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Two Notes on the Operation of Galitzin Seismographs

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TWO NOTES ON THE OPERATION OF GALITZIN SEISMOGRAPHS

I.—AN ISOPLETH DIAGRAM FOR THE RAPID EVALUATION OF THE DAMPING CONSTANT AND THE FREE PERIOD OF A GALITZIN SEISMOGRAPH PENDULUM

The chief feature of the Galitzin seismograph is that the motion of the pendulum is recorded galvanometrically. The displacement of the galvanometer coil is proportional to the angular velocity, and not to the displacement, of the pendulum. On this account, the interpretation of the record in terms of earth movement is a difficult matter except under certain conditions. The differential equation of the motion of the pendulum under the influence of an earth movement may be expressed—

$$\theta''+2\epsilon\theta'+n^2\theta+\frac{x''}{l}=0,$$

where θ is the angular displacement of the pendulum, ϵ is the coefficient of damping, n depends on the free period T of the pendulum $\left(n=\frac{2\pi}{T}\right)$, x is the displacement of the ground and l is the length of the equivalent simple pendulum. Similarly, the equation for the motion of the galvanometer coil may be written—

$$\phi''+2\,\epsilon_1\,\phi'+n_1^2\,\phi+k\cdot\theta'=0.$$

Here ϕ, ϵ_1, and n_1 have the same meanings as θ, ϵ and n, but with reference to the galvanometer; k is a constant depending on the strength of the inductive coupling and may be called the transmission coefficient.

The solution of these equations when x is a prescribed function of time is worked out fully by Prince Galitzin[1], and the relation between the galvanometer record and the earth movement is considerably simplified if both pendulum and galvanometer are made to have the same free period and to be critically aperiodic within very narrow limits. The attainment of these ideal conditions is aimed at, therefore, when an instrument is installed.

The first step is to make the galvanometer strictly aperiodic. This is simply a matter of adjusting the electrical resistance of the circuit, and need not be discussed here. When the condition is reached $\epsilon_1=n_1=\frac{2\pi}{T_1}$ and no appreciable changes are likely to take place. It remains to make the pendulum aperiodic and to give it a free period T equal to that of the galvanometer T_1. When these conditions are fulfilled $\epsilon_1=n_1=\epsilon=n$. For convenience the departure from aperiodicity may be written—

$$\mu^2=1-\frac{\epsilon^2}{n^2}\quad\text{(Galitzin calls } \mu^2 \text{ the damping constant[2])}$$

and the ideal is attained for $\mu^2=0$, and $n=n_1$.

The free period of the pendulum is altered by adjusting the tilt of the supporting frame, and may be measured directly if the damping system is removed. It is not difficult to bring T near to T_1 by this direct method, and having done this, the damping magnets can then be closed in until the pendulum just ceases to oscillate on being given a small displacement.

The next proceeding is to make an accurate determination of T and μ^2 to see how near the pendulum is to the ideal conditions. This is done by giving the pendulum a small impulse and by making observations of the movement recorded by the galvanometer. The displacement of the pendulum due to the impulse is given by

$$\theta=\frac{\theta_0'}{\mu n}e^{-\epsilon t}\sin\mu nt$$

---

[1] Vorlesungen über Seismometrie, 1914 (Leipzig).

[2] *The significance of* μ.—It may be recalled that when aperiodicity has not been attained the displacement of the pendulum after an initial impulse is a multiple of $e^{-\epsilon t}\sin\mu nt$. When the pendulum is overdamped μ is imaginary and the sine is hyperbolic. As long as μ is real it may be defined as the ratio of the periods of undamped and damped oscillations of the pendulum.

where θ_0' is the initial angular velocity. The motion takes the form shown by the full curve in Fig. 1.

