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An assessment of the feasibility of remote temperature sensing by passive microwave radiometry from an ocean data buoy.

by

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Abstract

An assessment is made of the feasibility of remote temperature sensing from an ocean data buoy, using passive microwave radiometry based on the 60 GHz O_2 band. Retrieval of mean temperatures of 100 mb layers from the ground up to the 300 mb level is attempted. Having summarised previous results based on statistical regression analysis, a method based on Backus-Gilbert theory is devised to explicitly incorporate instrumental measurement with climatological information.

Calculations are performed for a zenith pointing three channel system with an instrument noise of 0.5K per channel. This value for noise is dictated by buoy motion.

It is found that the weighting functions are unable to resolve the 100 mb layers, though resolution for the lowest layer comes close to 100 mb.

A climatological temperature covariance matrix is constructed from Ocean Weather Ship L data, and is augmented with sea surface measurements taken by meteorological instruments mounted on the buoy. In attempting to resolve layers above 500 mb, it is found that instrumental noise amplification becomes intolerably large, with the consequence that for these layers, climatological correlations with the lower atmosphere dominate overall retrieval accuracy. The addition of a fourth channel does not alter this conclusion.

It is concluded that, for it to operate under a variety of moderate cloud and rain conditions, the system cannot provide sufficiently accurate information to retrieve mean temperature profiles above the 700 mb level. In severe conditions, accurate information may be restricted to below the 800 mb level.

Contents

1. Introduction

At present, vertical temperature profiles of the atmosphere over oceanic regions are routinely obtained from both satellite-borne multichannel radiometers, and radiosondes launched from the ocean weather ship network. Neither system, however, is entirely satisfactory. On the one hand, satellite retrievals do not have the vertical resolution of radiosonde data, whilst on the other hand the cost of providing and maintaining sufficient weather ships to yield a satisfactory density of radiosonde data is prohibitive.

The study of temperature retrieval systems which may attain a good balance between vertical resolution and economic viability are, therefore, of considerable interest. In this report, one such system is discussed; a multifrequency passive microwave radiometer, mounted on an ocean buoy. The report will concentrate on the ability of this remote sensor to obtain accurate and well resolved tropospheric temperature profiles.

Before giving a detailed discussion of its performance, it is worth stating the requirements for such a system to be of practical use in operational weather forecasting. For numerical forecasting (on the UK 10-level model) the requirement is for accurate determination of mean temperatures of successive 100 mb layers. For traditional forecasting, considerable importance is generally attached to the 300 mb height chart, with 500 mb and 850 mb charts playing secondary roles.

In order to be considered as a satisfactory alternative to the radiosonde, it is therefore reasonable to require the radiometer to retrieve mean temperatures of 100 mb layers from the ground up to 300 mb (ie throughout the troposphere). If one follows GARP [1] requirements for radiosonde data, accuracy of the retrieved mean temperatures should be set at 1K rms. Furthermore, it is necessary that the system should be capable of operating under conditions of cloud and precipitation.

These latter requirements effectively rule out the use of a system based on the 15μ infra red CO_2 band, since emission at 15μ is strongly affected by cloud opacity. Such a system has been discussed by Rubinstein [2]. However, as has been pointed out by Pick [3], a more profitable approach would be to base the system on the 60 GHz (5 mm) band of O_2, firstly because radiometers of the required sensitivity and stability have been built and tested, and secondly because measurements are relatively insensitive to cloud and water vapour.

Groundbased microwave radiometry based on the 60 GHz band has been pioneered by Westwater and coworkers [4] [5] [6].

In Section 2.1, basic microwave emission models used in the ensuing calculations are described. In Section 2.2 consideration is given to the problems of buoy motion, and in Section 2.3 the pertinent results of Westwater et al will be summarised.

Westwater's analysis and results are based on statistical regression techniques, and in some respects these are unsatisfactory in assessing the capabilities of a radiometer. This is principally because of the implicit and somewhat inextricable interplay between instrumental measurement and a priori climatological information used in finding regression coefficients. A precise formulation of the problems this gives rise to, together with other unresolved problems, is summarised in Section 2.4.

In order to make explicit this interplay, the author has developed a technique based on Backus-Gilbert theory. An account of this technique is given in an appendix to this report, though the essential ideas are summarised in Sections 3.1 and 3.2. In Section 3.2, the relative importance of instrumental data over climatological data for 100 mb tropospheric layers is discussed. In Section 3.3 and 3.4 respectively, the effects of cloud and precipitation are included. Using the technique, the effect of altering the number of radiometer channels, and the noise equivalent temperature of each channel, is easily seen. In Section 3.5 some brief consideration will be given to the effects of sea spray, and sea water.

In Section 4, an objective study of retrieval accuracy is made; firstly by comparison with the GARP requirement, secondly by comparison with analysed radiosonde errors, and thirdly by comparison with the 12 hr forecast errors on the 10-level model. These latter errors are relevant since the 12 hr forecast is used as a background field for the objective analysis.

The uncertainties of Westwater's analysis are then resolved, and Section 4 finishes with some conclusions and recommendations.

2 Passive microwave sounding from the surface

2.1 Basic theory, and choice of absorption models

For microwave frequencies, the Planck function for atmospheric thermal emission is well approximated by the Rayleigh-Jeans limit, and for a non-scattering atmosphere in local thermodynamic equilibrium, the downward zenith brightness temperature, T^b_i, for channel number i at frequency ν_i, is given by

$$T^b_i = T^{b(ext)} \exp\left(-\int_0^{x_{top}} \alpha(\nu_i, x')\,dx'\right) + \int_0^{x_{top}} dx\, T(x)\,\alpha(\nu_i, x) \exp\left(-\int_0^{x} \alpha(\nu_i, x')\,dx'\right) \quad 1)$$

In 1), $T(x)$ is absolute temperature (K), $\alpha(\nu_i, x)$ is absorption coefficient (mb^{-1}), x is a suitable height coordinate, and x_{top} represents a height above which atmospheric emission is negligible.

The term $T^{b(ext)}$ is the brightness temperature of external sources. These include localised sources such as the sun or moon, as well as the isotropic 3^oK cosmic microwave background.

In the following, we shall take the antenna 3db beamwidth to be 20' of arc at 60 GHz. This is smaller than both the lunar and solar angular diameters, so that when either enters the field of view of the radiometer, a very significant change in brightness temperature will result in all but the most absorbing channels. Measurements made under these conditions would either have to be corrected for, or (more simply) ignored. However, as will be explained, only a zenith pointing radiometer will be considered, so that outside the tropics, problems of the sun or moon entering the field of view will not arise.

For window and weakly absorbing channels, the small contribution due to the cosmic microwave background must be included.

In the atmosphere, microwave absorption is due principally to molecular resonances of O_2 (60 GHz) and H_2O (22 GHz), and to continuum absorption by H_2O in all its three phases. In general, brightness temperature is a strong function of composition, and a weaker function of temperature. Thus, around 22 GHz, brightness temperature varies primarily because of variations in water concentration, whereas at 60 GHz, radiation from the homogeneous constituent O_2 depends mainly on temperature.

For calculations of $\alpha(\nu_i, x)$ throughout this report, the following theoretical models are used.

Absorption by O_2 is treated by means of a first order approximation to the impact theory of overlapping spectral lines, as given by Rosenkranz [7]. This model has been experimentally confirmed by Liebe [8]. For the single line H_2O vapour resonance, the basic Van Vleck [9] lineshape is used, with a semi-empirical correction made for continuum absorption using parameters derived from Becker and Autler [10]. For absorption by liquid water, the expression given by Staelin [11] is used. This expression assumes the validity of the Rayleigh approximation, so that attenuation is due purely to absorption, and is independent of drop size distribution. Westwater [12] shows that for a cloud with modal drop size of 100 μ , the error in applying the Rayleigh approximation is serious at 60 GHz. In particular rain and hail both scatter and absorb microwave energy. The attenuation coefficient must then be calculated from Mie theory. Attenuation due to rain has been given by Deirmendjian [13] assuming a gamma-function distribution of droplets.

Calculations of microwave attenuation are shown in figure 1. With a rainfall rate of 12 mm/hr, rain dominates all other sources of attenuation, except in the vicinity of the peak of the oxygen complex.

2.2 Problems of buoy motion

Since the radiometer would be required to operate on an ocean buoy, it is necessary to consider the effects of buoy motion on measurements of brightness temperature.

Ignoring variations of refracture index, the brightness temperature at some angle θ from the zenith is given by

$$T^b{}_i = T^{b(ext)} \exp\left(-\int_0^{x_{top}} \alpha(\nu_i, x') \sec\theta \, dx'\right)$$
$$+ \int_0^{x_{top}} dx \, T(x) \, \alpha(\nu_i, x) \sec\theta . \exp\left(-\int_0^{x} \alpha(\nu_i, x') \sec\theta \, dx'\right) \quad 2)$$

Equation 2) is the basis for a scanning radiometer, possibly an alternative to the multichannel instrument. However, at any scanning angle θ, the direction of sight varies about θ due to buoy motion.

Using data from Westwater [6] (pertaining to a specific ocean buoy), there is a probability of 0.5 that motion of the buoy about the vertical has a standard deviation less than 3°. Using 2), and assuming θ is a random variable with standard deviation of 3° about 1) 0°, 2) 20°, 3) 40°, then table 1 gives the standard deviation of brightness temperatures for these scanning angles, for three frequencies in the O_2 band.

It is seen that the 52.8 GHz channel is most sensitive to buoy motion. This may be understood in terms of the change in pathlength for changing angle of sight, which affects the weakly absorbing channels. The elimination of angle uncertainty would require the radiometer system to be supplemented with an accurate angle measuring device. To measure the 52.8 GHz channel brightness temperature at 40° from zenith to within .5K, angles would have to be measured to an accuracy of $\sim 20'$ of arc, which suggests that angle sensing is impracticable.

Alternatively, instrumental integration time should be much greater than a typical 'period of oscillation' of the buoy, so that a mean brightness temperature about θ will be sensed. Such a period is difficult to calculate not only as the buoy will in general be oscillating about all 3 axes, but also, since the state of the sea surface is continuously changing as the result of complex interactions between different scales of motion, the determination of a mean state may be meaningless to the accuracy required for these large scanning angles.

With a scanning mode, however, there is also the problem that measurements will be taken that are not simultaneous in space. Whilst temperature inhomogeneities may not be large, cloud inhomogeneities certainly are. Correction for cloud assumes uniform cover, and this assumption will be violated on a number of occasions with measurements taken at different angles.

There is a final reason why scanning is not desirable, which emerges from the analysis of Section 3.

The effect of scanning at a fixed frequency is to lower the weighting function in the atmosphere, so that less radiation measured by the radiometer is emitted from higher layers. A major problem with the instrument, however, is that even at zenith, insufficient radiation is received from these layers and therefore scanning cannot improve this situation.

Variation of brightness temperature about zenith is tolerably small that with an integration time of maybe half a minute, it is reasonable to assume the effective noise equivalent temperature of each channel to be no greater that 0.5 K. To attain some degree of independence from the buoy when conditions are more violent that the 3° motion, mounting by gimbals may be desirable, together with some electronic gating to prevent large angle information being included in the integration. We shall assume a noise of .5 K to be a representative figure for this study.

Further effects of an ocean buoy environment are discussed in Section 3.4.

2.3 Results of Westwater, in clear and cloudy conditions

For cloudless conditions, Westwater has applied standard linear statistical regression algorithms to extract temperature profiles from groundbased microwave radiation measurements, based on a zenith pointing three channel system with assumed instrumental noise of 0.5 K for each channel. Typical results of his are given in figure 2, for an ocean weather ship climatology, and particular choice (52.8 GHz, 54.0 GHz, 55.4 GHz) of channel frequencies. Climatology is assumed to be supplemented with current surface pressure and temperature values. It will be noticed that all retrievals exhibit a significant reduction in variance over climatology, even at high altitudes. Further, the improvement in accuracy of layer averages over point retrievals is substantial below about 750 mb. Finally, at high altitudes, there is little difference between point and layer average retrievals.

To allow for the effects of cloud in temperature retrievals, Westwater supplements measurements in the oxygen band with measurements from channels at 20.6 GHz and 31.65 GHz, the former lying in the wings of the water vapour line, the latter being affected by continuum H_2O absorption. Given a measurement in the oxygen band, Westwater estimates an equivalent clear air zenith brightness temperature at the frequency of the O_2 measurement by statistical regression of equivalent brightness temperature against a sum of linear and quadratic combinations of measurements of the O_2 and H_2O channels. It is assumed that measurements of the H_2O channels provide information about the presence of cloud in order that the algorithm be applied, though it may be feasible to apply a single algorithm to all cases, both clear and cloudy, without significant degradation in resulting effective noise levels.

As was mentioned in Section 2.2, when cloud exists, it is important that measurements are made as near simultaneous (both in space and time) as possible. Spatially, beamwidth varies inversely with frequency. With a beamwidth of 20' of arc at 60 GHz (40' at 30 GHz), any spatial atmospheric inhomogeneities may be considered small (having eliminated a scanning mode of operation).

To infer equivalent brightness temperature from cloud observations, the precise choice of frequencies in the oxygen band is significant. Westwater recommends the three O_2 channels to be centred on 52.8 GHz, 54.0 GHz, and 55.4 GHz, which for a variety of clear and cloudy conditions, yield the largest overall reduction in variance. Except where stated, calculations in Section 3 assume a three channel system centred on these frequencies.

Westwater's regression analyses are based on data from five separate weather ships. Envelope and average noise retrieval curves for the set of five ensembles, subdivided into clear and cloudy conditions, are illustrated in figures 3 and 4.

The main findings of Westwater et al [6] can be summarised as follows.

A three frequency zenith measurement (with instrumental noise of 0.5 K) made during clear conditions can be inverted by a statistical regression algorithm to infer 100 mb layer averages of temperature with rms accuracies that range from 0.5 K at the lowest altitudes, through 1.7 K at 500 mb, to 2.9 K at 300 mb. To correct for cloud emission, measurements are required near 21 and 31 GHz. With the five channel system, the rms temperature retrieval error for cloudy conditions is degraded (relative to clear air) by at most 0.3 K.

2.4 Problems to be resolved in assessing the instrument

Partly by the very nature of his analysis, Westwater does not resolve the following problems, necessary for a satisfactory assessment of the instrument.

1) How does the vertical resolution of the instrument compare with the chosen 100 mb layers?

2) For a given layer, is the accuracy of the retrieval principally due to the ability of the instrument to achieve a satisfactory resolution at that layer, or due to atmospheric climatological correlations with lower layers, whose temperature the instrument can retrieve more accurately?

3) What is the best possible retrieval accuracy obtainable in cloudy conditions, and how near does Westwater's algorithm come to this accuracy?

4) What is the effect of precipitation on the accuracy of temperature retrievals?

5) Can the accuracy of retrievals be significantly improved on by adding a fourth channel in the O_2 band?

