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RADAR ECHOES FROM AND TURBULENCE WITHIN CUMULUS AND CUMULONIMBUS CLOUDS

By R. F. JONES, B.A.

Summary.—Accelerometer records obtained on traverses of cumulus and cumulonimbus clouds by Spitfire aircraft have been analysed to give the frequency of gusts encountered of specified magnitudes. It appears that turbulence increases slowly with height in a cumulonimbus cloud until rather less than half way up the cloud, and thereafter remains at or near its maximum value until the top third of the cloud is reached. In the top third of the cloud the turbulence steadily decreases with height to a minimum value near the top of the cloud. The readings obtained are shown not to be inconsistent with statistics of American observations made in Florida and Ohio.

Radar observations made at the same time as the accelerometer records were obtained make it possible to relate the turbulence to the characteristics of the radar echo received from the part of the cloud traversed. It is found that the radar-echo characteristics, as shown on a height-range display tube, can be used to indicate the places in a cloud where turbulence is likely to be the most severe. Edges of the echoing volume in particular are shown to be regions of severe turbulence. Severe gusts frequently occur in sequences with the distance between individual gusts less than 400 yd.

Although most of the severe gusts occurred in the echo-producing regions of the clouds there was nevertheless a substantial number of severe gusts experienced in clouds, or parts of a cloud, which failed to give a detectable radar echo.

Introduction.—It has long been recognized that the type of clouds in which turbulence is most severe is the convective type, especially cumulus and cumulonimbus. Because of their characteristic appearance it has normally been possible for the pilot of an aircraft to recognize these clouds in sufficient time to avoid them. Occasionally, however, in day-time, intervening layers of cloud may obscure the cumulonimbus cloud from view, while at night, unless lightning occurs, the chance of avoiding the clouds is greatly reduced. The result has been that cumulonimbus clouds have frequently been entered unexpectedly, with the probability that the effects of turbulence have been greatly increased by the pilot's control of the aircraft.

The advent of radar and the discovery that radar echoes can be detected from clouds and precipitation which contain sufficient numbers of drops of great enough size, have led to the suggestion that the radar characteristics of a cloud may be a useful substitute for the visual ones in the detection of turbulent clouds.

To explore this possibility and to obtain reliable statistics of the degree of turbulence encountered in convective clouds in this country, a series of experiments has been undertaken at the Meteorological Office radar station at East Hill (near Dunstable, Bedfordshire) in co-operation with the Aerodynamics Flight of the Royal Aircraft Establishment at South Farnborough.

This report gives the results of the first phase of the experiments in which Spitfire aircraft were used. The second phase employed twin-seater Meteor aircraft.

Radar equipment.—The radar used comprised two separate equipments, a height-range indicator and a plan position indicator. Both equipments operated on a wave-length of 10 cm. with a peak-power output of 500 Kw. and a pulse length of 2 μ sec. The beam widths of the equipments, measured to half-power, are $1\frac{1}{2}°$ and $7\frac{1}{2}°$ in the vertical and horizontal planes respectively for the height-range indicator, and $7\frac{1}{2}°$ and $1\frac{1}{4}°$ respectively for the plan position indicator, i.e. the height-range indicator gives the better discrimination in the vertical plane and the plan position indicator in the horizontal plane.

Radar echoes from cumulus and cumulonimbus clouds.—The writer has given elsewhere[1, 2] a description and interpretation of the various types of weather echoes as viewed in the vertical cross-section, and has shown how it is possible, after some experience, to distinguish cumulonimbus and cumulus weather echoes from those associated with other cloud formations. The outstanding characteristic of the cumulonimbus type of echo, as seen on the height-range display tube, is the appearance of the echo in the form of a strong vertical column with sharply defined vertical edges, or, in more extensive formations, a number of strong columns separated by well marked troughs. Decay of a cumulonimbus is indicated by a tendency for the edges and tops of the echo columns to become less sharply defined, and in the later stages there is the appearance of a bright band near the freezing level, with the echo losing its well defined column structure.

It is important, however, to note that the echo received does not indicate the whole volume of the cloud, but only that part of the cloud in which the drop sizes and concentration are great enough to give a detectable signal at the radar receiver. The intensity of the signal received from a volume containing water drops has been shown by Ryde[3] to be proportional to Σnd^6 where n is the number of drops of diameter d per unit volume, so that one single raindrop of diameter 2 mm., say, can give an echo equivalent to that from a million cloud particles of diameter 0·2 mm.

With the equipment used at East Hill experience has shown that the minimum detectable signal is such that it is highly probable that, at ranges at which the experiments were performed (over 10 miles), echoes are only received from cumulus and cumulonimbus clouds which are precipitating, or which contain raindrops suspended in the vertical current. Clouds or parts of clouds in which the drop size has not reached raindrop size are not likely to give an echo with this equipment at the ranges used for these experiments.

The radar, therefore, will indicate the presence of precipitating cumulus and cumulonimbus clouds, and hence indicate the general areas where turbulence is likely to be severe, but for the maximum use to be made of the radar it is necessary to investigate whether the part of the cloud which gives an echo is the most turbulent, and also whether variations in the echo intensity have their counterpart in turbulence variations. It is not immediately obvious that large drops in a cloud (and hence a radar echo) and heavy turbulence must be related, but, in general terms, it might be reasonable to expect that the largest drops would occur in or very near the strongest up-currents, since in such currents the drops will be a longer time within the cloud and hence have a greater chance of growth. Turbulence, however, is presumably related to the way in which the vertical currents change in the horizontal, so that, for example, passage from a down-current to an up-current would be expected to give a severe bump if the edges of the up- and down-currents were sufficiently clearly defined with only a very narrow zone of transition. The large-scale currents themselves may be " solid " currents with little horizontal change in speed, and hence have relatively small turbulence compared with that encountered at the edges of the current.

Experimental procedure.—Spitfire aircraft from the Aerodynamics Flight of the Royal Aircraft Establishment investigated the turbulence in cumulus and cumulonimbus clouds under the control of the radar station at East Hill.

The instrumental equipment of the Spitfire included a vertical acceleration recorder with a time base; the recorder could be switched on and off by the pilot, and it was possible to get a total of about 20-min. record on any one sortie. All cloud flying was done at an indicated airspeed of 180 m.p.h. The pilot was briefed to keep his attitude both in pitch and roll as constant as possible when traversing the clouds. Owing to the stability characteristics of the aircraft only small corrections were necessary to maintain attitude in pitch, but considerably more effort was required to keep the aircraft laterally level.

On each sortie the aircraft, after identification on the radar display tubes, was controlled by R/T from East Hill and the pilot was given vectors along which to steer to put him close to a chosen echo. Normally the pilot would be able to recognize the cloud from part of which the echo was being received, and he was then given the height at which to fly through the cloud and the course to steer while in the cloud. This course was chosen so that the aircraft flew either directly towards or directly away from East Hill; this meant that the aircraft always investigated the cloud along the line of the radar beam of the height-range equipment. A vertical cross-section of the radar weather echo along the line of flight of the aircraft was, therefore, visible on the height-range display tube, as well as the echo from the aircraft as long as it was outside the weather echo. The pilot switched on his accelerometer immediately on entering cloud, and off again as soon as he was clear of cloud on the other side, the instants of switching on and off being communicated by R/T to East Hill. Frequent readings and photographs of the height-range tube were taken at East Hill during the aircraft traverse, and particular note made of the instants of entry into and departure from the echo-producing region. A number of such flights at different levels would be made through a single cloud. If time permitted other clouds giving rise to echoes were also investigated, while on some occasions the pilot was directed to an area clear of echo with instructions to make flights through the best developed cumulus he could find. This enabled some comparison to be made of the turbulence in clouds giving rise to an echo with that in clouds which gave no echo at East Hill.

