The dispersion of pollution in the atmosphere

General Characteristics

The concern shown in the first lecture about the qualitative nature of the airflow in the boundary layer is, in part, a reflection of the importance that these motions have in transporting and dispersing plumes of pollutants leaving chimney stacks or other sources. Ultimately the question must be:

"Given the sources, what is the best estimate we can make of the concentrations (the long-term average values and their shorter-term probability distributions) of the various pollutants at some specified receptor point given any required information about the terrain and the meteorology ?". It would be necessary to know certain details of the pollutants concerned: how much is being emitted per unit time, whether or not they are chemically reactive, whether the plume is buoyant, heavy or passive, whether the material is toxic, radioactive or explosive, whether the pollution is affected by precipitation or not, and so on.

Clearly there are lots of difficult questions here and many of them will have to be faced in the problem we have set ourselves. In the meantime the simplest situation will be considered, namely that of a passive, non-reactive, conservative pollutant being continuously emitted at constant rate from an elevated source.

Observing a visible plume of this kind one sees that normally

(a) the plume is carried away downwind

(b) the plume meanders both in the vertical and acrosswind

(c) the plume grows in size both in the vertical and the horizontal with distance (it looks rather like a narrow distorted cone with its apex at the stack exit) and ultimately impinges on the underlying ground.

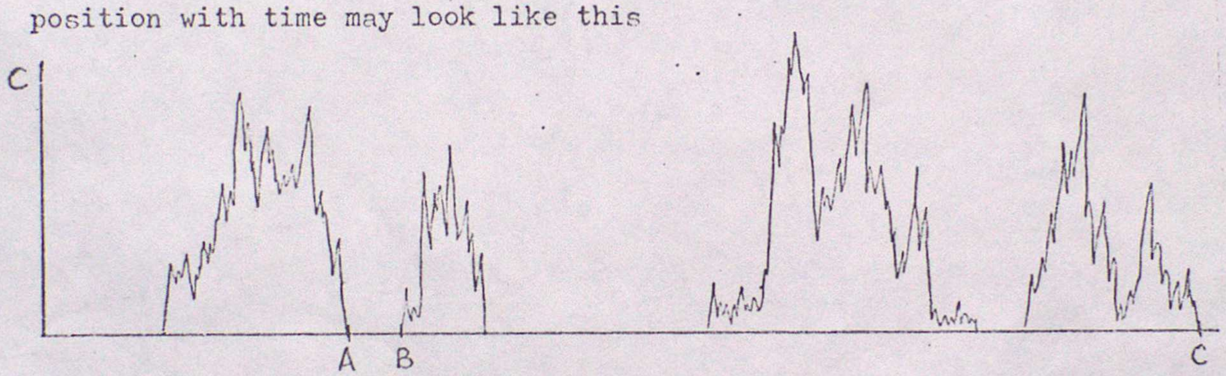
Sometimes the plume breaks up completely into discrete puffs which eventually may join up again as the individual puffs grow larger and larger. This break-up is more commonly observed with hot plumes, with plumes on warm summer days or when the wind is causing some kind of resonance effect within the chimney stack itself. Thus an observer at a fixed point may

(i) never be affected by the plume until a significant change in the meteorology occurs.

(ii) be almost always in the plume if he is directly downwind of the source

(iii) be affected by the plume in a very intermittent way.

In the last situation the record of concentration at the observer's position with time may look like this



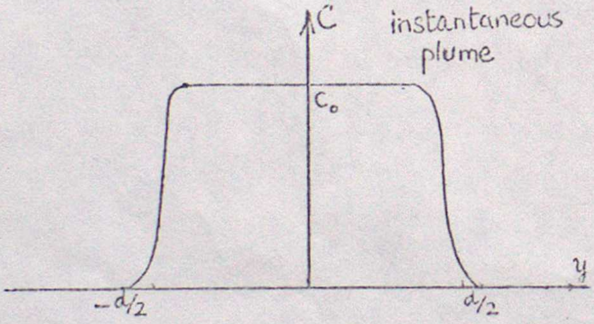
To find the effect of the pollutant on the receptor, the concentration may be estimated by analysing a continuous sample of air over a certain time interval. Clearly a very different picture would be obtained if that time interval were AB or BC, valid though each may be for its own interval. To get a representative average over a much longer time period, the interval should be at least AC if not very much longer.

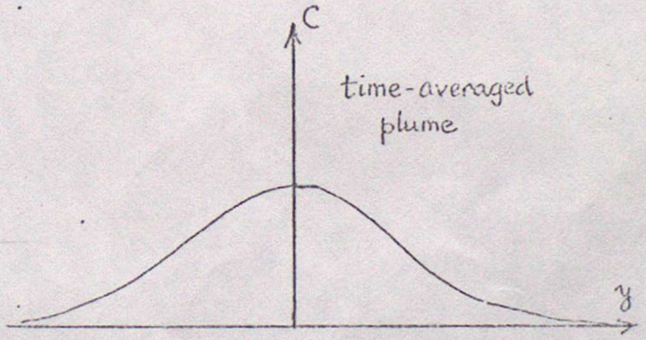
The time over which we measure the concentration is called the sampling time and it is clear that the answer we get depends in a very important way on this time.

At any moment and at a given distance downwind x from the source the plume will have a certain cross-wind dimension d. Measured some time later the width may be d', and again later d" and so on. Typically there may be something like a four-fold variation in the various values of d.

Due to the meandering of the plume as a whole, which comes about because of the larger eddies, the centre of the plume is constantly moving in the y-z plane (y - acrosswind, z - vertical). The concentration and the plume width averaged over many tens of minutes will reflect both the instantaneous width distribution d and the extent of the plume meandering. The time average width D is always as large as or larger than d.

Usually the ensemble-averaged concentration is nearly constant within the instantaneous plume, but varies in a nearly-Gaussian way within the time-averaged plume





Mathematically we could describe the width in terms of the "total" width, from where concentration C first becomes non-zero on the left of the plume to where it becomes just zero on the right. This might be adequate for the instantaneous plume, but is unsatisfactory for the time-averaged plume because of the long "tails" present in this distribution.

It is normal to define the width in terms of a root-mean-square width , defined mathematically as

$$\sigma_p^2 = \frac{\int_{-\infty}^{\infty} y^2 C(y)\, dy}{\int_{-\infty}^{\infty} C(y)\, dy}$$

For the nearly square "top-hat" distribution of the instantaneous plume where C(y) is a constant = C_o within the plume

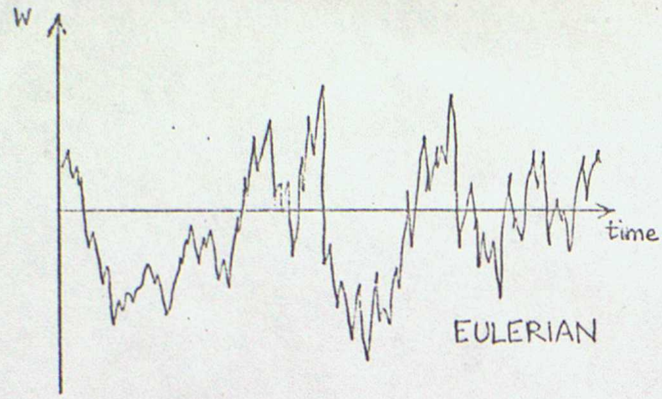
$$\sigma_p^2 = \frac{C_o \int_{-d/2}^{d/2} y^2 \, dy}{C_o \int_{-d/2}^{d/2} dy} \approx (0.29\, d)^2$$

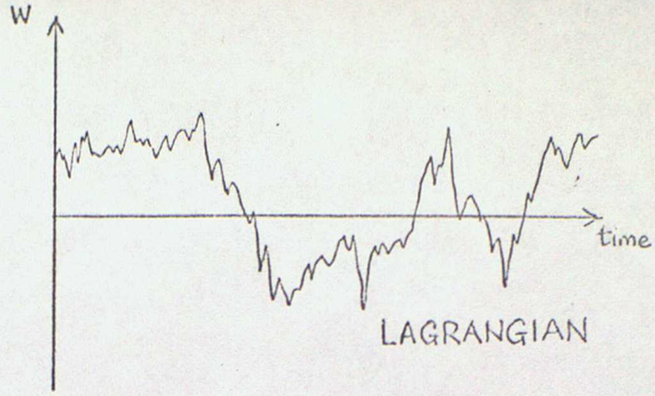
The variation of concentration with y in the time-averaged plume can be found very easily from mathematical tables giving the Gaussian or "normal" distribution.

Although the concentration within the plumes may approximate to these mathematical distributions in a statistical sense quite well, in practice the distribution of the pollutant is often very patchy within a plume, as can be seen by any observer. This is a very important property for pollutants either with an unpleasant odour or if they are explosive within certain concentration limits (like natural gas). Unfortunately the theory of this patchiness is almost non-existent.

