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Theory of mature tropical cyclones; A comparison between Kleinschmidt (1951) and Emanuel (1986)

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Abstract.

There exists a surprising degree of similarity between the steady-state models of mature tropical cyclones proposed by Kleinschmidt (1951) and Emanuel (1986) considering the debate there has been on this subject within this period. The authors both favour the thermodynamic disequilibrium between atmosphere and ocean as the cyclone energy source (which Emanuel calls an Air-Sea Interaction Instability theory[1]). Both also consider the cyclone as a thermodynamic energy cycle (specifically a Carnot heat engine in Emanuel's case) with a balance existing between the thermodynamic energy gained as air passes through the cyclone and the work done against frictional dissipation as air crosses the boundary layer. The authors both assume moist symmetric neutrality in the outflow structure and use this to obtain the slope of the outflow streamlines. In addition, although their calculations of an inflow layer structure are completely different, both authors obtain reasonable relations for the variation of azimuthal velocity with radius in this region. Kleinschmidt and Emanuel both develop expressions to calculate the central pressure depression at the top of the boundary layer and, although entirely different, these expressions give similar results for a 'typical' cyclone.

1.0 Introduction.

Tropical cyclones, which are also known as hurricanes and typhoons in some parts of the world, are some of the most spectacular and devastating features of the tropical atmosphere. Within the last ten years Emanuel has promoted the Air-Sea Interaction Instability theory as the energy source for steady-state tropical cyclones and proposed regarding steady-state cyclones as Carnot heat engines. The key assumption in both of these is

1. More recently renamed WISHE (Wind-induced surface heat exchange).

moist symmetric neutrality above a well mixed boundary layer. Some similar ideas have been expressed before Emanuel by authors other than Kleinschmidt (e.g. Malkus, 1960) but Kleinschmidt's model of a steady-state tropical cyclone bears considerable similarity to that of Emanuel. It may seem surprising that Kleinschmidt's work is comparable to some more modern thinking despite decades of debate on this subject, however, Kleinschmidt has been found to have contributed to other important concepts in modern day atmospheric dynamics e.g. PV-thinking (for a review see Thorpe 1993).

The aim of this work is to compare the theories developed by Kleinschmidt and Emanuel for mature tropical cyclones by examining two specific papers. Section 2 of this work compares the physical assumptions made when developing steady-state theoretical models of the cyclones. Section 3 considers the development and results of these models and Section 4 gives the conclusions of this work. Both papers consider the energy source of a cyclone and its energy budget. Both also consider the structure of the outflow and the inflow along the boundary layer. It should be noted that the papers also consider non-comparable aspects of tropical cyclones. In particular, Kleinschmidt also considers the mechanism of cyclogenesis.

2.0 Physical assumptions

The generally assumed structure and airflow of a mature tropical cyclone is shown in Fig. 1. This secondary circulation associated with the cyclone consists of inflow along the boundary layer into a region of enhanced convection surrounding the centre of the cyclone (known as the eyewall). The air ascends within convective cloud towers which are generally concentrated within the outwards sloping eyewall. Radial outflow occurs near the tropopause followed by a gentle subsidence of air at large radii.

To construct the simplest possible model, several assumptions are made about tropical cyclones (which both authors use) i.e. that they are axisymmetric and that hydrostatic balance and gradient wind balance apply. These (and the equation for the angular momentum per unit mass) may be written:

$$\alpha\frac{\partial p}{\partial z} = -g \text{ Hydrostatic balance} \tag{1}$$

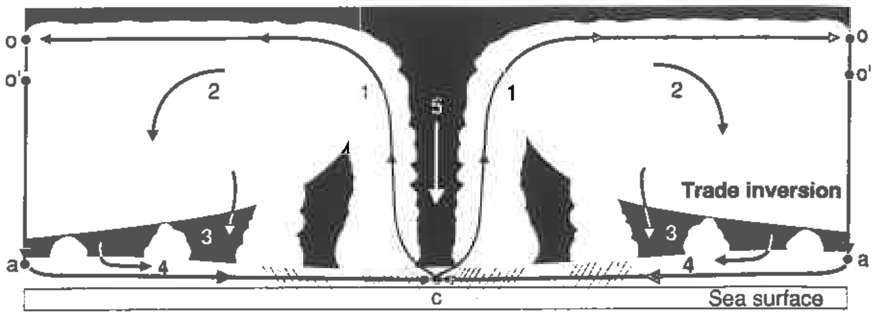

Fig. 1. schematic cross section of the secondary meridional circulation in a mature tropical cyclone. The cross section spans a width of about 1600km and a height of 15km. (After Emanuel 1988). Air spirals in towards the eye, (region 5) in the boundary or friction layer (the inflow, region 4), ascends along constant momentum surfaces in the eyewall cloud (the outflow, region 1), and slowly subsides and dries (in regions 2 and 3). The sea surface temperature may be considered constant.

$$\alpha\frac{\partial p}{\partial r} = \frac{v^2}{r} + fv \quad \text{Gradient wind balance} \tag{2}$$

$$M = rv + \frac{1}{2}f^2 r \quad \text{Angular momentum per unit mass} \tag{3}$$

where α = specific volume, p = pressure, z = height, g = acceleration due to gravity, r = radius from the cyclone centre, v = azimuthal velocity, f = Coriolis parameter and M = angular momentum per unit mass.

It is also generally assumed that the Coriolis parameter, f, can be considered constant. In addition, Emanuel states that he assumes that the flow above a well mixed boundary layer is inviscid and thermodynamically reversible. Kleinschmidt does not explicitly state these assumptions although they appear to have been assumed.

2.1 Energy source of a tropical cyclone.

The energy source of a tropical cyclone is a matter of some debate at this time with two current opposing theories. Firstly, the CISK (Conditional Instability of the Second Kind) hypothesis originally suggested by Charney and Eliassen (1964) holds that initial development results from the organized release of preexisting conditional instability and that tropical cyclone growth is the result of organized interaction between the cumulus scale and large-scale moisture convergence. An alternative source of the potential energy for the tropical cyclones is the Air-Sea Interaction Instability theory and this was advanced by Emanuel in the paper studied here. In this case the energy arises from the thermodynamic disequilibrium between the atmosphere and the underlying ocean yielding a potential for transfer of energy from sea to air even though the two media are usually at about the same temperature. Kleinschmidt's opinion in this matter can be seen by comparing quotes by the two authors:

Kleinschmidt:

'The heat removed from the sea by the storm is the basic energy source of the typhoon. In comparison to it, the latent heat of water vapour, which the air carries with it from the outside, plays no more than a secondary role.'

Emanuel:

'It is our hypothesis that tropical cyclones are developed and maintained against dissipation *entirely* by self-induced anomalous fluxes of moist enthalpy from the sea surface with virtually no contribution from preexisting CAPE.'

Thus Kleinschmidt regards the cyclone energy source as energy fluxes from the ocean as opposed to energy drawn in by the cyclone from it's environment. In this sense he favours a form of Emanuel's Air-Sea Interaction Instability opposed to a CISK type approach.

2.2 Structure of the outflow.

The authors write essentially identical statements describing the basic physical assumption they make in order to develop an outflow structure.

Kleinschmidt:

'Uniform moist potential temperature and uniform absolute momentum must prevail on every streamline in the outflow.'

Emanuel:

'Above the boundary layer saturated equivalent potential temperature (θ_e^*) is uniform along surfaces of constant angular momentum.'

Thus both Kleinschmidt and Emanuel are assuming moist symmetric neutrality in the outflow i.e. that lines of constant angular momentum and moist potential temperature are parallel. Kleinschmidt's term 'moist potential temperature' is identical to equivalent potential temperature (θ_e). Emanuel's θ_e* is equal in value to θ_e where the angular momentum surface intersects the boundary layer (within which θ_e is assumed to be well mixed in the vertical). Moist symmetric neutrality means (as explained by Emanuel) that the cyclone vortex is 'neutral to slantwise moist convection' and this implies that 'boundary layer air is neutrally buoyant when lifted along surfaces of constant angular momentum'.

To describe the outflow structure the slope of the outflow streamlines must be derived. Moist symmetric neutrality implies that the outflow lines may be defined equivalently by $M(\theta_e)$ or $\theta_e(M)$ surfaces although in fact Emanuel uses moist entropy (s*) instead of θ_e.

2.3 Structure of the inflow layer.

To complete the cyclone structure $\theta_e(r)$ or $M(r)$ must be found along the top of the boundary layer. Kleinschmidt obtains $v(r)$ along the top of the boundary layer by performing a momentum balance in this region and hence can obtain $M(r)$. The only assumption made about the boundary layer is that the density is constant throughout the entire layer. Emanuel determines $\theta_e(r)$ semi-empirically in the boundary layer by dividing the boundary layer into three regions, basically the eye, the eyewall and an outer region. The eye region he states to be mostly unsaturated and probably mechanically controlled by the flow outside the eye. He assumes that the distribution of θ_e in the eyewall region is dominated by cyclone scale fluxes while the region outside the radius of maximum winds is assumed to be controlled more by turbulent fluxes. For the two inner regions Emanuel uses Ooyama's (1969) formulation of the boundary layer and in the outer region he assumes constant surface relative humidity.

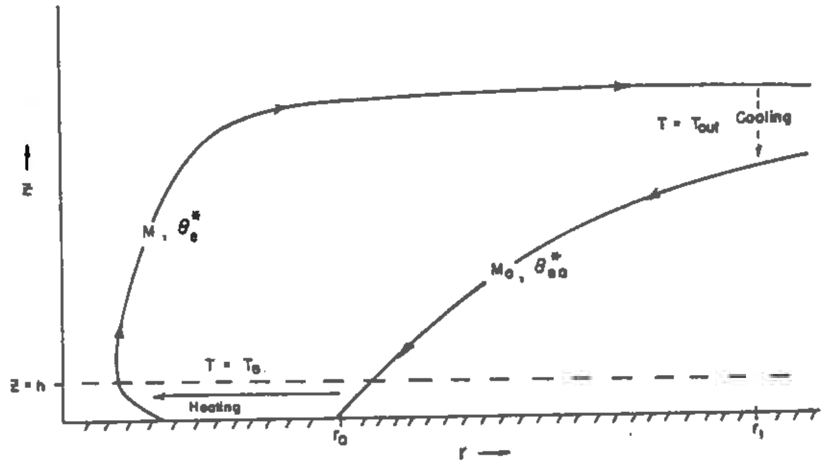

Fig. 2. The tropical cyclone as a Carnot heat engine. M is angular momentum with M_o the ambient value. $\theta_e{}^*$ is the saturated equivalent potential temperature with $\theta_{ea}{}^*$ the value at r_o. T_{out} is the outflow temperature and T_B is the absolute temperature at the top of the boundary layer. Latent and sensible heat are extracted from the ocean at the temperature T_B and ultimately given up at the outflow temperature T_{out}. The thermodynamic efficiency of the heat engine, ε, is therefore $(T_B - T_{out})/T_B$.

2.4 The energy budget

Both Kleinschmidt and Emanuel consider the tropical cyclone to contain a thermodynamic energy cycle and in Emanuel's case this is specifically referred to as a Carnot heat engine. Both authors assume that energy is gained by an air parcel from the ocean surface (between points a and c in Fig. 1) and that there is some energy loss through radiation as the parcel leaves the outflow (between points o and o'). This results on a net thermodynamic energy gain. Fig. 2 (taken from Emanuel's paper) illustrates Emanuel's Carnot cycle.

Both authors derive an energy budget by following a parcel of air around this secondary circulation. Kleinschmidt equates the thermodynamic energy gained, E, with the loss in kinetic energy as the inflow air crosses the boundary layer. Emanuel similarly equates a net heating ΔQ with the work done against frictional dissipation.

3.0 Development of a theoretical steady state model.

Both authors develop a steady-state analytical model based on thermodynamic disequilibrium as the energy source. The comparable aspects of their papers are their initial development of the outflow structure, their inflow structures, and their model of a tropical cyclone as a thermodynamic energy cycle. Kleinschmidt quotes a value for the central pressure depression for a 'typical' cyclone (measured at the top of the boundary layer) which he calculates using an equation he develops when considering the inflow structure. Emanuel obtains an expression for the central pressure as a function of the increase in surface ambient humidity between the core and the ambient environment. This expression can be obtained both from his work on the outflow structure and his energy budget work. This Section will compare the authors' inflow and outflow structures and examine their work on the energy budget of a tropical cyclone. It will also follow the authors' work towards a central surface pressure equation. The analysis will concentrate on the less well known work of Kleinschmidt and provide only brief notes on that of Emanuel.

3.1 Structure of the outflow.

<u>A summary of Kleinschmidt's approach:</u>

The equation for gradient wind balance (2) and that of hydrostatic balance (1) are rewritten using $\rho = 1/\alpha$ in place of α. Equation (2) is differentiated with respect to z and (1) with respect to r, giving on substitution

$$\left(f+\frac{2v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\rho^2}\left(\frac{\partial p}{\partial z}\frac{\partial \rho}{\partial r} - \frac{\partial p}{\partial r}\frac{\partial \rho}{\partial z}\right) \qquad (4)$$

It is assumed that in steady state uniform moist potential temperature and uniform momentum exist on every streamline which implies that the momentum is a function of moist potential temperature only. It is also assumed that density is a function of pressure and moist potential temperature only i.e.

$$M = M(\theta_e) \text{ and}$$

$$\rho = \rho(p, \theta_e)$$

$(\partial\rho)/(\partial r)$ and $(\partial\rho)/(\partial z)$ are calculated and substituted into (4) giving

$$\left(f+\frac{2v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\rho^2}\frac{\partial \rho}{\partial \theta_e}\left(\frac{\partial p}{\partial z}\frac{\partial \theta_e}{\partial r} - \frac{\partial p}{\partial r}\frac{\partial \theta_e}{\partial z}\right) \qquad (5)$$

and by using (1) again the following equation for $\tan\beta$, where β is the angle of inclination of the moist isentropes, is obtained

$$\tan\beta = -\frac{\frac{\partial\theta_e}{\partial r}}{\frac{\partial\theta_e}{\partial z}} = \frac{\rho}{g\frac{\partial\theta_e}{\partial z}\frac{\partial\rho}{\partial\theta_e}}\left(f+\frac{2v}{r}\right)\frac{\partial v}{\partial z} - \left.\frac{\partial z}{\partial r}\right|_p . \tag{6}$$

Now, from the equation for angular momentum per unit mass (3),

$$v = \frac{M}{r} - \frac{fr}{2} \tag{7}$$

which implies

$$\frac{\partial v}{\partial z} = \frac{1}{r}\frac{\partial M}{\partial z}, \tag{8}$$

or since $M = M(\theta_e)$,

$$\frac{\partial v}{\partial z} = \frac{1}{r}\frac{\partial M}{\partial\theta_e}\frac{\partial\theta_e}{\partial z} \tag{9}$$

From the gas law $p = \rho RT$,

$$\frac{1}{\rho}\frac{\partial\rho}{\partial\theta_e} = -\frac{1}{T}\left.\frac{\partial T}{\partial\theta_e}\right|_p \tag{10}$$

which assuming that θ_e is a linear function of temperature for constant pressure, i.e. $\theta_e = a(p) + b(p)T$[1], implies

$$\frac{1}{\rho}\frac{\partial\rho}{\partial\theta_e} = -\frac{1}{Tb}. \tag{11}$$

By substituting (7), (9) and (11) into (6) the following expression for $\tan\beta$ is obtained

$$\tan\beta = -\frac{2Tb}{gr^3}M\frac{\partial M}{\partial\theta_e} - \left.\frac{\partial z}{\partial r}\right|_p \tag{12}$$

1. N.B. This does not appear to be a very accurate assumption.

This can be integrated over r to obtain an expression for the height of a streamline (or momentum surface) as a function of radius,

$$z(r) = \frac{Tb}{gr^2} M \frac{\partial M}{\partial \theta_e} + H. \tag{13}$$

where H is an integration constant.

<u>A brief summary of Emanuel's approach:</u>

The equation for hydrostatic balance (1) is substituted into the equation for gradient wind balance (2) giving

$$g\left(\frac{\partial z}{\partial p}\right)\bigg|_p = \frac{M^2}{r^3} - \frac{1}{4} f^2 r. \tag{14}$$

Equation (1) may be alternatively expressed as

$$g\left(\frac{\partial z}{\partial p}\right)\bigg|_r = -\alpha \tag{15}$$

and so by eliminating z between (14) and (15) the following version of the thermal wind relation in radial coordinates may be obtained

$$\frac{1}{r^3}\left(\frac{\partial}{\partial p} M^2\right)\bigg|_r = -\left(\frac{\partial \alpha}{\partial r}\right)\bigg|_p \tag{16}$$

The assumptions are made that momentum is a function of moist entropy (s^*) only and that α is a function of pressure and moist entropy only i.e.

$$M = M(s^*)$$

$$\alpha = \alpha(p, s^*).$$

By using these assumptions, the gas law ($p\alpha = RT$) and the first law of thermodynamics, the following expression for the slope of the M (or s^*) surfaces may finally be derived:

$$\left.\frac{\partial r}{\partial p}\right|_M = \frac{r^3}{2M}\left[\frac{d}{dM}(s^*)\right]\left(\frac{\partial T}{\partial p}\right)\Big|_{s^*} \tag{17}$$

If this equation is integrated upward along M (or s^*) surfaces the result is

$$r^2|_M = \frac{M}{-\left[\frac{d}{dM}s^*\right][T - T_{out}(s^*)]}, \tag{18}$$

where T_{out} is an integration constant interpreted as the 'outflow temperature'. This is defined as the temperature that the air has along M surfaces as they flare out to large radii. See Fig. 2.

Emanuel continues this work to obtain the following pressure distribution for the top of the boundary layer:

$$-\frac{T_B - \bar{T}_{out}}{T_B} ln\left(\frac{\theta_e}{\theta_{ea}}\right) = ln\left(\frac{\pi}{\pi_a}\right) + \frac{1}{2} r \frac{\partial}{\partial r} ln\pi + \frac{1}{4}\frac{f^2}{C_p T_B}(r^2 - r_0^2) \text{ at } z = h, \tag{19}$$

where π is the Exner function $\pi = (p/p_0)^{R/C_p}$, π_a is the ambient value of π at $z = h$ and $\bar{T}_{out}$ is an average outflow temperature weighted with the saturated moist entropy of the outflow angular momentum surfaces. θ_e is equivalent potential temperature and θ_{ea} its value at r_o, T_B is the absolute temperature at the top of the boundary layer.