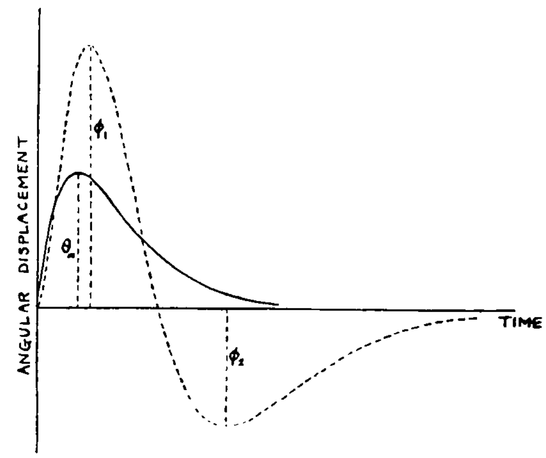


FIG. 1

The solution of the equation of motion of the galvanometer coil corresponding to the above form of pendulum movement is very complicated and is given fully by Galitzin. The broken curve in Fig. 1 shows the displacement followed by the coil; the measurements which are made are of the two maximum amplitudes ϕ_1 and ϕ_2 and the interval t_0 between the time of giving the impulse and the time when the coil crosses the zero line after the first half of the oscillation. Galitzin derives two expressions which enable μ^2 and T to be determined from these observations. Thus—

$$\xi = \frac{n_1 t_0 - a + c\xi^2}{b}; \qquad \text{where } \xi = \frac{T - T_1}{T}$$

$$a = 3 - 0{\cdot}15\,\mu^2$$
$$b = 1{\cdot}5 + 0{\cdot}225\,\mu^2$$
$$c = 0{\cdot}3 + 0{\cdot}0171\,\mu^2$$

$$\mu^2 = \frac{\beta - \alpha}{\alpha\psi_2 - \beta\psi_1}; \qquad \text{where } \beta = 2{\cdot}2937\,(1 + 0{\cdot}1732\,\xi^2)$$

$$\alpha = \frac{\phi_1}{\phi_2}$$
$$\psi_1 = -0{\cdot}0065377\,(1 + 5{\cdot}5981\,\xi + 1{\cdot}5556\,\xi^2)$$
$$\psi_2 = 0{\cdot}33988\,(1 + 0{\cdot}40192\,\xi - 0{\cdot}63417\,\xi^2)$$

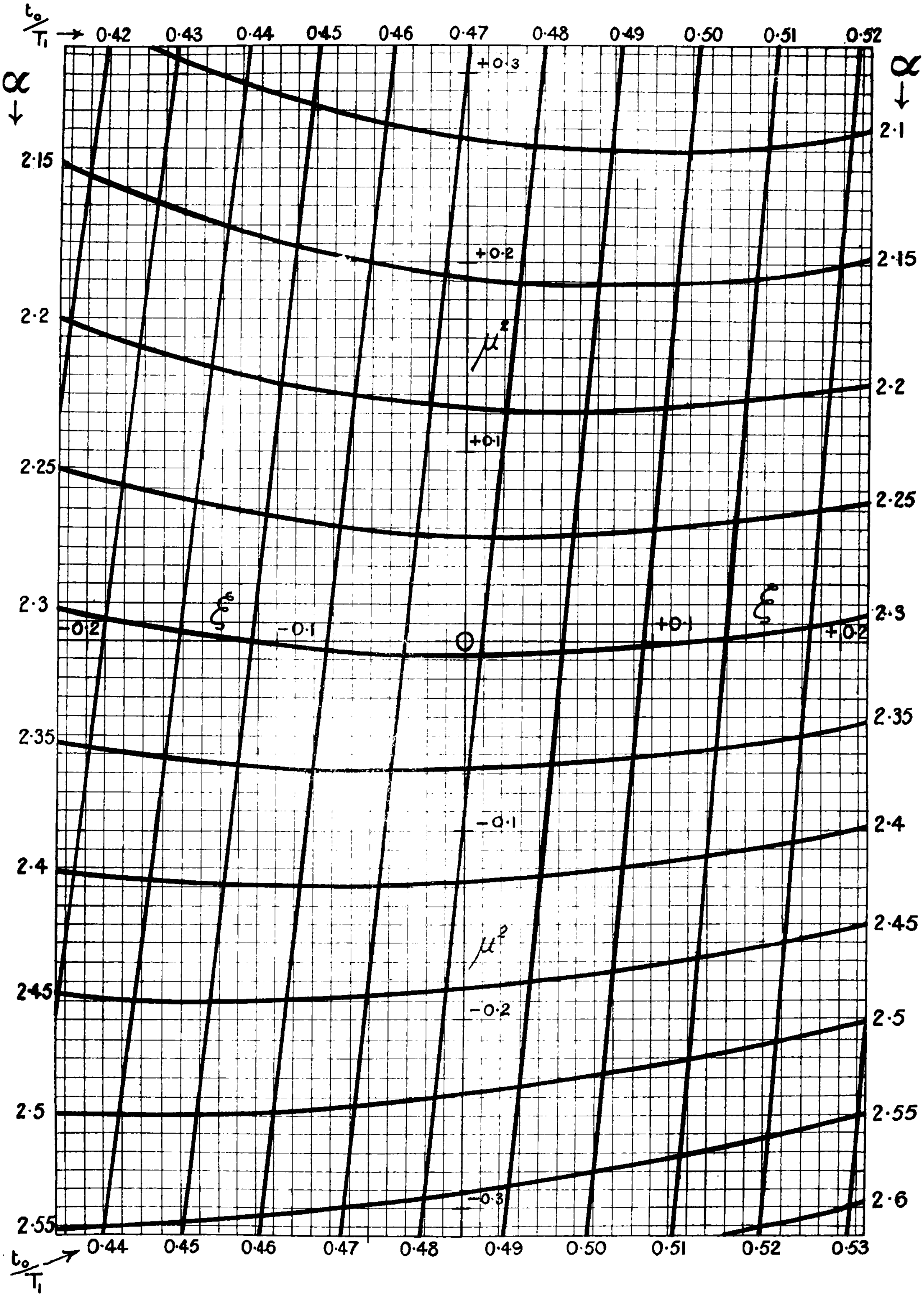
It will be seen that the computation of μ^2 and ξ is not very straightforward. The method suggested by Galitzin is that of successive approximation, μ^2 being calculated first on the assumption that $\xi = 0$ and then this value of μ^2 is used to determine the approximate value of ξ. The process is repeated using this value of ξ. It will be seen that the critical conditions are attained when, simultaneously