Representation of Particle Motion

It is clearly important to be able to estimate the concentration field downwind from the source. Since the "particles" of pollution respond to the turbulent fluctuations of the air, one might suppose that the whole diffusive process could be inferred in statistical terms by interpreting the fluctuations of a suitably response vane and anemometer. Herein lies a difficulty. The particles respond to eddies in a Lagrangian framework, whereas the anemometer and vane are in an Eulerian frame, they are fixed in space. The respective statistics are not in every respect identical. For example it is often the case that Lagrangian fluctuations occur more slowly than Eulerian fluctuations - they have a longer time-scale





This difference is in fact the most obvious one. Other differences are more subtle and a casual glance will not reveal them.

(a) Statistics of turbulence

Various statistics can be defined. All of them depend on the period of time in which the turbulent velocities are observed or sampled. The first is the mean velocity $\bar{w}$ (where w stands for any specified turbulent velocity component). The second is the variance $\overline{w'^2} = \overline{(w-\bar{w})^2}$ taken about the mean $\bar{w}$.

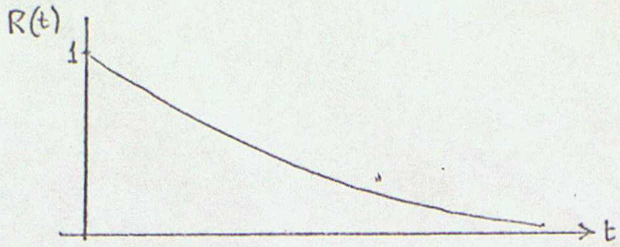
In practice some degree of time averaging or smoothing is implied - either consciously applied or implicit as a result of the finite inertia of the measuring instruments.

Let us call the sampling time T and the averaging time t_a.

A third statistic is the time-scale τ already referred to above. τ is best defined by the turbulent velocity correlation function R(t):

$$R(t) = \frac{\overline{w'(s)\, w'(s+t)}}{\overline{w'^2}}$$

where the turbulence is assumed to be sufficiently steady (in a statistical sense) that R(t) does not depend on s, although it may depend on the particular sampling period from which the w' are derived. It is clear that R(o) = 1, and R(t) for large t (assuming T is even larger) falls to nearly zero.



The time-scale τ is defined as the area under the correlation curve i.e.

$$\tau = \int_0^\infty R(t)\,dt$$

Again, τ is a function of T and t_a

Now $\overline{w}$ and $\overline{w'^2}$ have the same probability distribution whether the w's are measured in an Eulerian or Lagrangian frame provided the sampling and averaging times are equal when normalised by their respective time-scales.

(b) Eulerian and Lagrangian time-scales

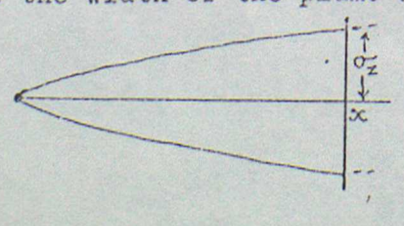
In an Eulerian frame, the time-scale clearly has something to do with the time taken for the dominant eddies to pass by the fixed point at which the fluctuations are being measured; that is $\tau_E \propto l/u$, where l is the size of these eddies and u the mean wind speed.

On the other hand, in a Lagrangian frame it can be argued that τ_L represents the eddy life time and this is $\propto l^{2/3}$ so that the ratio of the two time-scales $\beta = \frac{\tau_L}{\tau_E} \propto l^{-1/3}$

Since the dominant eddy size increases with increasing instability it follows that β is not a constant but is smallest in unstable conditions and largest in stable conditions. In near neutral conditions $\beta \approx 4$.

Plume dispersion

Consider a source at the origin embedded in a uniform airstream velocity u. We require the width of the plume at some point x downstream.



The width can be parametrized by the r.m.s. particle displacement σ_z from the axis. The individual particles arrive at x having followed tortuous paths from the source. The displacement of the particle from the axis z_p equals

$$z_p = \langle w \rangle_p \frac{x}{u}$$

where $\langle w \rangle_p$ is the velocity of that particle meaned over the time x/u. Then $\sigma_z^2 = \overline{\langle w \rangle_p^2} \frac{x^2}{u^2}$ averaged over the whole ensemble of particles; $\overline{\langle w \rangle_p^2}$ is the variance of the velocities meaned or averaged in a Lagrangian sense over time x/u and experienced or sampled over the total time T of the release in an Eulerian sense. Using the concepts outlined above, a Lagrangian averaging over time x/u is equivalent to an Eulerian averaging over time $t_a = x/\beta u$. Thus if $w(t)$ is the Eulerian experience of w then

$$\sigma_z = \langle \sigma_w \rangle_{\frac{x}{\beta u}, T} \frac{x}{u}$$

Height Variations

The above presupposes the turbulence is homogeneous. In the atmospheric boundary layer this it not true - especially in the first one or two hundred metres. This can complicate the problem of predicting σ_z, although the equation just derived may often be used to give a rough guide provided the plume has not intercepted either the ground or the top of the mixing layer. Generally the scales (both time and length scales) of vertical fluctuations increase almost linearly for the first tenth of the boundary layer and overland reach maxima some $\frac{2}{3}$rd of the way up before declining again as the top is approached.

Spectral Weighting Functions

The magnitude of $\langle \sigma_w \rangle_{\frac{x}{\beta u}, T}$ can be derived from the full w-spectrum of turbulence by applying two similar weighting functions (or band-pass filters), the first a low-pass filter representing the effect of averaging reducing the effect of the high-frequency eddies, the second a high-pass filter which removes the negligible effect of the largest eddies in the fluctuations taken over a finite sampling time. If $x/\beta u$ approaches T then the cloud becomes more like an individual large "puff" and the effect of alongwind fluctuations can no longer be ignored.

Effect of Topography

It would be inappropriate in this introductory talk to go into any detail on this topic. In fact this is an area of current interest and research and there is still a long way to go before a satisfactory understanding will be reached. Suffice it to mention just a few general qualitative points

(i) turbulence levels tend to be higher in hilly terrain resulting in more rapid dispersion.

(ii) when steep slopes exist (greater than 1 in 3) flow separation and recirculating airflow regions often occur, complicating enormously the prediction of concentrations.

(iii) stability is a crucial parameter in determining whether or not an elevated plume will impinge on a hillside or try to work its way round it.

(iv) in light wind conditions at night, gravitational flows dominate the air motions.

(v) in the absence of separation eddies and plumes may be temporarily distorted by isolated topography without dramatically changing the subsequent downwind distributions.

Dispersion in Unstable convective conditions

As we have already noted in the first lecture, vertical turbulence, at the larger scales, tends to become much more organised in convective conditions. The picture (perhaps somewhat oversimplified) is one of relatively strong and turbulent updraught regions which have high coherence through most of the depth of the mixing layer, and broader weakly turbulent downflow regions in between. The capping inversion is often humped over the updraught areas, and because of the resulting tightening of the wind and temperature gradients there, dynamic instability may occur (either on the scale of the inversion layer thickness, or on the scale of the humps themselves) which results in entrainment of stable warm air aloft into the boundary layer. Consequently the net heat flux may actually be downwards in this region.

Sometimes individual thermals break through the capping inversion and come to rest in the stable air above, gradually weakening the stability there, until a "quantum jump" in the depth of the boundary layer occurs incorporating this newly weakened air above the inversion. Thus although some pollution may temporally be lost from the boundary layer, much is ultimately returned as the depth grows.

Plumes within the boundary layer respond to the organised behaviour of up and downdraughts. Numerical and laboratory studies suggest that the height of the maximum concentration within a plume from an elevated source should first slowly decrease under the action of the general subsidence outside the thermals until it reaches ground level and then should subsequently increase as the plume gets advected into the strong updraught areas. Convincing evidence of this behaviour in the real atmosphere has yet to be found.

Dispersion in stable conditions

When potential temperature increases with height and the flux of heat is downwards, the air is stable and the amount of vertical turbulence is relatively small or virtually non-existent. Consequently in really stable conditions plumes often show little deepening and are subject only to slow horizontal two-dimensional eddies which fan the plume out into rather thin sheets.

Sometimes this lack of turbulence causes the wind gradients to tighten to such a degree that dynamic instability occurs and a rather short burst of vertical turbulence results which locally slackens the gradients, deepens the stable boundary layer and mixes pollution through the vertical.

Long releases and synoptic swinging

For releases of a pollutant over many hours or more, the plume is affected by changes in wind direction. Whilst such changes may be thought of as a manifestation of meso-scale or synoptic-scale eddies there is a case for treating them in a more heuristic manner, mainly because normal meteorological observations are capable of describing them but also because they are separated from the smaller-scale micrometeorological eddies by a marked reduction in energy transfer in a direct sense between the two scales of eddies and sometimes an energy "gap" in the turbulence spectrum, so that there is little direct dependence of one upon the other.