A means of answering these questions is formulated in the next Section, and after some discussion, these specific problems are answered in Section 4.

3 Investigation of retrieval techniques

3.1 Assessment of the vertical resolution of the proposed instrument

Unlike soundings taken from satellite platforms, the weighting functions

$$K_i(x) = \frac{1}{\Delta\nu}\int_{\nu_i - \Delta\nu/2}^{\nu_i + \Delta\nu/2} d\nu\, \alpha(\nu, x) \exp\left(-\int_0^x \alpha(\nu, x')\, dx'\right) \qquad 3)$$

of a groundbased radiometer are monotonically decreasing functions of height.

In 3), the effect of a finite bandwidth has been included. It is desirable to make the passbands reasonably sharp (within microwave instrumentation tolerances), and allow for a suitable signal to noise ratio by adjusting instrumental integration time. Following Pick [3], a bandwidth, $\Delta\nu$, of 220 MHz is chosen. In fact, since the absorption coefficients are slowly varying functions of frequency, the weighting functions are nearly monochromatic, and the precise choice of bandwidth is not critical.

Using the three channel zenith pointing system based on 52.8 GHz, 54.0 GHz and 55.4 GHz, weighting functions are illustrated in figure 5 for a (cloud free) 1976 US Standard Atmosphere [14], with water vapour density ρ (kg m^{-3}) given by

$$\rho = 10^{-2} \exp\left(-\frac{h}{2.2}\right) \qquad 4)$$

where h is altitude in km.

Studying figure 5, the resolving ability of the three channel system is not obvious. However, by taking any two weighting functions, subtracting the smaller from the larger, it is clearly possible to obtain a modified weighting function, $\tilde{K}$, which peaks for some height $\tilde{x}$, ie $\left. d\tilde{K}/dx \right|_{x=\tilde{x}} = 0$

Rubinstein [2] calls these $\tilde{K}$ differential weighting functions. Pick [3] has plotted typical differential weighting functions for three frequencies in the O_2 band, for a water free atmosphere (figure 6).

These differential weighting functions go some way to showing how the resolution of the instrument varies with height. What one is effectively trying to find, however, is a suitable linear combination of the available basic weighting functions that maximises the resolution about a given height.

Attempts to obtain high resolution from the basic set of low resolution weighting functions, result in an increase in sensitivity of the retrieved profiles to random noise. In other words, by subtracting two almost identical weighting functions, a composite function of high resolution will be obtained.

However since the brightness temperatures of the two channels are nearly equal, their subtraction will lead to a large noise to signal amplification. Hence, in finding a suitable linear combination of weighting functions, there must be a trade-off between the scale of features resolvable, and stability against noise.

The Backus-Gilbert theory [15] provides just such an analysis. Although it was originally developed for applications to inverse problems encountered in the physics of the solid earth, Conrath [16] has applied the theory to atmospheric temperature retrievals, in particular, satellite measurements of the 15 μ CO_2 band.

The mathematical details of Backus-Gilbert theory are summarised in Appendix A. We shall describe the qualitative features here.

If we form a composite weighting function from a linear combination of the basic set, we may describe its broad features by three numbers. Firstly we have the 'spread', which describes the resolution of the function about the level in question. If the spread is large, this may be because the function is very broad (in terms of the height coordinate), or otherwise its 'centre' is some height below or above the level in question. The centre may be figuratively thought of as the height of the 'centroid' of the composite weighting function. Finally, the 'resolving length' is defined as the spread about the centre.

In standard Backus-Gilbert theory, a trade-off curve is drawn which has spread as abscissa, and retrieval noise as ordinate.

In figure 7) trade-off curves are illustrated for the proposed instrument, for seven levels in the atmosphere, representing the midpoints of seven 100 mb layers from the ground upwards. These midpoints are given in Table 2 in pressure and scale-height coordinates. These latter coordinates, x , are defined in terms of the pressure, p , and its surface value p_s , by

$$x = -\ln\left(p/p_s\right)$$

In figure 7), the spread is given in customary scale-height coordinates. For none of the seven curves does the maximum resolution fall within the 100 mb of its corresponding layer. The minimum spread solution of curve 1 comes nearest to this requirement, having a spread of .13 scale heights (123 mb), with retrieval noise of 1.7 K.

Examining the resolution of composite weighting functions about higher layers, it may be calculated that none of their centres lies above about 500 mb. This is an indication that the instrument is not doing a good job at resolving the highest tropospheric layers.

It will be shown that the addition of cloud or rain has the effect of lowering the 'supremum' height of the centre, and this in turn has the effect of reducing the measurable information of the upper troposphere.

3.2 Incorporation of climatology with instrument measurements

So far we have used the Backus-Gilbert method to examine the radiometer's ability to retrieve temperature for varying vertical resolution. In practice, however, knowledge of atmospheric statistics will improve the accuracy of retrieval. Linear regression algorithms implicitly include both effects; so implicitly, however, that it is difficult to infer the extent to which achieved accuracy is a function of instrumental accuracy and resolution.

In Appendix B, a scheme is developed for incorporating climatology into a Backus-Gilbert variation.

The scheme is designed to tackle the problem of determining how instrumental measurement improves the accuracy of the temperature retrieval of a given atmospheric layer over climatology.

In the following, data from Ocean Weather Ship L has been used to construct a climatological covariance matrix of temperature. Layers were chosen from (scale height) $x = 0$ to $x = 1.2$ (300 mb) with depth $\Delta x = 0.05$. Radiosonde ascents for April and May of 1976 and 1977 were used, and in accordance with standard practice, temperatures were linearly interpolated between reported heights. The first row/column of the covariance matrix pertains to surface values.

Since conventional meteorological instruments would be mounted on an ocean buoy, one can assume a fairly precise value of surface temperature is known, say to .2 K (rms), and the covariance matrix is modified accordingly.

With this modified matrix, curves of retrieval noise, using instrument and climatology, may be plotted against the corresponding spread of the composite weighting function of the instrument. This is done in figure 8 for the 7 atmospheric layers. Inspection of figure 8 shows that instead of having a trade-off, there is a minimum retrieval noise value. Using these minimum noise values, the overall 'optimum retrieval accuracy', relative to climatology + surface value is illustrated in figure 9 for an assumed noise of 0.1 K, 0.5 K and 1.0 K.

For an instrumental noise of 0.1 K, the noise of the optimum retrieval is less than 1 K up to 400 mb above the surface. For reasons of buoy motion, discussed in Section 2, it is probably more realistic to assume a 0.5 K channel noise (which is done from now on). In this case, the profile noise is less than 1 K up to 250 mb above the surface. The improvement in accuracy over a priori knowledge is similar to the clear sky results of Westwater.

From figure 8, the optimum retrieval accuracy of the lowest 100 mb is .40 K, with an instrumental spread of .21 scale heights. Figure 7 shows that the instrumental noise for this spread is .36 K. This information tells us that the bulk of the weighting function is within the layer, with an overspill into the second layer, and the noise increase of 0.04 K is due to the imperfect climatological correlation between the first and second layers. In this case, instrumental information dominates the overall retrieval accuracy.

By comparison, the centre of the instrumental weighting function for the optimum retrieval of the highest layer, is, from figure 10, only 240 mb above the surface. The resolving length is equal to .60 scale heights, and the upshot is that little significant information detected by the radiometer is due to emission from this layer, so that the reduction in noise over climatology is due almost entirely to correlations with lower levels.

This is a significant finding; above about 500 mb, climatology is a more reliable source of information that measurements from a three channel radiometer.

The ineffectiveness of the instrument at high altitudes was demonstrated by producing a statistically 'freak' warming of the 350-300 mb layer by 5K. For each position of the Backus-Gilbert tradeoff curve, the corresponding change in brightness temperature to that of a standard atmosphere was completely swamped by the corresponding instrumental noise.

To obtain more information at higher levels, it would seem reasonable to add a fourth channel in the O_2 band, further from 60 GHz than the other three. Hence we consider the retrieval ability of the four channel system (50.0 GHz, 52.8 GHz, 54.0 GHz and 55.4 GHz).

It is found that with the addition of the fourth channel, the centre is now able to rise to a peak of about 650 mb above the surface, ie to the midpoint of the highest layer. This (minimum spread) solution is however, so noisy (instrumental retrieval noise is equal to 53 K) that the optimum retrieval noise is derived from the set of instrumental weighting functions whose centre lies at 250 mb above the surface. This is illustrated in figure 11. Compare this with the centre of the three channel radiometer for retrieval at the highest layer (Figure 10). The 50 GHz channel contributes a marginal amount of information to improving the accuracy of the retrieval. This improvement is illustrated in Figure 12, and is seen to be less than .1K.

3.3 Effect of cloud on retrieval

So far, only clear atmospheres have been studied. In the language of Backus-Gilbert theory the addition of cloud and precipitation lowers the supremum height of the centre function with the result that retrieval accuracies for higher layers suffer even more.

As Westwater discusses [6], if we assume that cloud droplets are sufficiently small to allow us to assume a Rayleigh absorption regime, the relevant parameter in a discussion of attenuation by cloud is liquid thickness.

For his analysis, Westwater uses conditions where the liquid thickness of each cloud of the regression ensemble is less than 4 mm of water.

The procedure developed above can be used for cloudy atmospheres, giving an estimate of what the best retrieval method can produce. We first introduce a cloud, about 250 m thick, with base at 200 m, and water density of .2 g/m^3, corresponding to a shallow stratus layer with liquid thickness of .05 mm. Using the Backus-Gilbert minimisation and incorporating climatology, as before. results are shown in Figure 13. The cloud has barely affected the retrieval accuracy - marginally improving it for lower levels, marginally degrading it for upper levels. The difference is less than .1K.

We increase the cloud geometric thickness to 2 km, and maintain the water density at .2 g/m^3. (This corresponds to moderate cloud cover with liquid thickness of .4 mm. In Westwater's regression sample, well over half the cloud sample had liquid thickness less than .4 mm.) Applying the Backus-Gilbert variation, results are also shown in Figure 13. It is seen that up to about 250 mb above the surface, the accuracy of retrieval is impaired very little; above this, the accuracy of retrieval is impaired by at most .3K.

As a final cloud example, we increase the geometric thickness to 3.5 km, and increase the water density to .6 g/m^3. (This corresponds to heavy cloud cover, with 2.1 mm liquid thickness. Only 2% of Westwater's sample had liquid thickness greater than or equal to this amount). Again, results are shown in Figure 13. At 250 mb above the surface, accuracy is impaired by about .2K relative to clear conditions; at 550 mb above the surface it is impaired by about .7K.

Since the moderate cloud cover model is close to a median in Westwater's sample. our results corroborate his, in that the sample rms noise will not be degraded by more than .3K. The effect of heavy cloud, however, should be noted. A cloud liquid thickness of 2 mm or greater can be expected in active frontal situations, or strongly convective conditions, and should not be thought of as exceptional.

For these cloud models, it is assumed that knowledge of the relevant cloud parameters is known a priori. It is, however, instructive to consider the errors of retrieval using clear sky weighting functions. The effect of this is to introduce a systematic error, given by the difference in brightness temperature between clear and cloudy conditions.

The combined systematic and random error is also illustrated in Figure 13. It is immediately clear that some knowledge of cloud conditions is essential to obtain a reasonable reduction in variance over climatology.

3.4 Effect of rain on retrieval

Let us now consider the effects of rain. As before, it should be emphasised that the Rayleigh assumptions break down, and one needs to consider not only liquid thickness, but also drop size distribution. This makes the correction for rain rather complex.

We use Deirmendjian's data [13] for a rainfall rate of 7.7 mm hr^{-1}, incorporating it in the .4 mm cloud model, and assume the rain to extend a little above the cloud base.

Figure 14 shows the effect of rain on the centre function. Compare this with Figure 8 for clear conditions. The Backus-Gilbert weighting functions cannot rise above the rain top. Rain has 'washed out' all information from higher layers, and retrieval accuracy of these higher layers is governed entirely by their statistical correlations with thelowest atmospheric layers. Retrieval accuracies are illustrated in Figure 15, and for layers 250 mb or more above the surface, deterioration compared with clear conditions is about .7K.

3.5 Further effects of the buoy environment on retrieval

In this section we raise further problems due to attentuation by water. In moderate conditions, the work of Lai and Shemdin [17] suggests that attenuation by sea spray should not be a problem. In rough conditions, with large swells, we have the problems of sea water wetting the antenna cover. Since the effect of a few mm of wateris to produce significant attentuation, this is an important consideration in the design of the instrument. The antenna must be adequately shielded from the effects of wetting by sea spray and sea water. Such wetting, however, may also occur as the result of rain, though as we have seen, the system in any case will not operate accurately under conditions of significant precipitation.

4. Conclusions

The results of section three corroborate those of Westwater, insofar as they show that optimum retrieval accuracies are significantly better than climatological accuracy, even up to 300 mb, in both clear and cloudy conditions. A significant improvement, however, does not imply a satisfactory improvement, and in this section we try to give an objective assessment of these retrieval accuracies.

The first criterion we may use is the GARP requirement of radiosonde (spot) data, which, as was stated in the introduction, is set at 1K.

Examining Figures 13 and 15, it is seen that for the 3 channel system with 0.5 K noise, optimum retrieval accuracies of the lowest two 100 mb layers, for all the considered atmospheric conditions, fall within this requirement. For moderate cloud cover, retrieval accuracy of the third layer falls within the 1 K requirement.

The 1 K error requirement, however, is presented for convenience and simplicity; for a more accurate assessment we compare optimum retrieval accuracies with known radiosonde errors.

These radiosonde errors have been tabulated by Atkins [18], both the 'raw' errors, and the residual errors after a standard correction, based on a 100 mb thickness analysis, has been applied. Her results are reproduced in Table 3 for the height errors of standard levels. Also in Table 3 are the rms errors of the update 12 hour forecast runs (for March 1978) of the 10-level model. These are used in an objective analysis scheme as a background field on which observations are plotted. Finally in Table 3, optimum retrieval errors of the radiometer system are given, converted to standard height errors, in clear, cloud, and rain conditions.

Inspection of Table 3 shows that for the 850 and 700 mb heights, the radiometer is not doing worse than the radiosonde, and is faring much better than the twelve hour forecast. For the 500 mb height, the radiometer is doing worse than the radiosonde, and for rain conditions it is not doing better than the twelve hour forecast. For the 300 mb height, the radiometer produces worse errors than the 12 hour forecast.

We are now in a position to answer the questions posed in section two.

1. Except for the lowest layer, the instrument is unable, even approximately, to resolve to within 100 mb.

2. Above about 700 mb, a significant contribution to the achieved accuracy is due to climatological correlation with lower layers. At 300 mb, accuracy is almost entirely due to this effect.

3. Westwater's algorithm comes near to attaining maximum accuracy for moderate cloud conditions, where retrieval is not more than 0.3K worse than clear retrieval. For heavy (eg frontal) cloud, retrieval is expected to suffer by more than this value.