$$\alpha = \beta = 2{\cdot}2937 \text{ and } t_0 = \frac{3T_1}{2\pi}$$

Now when an instrument is in the process of being adjusted to the critical condition it is a distinct advantage to be able to convert the observations of α $\left(\text{i.e. } \frac{\phi_1}{\phi_2}\right)$ and t_0 into μ^2 and ξ by some direct method. In this way the adjustment may be attained in quite a short space of time. With this object in view, an isopleth diagram has been constructed connecting μ^2, ξ, α and t_0. (Actually t_0/T_1 has been used as one of the variables so that the diagram is not restricted to instruments in which the galvanometers have the same free period.) The diagram has been made by giving t_0/T_1 and α a series of values and then making the calculations of μ^2 and ξ by Galitzin's method outlined above. The values of μ^2 and ξ have been plotted as abscissae and ordinates respectively and the isopleths of t_0/T_1 and α have been drawn.

Fig. 2

To face p. 4.



ISOPLETH DIAGRAM FOR OBTAINING DIRECTLY VALUES OF μ^2 AND ξ FROM OBSERVED VALUES OF t_0/T_1 AND α

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A reproduction of the diagram is given in Fig. 2. Having obtained a few observations of a and t_0 it is a simple matter to read off the corresponding values of μ^2 and ξ, T then being obtained by the relation

$$T = (1+\xi)\, T_1$$

The meshes formed by the isopleths have been made of such size that linear interpolation can be used without introducing any appreciable error in μ^2 or ξ. The diagram covers a range of values of μ^2 from about $+0{\cdot}3$ to $-0{\cdot}3$ and a range of values of ξ from about $0{\cdot}2$ to $-0{\cdot}2$.

It should be pointed out that by making approximations in the solution of the differential equation for the motion of the galvanometer much simpler expressions can be derived for μ^2 and ξ. Thus O. Somville[3] finds that provided the pendulum is very near to being critically aperiodic the following formulæ give sufficiently accurate values :

$$\mu^2 = \frac{2{\cdot}294 - a}{0{\cdot}795}$$

$$\xi = \frac{4\,\pi\, t_0}{3\,T_1} - 2 + \tfrac{1}{10}\,\mu^2$$

Similarly G. W. Walker[4] gives

$$\mu^2 = 2{\cdot}294 \left(\frac{2{\cdot}294}{a} - 1\right)$$

$$\xi = \tfrac{2}{3}\left(\frac{2\,\pi\, t_0}{T_1} - 3\right)$$

when ξ and μ^2 are not greater than $0{\cdot}1$.