Shortcomings of the radar in defining the vertical cross-section of the radar echo from cumulonimbus clouds.—It is assumed in this report that the vertical cross-section of the radar echo as seen on the height-range display tube is a true cross-section of the radar echo in the vertical plane enclosing the line of flight of the aircraft. This is not strictly true for two reasons:—

(i) the aircraft may not fly directly along the radar beam throughout the radar echo

(ii) the radar beam is not a narrow pencil beam but a divergent beam having a beam width (measured to half-power) of 7½° in the horizontal and 1½° in the vertical plane.

The effect of (i) was minimized by giving the pilot a course to steer through the cloud which would take him either directly towards or away from the station, and hence cause him to fly along the axis of the radar beam. Occasionally, however, the effect of wind drift might cause him to deviate slightly from the course given. This effect was not considered to have been serious in the great majority of the traverses made.

The effect of (ii), however, is potentially more serious because of the horizontal beam width of 7½°. At 20 to 30 miles range, at which range the majority of traverses were made, the beam width will vary from about 2½ miles at 20 miles range, with a depth of 2,600 ft., to 4 miles at 30 miles range with a depth of

4,200 ft. By keeping the radar response from the aircraft at its maximum value it was usually possible to be sure that the aircraft flew along the centre of the beam. The weather response, however, shown at a certain range and height on the display tube was the sum of all the responses at that range from the part of the cloud and precipitation within the beam. The response from a certain volume distribution of raindrops at the centre of the beam will, of course, be greater than that from the same volume distribution towards the edge of the beam, so that the portion of the cloud intercepted near the centre of the beam will frequently have a predominant effect on the response as seen on the height-range tube at that range. Nevertheless there may be occasions when a higher concentration of raindrops of greater size exists towards the edge of the beam than in the centre. On these occasions the response on the height-range tube will be partly, and perhaps sometimes principally, due to the response from a part of the cloud not in the centre of the beam and not therefore on the aircraft track.

No attempt has been made in this report to assess the effect of beam width on the results obtained, and indeed it is doubtful whether any assessment could be made. It may be noted, however, that the frequency with which apparent entry into an echoing volume was accompanied by a marked change in precipitation rate, reported by the pilot on R/T, and the correlation of major gusts with recognizable echo discontinuities (see below) makes it unlikely that serious errors have been introduced.

Accelerometer records.—Accelerations imparted to the aircraft were recorded photographically on 35-mm. film, the acceleration scale being 0·095 in. for 1·0*g* and the time scale about ½ in./sec., ¼ sec. time marks being recorded.

The analysis of the accelerometer records was made by the Aerodynamics Flight, and the results of the analysis shown in the form of a table for each traverse through a cloud, giving the number of gusts of various magnitudes, up and down gusts being shown separately. The numbers were determined at intervals of 0·1*g*, *g* being the acceleration due to gravity. Accelerations less than 0·1*g* were not measured, so that numbers of gusts giving the aircraft acceleration in the intervals 0·1 to 0·2*g*, 0·2 to 0·3*g*, 0·3 to 0·4*g*, etc., were found. In graphical presentations of the results the lower value of the acceleration range is used throughout this report. A gust giving the aircraft an acceleration *xg* is spoken of as a gust of *xg*.

For a " sharp-edged " gust, i.e. a gust the profile of which is such that the change in vertical speed takes place instantaneously at some point on the horizontal track of the aircraft, it is possible, from the characteristics of the aircraft, to calculate the vertical speed of the gust from the acceleration experienced by the aircraft. In practice, however, the gusts are not sharp-edged, but the vertical speed builds up to a maximum value over a variable horizontal distance. It has proved useful to the aerodynamist to assume an average distance of 100 ft. over which the gust builds up steadily to its maximum value, and this assumption introduces an " alleviation factor " (an empirical factor varying with the wing loading of the aircraft) which can be applied to the sharp-edged gust formula to obtain an equivalent vertical gust speed. For the Spitfire aircraft used an acceleration increment of 0·1*g* is equivalent to an increase in equivalent vertical air speed of 3·6 ft./sec. As different values of alleviation factor are used in America it is necessary in comparing these results with American results[4] to multiply the American values of gust speeds[5] in feet per second by about 1·7.

In a further analysis of the accelerometer records the points of occurrence of all gusts of 0·4*g* or greater, measured in seconds from the start of the accelerometer record, were recorded.

Data available.—Spitfire aircraft were available from August 1947 to November 1950 and during this period sorties were flown on 40 different days. The total number of cloud traverses for which accelerometer readings are available was 205, involving a total distance flown in cloud of about 1,175 miles. For most of these traverses there are also photographs of the radar height-range display tube, with records of the times of entry and departure of the aircraft from cloud and echoing volume. It is possible, therefore, to relate most of the accelerometer records to the radar echo received from the vertical cross-section of the cloud along the line of flight of the aircraft.

The data have been analysed in two ways :—

(i) The gust information from the accelerometer records has been used as purely statistical data to give the frequencies of gusts of various magnitudes and their variation with height. The results are given on pp. 6–15.

(ii) The turbulence deduced from the accelerometer records and its intensity has been compared with the recognizable features of the radar echo received at the time the accelerometer record was made. Results of this analysis are given on pp. 15–18.

Information obtained by method (i) can be compared with similar American results[4] to give an indication of the variation of turbulence with locality; the American observations, however, do not allow comparison by method (ii), since the exact position of the aircraft relative to the radar weather echo at any instant and the vertical cross-section of the radar echo at that instant were apparently not recorded in the United States project.

Frequency of occurrence of gusts of various magnitudes during all flights irrespective of height.—*Through clouds giving rise to a radar echo.*—Fig. 1 shows the result of plotting $\log N$, where N is the number of positive or negative gusts of magnitude $0 \cdot 1gn$ to $0 \cdot 1g\ (n+1)$ reported, n being an integer, against n for all flights, at all heights, which went through clouds from part of which a radar echo was received. This refers to a total distance of $1 \cdot 9747 \times 10^6$ yd.

It will be seen that the points plotted lie very close to the line

$$\log N = 4 \cdot 03 - 0 \cdot 328n,$$

and hence if N_0 is the number of gusts per 1,000 yd. of magnitude $0 \cdot 1gn$ to $0 \cdot 1g\ (n + 1)$

$$N_0 = 5 \cdot 426 \times 10^{-0 \cdot 328n}$$

alternatively the number of yards to be flown, N_d, before encountering a gust of magnitude $0 \cdot 1gn$ to $0 \cdot 1g\ (n+1)$ is

$$N_d = 184 \cdot 7 \times 10^{0 \cdot 328n},$$

and the average total number of gusts of all sizes greater than $0 \cdot 1g$ per 1,000 yd. likely to be experienced is

$$\sum_{n=1}^{\infty} 5 \cdot 426 \times 10^{-0 \cdot 328n} = 4 \cdot 81.$$

Through clouds which failed to give an echo.—Fig. 2 shows a similar graph for those flights which went through clouds which failed to give an echo. The total distance flown was $9 \cdot 9753 \times 10^4$ yd.