For releases over more than about 4 hours, the horizontal area affected by the plume depends more on wind direction changes than on small-scale turbulent lateral spreading. Statistics can be given for the probability of the wind turning through any specified angle in any specified time, and these can be useful in predicting likely areas of risk should an accident occur around a potentially hazardous source at some time in the future

Problems arising from airborne pollution

(i) At short range (0 - 10 km)

Sources of pollution are often situated in urban areas and strict control has to be applied to ensure the concentrations of potentially damaging pollutants are kept within acceptable limits for the great majority of the time and pose no significant threat to human health and property. This may involve careful siting of the source, suitable cleansing systems operating on the waste gases, and releasing the effluent from sufficiently high chimneys.

(ii) At medium range (10 - 100km)

The plumes will disperse to fill the mixing layer within this range and although the resulting ground-level concentrations will often be significantly less than at short range there still exists a potential risk to agriculture (including market gardening and forestry). Uptake of hazardous pollutants by sensitive receptors may be as much by an indirect route through deposition onto the ground and uptake by the plants roots, as by direct uptake from the air by the leaves. The effect of hills, valleys and coastlines are important and may determine optimum siting of new industrial sources.
Visibility is often adversely affected within this range through the action of hygroscopic aerosols.

(iii) At continental scale (100-2000Km)

Air concentrations are usually by now very small and potential damage occurs almost entirely through surface deposition - either by so-called "dry-deposition", the uptake by absorption, sedimention or impaction, or by "wet-deposition" when pollution is removed from the atmosphere by precipitation. The risk of damage to the environment now depends critically on other factors such as soil time, precipitation run-off characteristics etc. For example, fish-kill in freshwater lakes and rivers in Scandinavia occurs because of the poor buffering capacity of the local soils in spite of depositions relatively small compared to those in central Europe.

(iv) At global scale (> 2000 Km)

Atomic bomb debris with mixed half-life radionuclides pose potential threats to the world population everywhere. Other pollutants such as carbon dioxide and nitrogen oxides may modify the detailed structure of the atmosphere and cause global-scale warming (thereby affecting climate and agriculture) or permit greater penetration of ultra-violet rays from the sun to the ground with a resulting increase in skin cancers etc. Emphasis must be placed on the word "may" however: no proof of either of these effects has yet been forthcoming, but vigilance and continuous monitoring is clearly highly desirable.

Books

"Atmospheric Diffusion" (2nd Edition) by F Pasquill — Ellis Horwood. Chichester.

"Atmospheric Motion and Air Pollution" by R A Dobbins — Wiley Interscience New York

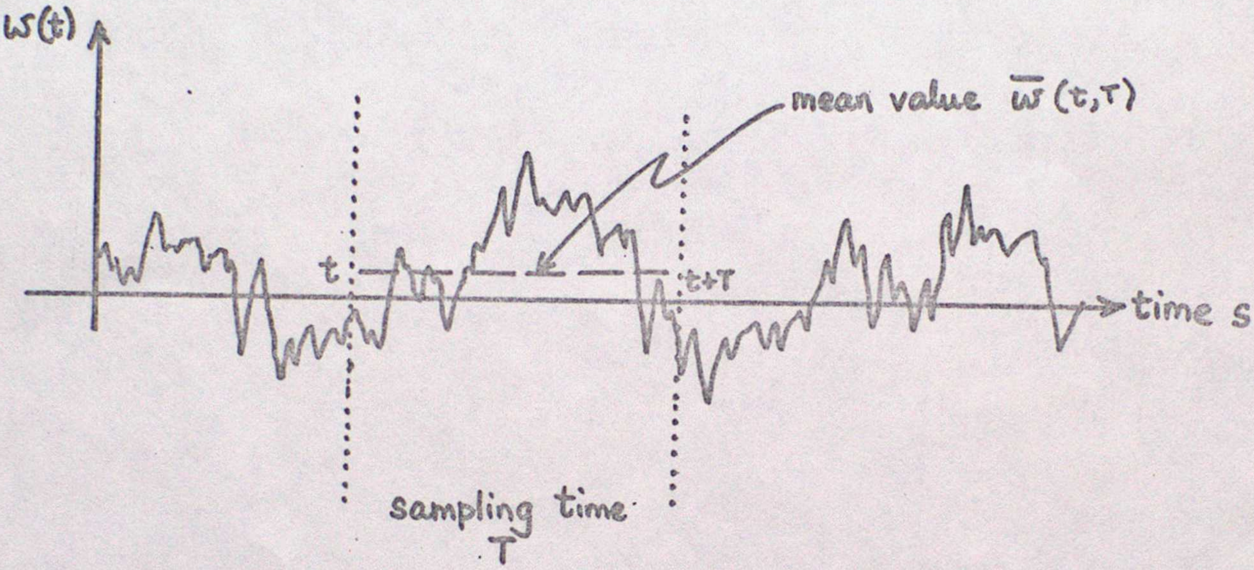
Part 2.

Notes on the dispersion of passive material in the atmosphere

Dispersion is caused by fluctuations in air velocity;

These fluctuations are called turbulence.

Consider a velocity trace:



1. Firstly, specify the starting time t and the length of the trace T being examined.

MEANS

2. Calculate the mean value of w over this period.

$$\overline{w} \equiv \frac{1}{T}\int_t^{t+T} w(s)\,ds$$

$\overline{w}$ will be different for different values of t and T.

3. In "stationary" turbulence (in which the statistics of the turbulence do not depend on t except in a random sense) then the ensemble-average $\overline{w}(T)$ is given by

$$\overline{w}(T) = \frac{1}{S}\int_{-S/2}^{S/2} \overline{w}(t,T)\,dt \qquad \text{where } S \text{ is very large.}$$

σ_w

4. The magnitude of the fluctuations of w about $\overline{w}$ is another important measure of the turbulence. This is often given by the root-mean-square difference σ_w:

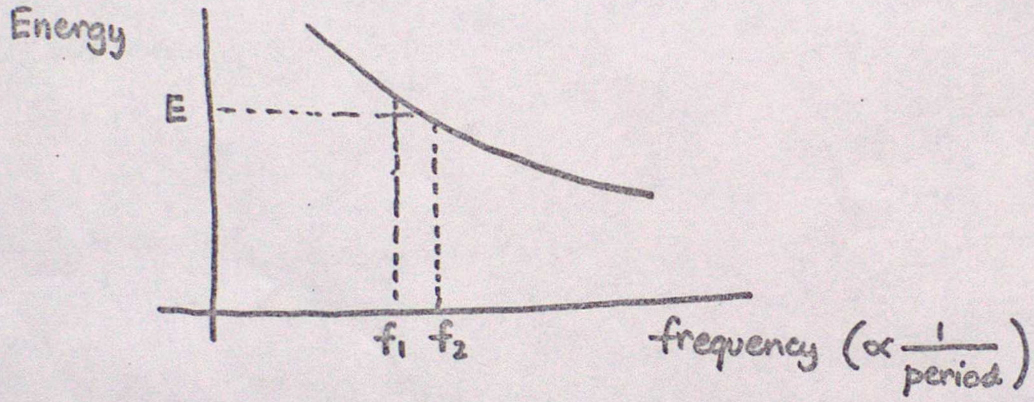
$$\sigma_w^2 \equiv \int_t^{t+T} \left(w(s) - \overline{w}(t,T)\right)^2 ds$$

1.

Looking back at the trace on the previous page we see there are some fluctuations which have a long time-scale (about as long as T, as it happens) and some have a very short time-scale.

In so far as we can separate these, we could associate a σ_w to each. σ_w^2 is an energy. We can say that fluctuations of a specified timescale (or period) have an amount of energy associated with them.

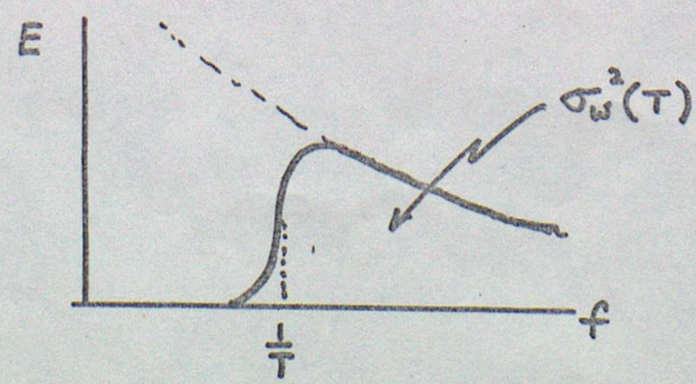
Actually the trace has a whole range of time-scales, or periods. We can form an ENERGY SPECTRUM



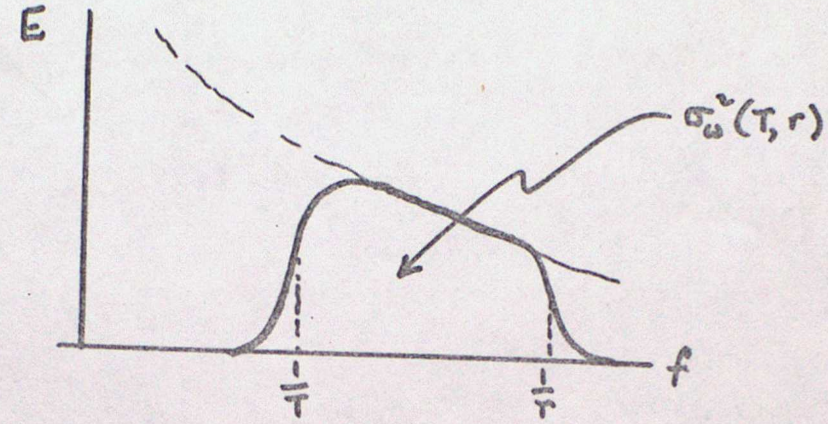
E is the amount of energy associated with fluctuations with frequencies between f_1 & f_2.

