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METEOROLOGICAL OFFICE

Geophysical Memoirs No. 121

(THIRD NUMBER, VOLUME XVII)

BUMPINESS IN CLEAR AIR AND ITS RELATION TO SOME SYNOPTIC-SCALE INDICES

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BUMPINESS IN CLEAR AIR AND ITS RELATION TO SOME SYNOPTIC-SCALE INDICES

SUMMARY

An analysis has been made of pilots' reports of bumpiness, or lack of it, along 772 385 km of track when aircraft were cruising in the neighbourhood of the United Kingdom during seven days in April and May 1972. An assessment has been made of the value of a pilot's report in short-term forecasting.

The Bushby–Timpson 10-level model, which has been described by Benwell *et alii* (1971), has been used to calculate analysed and forecast fields of some synoptic-scale turbulence indices. The value of each index as a predictor of bumps has been estimated by comparing it with pilots' reports made within 100 minutes of its time of validity.

A composite index is proposed which, on the data used, predicted bumps better than several well-known indices and almost as well as a recent report from another aircraft in the vicinity. The composite index was better than a pilot's report for predicting the absence of bumps.

1– INTRODUCTION

As part of an investigation into clear-air turbulence (CAT) in the spring of 1972 the Meteorological Office invited aircraft captains to complete maps showing bumpiness in clear air on any flights made on seven selected days. Special emphasis was put on reporting smooth as well as bumpy flight. The 'days' lasted 28 hours from 2200 to 0200 GMT and so covered three periods centred on the standard hours for radio-soundings. The decision whether or not to nominate a day was made 5 or 6 hours before it was due to begin. In general, 'days' were nominated because CAT seemed likely in the area shown on the reporting map. The last three 'days' were chosen because CAT seemed likely in the area where air traffic was expected to be densest. To illustrate the synoptic situations chosen, 300-mb contours for 1200 GMT each turbulence day are shown in Figures 1.1 to 1.7.

Figure 1.8a is an example of a reporting map showing the turbulence encountered on a flight from London to Belfast. The information and instructions that were printed on the back of each map are shown in Figure 1.8b. It can be seen from Figure 1.8b that the criteria for intensity of bumpiness were those laid down by the International Civil Aviation Organization (ICAO) and that they were readily available to the pilot when he made his report. A defect that may be inferred from this, however, is that no account is taken of the differing effects of a given gust on different types of aeroplane.

Most of the results from this investigation are presented in the form of graphs. For the reader who wishes to make his own assessment of the statistical significance of the results the data on which the graphs are based are available in an unpublished report by Sparks *et alii* (1976).

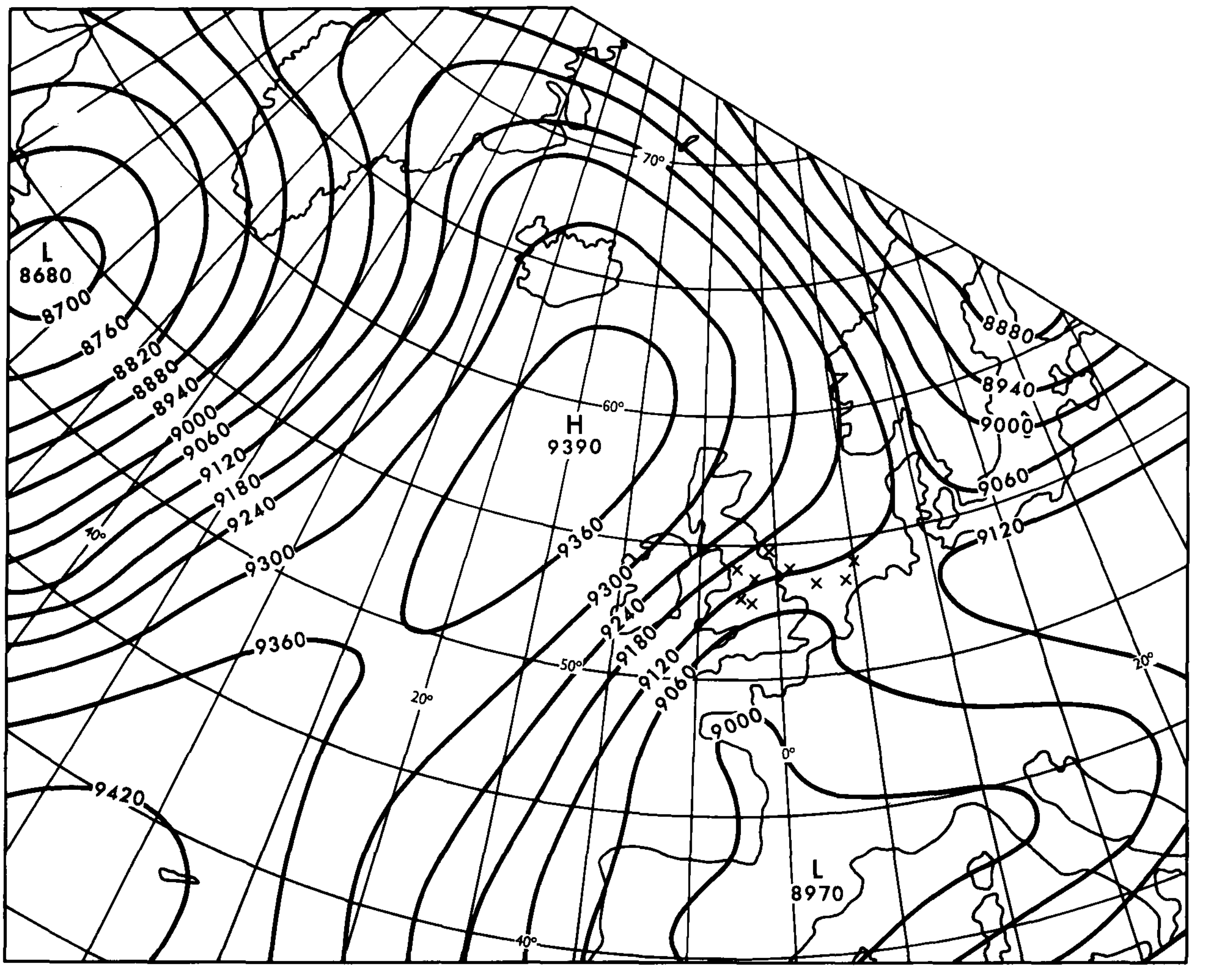


FIGURE 1.1. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 22 APRIL 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

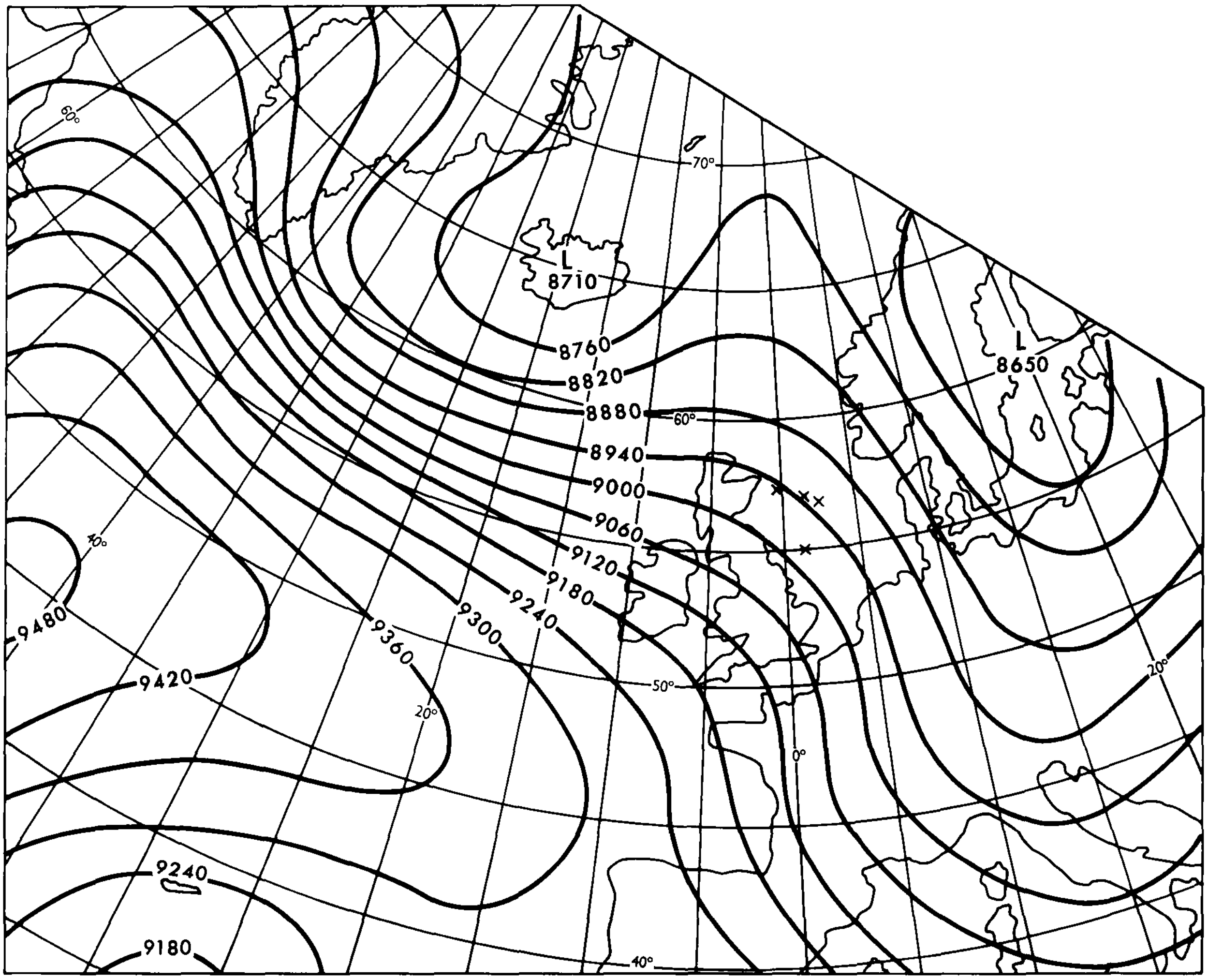


FIGURE 1.2. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 28 APRIL 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

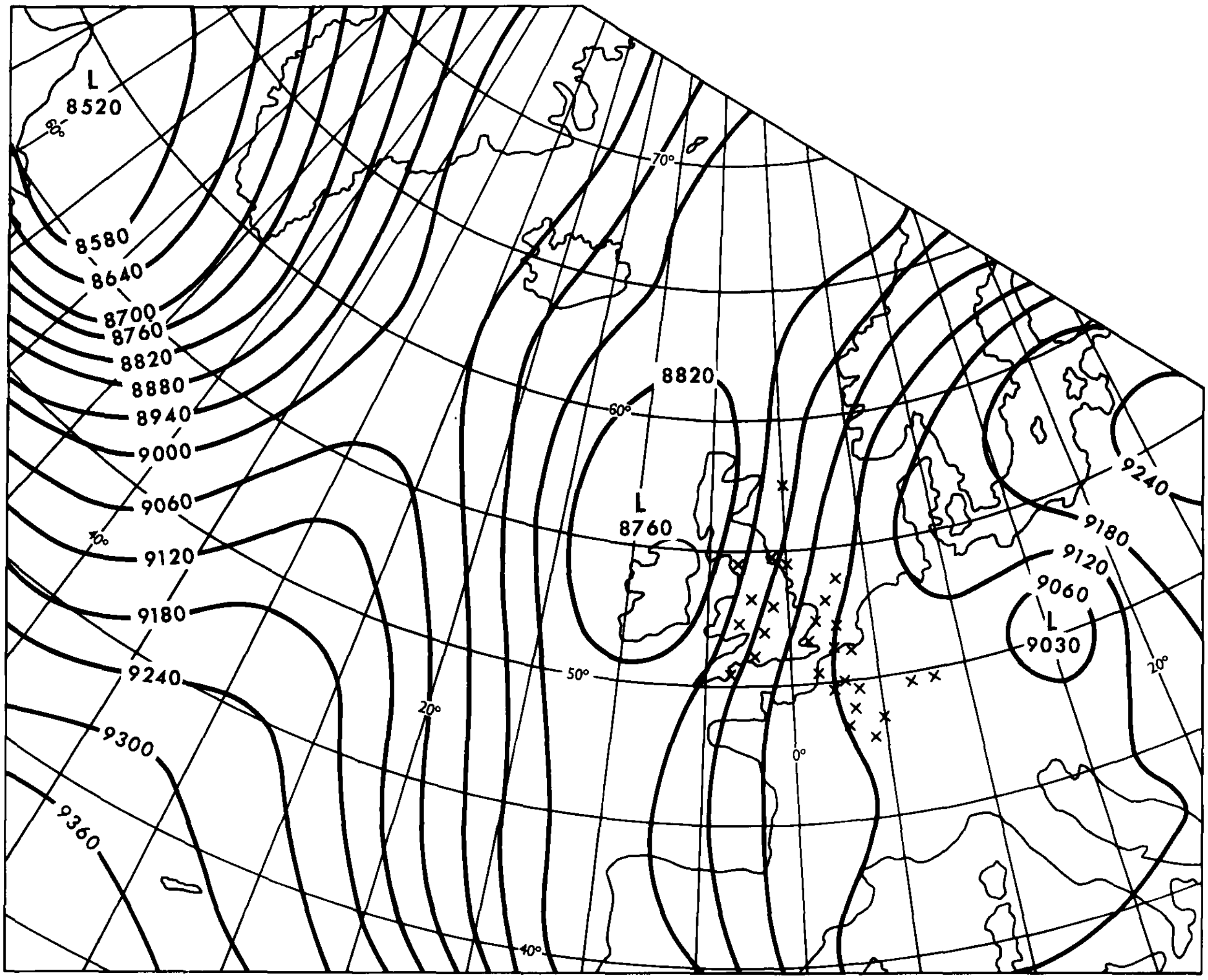


FIGURE 1.3. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 1 MAY 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

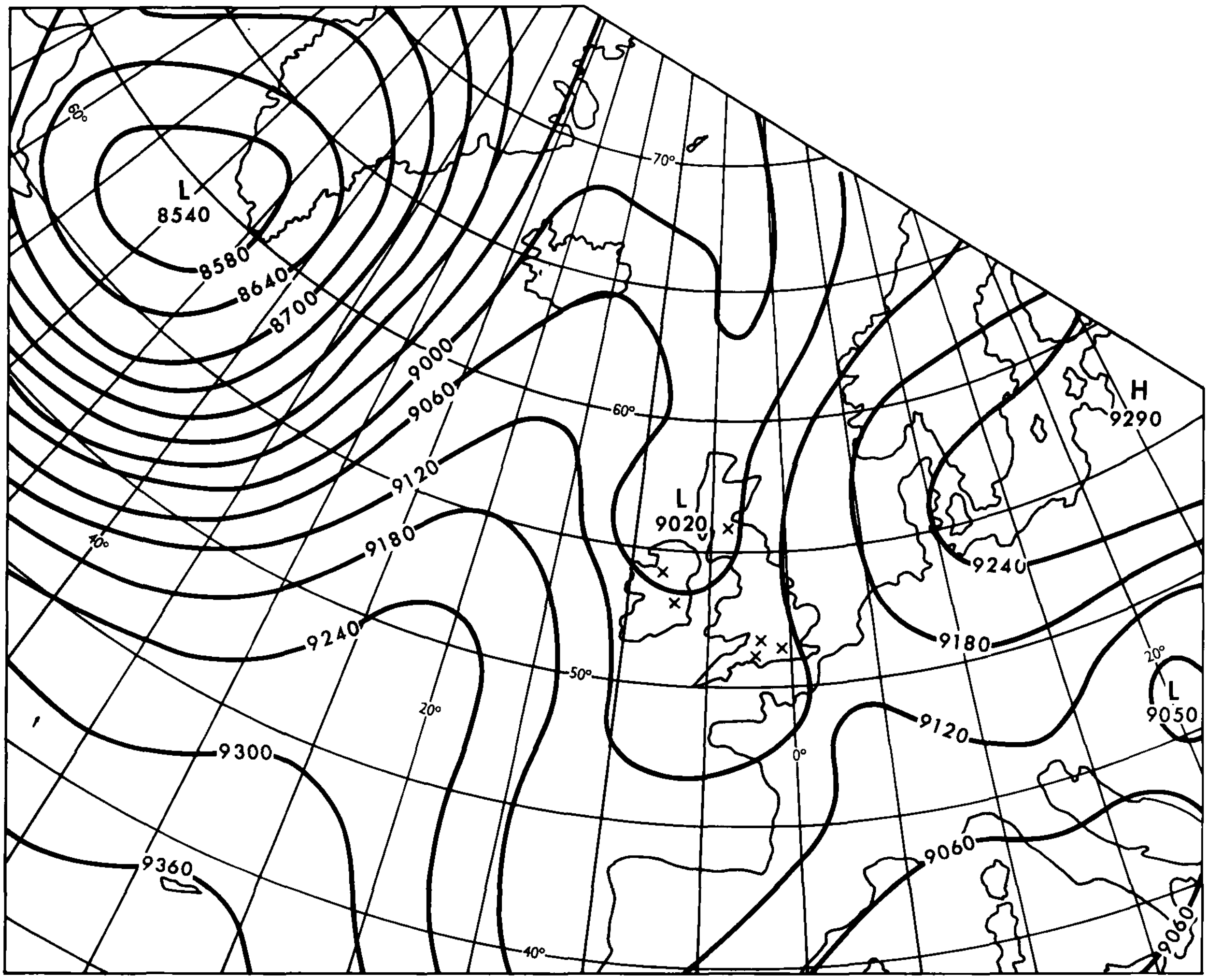


FIGURE 1.4. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 3 MAY 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