The use of these simple formulæ presupposes that the ideal conditions of tuning and damping have almost been reached, whereas the diagram based on the more exact theory is valid when the departure from critical conditions is large. The diagram is specially useful, therefore, when adjustments are being attempted after a rough tuning has been made by direct observations on the pendulum.

Since the departure from critical damping is dependent on the free period of the pendulum, it is best to complete the final adjustment of the period (using the impulse method and not the direct method for measuring the period[5]) before attempting fine adjustment of the damping. If the latter is attempted first then the critical condition of damping will be upset by final adjustment of the period.

For the benefit of workers who desire to construct a diagram on a scale larger than that reproduced the necessary numerical data are given in the accompanying tables.

VALUES OF μ^2.

a \ ξ	−·20	−·15	−·10	−0·5	0	+·05	+·10	+·15	+·20
2·1 ..	+·329	+·305	+·288	+·275	+·266	+·261	+·260	+·261	+·267
2·15 ..	+·245	+·225	+·210	+·199	+·193	+·189	+·190	+·192	+·199
2·2 ..	+·164	+·148	+·136	+·128	+·123	+·121	+·123	+·127	+·133
2·25 ..	+·087	+·074	+·065	+·059	+·056	+·056	+·059	+·064	+·071
2·3 ..	+·014	+·004	−·003	−·007	−·008	−·007	−·003	+·003	+·011
2·35 ..	−·057	−·064	−·068	−·070	−·069	−·066	−·062	−·055	−·046
2·4 ..	−·124	−·128	−·130	−·130	−·128	−·124	−·118	−·111	−·101
2·45 ..	−·188	−·191	−·190	−·188	−·184	−·179	−·172	−·164	−·154
2·5 ..	−·250	−·250	−·248	−·244	−·239	−·232	−·224	−·215	−·205
2·55 ..	−·311	−·307	−·303	−·297	−·291	−·283	−·274	−·265	−·254

[3] Constants des Sismographes Galitzin.—*Annales de l'Observatoire Royal de Belgique*, 1922 (Brussels).
[4] Modern Seismology, 1913 (London).
[5] Somville (loc. cit.) points out that the free period as obtained by direct measurement, with the damping magnets removed, differs appreciably from the value given by the impulse method, in which the damping magnets are in position, and shows that the discrepancy is due to the pendulum not being magnetically neutral.

VALUES OF ξ.

t_0/π \ μ^2	+0·3	+0·2	+0·1	0	−0·1	−0·2	−0·3
0·43 ..	−·156	−·168	−·179	−·191	−·202	−·215	−·227
0·44 ..	−·118	−·129	−·141	−·152	−·163	−·175	−·188
0·45 ..	−·080	−·091	−·102	−·113	−·124	−·135	−·147
0·46 ..	−·041	−·051	−·062	−·072	−·083	−·094	−·106
0·47 ..	−·001	−·011	−·021	−·031	−·041	−·052	−·063
0·48 ..	+·039	+·030	+·020	+·011	+·001	−·010	−·020
0·49 ..	+·080	+·071	+·062	+·053	+·044	+·034	+·024
0·50 ..	+·122	+·114	+·105	+·097	+·087	+·078	+·068
0·51 ..	+·164	+·156	+·148	+·140	+·131	+·122	+·114
0·52 ..	+·207	+·199	+·192	+·184	+·177	+·168	+·160

II—SOME PRACTICAL CONSIDERATIONS ARISING FROM THE VARIATION OF THE PERIOD OF THE PENDULUM OF A GALITZIN VERTICAL SEISMOGRAPH

§ 1—INTRODUCTION

The variation of the period of the pendulum of the Galitzin vertical seismograph with deviation from the normal position has been discussed by L. F. Richardson[1] and by J. Wilip[2]. They advocate the use of a modified design of instrument embodying two (or more) control springs instead of one, as a means of obtaining a more uniform period over the range of amplitudes usually encountered. T. Tamaru[3] also has dealt with the theory of vertical pendulums in which more than one spring is employed. The excuse for a further discussion of the same subject is that there must be a number of seismological stations where, although the merits of the two-spring system are fully appreciated, the original pattern of Galitzin vertical seismograph is retained in order to avoid the expense of alterations and therefore it may be of interest to deal with some of the practical limitations of the original type of instrument.

§ 2—VARIATION OF PERIOD WITH DISPLACEMENT

It is necessary first of all to outline briefly the theory of the instrument. In Fig. 1 the moving part of the pendulum is represented by OMB.