4. Moderate precipitation degrades retrieval by at least .7K for the middle layers of the troposphere. Retrieval requires knowledge of not only rainfall rate, but also drop-size distribtuion.

5. Adding a fourth channel to obtain better resolution at high altitudes does not significantly improve retrieval accuracy.

When considered as an all weather system, it is clear that a microwave radiometer mounted on an ocean buoy, is not capable of providing sufficiently accurate information to retrieve temperature profiles above the 700 mb level. In severe weather conditions, it may not reliably be able to sense above the 800 mb level.

Furthermore, since the instrument cannot resolve better than 100 mb (this resolution is not substantially bettered by increasing the number of channels), it is also incapable of sensing low level inversion structure.

The system would at best be capable of providing low level mean temperature profiles over oceanic regions, to supplement, rather than replace, radiosonde data from the existing ocean weather ship network.

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Scanning angle	52·8 GHZ	54·0 GHZ	55·4 GHZ
0°	0·5 K	0·3 K	0·0 K
20°	2·0 K	1·1 K	0·1 K
40°	4·4 K	1·8 K	0·2 K

Frequency

Table 1. Standard deviation of brightness temperature for buoy motion with 3^{o} standard deviation about scanning angle. Scanning angles are relative to zenith.

Level	Pressure	
	Mb above surface	Scale height
1	50	0·05
2	150	0·16
3	250	0·28
4	350	0·42
5	450	0·59
6	550	0·78
7	650	1·03

Table 2. Height of level numbers in pressure coordinates. The levels correspond to midpoints of 7 100mb layers.

Standard level (mb)	Uncorrected radiosonde	Corrected radiosonde	12 hour forecast	Radiometer (clear)	Radiometer (medium cloud)	Radiometer (medium cloud+rain)
850	8	8	22	2	2	3
700	9	9	23	5	6	9
500	14	11	28	19	24	30
300	25	17	40	47	54	65

Table 3. Comparison of r.m.s. radiosonde errors, numerical 12hr forecast errors and radiometer errors, in geopotential metres. A quantitative description of 'medium cloud' and 'rain' is given in the text.

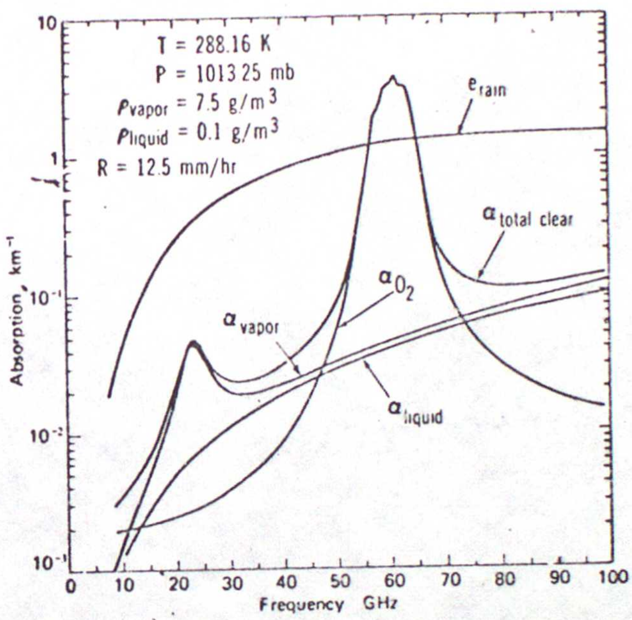


Fig. 1. Microwave atmospheric absorption in clear air, clouds, and rain. α = absorption coefficient, e = attenuation coefficient. (1 bar = 100 kPa). From Westwater et al [6].

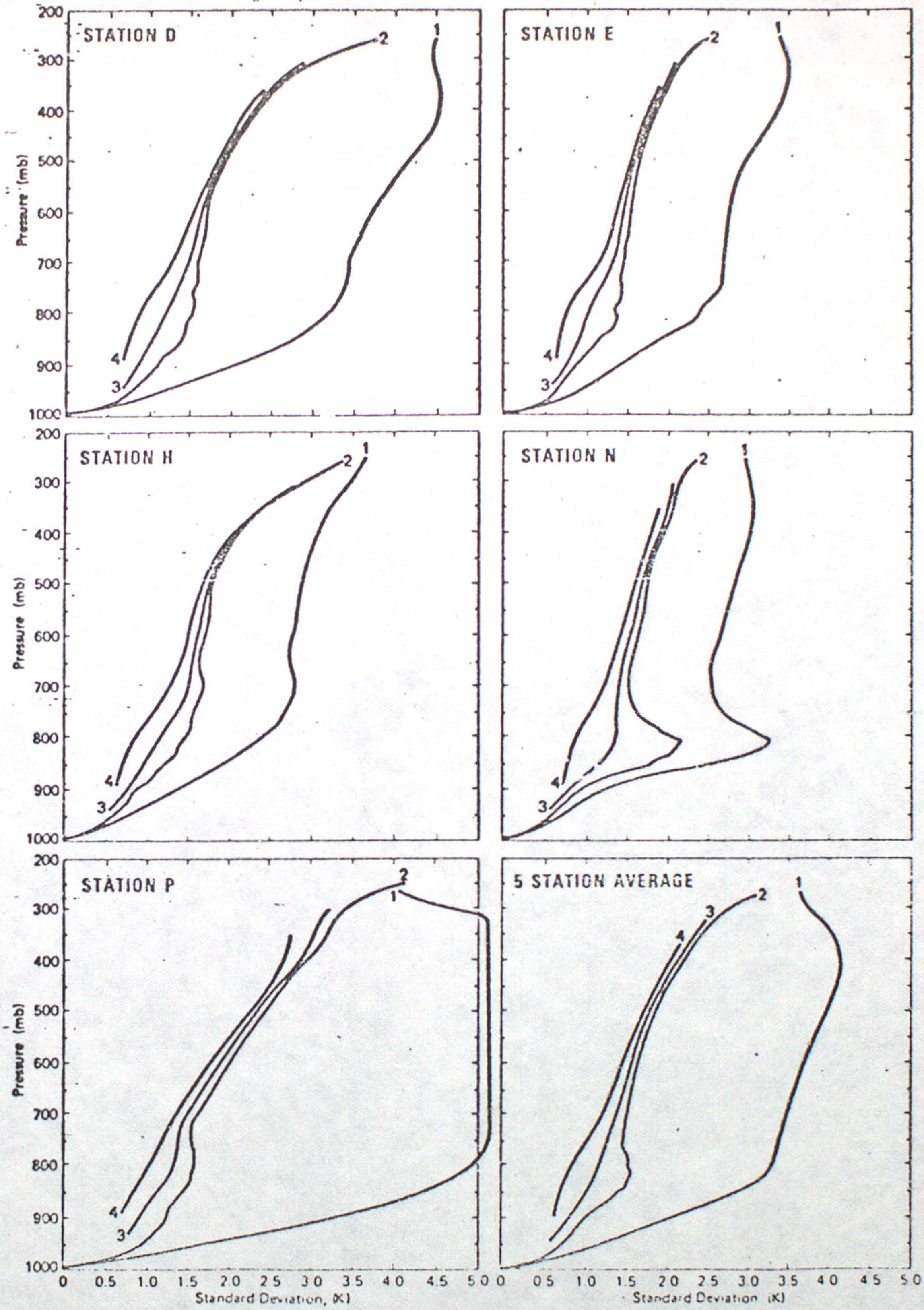


Figure 2 *Rms temperature retrieval accuracy for five weather ships. Instrumental noise = 0.5 K. Clear conditions. 1 = a priori statistics only, 2 = point retrieval, 3 = 100-mb layer retrieval, 4 = 200-mb layer retrieval. System C.*
From Westwater et al [6].

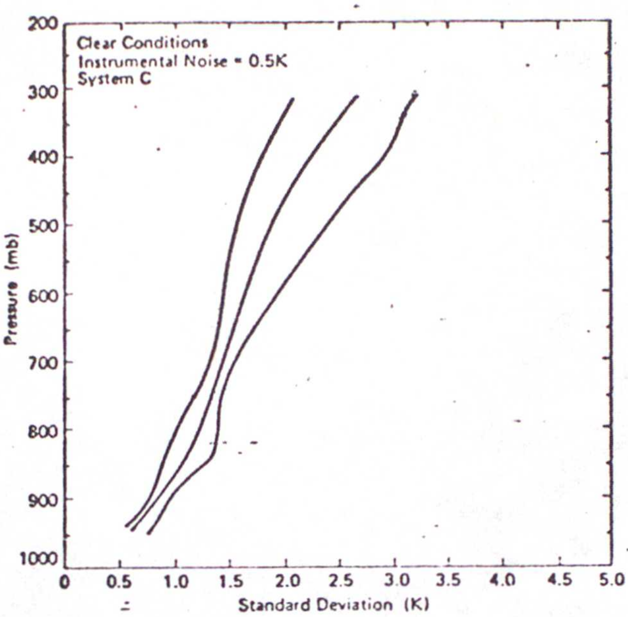


Figure 3 Envelopes of rms accuracies of temperature retrievals of layers 100-mb in thickness during clear conditions. Center curve is five-station rms average.

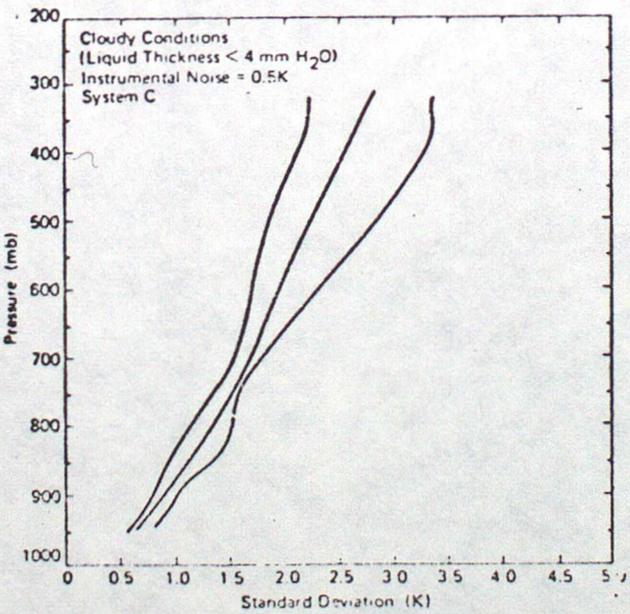


Figure 4 Envelopes of rms accuracies of temperature retrievals of layers 100-mb in thickness during cloudy conditions. Center curve is five-station rms average.

From Westwater et al [6]

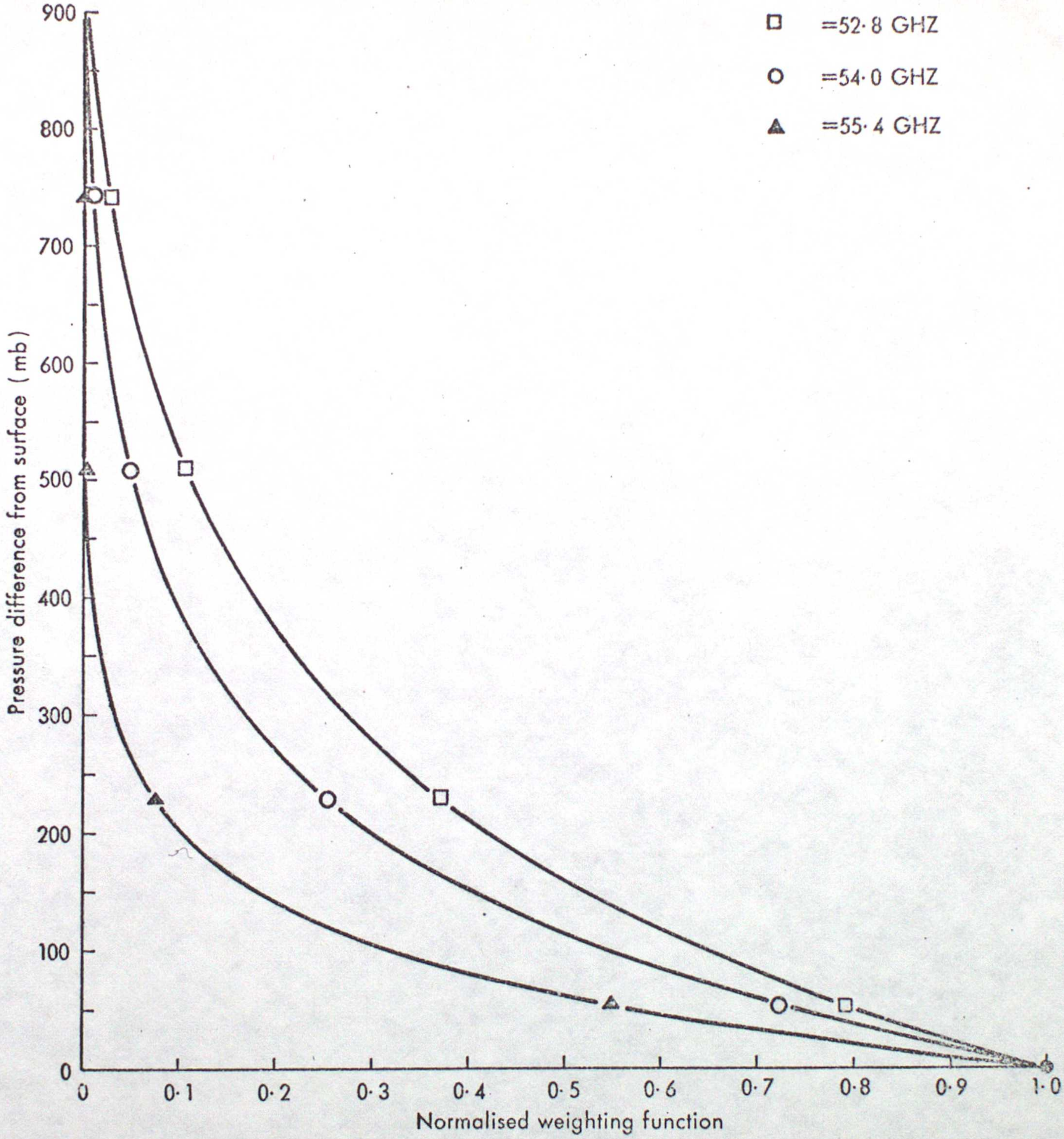


Figure 5. Basic instrumental weighting functions. Bandwidth = 220 MHz. Clear conditions.