With the notation above it is seen that the points fit reasonably the curves:

$$N_0 = 6 \cdot 78 \times 10^{-0 \cdot 363n}$$

$$\text{or } N_d = 147 \cdot 5 \times 10^{0 \cdot 363n}.$$

The comparison given in Fig. 4 of this curve with the curve of Fig. 1 indicates that gusts up to about $0 \cdot 3$ to $0 \cdot 4g$ are slightly more frequent in clouds which give no echo than in clouds which do, but that the larger and more

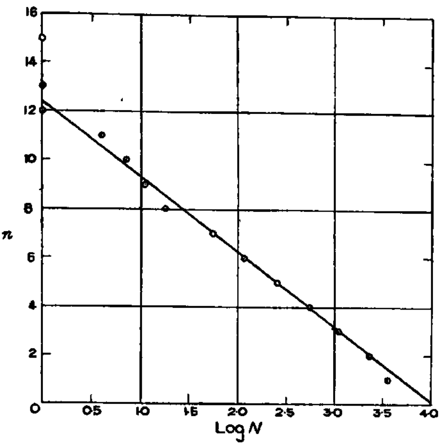

FIG. 1—FREQUENCY OF OCCURRENCE OF GUSTS OF VARIOUS MAGNITUDES IN CLOUDS GIVING RISE TO A RADAR ECHO

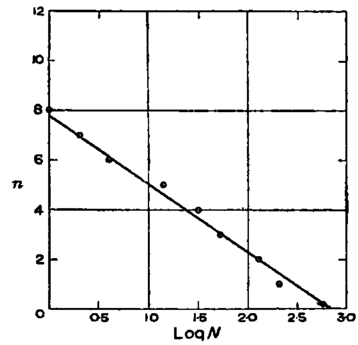

FIG. 2—FREQUENCY OF OCCURRENCE OF GUSTS OF VARIOUS MAGNITUDES IN CLOUDS WHICH FAILED TO GIVE A RADAR ECHO

dangerous gusts are less frequent. Thus, gusts of 1·0 to 1·1*g* occur about 1·8 times more frequently in clouds from part of which an echo is received than in clouds which give no echo.

Comparison with American results.—Table 13 of "The thunderstorm"[4] gives frequency distributions of maximum effective gust speeds in 1,000-yd. intervals for 6,633·3 miles of flying through thunderstorms over Florida. Only the maximum gust, positive or negative, in each 1,000 yd. is used for this table. Thus, when a large gust was recorded there were probably other smaller gusts in the same 1,000 yd. which would not be recorded, so that as a frequency table of all gusts it would be expected to be very nearly true for the higher gust values with a progressively greater departure from the true value (fewer gusts reported than actually occurred) as the gust value decreased. It is possible to make adjustments to the distributions reported to obtain an approximate frequency table for all gusts by making use of Fig. 35 of "The

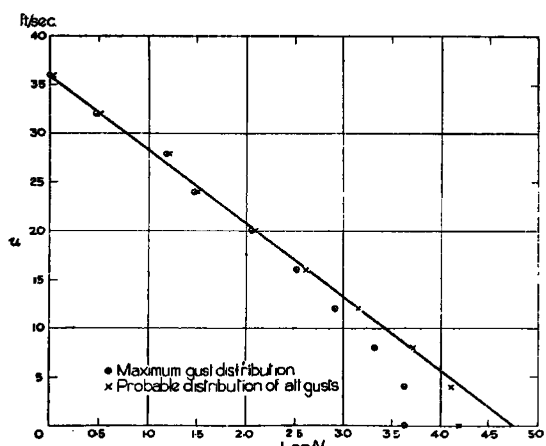

FIG. 3—FREQUENCY OF OCCURRENCE OF MAXIMUM GUSTS PER 1,000 YD. AND PROBABLE DISTRIBUTION OF ALL GUSTS EXPERIENCED ON FLIGHTS THROUGH FLORIDA THUNDERSTORMS[4]

thunderstorm "[4], which is a scatter diagram relating the speed of the maximum gust in a 1,000-yd. interval to the total number of gusts in that interval. Thus, when the maximum gust was 12–15·9 ft./sec., say, there were, on the average, about 6 gusts in the 1,000-yd. interval, i.e. 5 other gusts smaller than the maximum gust of the interval; 848 maximum gusts of 12–15·9 ft./sec. were recorded, and these were, therefore, associated with about 5 × 848 = 4,240 smaller gusts which were not recorded. If these 4,240 gusts were distributed among the gusts smaller than 15·9 ft./sec. in the same proportion as the maximum gusts the distributions of maximum gusts can be adjusted to allow for these gusts. Similarly for all the other gust intervals. The distribution so obtained, together with the maximum gust distribution of Table 13 of " The thunderstorm "[4] are shown in Fig. 3.

It will be seen that the frequencies of higher values of maximum gust speeds and the adjusted distribution, obtained as described above, fit reasonably

