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THE AERODYNAMIC TERM IN THE PENMAN FORMULA - USE OF HOURLY DATA

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1. INTRODUCTION

The aerodynamic term in the Penman formula is

$$\alpha_1 (e_a - e_d)\left(1 + \frac{u_2}{100}\right) \qquad (1)$$

where u_2 = wind speed at 2m in miles per day

ea = S.V.P. in mm of mercury

ed = actual V.P. in mm of mercury

α_1, is a constant

Changing the units, the term becomes

$$\alpha_2 (e_a - e_d)\left(u + \frac{100}{21}\right) \qquad (2)$$

where u = wind speed at 33 ft. in kt.

Expression (2) is suitable for use for London Airport data, where the effective height of the anemometer is 33 ft.

Let us assume that the true value of the aerodynamic term is given by expression (2) evaluated at hourly intervals, for some value of α_2.

We are going to study what errors are introduced when the same hourly data are used, but instead each variable is averaged over 24 hours and expression (2) is then evaluated using these 24-hour mean values. In other words, we are going to compare

$$A = \frac{1}{24}\sum_{h=1}^{24}\left[(e_a)_h - (e_d)_h\right]\left[u_h + \frac{100}{21}\right] = \text{"MEAN OF PRODUCTS"}$$

with

$$B = \frac{1}{24}\left[\sum_{h=1}^{24}(e_a)_h - \sum_{h=1}^{24}(e_d)_h\right]\left[\frac{1}{24}\sum_{h=1}^{24}u_h + \frac{100}{21}\right]$$

$$= \text{"PRODUCT OF MEANS"}$$

2. DATA

The data used were hourly values of temperature and vapour pressure (instantaneous values) and wind speed (mean during the preceding hour), for London Airport for the period January 1965 to January 1966.

3. RESULTS

Table 1 (available on request) shows the values of A, B and A-B for each day during the period.

Figure 1 shows 30-day and 365-day mean values of A and B.

Figure 2 shows for each month the ratio of the monthly mean values of A and B.

4. DISCUSSION

a. Errors in monthly means

Figure 1 shows that, in every month, A was on average greater than B. Figure 2 shows that the ratio between the two monthly means was fairly constant, ranging between 1.03 and 1.11 with a mean value of 1.076. There is a suggestion of a slight seasonal variation, winter months showing the lowest values of the ratio. March showed the highest value, and seems to be rather out of step with the other months.

Supposing we had used B instead of A in our calculations of evaporation. Use of B without correction would have given us monthly mean values of the aerodynamic term which were too low by between 3% and 11%. We would have got a better answer by applying a multiplying factor c = 1.076 to B; the annual mean would then have been correct, and monthly means correct to within ± 5%. If the variations from month to month can be shown to be regular seasonal changes rather than sampling errors (analyses of several years will be necessary to show this) the errors in the monthly means can be further reduced by making c a function of the time of year.

b. Errors in daily values

Supposing we had chosen c for each month such that $\overline{A} = c\overline{B}$, the bar denoting averages over the month; in other words, we had used the value of c given by Figure 2. What would then have been the root-mean-square error of the daily values?

The quantity we require is the standard deviation of the terms (A-cB), which we shall denote by $\sigma(A-cB)$. This is related to $\sigma(A-B)$, which we have calculated, by:

$$\sigma^2(A-cB) = \sigma^2(A-B) - (c-1)^2 \sigma^2(B)$$

thus the errors in (A-cB) will be a little lower than the errors in (A-B) which we shall now discuss.

The standard deviation of A-B, expressed as a percentage of the value of A, is given in Figure 3. Its value was around 7% throughout the year, varying in the range 4-9%. Errors were least in the winter months, and greatest during March.

In practice we would not have been able to choose the correct value of c, but would have used the average value 1.076, or some approximate value for the time of year. Hence the root-mean-square error would have been somewhat greater than that given above.

c. Relation with temperature range

There is some suggestion that errors are greater on days when the temperature range is large. There appeared to be quite a large number of such days in March, perhaps explaining the high mean and standard errors in that month. It would also help to explain why errors were less in winter, as the diurnal temperature range is less then. This suggests that we might be able to reduce the errors by making c depend on the diurnal range. However, this exercise would be of purely theoretical interest, as nobody is going to use B and take pains to make it fit A, when he could calculate A in the first place using exactly the same basic data.