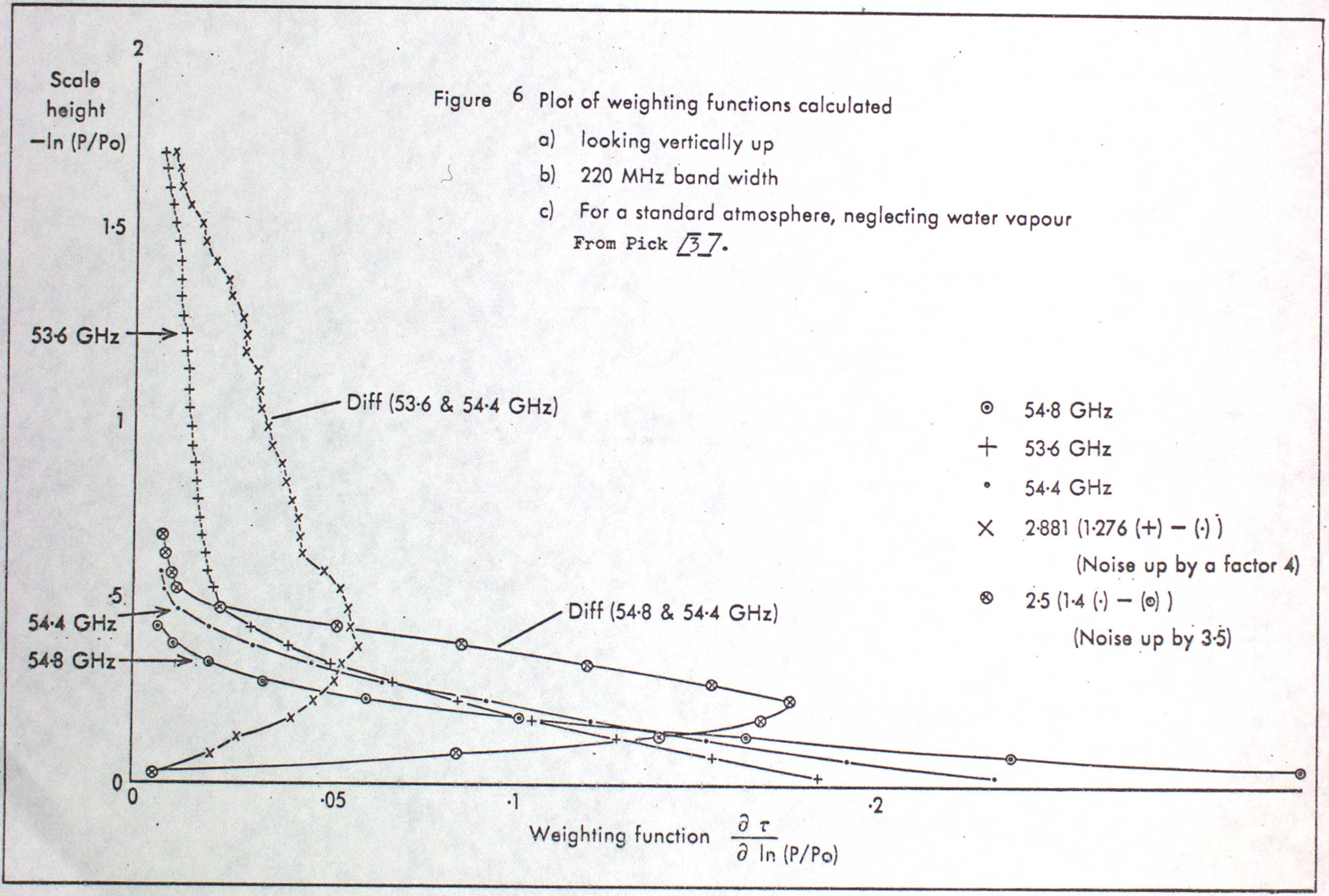


Figure 6 Plot of weighting functions calculated

a) looking vertically up

b) 220 MHz band width

c) For a standard atmosphere, neglecting water vapour

From Pick [3].

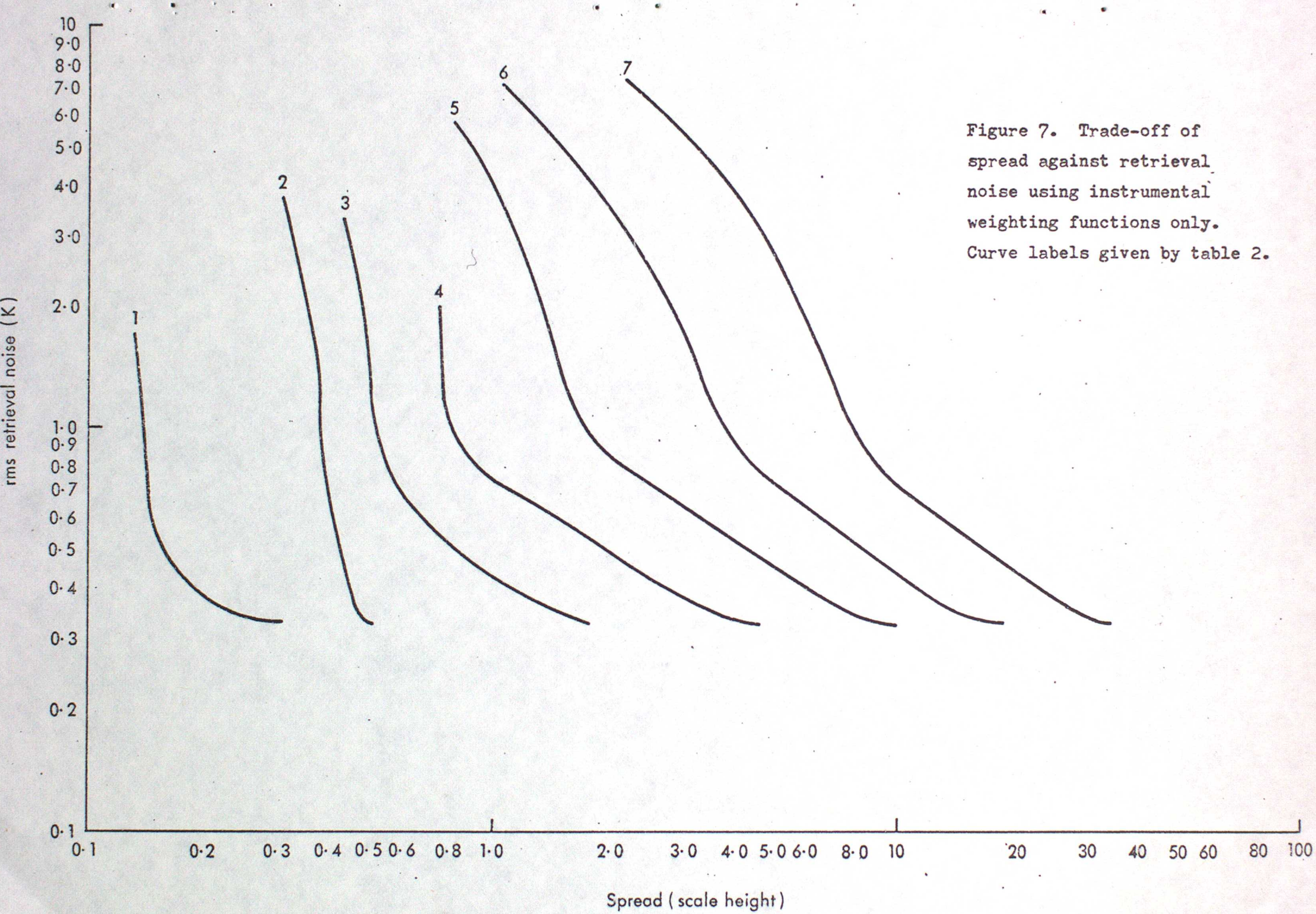


Figure 7. Trade-off of spread against retrieval noise using instrumental weighting functions only. Curve labels given by table 2.

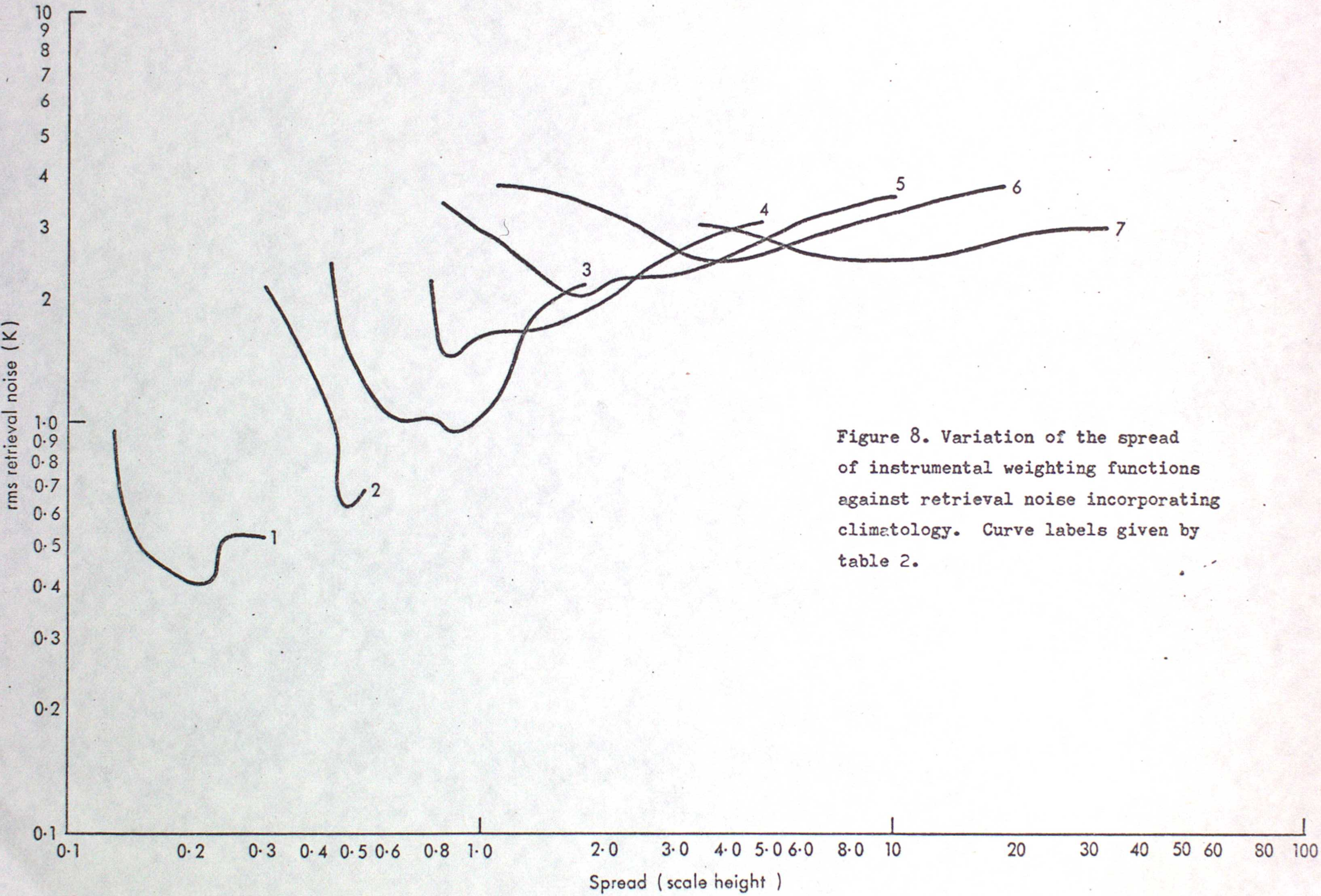


Figure 8. Variation of the spread of instrumental weighting functions against retrieval noise incorporating climatology. Curve labels given by table 2.

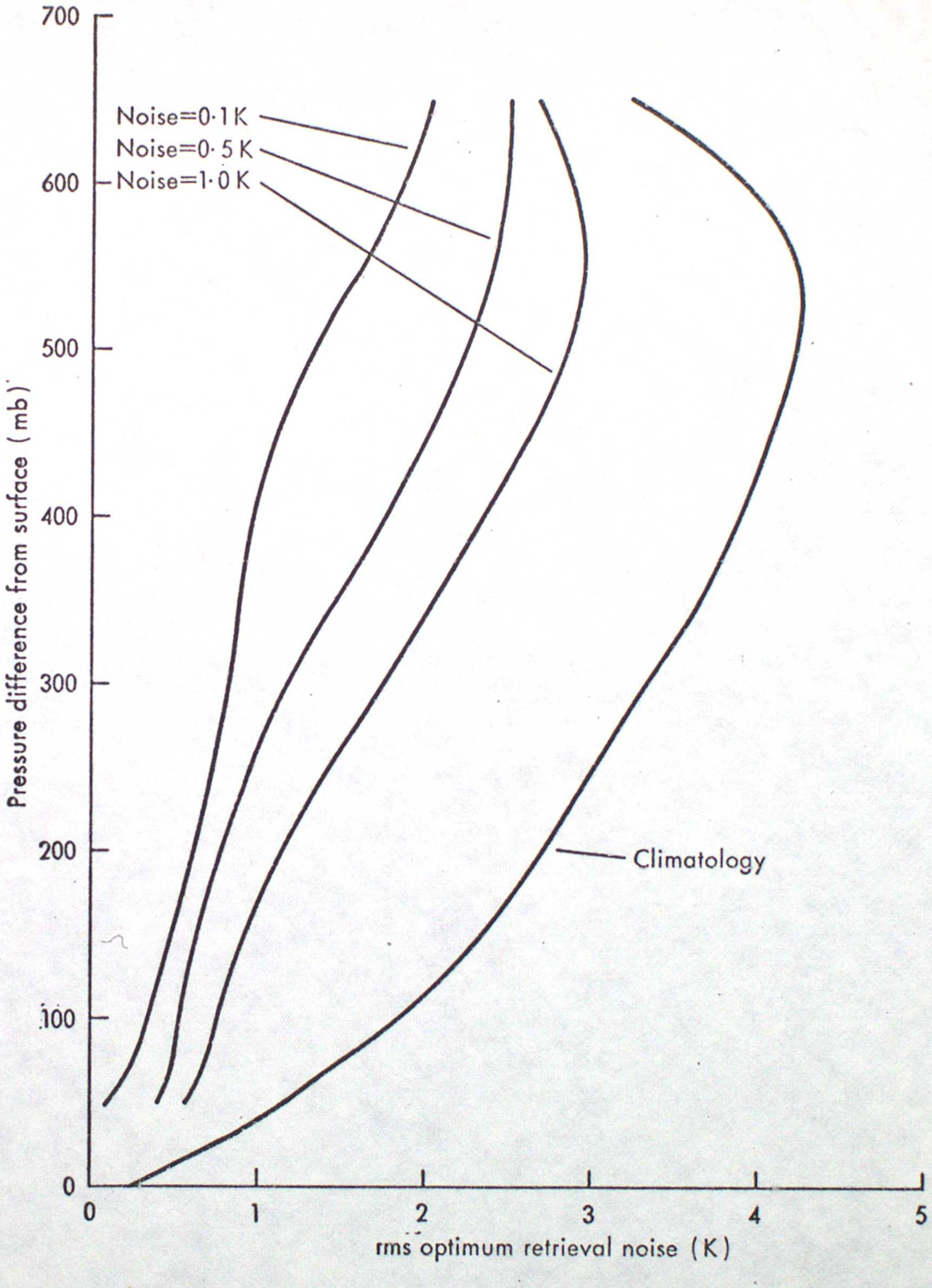


Figure 9. Optimum retrieval acc uracy of 100mb layers in clear conditions, with instrument noise of .1K, .5K and 1K per channel.