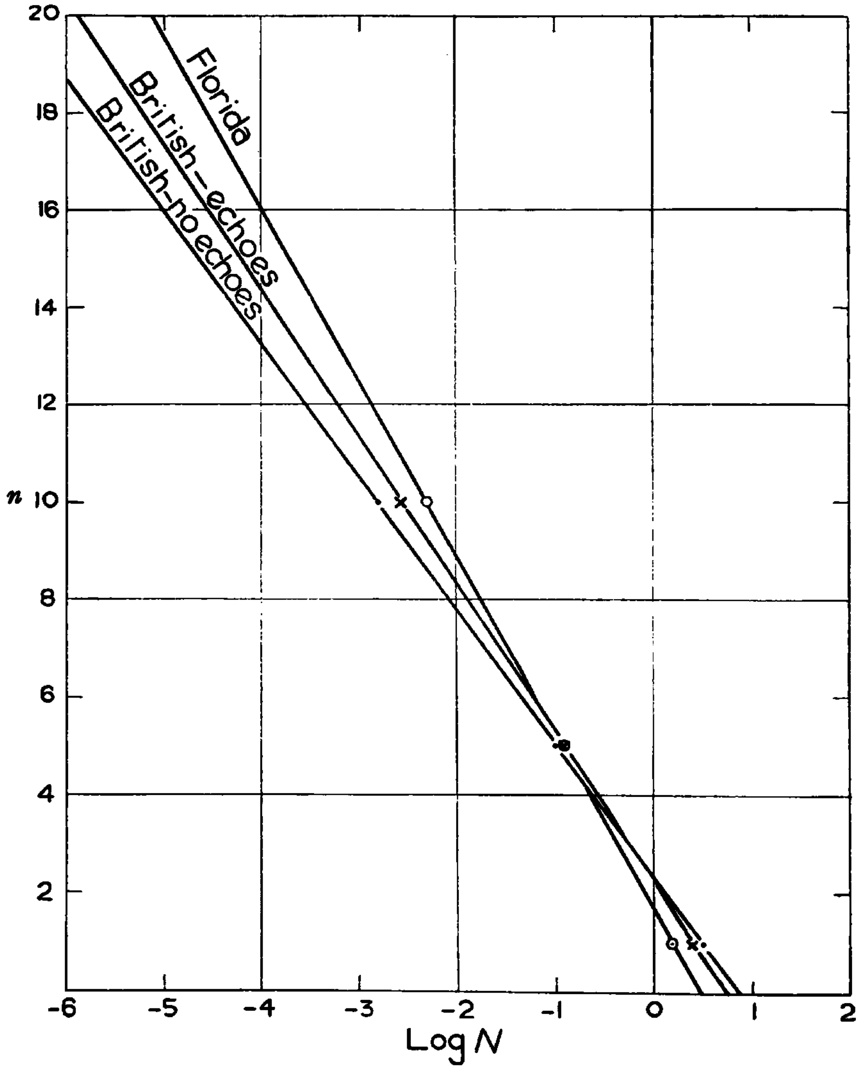

FIG. 4—COMPARISON OF GUST FREQUENCIES IN ENGLAND AND FLORIDA

the curve $N_u = a \times 10^{-bu}$ where a and b are constants and N_u is the number of gusts with speed u to $u + 4$ ft./sec. It is reasonable to hope, therefore, that this curve is a true representation of the gust frequencies. To convert the equation to this curve into the same form as that used for the results on p. 6 it is necessary to change the American gust speeds into accelerations which the Spitfire would have experienced, and change the interval over which the number of gusts is taken from 4 ft./sec. to $0 \cdot 1g$. According to Hislop[5] American speeds should be multiplied by $1 \cdot 7$ to give equivalent British speeds.

In British notation an acceleration of $0 \cdot 1g$ is equivalent to a vertical gust speed of $3 \cdot 6$ ft./sec., and therefore to $3 \cdot 6/1 \cdot 7 = 2 \cdot 12$ ft./sec. in American notation (see also p. 5).

The expression for the American results then becomes :—

$$N_0 = 2 \cdot 963 \times 10^{-0 \cdot 279n}$$

$$N_d = 338 \times 10^{0 \cdot 279n}.$$

This curve is also shown in Fig. 4, and it will be seen that gusts up to about $0 \cdot 6g$ are more frequent in England than in Florida, but that the gusts larger than about $0 \cdot 6g$ become more frequent in Florida. Thus gusts of $0 \cdot 3$ to $0 \cdot 4g$ are about $1 \cdot 3$ times as frequent in England as in Florida, but gusts of $1 \cdot 1$ to $1 \cdot 2g$ are about twice as frequent in Florida.

Therefore, comparing the severity of turbulence in the two areas by the frequency of occurrence of the rare, but very severe, gusts the turbulence is more severe in Florida than in England ; but comparing turbulence severity by the frequency of occurrence of all gusts greater than a certain value, then it can be shown that the turbulence is more severe in England as long as the critical value is $0 \cdot 4g$ or less. The severity is about the same if the critical value is taken as gusts of magnitude $0 \cdot 4$ to $0 \cdot 6g$, and turbulence is more severe in Florida if gusts greater than $0 \cdot 6g$ only are considered. The frequency of occurrence of all gusts of magnitude $0 \cdot 4g$ or greater is on the average $0 \cdot 5$/1,000 yd., or once in every 2,000 yd. of cumulonimbus cloud flying, for both Florida and England.

Variation of gust frequency with height.—*East Hill traverses.*—In an attempt to find whether there was any variation in turbulence with height the data were assessed according to the height of the traverses above sea level.

The heights at which traverses were made were divided into intervals of 5,000 ft., from 2,500 ft. upwards and the number of gusts of values $0 \cdot 1$ to $0 \cdot 2g$, $0 \cdot 2$ to $0 \cdot 3g$, etc., irrespective of sign, extracted. The data for each height interval are shown in Table I and are plotted in Fig. 5.

It is found in all cases that a curve of the form $N = a \times 10^{-bn}$ fits the points reasonably, where N is the number of gusts of magnitude $0 \cdot 1gn$ to $0 \cdot 1g\,(n+1)$, and a and b are constants for a particular height interval but vary with height intervals. For comparison the curves so found were reduced to give the number of gusts per 1,000 yd., $N_0 = a \times 10^{-bn}$. The way in which the turbulence varies with height can then be deduced from the variations in a and b with height.

The values of a and b for various height intervals are shown in Table II. The total number of gusts per 1,000 yd. greater than $0 \cdot 1g$ in any height interval is given by