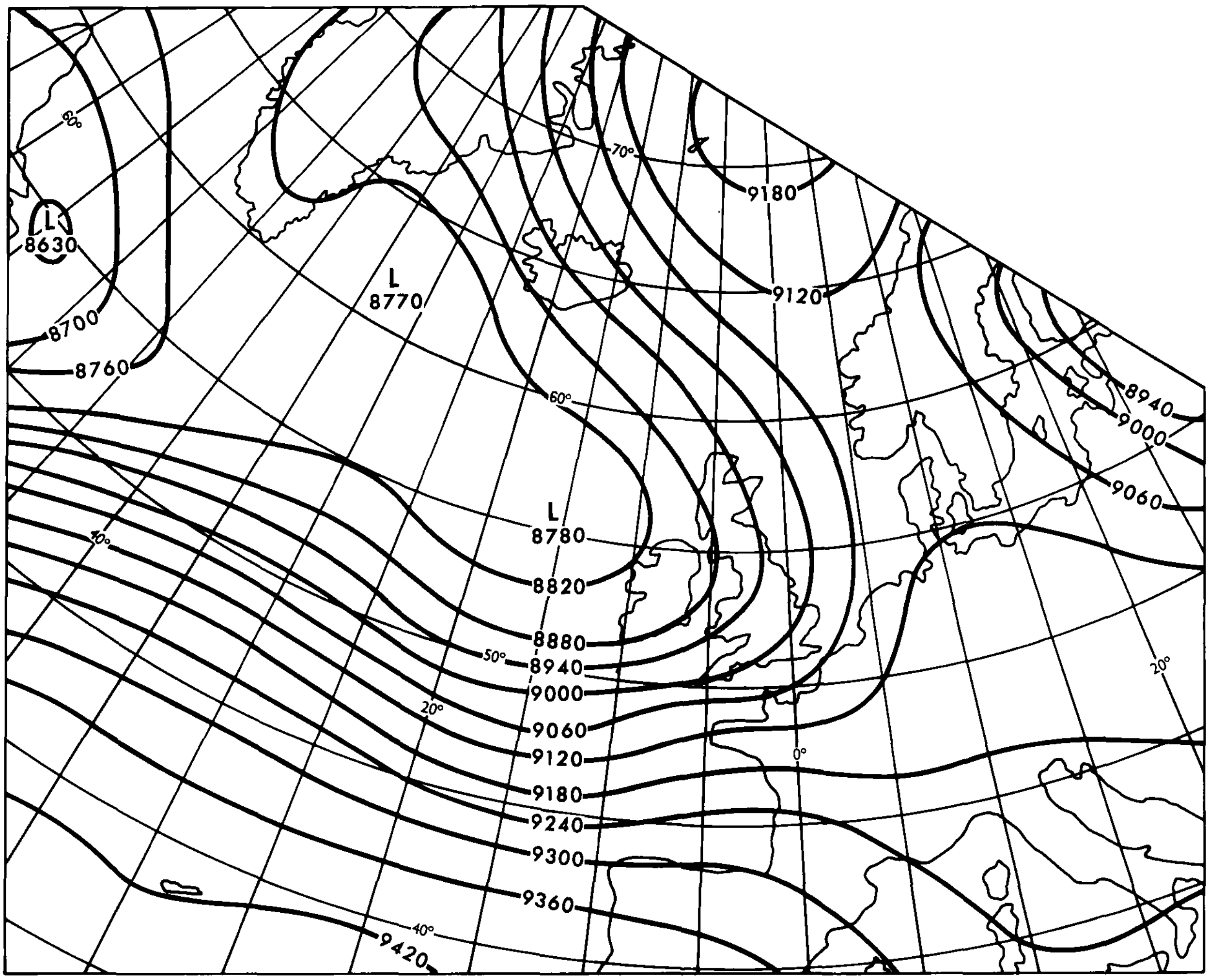


FIGURE 1.5. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 9 MAY 1972

No reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

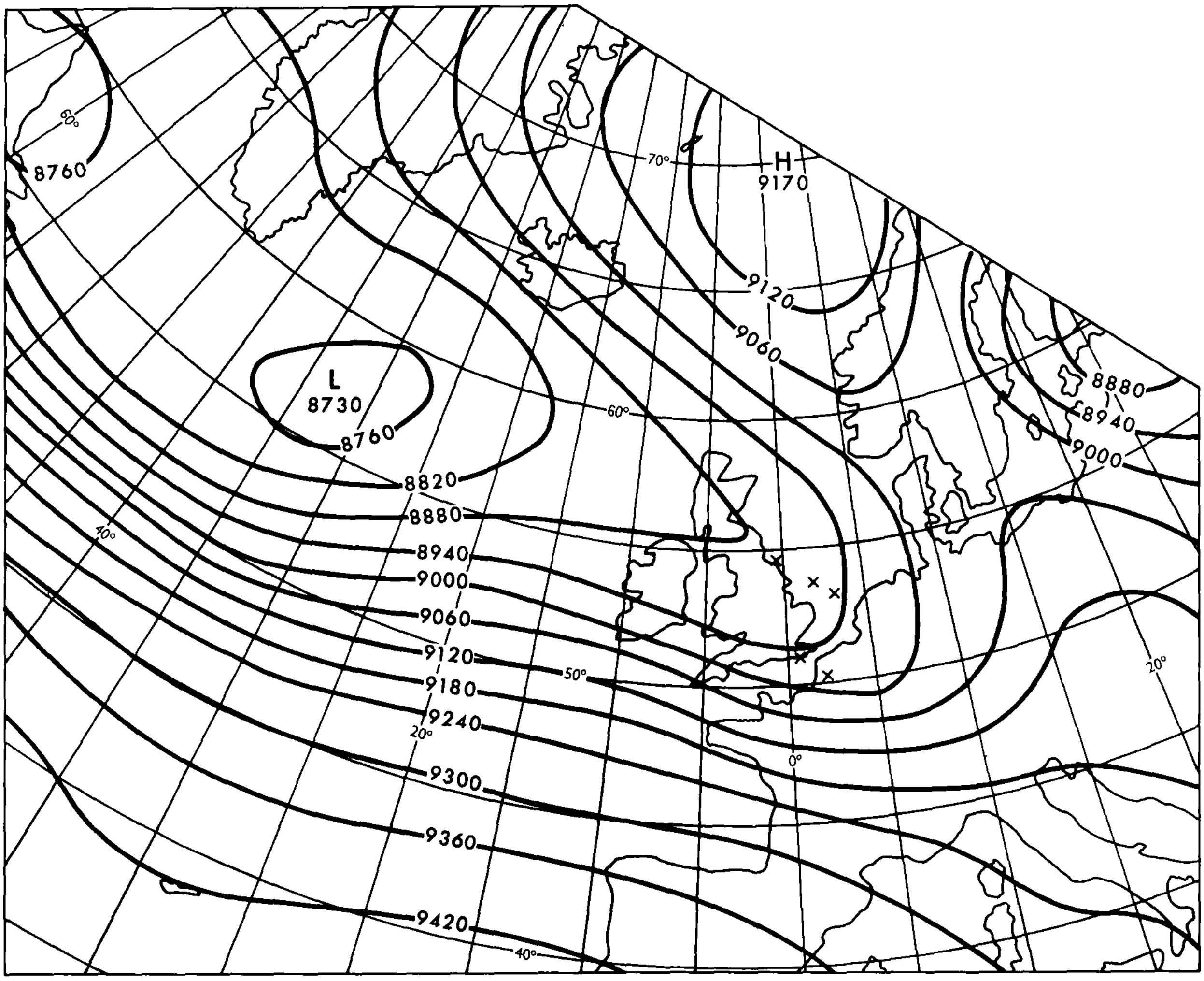


FIGURE 1.6. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 10 MAY 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

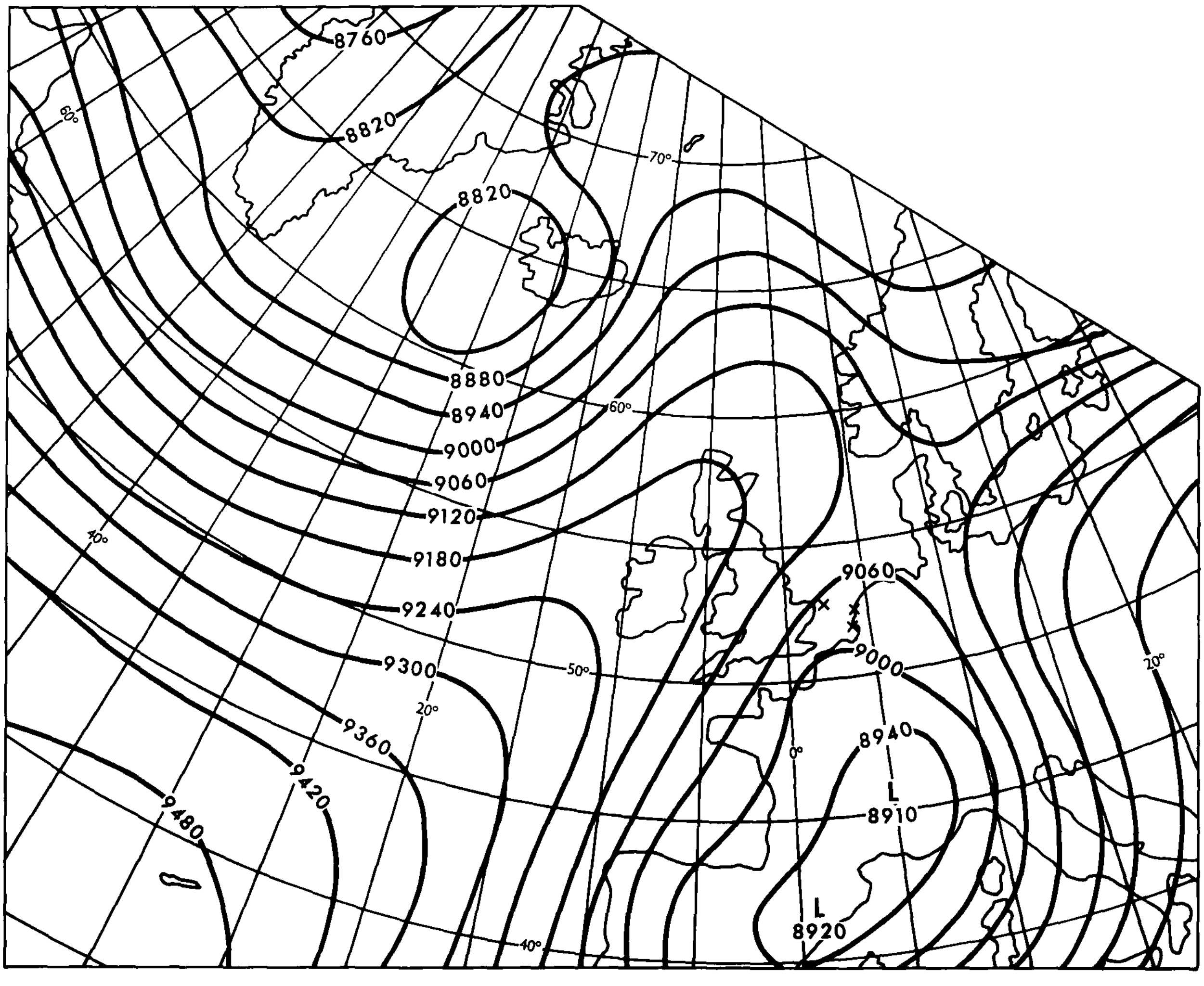


FIGURE 1.7. INITIALIZED FIELD OF 300-MILLIBAR GEOPOTENTIAL (gpm)

1200 GMT 16 MAY 1972

X = reports of moderate or severe bumps 200–400 mb, 1000–1319 GMT.

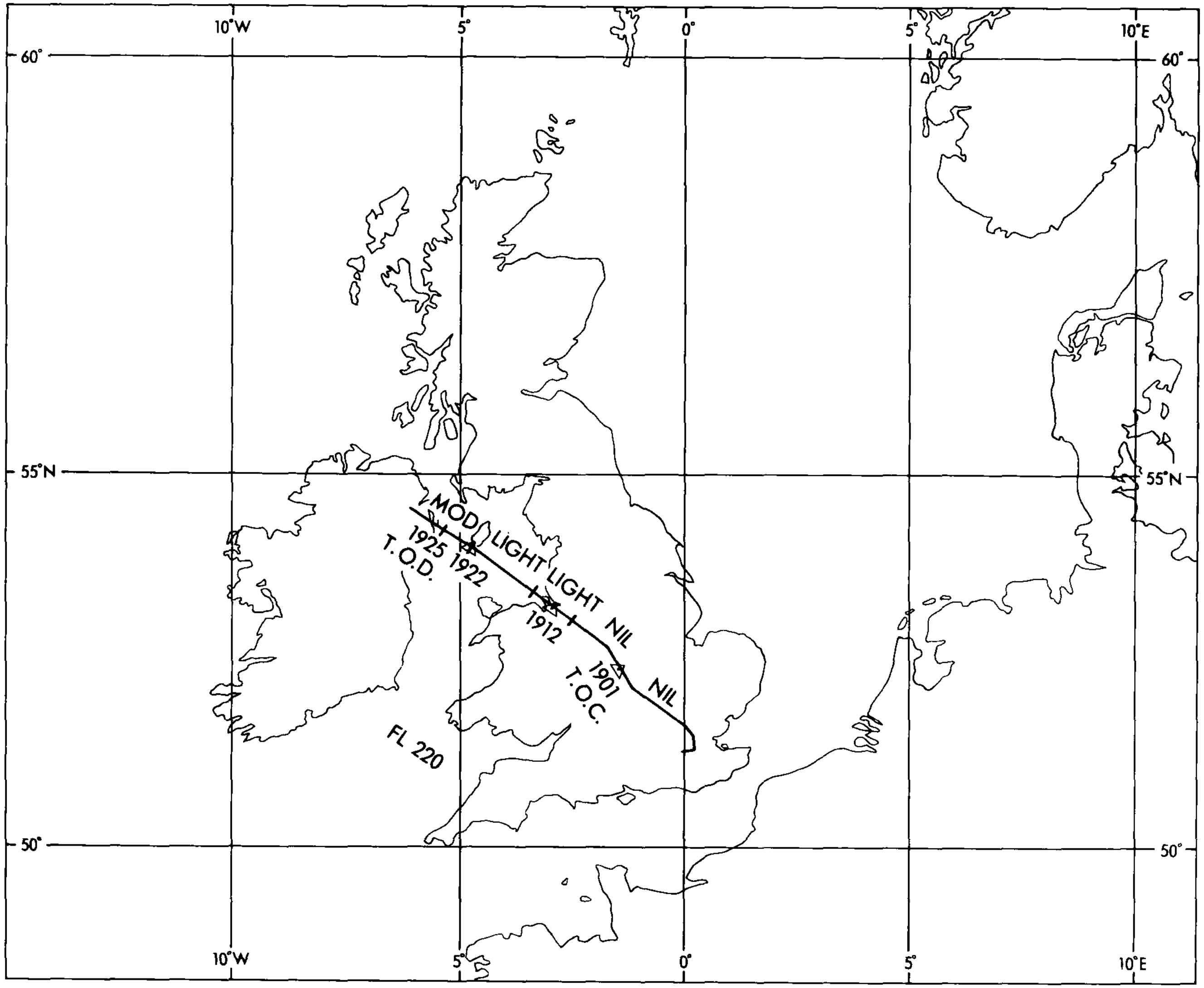


FIGURE 1.8a. EXAMPLE OF A REPORTING MAP SHOWING THE TURBULENCE ENCOUNTERED ON A FLIGHT FROM LONDON TO BELFAST

METEOROLOGICAL OFFICE CLEAR AIR TURBULENCE SURVEY

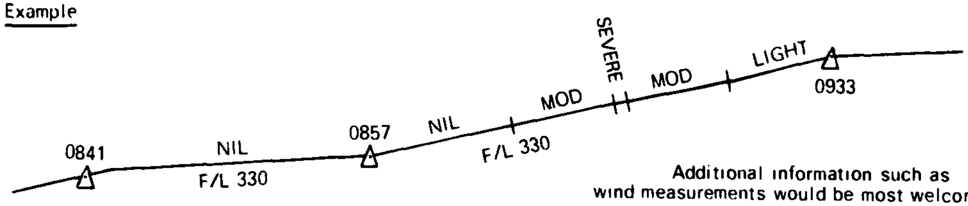
These maps are being issued on a few chosen days in order to help assess techniques of forecasting CAT. Captains are asked to describe the turbulence history of the cruise stage of their flight on the map overleaf and of the climb and descent stages on the Table on the right.