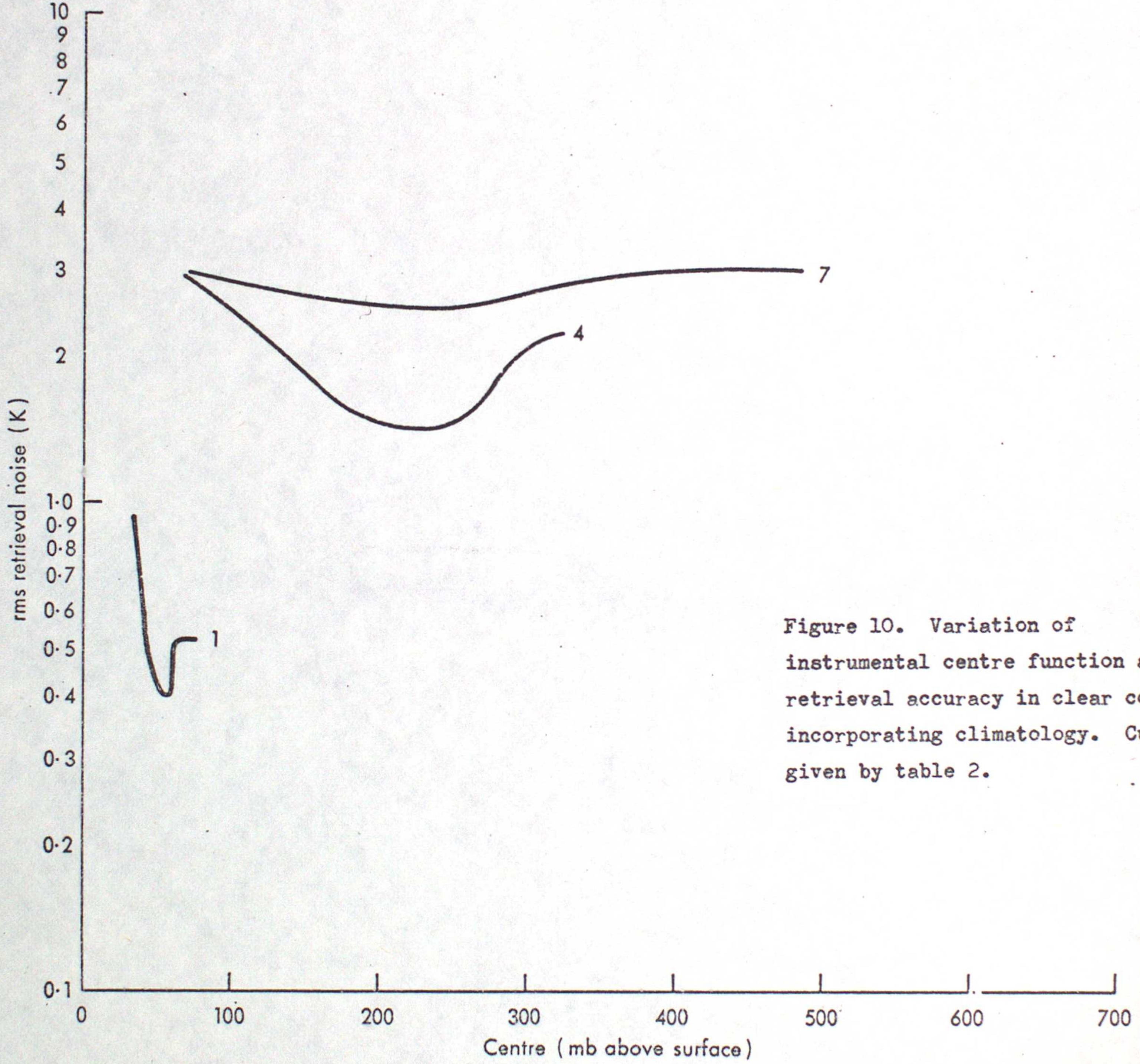


Figure 10. Variation of instrumental centre function against retrieval accuracy in clear conditions, incorporating climatology. Curve labels given by table 2.

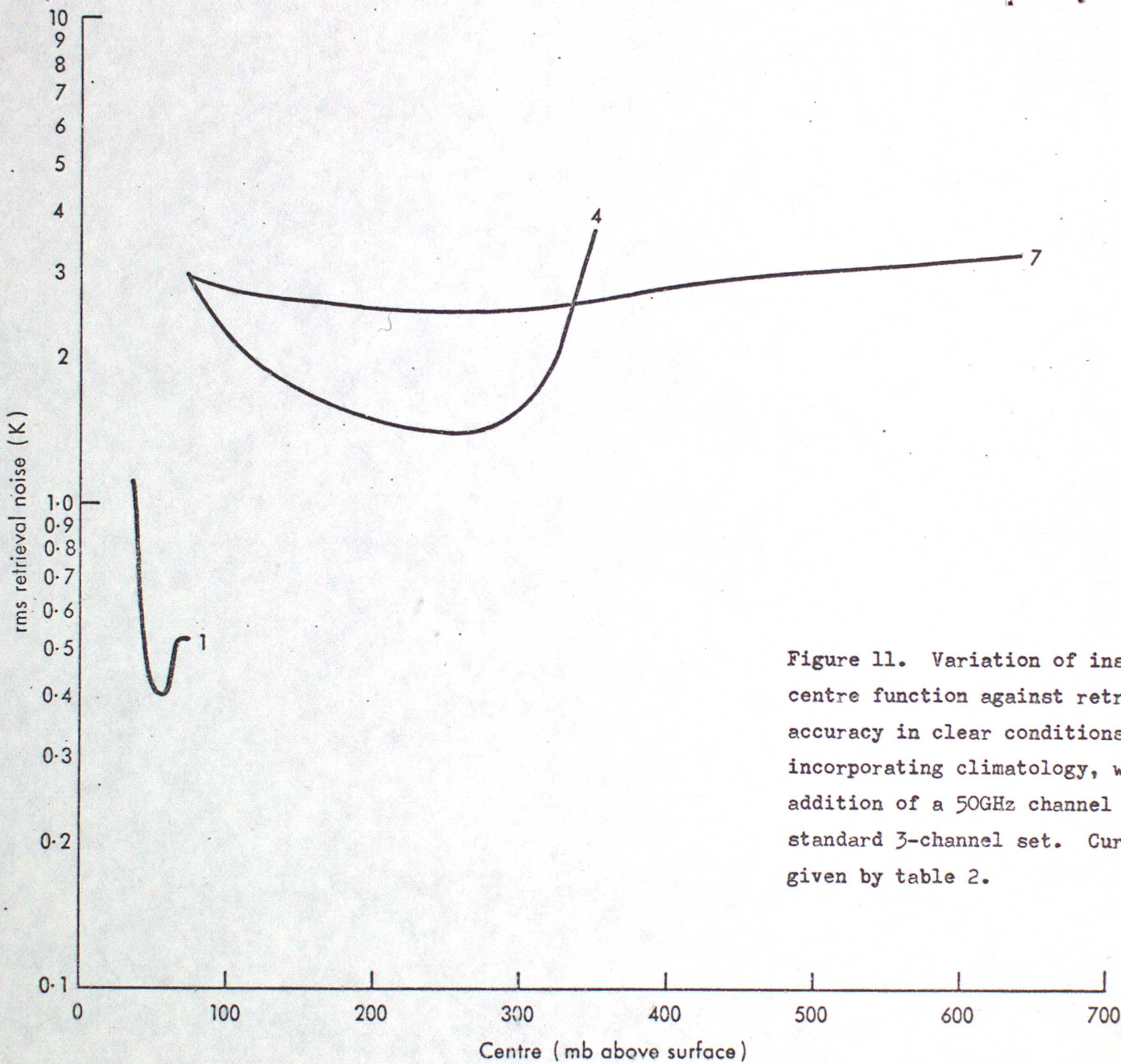


Figure 11. Variation of instrumental centre function against retrieval accuracy in clear conditions, incorporating climatology, with the addition of a 50GHz channel to the standard 3-channel set. Curve labels given by table 2.

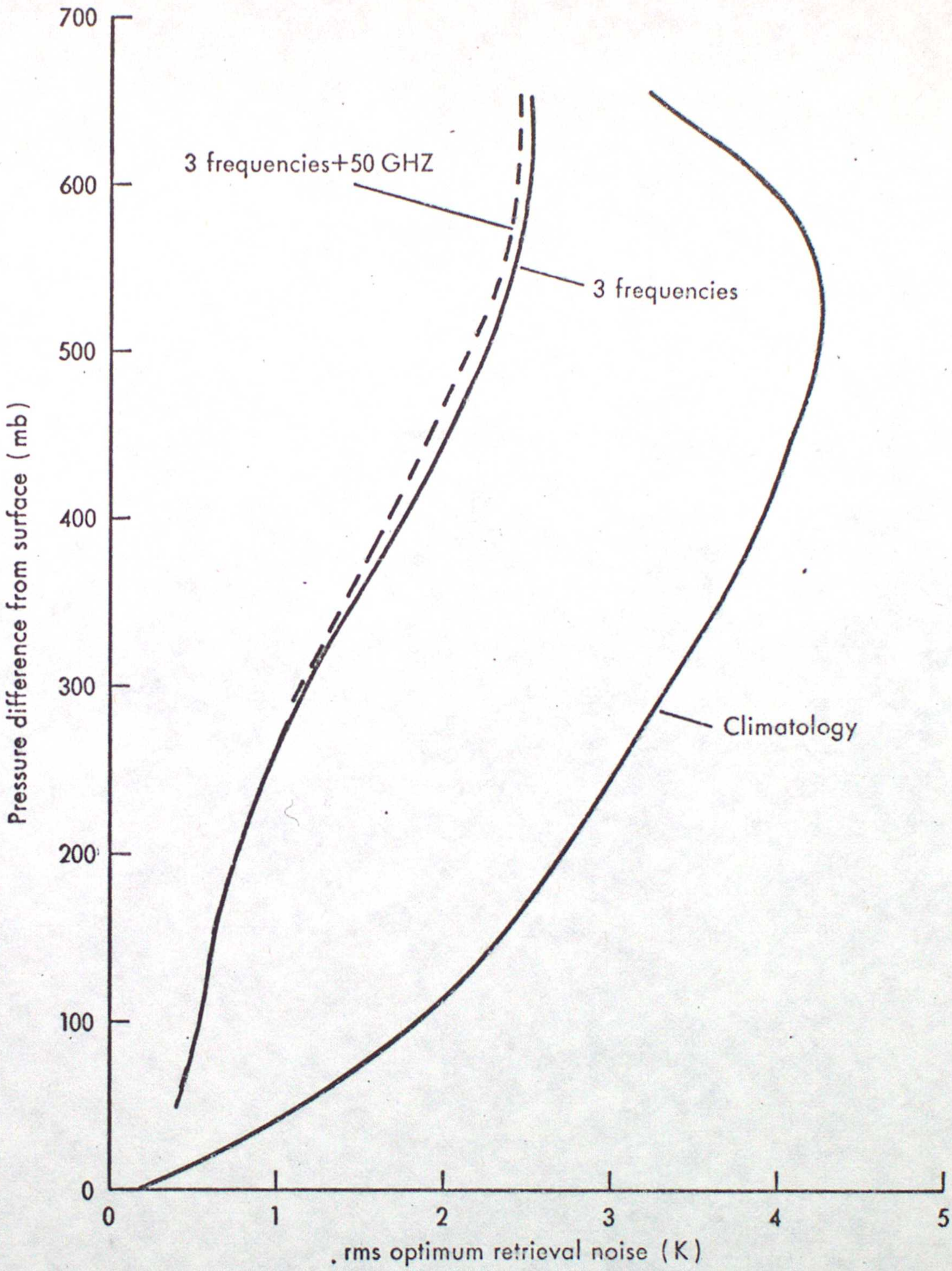


Figure 12. Improvement in optimum retrieval accuracy of 100mb layers, by adding a 50 GHz channel to the standard 3-channel set.

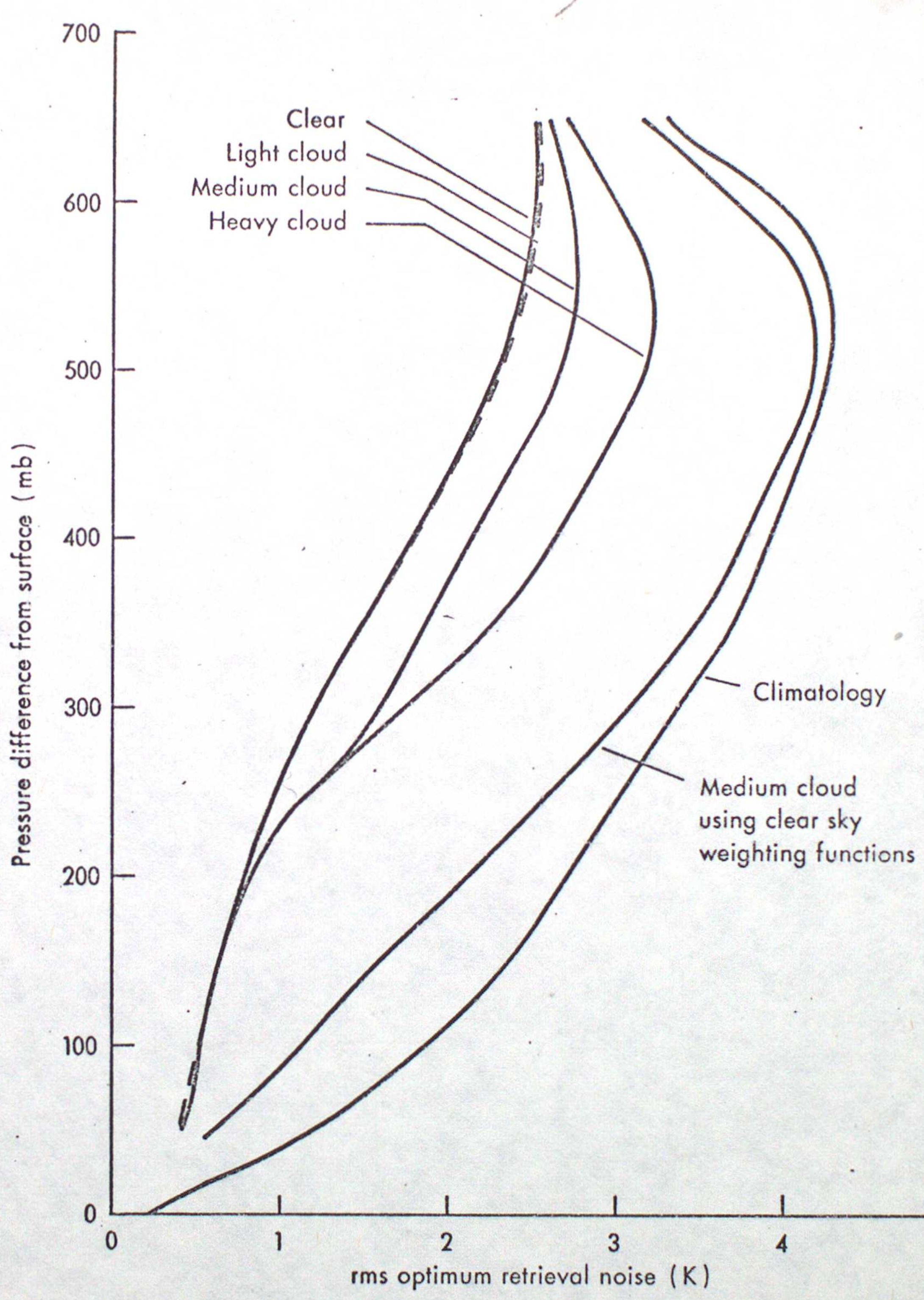


Figure 13. Optimum retrieval accuracy of 100mb layers for clear and cloudy conditions. Quantitative meaning of 'light', 'medium' and 'heavy' is given in the text.