Emanuel also derives an expression for the geometric area covered by the storm (not shown).

To conclude, both authors use the same basic assumptions and equations to derive an expression for the outflow streamline. Kleinschmidt derives the slope of the moist isentropes whereas Emanuel derives the slope of the momentum surfaces. These expressions (12 and 17) should be identical although this has not been explicitly proven. Both authors then proceed to integrate their equations (although over different variables). Emanuel alone continues this work to obtain a pressure distribution and the geometric area of the storm.

3.2 Structure of the inflow layer.

The approaches to and comparable results from the theories developed by the authors for the inflow structure are markedly different. Hence a comparison with an analytic model of the wind (and pressure) profiles in hurricanes (Holland, 1980) is also included to examine the possible validity of both authors' results.

<u>A summary of Kleinschmidt's approach:</u>

A momentum balance is assumed for a cylindrical ring-shaped section of the inflow layer of mean radius r and height h (the top of the boundary layer). The mass flow through a vertical side of the ring is

$$2\pi\rho r\int_0^h u\,dz$$

where u is the radial windspeed. From this the rate of mass increase in the ring of width Δr may be calculated:

$$2\pi\rho\Delta r\frac{\partial}{\partial r}\int_0^h ru\,dz.$$

The mass of the ring must be constant thus mass must flow out through the top of the ring at the same rate as it is gained through the sides. The rate of momentum loss due to mass loss through the top of the ring is calculated using the definition of angular momentum (3), and is given by

$$\left(2\pi\rho\Delta r\frac{\partial}{\partial r}\int_o^h r^2u\,(v+\frac{f}{2}r)\,dz\right)-2\pi\rho r\Delta r\,(V+\frac{f}{2}r)\,\frac{\partial}{\partial r}\int_o^h ru\,dz, \tag{20}$$

where v is the azimuthal velocity and V is the gradient wind at the top of the boundary layer. Note that an additional r has been included in the second term *c.f.* the equation in Kleinschmidt's paper (38).

The loss of momentum due to friction per unit volume is

$$-r\frac{d\tau}{dz},$$

where τ is the shear stress on the ocean surface in the direction of the gradient wind.

Integrating over the ring, the loss in momentum due to friction is

$$-2\pi\Delta r r^2 \int_o^h \left(\frac{d\tau}{dz}\right) dz = 2\pi\Delta r r^2 \tau_B \tag{21}$$

where τ_B is the component of the surface shear stress perpendicular to the radius. The shear stress at the lid is considered negligible.

Equations (20) and (21) are the basis for the momentum balance. The derivation following this equation is tedious and is not detailed here. It incorporates work related to the methods developed by Prandtl in his ideas on fluid dynamics. The final result is the following equation which Kleinschmidt calls 'the law of approximation for the gradient wind at the upper limit of the friction layer',

$$V = C\left(\frac{r_u^{\mu}}{r^{\mu-1}} - r\right). \tag{22}$$

r_u is the outer radius of the cyclone, defined as 'the location where the gradient wind no longer blows'. C and μ are predicted functions of several variables and are approximately constants outside the eye region. μ is approximately 2 which implies $V \sim 1/r$ outside the eye.

A brief summary of Emanuel's approach:

Emanuel's approach to the boundary layer structure is far more complex than that of Kleinschmidt's and is mainly concerned with determining the radial distribution of θ_e in the boundary layer. Using this he eventually obtains a complete two-dimensional structure for the steady-state tropical cyclone. To summarise, solutions for the pressure distribution (in terms of $\ln\pi$) are derived under the conditions that a calculated distribution of θ_e applies inside the cyclone core and that the outer region of the cyclone is characterized by constant surface relative humidity. In the outer region the result for the pressure distribution implies that, just outside the radius of maximum winds,

$$V \sim \frac{1}{r^{\beta}} \tag{23}$$

where β is a function of ε, the thermodynamic efficiency, q_a^*, the saturated ambient mixing ratio, RH_{as}, the ambient relative humidity and T_s, the temperature. The later three quantities are all measured at the top of the boundary layer.

Emanuel claims that for typical values of sea surface temperature and tropopause temperatures $\beta \approx 0.5$. He also claims that this implies that (23) is therefore 'close to the observed minus one-half power law typical of tropical cyclones'.

To summarise, Kleinschmidt and Emanuel both obtain relations for the variation with radius of the gradient wind at the top of the boundary layer.

Kleinschmidt: $V \sim \frac{1}{r^{\mu - 1}}$ with $\mu \approx 2$.

Emanuel: $V \sim \frac{1}{r^{\beta}}$ with $\beta \approx 0.5$.

There appears to exist a conflict in results which may perhaps be resolved by obtaining a third opinion. Holland (1980) develops an analytic model of the wind (and pressure) profiles in cyclones. He finds that the gradient wind may be expressed as

$$V = \left[AB\,(p_a - p_c)\,exp\frac{\left(-\frac{A}{r^B}\right)}{\rho r^B} + \frac{r^2 f^2}{4} \right]^{1/2} - \frac{rf}{2} \tag{24}$$

where p_a is ambient pressure, p_c is central pressure and A and B are scaling parameters. This implies that, in the region of maximum winds,

$$V \sim \left[\frac{1}{r^B} \right]^{1/2}$$

Now from physical reasoning B can be shown to lie between 1 and 2.5 and from a climatological approach between 1.5 and 2.5. Using the former B range in the above equation implies

$$V \sim \frac{1}{r^{0.5 to 1.25}}.$$

Interestingly this includes both Kleinschmidt's and Emanuel's predictions. It is noteworthy though that Emanuel's β parameter, as calculated for a 'typical' cyclone, is slightly outside the range of possible powers (to which r is raised) if Holland's B range is instead chosen from his climatological predictions. Hence, although the approaches to and results from the authors' inflow structures are very different, both find that $V \sim 1/r^{power}$. The powers suggested by each author are different but both are (just) encompassed as possible values by the conclusions of a third author.

3.3 The energy budget.

A summary of Kleinschmidt's approach:

The net thermodynamic energy gain, E, as a unit mass air parcel travels around the secondary circulation is calculated graphically from a tephigram by measuring the area between the environmental temperature curve outside the cyclone and the moist adiabat of the inner-most outflow surface. The azimuthal velocity, v_i', that an inflowing air parcel would have on reaching the inner radius of the eyewall, r_i, from the outer radius of the cyclone, r_u, in the absence of friction is calculated from momentum conservation,

$$v_i' = \frac{f}{2}\left(\frac{r_u}{r_i} - r_i\right). \qquad (25)$$

In the presence of friction a fraction q of this velocity exists as air reaching the eyewall leaves the boundary layer. Thus the loss of kinetic energy by a unit mass of air due to friction is

$$\frac{1}{2}v_i'(1-q^2),$$

or from (25)

$$\frac{f^2}{8}\left(\frac{r_u^2}{r_i}-r_i\right)^2(1-q^2).$$

It is assumed that this is the only energy loss, and that the work done to change the angular momentum (conserved in the outflow) back to its ambient value at large radii, is negligible (see Fig. 2).

Hence

$$\frac{f^2}{8}\left(\frac{r_u^2}{r_i}-r_i\right)^2(1-q^2) = E. \tag{26}$$

Assuming $r_i \ll r_u$,

$$r_u^4 = \frac{8Er_i^2}{f^2(1-q^2)}. \tag{27}$$

and since the maximum wind strength $v_i = qv_i'$ then from (25) and (27)

$$V_i^2 = 2E\frac{q^2}{1-q^2}. \tag{28}$$

The remaining equation to complete this analysis is taken from the outflow structure. If r_s is the radius at which a momentum surface of the outflow leaves the boundary layer (at height h_s) then from (13)

$$r_s^2 = -\frac{Tb}{g(H-h_s)}M\frac{dM}{d\theta_e}. \tag{29}$$

The main dimensions of a cyclone can now be determined from (27) to (29) i.e. the outer circumference, the mean radius of the ring wall and the maximum wind strength. However, Kleinschmidt is unable to complete his cyclone structure since he requires $M(\theta_e)$ to evaluate his outflow streamlines which he has calculated as θ_e surfaces (12). He declares that if this (or specifically $M(dM/(d\theta_e)))$ is known then every cyclone could be completely calculated from external data.

A brief summary of Emanuel's approach:

The balance between the heating, ΔQ, and the work done against frictional dissipation in the inflow and outflow is written symbolically as

$$\Delta Q = W_{PBL} + W_0, \tag{30}$$

where W_{PBL} is the work done in the boundary layer and W_0 is the work done in the outflow. W_0 is proportional to the kinetic energy required to bring the angular momentum of the outflow, M, back to its ambient value, M_0 - see Fig. 2. ΔQ is expressed as the sum of the latent heat acquired by the inflow from the ocean and the heat lost by radiational cooling at the outflow temperature.

From expressions for W_o and ΔQ an expression for W_{PBL} can be found,

$$W_{PBL} = C_p T_B \varepsilon ln\frac{\theta_e}{\theta_{ea}} + \frac{1}{2}frv - \frac{1}{4}f^2(r_0^2 - r^2).$$

The pressure distribution in the boundary layer can then be evaluated *via* Bernoulli's equation. When integrated inward from r_o at constant temperature Bernoulli's equation at $z = 0$ may be written

$$\frac{1}{2}v^2 + c_p T_B ln\pi + W_{PBL} = 0. \tag{31}$$

When the expression obtained above for W_{PBL} is substituted into (30) and $\alpha(\partial p/(\partial r))$ is substituted for the sum $v^2 + frv$, (from (2)), then an expression for the pressure distribution can be obtained:

$$ln\pi = -\frac{1}{2}r\frac{\partial}{\partial r}ln\pi - \varepsilon ln\frac{\theta_e}{\theta_{ea}} + \frac{1}{4}\frac{f^2}{C_p T_B}(r_0^2 - r^2), \text{at } z = 0 \tag{32}$$

where ε is the thermodynamic efficiency, $\varepsilon = (T_B - \overline{T}_{out})/T_B$ with $\overline{T}_{out}$ as defined in (19).

This expression for $\ln\pi$ is identical to that derived by Emanuel from his work on the outflow structure (19) implying that a steady state tropical cyclone may indeed be considered as a simple Carnot heat engine.

3.4 Development of a surface central pressure equation.

The surface central pressure of a tropical cyclone is often used as a measure of the intensity of the storm and as such it is a very important aspect of cyclone modelling. Appendix 1 lists the parameters for which values must be provided for each author's steady-state model in order to calculate the central pressure depression at the top of the boundary layer.

<u>A summary of Kleinschmidt's approach:</u>

Determination of the surface central pressure requires the use of (22) for the wind distribution at the upper limit of the boundary layer. The constants C and μ are calculated (how is not clear from Kleinschmidt's paper). This equation can then be substituted into the gradient wind balance equation (or a combination of the gradient wind balance and hydrostatic balance equations) to obtain an expression, which by integration over r, provides a surface central pressure depression calculated at the upper limit of the boundary layer. Kleinschmidt calculates a pressure depression of 24mb for a 'typical' cyclone and by following his approach similar values could be calculated.

An estimation of the values Kleinschmidt uses for C and μ was attempted using results that Kleinschmidt states he obtains from (22). Kleinschmidt sets the inner eyewall radius, r_i, to 15km and the outer eyewall radius to 50km. From (22) he finds $q = 5/6$ on entering the eyewall and this drops steadily to 2/3 within the eyewall. From (25) the azimuthal windspeeds at the inner and outer radii of the eyewall can be calculated (53.04 and 18.94ms^{-1} respectively). By substituting these values into (22) and solving the two simultaneous equations the following values for C and μ were obtained:

$$C = 2.856*10^{-5}\text{s}^{-1} \text{ and } \mu = 0.8$$

Equation (22) was substituted into (2) which was then integrated over r between r_u (the outer radius of the cyclone) and r_i. The value of r_u used was that calculated from (27) by Kleinschmidt i.e. 219km. The central pressure depression calculated depends on the density ρ which is unspecified by Kleinschmidt. However, using $\rho = 1.2\text{kgm}^{-3}$ a central pressure depression of 21mb is obtained which is in reasonable agreement with Kleinschmdit's value especially considering the lack of calculation equipment available to him in 1951.

<u>A brief summary of Emanuel's approach:</u>

From (19), at the storm's centre,

$$ln\frac{\pi_c}{\pi_a} = -\frac{T_B - \overline{T}_{out}}{T_B}ln\frac{\theta_{ec}}{\theta_{ea}} + \frac{1}{4}\frac{f^2 r_0^2}{C_p T_B}$$

where the subscript c denotes values at the storm's centre.

Expanding the natural logarithms and using the definition of π an expression for the pressure deficit at $z = h$ can be obtained:

$$p'_c \approx -\frac{C_p}{R}p_a\frac{T_B - \overline{T}_{out}}{T_B}\frac{\theta^*_{ec}{}'}{\theta_{ea}},$$

where primes denote departure from the base state and * indicates the saturation value.

Emanuel uses 'typical' cyclone values for θ_{ea}, T_B, $\overline{T}_{out}$ and p_0 (the ambient surface pressure at $z = 0$) to obtain a relation between p'_{cs} (the surface central pressure depression at $z = 0$) and $\theta^*_{ec}{}'$ by assuming

$$\frac{p'_c}{p_a} \approx \frac{p'_{cs}}{p_o}.$$

For comparison with Kleinschmidt, who obtains a surface central pressure depression at the top of the boundary layer ($z = h$), a 'typical' value for p_a (the ambient pressure at $z = h$) must instead be used. Hence assuming $\theta_{ea} = 345\text{K}$, $T_B = 295\text{K}$ and $\overline{T}_{out} = 200\text{K}$ (from Emanuel) and $p_a = 900\text{mb}$ then

$$p'_c \approx -2.9\theta^*_{ec}'.$$

Kleinschmidt's paper includes a graph of pressure *vs.* temperature on which is plotted both the environmental air sounding and the $\theta_e = 75°C$ moist adiabat which he considers to be that of the inner-most outflow streamline. From this graph it can be estimated that the above θ^*_{ec}' is approximately 14K for Kleinschmidt's 'typical' cyclone. This value is obtained by subtracting the environmental θ^*_e at 900mb from $\theta_e = 75°C$. Emanuel's expression would then imply a central pressure depression at the top of the boundary layer of 41mb for comparison with Kleinschmidt's own value of 24mb.

4.0 Conclusions

There appears to be a surprising degree of similarity between the steady state models of mature tropical cyclones proposed by Kleinschmidt in 1951 and Emanuel in 1986. The authors agree that the energy source of tropical cyclones is the thermodynamic disequilibrium between the atmosphere and ocean (Emanuel describes this as an Air-Sea Interaction Instability theory) and declare that the energy arising from preexisting conditional instability (CISK theory originating in 1964) is negligible. Both authors assume moist symmetric neutrality in the outflow and the initial work described by the authors for determining the outflow structure follows very similar lines. The authors derive expressions for the slope of the outflow streamlines using the same basic equations and assumptions. In Kleinschmidt's case the slope is that of the moist isentropes whereas Emanuel derives the slope of the momentum surfaces. The authors' use quite different approaches to obtain an inflow layer structure, however, both find that the gradient wind at the top of the boundary layer is related to radius by an expression of the form $v \sim 1/r^{power}$. Each author suggests a different power although both values are found to be reasonable. Both Kleinschmidt and Emanuel state that there exists an energy balance between the thermodynamic energy gained as air passes through the cyclone and the work done by the air against frictional dissipation although their development of this energy balance differs. Kleinschmidt equates this energy gain with the kinetic energy lost by the inflow air due to friction. Emanuel uses the thermodynamic energy gain and the work done in the outflow to obtain an expression for the work done in the boundary layer. Finally, both authors obtain expressions from which the central pressure depression at the top of the boundary layer can be calculated. These differ entirely in formation but can be compared for a 'typical' cyclone.

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Appendix 1

Externally provided parameters required to calculate the central surface pressure depression.

The tables below list the parameters for which values must be provided in order to calculate the central pressure depression at the top of the boundary layer for each author's steady-state model. The values used are included.

Kleinschmidt:

Symbol	Description	Value used
ϕ	Latitude	20°
f	Coriolis parameter	$1.5*10^{-5}$
r_i	Inner eyewall radius	15 km
r_u	Outer eyewall radius	50 km
q	Fraction of air velocity remaining when air which has travelled inwards along the top of the boundary layer is inside the eyewall	2/3
θ_e	Equivalent potential temperature of the moist adiabat along the inner-most outflow streamline	75°C
E	Net thermodynamic energy gain as a unit mass air parcel travels around the secondary circulation. Calculated graphically from a tephigram from the area between the environmental temperature curve and the moist adiabat of the inner-most outflow surface	1770 m^2s^{-2}
ρ	Air density. Value used not given in Kleinschmidt's paper	1.2 Kgm^{-3}
n	A parameter which rises with the vorticity. Found in the equations for C and μ (not given)	O(10)

Emanuel:

Symbol	Description	Value used
θ_{ea}	Ambient equivalent potential temperature at the top of the boundary layer	345K
T_B	Absolute temperature at the top of the boundary layer	295K
$\bar{T}_{out}$	Average outflow temperature weighted with the saturated moist entropy of the outflow angular momentum surfaces	200K
$\theta^*_{ec}{}'$	Saturated equivalent potential temperature perturbation at the centre of the cyclone along the top of the boundary layer. An estimated value was taken from Kleinschmidt's work for comparison with Emanuel's	14K
p_a	Ambient pressure. Used to calculate the central pressure depression at the top of the boundary layer	900mb

Appendix 2

Included below are the original German Kleinschmidt paper and it's English translation. For ease of understanding several points are noteworthy. The translator has translated as turbulence a word which would appear to be more appropriately be translated as vorticity from the context in which it is found. Kleinschmidt's notation differs slightly from more modern day papers. In particular the symbol l is used instead of the more common f for the Coriolis parameter and A instead of M is used to represent angular momentum.

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Grundlagen einer Theorie der tropischen Zyklonen.

Von

E. Kleinschmidt jun.

Mit 3 Textabbildungen.