$$\Sigma N_1 = \sum_{n=1}^{\infty} a \times 10^{-bn}$$

$$= \frac{a}{10^b - 1}.$$

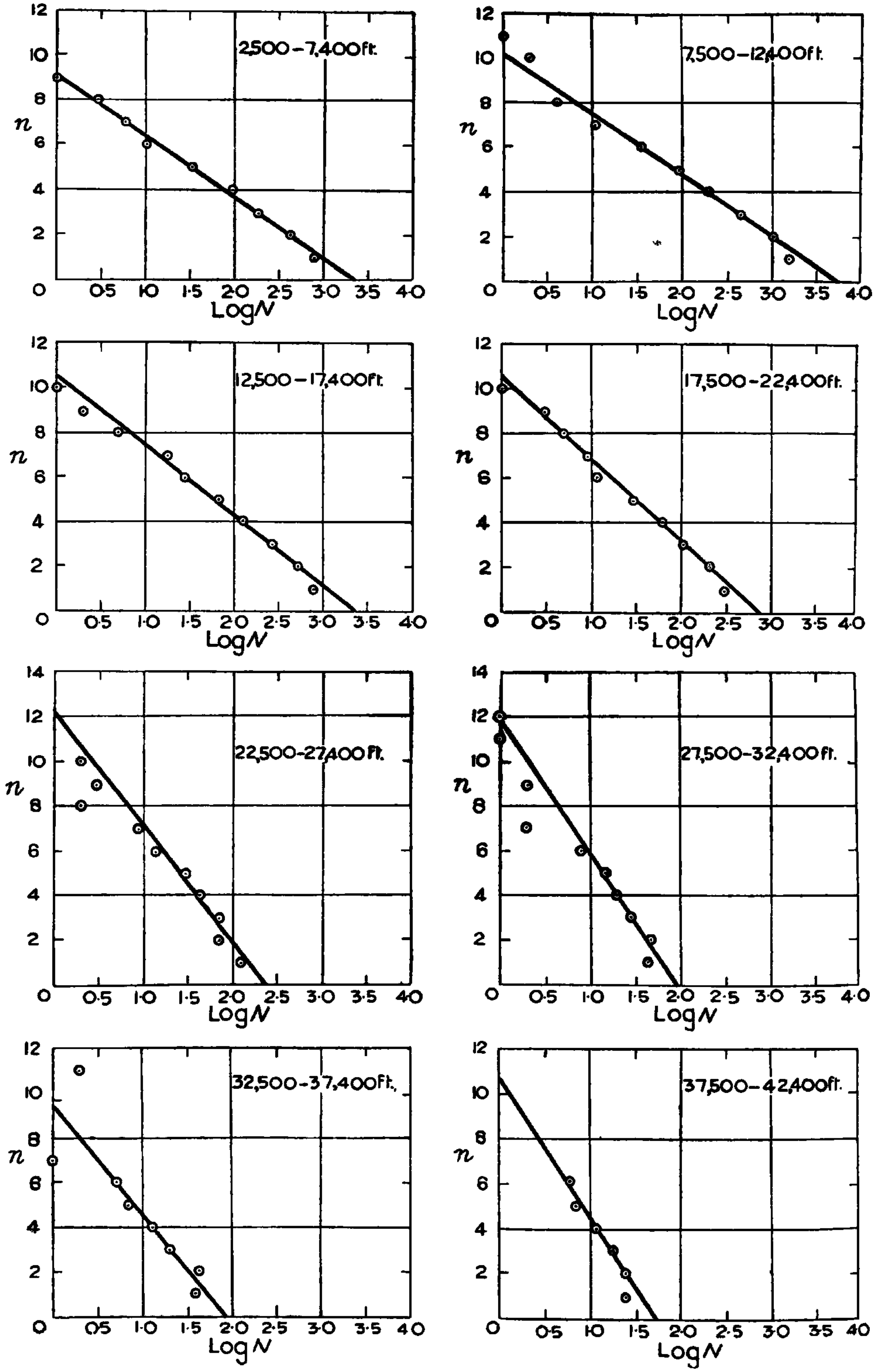

FIG. 5—VARIATION OF GUST FREQUENCY WITH HEIGHT

TABLE I—FREQUENCIES OF OCCURRENCE OF GUSTS OF VARIOUS MAGNITUDES IN VARIOUS HEIGHT INTERVALS ABOVE SEA LEVEL

Height above M.S.L.	±0·1 to 0·2g	±0·2 to 0·3g	±0·3 to 0·4g	±0·4 to 0·5g	±0·5 to 0·6g	±0·6 to 0·7g	±0·7 to 0·8g	±0·8 to 0·9g	±0·9 to 1·0g	±1·0 to 1·1g	±1·1 to 1·2g	±1·2 to 1·3g	±1·3 to 1·4g	±1·4 to 1·5g	±1·5 to 1·6g	Distance flown in height interval	No. of flights
ft.	*number of gusts*															yd.	
37,500–42,400	26	25	18	12	7	6	0	0	0	0	0	0	0	0	0	24,040	1
32,500–37,400	41	44	21	13	7	5	1	0	0	0	2	0	0	0	0	35,930	2
27,500–32,400	46	48	29	20	15	8	2	0	2	0	1	1	0	0	1	39,270	2
22,500–27,400	128	71	69	44	30	14	9	2	3	2	0	0	0	0	0	81,270	5
17,500–22,400	309	211	107	64	29	12	9	5	3	1	0	0	0	0	0	191,170	15
12,500–17,400	775	537	276	128	68	28	18	5	2	1	0	0	1	0	0	424,470	45
7,500–12,400	1,626	1,046	473	205	91	36	11	4	0	2	1	0	0	0	0	871,705	87
2,500– 7,400	799	425	183	91	34	11	6	3	1	1	0	0	0	0	0	399,664	48

TABLE III—FREQUENCIES OF OCCURRENCE OF GUSTS OF VARIOUS MAGNITUDES AT VARIOUS HEIGHTS RELATIVE TO THE RADAR-ECHO TOP

Ratio : echo top / aircraft height	±0·1g to 0·2g	±0·2g to 0·3g	±0·3g to 0·4g	±0·4g to 0·5g	±0·5g to 0·6g	±0·6g to 0·7g	±0·7g to 0·8g	±0·8g to 0·9g	±0·9g to 1·0g	±1·0g to 1·1g	±1·1g to 1·2g	±1·2g to 1·3g	±1·3g to 1·4g	±1·4g to 1·5g	±1·5g to 1·6g	Distance flown	No. of flights
	number of gusts															yd.	
0·40–0·59 (mean 0·53)	29	25	4	2	0	0	0	0	0	0	0	0	0	0	0	12,020	2
0·60–0·79 (mean 0·74)	27	28	13	7	6	1	2	0	0	0	0	0	0	0	0	20,480	5
0·80–0·99 (mean 0·91)	179	76	35	14	5	3	0	0	0	0	0	0	0	0	0	81,226	12
1·00–1·19	470	314	136	73	29	19	3	0	2	0	2	0	0	0	0	318,954	32
1·20–1·39	430	283	116	63	30	12	4	3	0	2	1	1	0	0	1	283,128	22
1·40–1·59	348	205	93	38	31	8	6	5	2	2	0	0	0	0	0	178,550	14
1·60–1·79	430	260	180	72	46	23	13	0	3	0	0	0	0	0	0	236,330	19
1·80–1·99	245	190	79	51	26	10	4	4	1	1	0	0	0	0	0	138,312	11
2·00–2·19	434	281	167	92	42	20	12	4	2	1	1	0	0	0	0	220,776	19
2·20–2·39	130	110	61	26	5	3	1	1	0	0	0	0	0	0	0	71,080	6
2·40–2·59	239	167	68	36	18	7	2	0	0	0	0	0	0	0	0	102,130	9
2·60–2·99	179	109	42	14	7	4	2	0	0	0	0	0	1	0	0	85,530	7
3·00–3·40 (mean 3·14)	130	70	27	17	7	3	1	0	0	0	0	0	0	0	0	76,760	7
4·00–4·20 (mean 4·04)	104	75	58	21	11	3	3	1	1	1	0	0	0	0	0	51,710	5

Similarly, the total number of gusts greater than 0·4*g* and 0·8*g* are given by ΣN_4 and ΣN_8 respectively, where

$$\Sigma N_4 = \frac{a}{10^{4b} - 10^{3b}}$$

$$\text{and } \Sigma N_8 = \frac{a}{10^{8b} - 10^{7b}}.$$

Values of ΣN_1, ΣN_4 and ΣN_8 for the various height intervals are also shown in Table II, and, together with *a* and *b*, are plotted in Fig. 6.