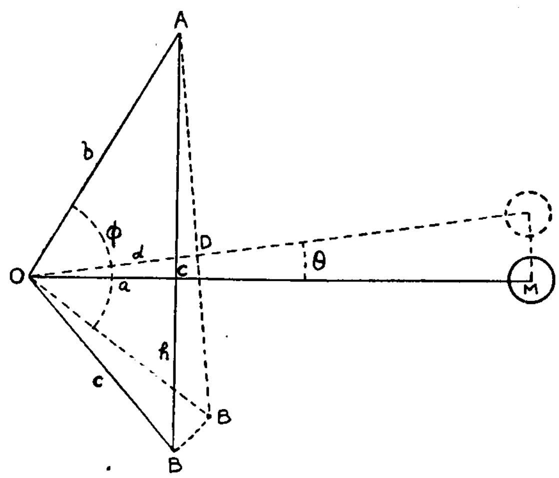

FIG. 1.

O is the turning axis and M the centre of mass. AB is the spiral spring which is fixed at A. The elastic constant of the spring is β given by

$$P-P_0=\beta\,(L-L_0)$$

where P_0 and L_0 are the tension and length of the spring respectively in the normal position, and P and L are the corresponding quantities when the boom is displaced through a small angle θ.

Referring to Fig. 1 it will be seen that the length of the spring in any position is given by:

$$L^2=b^2+c^2-2\,bc\cos\phi \quad \ldots \quad \ldots \quad (1)$$

where ϕ is the variable angle AOB, b is the fixed distance OA and c is the fixed length of the arm OB. The couple due to the spring is Pd where d is the length of the perpendicular from O to the axis of the spring and is equal to $\frac{bc\sin\phi}{L}$ (in the normal position $d_0=a$).

$$Pd=[P_0+\beta\,(L-L_0)]\,\frac{bc\sin\phi}{L} \quad \ldots \quad \ldots \quad (2)$$

[1] *Monthly Notices, R.A.S.*, Geophys. Suppl **1**, No 8, 1926.
[2] *Gerlands Beiträge zur Geophysik.*, *Leipzig*, **19**, Heft 4, 1928.
[3] *Physikalische Zeitschrift, Leipzig*, **4**, 1903, p. 638.

The couple due to the weight is $Mgr_0 \cos\theta$ ($\theta = \phi - \phi_0$), where r_0 is the distance of the centre of mass M from the axis of rotation. The net restoring couple is therefore :

$$C = Pd - Mgr_0 \cos\theta \qquad \ldots \qquad \ldots \qquad (3)$$

and the period is given by :

$$T = 2\pi\sqrt{\frac{I}{dC/d\phi}} \qquad \ldots \qquad \ldots \qquad (4)$$

I being the moment of inertia. This is the theory which is further developed by Richardson (loc. cit.).

By neglecting second-order quantities Galitzin obtained a simple formula for the period given by

$$\frac{4\pi^2}{T^2} = n^2 = \frac{\beta a^2}{I} - \frac{P_0 h}{I}\left(1 - \frac{h}{L_0}\right) \qquad \ldots \qquad \ldots \qquad (5)$$

This expression gives an apparently uniform period. Richardson, however, shows that the higher derivatives of the restoring couple are by no means negligible. The effect of this on the period can be seen by inserting numerical values, obtained from the dimensions of the pendulum, in the expressions (1) to (4). The theoretical curve connecting the period with the deviation has been worked out in the case of the Kew instrument. It is not reproduced here since it is exactly similar to the curve in Fig. 2A which was obtained experimentally.