Zusammenfassung. Bei den tropischen Zyklonen strömt die Luft in den bodennahen Schichten zum Zentrum, in der mittleren Troposphäre wieder nach außen. Der Zustrom wird durch die Reibung an der Meeresoberfläche beherrscht, der Abstrom erfolgt in einem Feld dynamischer Feuchtindifferenz. Wenn ein Taifun neu entstehen soll, muß anfangs eine dynamische Labilität vorhanden sein. Eine solche tritt nur unter ganz speziellen, schwer zu erfüllenden Bedingungen auf. Für den ausgereiften Taifun werden mit Hilfe der Impuls- und Energiesätze eine Reihe Gesetze aufgestellt, die die wichtigsten Erscheinungen — Auge des Sturms, Sturmzone, Wolkenschirm usw. — erklären und ihr gegenseitiges Größenverhältnis zu berechnen gestatten. Nach diesen Gesetzen wird das Modell eines Taifuns entworfen.

Summary. In tropical cyclones the air near the ground flows towards the centre, in the middle troposphere it flows outwards. The inflow is governed by the friction on the surface of the sea, the outflow takes place in a field of dynamical moist-indifferent equilibrium. If a typhoon is to be developed, there must first be a dynamical instability. This occurs only under special conditions which cannot easily be met. For the fully developed typhoon a number of laws are stated by means of the principles of momentum and energy which explain the most important phenomena — eye of the storm, zone of the storm, cloud screen, etc. — and allow the calculation of their mutual ratio of magnitude. According to these laws the model of a typhoon is designed.

Résumé. Dans les cyclones tropicaux l'air des couches inférieures s'écoule vers le centre, celui de la troposphère moyenne vers l'extérieur. L'apport est dominé par le frottement à la surface de la mer; le flux expulsé a lieu dans un champ d'indifférence pseudolabile au point de vue dynamique. Pour qu'un typhon prenne naissance il faut qu'il y ait au préalable de la labilité dynamique, laquelle n'apparaît que dans des conditions toutes particulières, difficiles à remplir. Dans le cas du typhon bien développé on se sert des théorèmes de l'impulsion et de l'énergie pour établir une série de lois qui permettent d'expliquer les principaux phénomènes (oeil de la tempête, région à vents violents, panache nuageux, etc.) et de calculer leurs ordres de grandeur respectifs. L'auteur propose un modèle de typhon conçu d'après ces lois.

Vorbemerkungen.

Über den Mechanismus der tropischen Zyklonen gab es bisher mehrere verschiedene Ansichten. Eine Gegenüberstellung findet sich bei Riehl [*1*]. Zu entscheiden, was an diesen Theorien richtig und was falsch ist, war lange Zeit unmöglich, da den Taifunen mit den aerologischen Meßgeräten nur sehr schwer beizukommen ist. Erst in den letzten zehn Jahren gelang es, Daten aus dem Innern tropischer Wirbelstürme zu erhalten. Zweimal konnte sogar der Aufstieg einer Radiosonde im windschwachen Auge des Sturmes durchgeführt werden [*2*, *3*], so daß wir heute über den thermischen Aufbau der Taifune gut unterrichtet sind. Auch sonst fehlt es nicht an neuen Hilfsmitteln; die Vermessung der Starkregenfelder durch Radar [*4*, *5*] sowie die unmittelbare Erkundung durch Flugzeuge [*6*] haben viele neue Tatsachen erbracht.

Die im folgenden entworfene Theorie hat das Ziel, möglichst viele Einzelheiten auf wenige einfache Prinzipien zurückzuführen. Daß ein so vielseitiges Gebilde wie der tropische Wirbelsturm immer einige Seiten aufweist, die nur durch verwickeltere Theorien erklärt werden können, liegt auf der Hand. Mit solchen können wir uns hier nicht befassen. Es geht nur um die Erklärung der grundlegenden Vorgänge.

Wie fast bei jeder Theorie atmosphärischer Vorgänge, muß eine Reihe vereinfachender Annahmen gemacht werden; ihre Berechtigung läßt sich an den später berechneten Modellen nachweisen. *1.* Der Durchmesser des Taifuns ist klein gegenüber der Entfernung vom Äquator, so daß die Coriolisparameter als unabhängig von der geographischen Breite gelten können. *2.* Der horizontale Anteil des Drehvektors der Erde wird vernachlässigt. Es werden also Verhältnisse angenommen, wie sie auf einer rotierenden Scheibe herrschen. *3.* Die Asymmetrien, die man häufig bei Taifunen beobachtet, sind Äußerlichkeiten und haben keinen entscheidenden Einfluß; der Idealtaifun ist somit kreissymmetrisch. *4.* Oberhalb der bodennahen Reibungsschicht ist das statische und das Gradientwindgleichgewicht in guter Näherung erfüllt. Legen wir um die Achse des Taifuns ein System von Zylinderkoordinaten, so gilt also

$$l\,v + \frac{v^2}{r} = \frac{1}{\varrho}\,\frac{\partial p}{\partial r} \qquad (1)$$

und

$$-g = \frac{1}{\varrho}\,\frac{\partial p}{\partial z}\,. \qquad (2)$$

(Es bedeutet: r den Abstand von der Achse, z die Höhe, $l = 2\,\omega \sin\varphi$ den Coriolisparameter, v die tangentiale Horizontalgeschwindigkeit, p den Druck, ϱ die Dichte, g die Schwerebeschleunigung.) Aus (1) und (2) folgt

$$l\,v + \frac{v^2}{r} = g\,\frac{\partial_p z}{\partial r}\,, \qquad (3)$$

wo $\partial_p z/\partial r$ die Neigung der isobaren Flächen ist.

Für die kreissymmetrischen Felder folgt aus unserer *4.* Bedingung eine wesentliche Beschränkung. Während nämlich Tiefs beliebiger Stärke existieren können, ist das bei kreisförmigen Hochs nicht der Fall. Denn (3) ist eine quadratische Gleichung in v. Sie hat keine reelle Lösung, wenn ihre Diskriminante

$$D = l^2 r^2 + 4\,g\,r\,\frac{\partial_p z}{\partial r} \qquad (4)$$

negativ ist. In diesem Fall gibt es keinen Gradientwind, der das Hoch am Auseinanderfließen hindern könnte. Ist D dagegen positiv, d. h.

$$\frac{\partial_p z}{\partial r} > -\frac{l^2 r}{4\,g}\,, \qquad (5)$$

so gibt es zwei Lösungen v. Von diesen kommt nur eine in Frage, nämlich

$$v = -\frac{l\,r}{2} + \frac{\sqrt{D}}{2}\,. \qquad (6)$$

Bei der anderen Lösung wäre der absolute Rotor des Windfeldes der Erdrotation entgegengerichtet, was in Taifunen nicht vorkommt.

Im Grenzfall, wenn in (5) an Stelle von $>$ das Gleichheitszeichen steht, bildet die isobare Fläche ein nach unten offenes Paraboloid:

$$z = z_{r=0} - \frac{l^2 r^2}{8\,g}\,. \qquad (7)$$

Dieses „Grenzhoch" ist in tropischen Breiten sehr flach. Selbst auf einer Breite von 20° sinkt das Paraboloid von der Achse bis zum Abstand von 500 km noch nicht einmal um 8 m ab, und der antizyklonale Wind in dieser Entfernung beträgt 12,5 m/s! Je näher am Äquator, um so weniger ausgeprägt können die kreisrunden Antizyklonen sein, entsprechend dem Faktor l^2 in (7).

Die Bedingung (5), wonach das Grenzhoch nicht überschritten werden darf, nennen wir im folgenden Grenzhochbedingung. Sie ist wichtig im Hinblick auf den warmen Kern der tropischen Zyklonen. Entsprechend den hohen Temperaturen des Kerns nimmt die am Boden vorhandene Depression nach oben rasch ab. Diese Abnahme darf nun nicht wesentlich über den Druckausgleich hinausführen. Deshalb können die Temperaturen im Kern nie eine gewisse obere Grenze überschreiten, die jeweils durch die Stärke des Bodentiefs gegeben ist. Im Vorstadium des Taifuns, wenn am Boden nur ein flaches Tief von wenigen Millibar liegt, kann die mittlere Temperatur der Troposphäre über dem zentralen Bereich um nicht viel mehr als 1° C höher liegen als im Umkreis. Andernfalls würden die Kernmassen unweigerlich infolge ihres statischen Auftriebes aufsteigen und sich oben ausbreiten.

Der Flächensatz und die dynamische Feuchtlabilität.

Die Bewegung der Luft im Taifun besteht aus dem zirkularen Wind v und einer Komponente senkrecht dazu. Diese zweite Komponente stellen wir durch einen Strom dar, dessen Stromlinien in den rz-Ebenen liegen und den wir Vertikalstrom nennen. In seinem unteren Teil bildet er den Zufluß, im oberen Teil den Ausfluß aus dem Taifun. Die Bewegung im Vertikalstrom erfolgt nahezu unbeschleunigt, da die rz-Komponenten der reellen Kräfte (Druck, Schwere, Reibung) und die der Scheinkräfte (Zentrifugal- und Corioliskraft) in fast völligem Gleichgewicht stehen.

In Richtung der Komponente v wirkt weder eine Druck- noch die Schwerkraft. Deshalb lautet hier die Eulersche Bewegungsgleichung

$$\frac{dv}{dt} = -l\frac{dr}{dt} - \frac{v}{r}\frac{dr}{dt}. \tag{8}$$

Das Integral dieser Differentialgleichung ist

$$r\left(v + \frac{l\,r}{2}\right) = A, \tag{9}$$

wo A eine Konstante ist. Die Gleichung schreibt dem einzelnen Luftquantum, das sich in wechselndem Abstand von der Achse bewegt, eine ganz bestimmte Änderung seiner Tangentialgeschwindigkeit vor, unabhängig von der speziellen Gestalt des kreisförmigen Druckfeldes. Die linke Seite von (9) ist der absolute, d. h. in einem Inertialsystem gemessene Drehimpuls der Masseneinheit bezüglich der Taifunachse. Die Gleichung selbst ist der sogenannte Flächensatz, der die Konstanz dieses Drehimpulses fordert.

Für den in Bodennähe erfolgenden Zufluß gilt weder (8) noch (9). Denn hier spielt die Bodenreibung eine entscheidende Rolle. Der einströmenden Luft, die ursprünglich dank ihrer achsenfernen Lage einen sehr hohen absoluten Drehimpuls besaß, auch wenn sie dabei ganz ruhig lag, wird ein großer Teil dieses Impulses durch die Reibung entzogen. Anders dagegen im Abstrom des Taifuns, der in nahezu reibungsfreien Höhen verläuft. Hier muß sich das einzelne Teilchen, mindestens in gewisser Näherung, nach dem Flächensatz richten.

Man hat geglaubt, daß diese Tatsache schwer oder gar nicht mit den Beobachtungen in Einklang zu bringen sei [7]. Entnimmt man nämlich v der Gl. (9) und setzt es in (1) ein, so erhält man für jede Stromlinie im reibungsfreien Raum

$$\frac{1}{\varrho}\frac{\partial p}{\partial r} = \frac{A^2}{r^3} - \frac{l^2 r}{4} \quad \text{mit } A = \text{const.} \tag{10}$$

Nimmt man nun an, daß das Ausströmen horizontal erfolgt, so ist (10) eine Differentialgleichung für das radiale Druckprofil in der Höhe des Ausflusses, wobei natürlich die Achse als singuläre Stelle ausgenommen werden muß. Hier befindet sich ja auch das Auge, dessen Massen am Vertikalfluß nicht teilnehmen.

Die Massen des Vertikalstroms besitzen beim Verlassen der Bodenreibungsschicht immer noch einen hohen absoluten Drehimpuls A. Infolgedessen müßte, damit sie wieder ausströmen könnten, das radiale Druckprofil vom Rand des Auges bis zum Außenrand des Taifuns eine starke negative Krümmung besitzen. Das ist nun sicher nicht der Fall. Gerade die neuen Messungen aus dem Innern [2, 3] beweisen, daß das Druckfeld in keiner Höhenschicht (außer in Bodennähe) auch nur entfernt der Gl. (10) entspricht.

Der Widerspruch löst sich dadurch, daß die Luft — man vergleiche dazu Abb. 2 — sofort nach dem Verlassen der Reibungsschicht auszuströmen beginnt, aber nicht horizontal, sondern steil nach oben (in der Natur anfangs etwa unter 45°). Infolgedessen muß die einzelne Höhenschicht nur über ein kurzes Stück das von (10) geforderte Druckgefälle aufweisen. Da fernerhin der Kern des Taifuns warm ist, nimmt der horizontale Druckgradient nach oben ab. Auf diese Weise ist es möglich, daß (10) auf jeder Stromlinie des Abstroms tatsächlich erfüllt ist. Diesen Zustand bezeichnet man als dynamisch feuchtindifferent. Denn ein längs der Stromlinie verschobenes Teilchen befindet sich überall im Gleichgewicht. Das dynamisch indifferente Gleichgewicht geradliniger Stromfelder hat der Verfasser in [8] untersucht. Die dortigen Ergebnisse lassen sich sinngemäß auf Kreisstromfelder übertragen.

Die mächtige Wolkenmasse, die sich bei jeder tropischen Zyklone etwa zwischen 4 und 10 km Höhe schirmartig ausbreitet, befindet sich im feuchtindifferenten Zustand. In ihr erfolgt das Ausströmen aus dem Taifun. Genau genommen herrscht eine schwache dynamische Labilität, die die Energie zur Überwindung der inneren Reibung des Ausflusses liefert. Davon können wir hier aber absehen.

Dynamische Indifferenz verlangt statische Stabilität. Die Wolke muß also feuchtstabil geschichtet sein. Tatsächlich zeigen die Radiosonden im Bereich des Vertikalstroms sowohl einzeln [3] wie gemittelt [9] oberhalb 5 km eine deutliche Feuchtstabilität. Bemerkenswert ist, daß auch im Monatsmittel über dem Karibischen Meer der September diese Stabilität ab 550 mb deutlich aufweist, während sie im Februar erst bei etwa 350 mb beginnt [10]. Hier dürfte mit ein Grund liegen, warum die Taifune nur im Sommer und Herbst aufzutreten pflegen. Denn das Ausströmen in der statisch feuchtstabilen Schicht ist der primäre Vorgang in den tropischen Zyklonen. Wenn die Schichtung bis hoch hinauf feuchtlabil ist, entstehen allenfalls Cumulonimben, aber kein Taifun.

In den untersten 3 bis 5 km herrscht über den tropischen Meeren fast immer eine merkliche Feuchtlabilität. Diese Schicht muß in der Sturmzone des Taifuns vollständig zur bodennahen Reibungsschicht gerechnet werden. Denn die stürmische Bewegung veranlaßt eine lebhafte Konvektion mit heftigen Regenschauern. Durch den starken Vertikalaustausch wird die Ekmansche Reibungsspirale verzerrt. Die Reibungshöhe liegt in der Nähe der Cumulus-Köpfe und steigt gegen das Innere des Taifuns an (Abb. 2). Die Reibungsschicht hängt in dem Nimbostratus-Ringwall, der das Auge umschließt, mit der dynamisch indifferenten Abstromwolke zusammen.

Die von Wexler [5] ausgewerteten Radaraufnahmen der starken Schauerregen zeigen, daß die Konvektionsströme in Banden angeordnet sind, die mit den Stromlinien des Sturms spiralförmig gegen das Innere verlaufen. Diese Erscheinung beruht auf der starken Windscherung der Reibungsschicht; sie wurde von SOBHAG MAL [11] experimentell nachgewiesen.

Der Ausgangszustand für tropische Zyklonen.

Ehe wir an die Durchrechnung der ausgebildeten Zyklone gehen, untersuchen wir, aus welcher Anfangslage heraus ein Taifun sich entwickeln kann. Die wichtigste Voraussetzung ist nach dem letzten Abschnitt eine dynamische Feuchtlabilität in der statisch stabilen Schicht. Es erhebt sich die Frage, wann eine solche vorhanden ist.

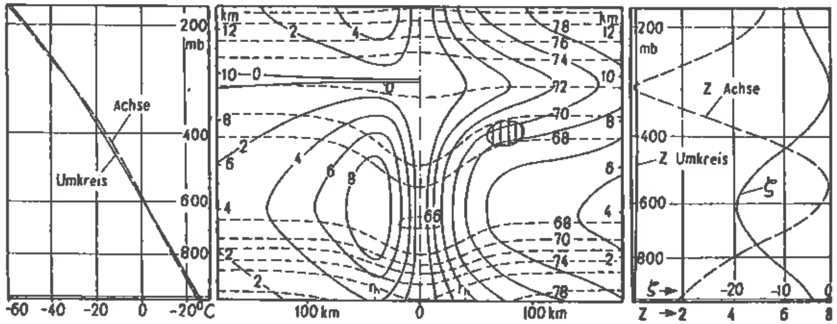

Abb. 1. Mittleres Feld: Schnitt durch eine Störung mit dynamischer Feuchtlabilität. Gestrichelt: Linien gleicher feuchtpotentieller Temperatur; außerdem linke Hälfte Linien gleicher Gradientwindstärke (m/s), rechte Hälfte Wirbellinien des absoluten Windfeldes. Schraffiert: Zone dynamischer Labilität. Rechtes Feld: Z-Verteilung, Z in 10^{-7} cgs, und Einsenkung ζ der Druckflächen in m.

Abb. 1 zeigt im Mittelteil einen Vertikalschnitt. In eine sonst windstille Atmosphäre ist eine kreisrunde, nach außen abklingende „Störung" eingelagert. Mit dem Wort Störung ist gemeint, daß hier Massen mit anderen Z-Werten liegen als in der ungestörten Atmosphäre. (Die ERTELsche Größe Z und ihre Bedeutung für das Druckfeld hat der Verfasser in [12, 13] behandelt.) Auf der linken Seite zeigt Abb. 1 die Temperatur auf der Achse der Störung und im windstillen Außenraum. Im Vertikalschnitt sind die Linien gleicher feuchtpotentieller Temperatur gestrichelt eingetragen, ferner links die Linien gleicher Gradientwindstärke v, rechts die absoluten Wirbellinien des Gradientwindfeldes. Deren Neigungswinkel α ist gegeben durch

$$\operatorname{tg} \alpha = \frac{l + v/r + \partial v/\partial r}{-\partial v/\partial z}. \tag{11}$$

Führt man hier mit Hilfe von (9) den absoluten Drehimpuls ein, so findet man, daß

$$\operatorname{tg} \alpha = -\frac{\partial A}{\partial r} \bigg/ \frac{\partial A}{\partial z} \tag{12}$$

ist. Also sind die Wirbellinien gleichzeitig Linien gleichen absoluten Drehimpulses.