SAMPLING & AVERAGING

If the trace is sampled over a period T, then fluctuations with a long period (or low frequency), may affect $\overline{w}$ but hardly affect σ_w, or the energy. The samples' energy comes from the higher frequencies



Similarly if we SMOOTHED the trace first by averaging $w(s)$ over running intervals of length r, say, then we would lose the contribution of the higher frequencies. All wind measuring instruments lose some of the higher frequencies because they cannot respond fast enough.

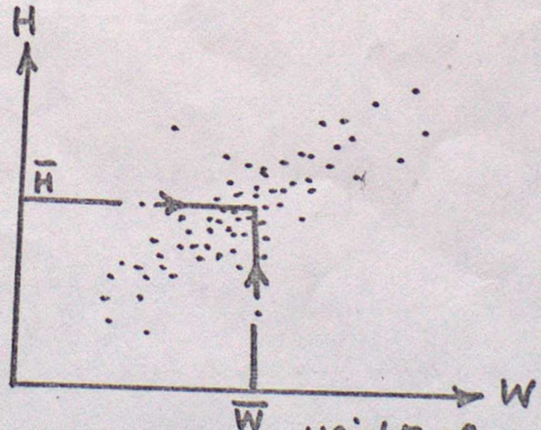


CORRELATION

Correlation is another useful measure of turbulence. To remind ourselves about correlation consider a more everyday example :–

Consider the plot :

Results of sample of 50 thirty year old men. } height of men aged 30.



weight of men aged 30

There is obviously a relation between W and H. We say they are correlated.

The correlation coefficient r is defined as :

$$r = \frac{\frac{1}{N}\sum_{i=1}^{N}(W_i-\bar{W})(H_i-\bar{H})}{\sigma_W \cdot \sigma_H}$$

where N is the number in the sample ($N=50$, here).

$$\sigma_W^2 = \frac{1}{N}\sum_{i=1}^{N}(W_i-\bar{W})^2, \quad \sigma_H^2 = \frac{1}{N}\sum_{i=1}^{N}(H_i-\bar{H})^2 .$$

A correlation coefficient always lies between -1 and $+1$.

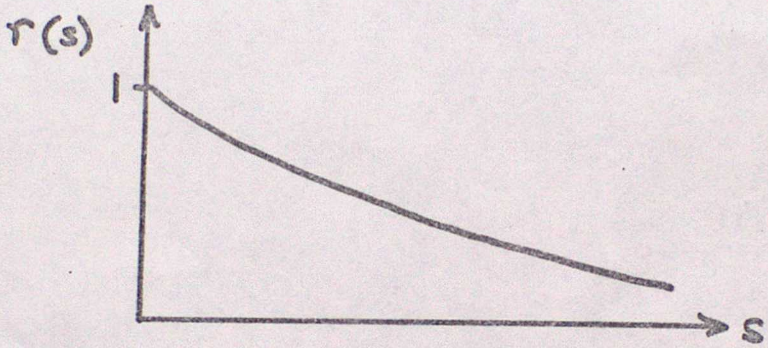
If $r = 1$, the two variables are perfectly related & increase together

$r = -1$, ... but change in opposite directions

$r = 0$, the two variables are unrelated

if $0 < r < 1$ the two variables are partly related.

This is true for $w(t)$ and $w(t+s)$ where s is some specified time interval.



– The smaller s is the more closely are the two velocities correlated.

$r(0) \equiv 1$ by definition.

Small-scale turbulence : $r(s)$ falls off quickly with s
Large-scale turbulence : $r(s)$ .. .. slowly .. ..

The <u>area</u> under the curve has dimensions of TIME.

– we call this time the time-scale τ of turbulence.

$$\tau = \int_0^\infty r(s)\,ds.$$

– τ tells us something about the typical scale of the turbulent eddies.

Mathematically $r(s)$ is related uniquely to the energy spectrum $E(f)$.

Eulerian Statistics

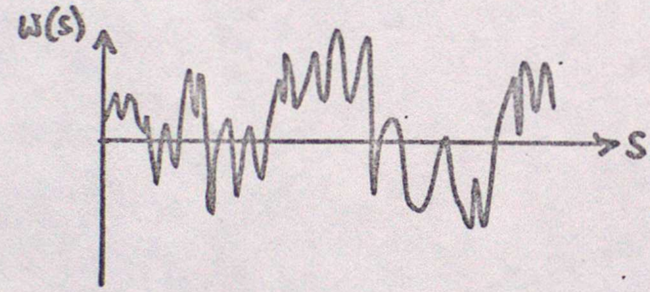
In meteorology, measurements made at a FIXED point are called EULERIAN:

Most measurements are Eulerian.

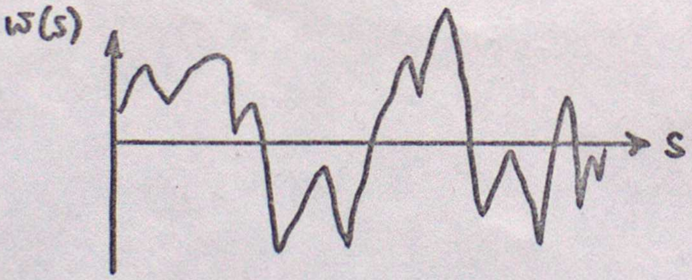
Lagrangian Statistics

Measurements made following the air motion are called LAGRANGIAN.

Measurements made from a free-floating balloon are approximately LAGRANGIAN.



EULERIAN



LAGRANGIAN

Often (but not always) the most obvious difference between an Eulerian w-trace and a Lagrangian w-trace is that the former has a shorter time-scale than the latter. Otherwise σ_w etc is the same.

$$\frac{\tau\ (\text{Lagrangian})}{\tau\ (\text{Eulerian})} \equiv \frac{\tau_L}{\tau_E} = \beta \approx 4 \quad (\text{typically}).$$

5.

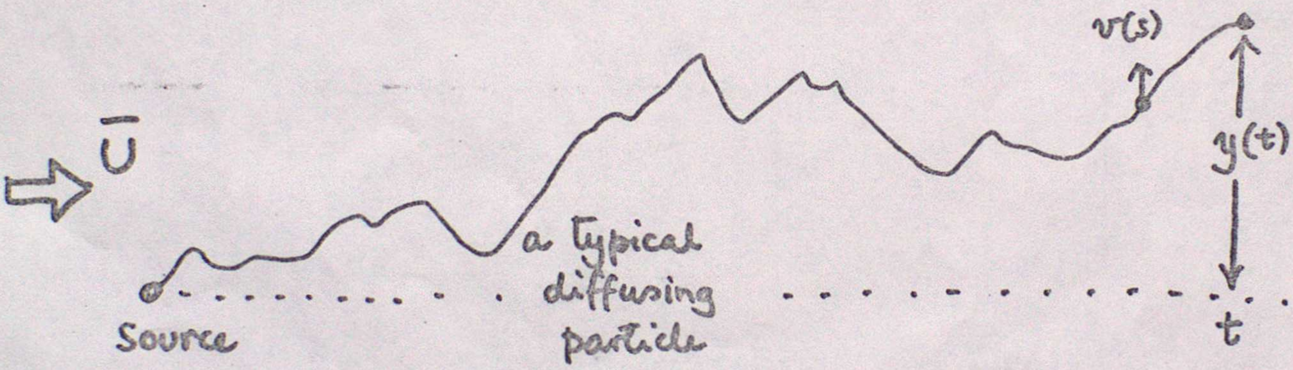
Diffusion

Particles follow the air motion

– They therefore respond to Lagrangian properties.

The great difficulty we face is to describe particle (or plume) behaviour, which is LAGRANGIAN, in terms of measurements, which are EULERIAN.

One hypothesis says that we can use Eulerian data as Lagrangian data provided we allow for the different time-scales.



$$y(t) = \int_0^t v(s)\,ds = \overline{v(s)}\,.\,t$$

a Lagrangian average.