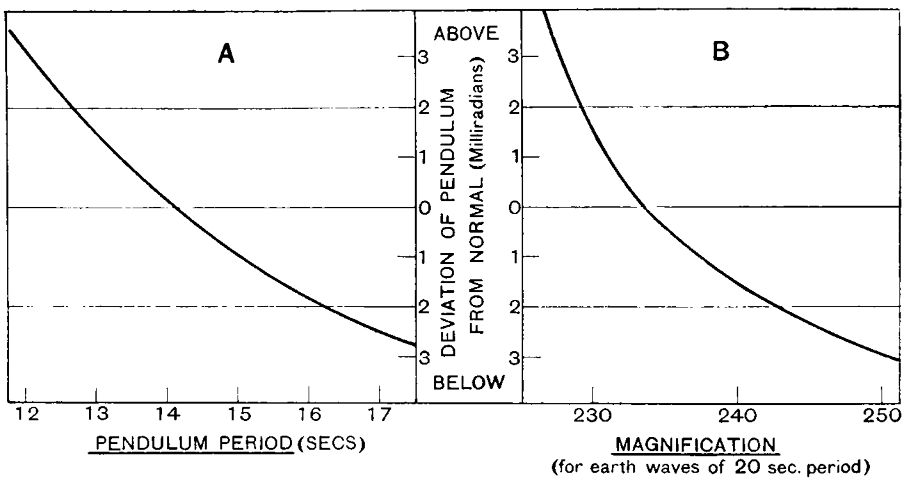

FIG. 2.

It will be seen that the variation of the period is extremely large, especially when the pendulum is displaced below its normal position. In practice when the displacement reaches about 0·02 radian below the normal position the pendulum becomes unstable. This is the most serious consequence of the variation in period since it causes a complete loss of record until the pendulum is readjusted to its normal position. The amount of the variation can be reduced by using a lower working period but this is considered disadvantageous for recording distant earthquakes.

3—Effects of Variation of Period on Magnification

One effect of the variation of the period is to make the determination of the angle of emergence of a seismic ray very difficult and inaccurate. In order to achieve reliable results in this direction it is essential that the periods of all three components of the seismograph installation should be the same. With the period of the vertical pendulum varying as it does, it is impossible to obtain an accurate measure of the amplitude of an impulse and it is seldom attempted.

The amplitudes of the sinusoidal waves in the main phase of an earthquake are, however, measured as a routine procedure at a number of stations, and it is of interest to see what reliance can be placed on measurements obtained from records of the Galitzin vertical seismograph. The magnification given by the galvanometer record for a sinusoidal wave is expressed by the formula :

$$M = \frac{T_p K A}{\pi l} \quad \frac{1}{(1+u_1^2)(1+u^2)\sqrt{1-\mu^2 f(u)}} \quad \cdots \quad \cdots \quad (6)$$

where T_p is the period of the earth wave, K is the transmission coefficient depending chiefly on the strength of the inductive coupling between the pendulum and the galvanometer circuit, $u_1 = T_p/T_1$ (T_1 being the galvanometer period), $u = T_p/T$; μ^2 is the damping constant and is equal to $1 - \frac{\epsilon^2 T^2}{4\pi^2}$, ϵ being the ordinary damping coefficient and $f(u)$ is $\left[\frac{2u}{1+u^2}\right]^2$. Now when the boom of the vertical pendulum shifts its equilibrium position (usually on account of temperature change), causing an alteration in the period T, it is clear that the magnification will not be the same as for the true equilibrium position. In the above formula both u and μ^2 are altered. Differentiating M with respect to T we get :

$$\frac{dM}{dT} = -\frac{2\,C\,u^2\,(1-u^2)}{T\,(1+u^2)^3\,[1-\mu^2 f(u)]^{3/2}} \quad \cdots \quad \cdots \quad (7)$$

where C replaces $\frac{1}{T_p}\ \frac{\pi l}{K A}\ (1+u_1^2)$.

We can express the proportional change in the magnification as a factor (γ) of the proportional change in the period thus :

$$\frac{\delta M}{M} = -\frac{2\,u^2\,(1-u^2)}{(1+u^2)\,[1-\mu^2 f(u)]}\ \frac{\delta T}{T} = \gamma\,\frac{\delta T}{T} \quad \cdots \quad \cdots \quad (8)$$

The ratio of the corresponding changes therefore is dependent on the period of the earth wave (since $u = \frac{T_p}{T}$) as is the magnification itself. In the following table values of γ have been calculated for some given values of u, also the earth-wave periods T_p, corresponding to these values of u and a pendulum period in the normal position of 14 seconds. To simplify the calculations μ^2, in the normal position, has been taken as zero (i.e., the damping as being critically aperiodic).

$u\left(=\frac{T_p}{T}\right)$	0·2	0·5	0·6	0·8	1·0	2·0	3·0	4·0	infinite
T_p (for T 14 secs.)	2·8	7	8·4	11·2	14	28	42	56	infinite
γ	−0·07	−0·24	−0·25	−0·17	0	+0·96	+1·44	+1·66	+2·0

These figures are shown graphically in Fig. 3. It will be seen that so long as u is less than unity an increase in pendulum period causes a decrease in magnification. When u is less than 1·2 the change in magnification is not more than one quarter of the change in pendulum period, but when u is greater than 2 the change in M is even greater than the change in T. It must be remembered, however, that the above values of γ do not apply strictly to large increments.