Gewöhnlich verlaufen die Wirbellinien steiler als die Feuchtisentropen. Wo sie flacher liegen, nimmt der Drehimpuls längs der Feuchtisentropen nach außen ab, und damit ist die Lage dynamisch labil. Das ist in dem schraffierten Bereich des Vertikalschnittes der Fall, also in einer Zone, die sich ringförmig in einigem Abstand um die Achse zieht.

Die Berechnung des in Abb. 1 gezeigten Modells geschieht in folgender Weise: Die Höhenlage der isobaren Flächen ist gegeben in der Form

$$z(p, r) = z_\infty(p) + \frac{\zeta(p)}{1 + r^2/r_h^2}. \tag{13}$$

Dabei ist z_∞ die Höhe der Druckfläche in der ungestörten Atmosphäre ($r = \infty$), ζ die durch die Störung bewirkte Überhöhung auf der Achse, r_h der Radius, bis zu dem die Störung auf die Hälfte abklingt. Dieser Radius sei für alle p derselbe, die Störung sei also zylindrisch aufgebaut.

Das zugehörige Gradientwindfeld läßt sich aus Gl. (3) berechnen, wenn man dort z aus (13) einsetzt. Man findet

$$v = -\frac{l r}{2} + \frac{1}{2}\sqrt{l^2 r^2 - 8 g \zeta(p)\, r^2 r_h^2 / (r^2 + r_h^2)^2}. \tag{14}$$

Dies setzen wir in (11) ein. Dabei ist zu bemerken, daß v durch (14) nicht als Funktion von r und z, sondern von r und p gegeben ist, wobei p selbst von r und z abhängt. Beachtet man dies, so erhält man nach einiger Rechnung als Steigung der absoluten Wirbellinien in unserem Modell

$$\operatorname{tg} \alpha = \frac{l^3 (r^2 + r_h^2)^3 - 8 g \zeta r_h^4}{2 g r r_h^2 (r^2 + r_h^2) \dfrac{\partial \zeta}{\partial p} \dfrac{\partial p}{\partial z}} - \frac{\partial p}{\partial r} \bigg/ \frac{\partial p}{\partial z}. \tag{15}$$

In dieser Formel läßt sich $\frac{\partial \zeta}{\partial p} \frac{\partial p}{\partial z}$ noch durch die Temperaturstörung ausdrücken. Bezeichnen wir die *isobare* Temperaturdifferenz zwischen Achse und Außenraum mit ΔT, so ist nämlich

$$\frac{\partial \zeta}{\partial p} \frac{\partial p}{\partial z} = \frac{\Delta T}{T}. \tag{16}$$

Beweis: Für $r = 0$ (Index 0) geht (13) über in

$$z_0(p) = z_\infty(p) + \zeta(p).$$

Daraus folgt sofort

$$\frac{\partial \zeta}{\partial p} \frac{\partial p}{\partial z} = \left(\frac{\partial z_0}{\partial p} - \frac{\partial z_\infty}{\partial p}\right) \cdot \frac{\partial p}{\partial z}$$

und weiter aus der statischen Grundgleichung (2)

$$\frac{\partial \zeta}{\partial p} \frac{\partial p}{\partial z} = \left(\frac{1}{\varrho_0} - \frac{1}{\varrho_\infty}\right) \cdot \varrho.$$

Schließlich liefert die Gasgleichung $p/\varrho = RT$ und die Definition $\Delta T = T_0 - T_\infty$ die zu beweisende Beziehung (16).

Auf demselben Wege gelangt man zum nächsten Ziel, zu der Gleichung für die Steigung der Feuchtisentropen. Differenziert man (13) nach p, berücksichtigt wieder statische Grundgleichung und Gasgleichung, so findet man zunächst, daß das isobare Temperaturfeld in derselben Weise von r abhängt wie die Höhe der Druckflächen:

$$T(p, r) = T_\infty(p) + \frac{\Delta T(p)}{1 + r^2/r_h^2}. \tag{17}$$

Der *isobare* Temperaturgradient ist demnach

$$\frac{\partial_p T}{\partial r} = -\frac{2 r r_h^2 \Delta T}{(r^2 + r_h^2)^2}. \tag{18}$$

Nun machen wir davon Gebrauch, daß man die feuchtpotentielle Temperatur für konstanten Druck sehr genau als lineare Funktion von T darstellen kann:

$$\Theta = a(p) + b(p) \cdot T, \tag{19}$$

jedenfalls in den kleinen Bereichen, die durch die Temperaturkurven der Abb. 1 begrenzt werden. Aus (18) und (19) ergibt sich der isobare Gradient von Θ:

$$\frac{\partial_p \Theta}{\partial r} = b(p) \frac{\partial_p T}{\partial r} = -b(p) \frac{2 r r_h^2 \Delta T}{(r^2 + r_h^2)^2}. \tag{20}$$

Der *horizontale* Gradient hängt mit dem isobaren zusammen durch die Formel

$$\frac{\partial \Theta}{\partial r} = \frac{\partial_p \Theta}{\partial r} + \frac{\partial \Theta}{\partial z} \cdot \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \tag{21}$$

Hier setzen wir (20) ein und dividieren dann durch $-\partial\Theta/\partial z$ (welche Größe wir weiterhin mit Γ bezeichnen). So erhalten wir die Formel für den Steigungswinkel β der Feuchtisentropen:

$$\operatorname{tg} \beta = \frac{b \Delta T}{\Gamma} \frac{2 r r_h^2}{(r^2 + r_h^2)^2} - \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \tag{22}$$

Die Bedingung für dynamische Labilität,

$$\operatorname{tg} \alpha < \operatorname{tg} \beta,$$

läßt sich nun mit Hilfe von (15), (16) und (22) ausführlich hinschreiben. Sie lautet, wenn wir noch die dimensionslose Größe $x = r/r_h$ einführen:

$$\boxed{\frac{T \Gamma}{4 g b (\Delta T)^2} \left(l^2 r_h^2 \frac{(1 + x^2)^4}{x^2} - 8 g \zeta \frac{1 + x^2}{x^2} \right) < 1.} \tag{23}$$

Das ist die eigentliche Bedingung für die Entstehung tropischer Zyklonen.

Zu der Forderung dynamischer Labilität tritt die der statischen Stabilität:

$$\Gamma > 0. \tag{24}$$

Ferner fügen wir noch die *Grenzhochbedingung* hinzu. Sie ist in der Atmosphäre eo ipso erfüllt, muß aber bei der Konstruktion eines Modells ausdrücklich beachtet werden. Bildet man $\partial_p z/\partial r$ durch Ableitung von (13) nach r und setzt es in die Grenzhochbedingung (5) ein, so zeigt sich, daß diese Bedingung in unserem Modell, d. h. bei dem Ansatz (13), am schwersten auf der Achse zu erfüllen ist. Für $r = 0$ geht sie aber über in

$$\zeta < \frac{l^2 r_h^2}{8 g}. \tag{25}$$

Diese spezielle Form der Grenzhochbedingung hat, wie man leicht feststellt, zur Folge, daß der Klammerausdruck in (23) stets positiv ist. Ferner sieht man, daß sein Wert sowohl für $r \to 0$, wie für $r \to \infty$ über alle Grenzen wächst. Also ist der labile Bereich stets ein ringförmiges Gebilde. Eine gewisse Umgebung der Achse bleibt immer dynamisch stabil. Hier ist schon in der Ausgangslage das Auge des Taifuns vorgebildet.

Wir fragen nun: Wie müssen wir die Einzelgrößen wählen, damit (23) erfüllt ist? Die Größen T, g und b bewegen sich in engen Grenzen, können also nicht viel zur Labilität beitragen. Auch werden für sie im folgenden immer nur Mittelwerte eingesetzt. Ferner ist zu beachten, daß der Wert von x, also der normierte Abstand von der Achse, keinen Einfluß auf die Wahl der übrigen Größen hat. Denn in jedem Fall, d. h. für jedes r_h, durchlaufen die beiden Brüche in x von $r = 0$ bis $r = \infty$ denselben Wertevorrat.

Von den übrigen Größen hängen drei, nämlich Γ, ΔT und ζ in bestimmter Weise voneinander ab. Ein hohes ΔT, an sich wünschenswert, hat laut statischer Grundgleichung ein rasches Anwachsen von ζ nach oben zur Folge, so daß die durch (25) gegebene Grenze bald erreicht wird, wenn ζ nicht einen genügend hohen negativen Wert besitzt. Man kann zwar ΔT nach oben rasch abnehmen lassen, doch ist das um so weniger möglich, je geringer die mittlere statische Stabilität ist. Der Zusammenhang ist derart, daß im günstigsten Fall

$$\zeta \text{ angenähert proportional } -\frac{(\Delta T)^2}{\Gamma} \tag{26}$$

sein muß. Der „günstigste Fall" ist der, wo eine Depression nach oben gerade bis zum Grenzhoch, d. h. praktisch bis zum Druckausgleich abnimmt. In unserem Modell (Abb. 1) ist dieser Fall angenommen. Der Druckausgleich liegt in 9700 m Höhe, darüber beginnt eine neue, schwächere Depression. Würde man nicht den günstigsten Fall annehmen, so wäre eine dynamische Labilität kaum zu erreichen. Denn gerade der Druckausgleich ist es, der in Abb. 1 den flachen Verlauf der Wirbellinien in der Höhe des labilen Ringes bewirkt.

Aus der Forderung (26) folgt, daß der zweite Summand auf der linken Seite von (23) sich ebenfalls in ziemlich engen Grenzen bewegen muß und daß infolgedessen die dynamische Labilität hauptsächlich von der Wahl des Ausdrucks

$$\frac{\Gamma l^2 r_h^2}{(\Delta T)^2}$$

abhängt. Zwischen dessen Einzelgrößen besteht keinerlei Abhängigkeit mehr. Hiernach gibt es außer dem Druckausgleich in der Höhe, der geradezu notwendig ist, noch drei günstige Momente für die Bildung von

Taifunen: 1. Geringe statische Feuchtstabilität, 2. starker radialer Temperaturgradient, 3. niedere geographische Breite. Schon auf 20° Breite muß man einen abnorm steilen Temperaturanstieg zum Kern annehmen, um einen labilen Ring zu erhalten. Natürlich darf man aus der Formel (23) nicht den Schluß ziehen, der Äquator sei der günstigste Ort für die Bildung von Taifunen; denn am Äquator fehlt die Corioliskraft, die für die weitere Entwicklung des Taifuns unentbehrlich ist.

Dem Modell liegen folgende Daten zugrunde: $\varphi = 8°$, l also $= 2 \cdot 10^{-5}\,s^{-1}$, ferner im 400 mb-Niveau, wo der labile Ring liegen soll: $T = 257°$, mittleres $\Gamma = 0{,}9°\ \Theta/1000$ m, $\zeta = -7{,}2$ m, $\Delta T = 1{,}5°$ C, $r_h = 40$ km. Mit diesen Werten geht (23) über in

$$0{,}00076\,\frac{(1+x^2)^4}{x^2} + 0{,}654\,\frac{1+x^2}{x^2} < 1$$

und ist erfüllt für $1{,}45 < x < 2{,}305$, d. h. für 58 km $< r <$ 92 km.

Rechts in Abb. 1 ist dargestellt die Einsenkung der Druckflächen (ζ) und die Z-Verteilung auf der Achse und im Außenraum. Die für drohende Taifungefahr typische Z-Verteilung ist hiernach: engbegrenzte Masse von hohem Z mit dem Schwerpunkt in etwa 5 km Höhe, darüber rascher Übergang zu unternormalem Z. Das letzte ist notwendig, denn nur unternormales Z kann die Wirkung der hochzyklonalen Masse bis zum Druckausgleich kompensieren. Die erneute Zunahme von Z in unserem Modell oberhalb 10 km ist nicht wesentlich, kann also auch fehlen.

Für die Vorhersage brauchbarer ist eine Aussage über Druck- und Temperaturfeld: Schwache Feuchtstabilität, engbegrenztes Tief im 500 mb-Niveau mit 10 bis 20 m Einsenkung der Druckflächen, darüber rasche Abnahme der Depression, möglichst über den Druckausgleich hinaus. Die Hauptpunkte dieser Regel, nämlich das Tief in der mittleren, das (flache) Hoch in der oberen Troposphäre, sind bereits empirisch gefunden [*14*]. Wenn RIEHL anderseits betont, daß die Anfangsstörung immer einen kalten Kern habe, so stimmt das auch, aber nur zwischen dem Boden und etwa 5 km Höhe. Darüber muß die Störung einen warmen Kern haben, und hier oben nimmt der Taifun seinen Ausgang.

Im übrigen ist die Taifunbedingung (23) zu subtil, als daß sie durch das Druck- oder das Z-Feld hinreichend beschrieben werden könnte. Die Durchrechnung eines Modelles zeigt, wie stark sich schon kleinste Änderungen der Daten auswirken, wie sie den Labilitätsbereich weit ausdehnen oder ganz zum Verschwinden bringen können. Überhaupt ist der vollkommene Druckausgleich oder gar das Grenzhoch ein derart spezieller Fall bei den kleinräumigen kreisrunden Störungen, daß man sich nicht über die Seltenheit der Taifune zu wundern braucht.

Es wäre nun zu untersuchen, wie weit gewisse Wetterlagen, die als taifun-anfällig bekannt sind, wie das RODEWALDsche Dreimasseneck [*15*], mit obigen Bedingungen übereinstimmen. Das ist im Rahmen dieser Arbeit nicht möglich. Nur auf eines sei hingewiesen: Wenn sich ein Taifun entwickeln soll, muß zu Beginn der dynamisch labile Bereich

auch wirklich einen geschlossenen Ring bilden. Andernfalls sind die Anfangswiderstände zu groß, um den eigentlichen Taifun in Gang zu bringen. Ein geschlossener Ring ist nur möglich in der Nähe des thermischen Äquators, wo die absoluten Wirbellinien gelegentlich senkrecht verlaufen. In den Westwindzonen treten zwar auch dynamisch feuchtlabile Bereiche auf. Sie sind aber nicht ringförmig geschlossen. Denn in ihnen steigen alle Wirbellinien nach einer Richtung, nämlich zur höheren Breite hin an. Eine aktive Weströmung ist deshalb, was ebenfalls RIEHL schon empirisch gefunden hat, geradezu taifun-feindlich. Aus ihren dynamisch labilen Bereichen entstehen die Warmfronten der gemäßigten Breiten.

Wie kommt es nun zu einer gefährlichen Ausgangslage? Es gibt da *zwei Möglichkeiten*. Entweder entsteht die Lage auf advektivem Wege; wenn mit der Passatströmung Massen von hohem Z nach Westen ziehen, während in die darüberliegende schwache Weströmung Massen von geringem Z eingelagert sind, muß es gelegentlich zur gefährlichen Z-Verteilung kommen. Die andere Möglichkeit ist das Wirken nichtadiabatischer Vorgänge; hier könnte die verbreitete Schauertätigkeit, die der Taifunbildung vorangeht, eine Rolle spielen.

Der stationäre Abstrom im voll entwickelten Taifun.

Über die Entwicklung der tropischen Zyklone aus der Ausgangslage des letzten Abschnitts wird in dieser Arbeit nichts Näheres ausgesagt. Wir wenden uns gleich dem Zustand des ausgebildeten Taifuns zu. Auf die Untersuchung des instationären Zwischenstadiums kann um so leichter verzichtet werden, als der Endzustand zwar in starkem Maße vom Aufbau der Außenatmosphäre, aber gar nicht von der speziellen Form der Anfangsstörung abhängt. Von dieser wird nur verlangt, daß sie eine dynamische Feuchtlabilität aufweist, die genügend Energie entwickeln kann, um die Anfangswiderstände zu überwinden. Wenn erst die eigentliche Energiequelle — die von der sturmgepeitschten See an die Luft abgegebene Wärme — erschlossen ist, dann strebt der Taifun einem nahezu stationären Zustand zu, der mit der ursprünglichen Labilität nichts gemein hat. Da der ausgereifte Taifun ein in sich stabiles Gebilde ist, stellt seine Entstehung nichts als den Übergang von einem instabilen in einen stabilen Zustand dar.

Wir beginnen bei dem primären Teil des Taifuns, dem Abstrom in der oberen, statisch stabilen Schicht. Im stationären Zustand muß auf jeder Stromlinie des Abstroms einheitliche feuchtpotentielle Temperatur und einheitlicher absoluter Drehimpuls herrschen. Also ist A im Abstrom eine reine Funktion von Θ. Was wir suchen, ist wieder eine Gleichung für die Steigung der feuchtisentropen Flächen, jetzt aber unter der Bedingung $A = A(\Theta)$. Dazu differenzieren wir (1) nach z, (2) nach r, bilden die Differenz und erhalten

$$\left(l + 2\,\frac{v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\varrho^2}\left(\frac{\partial p}{\partial z}\,\frac{\partial \varrho}{\partial r} - \frac{\partial p}{\partial r}\,\frac{\partial \varrho}{\partial z}\right). \qquad (27)$$

Die Dichte ϱ fassen wir als Funktion des Drucks und der feuchtpotentiellen Temperatur auf: $\varrho = \varrho\,(p, \Theta)$. Für die räumlichen Ableitungen von ϱ gilt

$$\frac{\partial \varrho}{\partial r} = \frac{\partial \varrho}{\partial \Theta}\frac{\partial \Theta}{\partial r} + \frac{\partial \varrho}{\partial p}\frac{\partial p}{\partial r} \quad \text{und} \quad \frac{\partial \varrho}{\partial z} = \frac{\partial \varrho}{\partial \Theta}\frac{\partial \Theta}{\partial z} + \frac{\partial \varrho}{\partial p}\frac{\partial p}{\partial z}.$$

Setzt man dies in (27) ein, so folgt

$$\left(l + 2\frac{v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\varrho^2}\frac{\partial \varrho}{\partial \Theta}\left(\frac{\partial p}{\partial z}\frac{\partial \Theta}{\partial r} - \frac{\partial p}{\partial r}\frac{\partial \Theta}{\partial z}\right) \tag{28}$$

und weiter mit Hilfe von (2) die Gleichung

$$\operatorname{tg}\beta = -\frac{\partial \Theta}{\partial r}\Big/\frac{\partial \Theta}{\partial z} = \frac{\varrho}{g\,\dfrac{\partial \varrho}{\partial \Theta}\,\dfrac{\partial \Theta}{\partial z}}\left(l + 2\frac{v}{r}\right)\frac{\partial v}{\partial z} - \frac{\partial p}{\partial r}\Big/\frac{\partial p}{\partial z}. \tag{29}$$

Ihr entspricht für die Ausgangslage die Gl. (22). Jetzt führen wir den absoluten Drehimpuls ein. Aus (9) folgt

$$v = \frac{A}{r} - \frac{l\,r}{2}, \quad \frac{\partial v}{\partial z} = \frac{1}{r}\frac{\partial A}{\partial z} \tag{30, 31}$$

oder, da A allein von Θ abhängt,

$$\frac{\partial v}{\partial z} = \frac{1}{r}\frac{dA}{d\Theta}\frac{\partial \Theta}{\partial z}. \tag{32}$$

Ferner bedenken wir, daß $\partial\varrho/\partial\Theta$ die Ableitung bei konstantem Druck ist, so daß aus der Gasgleichung $\varrho = p/RT$ folgt:

$$\frac{1}{\varrho}\frac{\partial \varrho}{\partial \Theta} = -\frac{1}{T}\frac{\partial_p T}{\partial \Theta},$$

wofür nach (19) geschrieben werden kann

$$\frac{1}{\varrho}\frac{\partial \varrho}{\partial \Theta} = -\frac{1}{T\,b}. \tag{33}$$

Setzen wir (30), (32) und (33) in (29) ein, so erhalten wir das Gesetz für die Steigung der Stromlinien im Abstrom:

$$\operatorname{tg}\beta = -\frac{2\,T\,b}{g\,r^3}A\frac{dA}{d\Theta} - \frac{\partial p}{\partial r}\Big/\frac{\partial p}{\partial z}. \tag{34}$$

$A\,dA/d\Theta$ hat auf jeder Stromlinie einen konstanten negativen Wert. Da T und b näherungsweise ebenfalls als konstant gelten können, nähern sich die Stromlinien ungefähr mit r^{-3} den isobaren Flächen, deren Steigung durch das zweite Glied der rechten Seite gegeben ist.