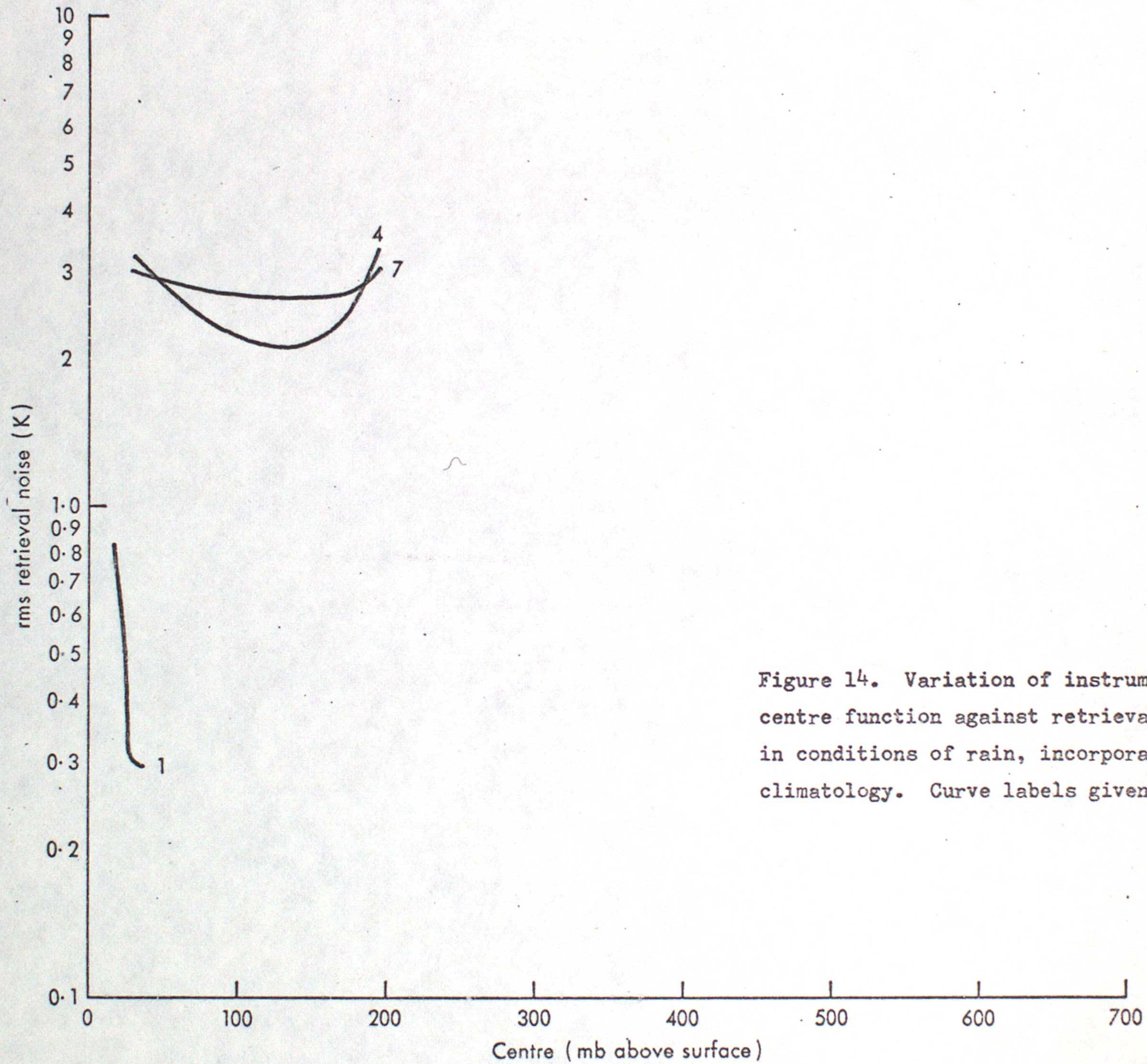


Figure 14. Variation of instrumental centre function against retrieval accuracy in conditions of rain, incorporating climatology. Curve labels given by table 2.

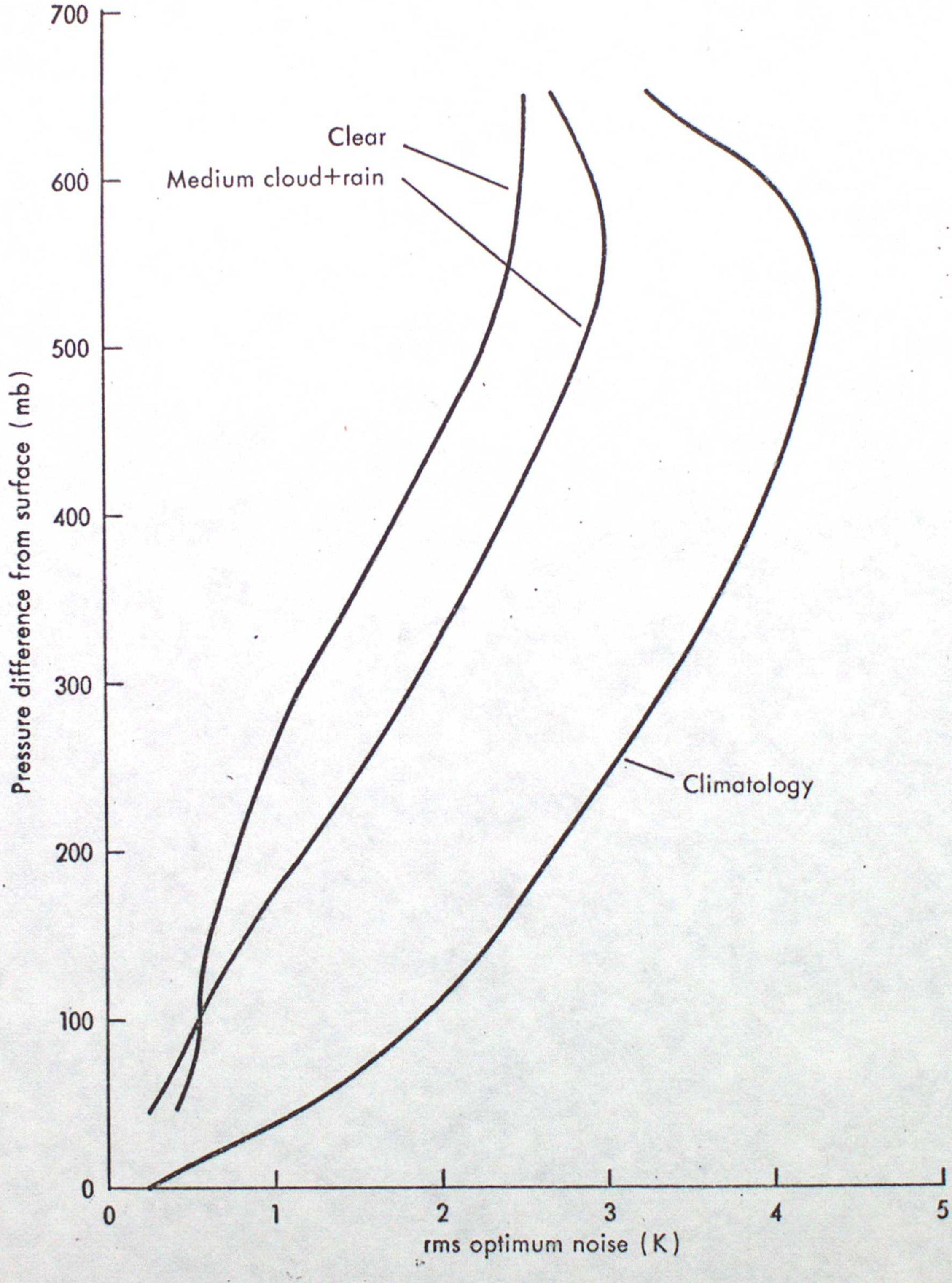


Figure 15. Optimum retrieval accuracy of 100mb layers in clear and rainy conditions.

Appendix A

Conventional Backus-Gilbert Theory

For fuller details of this theory, the reader should consult Conrath [A1].

Consider an n channel radiometer, and let $i, j = 1, 2 \ldots n$. For heights x' in the atmosphere, denote the vector of weighting functions by $K_i(x')$, and the covariance matrix of measurement errors by V_{ij}. It is assumed that $K_i(x')$ is temperature independent.

In the Backus-Gilbert scheme, for a height x, we require to find a vector of functions $a_i(x)$ which form the composite weighting function

$$A(x,x') = \sum_{i=1}^{n} a_i(x) K_i(x') , \qquad \text{A1)}$$

such that the function $Q(x)$, given by

$$Q(x) = q\, s(x) + (1-q) r\, \sigma^2(x) \qquad \text{A2)}$$

is minimised.

The spread, $s(x)$, of $A(x,x')$ about x is given by

$$s(x) = 12 \int_0^{x_{top}} (x-x')^2 A^2(x,x')\, dx' \qquad \text{A3)}$$

Atmospheric emission is assumed to be negligible above x_{top}. The normalising factor 12 is chosen such that when $A(x,x')$ is a rectangular function of width l, centred on x, then $s(x) = l$. Further, the rms retrieval error $\sigma^2(x)$, is given by

$$\sigma^2(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} V_{ij}\, a_i(x) a_j(x) \qquad \text{A4)}$$

The coefficient r ensures that A2) is dimensionally correct. By varying the parameter q between zero and unity, emphasis can be shifted from minimisation of error to minimisation of spread. Thus one can conduct a trade-off between vertical resolution and accuracy.

Two additional functions further characterise the behaviour of

They are the centre function

$$c(x) = \int_0^{x_{top}} x' A^2(x,x')\,dx' \Big/ \int_0^{x_{top}} A^2(x,x')\,dx' \quad \text{A5)}$$

and the resolving length

$$w(x) = 12 \int_0^{x_{top}} \left(c(x) - x'\right)^2 A^2(x,x')\,dx' \qquad \text{A6)}$$

Typical Backus-Gilbert weighting functions for the 3-channel instrument described in the main body of the text, are illustrated in Figures A1 to A7.

Appendix B

Incoporation of climatology into Backus-Gilbert theory

Let us now consider the problem of determining how instrumental information can improve the accuracy of temperature retrievals for a specific atmospheric layer, in contrast with climatology.

Let the climatological temperature covariance between any two levels x, x' in the atmosphere be $E(x,x')$, and consider a layer Λ, of depth d, and midpoint y.

Consider also a Backus-Gilbert scheme for the instrumental weighting functions, and suppose for some value q of the trade-off parameter, the composite weighting function about y is $A(y,x;q)$, with retrieval noise f. The function A is used to produce a weighted mean of temperature over the atmosphere. Denoting climatological averages by $\langle \quad \rangle$, then the climatological variance of this weighted mean of temperature, $P(y;q)$, is given by

$$\left\langle \left[\int_0^{x_{top}} dx \left(T(x) - \langle T(x) \rangle \right) A(y,x;q) \right]^2 \right\rangle \tag{B1}$$

$$= \left\langle \int_0^{x_{top}} dx \int_0^{x_{top}} dx' \left(T(x) - \langle T(x') \rangle \right) \left(T(x') - \langle T(x') \rangle \right) A(y,x;q) A(y,x';q) \right\rangle \tag{B2}$$

$$= \int_0^{x_{top}} dx \int_0^{x_{top}} dx' \left\langle \left(T(x) - \langle T(x) \rangle \right) \left(T(x') - \langle T(x') \rangle \right) \right\rangle A(y,x;q) A(y,x';q) \tag{B3}$$

$$= \int_0^{x_{top}} dx \int_0^{x_{top}} dx' \, E(x,x') A(y,x;q) A(y,x';q) \tag{B4}$$

The distribution of $A(y,x;q)$ about y may be satisfactorily described by its centre function C, and resolving length W, so that

$$\rho(y;q) \simeq \frac{1}{W^2}\int_{C-W/2}^{C+W/2} dx \int_{C-W/2}^{C+W/2} dx'\, E(x,x') \qquad \text{B5)}$$

This approximation shows how information other than spread is incorporated into our scheme.

The climatological covariance matrix, $\underline{E}_s$, between the (vertical) mean temperature of Δ, and the (vertical) mean determined

B4)

by $A(y,x;q)$ is given by

$$\underline{E}_s = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \qquad \text{B6)}$$

where

$$e_{11} = \frac{1}{d^2}\int_{y-\frac{d}{2}}^{y+\frac{d}{2}} dx \int_{y-\frac{d}{2}}^{y+\frac{d}{2}} dx'\, E(x,x') \qquad \text{B7)}$$

$$e_{12} = e_{21} = \frac{1}{d}\int_{y-\frac{d}{2}}^{y+\frac{d}{2}} dx \int_{0}^{x_{top}} dx'\, E(x,x')A(y,x;q) \qquad \text{B8)}$$

and

$$e_{22} = P(y;q) \qquad \text{B9)}$$

If $P(y;q)$ is given by B5) then e_{12} is approximated accordingly.

Radiometric observations do not relate directly to Δ, so that the instrumental covariance matrix of retrieval noise, $\underline{E}_I$, is given by

$$\underline{E}_I = \begin{pmatrix} \infty & 0 \\ 0 & f^2 \end{pmatrix} \qquad \text{B10)}$$

The matrices $\underline{E}_I$ and $\underline{E}_s$ represent error estimates of two independent information sources for the temperature profile. They may be combined to obtain the optimum estimate [B1] with covariance.

$$\underline{E}_{I+S} = \left(\underline{E}_I^{-1} + \underline{E}_s^{-1}\right)^{-1} \qquad \text{B11)}$$

Using B6)-B10), B11) becomes

$$\underline{E}_{I+S} = \frac{1}{(e_{22}f^{-2}+1)} \begin{pmatrix} e_{11}+f^{-2}|\underline{E}_s| & e_{12} \\ e_{21} & e_{22} \end{pmatrix} \qquad \text{B12)}$$

where $|\ |$ denotes the determinant.