TABLE II—VALUES OF THE COEFFICIENTS a, b IN THE EQUATION $N_0 = a \times 10^{-bn}$ AND OF ΣN_1, ΣN_4, ΣN_8

Height above M.S.L. (mean of 5,000-ft. layer)	a	b	ΣN_1	ΣN_4 gusts per 1,000 yd.	ΣN_8
ft.					
40,000	2·19	0·162	4·85	1·582	0·356
35,000	2·69	0·204	4·48	1·095	0·167
30,000	2·32	0·165	5·01	1·602	0·351
25,000	2·69	0·192	4·84	1·282	0·219
20,000	3·66	0·270	4·25	0·658	0·056
15,000	5·54	0·322	5·04	0·545	0·028
10,000	6·60	0·369	4·95	0·387	0·013
5,000	5·60	0·367	4·21	0·334	0·011

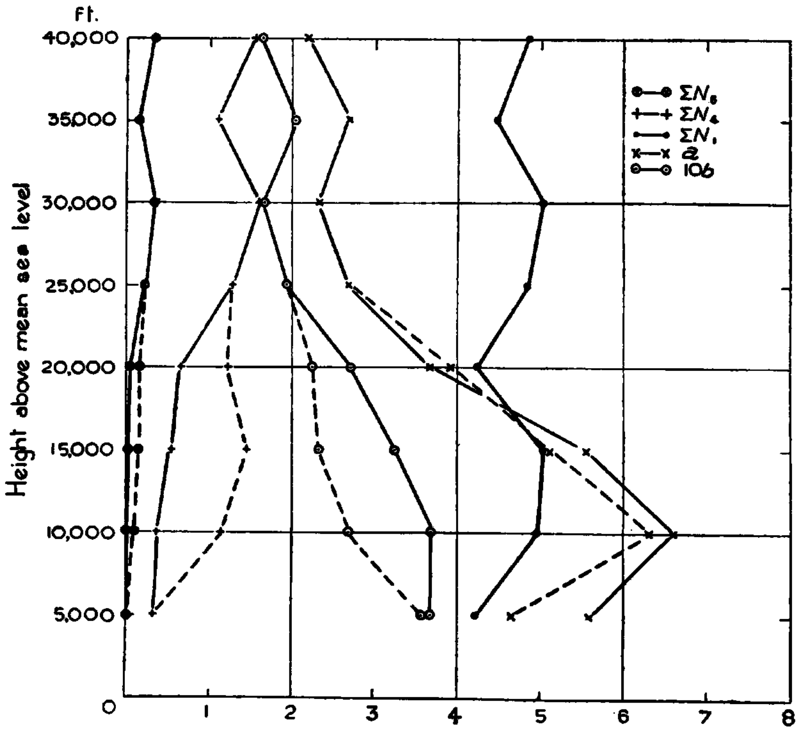

FIG. 6—VARIATION OF GUST FREQUENCY WITH HEIGHT ABOVE SEA LEVEL

Dotted curves refer only to clouds through which traverses were made at heights greater than 22,500 ft.

While the total number of gusts greater than $0 \cdot 1g$ experienced varies little with height (four to five gusts per 1,000 yd. at all heights) there is clearly a tendency for the frequency of larger gusts to increase steadily with height, with a marked increase in frequency at heights above 22,500 ft. Thus, in the height interval 22,500–27,400 ft. gusts greater than $0 \cdot 8g$ in magnitude were four times more frequent than in the height interval 17,500–22,400 ft., and these in turn were four to five times more frequent than in the intervals from 2,500 to 12,500 ft. It is to be noted, however, that there is a substantial decrease in the number of flights and distance flown in the 22,500–27,400-ft. interval compared with the lower intervals (see Table I) with, therefore, some decrease in the reliability of the statistics. The small number of traverses flown above 22,500 ft. is an indication of the infrequency with which cumulonimbus tops exceeded this height during the experiments.

The traverses made at heights greater than 22,500 ft. suggest that the cumulonimbus clouds which do exceed this height have a greater frequency of large gusts at all heights than the smaller clouds. This is confirmed by extracting those traverses made at lower levels through the same clouds which furnish the statistics for levels above 22,500 ft. The curves of *a* and *b* obtained for these clouds are shown dotted in Fig. 6, indicating (from the curve for *b*—the curve for *a* being little altered) that the larger gusts are more frequent at all levels in these larger clouds. The slope of the dotted curve for *b* indicates, nevertheless, that the frequency of large gusts, even in the big clouds, increases with increasing height.

Comparison with American results.—American results[4] extend up to 26,000 ft. above M.S.L., and were made through clouds of great vertical extent in an area where such clouds are of frequent occurrence. For the larger gusts[4] there appeared to be a tendency for maximum frequency in the neighbourhood of the freezing level, but no great significance was attached to this and it was assumed[4] that " lacking more conclusive data, the mean gust speeds increase with height in a thunderstorm cell up to an altitude of 5,000–10,000 ft. below the top of the cloud ".

The results here presented do not conflict with this view, but, with the greater variability in vertical extent of the cloud experienced in England, the analysis presented above does not enable the relationship of turbulence maximum, if any, to distance from cloud top to be determined. The British results were, therefore, re-assessed in terms of the height of the traverse relative to the radar echo top, which is assumed for this purpose to be sufficiently close to the true cloud top. This analysis then transfers the emphasis from height above sea level to height within the cloud with a view to discovering the most turbulent portion of the cumulonimbus cloud whatever its vertical extent.

The results of this analysis follow.

Variation of gust frequency with height relative to the radar-echo tops.—*East Hill data.*—The results of this analysis for various intervals of the ratio, height of echo top to height of traverse, are shown in Tables III (p. 11) and IV. The method of analysis was precisely the same as that given on p. 9, and the quantities *a*, *b*, ΣN_1, ΣN_4, ΣN_8 have the same significance. These quantities are plotted in Fig. 7.

The results show a zone of maximum turbulence, if gusts greater than $0 \cdot 4g$ are considered, when the ratio of echo-top height to aircraft height is between $1 \cdot 60$ and $2 \cdot 20$. For the smaller gusts the zone of maximum frequency is spread over a greater range, from $1 \cdot 60$ to $2 \cdot 60$, which includes the range over which the larger gusts are also most frequent. High values of frequency of both large

TABLE IV—VALUES OF THE COEFFICIENTS a, b IN THE EQUATION $N_0 = a \times 10^{-bw}$ FOR VARIOUS INTERVALS OF THE RATIO, HEIGHT OF ECHO TOP TO HEIGHT OF TRAVERSE, AND OF THE VALUES ΣN_1, ΣN_4, ΣN_8

Ratio : Height echo top / Height traverse	a	b	ΣN_1 (gusts per 1,000 yd.)	ΣN_4 (gusts per 1,000 yd.)	ΣN_8 (gusts per 1,000 yd.)	Distance flown	Average distance per traverse	No. of flights
						yd.	yd.	
4·00–4·20 (mean 4·04)	5·21	0·283	5·670	0·803	0·0593	51,710	10,342	5
3·00–3·39 (mean 3·14)	4·03	0·332	3·520	0·355	0·0167	76,760	10,966	7
2·60–2·99	5·10	0·338	4·330	0·419	0·0186	85,530	12,219	7
2·40–2·59	6·18	0·315	5·800	0·658	0·0362	102,130	11,348	9
2·20–2·39	6·73	0·348	5·480	0·495	0·0201	71,080	11,847	6
2·00–2·19	5·32	0·290	5·600	0·756	0·0522	220,776	11,620	19
1·80–1·99	4·78	0·285	5·150	0·719	0·0521	138,312	12,574	11
1·60–1·79	4·53	0·278	5·050	0·741	0·0572	236,330	12,438	19
1·40–1·59	3·70	0·285	3·990	0·557	0·0403	178,550	12,754	14
1·20–1·39	3·96	0·321	3·616	0·394	0·0205	283,128	12,869	22
1·00–1·19	4·43	0·335	3·805	0·376	0·0172	318,954	9,967	32
0·80–0·99	5·02	0·368	3·766	0·296	0·0100	81,226	6,769	12