Integriert man (34) über r, so erhält man die Höhenfunktion $z\,(r)$ einer Stromlinie. Da gegen das Innere hin die Senkung der Druckflächen viel kleiner ist als der Abfall der Feuchtisentropen, so gilt für die inneren Teile der Stromlinie — T und b sind als konstant vorausgesetzt —

$$z\,(r) = \frac{T\,b}{g\,r^2}A\frac{dA}{d\Theta} + \mathrm{H} \tag{35}$$

mit H als Integrationskonstante. In den äußeren Teilen ist $\operatorname{tg}\beta$ nach (34) so klein, daß die Stromlinie praktisch horizontal verläuft. Da das Temperaturfeld des Abstroms stetig in die Außenatmosphäre übergehen muß, ist H ungefähr die Höhe, in der im Außenraum die feuchtpotentielle Temperatur der Stromlinie angetroffen wird.

$A\,dA/\Theta$ ist, wie schon erwähnt, eine negative Größe. Denn A nimmt gegen das Auge hin ab, weil die zuströmenden Massen durch die Bodenreibung Drehimpuls verlieren, und Θ wird in der Sturmzone durch das Verdampfen des Meerwassers ständig erhöht. Im übrigen sei hier schon bemerkt, daß es die Unsicherheit in der Bestimmung von $A\,dA/d\Theta$ ist, was vorläufig den Versuch, einen Taifun vollständig aus den äußeren Umständen zu berechnen, scheitern läßt.

Der stationäre Zustrom in der bodennahen Reibungsschicht.

Die in den Taifun einströmenden Massen bringen von außen einen hohen absoluten Drehimpuls mit, auch wenn sie ursprünglich fast still

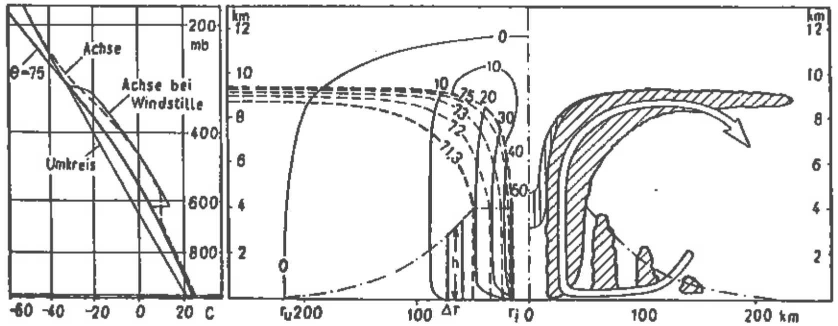

Abb. 2. Schnitt durch einen Taifun mäßiger Stärke. Gestrichelt: Linien gleicher feuchtpotentieller Temperatur. Ausgezogen: Linien gleicher Gradientwindstärke (m/s).

lagen. Einen großen Teil davon verlieren sie durch die Bodenreibung. Wir stellen für einen ringförmigen Abschnitt der Reibungsschicht die Bilanz des absoluten Drehimpulses auf. Der Querschnitt des Rings, von der Breite Δr und der Höhe h ist in Abb. 2 eingezeichnet. Der Einfachheit halber werde die Dichte ϱ in der ganzen Reibungsschicht als konstant angesehen. Auch muß die oben erwähnte Tatsache, daß der Austausch in der Reibungsschicht bandenförmig geordnet verläuft, hier unterschlagen werden. Es werde vielmehr isotrope Turbulenz angenommen.

Der Massenfluß durch eine senkrechte Wand des Rings ist

$$2\,\pi\,\varrho\,r\int_0^h u\,dz \quad (u = \text{Radialgeschwindigkeit}). \tag{36}$$

Differenzieren wir diesen Ausdruck nach r und multiplizieren mit Δr, so erhalten wir den sekundlichen Massenverlust im Ring, der auf den Fluß durch die senkrechten Wände zurückzuführen ist:

$$2\,\pi\,\varrho\,\Delta r\,\frac{\partial}{\partial r}\int_0^h r\,u\,dz. \tag{37}$$

Da sich die Masse im Ring nicht ändern soll, muß durch den Deckel dieselbe Masse pro Sekunde einfließen.

Der absolute Drehimpuls ist durch (9) gegeben. Bezeichnen wir mit V den Gradientwind am oberen Rand der Reibungsschicht, so beträgt der Verlust an Drehimpuls infolge des Flusses durch Wände und Deckel des Rings

$$2\pi\varrho\,\Delta r\,\frac{\partial}{\partial r}\int_0^h r^2 u\left(v+\frac{l}{2}r\right)dz-2\pi\varrho\,\Delta r\left(V+\frac{l}{2}r\right)\frac{\partial}{\partial r}\int_0^h r\,u\,dz. \tag{38}$$

Ferner verliert die Volumeinheit durch die Scheinreibung sekundlich an Drehimpuls

$$-r\,\frac{d\tau}{dz}.$$

Dabei ist τ die Schubspannung an einer horizontalen Fläche in Richtung des Gradientwinds. Der ganze Ring verliert durch Reibung

$$-2\pi\,\Delta r\,r^2\int_0^h\frac{d\tau}{dz}\,dz=2\pi\,\Delta r\,r^2\,\tau_B, \tag{39}$$

wo τ_B die zum Radius senkrechte Komponente der Bodenschubspannung ist. Die Schubspannung am Deckel des Rings konnte, da gegen τ_B sehr klein, in (39) weggelassen werden.

Die Formeln (38) und (39) enthalten alle Einzelposten der gewünschten Bilanz. Ehe wir diese aber hinschreiben, soll (38) noch eine andere Darstellung erfahren. Sie schließt sich an die Methoden an, die Prandtl in seiner Strömungslehre [*16*] entwickelt hat. Das Folgende kann deshalb kurz gefaßt werden.

Für die Strömung in der Reibungsschicht wird angesetzt

$$v=V\left(\frac{z}{h}\right)^{\frac{1}{n}},\quad u=-v\left(1-\frac{z}{h}\right)\operatorname{tg}\gamma. \tag{40}$$

γ ist der Ablenkungswinkel des Bodenwindes gegen den Gradientwind, n ist eine Zahl von der Größenordnung 10, die mit der Turbulenz anwächst. Die Windverteilung (40) ersetzt bei turbulenter Strömung die Ekman-Spirale, die ja für laminare Strömung abgeleitet ist. Prandtl hat für den Fall einer gleichmäßigen und geradlinigen Strömung gezeigt, wie h und γ mittels der Impulssätze berechnet werden können. Eine entsprechende Rechnung für den Zustrom der Taifune würde den Rahmen dieser Arbeit sprengen. Es muß hier genügen, daß mit Hilfe einiger einfacher Annahmen, die sich auf Prandtls Ergebnisse stützen, das Wesentliche der bodennahen Strömung deutlich wird.

Die Zahl n soll über den ganzen Bereich des Zustroms als konstant gelten. Ebenso wird γ, im Einklang mit der Beobachtung, als etwa konstant angesehen. Das gilt jedoch nicht im innersten Bereich der Sturmzone. Dort geht γ mit Annäherung an das Auge gegen Null. Setzt man (40) in (38) ein, so lassen sich die Integrale von (38) ausführen. Die Forderung, daß in unserem Ring der Drehimpuls zeitlich konstant sein soll, erhält dabei die Form

$$\frac{V^2h+rV\frac{\partial}{\partial r}(Vh)-l\,r\,V\,h}{(n+1)\,(2n+1)}-\frac{2V^2h+r\frac{\partial}{\partial r}(V^2h)}{(n+2)\,(2n+2)}=\frac{-r\,\tau_B}{\varrho\operatorname{tg}\gamma\,n^2}. \tag{41}$$

Für die Reibungshöhe machen wir die Annahme

$$h=m\,\frac{V}{l}\quad(m=\text{Zahl}). \tag{42}$$

Damit wird aus (41) eine Differentialgleichung für V allein, nämlich

$$\frac{\partial V}{\partial r}(n-2{,}5)+\frac{V}{r}(n-1)+l\,(n+2)=l\,\frac{\tau_B\,(n+1)\,(n+2)\,(2n+1)}{\varrho\,V^2\operatorname{tg}\gamma\,m\,n^2}. \tag{43}$$

Nun ist die Bodenschubspannung τ_B näherungsweise proportional ϱV^2. Also bilden die Größen auf der rechten Seite von (43) insgesamt eine Konstante K und das Integral der Differentialgleichung läßt sich geschlossen angeben:

$$\left.\begin{aligned}&V=C\left(\frac{r_u^{\mu}}{r^{\mu-1}}-r\right)\\ \text{mit}\quad&C=l\,\frac{n+2-K}{2n-3{,}5}\quad\text{und}\quad\mu=\frac{2n-3{,}5}{n-2{,}5}.\end{aligned}\right\} \tag{44}$$

Das ist ein Näherungsgesetz für den Gradientwind an der Obergrenze der Reibungsschicht. Der Außenradius r_u des Taifuns ist hiernach definiert als der Ort, wo kein Gradientwind mehr weht. Da μ ungefähr $=2$, so nimmt V nach innen etwa mit $1/r$ zu. Dasselbe gilt nach (42) für die Reibungshöhe h.

Es ist wohl zu beachten, daß das Gesetz sicher seine Gültigkeit verliert, wenn man zu nahe an das Auge gelangt. Unter dem inneren Ringwall wird γ ständig kleiner, somit der Wert der „Konstanten" K immer größer. Also wächst V gegen das Auge zu nicht mehr mit $1/r$, sondern langsamer. Das feuchtadiabatische Aufsteigen der Massen, für die der Weg in die dynamisch labile Oberschicht freigeworden ist, ist hier der alles beherrschende Vorgang.

Der Energiehaushalt der tropischen Zyklonen.

In der stationären Vertikalströmung des Taifuns werden laufend große Energiemengen umgesetzt. Ihnen gegenüber spielen Energieumsätze, die vielleicht im Auge oder im umgebenden Außenraum vor sich gehen, keine Rolle. Wir können uns also bei einer Energiebilanz auf die Vorgänge im Vertikalstrom beschränken.

Wie sich im letzten Abschnitt ergab, bezieht der Zustrom seinen Nachschub nicht von außen, sondern von oben. Das gilt allerdings nur bei statisch indifferenter Schichtung (wir hatten $\varrho=\text{const.}$ vorausgesetzt).

Bei der oberhalb des Reibungsstroms herrschenden trocken-stabilen Schichtung müßte der Massennachschub zum großen Teil doch von außen erfolgen. Es ist nun sehr bemerkenswert, daß rings um den von SIMPSON bearbeiteten Taifun vom Oktober 1946 [*3*] zwischen 6 und 8 km Höhe eine Zone großer Trockenheit liegt, die sich auch noch in tieferen Lagen bemerkbar macht und die nur dadurch hervorgerufen sein kann, daß die Luft absinkt. Es scheint also doch ein geschlossener Kreislauf im Taifun zu existieren[1]. Nach dem Verlassen des Abstroms sinken die Massen wieder langsam nach unten bis in Bodennähe, wobei sie durch Ausstrahlung Wärme verlieren müssen, um das statische Gleichgewicht zu behalten. Wenn auch außerdem ständig neue Luft aus dem Außenraum einbezogen wird, so handelt es sich doch im Zustrom um Massen, die aus einer gewissen Höhe kommen und die, verglichen mit der normalen, über dem Wasser liegenden tropischen Luft, zunächst eine geringe pseudopotentielle Temperatur besitzen.

Während des Zuströmens wird nun die pseudopotentielle Temperatur stark erhöht, einmal durch direkten Wärmeübergang vom Wasser zur Luft, zum anderen — und das ist der wichtigere Vorgang — durch das *Verdunsten des Meerwassers*. Besonders im mittleren Teil der Sturmzone bedingt die ursprüngliche Trockenheit der zuströmenden Luft im Verein mit der ungeheuren Turbulenz ein überaus hohes Dampfdruckgefälle über dem Wasser. Auch das Verdampfen des aufgewehten Gischts und des Regens trägt zur Erhöhung der pseudopotentiellen Temperatur bei, zwar nicht unmittelbar, da die Verdampfungswärme der Luft entzogen wird, aber indirekt, weil die Abkühlung den Wärmeübergang vom Meer zur Luft verstärkt. Nach den Höhenmessungen aus dem Taifun vom Oktober 1946 darf man annehmen, daß die im innersten Teil der Sturmzone die Meeresoberfläche verlassenden Luftmassen nahezu die Temperatur des Wassers besitzen und mit Wasserdampf gesättigt sind.

Die durch den Sturm dem Meer entzogene Wärme ist die wesentliche Energiequelle des Taifuns. Neben ihr spielt die latente Wärme des Wasserdampfes, den die Luft etwa von außen mitbringt, nur eine untergeordnete Rolle. Mit dieser Feststellung wird deutlich, warum die tropischen Zyklonen vorwiegend über den warmen Meeresteilen der Tropen auftreten. Es wird auch klar, warum jeder Taifun, der das Festland betritt, zum Sterben verurteilt ist.

Um eine einfache Energiebilanz zu gewinnen, verfolgen wir 1 g Luft, das vom Außenradius an durch den Zustrom, anschließend auf der Innenfläche durch den Abstrom läuft und das dann wieder zu seinem Ausgangspunkt hinabsinkt. Den Energiegewinn E aus dem damit verbundenen thermodynamischen Kreisprozeß findet man in bekannter Weise, indem man die Temperaturkurve der Außenatmosphäre und die Feuchtadiabate der innersten Abstromfläche in ein Tephigramm einträgt und die zwischen den Kurven liegende Fläche ausmißt.

[1] Zusatz bei der Korrektur: Diese Ansicht hat schon T. BERGERON ausgesprochen in: De tropiska orkanernas problem, Svenska Fysikersamfundets publ. Kosmos **27**, 122 (1949).

Dem Gewinn E steht ein Verlust an kinetischer Energie gegenüber. In Abb. 3 ist durch den dicken Linienzug dargestellt, wie sich v während des Kreislaufes ändert: von der Windstille am Außenradius (Punkt 1) nach einer Funktion der Form (44) bis zum Eintritt in den Ringwall (Punkt 2), dann nach einer schwächer ansteigenden Funktion bis zum stärksten Sturm auf dem Innenradius (Punkt 3); im Abstrom längs der Linie konstanten absoluten Drehimpulses bis Punkt 4. Während des Absinkens muß der übriggebliebene Impuls künstlich entzogen werden, um den Kreis zu schließen. Wäre keine Bodenreibung wirksam, würde das Teilchen auch im Zustrom seinen Drehimpuls behalten und nach Punkt 3′ gelangen. Seine Geschwindigkeit wäre dort nach dem Flächensatz:

$$V_i' = \frac{l}{2}\left(\frac{r_u^2}{r_i} - r_i\right). \tag{45}$$

Tatsächlich ist, wenn das Teilchen auf dem Innenradius die Reibungsschicht verläßt, infolge der Bodenreibung nur noch ein Bruchteil q dieser Geschwindigkeit vorhanden. Also verliert das Teilchen an kinetischer Energie durch Reibung

$$\frac{1}{2}\, V_i'^2 (1 - q^2) \text{ oder nach (45)}$$

$$\frac{l^2}{8}\left(\frac{r_u^2}{r_i} - r_i\right)^2 (1 - q^2).$$

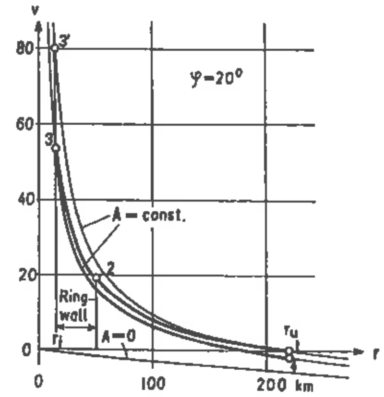

Abb. 3. Windplan des Vertikal-Kreislaufs im Taifun der Abb. 2.