5. CONCLUSIONS

1. In using the product of the means rather than the mean of the products, we should increase our estimate by about 8%, perhaps somewhat less in winter.

2. Having made this correction, the standard error of the daily estimates is about 7%, perhaps somewhat less in winter.

NOTE This memo is circulated for discussion purposes only. Any comments or suggestions should be sent to the author.

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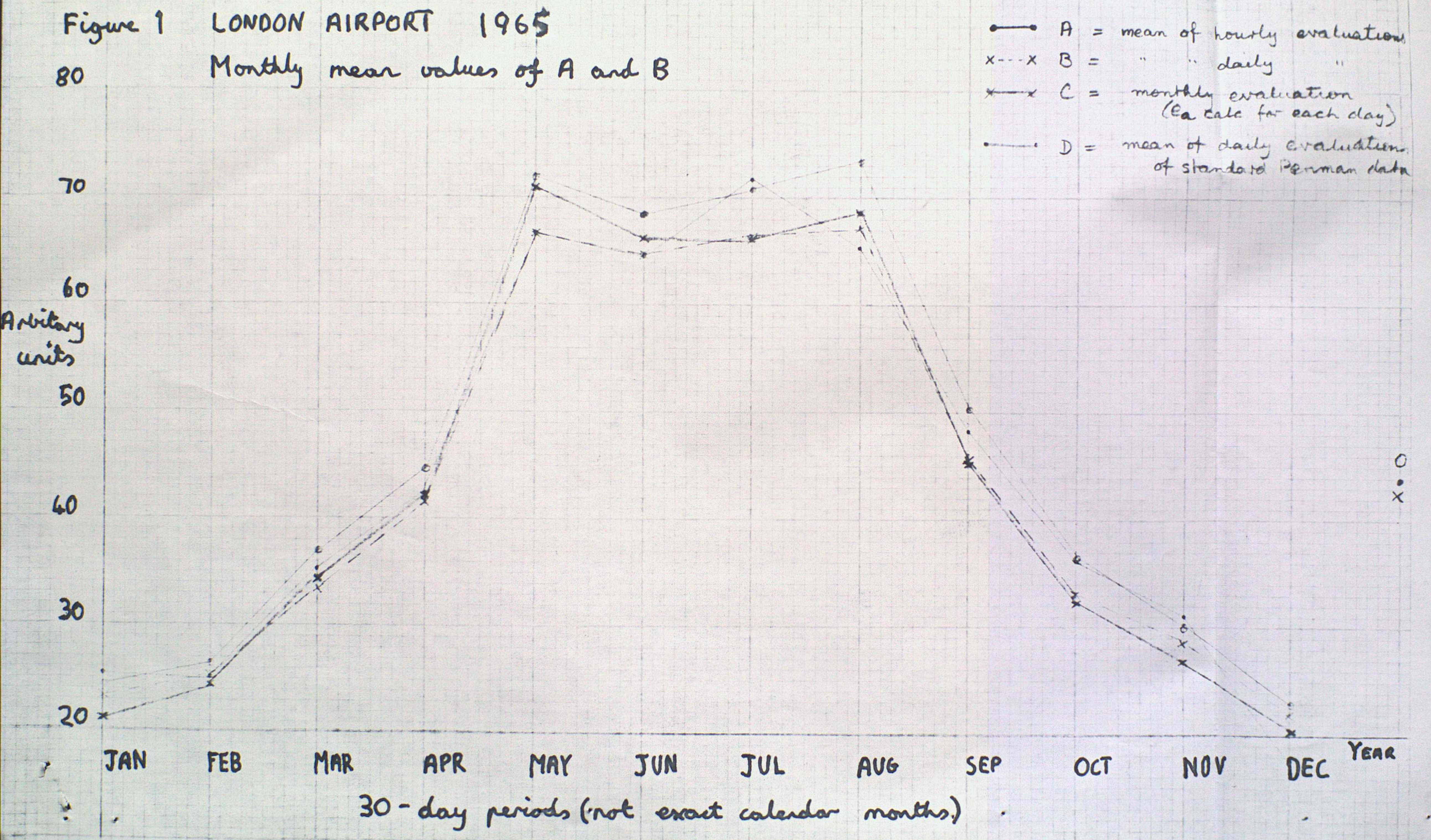