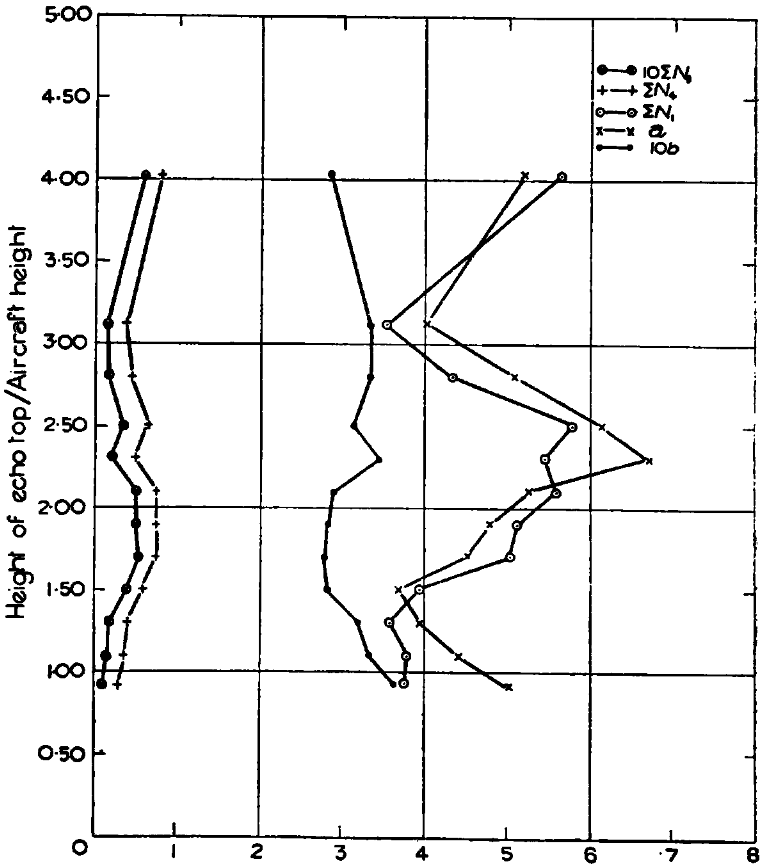

FIG. 7—VARIATION OF GUST FREQUENCY WITH HEIGHT RELATIVE TO THE HEIGHT OF THE RADAR-ECHO TOPS

and small gusts are also found when the ratio is 4·00 to 4·20, but these frequencies are based on a small number of flights and may not be significant. The question is important, however, in that the recommended practice is for aircraft to fly through thunderstorms at the lowest height consistent with safety, i.e. at high values of the ratio echo-top height to aircraft height. Apart from this doubtful point it appears probable that the turbulence increases upwards in the cloud until rather less than half way up the cloud ($1/2{\cdot}2 = 0{\cdot}45$), and thereafter remains near its maximum until about two-thirds ($1/1{\cdot}60$) of the way up the cloud. In the top third of the cloud (strictly the top third of the part of the cloud giving an echo) the turbulence gradually decreases upwards until near the echo top—which is probably very close to the cloud top in an active cloud—turbulence is at a minimum.

Comparison with American results.—The mean maximum radar-echo top observed in 199 observations of the American Thunderstorm Project[4] in Ohio (not necessarily of the storms or parts of the storms actually traversed) was 37,000–38,000 ft. The proportions 1/2·2 and 1/1·6 of 37,500 ft. are 16,900 and 23,500 ft., i.e. if the deductions above apply in Ohio, the turbulence measured there would be expected to show a maximum between 16,900 and 23,500 ft. The maximum turbulence would, however, appear to have been between 10,000 and 15,000 ft. above M.S.L. in Ohio[4], although between 16,000 and 21,000 ft. in Florida[4].

If, however, the Ohio results for each level of flight are smoothed in the same way as the combined Florida results (i.e. by forming an equation of the form $N = a \times 10^{-bn}$ for each level where n is now in feet per second of equivalent gust speed and N is the total number of gusts of speed n to $n + 4$) then it is found that the factor b, which is the dominating factor in the frequency of the large gusts, varies little from 10,000 ft. upwards being 0·123, 0·108, 0·099, 0·105, 0·101 at 5,000, 10,000, 15,000, 20,000 and 25,000 ft. respectively; 10,000 and 25,000 ft. would give ratios of mean maximum echo-top height to aircraft height of 3·75 and 1·5 respectively. A further point is that the mean echo-top height of the storms actually flown is likely to be less than the mean maximum echo top of the 199 storms measured.

Cloud diameters.—An interesting point arising from the data as analysed in this section is the constancy of the average distance flown per traverse for differing ratios of echo-top height to height of traverse (see Table IV). This suggests that on the average the cumulonimbus cloud may be regarded as a cylinder over a large part, from about 0·3 or less to about 0·8, of its vertical extent. The average diameter is 11,900 yd., approximately 6·8 miles. The average diameter of the Florida and Ohio storms[4] would appear to have been 9·0 miles (traverses 1,363, total miles flown 12,310·5).

Turbulence in relation to the radar echo.—The turbulence data already obtained can be assessed in relation to the radar echo to see whether easily recognizable features of the radar echo are associated with marked changes in frequency or intensity of gusts. Various aspects of this problem are dealt with below.

General appearance of the echo in relation to the turbulence encountered.—Experience has shown, as described on p. 3, that the form a cumulonimbus echo takes when seen on the height-range display tube can give a useful idea of the stage of development the cumulonimbus has reached. Echoes of high intensity with sharply defined edges and tops come from developing cumulonimbus clouds, and when the top and sides of the echo become diffuse the cloud has probably passed its maximum development. A sharply defined radar-echo edge, as seen on the display tube, is a characteristic of the echo—

not related to the focussing capabilities of the display tube—and is an indication that the increase in radar-echo intensity from zero to a high value takes place over a horizontal distance too short to be discriminated by the radar, i.e. over a distance of less than half a pulse length—about 300 yd. on the equipment used. Turbulence characteristics would be expected also to be indicated by the appearance of the echo since it is reasonable to expect greater turbulence in a developing cloud than in one which is decaying. In confirmation of this idea it was found that of 44 gusts of $\pm$ 0·8*g* or greater recorded, 34 were encountered in clouds from part of which an echo with sharply defined edges and tops was received (20 with single-column echoes and 14 with echoes having more than one column), 8 were encountered in clouds of which the associated echo showed sharp edges but a diffuse top, and only 1 in a cloud associated with a generally diffuse echo. One gust of 0·8*g* was also encountered in a cumulus cloud from which no radar echo was received.

Conversely only 6 gusts, out of a total of 213 gusts of $\pm$ 0·6*g* or greater recorded in clouds containing an echo, were encountered in a cloud associated with an echo containing a clearly visible bright band, although about 12 per cent. of the traverses were made through such clouds.

Although it should be remembered that equal distances were not flown in clouds associated with the various types of echo it seems clear that a useful deduction on the likelihood of encountering severe gusts in a cloud can be made from the appearance of the associated radar echo on the height-range display tube.