Some measurements obtained in a series of standardisation tests on the Kew vertical seismograph show how much the magnification (for sinusoidal waves of fixed period) is affected by large changes in the pendulum period due to the deviation of the equilibrium position from normal. The variations are shown in Fig. 2B in which magnifications for waves of 20-seconds period are plotted against angular deviation of the pendulum ; in Fig. 2A the corresponding pendulum periods are plotted. In the operation of the Kew seismograph the former practice was to limit the drift of the pendulum (due to temperature change) to $\pm 3{\cdot}7$ milliradians, an adjustment being made if this amount was reached. The magnification factors corresponding to the period of the pendulum in the normal equilibrium position were assumed to apply over the whole range of this drift. Referring to Fig. 2 it will be seen that in the normal position the pendulum period is about 14 seconds and the magnification (for $T_p = 20$ seconds) is 233. If the pendulum rises 3·7 milliradians the period falls to 12 seconds, i.e., a change of about -16 per cent, while the magnification decreases by only 2 per cent. If the pendulum falls through the same angle the period increases by 30 per cent, and the corresponding increase in magnification is 10 per cent. Thus

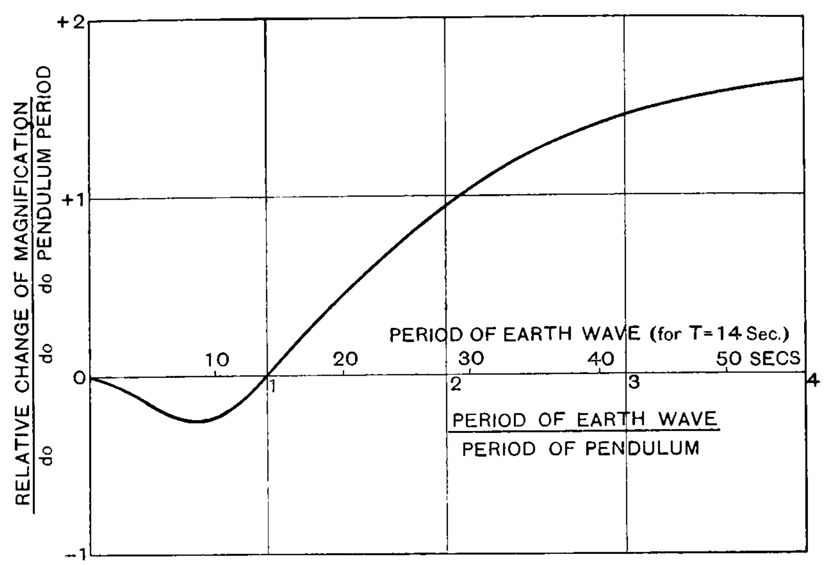

FIG. 3.

in applying the magnification corresponding to the normal position to any other position within the range of drift permitted an error of 10 per cent may occur. In view of the asymmetry of the variation of the period it is clearly better to reduce the permissible drift below normal and, if necessary, allow a larger drift above normal. Thus, if the drift is limited to 2 milliradians above normal and 1 milliradian below, the error of the magnification factor for earth waves of 20-seconds period does not exceed about 2 per cent, or for earth waves of 30-seconds period about 3 per cent. These limits are now in use at Kew.

In addition to the variation due to change in the equilibrium position of the pendulum there is the fact that the oscillations of large amplitude about any mean position are not simple harmonic. While we cannot avoid this last consequence, except by incorporating additional springs, we can improve the accuracy of measurements of small amplitudes by keeping the drift of the pendulum within narrower limits.

§ 4—ADJUSTMENT OF PENDULUM ZERO

The usual cause of the drift of the pendulum from its normal position is the change in the elastic constant of the spring due to change of temperature. The temperature effect in the case of the steel spring as ordinarily used is very great. Thus for the spring originally fitted to the seismograph at Kew the elastic constant β is $21{\cdot}2 \times 10^5$ gm./cm, and the temperature coefficient of β is about $-2{\cdot}2 \times 10^4$ per degree C. The steel spring[4] has recently been replaced by one made of elinvar alloy, of which the temperature coefficient of the elastic constant is only about one-tenth of that of the original spring. This is a decided improvement since large drifts occur much less frequently and the pendulum requires less adjustment.