Auf dem Abstrom tritt kein derartiger Verlust ein und der künstliche Energieentzug zwischen Punkt 4 und dem Ausgangspunkt ist ohne Bedeutung. Also lautet die Energiebilanz

$$\frac{l^2}{8}\left(\frac{r_u^2}{r_i} - r_i\right)^2 (1 - q^2) = E. \tag{46}$$

Daraus folgt, da $r_i \ll r_u$, in guter Näherung

$$\boxed{r_u^4 = \frac{8\, E\, r_i^2}{l^2 (1 - q^2)}.} \tag{47}$$

Für die maximale Windstärke $V_i = q \cdot V_i'$ folgt aus (45) und (47) in ebensolcher Näherung

$$\boxed{V_i^2 = 2\, E\, \frac{q^2}{1 - q^2}.} \tag{48}$$

Der Vergleich zwischen (44) und (45) legt die Annahme nahe, daß q in allen Taifunen nahezu denselben Wert hat. Somit hängt nach (48) die größte Windstärke im Taifun nur von E, d. h. nur vom Aufbau der Außenatmosphäre und von der Wassertemperatur ab.

Die Berechnung eines Taifuns.

Bezeichnen wir mit r_s den Radius, bei dem eine Stromlinie des Abstroms die Reibungsschicht (Höhe h_s) verläßt, so ist nach (35)

$$\boxed{r_s^2 = -\frac{T\,b}{g\,(H-h_s)}\;A\,\frac{dA}{d\Theta}.} \tag{49}$$

Diese Gleichung zeigt, daß der Ringwall, in dem die Massen in die Oberschicht übertreten, sowohl nach innen wie nach außen begrenzt sein muß. Denn $r_s = 0$ ist nur möglich für $A = \text{const.}$, d. h. entweder am Äquator, wo die Corioliskraft fehlt und $A = 0$ sein kann, oder bei fehlender Bodenreibung. Es ist also letzten Endes die Bodenreibung im Verein mit der Corioliskraft, die die einströmenden Massen nicht bis zur Achse vordringen läßt. Nach außen darf der Ringwall ebenfalls nicht zu weit reichen. Sonst würde nach (49) $dA^2/d\Theta$ Werte annehmen, die nur durch eine übermäßig starke Bodenreibung (dA^2 sehr groß) oder durch verschwindende Wärmezufuhr ($d\Theta$ sehr klein) zu erreichen wären. Beides ist offenbar nicht der Fall. Die Erfahrung zeigt, daß es immer nur ein verhältnismäßig schmaler Ring ist, in dem (49) erfüllt werden kann.

Durch die drei Gleichungen (47) bis (49) werden die Hauptmaße eines Taifuns bestimmt: äußerer Umfang, mittlerer Radius des Ringwalls und maximale Windstärke. Die bestimmenden Größen sind fast alle durch die äußeren Umstände gegeben oder sie liegen stets innerhalb enger Grenzen und sind der Erfahrung zu entnehmen. Einzig für $A\;dA/d\Theta$ liegt vorläufig keinerlei Anhalt vor. Wenn es gelänge, noch eine vierte Formel aufzustellen, die diese Größe mit den Hauptmaßen des Taifuns verbindet, so wäre jeder Taifun vollständig aus den äußeren Daten zu berechnen.

So lange wir diese Formel nicht kennen, müssen wir zur Konstruktion unseres Modells wenigstens *ein* Hauptmaß ebenfalls der Erfahrung entnehmen. Wir setzen daher den Innenradius mit 15 km fest. Die übrigen Daten sind: $\varphi = 20°$, also $l = 5 \cdot 10^{-5}\,\mathrm{s}^{-1}$, $q = {}^2/_3$, $\Theta_i = 75°$. Die Temperaturkurve für den Umkreis (Abb. 2) und die Feuchtadiabate $\Theta = 75°$ umschließen eine Energiefläche $E = 1770\ \mathrm{m^2/s^2}$. Mit diesen Werten findet man aus (47) und (48)

$$r_u = 219\ \mathrm{km}, \qquad V_i = 53{,}3\ \mathrm{m/s}.$$

Den äußeren Radius des Ringwalls setzen wir ebenfalls willkürlich mit 50 km fest. Für die Geschwindigkeitsverteilung an der Obergrenze der Reibungsschicht bestimmen wir die Konstanten von (44) so, daß V beim Eintritt in den Ringwall ${}^5/_6$ des reibungslosen Windes ausmacht (Abb. 3). Im Ringwall selbst geht dieser Bruchteil stetig auf ${}^2/_3$ zurück.

Zur Berechnung des Abstroms wird aus (35) und (49) $A\;dA/\Theta$ eliminiert. Man erhält

$$z = H - \frac{r_s^2}{r^2}\,(H - h_s). \tag{50}$$

H ist zunächst nur für die innerste Stromlinie bekannt ($H_i = 9340$ m), da man das Θ der folgenden Stromlinien noch nicht kennt. Um es zu erhalten, drücken wir $dA/d\Theta$ durch die Gradienten von A und Θ längs der Obergrenze der Reibungsschicht aus:

$$\frac{dA}{d\Theta} = \frac{dA}{dr}\Big/\frac{d\Theta}{dr} \tag{51}$$

Hieraus und aus (49) folgt

$$\frac{d\Theta}{dr} = -\frac{T\,b}{g\,(H-h_s)\,r^2}\;A\,\frac{dA}{dr}. \tag{52}$$

Hier setzen wir für H und h_s je einen Mittelwert (9 km und 4 km) ein, berechnen $A\;dA/dr$ aus der Windverteilung der Abb. 3 und erhalten so den Gang von Θ im Ringwall. Es ergibt sich, daß Θ vom inneren zum äußeren Rand um 3,7° abnimmt. Damit ist alles vorhanden, um die Stromlinien des Abstroms zu berechnen. Die Windverteilung im Abstrom ergibt sich aus dem Wind in Reibungshöhe und dem Flächensatz.

Die Bodendepression erhält man, wenn man den Wind in Reibungshöhe in (1) oder (3) einsetzt und über r integriert. Sie beträgt in unserem Modell 24 mb. Das entspricht einem mäßigen Wirbelsturm. Da nach (3) die Depression im wesentlichen mit v^2 geht, v^2 aber nach (48) proportional E ist, so ist die Depression selbst proportional E. Man muß also annehmen, daß in den kräftigsten Taifunen die verfügbare thermodynamische Energie doppelt so groß wie in unserem Modell oder noch größer ist.

Das Auge der tropischen Zyklone.

Aufbau und Umfang des Taifuns werden — das ging aus den letzten Abschnitten hervor — allein durch die äußere Umgebung und die Vorgänge im Vertikalstrom bestimmt. Die Luftmassen im Auge haben sich diesem Aufbau anzupassen, d. h. sie haben sich mit dem Vertikalstrom ins Gleichgewicht zu setzen. Wie sieht dieses Gleichgewicht aus?

Nehmen wir zunächst an, im Auge herrsche völlige Windstille. Auf der Grenzfläche zum Abstrom besteht dann ein Sprung der Windstärke von 0 auf v. Dem entspricht nach MARGULES ein Temperatursprung

$$\delta T = T\,\frac{l\,v + v^2/r}{g\,\mathrm{tg}\,\beta}. \tag{53}$$

Die Steigung der Grenzfläche, $\mathrm{tg}\,\beta$, ist aus (34) zu entnehmen. Da bei Windstille die isothermen Flächen horizontal liegen, ist mit (53) auch die Temperatur auf der Achse gegeben. Sie ist links in Abb. 2 ebenfalls eingetragen. Im unteren Teil des Auges steht die Grenzfläche senkrecht, also ist $\delta T = 0$. Mit Beginn des Abstroms springt $\mathrm{tg}\,\beta$ ungefähr auf 1 und δT auf 7,9° C. Nach oben nimmt dann δT nur sehr langsam ab (etwa mit T) und geht erst kurz vor der Obergrenze des Taifuns rasch auf Null zurück.

Hieraus sieht man, daß im Auge keine Windstille herrscht. Nicht etwa, weil die statische Instabilität an der Obergrenze sich nicht halten kann, sondern weil sie schon gar nicht entstehen kann. Denn die Luftmassen im Auge behalten während der Entstehung des Taifuns ihr Z fast unverändert. Selbst wenn die Strahlung das Z etwas verändern sollte, so kann es doch nie zu negativen Werten kommen. Vielmehr folgt die Temperatur auf der Achse etwa dem Verlauf der gestrichelten Linie in Abb. 2. Im unteren Teil des Abstroms, wo v sehr groß und r klein ist, muß sich δT allerdings nach (53) richten. Die hohe Temperatur entsteht dadurch, daß die Luft während der Entwicklung des Taifuns tiefer steigt und sich adiabatisch erwärmt. Die darüber liegende Luftsäule streckt sich; dabei entsteht eine mäßige zyklonale Rotation. Erst einige Kilometer über der Obergrenze ist nichts mehr von der Streckung zu spüren.

Der Radiosondenaufstieg vom 8. Oktober 1946 [*3*] zeigt den fraglichen Temperaturaufbau sehr deutlich, die Sonde vom 19. Oktober 1944 [*2*] ebenfalls bis 250 mb. Darüber dürfte sie erheblich zu warm sein. Beide Radiosonden zeigen die Zunahme der Temperatur beim Beginn des Abstroms, allerdings nicht als scharfen Sprung, sondern als mächtige Isothermie. Vermutlich erstreckt sich diese Übergangsschicht über einen großen Teil des Abstroms (senkrecht schraffiert in Abb. 2). Ein scharfer Sprung in Temperatur und Wind wird durch den Austausch verhindert.

Auffallend sind die hohen Feuchtewerte, die im Auge gemessen wurden. Sie zeigen, daß der Austausch zwischen Abstrom und Auge durch die Isothermie keineswegs unterbunden ist. In einem eben erst entstandenen Taifun ist es oberhalb der Isothermie sicher sehr trocken. Aber bei längerem Bestehen erfolgt eine Anreicherung von Wasserdampf im Auge, die bis zur Bildung von hohen Wolkenschichten führt.

Herrn Professor L. PRANDTL verdanke ich den Hinweis, daß die Impulsvernichtung in der Bodenreibungsschicht die Dimensionen des Taifuns wesentlich mitbestimmen muß. Herrn HERBERT RIEHL (Chicago) danke ich für die freundliche Übersendung von Beobachtungsmaterial aus dem Bereich von Taifunen.

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Principles of the Theory of tropical Cyclones

by
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With 3 illustrations.

Summary

In tropical cyclones the air near the ground flows towards the centre, in the middle troposphere it flows outwards. The inflow is governed by the friction on the surface of the sea, the outflow takes place in a field of dynamical moist-indifferent equilibrium. If a typhoon is to be developed, there must first be a dynamical instability. This occurs only under special conditions which cannot easily be met. For the fully developed typhoon a number of laws are stated by means of the principles of momentum and energy which explain the most important phenomena - eye of the storm, zone of the storm, cloud screen, etc. - and allow the calculation of their mutual ratio of magnitude. According to these laws the model of a typhoon is designed.

PRELIMINARY REMARKS

Up till now there have been several different viewpoints on the mechanism of tropical cyclones. A comparison can be found in Riehl [1]. Deciding what was right and what was wrong about these theories was impossible for a long time since it was very difficult to get close to the typhoons using aerological measuring equipment. Only in the last ten years was success achieved in obtaining data from the inside of tropical cyclones. On two occasions it had been possible to fly a radiosonde in the low wind eye of the storm [2,3] with the result that today we are well informed about the thermal structure of typhoons. But even without this there is no lack of new aids; the surveying of the heavy rain fields using radar [4,5] and the direct reconnaissance using aircraft [6] have produced many new results.

The theory sketched out below is aimed at attributing as many details as possible to a small number of principles. The fact that such a versatile structure as the tropical cyclone can always throw up aspects which can only be explained by fairly involved theories is obvious. We cannot deal with these here where we want only to explain the basic processes.

As is the case with every theory about atmospheric processes a series of simplifying assumptions must be made; their justification is proved by the models calculated later. 1 - the diameter of the typhoon is small compared to the distance from the equator with the result that the Coriolis parameters can be regarded as independent of the geographical latitude. 2 - the horizontal portion of the rotating vector of the earth can be disregarded. Therefore ratios as they apply to a rotating disk are assumed. 3 - the asymmetries frequently observed in typhoons are external characteristics and have no significant influence; the ideal typhoon is therefore rotationally symmetrical. 4 - above the friction layer near the ground the static and the gradient wind balance is achieved to a great degree. If we lay a system of cylindrical coordinates around the axis of the typhoon, the following applies;

$$l v + \frac{v^2}{r} = \frac{1}{\varrho}\frac{\partial p}{\partial r} \qquad (1)$$

and

$$-g = \frac{1}{\varrho}\frac{\partial p}{\partial z}. \qquad (2)$$

(r = the distance from the axis, z = the height, $l = 2\,\omega \sin\varphi$ the Coriolis parameter, v = the tangential horizontal speed, p = the pressure, ϱ the density, g = the gravitational acceleration). From (1) and (2) follows

$$l v + \frac{v^2}{r} = g\frac{\partial_p z}{\partial r}, \qquad (3)$$

where $\partial_p z/\partial r$ is the inclination of the isobar surfaces.

From our 4th condition follows a fundamental restriction for the rotationally symmetrical fields. Whereas lows of any depth may exist, this is not the case with circular highs. For (3) is a quadratic equation in v. It has no real solution if its discriminant

$$D = l^2 r^2 + 4 g r \frac{\partial_p z}{\partial r} \qquad (4)$$

is negative. In this case there is no gradient wind which might prevent the high from breaking up. If D is positive in contrast, i.e.

$$\frac{\partial_p z}{\partial r} > -\frac{l^2 r}{4 g}, \qquad (5)$$

then there are two solutions v. Only one of these can be considered, namely

$$v = -\frac{l r}{2} + \frac{\sqrt{D}}{2}. \qquad (6)$$

In the other solution the absolute rotor of the wind field of the earth's rotation would be inverted, and this does not happen in typhoons.

In the borderline case, when the equal sign appears in place of > in (5), the isobar surface forms a paraboloid which is open to the bottom:

$$z = z_{r=0} - \frac{l^2 r^2}{8g}. \qquad (7)$$

This "borderline high" is very flat in tropical latitudes. Even at a latitude of 20° the paraboloid does not even drop by as much as 8m from the axis up to a distance of 500 km and the anticyclonal wind at this distance amounts to 12.5 m/s ! The closer to the equator the more marked the circular anticyclones may become, which is line with the factor l^2 in (7).

In the rest of the text we shall call condition (5), which means that the borderline high may not be exceeded, the borderline high condition. It is important in relation to the warm core of tropical cyclones. The upward depression on the ground rapidly decays in line with the high temperatures of the core. This drop must not fall greatly beyond the pressure compensation. For this reason the temperatures in the core can never exceed a certain limit which is set in each case by the strength of the surface low. In the preliminary build-up of a typhoon, when only a flat low of a few millibars is present on the surface, the average temperature of the troposhpere cannot be above the central area by much more than 1°C than in the surrounding area. If this was not the case the core masses would undoubtedly rise as a result of its static lift and would expand aloft.

THE LAW OF AREAS AND THE DYNAMIC MOIST INSTABILITY

The movement of air in the typhoon consists of the circular wind v and one component which is perpendicular to it. We shall illustrate this second component by means of a flow whose flow lines are to be found in the rz-levels and which we shall call the vertical flow. In its lower section it forms the inflow, in the upper part it forms the outflow from the typhoon. Movement in the vertical flow takes place almost without acceleration since the rz-components of the actual forces (pressure, gravity, friction) and those of the apparent forces (centrifugal and Coriolis force) are in almost complete equilibrium.

In the direction of the components v the pressure or the gravitational force have almost no effect. For this reason Euler's movement equation is as follows

$$\frac{dv}{dt} = -l\frac{dr}{dt} - \frac{v}{r}\frac{dr}{dt}. \qquad (8)$$

The integral of this differential equation is

$$r\left(v + \frac{lr}{2}\right) = A, \qquad (9)$$

where A is a constant. The equation dictates for the individual air quantum, which is moving at varying distances from the axis, a very definite change in its tangential speed, irrespective of the special shape of the circular pressure field. The left side of (9) is the absolute, i.e.the measured angular momentum of the unit of mass in an inertial system in relation to the axis of the typhoon. The equation itself is the so-called law of areas which requires the constancy of this angular momentum.

Neither (8) nor (9) apply to the inflow which occurs close to the ground. In this case the ground friction plays a decisive role. The incoming air, which initially possessed a very high absolute angular momentum due to its position far away from the axis, even when it remained quite calm when doing so, loses a great deal of this momentum due to friction. Things are different to this in the outflow of the typhoon which runs at almost frictionless altitudes. In this case the individual particle, at least to a certain degree, must comply with the law of areas.

It was believed that this fact could not be reconciled at all or only with difficulty with the observations [7]. If v is removed from equation (9) and it is inserted into (1), the following equation is obtained for every streamline in the frictionless area;

$$\frac{1}{\varrho}\frac{\partial p}{\partial r} = \frac{A^2}{r^3} - \frac{l^2 r}{4} \quad \text{with } A = \text{const.} \qquad (10)$$

If it is now assumed that the outflow takes place horizontally, then (10) is a differential equation for the radial pressure profile at the height of the outflow; however the axis must be removed as a singular position. Here too can be found the eye whose masses are not part of the vertical flow.

When they leave the ground friction layer, the masses of the vertical flow still possess a high absolute angular momentum A. As a result of this, so that they could flow out once again, the radial pressure profile from the edge of the eye to the external edge of the typhoon would have to possess a very negative curvature. This is definitely not the case. It is precisely the new pressures from the inside [2,3] which prove that the pressure field does not correspond even remotely to equation (10) (except when close to the ground) at any contour.

The inconsistency is solved by the fact that - on this point compare figure 2 - the air begins to flow out immediately after leaving the friction layer, and does not do so horizontally, but rises steeply (in nature at an angle of 45° initially). Consequently the individual contour must possess the drop in pressure demanded by (10) over no more than a short distance. Since the core of the typhoon is warm in addition to this, the horizontal pressure gradient decreases towards its top. In this way it is possible that (10) is in fact met on every streamline of the outflow. This condition is known as dynamic moist-indifferent equilibrium because a particle displaced along the streamline can be found to be at equilibrium everywhere. The dynamically indifferent equilibrium of linear flow fields has been investigated by the author in [8]. The results given there can be transferred accordingly to cyclic flow fields.

The thick cloud mass, which spreads like an umbrella at an altitude between 4 and 10 km in every tropical cyclone, is in the moist indifferent condition. In it the outflow from the typhoon takes place. In precise terms a weak dynamic instability which provides the energy to overcome the internal friction of the outflow prevails. However we can disregard this here.