No correlation was found to exist between the magnitude of the maximum gusts encountered within a cloud containing an echo and the vertical extent of the echo. It should be remembered, however, that the analysis on p. 9 indicated the frequency of large gusts to be greater in clouds of greater vertical extent than 22,500 ft.

Places of occurrence of gusts of $\pm$ *0·4g or greater relative to the echo.*—From all the accelerometer records obtained, the points of occurrence (measured in seconds from the start of the run) of the gusts of $\pm$0·4*g* or greater were extracted. For those traverses, therefore, for which detailed correlation with radar-echo photographs was possible, the points of occurrence of these larger gusts relative to the outstanding features of the weather echo could be determined. This has been done for 561 such gusts out of a total recorded of 1,023.

The results of this analysis are shown in Table V. Salient features of this table are as follows:—

(i) The edges of the volume of the cloud from which a radar echo is received are places where the turbulence is likely to be severe. Of the 441 gusts greater than $\pm$ 0·4*g* which occurred within the echo-producing region of the cloud, 243 or 55 per cent. (including those in a narrow column of echo) occurred close to the edges of the echo-producing region. There was a marked preponderance of large positive gusts at the leading edge of the echo and of large negative gusts at the trailing edge, especially for gusts greater than $\pm$ 0·6*g*. The leading edge of the echo is the edge of the echo-producing region first encountered by the aircraft on its traverse. If it is assumed that turbulence is likely to be severe at places where the upward currents change rapidly in value, or even in sign, as seems reasonable, then the inference is strong that the echo-producing region of the cloud is a region where the upward currents are of very different value from those elsewhere in the cloud. The preponderance of large up-gusts at the leading edge and of large down-gusts at the trailing edge suggests that the part of the cloud from which a column of echo is received is frequently associated with a strong up-current.

(ii) Discontinuities in the weather echo as revealed by fluctuations in the echo top, e.g. peaks and troughs, were also indications of places in the cloud where the turbulence was likely to be severe.

(iii) A substantial number of large gusts (21 per cent. of the total located), with a maximum gust of $-1 \cdot 0g$, were encountered in parts of the cloud either above or to the side of the echo-producing region. As regards the gusts experienced in cloud above the echo-producing region, however, the analysis on p. 13 shows the average turbulence experienced to be less than in flights which went through the echo-producing region. It should also be remembered that 53 gusts of $\pm 0 \cdot 4g$ or greater, with a maximum one of $0 \cdot 8$ to $0 \cdot 9g$, also occurred in clouds from no part of which echo was received.

TABLE V—PLACES OF OCCURRENCE OF GUSTS OF $\pm 0 \cdot 4g$ OR GREATER RELATIVE TO THE WEATHER ECHO

Gust value	Positions relative to weather echo: Clear of echo (none beneath)	Leading edge of echo	Peak of echo	Trailing edge of echo	Above echo	In part of echo with decreasing top	In part of echo with increasing top	In narrow column of echo	Trough between columns	Weak and diffuse echo	Total
g											
+1·0	0	1	0	0	0	0	0	0	0	0	1
+0·9	0	0	0	0	0	0	0	1	0	0	1
+0·8	0	1	0	1	0	0	1	4	0	0	7
+0·7	1	1	6	1	2	0	2	0	0	0	13
+0·6	5	11	7	3	0	0	1	2	1	0	30
+0·5	13	15	12	15	1	1	4	13	2	0	76
+0·4	18	33	24	18	11	14	10	5	7	2	142
Total	37	62	49	38	14	15	18	25	10	2	270
−0·4	29	33	31	36	11	11	8	5	14	1	179
−0·5	13	10	13	16	3	2	7	1	3	1	69
−0·6	5	1	3	9	4	1	1	1	1	0	26
−0·7	1	0	0	4	1	1	1	1	1	0	10
−0·8	1	0	1	1	0	0	1	0	0	0	4
−0·9	0	0	0	0	0	0	1	0	0	0	1
−1·0	0	0	0	0	1	0	1	0	0	0	2
Total	49	44	48	66	20	15	20	8	19	2	291
All gust values	86	106	97	104	34	30	38	33	29	4	561

Thus, it will be seen that most of the large gusts occur within the echo-producing regions of a cumulonimbus cloud and are associated with recognizable changes in the echo structure. The inference is strong, therefore, that the changes in the vertical currents within the most active part of the cloud are reflected in changes in appearance of the radar echo received from that part.

It cannot be assumed, however, that by avoiding echo-producing regions no severe gusts will be encountered.

The occurrence of large gusts of opposite sign close to each other.—A noticeable feature when assessing the gusts greater than $\pm 0 \cdot 4g$ was the frequency with which a large gust was followed within a very short time by one or more further large gusts, frequently of opposite sign. There was thus a tendency for the large gusts to occur in sequences and examination of the points of occurrence of 643 gusts larger than $\pm 0 \cdot 4g$ revealed that 385 (60 per cent.) occurred in 125 sequences of two or more gusts in which the individual gusts were not further than 4 sec. (equivalent to about 400 yd.) apart. The total number of gusts in a sequence ranged from 2 to 14 with a total length varying from 0·5 to 26 sec. (50 to 2,600 yd.). The mean length of a sequence was 4·0 sec. (400 yd.).

It was possible to assess the positions of 111 of these sequences relative to the weather echo (the individual gusts, of course, appear in Table V). While a few of the longer sequences extended from one feature to another of the weather echo, e.g. from clear of echo to leading edge of echo, most of the sequences were short enough for their position to be ascribed to various parts of the echo as shown in Table VI.

TABLE VI—POSITIONS OF SEQUENCES OF GUSTS OF $\pm 0 \cdot 4g$ OR GREATER RELATIVE TO THE WEATHER ECHO

Leading or trailing edge, narrow column	Peak	Trough	Rising or falling echo top	Clear of echo (none beneath)	Above echo	Long sequences
48	19	8	8	10	12	6

The high frequency of sequences associated with echo discontinuities suggests that the transition from a vertical current of one value to one of a very different value is often accompanied by a turbulent zone, the width of which may vary from 50 yd. to, in exceptional cases, 2,600 yd. with a mean value of 400 yd. An example of three large gusts and three small gusts occurring in 4 sec. is shown in Fig. 8.

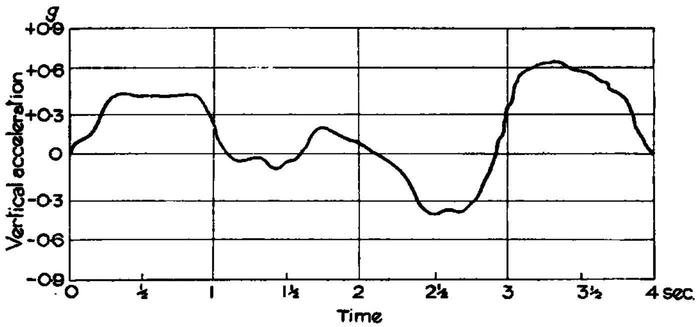

FIG. 8—SEQUENCE OF THREE MAJOR GUSTS FROM AIRCRAFT ACCELEROMETER RECORD

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