000904

Reports of no turbulence will be as important as reports of turbulence.

For all points of the cruise stage within the area of the map, indicate the track, flight level and turbulence encountered. Mark the time (GMT) at convenient intervals.

Example



Additional information such as wind measurements would be most welcome

Turbulence Criteria (Based on ICAO Doc.8812, AN-CONF/6, 1969)

Description	Effects
'Moderate'	Moderate changes in aircraft attitude and/or altitude but the aircraft remains in positive control at all times. Variations in air speed are usually small. Loose obj ects move about. Occupants feel strain against seat belts. Peak changes in accelerometer readings at c.g. of 0 5g to 1.0g.
'Severe'	Abrupt changes in aircraft attitude and/or altitude; Variations in airspeed are usually large. Loose objects tossed about. Occupants are forced violently against seat belts. Peak changes in accelerometer readings at c.g. of more than 1.0g.

'Light' and 'extreme' may be reported when effects are less or greater than these.

Captains are asked to hand maps to a British meteorological office or to their Company representative at their destination to be sent to The Director-General, Meteorological Office(Met O 9),
London Road
Bracknell, Berks, U K

Date 1:5:72

Aircraft type BAC 1-11

Company/Unit BCAL

Flight number BR 935

Departed LGW at 1835 G M T.

Turbulence on climb

Area and time (GMT)	Between flight levels	Intensity of Turbulence
S of EGLL	0-220	NIL
E of EGLL		

Turbulence on descent

Area and time (GMT)	Between flight levels	Intensity of turbulence
	NIL	

FIGURE 1.8b. INFORMATION AND INSTRUCTIONS PRINTED ON THE REVERSE OF THE MAP SHOWN IN FIGURE 1.8a

2– QUALITY CONTROL AND DIGITIZATION OF THE REPORTS FROM AIRCRAFT PILOTS

3050 maps were received, reporting pilots' assessments of the bumpiness experienced along their aircrafts' tracks. Some gave all the information which had been asked for. Others had to be discarded because they gave too little information or because the aircraft were in cloud. Some were incomplete or ambiguous but could be made useful by making reasonable assumptions. If a time were missing, for example, a ground speed suited to the wind and type of aircraft would be assumed so that the time could be deduced from an earlier known time and position. When such assumptions were made the observations were graded as second quality. Some reports were graded second quality along parts of the route and first quality along other parts. The reports in these two categories were reduced and analysed both separately and together, but since no difference was detected between the two analyses only the combined analysis is presented.

Although some reports were made of bumpiness during climb and descent, as a body the reports did not give enough detail to be useful for these stages. Only the cruise stages of flights were analysed.

The turbulence history of each flight was divided into elementary observations and digitized. Each elementary observation had a unique numerical value for the time, height, grid square*, length of track in the grid square, intensity of bumpiness along that length of track and quality of observation. Each digitized elementary observation was then checked/.

These elementary observations form the sets of data which were compared with the fields of dynamical indices. Stored in printed form and on magnetic tape they are available for scrutiny in the archives of the Meteorological Office, together with the reporting maps.

3–ANALYSIS OF PILOTS' REPORTS

3.1 *Introduction*

The pilots' reports of bumpiness provide the standard against which the performance of the synoptic-scale indices must be judged. The distribution and self-consistency of the reports are, therefore, worthy of study. The reports have been analysed to show, for each level, the fraction of the distance flown which was bumpy and how often periods of bumpiness were reported. An attempt has been made to illustrate the region in space and time for which a pilot's report of bumpiness is representative.

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* This refers to the grid of the forecast model—see Figure 2.1 and Benwell *et alii* (1971).
/ The digitization and checking are described in the full report by Sparks *et alii* (1976).

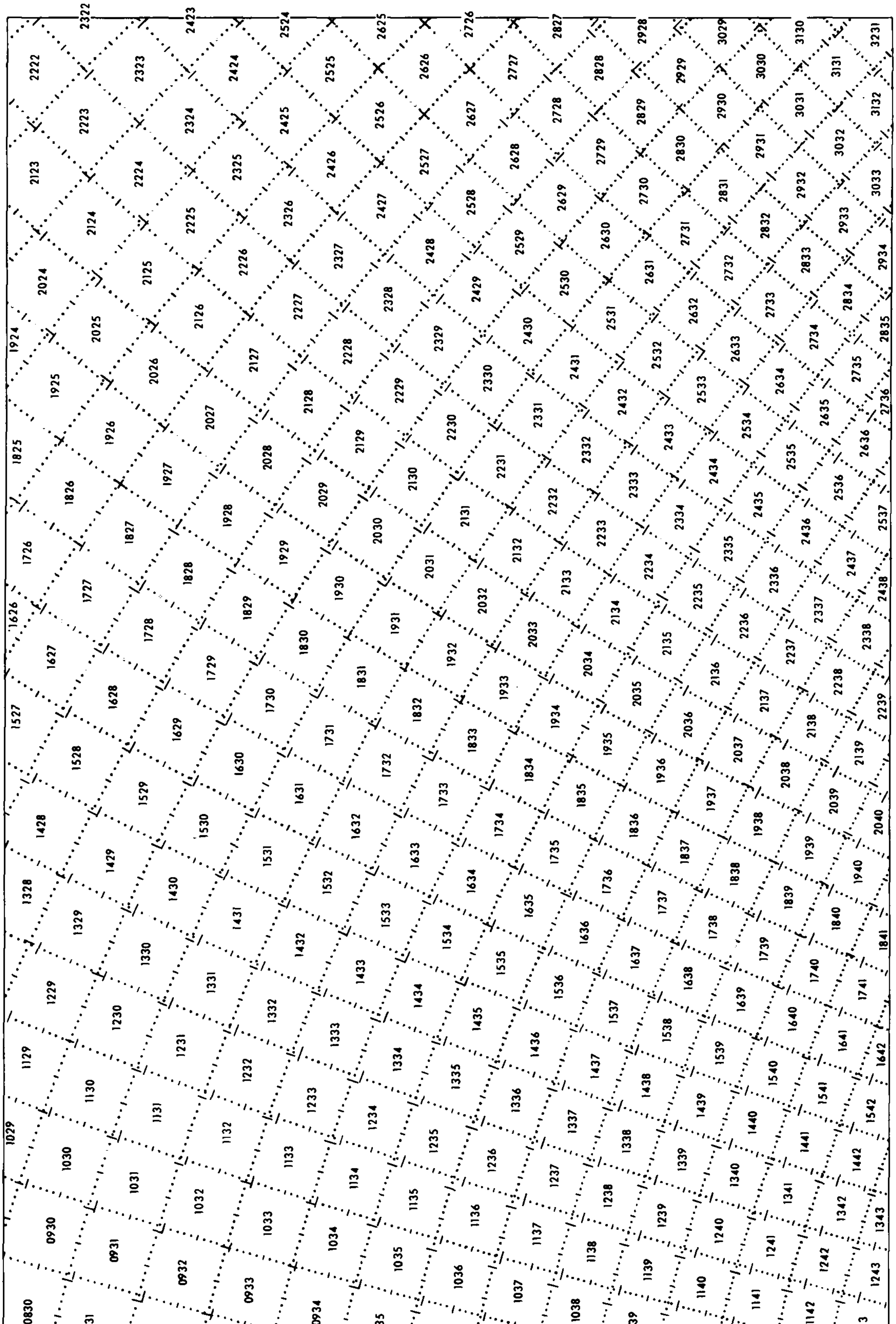


FIGURE 2.1. GRID SQUARES OF THE FORECAST MODEL ON THE PROJECTION OF THE REPORTING MAP SHOWN IN FIGURE 1.8a

3.2 *The proportion of flying which was bumpy*

The fraction of distance flown which was bumpy was found separately for flight over land, flight near the coast and flight over the sea, well away from land. (Figure 3.1 shows the areas described as falling into each category.) The results are plotted in Figure 3.2. The distances and proportions refer to flight in overlapping 100-mb layers centred on the height shown. Some features are immediately apparent:

(i) over the sea, reports of CAT were restricted to levels near the jet stream;

(ii) at jet-stream levels, the proportion of CAT of each intensity was about the same whatever the terrain;

(iii) over land and near the coast, the proportions of CAT at the jet-stream level and below it were about the same, except that below 550 mb severe turbulence was not found;

(iv) the maximum proportion of severe CAT over the sea was at a higher level than the maximum proportion of slight CAT over the sea;

(v) over all three types of terrain the proportion of flight with slight bumpiness increased with height between 150 and 100 mb whereas, more predictably, the proportion with severe bumpiness decreased above 250 mb.

In order that the effects of sampling errors may be assessed, the data are also plotted on Figure 3.3. This shows which points are based on at least 23 reports and on at least 14 reports of bumpiness of each intensity. It has been shown by Cornford (1967) that when events of low probability occur randomly, samples of 23 (14) or more events give frequencies of occurrence which are more than 50 per cent greater (less) than the mean frequency found from a large number of similar samples in only 5 per cent of trials. In the present case this approach provides only a rough guide to the likely 'true' proportions of flight in different intensitites of bumpiness, because all encounters are not of the same length. Neither can it show the significance of points denoting reports of no bumpiness which become more significant the longer the distance they represent. In Figure 3.3 all points represent at least 12 000 km of flight, while those for severe bumpiness represent at least 30 000 km of flight.

Overall, despite the fact that these were days when CAT was forecast to occur in the map area and so form a biased sample (there were many other days in the same period when little or no CAT was forecast), the frequency of CAT was found to be similar to that reported by Dreyling (1973) and typical of airline experience.

3.3 *Frequency of encounters with bumpiness*

For many purposes it is the time or distance in bumpy flight which matters. Operationally, though, it is more often the fact that bumpiness begins at all which is important. So the pilots' reports and data on indices have been arranged to show the frequency with which bumpiness was met.

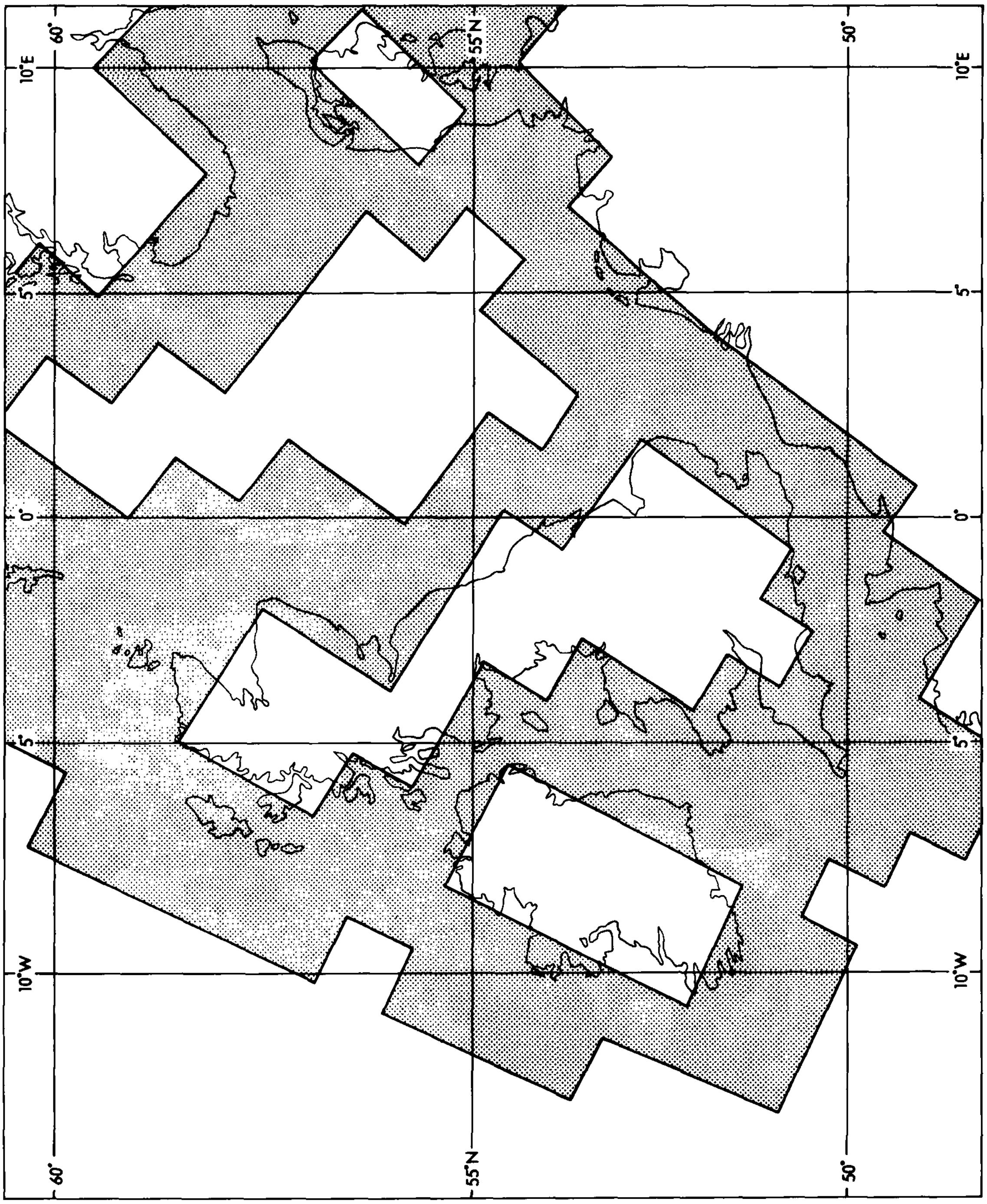


FIGURE 3.1. AREAS DEFINED AS 'LAND', 'SEA' AND 'NEAR COASTS'

The coastal area is shaded.