From B12) the resulting error of the combined instrument/climatology estimate of the mean temperature of Δ is

$$\sigma(\Delta;q) = \left(e_{11} - \frac{e_{12}^2}{e_{22}+f^2}\right)^{\frac{1}{2}} \leq e_{11}^{\frac{1}{2}} \qquad \text{B13)}$$

By varying q between 0 and 1, as before, we obtain a curve of instrumental spread against $\sigma(\Delta; q)$. Unlike the original Backus-Gilbert trade-off curve, this has a noise minimum for some value q_0 of q.

The precise value of q_0 depends on the degree of climatological correlation between the atmospheric layers. If, for example, the temperature of the layers were very highly correlated, then the optimum q would correspond to the minimum noise solution of the Backus-Gilbert variation, that is $q_0 = 0$. Conversely, if the temperature of the layers were very poorly correlated, then the optimum would correspond to the minimum spread solution (about the midpoint of Δ), ie $q_0 = 1$. In reality, will lie somewhere in between these values.

If Δ is one of a number of layers in the atmosphere, the optimum retrieval noise (the noise at $q = q_0$) may be plotted as a function of height, and the reduction in variance over climatology observed. It is important to realise that this retrieval noise relates precisely to the layer Δ in question.

Finally, the errors of retrieval using clear sky weighting functions in cloudy conditions may be calculated by including in B13 a systematic error, ΔT, given by the difference in brightness temperature between clear and cloudy conditions, so that

$$\sigma(\Delta, q) = \left(e_{11} - \frac{e_{12}^2}{e_{22} + f^2 + \Delta T^2} \right)^{\frac{1}{2}} \qquad \text{B14)}$$

References to Appendices

A1 Conrath, B.J., (1972). J. Atmos. Sci. 29 1262.

B1 Deutsch, R., (1964). Estimation Theory. Prentice Hall, Inc., Englewood Cliffs, New Jersey.

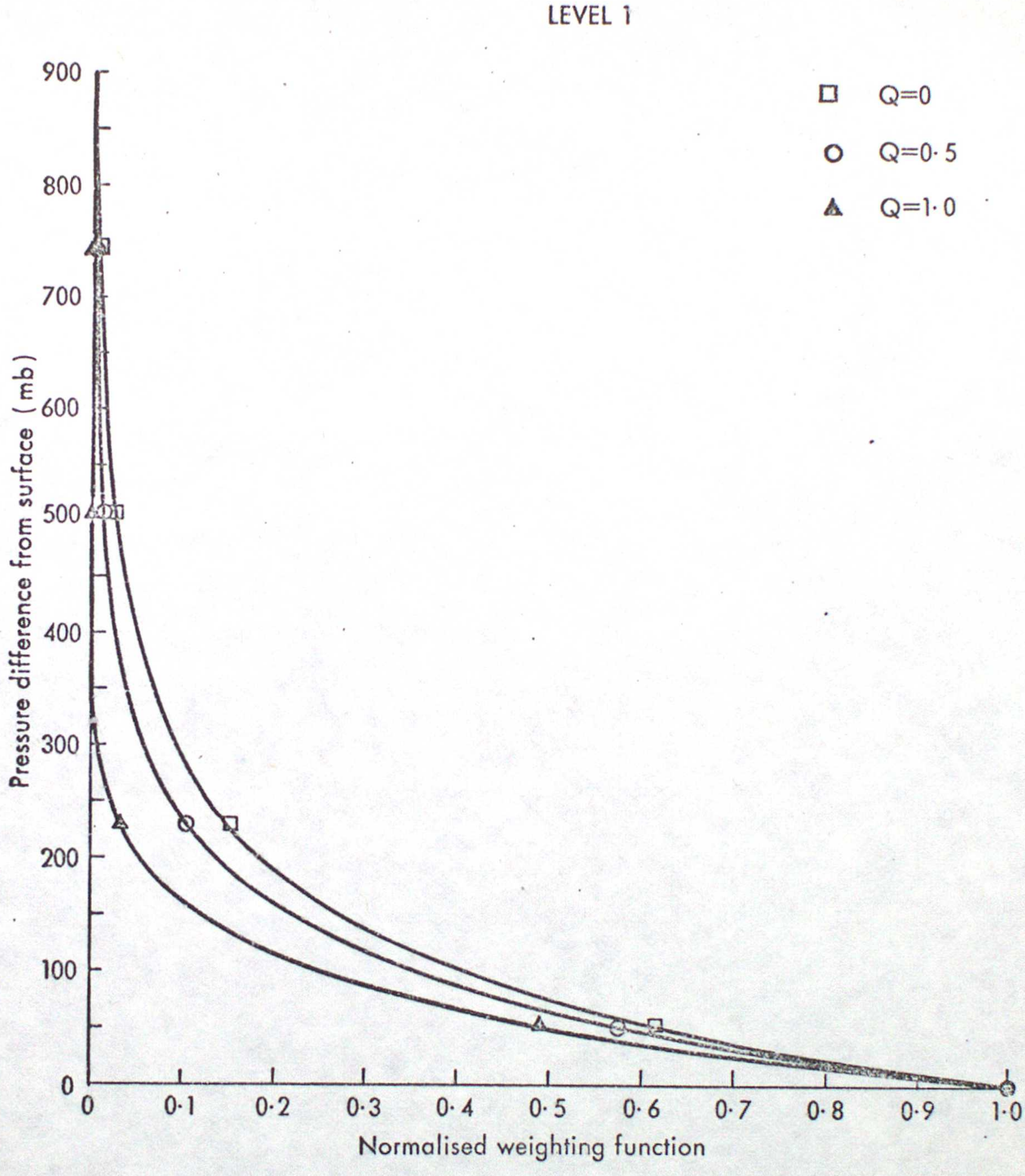
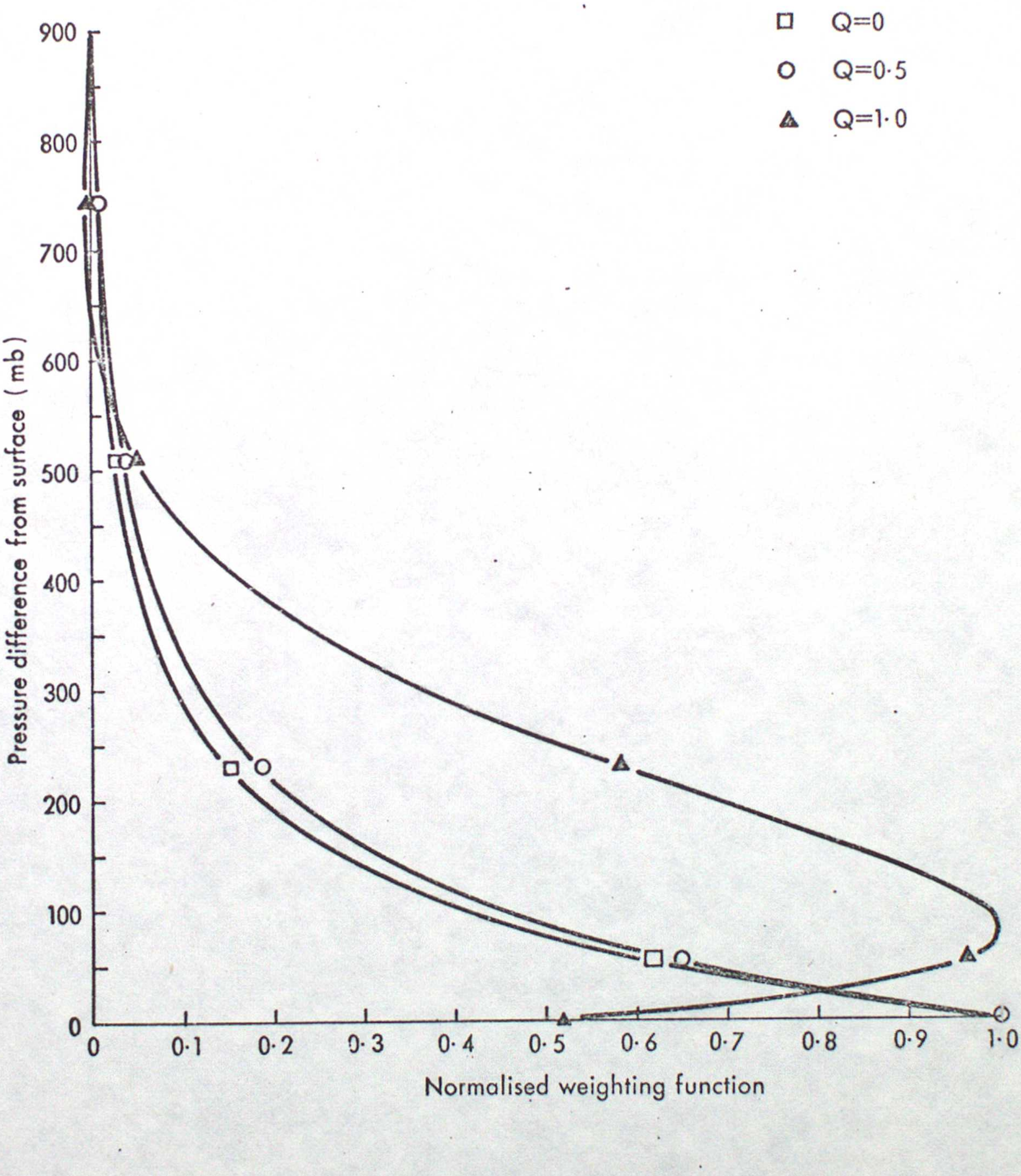
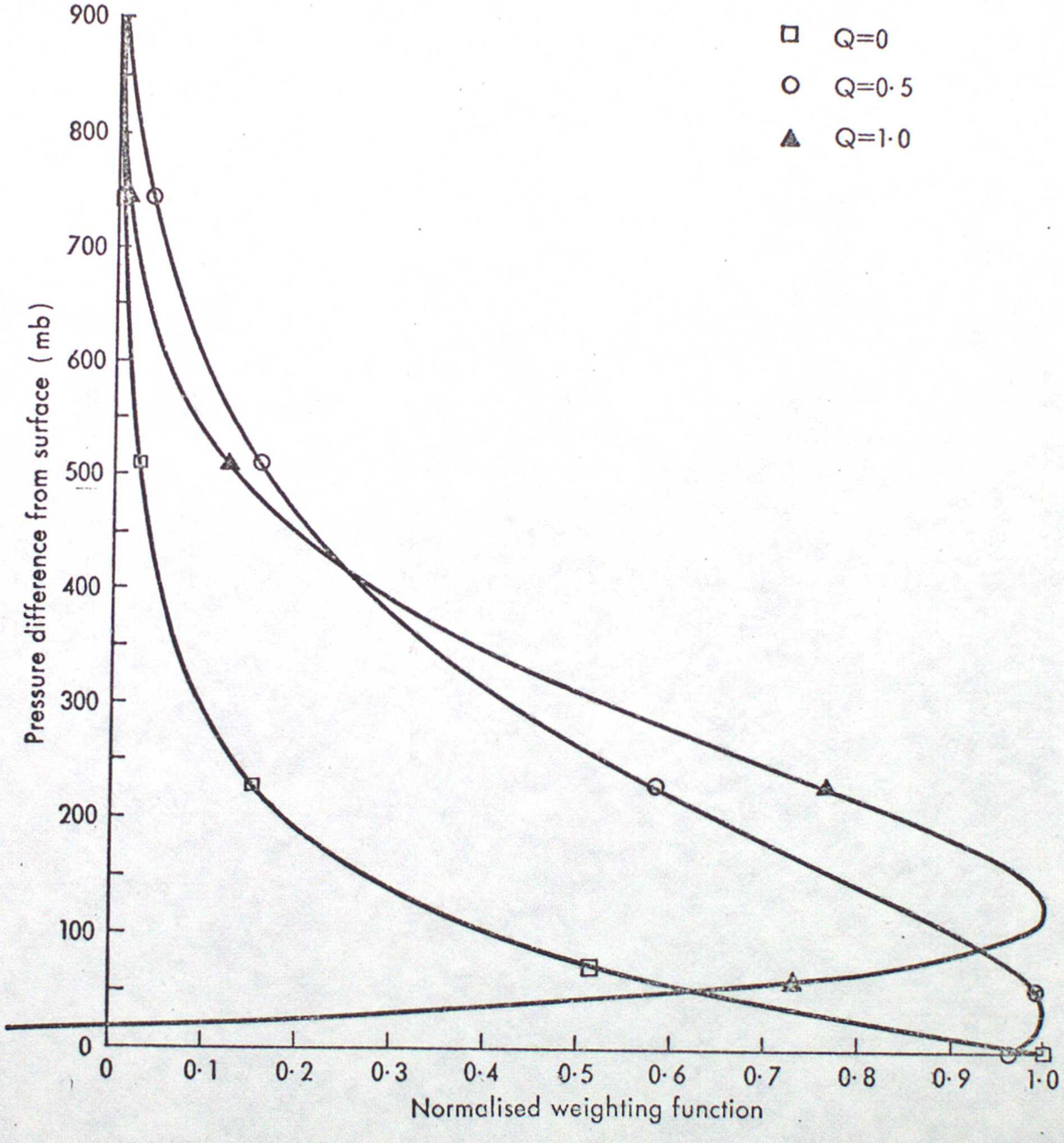


Figure A1-A7. Modified weighting function for seven levels and three values of the Backus-Gilbert parameter. q = 0 corresponds to minimum noise, q = 1 corresponds to minimum spread (relative to the level in question). The pressure coordinates of the level numbers are given in table 2.