Dynamic indifference demands static stability. The cloud must therefore be layered in a moist-stable manner. In fact the radiosondes in the area of the vertical flow show individually [3] and on average [9] a distinct moisture stability above 5 km. What is worth noting is the fact that even in the middle of the month over the Caribbean September clearly showed this stability from 550 mb upwards, whereas it began in February only at around 350mb [10]. In this case this might well be one of the reasons why typhoons usually occur only in summer and autumn since the outflow in the statically moist-stable layer is the primary process in tropical cyclones. If the strata are moist-stable very high up, then cumulo-nimbus is the result, but not typhoons.

In the bottom 3 to 5 km there is almost always a marked moist instability over tropical seas. In the zone of the storm of the typhoon this layer must be counted as part of the friction layer close to the ground. This is the case because the stormy movement creates a lively convection with heavy rain showers. The Ekman friction spiral is distorted by the strong vertical exchange. The altitude of the friction is in the proximity of the cumulus tops and rises against the inside of the typhoon (fig. 2). The friction layer is joined in the nimbostratus ringwall, which encompasses the eye, with the dynamically indifferent outflow cloud.

The radar images of heavy showers evaluated by Wexler [5] show that the convection currents are arranged in bands which run in a spiral shape with the streamlines of the storm towards the inside. This phenomenon is based on the strong wind shear of the friction layer; it was proved in experiments by Sobhag Mal [11].

THE INITIAL STATE FOR TROPICAL CYCLONES

Before we come to calculating the fully developed cyclone, we shall examine the initial situation from which a typhoon can develop. On the basis of the last section the most important prerequisite is a dynamic moist instability in the statically stable layer. This raises the question of when such an instability is present.

In its middle section figure 1 shows a vertical view. In an atmosphere which is otherwise calm there is stored a circular "disturbance" which is decaying at the outer edges. The word disturbance means that in this case the masses have different Z-values than in the undisturbed atmosphere. (Ertel's dimesnion Z and its significance have been dealt with by the author in [12, 13]). On the left side figure 1 shows the temperature along the axis of the disturbance and in the calm outer area. In the vertical section the lines of equal moist-potential temperature have been entered (broken lines), further to the left are the lines of equal gradient wind strength v, and on the right the absolute turbulence lines of the gradient wind field. Their angle of inclination α is given by

$$\operatorname{tg} \alpha = \frac{l + v/r + \partial v/\partial r}{-\partial v/\partial z}. \qquad (11)$$

If the absolute angular momentum is inserted here with the help of (9), then it can be found that

$$\operatorname{tg} \alpha = -\frac{\partial A}{\partial r} \Big/ \frac{\partial A}{\partial z} \qquad (12)$$

This means that the turbulence lines are at one and the same time lines of equal absolute angular momentum.

Normally the turbulence lines run at a steeper angle than the moist isentropes. In positions where they lie flatter, the momentum along the moist isentropes decreases at the external edges and in this way the position is dynamically instable. This is the case in the hatched area of the vertical section, i.e. in a zone which is drawn like a ring at a certain distance around the axis.

The calculation of the model shown in figure 1 was done in the following way; the altitude of the isobar surfaces was given in the form

$$z(p, r) = z_{\infty}(p) + \frac{\zeta(p)}{1 + r^2/r_h^2} \qquad (13)$$

In this equation z_{∞} is the altitude of the pressure surface in the undisturbed atmosphere ($r = \infty$), ζ is the increase along the axis caused by the disturbance, r_h is the radius up to which the disturbance falls to half. This radius is the same for all p, the disturbance is cylindrical in structure.

The related gradient wind field can be calculated from equation (3) if the z from (13) is inserted. The following is found;

$$v = -\frac{lr}{2} + \frac{1}{2}\sqrt{l^2 r^2 - 8 g \zeta(p) r^2 r_h^2 / (r^2 + r_h^2)^2}. \qquad (14)$$

We shall insert this into (11). In doing so it should be noted that v is given by (14) not as a function of r and z, but of r and p, where p itself depends on r and z. If this is taken into account, then the following equation is obtained after some calculation as the increase of the absolute turbulence lines in our model;

$$\operatorname{tg} \alpha = \frac{l^2 (r^2 + r_h^2)^3 - 8 g \zeta r_h^4}{2 g r r_h^2 (r^2 + r_h^2) \frac{\partial \zeta}{\partial p} \frac{\partial p}{\partial z}} - \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \qquad (15)$$

In this formula $\frac{\partial \zeta}{\partial p}\ \frac{\partial p}{\partial z}$ can also be expressed by the temperature disturbance. If we designate the isobar temperature difference between the the axis and the external area as ΔT, then the following is obtained;

$$\frac{\partial \zeta}{\partial p} \cdot \frac{\partial p}{\partial z} = \frac{\Delta T}{T}. \qquad (16)$$

Proof: for r = 0 (index) equation (13) changes to

$$z_0(p) = z_\infty(p) + \zeta(p).$$

From this can be deduced immediately

$$\frac{\partial \zeta}{\partial p} \cdot \frac{\partial p}{\partial z} = \left(\frac{\partial z_0}{\partial p} - \frac{\partial z_\infty}{\partial p}\right) \cdot \frac{\partial p}{\partial z}$$

and also from the static basic equation (2)

$$\frac{\partial \zeta}{\partial p} \cdot \frac{\partial p}{\partial z} = \left(\frac{1}{\varrho_0} - \frac{1}{\varrho_\infty}\right) \cdot \varrho.$$

Finally the gas equation produces p/ϱ = RT and the definition ΔT $T_0 - T_\infty$ the equation to be proved (16).

In the same way we reached our next goal, the equation for the increase in the moist isentropes. If (13) is differentiated in accordance with p and the static basic equation and the gas equation are taken into account once again, then what is found initially is the fact that the isobar temperature field depends on r in the same way as the altitude of the pressure surfaces;

$$T(p, r) = T_\infty(p) + \frac{\Delta T(p)}{1 + r^2/r_h^2}. \qquad (17)$$

The isobar temperature gradient on this basis is

$$\frac{\partial_p T}{\partial r} = - \frac{2 r r_h^2 \Delta T}{(r^2 + r_h^2)^2}. \qquad (18)$$

Now we shall make use of the fact that the moist potential temperature can be shown very precisely for constant pressure as a linear function of T;

$$\Theta = a(p) + b(p) \cdot T, \qquad (19)$$

at least in the small areas which are limited by the temperature curves in figure 1. From (18) and (19) comes the isobar gradient of Θ:

$$\frac{\partial_p \Theta}{\partial r} = b(p) \frac{\partial_p T}{\partial r} = - b(p) \frac{2 r r_h^2 \Delta T}{(r^2 + r_h^2)^2}. \qquad (20)$$

The horizontal gradient is related to the isobar one by the formula

$$\frac{\partial \Theta}{\partial r} = \frac{\partial_p \Theta}{\partial r} + \frac{\partial \Theta}{\partial z} \cdot \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \qquad (21)$$

In this case we inserted (20) and then divided by $-\partial\Theta/\partial z$ (a magnitude which we shall call Γ from now on). Thus we obtained the formula for the angle of increase β of the moist isentropes;

$$\operatorname{tg} \beta = \frac{b \Delta T}{\Gamma} \frac{2 r r_h^2}{(r^2 + r_h^2)^2} - \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \qquad (22)$$

The condition for dynamic instability,

$$\operatorname{tg} \alpha < \operatorname{tg} \beta,$$

can now be noted in detail with the help of (15), (16) and (22). If we also introduce the dimensionless magnitude $x = r/r_h$, it is as follows;

$$\boxed{\frac{T \Gamma}{4 g b (\Delta T)^2} \left(l^2 r_h^2 \frac{(1+x^2)^4}{x^2} - 8 g \zeta \frac{1+x^2}{x^2}\right) < 1.} \qquad (23).$$

This is the actual condition for the creation of tropical cyclones.

To the requirement of dynamic instability can be added that of static stability:

$$\Gamma > 0. \qquad (24).$$

On top of this we also added the limiting high condition. It is fulfilled in the atmosphere by itself, but must be explicitly taken into account when devising a model. If $\partial p/\partial r$ is formed by deducing (13) according to r and it is inserted into the limiting high condition (5), then it can be shown that in our model, i.e. in the formulation in (13), this condition is the most difficult to satisfy along the axis. For r = 0 it changes however to

$$\zeta < \frac{l^2 r_h^2}{8 g}. \qquad (25)$$

As can be easily determined, this special form of the limiting high condition means that the expression in brackets in (13) is always positive. Furthermore it can be seen that its value for $r \to 0$ and for $r \to \infty$ goes beyond all boundaries. Therefore the instable area is always a ring-shaped structure. A certain environment of the axis always remains dynamically stable. And here as early as this stage the eye of the typhoon has been formed in the initial situation.

The question is now; how must we select the individual dimensions so that (23) is met ? The dimensions T, g and b move within narrow limits, and therefore cannot contribute much to the instability. Average values only shall be inserted for them in the rest of the text. Furthermore it must be borne in mind that the value of x, that is the standard distance from the axis, has no influence on the choice of the other dimensions. In each case, i.e. for every r_h, both fractions pass through the same stock of values in x from r = 0 to r = ∞.

Of the other dimensions three, namely Γ, ΔT and ζ depend in particular on one another. A high ΔT, which is desirable in itself, leads according to the static basic equation to a rapid rise of ζ with the result that the limit set by (25) is soon reached if ζ does not possess an adequately high negative value. The ΔT can be allowed to diminish rapidly as it rises, but this becomes increasingly less possible the smaller the average static stability is. The correlation is such that in the best case must be approximately proportional;

$$\zeta \text{ approx. proportional } - \frac{(\Delta T)^2}{\Gamma} \qquad (26).$$

The "best case" is the one in which the depression lessens right up to the limiting high ,i.e. in practice up to the pressure compensation. In our model (figure 1) this case is assumed. The pressure compensation occurs at an altitude of 9700m, above this begins a new, weaker depression. If the best case were not assumed, then a dynamic instability could not be achieved, because it is precisely this pressure compensation which in figure 1 brings about the flat curve of the turbulence lines at the altitude of the instable ring.

From the requirement of (26) it follows that the second summand on the left side of (23) must also move in rather narrow limits and that as a result of this the dynamic instability depends mainly on the choice of the expression

$$\frac{\Gamma l^2 r_h^2}{(\Delta T)^2}$$

There is no longer any dependency between these individual dimensions. On this basis, apart from the pressure compensation at altitude, which is necessary for this, there are three other favourable times for the development of typhoons: 1. small static moist stability, 2. strong radial temperature gradient, 3. low geographical latitude. Even at 20° latitude an abnormally steep temperature rise towards the core must be assumed in order to obtain an instable ring. Naturally the conclusion must not be drawn from formula (23) that the equator is the best location for the formation of typhoons because at the equator there is no Coriolis force, which is vital for the further development of the typhoon.

The model is based on the following data; $\varphi = 8°$, 1 therefore = $2 \cdot 10^{-5} s^{-1}$, also in the 400 mb level, where the instable ring is supposed to be located: $T = 257°$, average $\Gamma = 0.9° \Theta/1000$ m, $\zeta = -7.2$m, $\Delta T = 1.5°C$, $r_h = 40$ km. With these values (23) changes to

$$0{,}00076\,\frac{(1+x^2)^4}{x^2} + 0{,}654\,\frac{1+x^2}{x^2} < 1$$

and is fulfilled for $1.45 < x < 2.305$, i.e. for 58 km < r < 92km.

On the right in figure 1 can be seen the impression of the pressure surfaces (ζ) and the Z-distribution along the axis and in the outer area. On this basis the Z-distribution typical of the impending danger of a typhoon is as follows; narrowly limited mass of high Z with focal point at an altitude of approx. 5 km, above this rapid transition to sub-normal Z. The latter is necessary because only sub-normal Z can compensate the effect of high cyclone masses until pressure compensation. The renewed increase in Z in our model above 10 km is not fundamental, and can also be omitted therefore.

What is more useful for prediction is information on the pressure and temperature field; weak moist stability, narrowly restricted low in the 500 mb level with 10 to 20 m indentation of the pressure surfaces, above this rapid drop in the depression, as far as possible beyond the pressure compensation. The main points of these rules, i.e. the low in the middle, the (flat) high in the upper troposphere, have already been found empirically [14]. When Riehl asserts on the other hand that the initial disturbance always has a cold core, this is also true, but only between the ground and an altitude of approx. 5km. Above this the disturbance must have a warm core and it is up here that the typhoon has its start.

Moreover the typhoon condition (23) is too subtle as to be capable of being described adequately by the pressure or the Z field. The calculation of a model shows how great an effect even the smallest of changes in data can have, how they can extend greatly the instability area or cause it to disappear completely. Overall the complete pressure compensation or even the limiting high is such a type of special case in these circular disturbances in small areas that there is no need to be surprised at the infrequency of typhoons.

What is to be examined now is to what extent certain weather conditions which are known to cause typhoons, such as Rodewald's mass triangle [15], agree with the above mentioned conditions. This is not possible within the framework of this paper.One thing should be pointed out; if a typhoon is to be developed, the dynamically instable area must also form a closed ring at the start. If it does not, the initial resistances are too big to set the actual typhoon in motion. A closed ring is only posible in the vicinity of the thermal equator where the absolute turbulence lines occasionally run vertically. In the western wind zones dynamically moist instable areas do occur, but they are not closed in a ring pattern because in them all the turbulence lines rise in one direction, namely towards the higher altitude. For this reason an active western wind, something which Riehl also discovered empirically, is typhoon-unfriendly. From its dynamically instable areas come the warm fronts of the temperate latitudes.

What then brings about a dangerous initial situation ? There are two possibilities; either the situation arises in an advective way; if masses of high Z move to the west with the trade wind, whereas masses of low Z are stored in the weak western wind lying above it, then this must lead to a dangerous Z distribution from time to time. The other possibility is the effect of non-adiabatic processes; in these circumstances the widespread showers which precede the typhoon might play a role.

THE STATIONARY OUTFLOW IN THE FULLY-DEVELOPED TYPHOON

Nothing more shall be said in this paper about the development of the tropical cyclone from the initial situation described in the last section. We shall turn immediately to the condition of the developed typhoon. It is all the easier not to investigate the instationary intermediate stage since the final stage depends to a great extent on the structure of the external atmosphere, but not at all on the special shape of the initial perturbation. All that is demanded from the initial perturbation now is that it displays a dynamic moist instability which can develop enough energy to overcome the initial resistances. When the actual energy source - the heat given off by the storm-whipped sea to the air - is tapped, then the typhoon becomes an almost stationary condition which has nothing in common with the original instability. Since the fully-blown typhoon is a stable structure in itself, its formation represents no more than the transition from an instable to a stable condition.

We shall begin with the primary part of the typhoon, the outflow in the upper, statically stable layer. In the stationary condition uniform moist potential temperature and uniform absolute momentum must prevail on every streamline of the outflow. This means that in the outflow A is a pure function of (Θ). What we are looking for is once again an equation for the rise of the moist isentropic surfaces, but now in the condition A = A (Θ). To do this we differentiate (1) according to z and (2) according to r, form the difference and obtain

$$\left(l + 2\frac{v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\varrho^2}\left(\frac{\partial p}{\partial z}\frac{\partial \varrho}{\partial r} - \frac{\partial p}{\partial r}\frac{\partial \varrho}{\partial z}\right). \qquad (27)$$

We consider the density ϱ as a function of the pressure and the moist potential temperature: $\varrho = \varrho\,(p, \Theta)$. For the spatial differenciations of ϱ the following applies:

$$\frac{\partial \varrho}{\partial r} = \frac{\partial \varrho}{\partial \Theta}\frac{\partial \Theta}{\partial r} + \frac{\partial \varrho}{\partial p}\frac{\partial p}{\partial r} \quad \text{und} \quad \frac{\partial \varrho}{\partial z} = \frac{\partial \varrho}{\partial \Theta}\frac{\partial \Theta}{\partial z} + \frac{\partial \varrho}{\partial p}\frac{\partial p}{\partial z}.$$

If this is inserted into (27), then it follows

$$\left(l + 2\frac{v}{r}\right)\frac{\partial v}{\partial z} = \frac{1}{\varrho^2}\frac{\partial \varrho}{\partial \Theta}\left(\frac{\partial p}{\partial z}\frac{\partial \Theta}{\partial r} - \frac{\partial p}{\partial r}\frac{\partial \Theta}{\partial z}\right) \qquad (28)$$

and further with the help of (2) the equation

$$\operatorname{tg}\beta = -\frac{\partial \Theta}{\partial r} \Big/ \frac{\partial \Theta}{\partial z} = \frac{-\varrho}{g\,\frac{\partial \varrho}{\partial \Theta}\frac{\partial \Theta}{\partial z}}\left(l + 2\frac{v}{r}\right)\frac{\partial v}{\partial z} - \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \qquad (29)$$

Equation (22) corresponds to this for the initial position. Now we introduce the absolute momentum. From (9) follows

$$v = \frac{A}{r} - \frac{l\,r}{2}, \quad \frac{\partial v}{\partial z} = \frac{1}{r}\frac{\partial A}{\partial z} \qquad (30,\ 31)$$

or, since A depends only on Θ,

$$\frac{\partial v}{\partial z} = \frac{1}{r}\frac{dA}{d\Theta}\frac{\partial \Theta}{\partial z}. \qquad (32)$$

In addition we consider that $\partial\varrho/\partial\Theta$ is the derivation at a constant pressure with the result that from the gas equation ϱ = p/RT follows;

$$\frac{1}{\varrho}\frac{\partial \varrho}{\partial \Theta} = -\frac{1}{T}\frac{\partial_p T}{\partial \Theta},$$

for which according to (19) the following can be written

$$\frac{1}{\varrho}\frac{\partial \varrho}{\partial \Theta} = -\frac{1}{T\,b}. \qquad (33)$$

If we insert (30), (32) and (33) into (29), then we obtain the law for the increase of the streamlines in the outflow;

$$\operatorname{tg}\beta = -\frac{2\,T\,b}{g\,r^3}\,A\,\frac{dA}{d\Theta} - \frac{\partial p}{\partial r} \Big/ \frac{\partial p}{\partial z}. \qquad (34)$$

A dA/dΘ has on each streamline a constant negative value. Since T and b can also be considered roughly as constant, the streamlines with about r^{-3} get close to the isobar surfaces whose rise is provided by the second term on the right side.