The root mean square average value of $y(t)$ of an ensemble of particles is

$$= \left[\,\overline{y^2(t)}\,\right]^{1/2} = \sigma_y$$

$$\sigma_y = \left[\sigma_v\right] t$$

sampled over the time of release T,
v averaged in a Lagrangian sense over t

i.e. $\sigma_y = \left[\sigma_v\right]_{T,\,t/\beta}\, t$: both sampling & averaging now Euleria

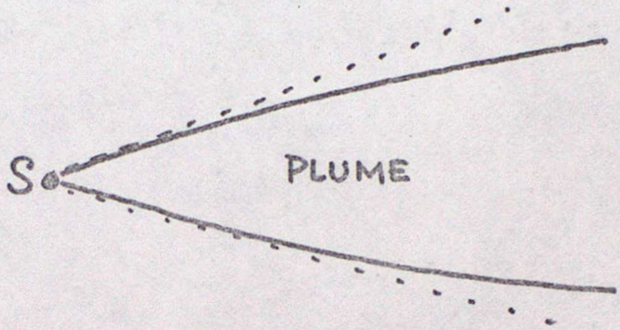
Often rather than measuring v, the horizontal velocity fluctuation, the angle θ of a wind vane is measured.

Then $\quad \sigma_y = \sigma_{\theta_{T,\,t/\beta}} \cdot x$

where x is the downwind distance where σ_y is required.
The total width of the plume is about $4.3\,\sigma_y$.

$$\beta \approx \frac{0.6}{\sigma_{\theta_{T,0}}}$$

Put another way:



$$\sigma_y = \sigma_{\theta_{T,0}}\, x\, f(x)$$

where over rather level countryside $f(x)$ takes the following values:

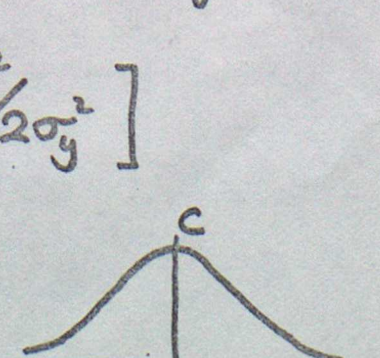
x (km)	0.1	0.2	0.4	1	2	4	10	>10
$f(x)$	0.8	0.7	0.65	0.6	0.5	0.4	0.33	$0.33\left(\frac{10}{x}\right)^{1/2}$

Over rough terrain, like hills, $f(x)$ falls off more slowly.

Gaussian plume distribution

We find that the following form describes the variation of concentration across a plume averaged over time T:

$$C(y) = \frac{A}{\sigma_y} \exp\left[-\, y^2/2\sigma_y^2\right]$$



VERTICAL DISPERSION

Dispersion in the vertical can also be estimated. Turbulence varies in character with height, however, and this makes it more difficult than horizontal dispersion.

There are several ways of tackling the problem (see Pasquill & Smith's "Atmospheric Diffusion", 1983)

We will mention here one fairly simple way, due originally to Pasquill, but modified since by Smith and others.

Pasquill looked at field data on diffusion and saw that he could characterise the rate of deepening of a plume from a continuous ground-level source in terms of stability. He required a simple scheme which could be used by a "layman" out in the field without sophisticated instrumentation and without a computer.

He expressed stability in terms of the wind speed at a height of 10 metres (the standard height at met. stations) and a subjective assessment of the incoming solar radiation at the site, divided into one of three classes: weak, moderate & strong.

Smith later replaced this rather crude assessment by "heat flux" and showed how this could be estimated (see later).

FIGURE 4. The revised scheme for P based on lines of constant μ.

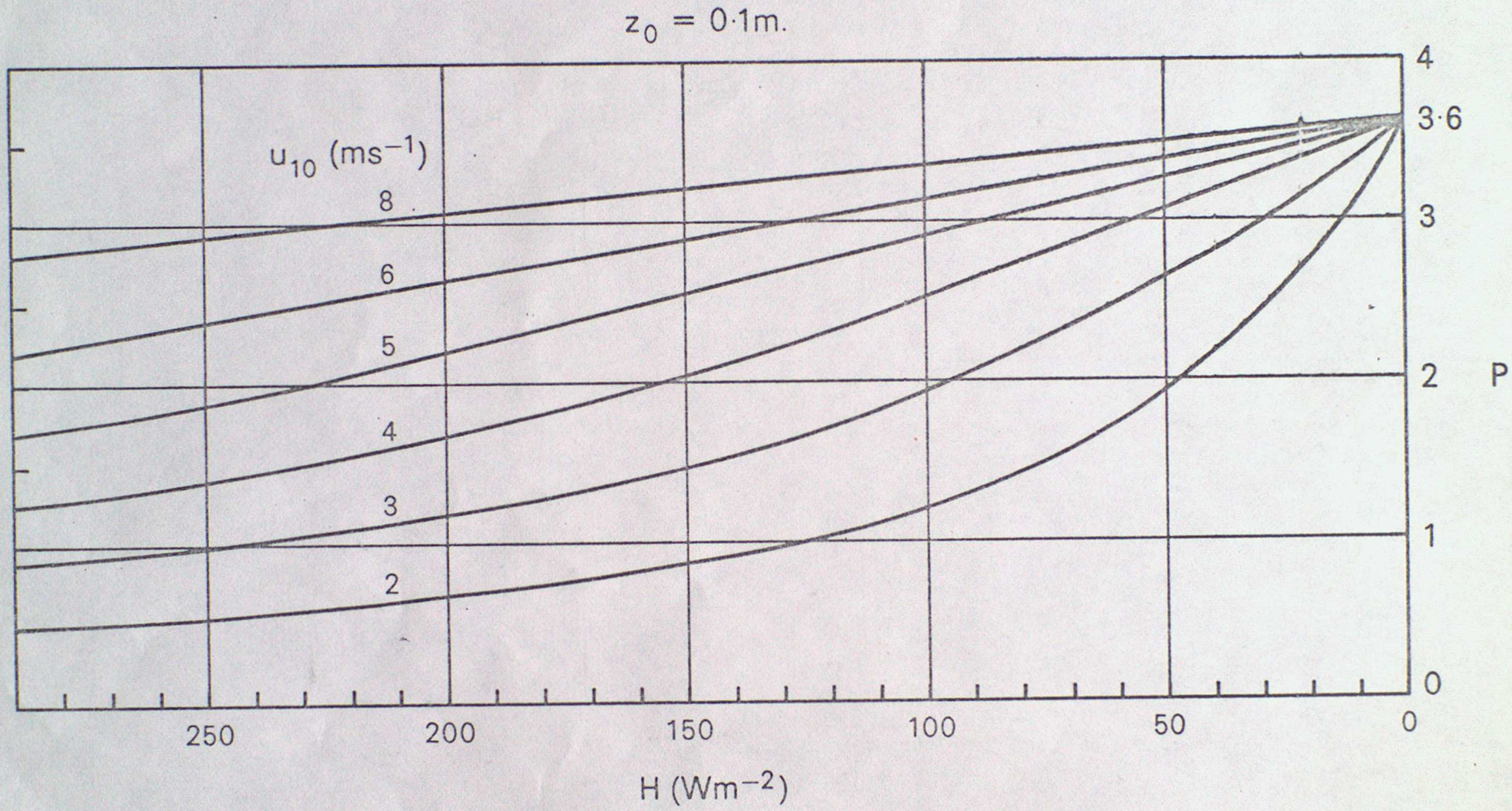


FIGURE 1

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9

Pasquill then defined 7 stability categories, related to the rate of plume deepening, which he called

A (The most unstable conditions - The most rapid rate of deepening)

B (unstable)

C (slightly unstable)

D (neutral stability)

E (slightly stable)

F (stable)

G (very stable)

Smith later replaced these with a continuous scale

$0 \leq P \leq 1$ corresponds to A

$1 < P \leq 2$ .. .. B

etc.

Strictly neutral corresponds to $P = 3.6$.

The first Figure (next page) shows how Smith obtains P from the heat flux H and the wind speed u_{10} for unstable conditions

e.g. If $H = 100\ Wm^{-2}$ and $u_{10} = 4\ ms^{-1}$

then $P = 2.5$ (or Pasquill Category C).

<u>At NIGHT</u> Smith uses the formula

$$P = 3.6 + \frac{120 - 13.3c}{27 - 2c} \exp\left[-\frac{3}{8}u_{10}\right]$$

where c = cloud amount, measured in $\frac{1}{8}$ths of sky covered.

