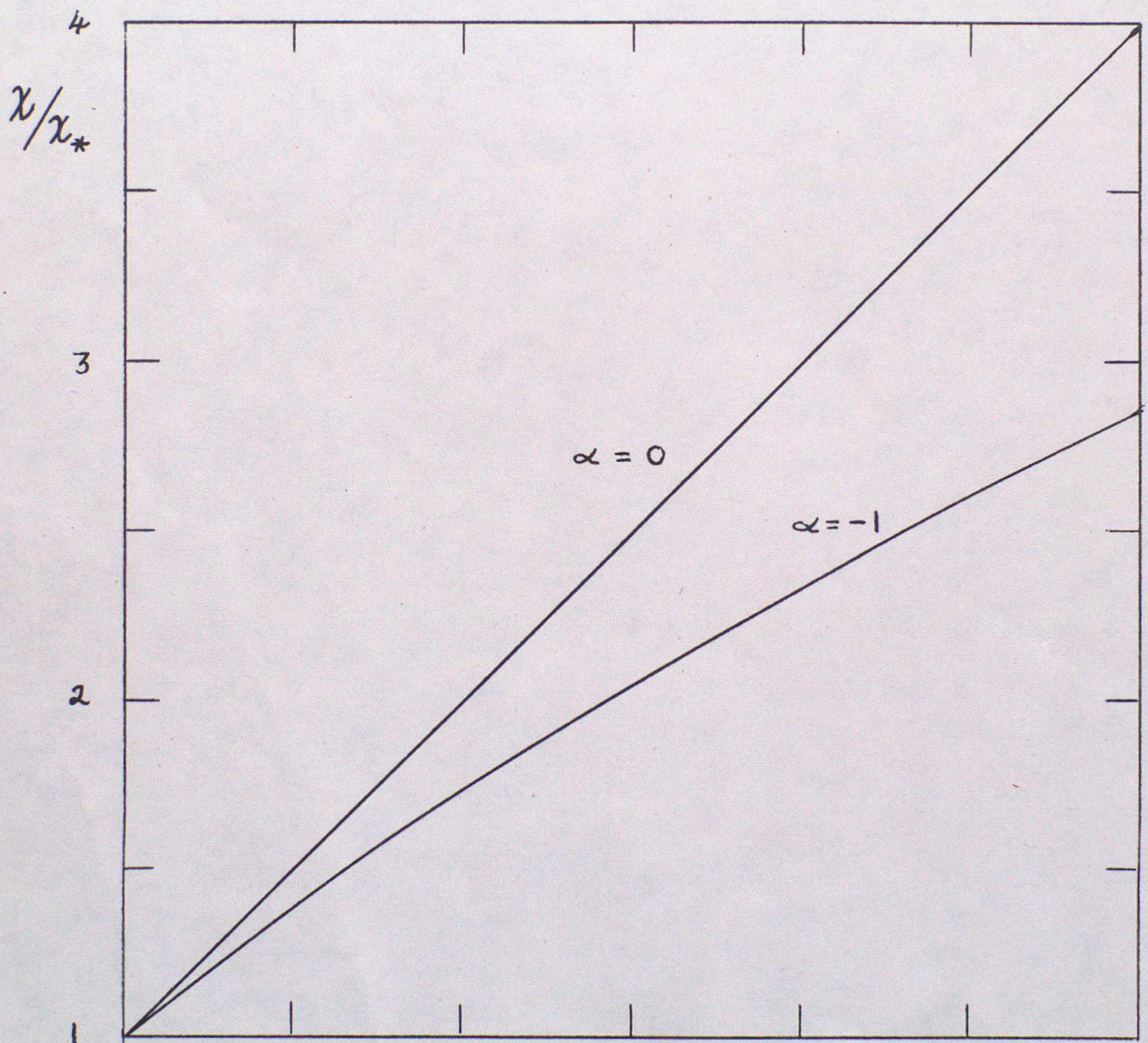


Fig. 4.