[4] F. J. Scrase; *London Inst. Physics, J. Sci. Instr.* **6**, No. 12, 1929, p. 385.

On referring back to the formulæ for the period of the pendulum it will be seen that a change in β affects the period. We did not take account of this before, since we assumed β was constant. This effect, however, is small compared with the variation in period due to displacement but it persists after the pendulum has been readjusted to its normal position provided the temperature remains different from the original temperature.

Now there are two ways of correcting the drift of the pendulum due to the effect of temperature on the elastic constant of the spring. The first is by altering the moment of the moving system and the second by altering the length of the spring. Some calculations show that it is better to adopt the former method since it causes less alteration to the working period.

The turning moment is adjusted by moving a small weight along a screw thread attached to the pendulum frame. If the whole of this adjustment has been utilised then the main mass of the pendulum can be moved. To take the case of the Kew instrument, a rise of 1°C. causes a reduction by 0·022 per cent in β, the elastic constant of the steel spring. In order to restore the balance in the normal position the small weight (100 gm.), which is about 50 cm. from the turning axis must be moved one cm. To find the effect on the period in the normal position we can use Galitzin's formula (5) which can be taken as valid in this position. The rise in temperature and the necessary adjustment affect β, P_0 and I. P_0 is affected in the same ratio as β since the length of the spring L_0 is kept constant and, assuming that the unstretched length is unchanged, the extension is unaltered. The change in I is $m\ (r_1^2-r_2^2)$. i.e. $100\ (50^2-49^2)$ or 10^4 gm. cm.^{2} and since I is about 156×10^5 gm. cm.^{2} the percentage decrease is 0·06. Thus the net effect on $n^2\ (=4\ \pi^2/T^2)$ is an increase of about 0·04 per cent. The period T therefore is decreased by about 0·02 per cent for 1°C. rise in temperature. So that even for a change of 15°C. (the annual range of temperature in the seismograph room at Kew) the alteration in the working period due to this method of maintaining the pendulum near its normal position is negligible compared with the variation with the deviation from normal. Moreover, if necessary, the mean position of the adjustable weight could be so arranged that the change in I would counterbalance those of β and P_0. For the relative change of β or P_0 is equal to the relative change of the moment which is $mg\ (r_1-r_2)/Mgr_0$ and this can be made equal to $m\ (r_1^2-r_2^2)/I$, m being the mass of the small weight and $\frac{1}{2}(r_1+r_2)$ its mean distance. In our case equality would occur if the adjustable weight were fitted at about 20 cm. instead of 50 cm. from the turning axis.

The second method of adjustment can be examined in a similar manner. In this case we keep P_0, I and the turning moment constant but alter L_0 in order to counteract the change in β. A decrease of 0·02 per cent in β (corresponding to a rise of 1°C.) necessitates an increase in L_0 of half this amount to support the same load in the case of the Kew instrument for if $P_0 = \beta\ (L_0-l_0)$, l_0 being the unstretched length of the spring, then $\frac{\delta L_0}{L_0} = -\frac{P_0}{\beta L_0}\ \frac{\delta\beta}{\beta}$. Using the Galitzin formula again we find that these changes cause an increase in the period of 0·75 per cent for a rise of 1°C. The annual range of temperature therefore may cause a change of 11 per cent in the working period and this effect is additional to the variation of the period with deviation from the normal. It is clear then that this method of adjustment is not to be recommended.

§ 5—Summary

The single-spring system employed in the Galitzin vertical seismograph involves a variation in the period of the pendulum with deviation from the normal position. The variation is much more pronounced at high periods than at low periods. It can only be eliminated by fitting additional springs.

The relative change in magnification is less than that of the pendulum period for earth waves having periods less than double that of the pendulum. Since the variations are symmetrical about the normal equilibrium position the limit for the drift of the pendulum should be less when the deviation is below normal than when it is above normal.

In correcting the drift of the pendulum it is better to alter the turning moment (by moving the adjustable weight or, when necessary, the main weight) than to alter the length of the spring. The latter method causes an appreciable change in the period at the normal equilibrium position, and this effect is in addition to the change in period with deviation.

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