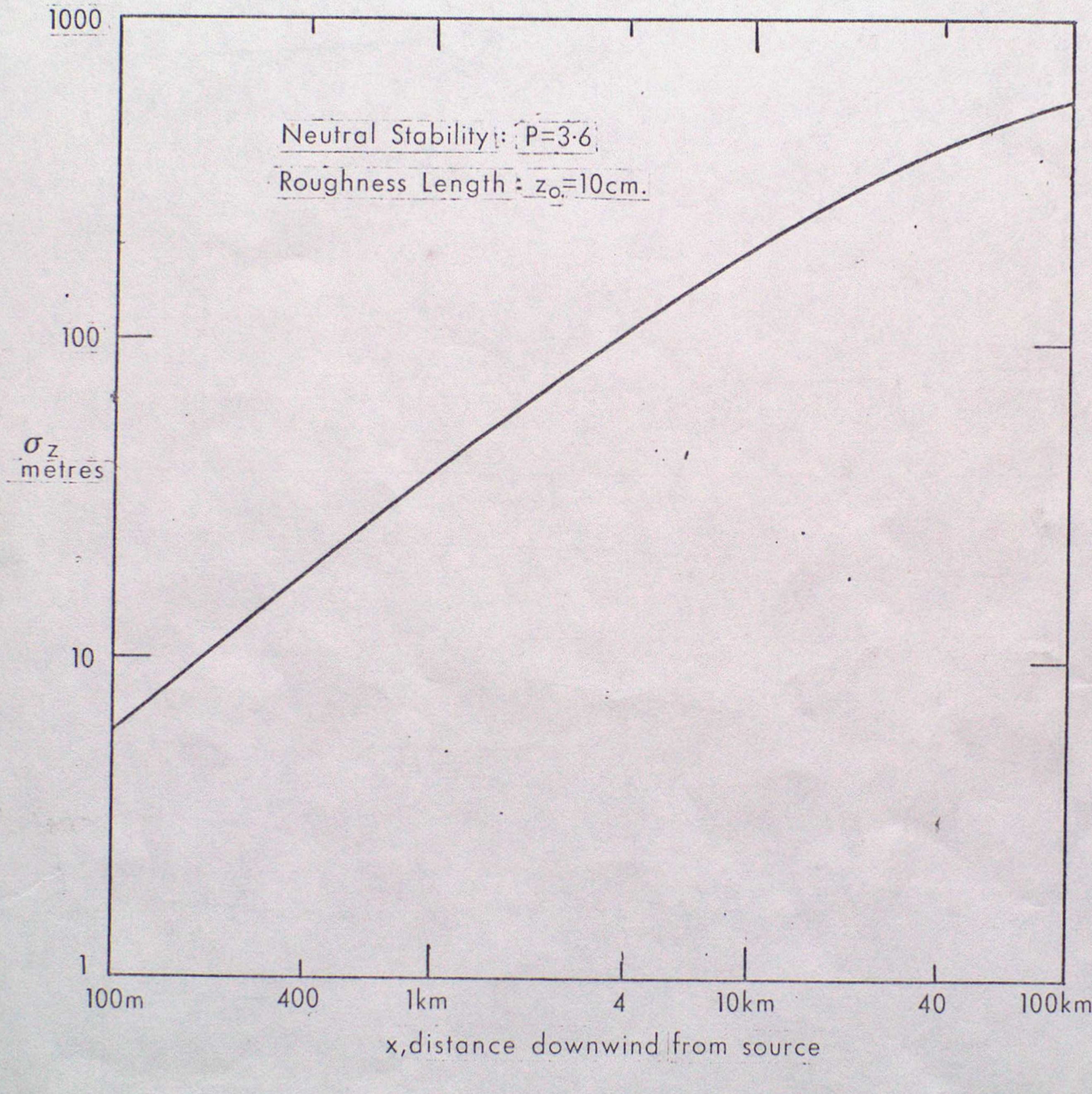
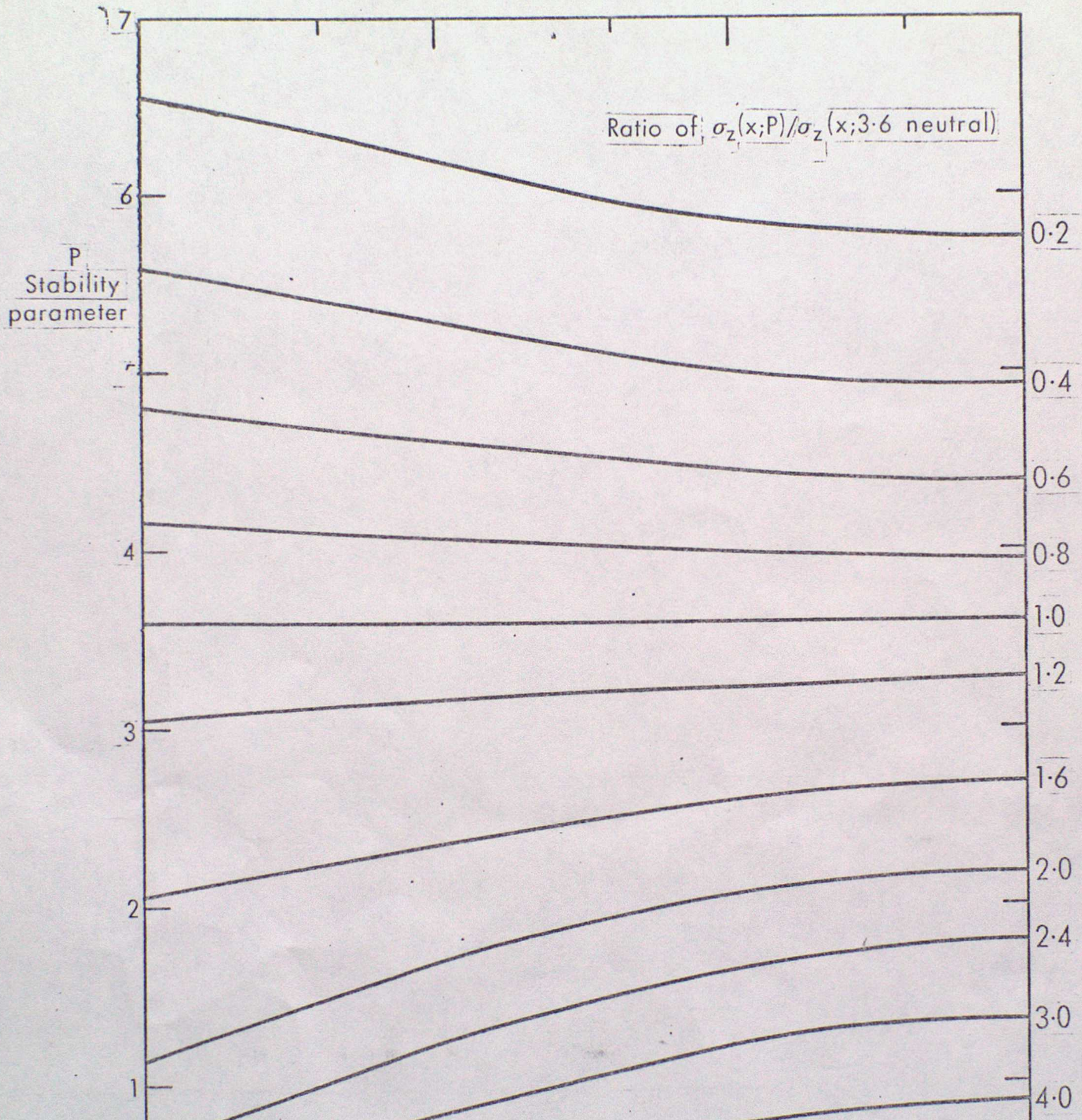