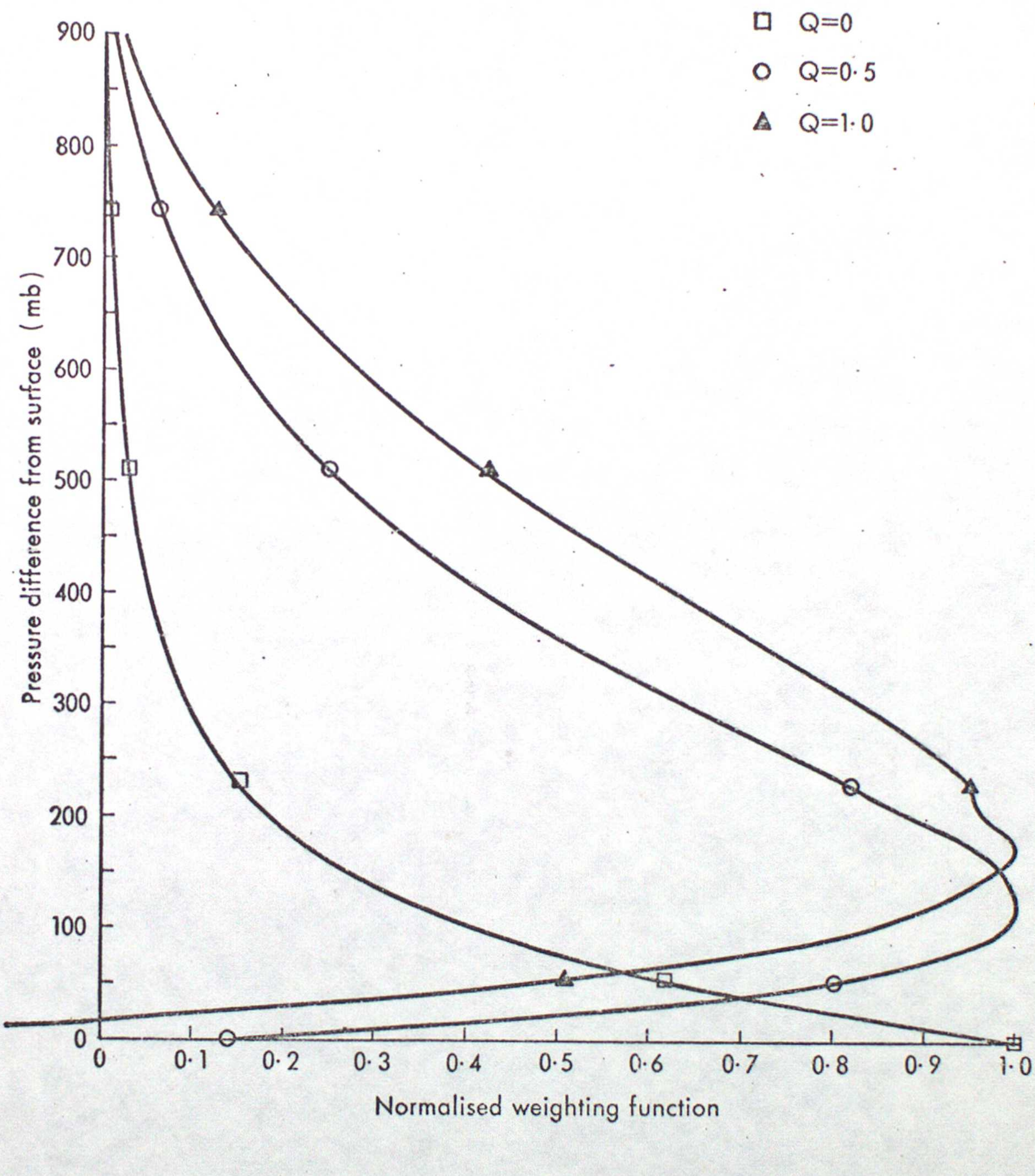
LEVEL 2


Q=0
Q=0·5
Q=1·0
Pressure difference from surface (mb)
900
800
700
600
500
400
300
200
100
0
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function

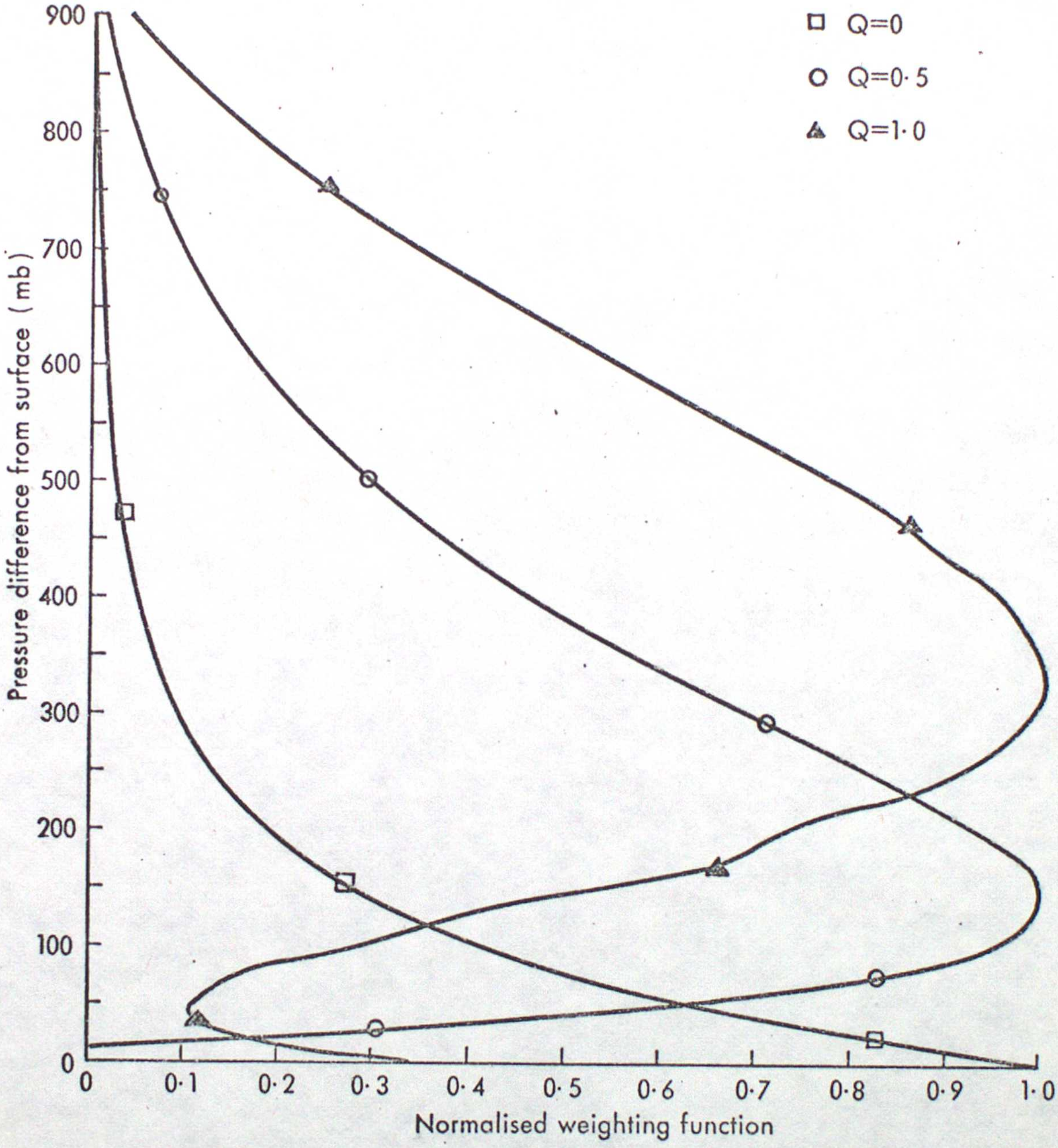
LEVEL 3

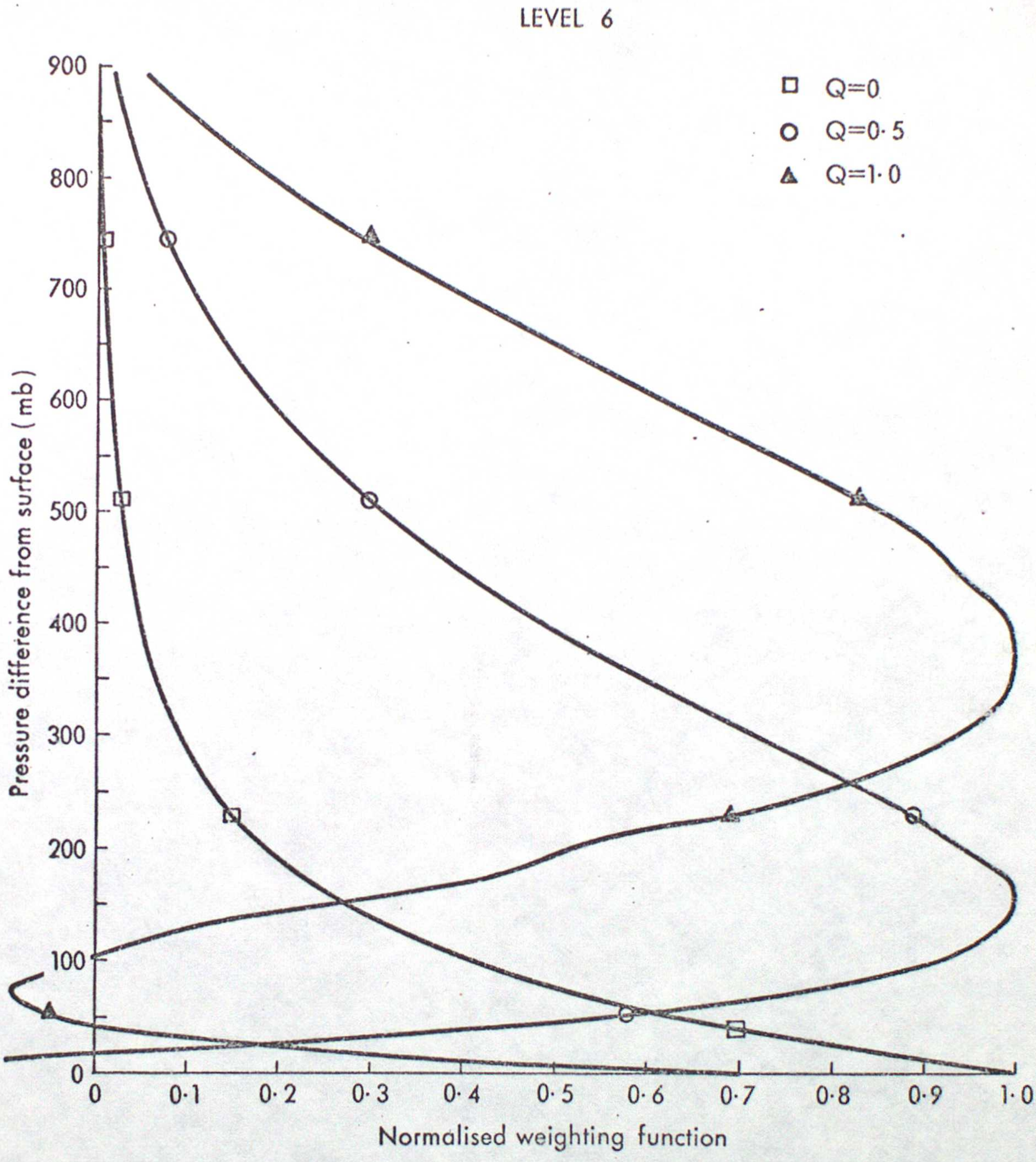

900
800
700
600
500
400
300
200
100
0
Pressure difference from surface (mb)
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function
Q=0
Q=0·5
Q=1·0

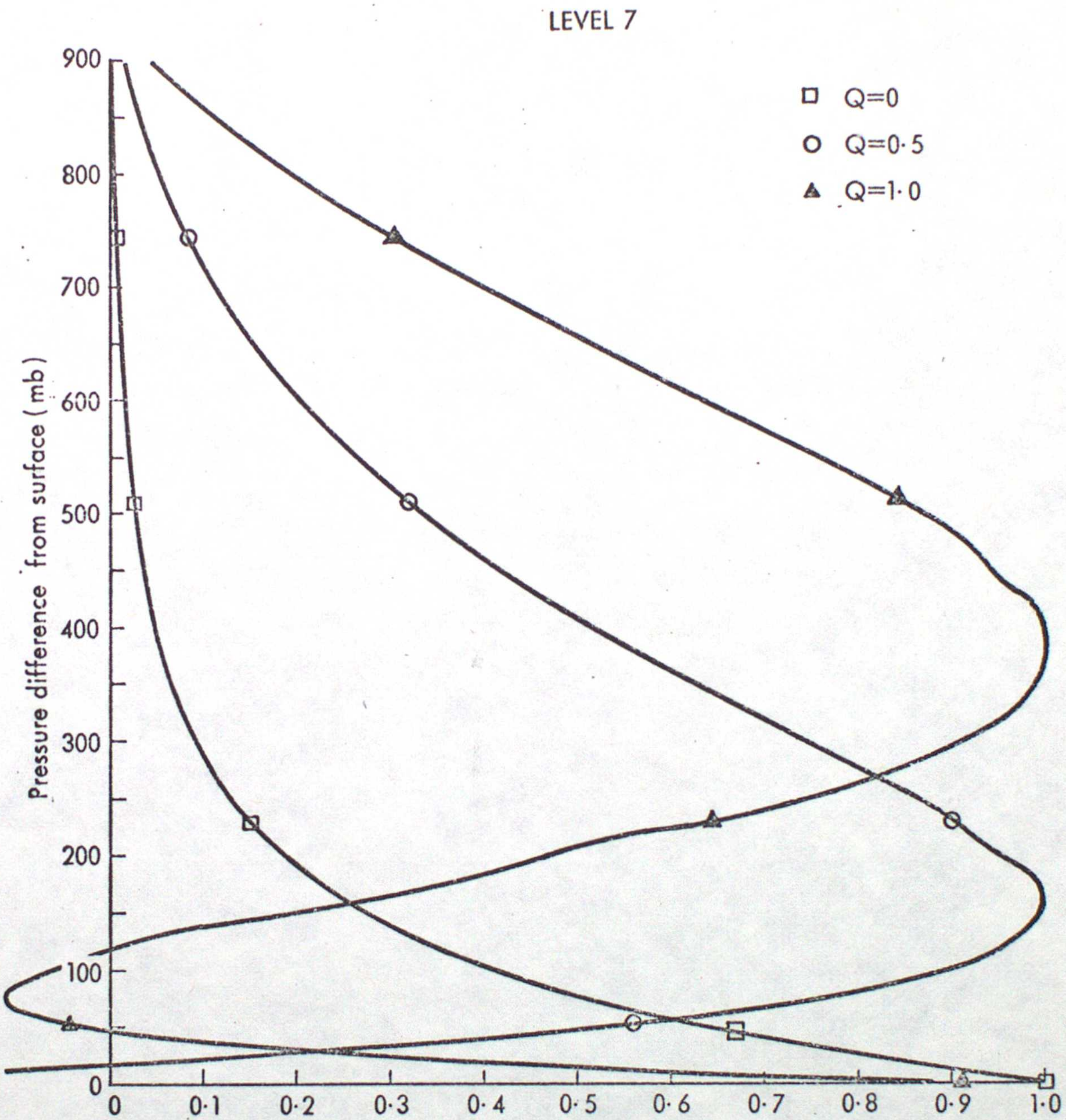
LEVEL 4


Q=0
Q=0·5
Q=1·0
900
800
700
600
500
400
300
200
100
0
Pressure difference from surface (mb)
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function

LEVEL 5


Q=0
Q=0·5
Q=1·0
900
800
700
600
500
400
300
200
100
0
Pressure difference from surface (mb)
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function


LEVEL 6
Q=0
Q=0·5
Q=1·0
900
800
700
600
500
400
300
200
100
0
Pressure difference from surface (mb)
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function


LEVEL 7
Q=0
Q=0·5
Q=1·0
Pressure difference from surface (mb)
900
800
700
600
500
400
300
200
100
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
Normalised weighting function