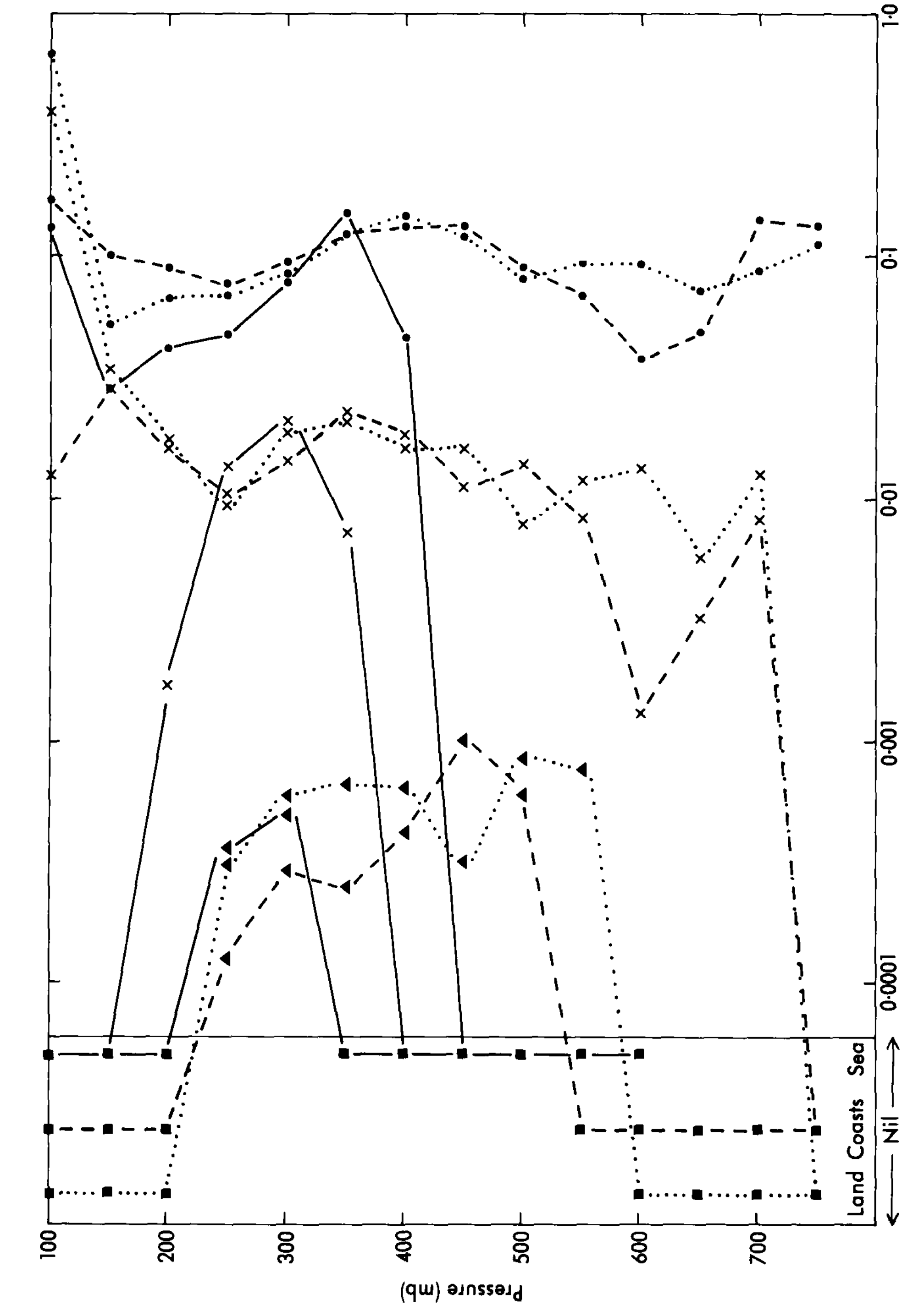


FIGURE 3.2. DISTRIBUTION OF BUMPS ——— SEA – – – NEAR COASTS ..... LAND

● slight, moderate and severe ▲ severe
× moderate and severe ■ nil

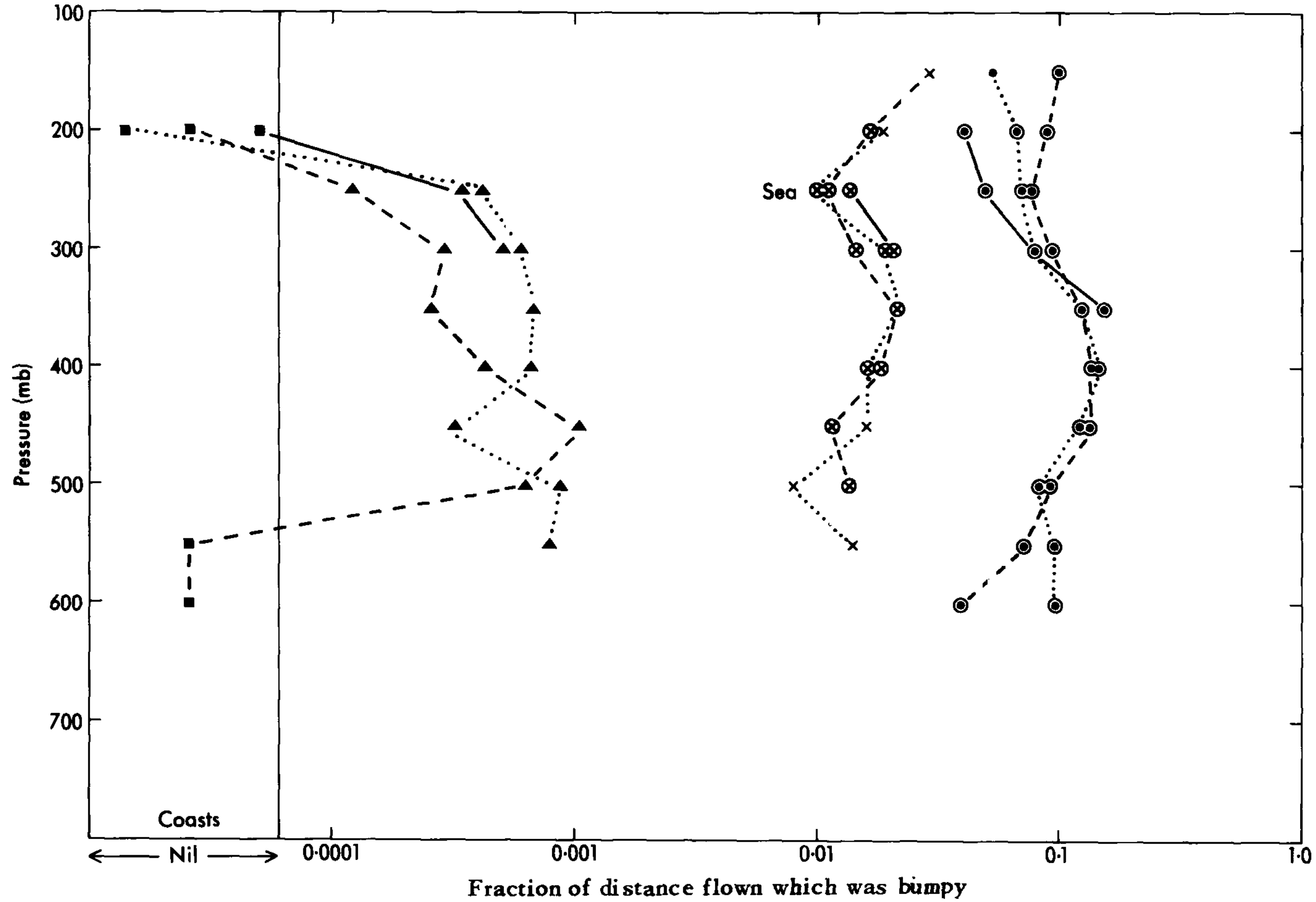


FIGURE 3.3. DISTRIBUTION OF BUMPS SEA NEAR COASTS LAND

● slight, moderate and severe ▲ severe

× moderate and severe ■ nil

All points denoted ● or × are based on at least 30 000 km of flight but less than 14 reports.
All points denoted ▲ or ■ are based on at least 12 000 km of flight and at least 14 observations of bumpiness of the intensity shown. Points in a circle are based on 23 observations (◉ or ⊗).

Figure 3.4 shows how the frequency of encounters varied with height and terrain. The frequencies are normalized to numbers of encounters per 100 km. In general, the Figure may be used to show the probability of meeting one or more patches of bumpiness in 100 km of flight. (This is not true for the higher frequencies. From the Poisson distribution we find that when the mean frequency is a, the probability P of 1 occurrence or more is as follows,

$a = 0.1$, $P = 0.1$; $a = 0.5$, $P = 0.4$; $a = 1$, $P = 0.65$; $a = 2$, $P = 0.87$).

There is a striking similarity between the patterns of Figures 3.2 and 3.4 and one may infer that there was no great difference in the horizontal extent of bumpy patches at different heights or over different types of terrain.

3.4 *Maps of pilots' reports*

Pilots' reports for the periods 1000 to 1319 GMT each turbulence day were plotted on maps. A map was constructed for each 100-mb layer to show the number of pilots who had flown through each grid square and the highest intensity of bumpiness each had reported in that grid square. The reports plotted were restricted to those within the area outlined in Figure 3.5.

Figure 3.5 shows the distribution of reports for the layer 300–400 mb on 10 May 1972. There are two numbers in each grid square, the lower is the total number of aircraft captains who flew through the square and the upper shows the bumpiness they reported. The upper number combines the number of reports of no bumps, A; slight bumps, B; moderate bumps, C; and severe bumps, D, in the form D × 1000 + C × 100 + B × 10 + A.

The map for 10 May shows that slight bumpiness was very widespread and suggests that moderate bumps may have been confined to an area near the middle of the map but the number of reports in each square is so small that this could be due to sampling errors. This is typical of most of the maps. Generally the boundaries between turbulent areas and non-turbulent areas could not be drawn with confidence.

Figure 3.6, however, shows an occasion with an unusually well-defined bumpy area. The reports are from the layer 300–400 mb on 1 May 1972. The map shows a large area of turbulence, much of it moderate, over England and France. There are no reports of bumpiness over Germany and Denmark or to the north-east of Scotland even though there are pilots' reports from these areas. The north-east and south-west boundaries of the turbulent area are unknown because of lack of reports. Within the turbulent area, not every pilot experienced bumps in every grid square. The probability of encountering bumps in any one grid square in the region was about 0.4 and the probability of encountering moderate or severe bumps in any one grid square was about 0.14.

Although individual patches of turbulence are not large or long-lived, they may be assembled in rather persistently large areas. Such areas must be regarded as bumpy even though not every pilot flies in one will experience moderate or severe bumps. This is well illustrated by Figure 3.6.

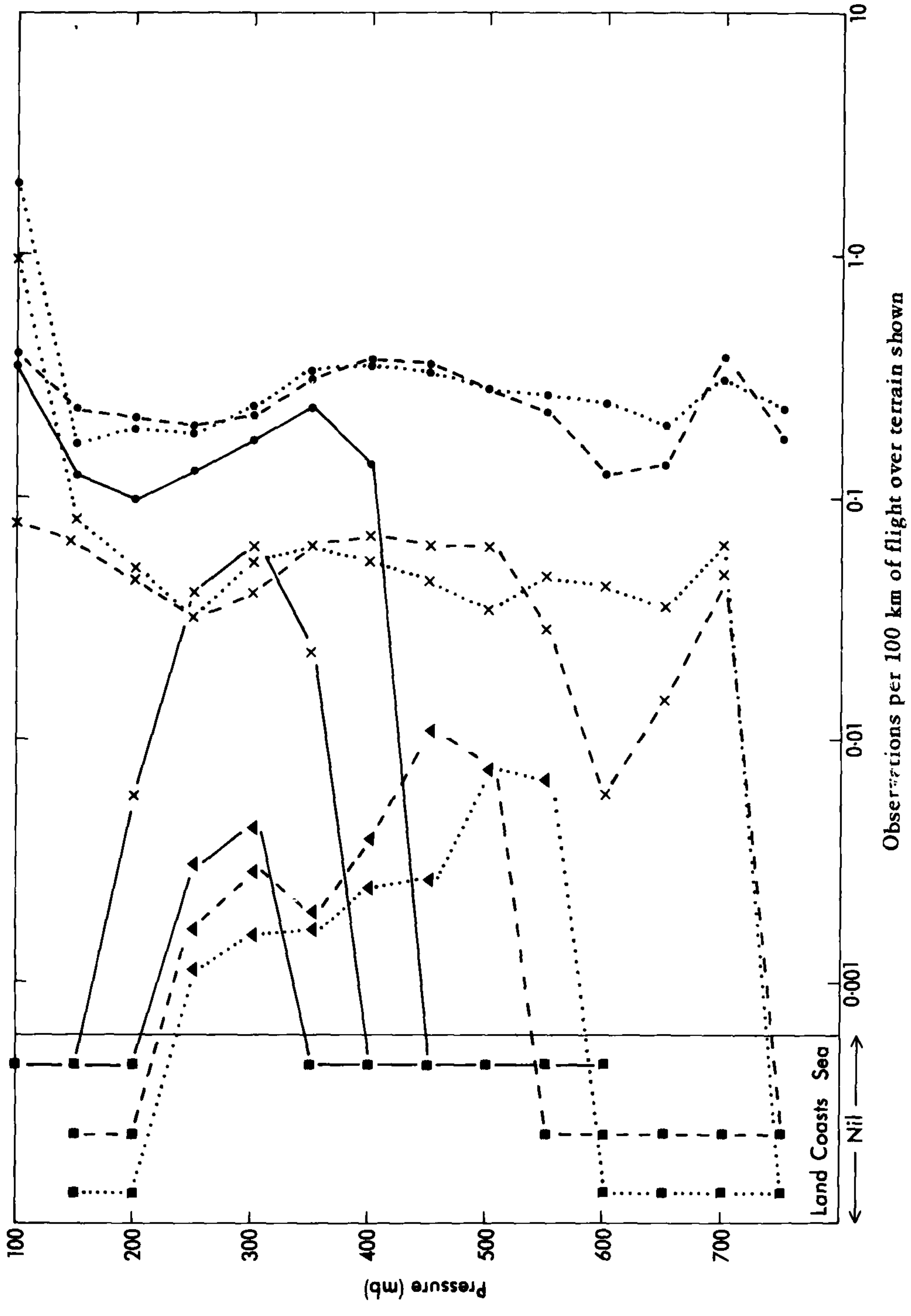


FIGURE 3.4. DISTRIBUTION OF BUMPS SEA NEAR COASTS LAND

● slight, moderate and severe ▲ severe
× moderate and severe ■ nil

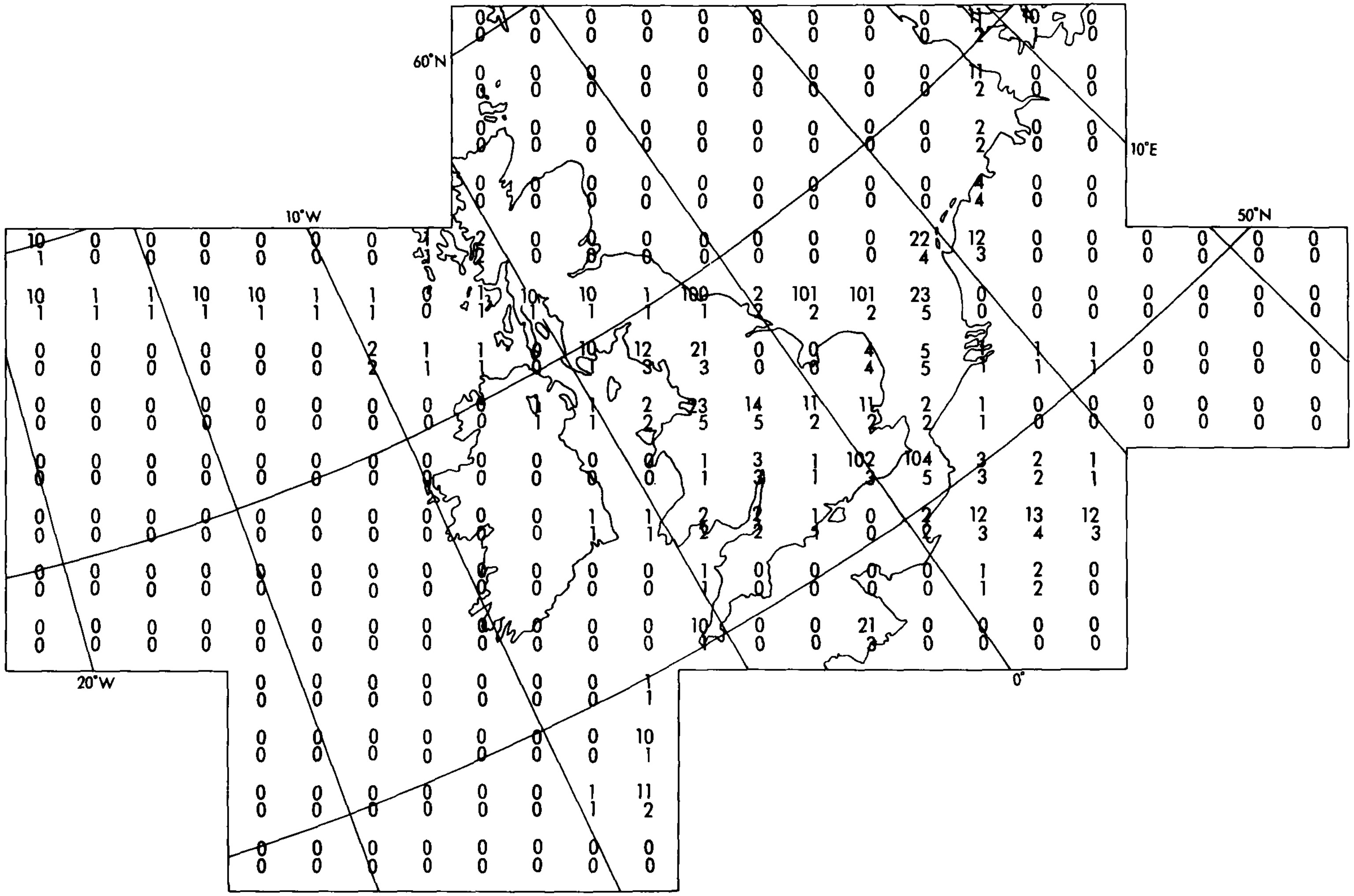


FIGURE 3.5. REPORTS OF CLEAR-AIR TURBULENCE 10 MAY 1972, 1000-1319 GMT, 300-400 mb

The lower number shows the total of aircraft which flew through the square. The upper number shows the bumpiness which was reported in the form 1000D + 100C + 10B + A, where D is severe bumpiness, C moderate, B slight and A nil bumps.