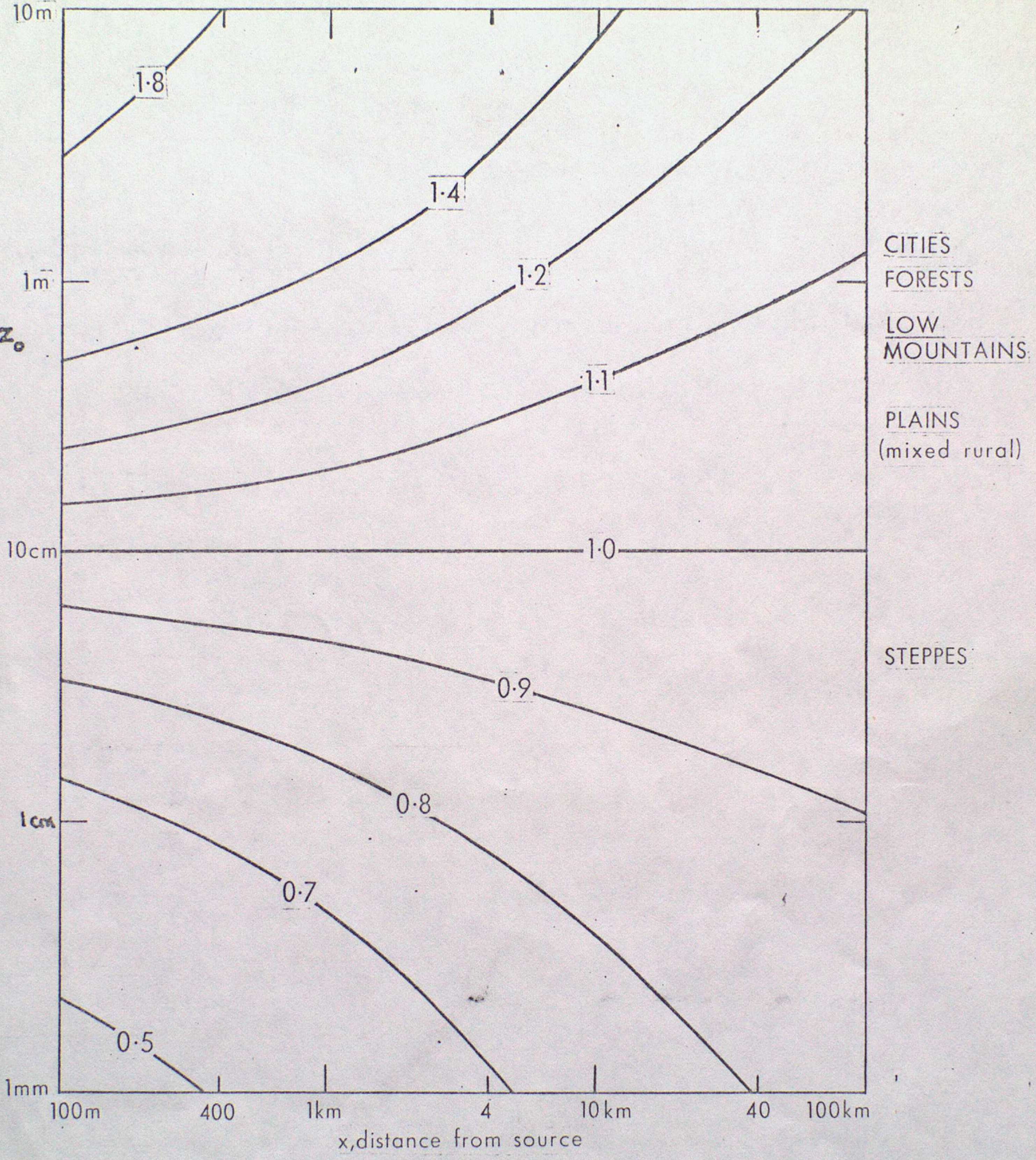
FIGURE 2

Carter DO ~~[illegible]~~ 1980 IL



Ratio of $\sigma_z(x;P)/\sigma_z(x;3{\cdot}6\ \text{neutral})$
P
Stability
parameter
0
1
2
3
4
6
7
0·2
0·4
0·6
0·8
1·0
1·2
1·6
2·0
2·4
3·0
4·0
6·0
100m
400
1km
4
10km
40
100km
x,distance downwind from source

FIGURE 3



CONTOURS OF
$\sigma_z(x;z_0)/\sigma_z(x;z_0=0{\cdot}1\text{m})$

These curves are virtually independent of heat flux

FIGURE 4

[illegible] 1980 13

The next Figure (Fig 2) gives the depth of the cloud, represented by the root-mean-square particle height σ_z

$$\sigma_z^2 = \int_0^\infty z^2 C(z)\,dz \Big/ \int_0^\infty C(z)\,dz$$

($C(z)$ = concentration at ht. z)

[Usually the total depth $h \approx 2.15\,\sigma_z$], in terms of x, the distance downwind from the source for neutral stability and for a surface roughness length of 10 cms.

For example, suppose we want h at $x = 4$ km downwind. According to Fig 2, $\sigma_z(4\text{ km}) \approx 100$ m.

But if we know that P is actually 2.5 and z_0 is 1 cm, we have to modify σ_z's. estimate.

Look at Fig. 3. This allows us to correct for P. It says that at $x = 4$ km & with $P = 2.5$

σ_z has to be multiplied by 1.6 (approx.)

i.e. For $P = 2.5$, $z_0 = 10$ cm, $x = 4$ km

$$\sigma_z = 100 \times 1.6 = 160 \text{ m.}$$

Finally, correcting for z_0, turn to Fig 4. It says that σ_z has to be multiplied by 0.82 at $x = 4$ km.

⇨ $\therefore\ h = 2.15 \times 160 \times 0.82 = 282$ metres.

Try an example for yourself!

This method only gives a general estimate of h.
It may not be too good in very unstable conditions because local topography may set off large roll-motions in the boundary layer which are scheme cannot simulate.

The method is useful however and very popular.

* N.B. It really should be applied only for sources close to the ground.

To estimate the Heat Flux H.

The following scheme is quite successful at estimating H.
It can be programmed for a programmable hand-held calculator like the Texas Instruments TI.59.

The basic formulae are given without proof or proper explanation:

A = solar latitude $= 23.3 \sin(0.9863(n-81))$

where n = Julian day number (Jan 1^{st} ⇒ $n=1$ etc).

Sine(Solar elevation) $= s = \sin\varphi \sin A + \cos\varphi \cos A \cos(15(t-12))$

where φ = latitude (geographical)

t = time of day in hours. (local Solar time)

Let $s_m = \sin\varphi \sin A + \cos\varphi \cos A$.

If the cloud amount $c \leq 6/8^{ths}$ then the nett incoming radiation R_n is given by $R_n = -45 + 102.5s + 742.2s^2 - 172.1s^3$ in slightly polluted, maritime areas like Southern England.

If $c \geq 7/8^{ths}$ then $R_n = 6.2 + 41.6s + 155s^2 + 99.6s^3$.

The available energy for sensible & latent heat fluxes is

$$\mathcal{R} = \left(1.2 - 0.4\frac{s}{s_m}\right) R_n$$

Then the sensible heat flux $H = \dfrac{r_a + r_{ST} - r_i}{(1+\frac{\bar{\Delta}}{\gamma})r_a + r_{ST}} \mathcal{R}$

(Monteith's formula)

where :-

r_a = aerodynamic resistence $= \dfrac{260}{u_{10}}$

r_i = climatological resistence $= \dfrac{\rho c_p}{\gamma} \dfrac{(e_w(T) - e_w(T_d))}{\mathcal{R}}$

ρ = air density
c_p = specific heat at constant pressure
T = air temperature
} $\rho c_p = 1305 - 4.3T$

$\gamma = 0.646 - 0.0006T$

$e_w(T) = 6.11 + 0.4T + 0.025T^2$

T_d = dew point temperature (given in standard met. observations)

$\Delta = \dfrac{d}{dT} e_w(T) = 0.4 + .05T$

$\bar{\Delta}$ = average between Δ at temperature T and Δ at the soil temperature T_s

where $T_s \approx T + a \dfrac{\mathcal{R}}{\rho c_p} r_a$

where $a = \left(\dfrac{r_{ST}}{1000}\right)(8.17 - 0.143T) - \left(\dfrac{r_{ST}}{1000}\right)^2 (16.7 - 0.515T)$

unless $r_{ST} \geq 400$ when $a = 1$.

r_{ST} = stomatal resistence

given in standard met. observations.

If W = present weather
0,1 no precip. or fog
2 precip in last hour
3 blowing snow
4 fog
5-9 precipitation.

If M = state of ground
0 surface dry
1 " moist
2 " wet
3-7 " snow, ice, etc covered,
8-9 " covered with loose dry snow

If $M = 2$ or $M = 3-9$ & $T < 0$, or $W \geq 5$ then $P = 3.6$
If $M = 8, 9$ and $T \geq 5$ then $P = 4.5$
Otherwise if $M = 3-9$ and $T \geq 0$
$u < 1.5$ then $P = 6.5$
$1.5 < u < 3$ then $P = 5.5$
$u > 3$ then $P = 4.5$

Otherwise if the number of days since it last rained more than 1mm is d and $d \geq 10$ days and $M = 0$ then $r_{ST} = 2000$

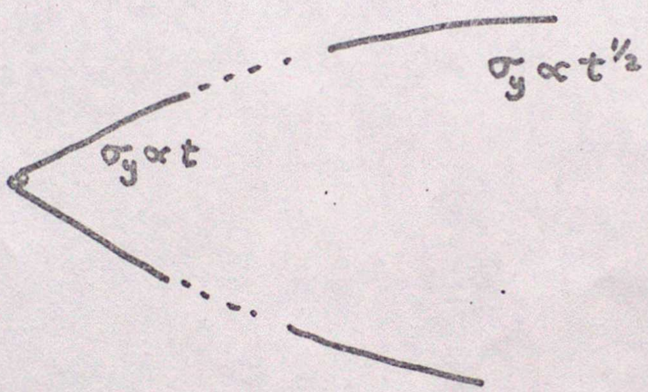
otherwise between Oct and March (winter) $r_{ST} = 80$

" April and Sept. (summer) $r_{ST} = 50$

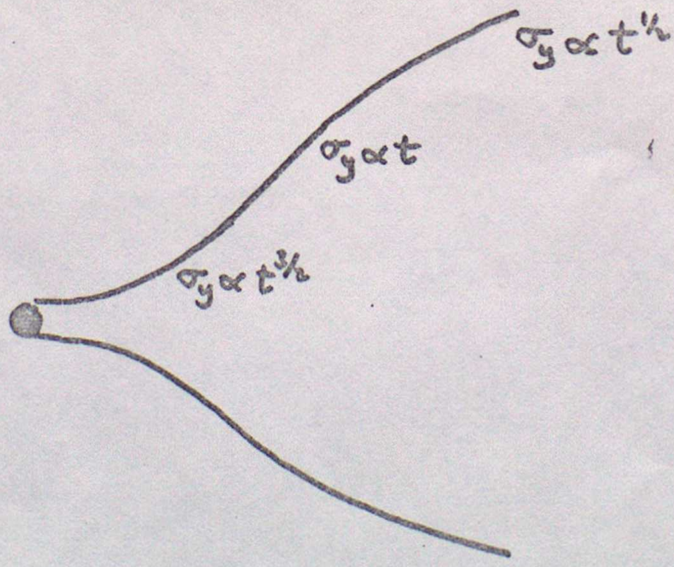
Fluctuating Plumes and Puffs

If material is released into the air almost instantaneously it forms a cloud or puff.