If (34) is integrated over r, then the altitude function z(r) of a streamline is obtained. Since the reduction of the pressure surfaces is much smaller towards the inside than the fall in the moist isentropes, then the following equation applies to the inner parts of the streamline - provided T and b are constant -

$$z\,(r) = \frac{T\,b}{g\,r^2}\,A\,\frac{dA}{d\Theta} + H \qquad (35)$$

with H as the integration constant. In the outer parts tg β is so small according to (34) that the streamline runs almost horizontally. Since the temperature field of the outflow has to transfer into the outer atmosphere constantly, H is about the altitude at which in the external area it meets the moist potential temperature of the streamline.

A dA/Θ is, as already mentioned, a negative dimension because A decreases towards the eye because the incoming masses lose their momentum because of the ground friction and Θ is constantly increased in the storm area by the evaporation of the sea water. On top of this it should be noted here that it is the uncertainty in the determination of A dA/dΘ which temporarily caused the experiment to calculate a typhoon completely from the external conditions to fail.

THE STATIONARY INFLOW IN THE FRICTION LAYER CLOSE TO THE GROUND

The masses flowing into the typhoon from outside bring with them a high absolute momentum, even if they were almost calm originally. A large proportion of these masses are lost through ground friction. We shall establish the balance of the absolute momentum for a ring-shaped section of the friction layer. The cross-section of the ring, from latitude Δr and altitude h, has been entered in figure 2. For the sake of simplicity the density ϱ in the entire friction layer shall be regarded as constant. The above mentioned fact that exchanges in the friction layer take place in a band-type arrangement must be omitted here. What is assumed is isotropic turbulence.

The mass flow through a vertical wall of the ring is

$$2\pi\varrho\, r\int_0^h u\,dz \quad (u = \text{radial speed}) \qquad (36)$$

If we differentiate this expression according to r and multiply with Δr, then we obtain the mass loss per second in the ring which can be attributed to the flow through the vertical wall:

$$2\pi\varrho\,\Delta r\,\frac{\partial}{\partial r}\int_0^h r\,u\,dz. \qquad (37)$$

Since this mass in the ring is not to change, the same mass per second must flow through the lid.

The absolute momentum is given by (9). If we call the gradient wind at the upper edge of the friction layer V, then the loss of momentum as a result of the flow through the wall and lid of the ring is

$$2\pi\varrho\,\Delta r\,\frac{\partial}{\partial r}\int_0^h r^2 u\left(r-\frac{l}{2}r\right)dz-2\pi\varrho\,\Delta r\left(V+\frac{l}{2}r\right)\frac{\partial}{\partial r}\int_0^h r\,u\,dz. \qquad (38)$$

In addition the volume unit loses momentum every second by the apparent friction as follows;

$$-r\,\frac{d\tau}{dz}.$$

In this case τ is the shearing strain on a horizontal surface in the direction of the gradient wind. The entire ring loses the following as result of friction;

$$-2\pi\Delta r\,r^2\int_0^h\frac{d\tau}{dz}\,dz=2\pi\Delta r\,r^2\tau_B, \qquad (39)$$

where τ_B is the component of the ground shearing strain perpendicular to the radius. The shear strain at the lid of the ring could be omitted since in (39) since it is very small in contrast to τ_B .

Formulae (38) and (39) contain all the individual items of the desired balance. But before we write these down, (38) should be shown in a different manner. It is related to the methods which Prandtl developed in his ideas on fluid mechanics [16]. The following can therefore be stated in brief.

For the flow in the friction layer the following equation was applied

$$v=V\left(\frac{z}{h}\right)^{\frac{1}{n}},\quad u=-v\left(1-\frac{z}{h}\right)\operatorname{tg}\gamma. \qquad (40)$$

γ is the angle of deflection of the ground wind against the gradient wind, n is the number of an order of magnitude of 10 which rises with the turbulence. During turbulent flow the wind distribution (40) replaces the Ekman spiral which was derived for laminar flow. In the case of an even and linear flow Prandtl showed how h and γ can be calculated by means of the principles of momentum. A corresponding calculation for the inflow of typhoons would go beyond the framework of this paper. Here it is enough to state that with the help of a few simple assumptions based on Prandtl's results the basics of the flow close to the ground become clear.

The number n should apply across the entire range of the inflow as constant. γ should also be regarded as being roughly constant, which is in line with the observations. However this does not apply in the innermost area of the storm zone. Here γ goes towards zero as the eye of the storm gets closer. If (40) is inserted into (38), then the integrals of (38) can be applied. The requirement for the momentum in our ring to be constant in time then looks as follows

$$\frac{V^2h+rV\frac{\partial}{\partial r}(Vh)-lrVh}{(n+1)(2n+1)}-\frac{2V^2h+r\frac{\partial}{\partial r}(V^2h)}{(n+2)(2n+2)}=\frac{-r\tau_B}{\varrho\operatorname{tg}\gamma\,n^2}. \quad (41) \qquad (41)$$

For the friction altitude we make the assumption

$$h=m\frac{V}{l}\quad(m=\text{Zahl}). \qquad (42)$$

In this way from (41) comes a differential equation for V alone,

$$\frac{\partial V}{\partial r}(n-2{,}5)+\frac{V}{r}(n-1)+l(n+2)=l\,\frac{\tau_B(n+1)(n+2)(2n+1)}{\varrho V^2\operatorname{tg}\gamma\,m\,n^2}. \qquad (43)$$

Now the ground shear strain τ_B is approximately proportional to ϱV^2 . Therefore the dimensions on the right side of (43) form overall a constant K and the integral of the differential equation can be given as the following unified whole;

$$V=C\left(\frac{r_u^{\mu}}{r^{\mu-1}}-r\right)$$

with

$$C=l\,\frac{n+2-K}{2n-3{,}5}\quad\text{und}\quad\mu=\frac{2n-3{,}5}{n-2{,}5}. \qquad (44)$$

This is the law of approximation for the gradient wind at the upper limit of the friction layer. The external radius r_u of the typhoon is defined on this basis as the location where the gradient wind no longer blows. Since μ approximately = 2, then V increases towards the inside with about 1/r. In accordance with (42) the same applies to the friction altitude h.

It must be borne in mind that the law does lose its validity if the eye of the storm is approached too closely. Under the inner ring wall γ becomes steadily smaller so that the value of the "constants" K becomes bigger and bigger. Therefore V does not grow more in the face of the eye with 1/r, but does so more slowly. The moist-adiabatic rise of the masses, whose path into the dynamically instable upper layer has been cleared, is the prime dominant process in these circumstances.

THE ENERGY BUDGET OF TROPICAL CYCLONES

In the stationary vertical flow of the typhoon large energy masses are constantly being shifted. In contrast to these, transfers of energy which occur in the eye of the storm or in the outer area play no role. Therefore in drawing up an energy balance we can restrict ourselves to the processes in the vertical stream.

As was shown in the last section, the inflow does not draw its supplies from outside, but from above. However this applies only in statically indifferent strata (we had assumed ϱ = constant).

In the dry stable strata which prevail above the friction flow the supply of masses must come from outside for the greater part however. It is now very noticeable that right round the typhoon of October 1946 [3] studied by Simpson there was between 6 and 8 km a very dry zone which even made its present felt in lower situations and which can only have been caused by the fact that the air was dropping. Therefore a closed circulation does appear to exist in the typhoon[1]. After they ~~have~~ leave the outflow, the masses once again drop down slowly to close to the ground at which point they must lose heat due to radiation in order to maintain the static equilibrium. If in addition new air is constantly sucked in from the outside, then in the inflow this air is masses which come from a certain altitude and which, compared with the normal tropical air lying above the water, possess a small pseudo-potential temperature.

During the inflow process the pseudo-potential temperature is greatly increased, on the one hand by the direct heat transfer from the water to the air, and on the other - and this is the more important process - by the evaporation of the sea water. In the middle section of the storm zone in particular the original dryness of the incoming air together with the huge turbulence brings about a large vapour pressure fall above the water. Even the vaporisation of the whipped-up spray and the rain contributes to the increase in the pseudo-potential temperature, and indeed not directly, since the evaporation heat of the air is removed, but indirectly because the cooling reinforces the heat transfer from the sea to the air. On the basis of the altitude measurements from the typhoon of October 1946 it can be assumed that the air masses, in the innermost part of the storm zone, leaving the sea surface have almost the temperature of the water and are saturated with water vapour.

The heat removed from the sea by the storm is the basic energy source of the typhoon. In comparison to it, the latent heat of the water vapour, which the air carries with it from the outside, plays no more than a secondary role. From this fact it becomes clear why tropical cyclones occur primarily over the warm areas of the sea. It also becomes clear why every typhoon which hits the mainland is condemned to death.

In order to obtain a simple energy balance we shall follow 1 g of air which runs from the outer radius through the inflow, then subsequently along the inner surface through the outflow and drops once again to its starting point. The energy gain E from the thermodynamic circular process associated with this can be found in the familiar way by recording in a tephigram the temperature curve of the outer atmosphere and the moist adiabats of the innermost outflow surface and by measuring the surface located between the curves.

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1) Addendum found during proof-reading; this opinion was already expressed by T. Bergeron in: De tropiska orkanernas problem, Svenska Fysikersam-fundets publ. Kosmos 27, 122 (1949).

The gain E contrasts with a loss of kinetic energy. In figure 3 the thick line illustrates how v changes during the circulation: from the calm at the outer radius (point 1) after a function of the form (44) up to the entry into the ring wall (point 2), then after a slowly rising function up to the most violent storm on the inner radius (point 3); in the outflow along the line of constant absolute angular momentum up to point 4. While the energy is dropping, the remaining momnetum must be artificially removed in order to close the circle. If there was no ground friction effect, the particle would also retain its angular momentum in the inflow and would reach point 3'. In accordance with the law of areas its velocity would then be:

$$V_i' = \frac{l}{2}\left(\frac{r_u^2}{r_i} - r_i\right). \qquad (45)$$

What does in fact happen is that when the particle on the inner radius leaves the friction layer, all that is left is a fraction q of this velocity as a result of the ground friction. Therefore the particle loses kinetic energy due to friction as follows:

$\frac{1}{2} V_i'^2 (1 - q^2)$, or as per (45)

$$\frac{l^2}{8}\left(\frac{r_u^2}{r_i} - r_i\right)^2 (1 - q^2).$$

On the outflow no such loss occurs and the artificial removal of energy between point 4 and the starting point is of no significance. Therefore the energy balance is;

$$\frac{l^2}{8}\left(\frac{r_u^2}{r_i} - r_i\right)^2 (1 - q^2) = E. \qquad (46)$$

From this it follows since $r_i \ll r_u$, as a good approximation

$$\boxed{r_u^4 = \frac{8 E r_i^2}{l^2 (1 - q^2)}.} \qquad (47)$$

For the maximum wind strength $V_i = q \cdot V_i'$ it follows from (45) and (47) as a good approximation

$$\boxed{V_i^2 = 2 E \frac{q^2}{1 - q^2}.} \qquad (48)$$

The comparison between (44) and (45) makes the assumption that in all typhoons q has almost the same value. In this way, according to (48), the greatest wind strength in the typhoon depends only on E, i.e. only on the structure of the outer atmosphere and on the water temperature.

<u>THE CALCULATION OF A TYPHOON</u>

If we call the radius at which a streamline of the outflow leaves the friction layer (altitude h_s) r_s , according to (35) the following is true:

$$\boxed{r_s^2 = -\frac{T b}{g (H - h_s)} \cdot A \frac{dA}{d\Theta}.} \qquad (49)$$

This equation shows that the ring wall in which the masses cross over into the upper layer must be limited to the inside and towards the outside because $r_s = 0$ is only posible for A = const., i.e. either at the equator, where the Coriolis force is absent and A = 0, or if there is no ground friction. In the final analysis it is the ground friction in conjunction with the Coriolis force which does not allow the inflowing masses to push forward as far as the axis. The ring wall must also not extend too far to the outside, otherwise according to (49) $dA^2/d\Theta$ would assume values which could only be obtained by an excessively high ground friction (dA^2 very large) or by minimal heat intake ($d\Theta$ very small). In both sets of circumstances this is obviously not the case. Experience shows that it is always only a reltively narrow ring in which (49) can be fulfilled.

The main dimensions of a typhoon are determined by the three equations (47) to (49); outer circumference, mean radius of the ring wall and maximum wind strength. The decisive dimensions have almost all been set by the outer conditions or they are always located within narrow boundaries and can be drawn from experience. There is temporarily no evidence at all available for $A\, dA/d\Theta$. If a fourth formula could be established, then every typhoon could be calculated completely from the extenal data.

For as long as we do not know this formula, we must draw at least one main dimension from experience to construct our model. Therefore we set the inner radius at 15 km. The other data is; $\varphi = 20°$, i.e. $l = 5\cdot10^{-5}\,s^{-1}$, $q = 2/3$, $\Theta_i = 75°$. The temperature curve for the surroundings (figure 2) and the moist adiabates $\Theta = 75°$ include an energy surface $E = 1770\ m^2/s^2$. With these values the following can be found from (47) and (48)

$$r_u = 219\ \text{km}, \qquad V_i = 53.3\ \text{m/s}.$$

We shall also set the outer radius of the ring wall arbitrarily at 50 km. For the velocity distribution at the upper limit of the friction layer we determine the constants of (44) with the result that V makes up 5/6 of the frictionless wind when it enters the ring wall (fig. 3). In the ring wall itself this fraction drops steadily to 2/3.

To calculate the outflow $A\, dA/\Theta$ is eliminated from (35) and (49). This produces;

$$z = H - \frac{r_s^2}{r^2}(H - h_s). \tag{50}$$

Initially H is known for the innermost streamline ($H_i = 9340$ m) since the Θ of the following streamlines is not known. In order to obtain it, we shall express $dA/d\Theta$ by the gradients of A and along the upper limit of the friction layer:

$$\frac{dA}{d\Theta} = \frac{dA}{dr} : \frac{d\Theta}{dr} \tag{51}$$

From this and from (49) follows

$$\frac{d\Theta}{dr} = -\frac{T\,b}{g\,(H - h_s)\,r^2}\,A\,\frac{dA}{dr}. \tag{52}$$

Here we insert a mean value (9 km and 4 km) for H and h_s respectively, calculate $A\, dA/dr$ from the wind distribution of figure 3 and thus obtain the progress of Θ in the ring wall. The result is that Θ drops by 3.7° from the inner to the outer radius. Thus everything is here to allow us to calculate the streamlines of the outflow. The wind distribution in the outflow is a result of the wind at the friction altitude and the law of areas.

The ground depression is obtained if the wind is inserted into the friction altitude in (1) or (3) and integrated over r. In our model it amounts to 24 mb. This corresponds to a moderate cyclone. Since the depression does basically with v^2 according to (3), but v^2 is proportional to to E according to (48), then the depression itself is proportional to E. It must also be assumed that in powerful typhoons the available thermodynamic energy is twice as large as in our model or even greater.

THE EYE OF THE TROPICAL CYCLONE

The structure and the size of the typhoon - this emerged form the last sections - are determined purely by the external environment and the processes in the vertical stream. The air masses in the eye have to adapt to this structure, i.e. they have to come into equilibrium with the vertical stream. What does this equilibrium look like ?

Let us assume initially that a total calm prevails in the eye. On the boundary surface to the outflow there is then a leap in the wind strength from 0 to v. This corresponds to a temperature leap after Margules of

$$\delta T = T\,\frac{l v + v^2/r}{g\,\mathrm{tg}\,\beta}. \tag{53}$$

The rise of the boundary surface, $\mathrm{tg}\,\beta$, can be taken from (34). Since the isothermal surfaces lie horizontally during a calm, the temperature on the axis is also given by (35). It is entered in figure 2 on the left. In the lower part of the eye the boundary surface is vertical, i.e. $\delta T = 0$. As the outflow begins $\mathrm{tg}\,\beta$ jumps to about 1 and δT to 7.9°C. δT then declines only very slowly as it rises (approx. with T) and returns rapidly to zero just short of the upper limit of the typhoon.

From this it can be seen that there is no wind calm in the eye. This is not because the static instability at the upper limit cannot be held, but because it cannot develop at all. This is because the air masses in the eye retain their Z almost unchanged during the development of the typhoon. Even if the radiation were to change the Z in any way, this would still never lead to negative values. It is more the case that the temperature on the axis follows ~~the~~ approximately the curve of the dotted line in figure 2. In the lower part of the outflow, where v is very large and r very small, δT must be based on (53). The high temperature is created by the fact that the air rises higher during the development of the typhoon and heats up adiabatically. The air column above this extends; in this process a moderate cyclonal rotation occurs. It is not until a few kilometres above the upper limit that there is no more of this expansion to be seen.

The radiosonde flight of 8 October 1946 [3] showed the questionable temperature structure very clearly, and the same was true of the sonde of 19 October 1944 [2] up to 250 mb. Above it must have been far too warm. Both radiosondes showed the increase in temperature at the start of the outflow, not as a sharp leap, but as a thick isothermal layer. Presumably this transitional layer extends over a large part of the outflow (vertically hatched area in figure 2). A sharp jump in temperature and wind is prevented by the exchange.

What is striking are the high moisture values measured at the eye. They show that the exchange between outflow and eye is not at all elminated by the isothermal layer. In a typhoon which has just been formed it is very dry above the isothermal layer. But if it lasts for a longer period the eye is enriched by water vapour and this leads to the formation of high cloud layers.

I would like to thank Professor L. Prandtl for the information that the destruction of the momentum in the ground friction layer must also play a fundamental part in determining the dimensions of the typhoon. I also thank Herbert Riehl (Chicago) for the friendly transmission of observation material from the area of the typhoon.

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Captions to the figures.

Figure 1; average field; section through a dusturbance with dynamic moist instability; dotted lines - lines of equal moist potential temperature. On top of this on the left half are lines equal gradient wind strength (m/s), on right half turbulence lines of the absolute wind field. Hatched; zone of dynamic instability. Right fieldl Z-distribution, Z in 10^{-7} egs, and indentation ζ of the pressure surfaces in m.
1 = axis 2 = environment

Figure 2; section through a typhoon of moderate strength. Dotted lines ; lines of equal moist potential temperature. Straight lines; lines of equal gradient wind strength (m/s).
1 = axis 2 = environment 3 = axis during wind calm.

Figure 3; wind plan of the vertical circulation in the typhoon of figure 2.

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