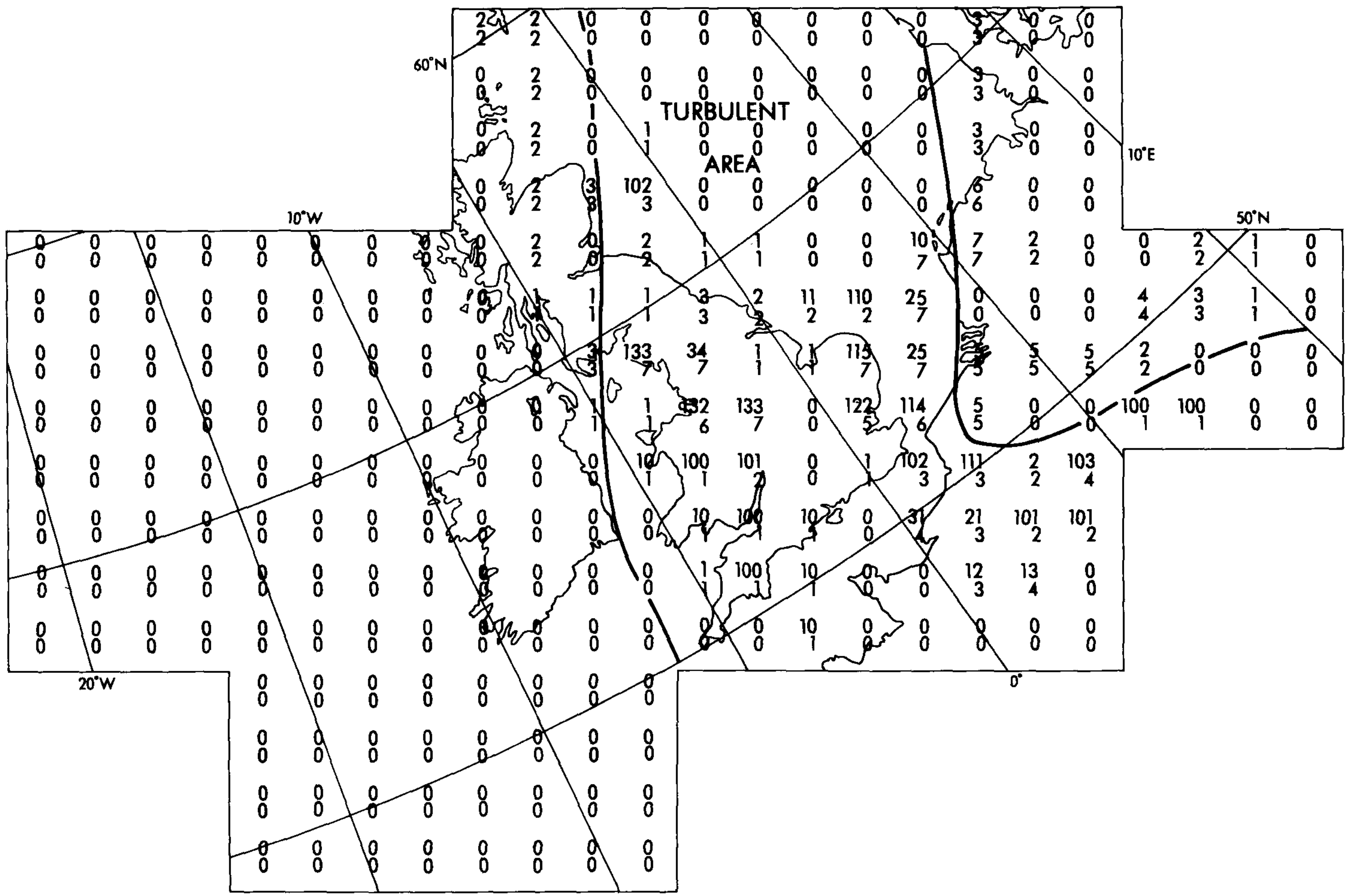


FIGURE 3.6. REPORTS OF CLEAR-AIR TURBULENCE 1 MAY 1972, 1000–1319 GMT, 300–400 mb

The lower number shows the total of aircraft which flew through the square. The upper number shows the bumpiness which was reported in the form 1000D + 100C + 10B + A, where D is severe bumpiness, C moderate, B slight and A nil bumps.

3.5 *The comparison of reports from different pilots*

3.5.1 It was seen in the previous paragraph that even when pilots' reports from layers 100 mb deep and periods 200 minutes long were grouped together it was not generally possible to identify turbulent areas, with much confidence, by plotting the reports on maps. In order to compare reports from thinner layers and shorter periods other techniques had to be used. Each pilot's report was located in a 'unit' region of space and time. A unit region had area one grid square (side ≈ 100 km), depth one nominal flight level (± 150 m) and duration one observing period (≈ 72 minutes). By combining reports from all unit regions which had a particular characteristic we were able to obtain a sufficiently large sample to determine the relationship between that characteristic and bumpiness. When a pilot made more than one report from the same unit region the highest intensity of turbulence he reported was used to characterize his experience in that region. The percentage frequency with which pilots reported bumps in regions with a particular characteristic was used as an estimate of the probability that bumps would be encounted in any region with that characteristic. This does not give a completely satisfactory measure of the probability of bumps in a region, firstly because the probability is unlikely to be uniform throughout the regions and secondly because not all aircraft fly the same distance through a region. Some may fly along a diagonal and others may only pass through a corner of the region. However, it has been shown by Sparks *et alii* (1976) that if the probability of encountering bumps in unit distance (100 km) of flight is assumed constant throughout the regions and aircraft fly through the regions on random straight paths the mean probability of encountering bumps during a flight through a region can be calculated. Conversely, if the mean probability of encountering bumps in a region is known the probability of encountering bumps in 100 km of flight, under the same conditions, can be calculated. This relationship is shown in Figure 3.7.

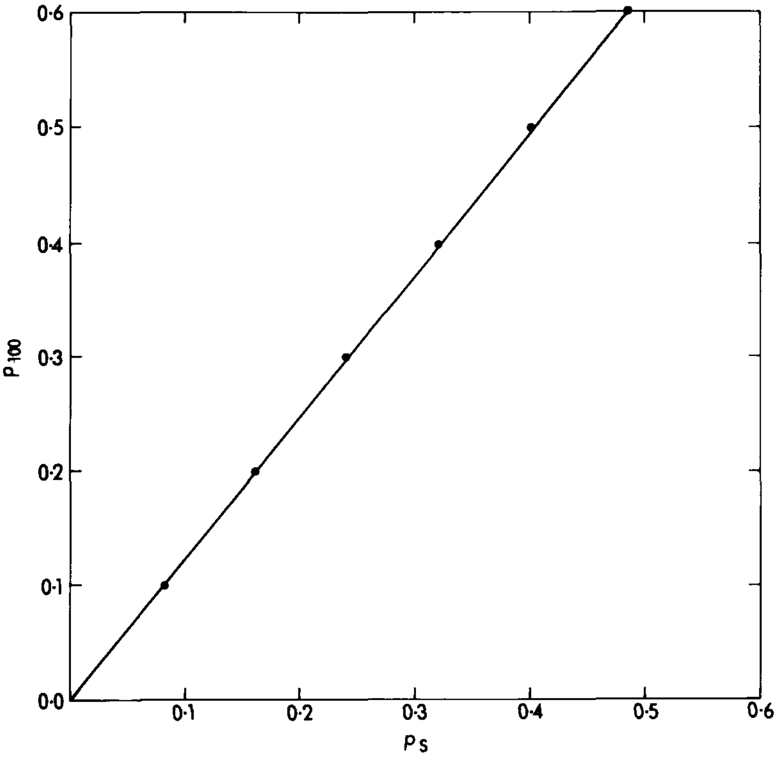


FIGURE 3.7. THE RELATIONSHIP BETWEEN THE PROBABILITY OF ENCOUNTERING BUMPS IN 100 km OF FLIGHT (p_{100}) AND THE PROBABILITY OF ENCOUNTERING BUMPS IN A GRID SQUARE OF SIDE 100 km (p_S)

We consider first pairs of reports from the same unit region. When more than two pilots reported from the same region the pairs were selected at random and each report was used only once. Thus a region which contained three reports yielded only one pair. Two pilots who reported from the same unit region would normally have been flying along the same airway and would have had a minimum separation of 5 minutes at any point (ICAO 1971). If the aircraft are assumed to have had random separations in time within the observing period between the minimum of 5 minutes and the maximum of 72 minutes then their mean separation in time would have been about 27 minutes. The time interval between aircraft was the most important separation between reports from the same unit region because most aircraft would have flown very nearly the same route along an airway and would have been at the same nominal flight level.

Table 3.1 shows the distribution of pairs of pilots' reports from the same region. The significance of the distribution of reports in the table cannot be tested as it stands using the χ^2 statistic, but if the reports of moderate and severe bumps are grouped with the reports of slight bumps χ^2 has the value 13.4 which has a probability of less than 0.001 of occurring by chance if there were no association between the reports.

TABLE 3.1 THE RELATIONSHIP BETWEEN PAIRS OF TURBULENCE REPORTS FROM THE SAME UNIT REGION

(size 1 grid square × 1 flight level × 1 period)

		PILOT 'A'			
		NIL	SLIGHT	MOD + SEV	TOTAL
PILOT 'B'	NIL	448	68	14	530
	SLIGHT	82	27	4	113
	MOD + SEV	13	5	4	22
	TOTAL	543	100	22	665

This Table is a combination of data from all unit regions from which there were reports by at least two different pilots.

The significance of the four pairs of reports of moderate or severe bumps may be tested using the Poisson distribution. If the reports were independent the average number expected in that box with 665 pairs would be about 0.73. The Poisson distribution gives the probability of 4 or more pairs as 0.0007 when the average number is 0.73.

Table 3.1 shows that turbulence reports from different pilots in the same unit region are associated closely enough for a report from one pilot to be a useful guide to the turbulence to be expected by another pilot. The way in which that association changes when the pilots have a larger separation in space or time was investigated by pairing reports from pilots in different unit regions with various separations between the regions.

In order to present the results of this investigation in a convenient form a simple measure of the usefulness of the association between one pilot's report and another pilot's experience was defined.

In the absence of any information (e.g. a pilot's report, the value of a meteorological index etc.) which is specific to a small region of the atmosphere the only estimate that can be made of the percentage probability that a pilot would encounter bumps in that region is the percentage frequency with which bumps were reported in regions of the same size in the whole data set. This percentage frequency we call the background frequency.

For regions of one grid square (100 km × 100 km) the background frequency for moderate or severe bumps was in the range 3.2 to 4.0 per cent, depending on how the data were selected, and for all bumps was in the range 14 to 19 per cent.

A pilot's report (or a meteorological index) is useful if it can provide an estimate of the conditional probability of encountering bumps which is significantly different from the background frequency. The ratio of this conditional probability to the background frequency has been used as a measure of the usefulness of the report (or index). The ratio is denoted by R_B when it is calculated for all bumps and R_M when it is for moderate or severe bumps.

For some parts of the analysis subsets of the data have been used and, where appropriate, a background frequency has been calculated for each subset. For example, reports paired in the same unit region form a subset. From Table 3.1 it can be seen that the background frequency of moderate or severe bumps in that subset was 3.31 per cent and the frequency with which moderate or severe bumps were reported when another pilot had reported moderate or severe bumps was 18.2 per cent. This gives a value of 5.5 for R_M.

3.5.2 Figure 3.8 shows how R_M varied with the intensity of bumps reported by another pilot in the same volume of the atmosphere and with the time interval between the reports. The volumes were one grid square by one flight level and the time intervals were means calculated on the assumption that reports were at random times within each observing period. The Figure shows that a recent report of moderate or severe bumps has considerable value because it indicates that another pilot flying in the same volume has a probability of encountering moderate or severe bumps about five times the background level. This probability falls as the time interval increases and the figure suggests that a report of moderate bumps is no more valuable than a report of slight bumps after about five hours.

R_M for reports of slight bumps shows no consistent variation with time. A single report of slight bumps, however, indicates that another pilot flying in the same volume would have a probability of encountering moderate or severe bumps about 1.5 times the background level for at least four hours after the report.

The Figure also shows that a single report of no bumps implies that other pilots in the same volume may be expected to report moderate or severe bumps with a frequency below the background level. This frequency shows a slight increase with the time interval between the reports.

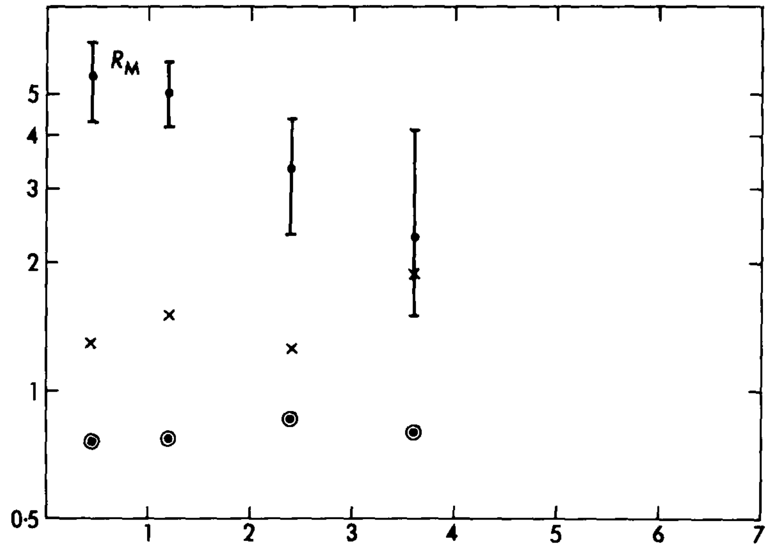


FIGURE 3.8. THE RELATIONSHIP BETWEEN REPORTS OF BUMPS AND THEIR SEPARATION IN TIME

$$R_M = \frac{\text{percentage of reports of moderate or severe bumps}^*}{\text{background percentage}^\dagger \text{ of moderate or severe bumps } (= 3.2)}$$

* both pilots of a pair being in the same grid square and at the same flight level. The separation in time was as plotted.

† see text para. 3.5.1 for definition.

• when the other pilot of the pair reported moderate or severe bumps

x when the other pilot of the pair reported slight bumps

when the other pilot of the pair reported no bumps

The vertical bars through the points show how R_M would have changed if either one more pair, or one less pair (of pilots) had both reported moderate or severe bumps.

While R_M is the ratio of the percentage of reports of moderate and severe bumps, under a stated condition, to the background level of moderate and severe bumps, R_B is the ratio of the percentage of reports of ALL bumps (slight, moderate and severe), under the stated condition, to the background level of ALL bumps. The variation of R_B with the intensity of bumps reported and the time between the reports is shown in Figure 3.9. The frequency with which bumps were encountered varied significantly with the intensity of bumps reported by other pilots but the relationships showed no dependence on the time interval between reports over the range investigated.