Puffs grow differently to the more continuous plume. The latter grows like this:

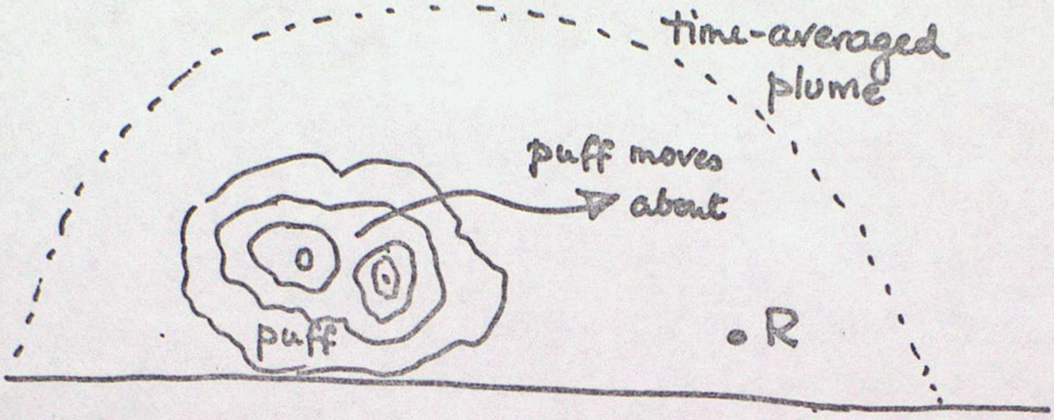


The former grows like this:

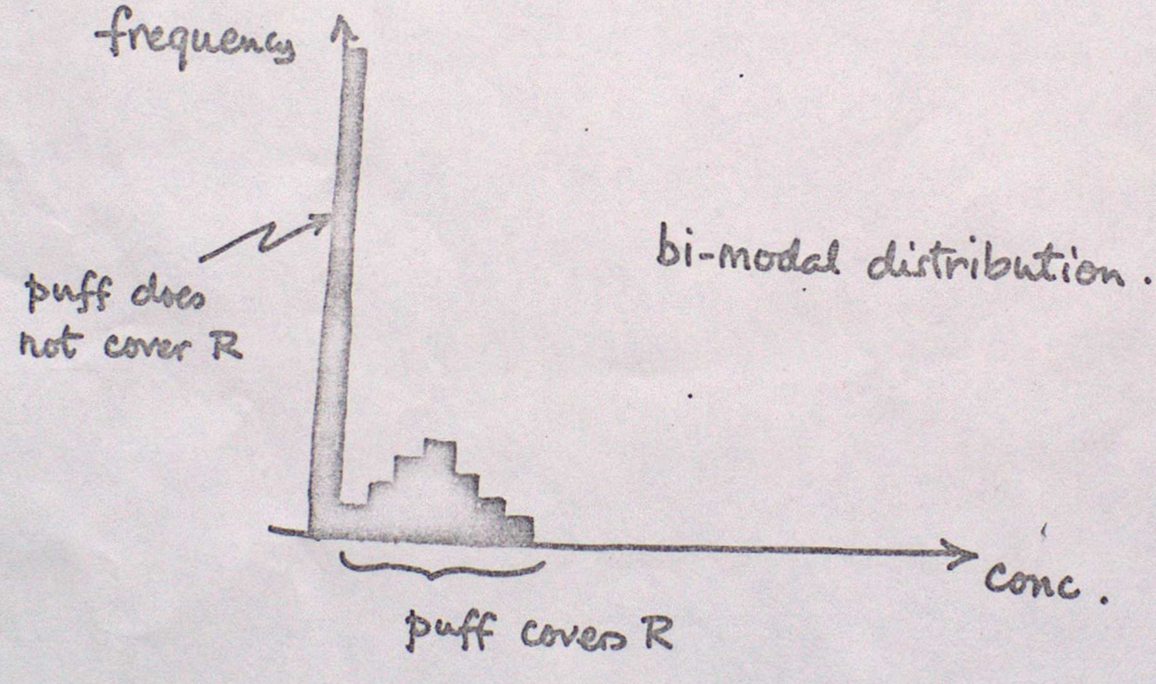


The reason is this: only eddies smaller than the puff are effective at making it grow in size. Bigger eddies just move the whole puff around. As the puff grows more and more eddies become effective (hence the acceleration) until <u>all</u> eddies become effective when it acts like the plume.

The instantaneous puff is always smaller than the time-averaged plume.



Consequently at any receptor R, sometimes the puff engulfs R and sometimes it does not



Mixing depth

The top of the mixing layer is often an effective lid to further vertical mixing.

h, or σ_z, is therefore limited in magnitude.

Clouds sitting on the top of a boundary layer can however enable pollution to escape.

Plume Rise Formula

Let Q = energy generation capacity of a power station (in megawatts)

Q_H = waste heat flowing out of the stack (in MW)

Roughly $Q_H = \frac{1}{7} Q$

Now by definition $Q_H = \rho c_p v_i (T_s - T_a) \times 10^{-6}$

where $\left.\begin{array}{l}\rho = \text{density of air} \\ c_p = \text{specific heat at constant pressure}\end{array}\right\} \rho c_p \approx 1290$

v_i = volume flux of gases leaving the stack

The stack often has 2 flue pipes each of diameter $d \approx 6$ metres, and the efflux velocity V_s is often about $16\ \mathrm{m s^{-1}}$ for a 1000 MW station.

Thus $v_i \equiv 2 \times \frac{\pi d^2}{4} \times V_s \approx 900$

for $Q = 1000$ MW.

T_s is the temperature of the stack gases

T_a " " " " " air

Usually for power stations $T_s - T_a \approx 100\,^\circ\mathrm{K}$

For such a power station:

$$Q_H = 1290 \times 900 \times 100 \times 10^{-6} = 116 \text{ MW}$$

There have been a great number of plume rise formulae derived. They often look very different. Numerically, the best are not so different.

In the U.S.A. the formulae due to G. Briggs are favoured. In the U.K. the following formula by Dr D.J. Moore of C.E.R.L. is often used:

Moore's formula

Plume rise Δh is given by :

$$\Delta h = 2.25\, Q_H^{1/4}\, x^{*3/4}\, \frac{f}{U^*}\left(\frac{T'}{110}\right)^{1/8}\left[1 + \frac{27d + 1.5x_m}{x^*} + \frac{54\, d\, x_m}{x^{*2}}\right]^{1/4}$$

where $x_m = \dfrac{U V_s T_a}{g(T_s - T_a)}$, $x_s = \dfrac{120\, U^*}{\sqrt{\Delta\theta^*}}$

$x_N = \min(4224,\ 1920 + 19.2 h_s)$

d = flue pipe diameter

V_s = efflux velocity (ms^{-1})

T_a = air temp (°K)

T_s = temp. of stack gases (°K)

g = 9.81 , acceleration due to gravity .

U = wind speed at stack height (ms^{-1})

h_s = stack height (m)

$U^* = \max[0.2, U]$ (ms^{-1})

$\Delta\theta$ = increase in potential temp of air over 100 m

= " " temperature " " " " + 0.98°

$\Delta\theta^* = \max[\Delta\theta, 0.08]$

$h^* = \min[h_s, 120]$

$f = 0.16 + 0.007 h^*$

or $= 1$ if $\dfrac{\Delta\theta}{U^2} > 0.0025$

$T' = \max(12, T_s - T_a)$

$x_T = \dfrac{x_s\, x_N}{(x_s^2 + x_N^2)^{1/2}}$, $x^* = \dfrac{x\, x_T}{(x^2 + x_T^2)^{1/2}}$

Example Calculate plume rise at $x = 1000$ metres downwind.

Suppose $V_s = 16\ ms^{-1}$ $\qquad T_a = 15°C = 288°K$

$Q_H = 116\ MW$ $\qquad T_s = 388°K \quad \therefore T' = 100$

$h_s = 150m$ $\qquad U = 5\ ms^{-1}$

$\Delta\theta = 0.5° \rightarrow \therefore f = 1$

Then $U^* = 5$, $\Delta\theta^* = 0.5$, $h^* = 120$, $T' = 100$

$x_N = 4224$

$x_S = 848.5$

$x_m = 23.5$

$\therefore x_T = 831.9$ and $x^* = 639.5$

Hence $\Delta h = 2.25 \times 116^{1/4} \times 639.5^{3/4} \times \frac{1}{5} \times \left(\frac{100}{110}\right)^{1/8} \left(1 + \frac{27 \times 6 + 1.5 \times 23.5}{639.5} + \frac{54 \times 6 \times 23.5}{639.5^2}\right)^{1/\ldots}$

$= 199$ metres