Figure 1 LONDON AIRPORT 1965
Monthly mean values of A and B
A = mean of hourly evaluations
B = " " daily "
C = monthly evaluation (ea calc for each day)
D = mean of daily evaluations of standard Penman data
80
70
60
50
40
30
20
Arbitary units
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
YEAR
30-day periods (not exact calendar months)

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Figure 2 LONDON AIRPORT, 1965

MONTHLY MEAN VALUE OF A
" " " " B

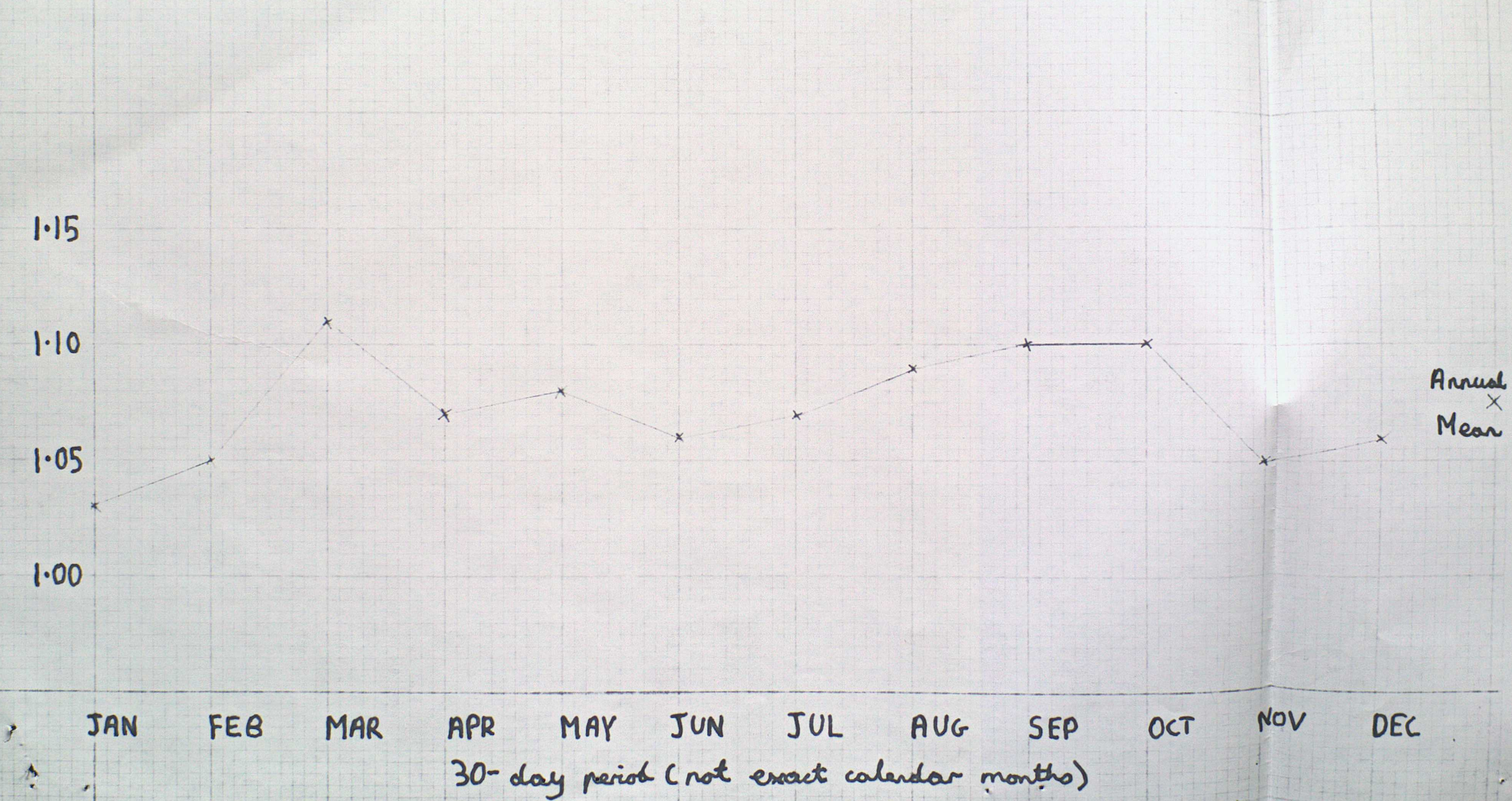

Figure 3

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LONDON AIRPORT, 1965. Standard deviation of daily differences A – B, expressed as a percentage of A