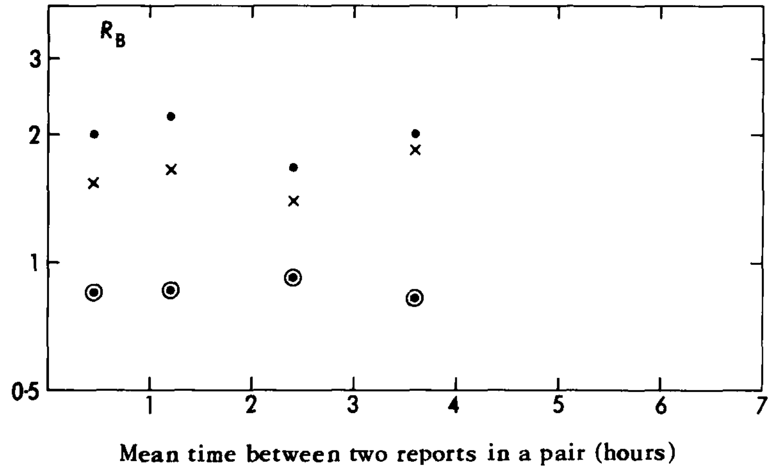


FIGURE 3.9. THE RELATIONSHIP BETWEEN REPORTS OF BUMPS AND THEIR SEPARATION IN TIME

$$R_B = \frac{\text{percentage of reports of moderate or severe bumps}^{*}}{\text{background percentage}^{\dagger}\ \text{of moderate or severe bumps } (= 17.1)}$$

* both pilots of a pair being in the same grid square and at the same flight level. The separation in time was as plotted.

† see text para. 3.5.1 for definition.

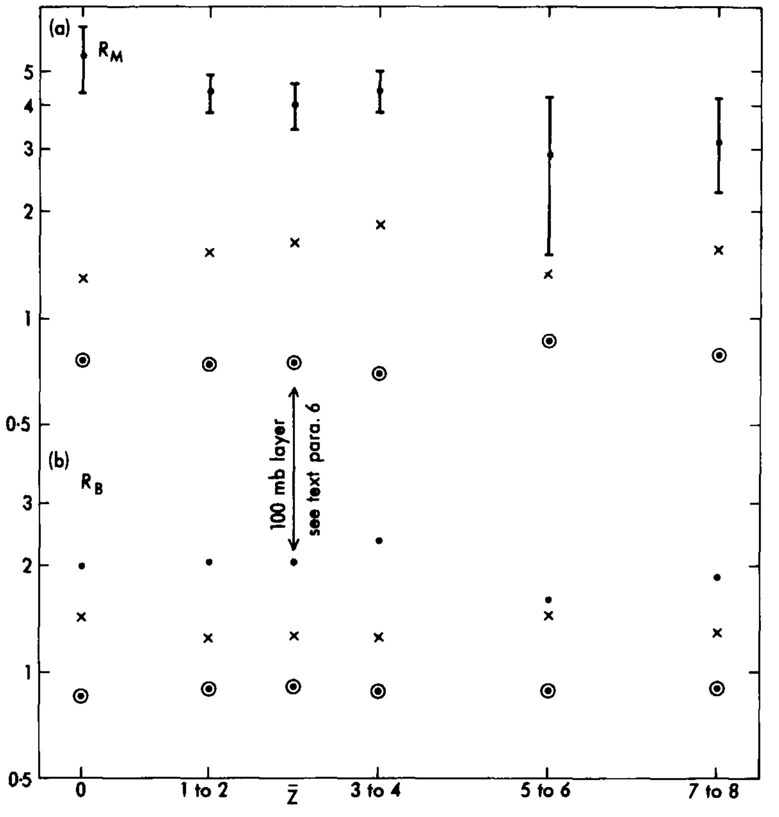
• when the other pilot of the pair reported moderate or severe bumps

x when the other pilot of the pair reported slight bumps

when the other pilot of the pair reported no bumps

The vertical bars through the points show how R_B would have changed if either one more pair, or one less pair (of pilots) had both reported moderate or severe bumps.

3.5.3 Figure 3.10 shows how the value of a report varied with the intensity of bumps reported and the vertical separation between the aircraft. In a first analysis the vertical separation was increased in steps of 1000 ft (300 m) but the sampling errors were so large that the data were regrouped into 2000-ft (600-m) layers. Both R_M and R_B showed consistent and significant dependence on the intensity of bumps reported by other pilots. R_M decreased from about five to three as the vertical separation increased from zero to 8000 ft (2400 m) but none of the other relationships showed any significant dependence on the vertical separation between the pilots.



Vertical separation between the two reports in a pair (ft x 10^3)

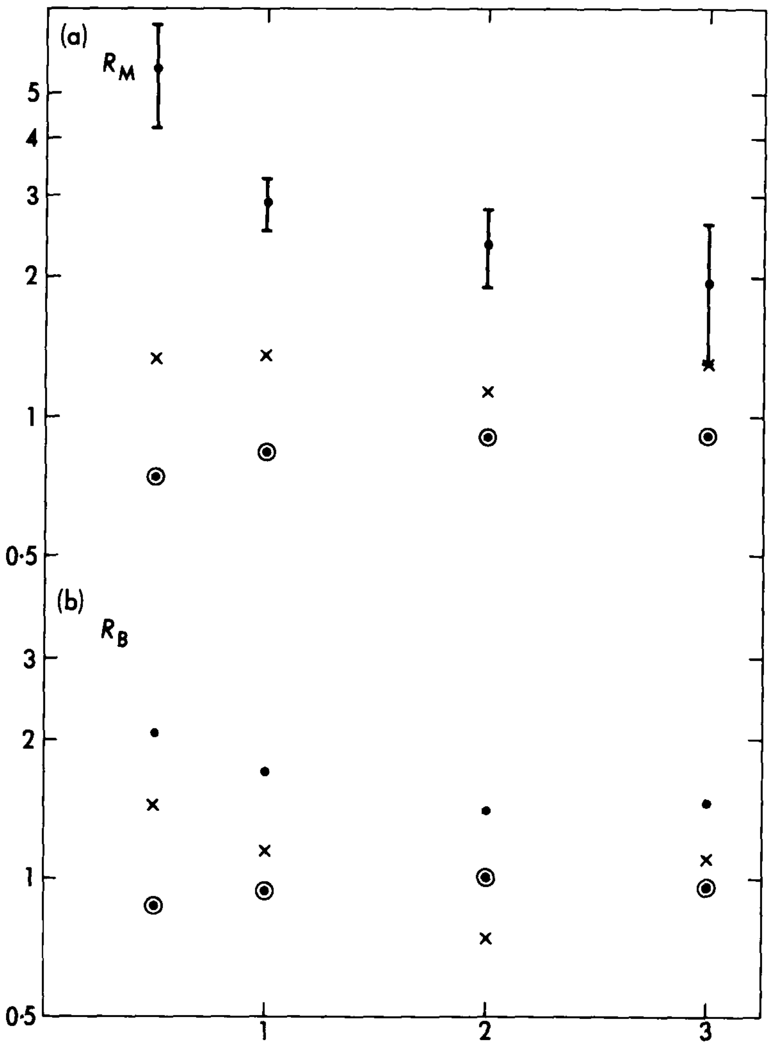
FIGURE 3.10. THE RELATIONSHIP BETWEEN REPORTS OF BUMPS AND THEIR VERTICAL SEPARATION

See Figures 3.7 and 3.8 for definition of symbols.

Mean background level of moedrate or severe bumps was 3.3 per cent.
Mean background level of all bumps was 19.0 per cent.

Both pilots of a pair were in the same grid square and the same period.
Z is the mean vertical separation between aircraft in the same 100-mb layer.

3.5.4 The relationship between pilots' reports of bumps and their horizontal separation is shown in Figure 3.11. Neither R_M nor R_B differ significantly from 1, for any intensity of turbulence reported by the 'other' pilot, when the separation between the pilots is 200 or 300 km. The data suggest that a single report of bumps has no value to another pilot more than about 150 km away from the location of that report. However, that figure may be rather misleading. Turbulence is often located in strong baroclinic zones and, since the intersection between a sloping baroclinic zone and a fixed flight level usually has a much greater length than width, we should expect turbulent zones to be elongated. Unless the major axis of such a zone lay along either the X or Y axes of the grid of the 10-level model the method of analysis used would have underestimated its horizontal extent.



Average horizontal separation between the two reports in a pair (ft x 10^3)

FIGURE 3.11. THE RELATIONSHIP BETWEEN REPORTS OF BUMPS AND THEIR HORIZONTAL SEPARATION

See Figures 3.7 and 3.8 for definition of symbols.

Mean background level of moderate or severe bumps was 4.0 per cent.

Mean background level of all bumps was 18.1 per cent.

4 – THE SYNOPTIC-SCALE INDICES

Roach's (1970) indices and other, more common, meteorological parameters derivable from the 10-level model have been tested as locators and predictors of turbulence.

Roach has argued as follows. Clear-air turbulence is often associated with instabilities in the flow which occur when a stable layer allows strong wind shears to develop across it. Miles and Howard (1964) have shown that a condition for this Kelvin–Helmoltz Instability (KHI) is that the Richardson number Ri should not exceed a quarter. However, the synoptic network of radio-soundings resolves only layers which are thicker than those in which KHI occurs, so measured values of Ri when CAT occurs may exceed ¼ by some indeterminate amount. Consequently, Roach thought that better indices might be either the rate at which Ri is decreasing (which has the additional advantage of introducing a predictive element) on the rate at which large-scale turbulent energy is being converted into turbulence on the scales which bump aircraft. A similar index to the first, using the rate of change of Ri, has been mentioned by Penn (1970) who planned to compare it with reports from aircraft but no results are known to the authors. An attempt to relate the dissipation of turbulent energy (ϵ) to aircraft response has also been made by MacCready (1964). In preference to Ri, Roach found ln Ri more convenient to handle and defined two indices,

(i) $$\Phi = -\frac{D}{DT}\ln Ri$$

where $$\ln Ri = \ln\left\{-\frac{1}{\rho\theta}\frac{\partial\theta}{\partial p}\left[\left(\frac{\partial u}{\partial u}\right)^2+\left(\frac{\partial v}{\partial p}\right)^2\right]\right\}$$

and $\frac{D}{DT}$ denotes differentiation following the air motion, ρ is density,

θ is potential temperature, u and v are components of the horizontal wind and p is pressure.

(ii) $$\epsilon = \frac{(\Delta V)^2}{24}$$

where ΔV is the magnitude of the vector shear between horizontal winds at the base and top of the layer of depth Δz across which Φ has been evaluated. Roach found that if Φ is expressed in analytical form, by expanding the right-hand side of (i) in terms of the equations of motion and thermodynamics, it is impracticable to derive representative values of some of the significant terms from conventional synoptic data. Following Roach, Brown (1973) has investigated an approximate form of the index:

$\Phi \approx (0.3\,\zeta_a^2 + D_s^2 + D_T^2)^{1/2} \equiv \Phi_B$, say. Whereas Φ is difficult to calculate, Φ_B can be found readily.

Here $\zeta_a = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f\right)$, the vertical component of the absolute vorticity of the flow at a level representative of the layer for which Φ is required.

$D_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$, the shearing deformation of the horizontal flow.

$D_T = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$, the stretching deformation in the horizontal.

Brown has also suggested the use of ϵ'_{500} where $\epsilon'_{500} = 500\ \epsilon/\Delta z$, where Δz is in geopotential metres. This makes the index less dependent on Δz, the thickness of the layer for which ΔV and Φ are evaluated.

For convenience it was $\Phi_B = 10^5\ \Phi'_B$ and ln ϵ_{500}, where $\epsilon_{500} = 10^7\ \epsilon'_{500}$, which were used in the present work.

It should be pointed out that Φ'_B contains only quantities relating to the horizontal flow field, whereas Φ contains vertical quantities. The best justification for the assumptions made in deriving Φ'_B from Φ would be that Φ_B was closely related to the occurrence of CAT. To test this is one of the purposes of this paper.

The other meteorological parameters that have been tested as locators and predictors of turbulence are wind speed, Richardson number, vertical wind shear, horizontal wind shear, vertical velocity, the horizontal gradient of vertical velocity, deformation and vorticity. The symbols and units used for the indices are given in Table 4.1.

TABLE 4.1. THE SYNOPTIC-SCALE INDICES TESTED AS LOCATORS OF TURBULENCE

Index	Symbol	Practical units used
Richardson number	Ri used as $\ln Ri$	dimensionless
Wind speed	V	$m\ s^{-1}$
Modified wind speed	$\bar{V}$	$m\ s^{-1}$
Vertical wind shear	S_V	$m\ s^{-1}\ km^{-1}$
Modified vertical wind shear	$\bar{S}_V$	$m\ s^{-1}\ km^{-1}$
Horizontal wind shear	S_H	$m\ s^{-1}\ 100\ km^{-1}$
Modified horizontal wind shear	S_H	$m\ s^{-1}\ 100\ km^{-1}$
Vertical velocity	$w = \frac{dp}{dt} \times \frac{dh}{dp}$	geopotential metres per hour
Horizontal gradient of vertical velocity	∇w	$h^{-1} \times 10^{-4}$
Deformation	D	$s^{-1} \times 10^{-5}$
Vorticity	ζ_a	$s^{-1} \times 10^{-5}$
Roach/Brown Φ	Φ_B	$s^{-1} \times 10^{-5}$
Roach/Brown ϵ	ϵ_{500}	$m^2\ s^{-3} \times 10^{-7}$
Empirical index	E	dimensionless

It must be recognized that all the indices are dependent on the horizontal and vertical scales represented by the various quantities from which they are calculated. Each index was calculated at grid points of the fine mesh 10-level model and is representative of an area about 100 km square and a layer about 100 mb thick.

The interpolation schemes by which the indices were calculated are given in Appendix 1.

5 - COMPARISON OF BUMPINESS WITH VALUES OF THE INDICES

5.1 A useful index is one which discriminates as precisely as the atmosphere and the pilots' reports allow between volumes of air which cause bumps and volumes in which flight is smooth.

Ideally, a high value of an index should encompass regions in which all pilots would experience bumps and outside which there should be no bumps. However, that was found to be impossible, even for the smallest regions we were able to consider, because two pilots flying in the same region often have different experiences of turbulence.

The closest that an index can be expected to approach the ideal depends on the volume and period it represents. The constraints of the 10- level model dictate that the smallest volumes for which an index can be calculated are 1 grid-length square and 100 mb thick. The period of validity of the index can be chosen arbitrarily but it was found that if it was made too short there were insufficient reports within the period to obtain significant results. For most of this analysis the period of validity was chosen as 200 minutes centred at the time of the midday radio soundings. The observations made around midnight were not used because the evaluation of the indices and their comparison with reports of bumps requires a considerable amount of computer time and the number of reports near midnight did not warrant it.

Since it was not possible to calculate exactly the performance to be expected from an ideal index in predicting bumps the performances of the indices have been compared with that of a pilot's report.

The predictive value (R_M or R_B) of any one report from a volume with dimensions 1 grid square by 100 mb was obtained by grouping the reports from such values and selecting pairs, as in Section 4. The results of that analysis are plotted in Figure 3.10 against the mean vertical separation $\overline{Z}$* between reports paired at random from within 100-mb layers. The directly calculated values of R_M and R_B are shown against $\overline{Z}$ in Figure 3.10 and are in good agreement with the values that would be estimated by interpolation from the other values plotted on the figure.

The predictive value of a pilot's report in a volume 1 grid square by 100 mb is presented in a slightly different way in Figure 5.1. The abcissa shows the percentage of pilots who reported bumps less than or equal to the stated intensity. A pilot's report is a good index for locating

* At around 300 mb a layer 100 mb deep is about 7500 ft (2.3 km) thick. The mean vertical separation between aircraft randomly arranged in the layer, paired randomly, is $\overline{Z} \approx 2500$ ft (0.8 km).

bumpy air since both R_M and R_B are large for only a small proportion of the reports. However, a single pilot's report is not of great value for locating smooth air because the minimum values of R_M and R_B are only slightly below unity.

The performances of the meteorological parameters in predicting turbulence are shown in the same form as Figure 5.1 so that they can be readily compared with the value of a pilot's report.

The meteorological indices were calculated from both initialized fields and 12-hour forecast fields. Every index tested performed at least as well when calculated from the forecasts as when calculated from initialized fields; therefore, only the results from the forecasts are presented.

The association between Φ_B and bumpiness is shown in Figure 5.2. The relationship is clearly of little value but it is worth while looking at this Figure quite closely because there are nine similar Figures some of which do show significant relationships. Concentrating on R_B it can be seen that just under 5 per cent of pilots' reports were from regions in which Φ_B was forecast to be $\leqslant 5 \times 10^{-5}$ s $^{-1}$ and that of these reports about 17.4 per cent (i.e. $R_B \times$ Background level $= 1.16 \times 15$ per cent) were of bumps at least slight. Similarly about 10 per cent of pilots' reports were from regions in which $\Phi_B > 14 \times 10^{-5}$ s $^{-1}$ and 14.3 per cent of these reports were of bumps at least slight.

The associations with bumps of the two components of Φ_B, deformation (D) and absolute vorticity (ζ_a) are shown in figures 5.3 and 5.4. There is no evidence of a useful relationship between D and bumpiness but the relationship between ζ_a and bumpiness is more interesting. R_B increases as ζ_a approaches zero and exceeds 2.7 in the 1 per cent of reports from regions in which ζ_a was forecast to be negative. There is also an indication that bumpiness is above the background level when $\zeta_a > 24 \times 10^{-5}$ s^{-1}.

Closely allied to vorticity is horizontal wind sheer S_H. Figure 5.5 shows its relationship with bumpiness. Negative values of S_H indicate anticyclonic shear. It can be seen that high values of shear, both negative and positive are associated with bumpiness.

Figures 5.6 to 5.8 show the relationships between bumpiness and the indices S_V(vertical wind shear), ln ϵ_{500} and ln Ri. Since ϵ_{500} and Ri are both highly dependent on S_V it is not surprising that the three relationships are similar. The association between bumps and ln Ri is perhaps slightly better than with the other two indices. In the relationship between R_B and ln Ri there is no suggestion of a threshold below which there is turbulence. The Figure shows a continuous relationship with bumps becoming steadily less probable as ln Ri increases. The relationship between R_M and ln Ri shows much more random variation between classes, reflecting the smaller number of reports of moderate or severe bumps. The very low incidence of moderate bumps in regions with ln $Ri > 5$ is encouraging.

The association between wind speed and bumps is shown in Figure 5.9 to be fairly consistent and in the expected sense. Indeed this simple index seems almost as good as any other as a predictor of slight bumps.

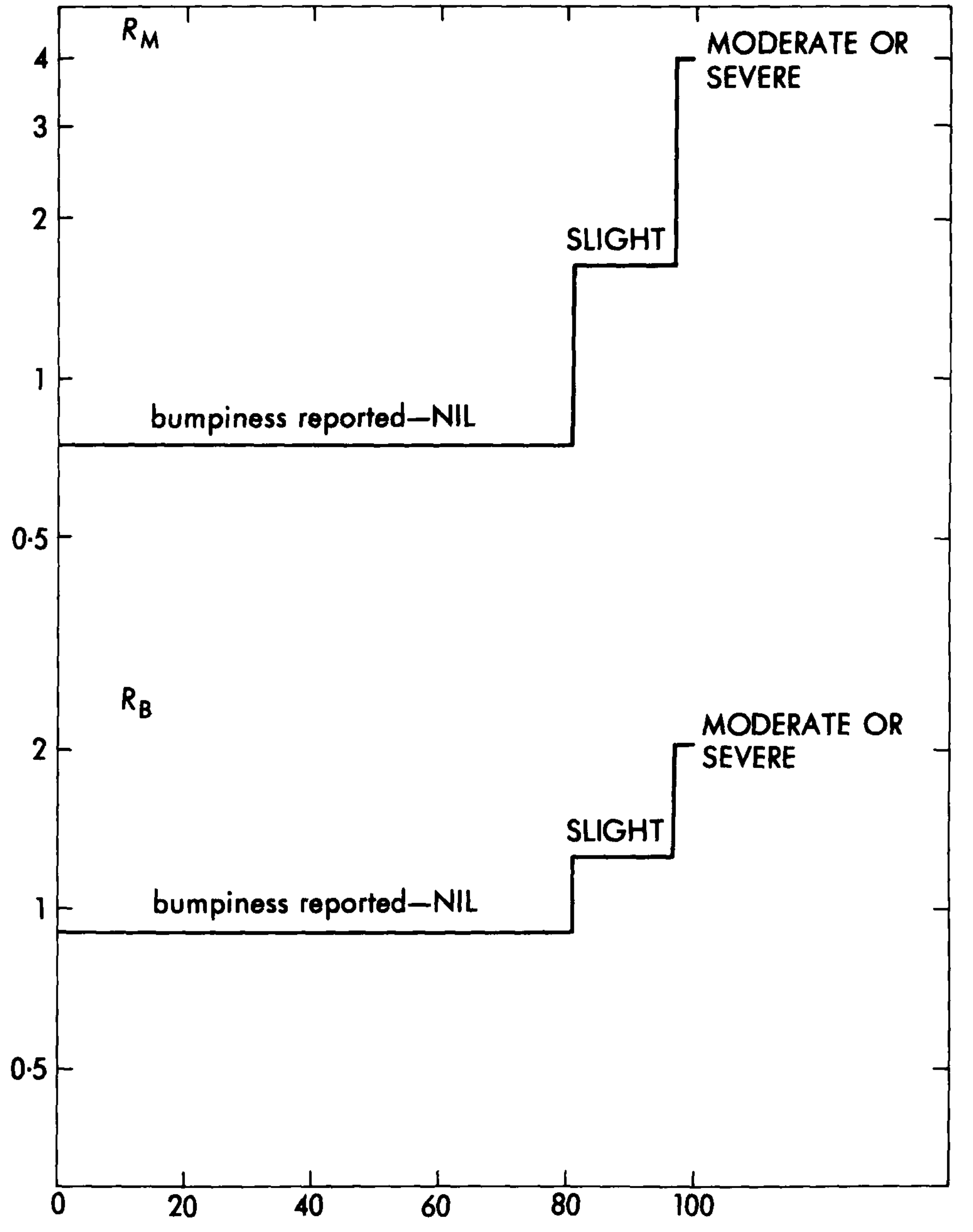


FIGURE 5.1. THE VALUE OF A SINGLE PILOT'S REPORT AS AN INDEX OF BUMPINESS IN A REGION 1 GRID SQUARE BY 100 mb BY 72 MINUTES

R_M is defined in Figure 3.8.
Background level of moderate to severe bumps was 3.2%.

R_B is defined in Figure 3.9.
Background level of bumps was 18.8%.

Cumulative percentage of observations with bumpiness reported less than or equal to the value shown

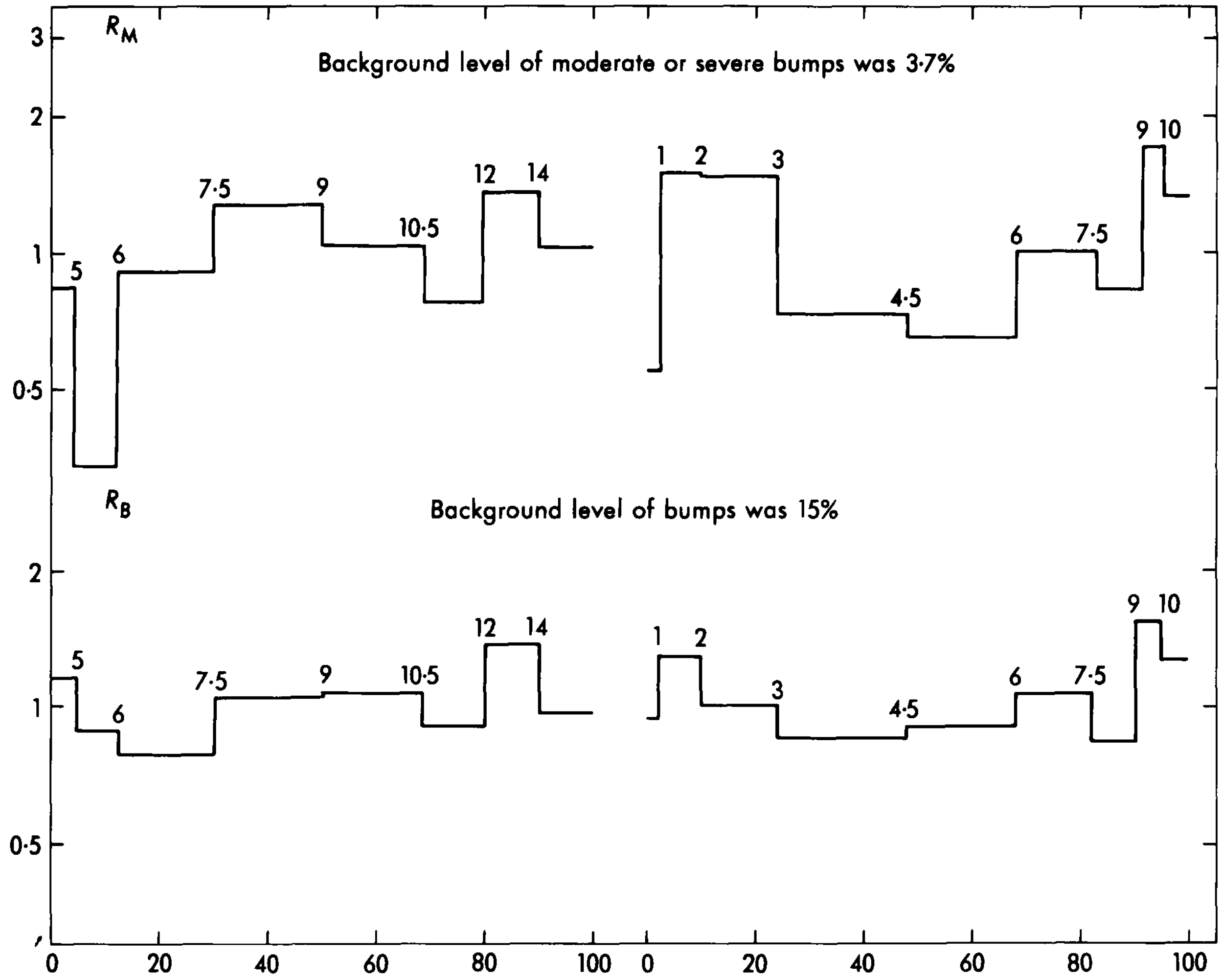


FIGURE 5.2. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF Φ_B AND BUMPINESS

FIGURE 5.3. THE RELATONSHIP BETWEEN 12-HOUR FORECASTS OF *D* AND BUMPINESS

Cumulative percentage of observations with index less than or equal to value shown
Values of indices are in $s^{-1} \times 10^{-5}$.

Finally, two indices involving the predicted fields of vertical velocity have been tested. It was expected that there would be a positive correlation between the magnitude of the vertical velocity $|w|$ and bumpiness since both are associated with geostrophic motions. Figure 5.10 shows that expectation to be correct. Figure 5.11 shows that the magnitude of the horizontal gradient of vertical velocity $|\nabla w|$ is related to bumpiness. This was expected because it has been shown by Miller that $|\nabla w|$ is important in frontogenesis.

Figures 5.1 to 5.11 show that no single meteorological index is as good for a positive prediction of moderate bumps as a recent pilot's report of moderate bumps. Some of the indices are, however, better than a single pilot's report for predicting that a particular region will not produce moderate bumps. Also the indices are comparatively good at predicting slight bumps.

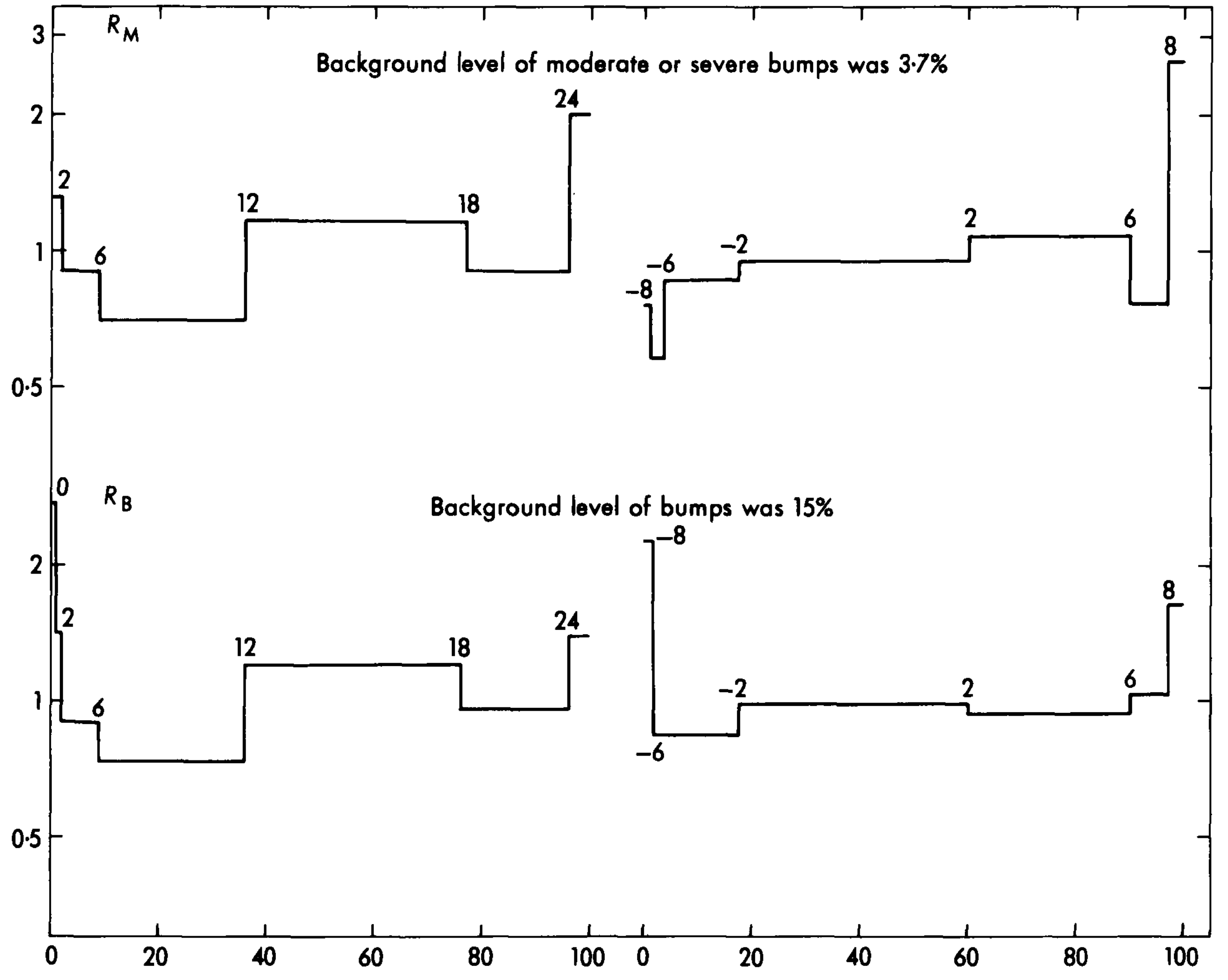


FIGURE 5.4. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF ζ_a AND BUMPINESS

FIGURE 5.5. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF S_H AND BUMPINESS

Cumulative percentage of observations with index less than or equal to value shown
Values of indices are in $s^{-1} \times 10^{-5}$.

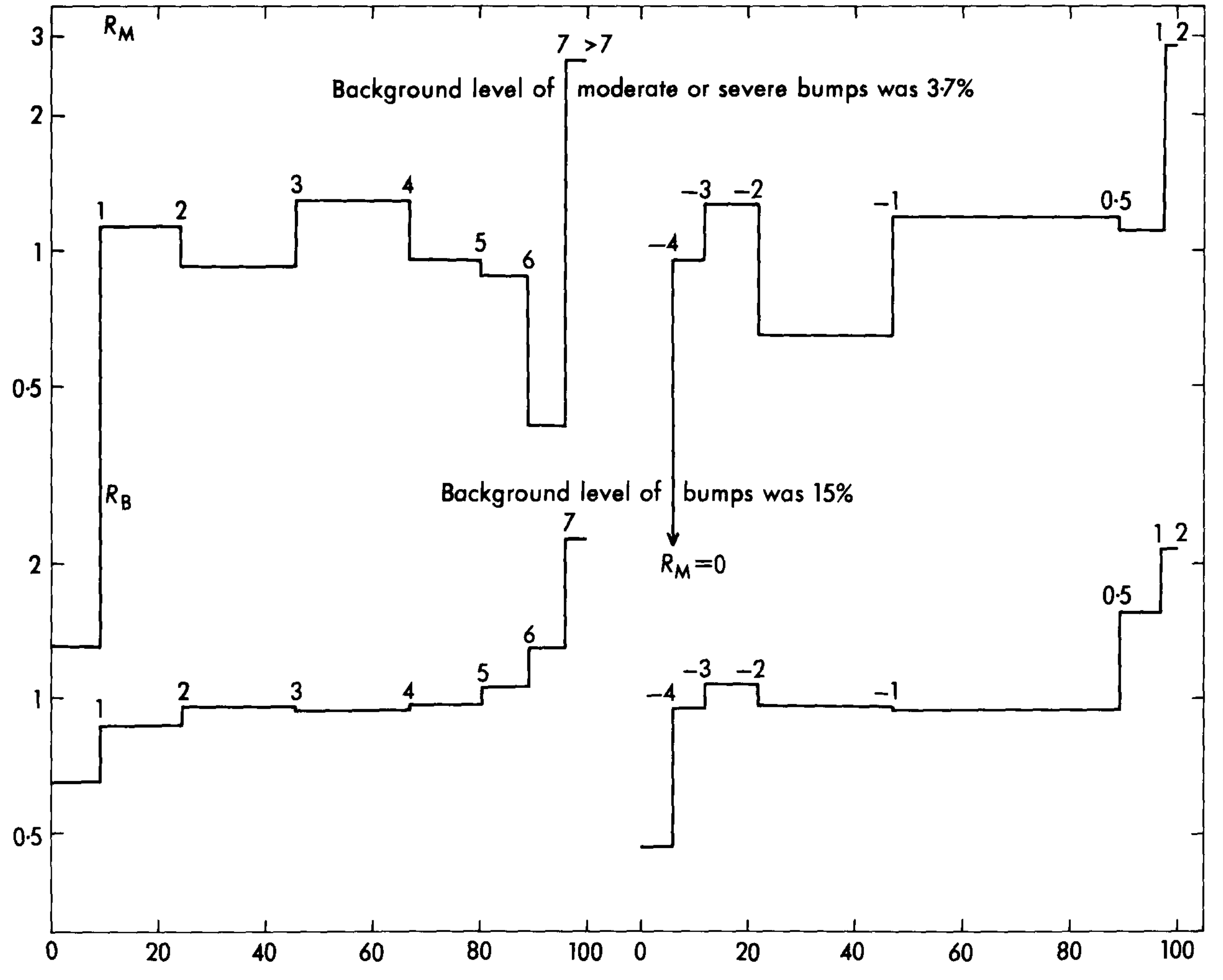


FIGURE 5.6. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF S_V AND BUMPINESS

FIGURE 5.7. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF ϵ_{500} AND BUMPINESS

Cumulative percentage of observations with index less than or equal to value shown

Values of S are in $s^{-1} \times 10^{-3}$.

Values of ϵ_{500} are in $W_g^{-1} \times 10^{-7}$.

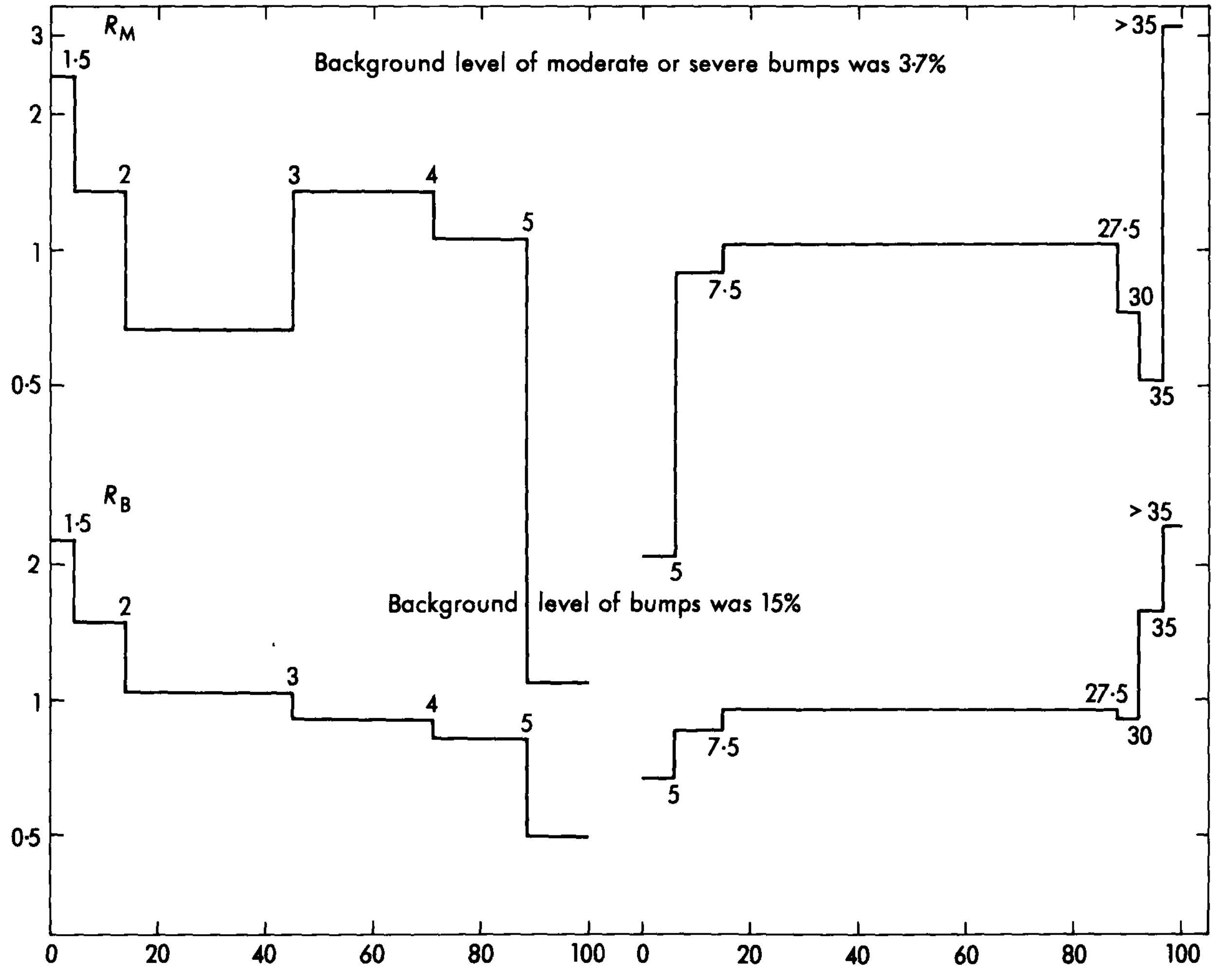


FIGURE 5.8. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF ln *Ri* AND BUMPINESS

FIGURE 5.9. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF *V* AND BUMPINESS
Values of *V* are in m s^{-1}.

Cumulative percentage of observations with index less than or equal to value shown

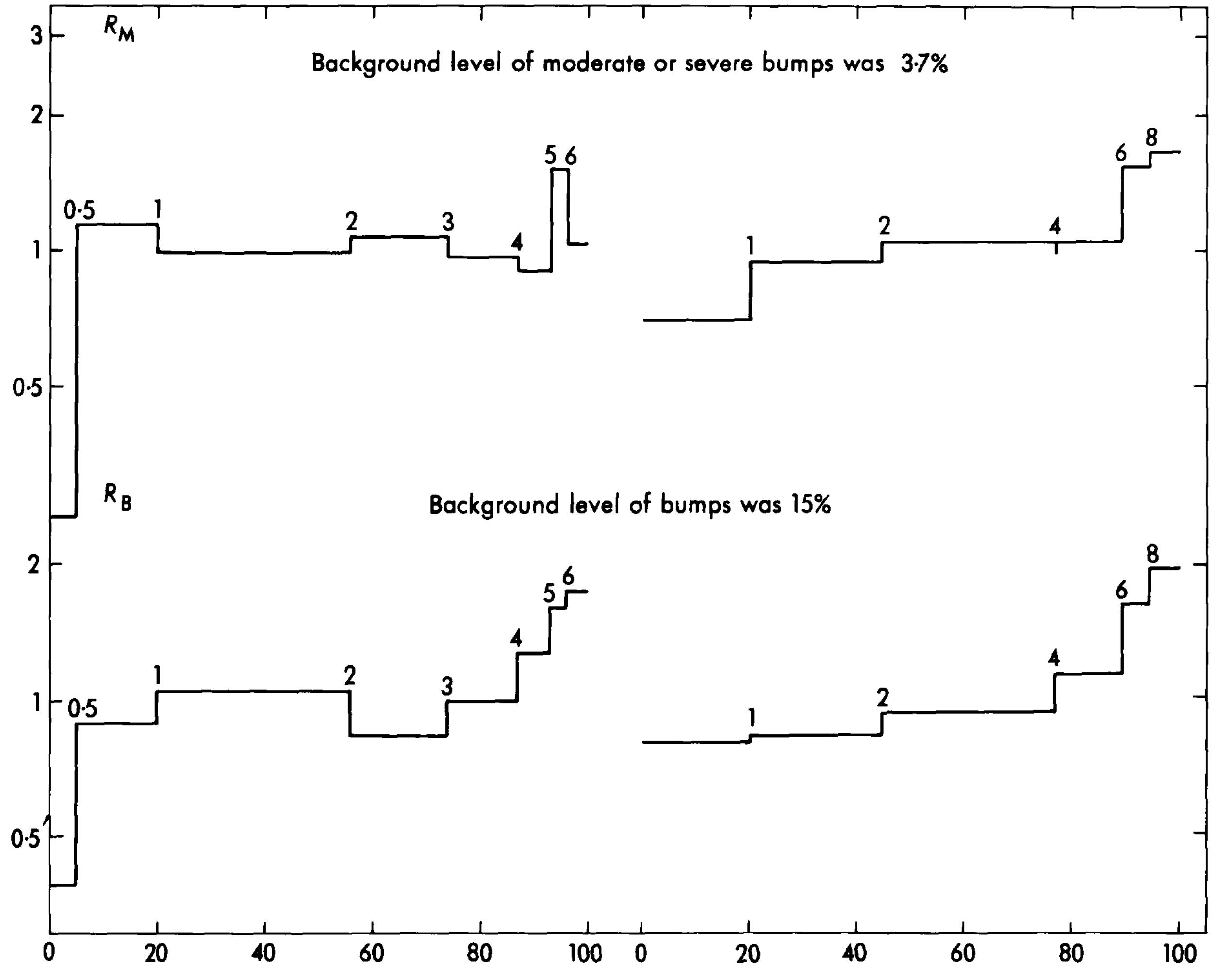


FIGURE 5.10. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF $|w|$ AND BUMPINESS

The values of $|w|$ in mb h^{-1} were multiplied by the thickness of a 1-mb layer to give $|w|$ in dam h^{-1} (1 dam = 10 m).

FIGURE 5.11. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF $|\nabla w|$ AND BUMPINESS

The values of $|\nabla w|$ are in $h^{-1} \times 10^{-4}$.

Cumulative percentage of observations with index less than or equal to value shown

Eight of the ten indices tested showed potentially useful associations with bumpiness but it is clear that they are not eight independent predictors. The problem of finding the best combination of indices is considered in the next paragraph.

5.2 *Combination of the indices*

Two methods were used to combine the indices; multiple linear regression and discriminant analysis. The two methods gave almost identical results but since they provide different insights the results of both procedures are reproduced.

Both methods find the 'best' linear relationship between the indices and bumpiness on the assumption that each index is itself linearly related to bumpiness. Many of the indices have relationships with bumpiness that are far from linear (e.g. wind speed, Figure 5.9).

The linearity of the relationship between wind speed and bumpiness was improved by calculating a modified wind index ($\dot{V}$) using the formula

$$\dot{V} = \left(\frac{V - \overline{V}}{\sigma_V}\right)^3$$

where V = wind speed

$\overline{V}$ = mean wind speed

σ_V = standard deviation of wind speed.

Other indices modified in a similar way were S, ln ϵ_{500} and $|w|$.

This transformation did not improve the linearity of the relationships between ln Ri or $|\nabla w|$ and bumpiness.

Horizontal wind shear which has a U-shaped relationship with bumpiness (see Figure 5.5.) was modified using the formula:

$$\text{Modified horizontal shear} = S_H = \left(\frac{S_H - \overline{S}_H}{\sigma_{S(H)}}\right)^4$$

where S_H is the horizontal wind shear

$\overline{S}_H$ is the mean horizontal wind shear

$\sigma_{S(H)}$ is the standard deviation of S_H

A modified vorticity was calculated in the same way.

It was noticed that, after modification, a few outliers could have a disproportionate effect on the slope of the regression line. This was overcome arbitrarily by restricting the original index to 3 standard deviations about its mean and assigning a value of 3 standard deviations from the mean to any outliers. Thus the modified indices derived by cubing had a maximum range from −27 to +27 and the other modified indices had maxima of 81.

The matrix of correlations between the unmodified indices calculated at standard pressure levels is shown in Table 5.1. The indices Φ_B, D, ζ_a and S_H form a fairly highly inter-correlated group. None of this group of indices is highly correlated with the remaining three but the vertical velocity $|w|$ is highly correlated with the horizontal gradient of vertical velocity $|\nabla w|$. Thus from the seven standard-level indices we have only three reasonably independent predictors.

TABLE 5.1. CORRELATION MATRIX FOR INDICES CALCULATED AT STANDARD LEVELS

VARIABLE	Φ_B	D	ζ_a	S_H	$\|w\|$	$\|\nabla w\|$	V
Φ_B	1.000	0.719	0.793	0.634	0.020	0.039	0.147
D		1.000	0.219	0.148	0.107	0.104	0.236
ζ_a			1.000	0.862	−0.094	−0.083	−0.012
S_H				1.000	−0.141	−0.173	−0.034
$\|w\|$					1.000	0.750	0.228
$\|\nabla w\|$						1.000	0.228
V							1.000

Table 5.2 shows the correlation matrix for the indices that are representative of the layers between standard levels. The three indices, S, ln ϵ_{500} and ln Ri, are very highly inter-correlated suggesting that almost all the turbulence information contained in Ri and ϵ_{500} is in the wind-shear terms.

TABLE 5.2. CORRELATION MATRIX FOR INDICES CALCULATED FOR LAYERS BETWEEN STANDARD LEVELS

VARIABLE	ϵ_{500}	ln Ri	S_V
ϵ_{500}	1.000	−0.788	0.857
ln Ri		1.000	−0.826
S_V			1.000

The indices, both modified and unmodified, were used as predictors and a multiple regression program was allowed to select the best combination. In order to run the program we had to assign numerical values to the turbulence intensities. We chose the simple scale NIL = 0, SLIGHT = 1, MODERATE = 2 and SEVERE = 3. The program used is a step-wise process which takes one predictor at a time into the regression equation and stops when the remaining variables can make no significant contribution.

Four predictors were taken into the equation. They were, in order of statistical significance: horizontal gradient of vertical velocity $|\nabla w|$ modified wind speed ($\dot{V}$), modified horizontal wind shear ($\dot{s}_H$) and modified vertical wind shear ($\dot{s}_V$). The inter-correlations between these variables and their correlations with bumpiness are shown in Table 5.3.

TABLE 5.3. CORRELATION FOR INDICES INCLUDED IN *E*

VARIABLE	$\dot{s}_V$	$\dot{V}$	$\dot{s}_H$	∇w	BUMPS
$\dot{s}_V$	1.000	0.416	0.082	0.276	0.120
$\dot{V}$		1.000	0.164	0.132	0.146
$\dot{s}_H$			1.000	0.178	0.096
$\lvert\nabla w\rvert$				1.000	0.139
BUMPS					1.000

The regression produced an empirical index which we call *E*. This index, which is dimensionless is given by:

$$E = 0.1738\ |\nabla w| + 0.1464\ \dot{V} + 0.0578\ \dot{s}_V + 0.0242\ \dot{s}_H + 1.1804.$$

The means and standard deviations of the predictors used in the regression equation, their coefficients in the equation and the statistical significance of the coefficients are shown in Table 5.4. It is clear that the dominant predictors are $|\nabla w|$ and $\dot{V}$.

TABLE 5.4. INDICES INCLUDED IN *E*

INDEX	MEAN	S.D.	COEFFICIENT	STD.ERROR	*F* TO REMOVE
CONSTANT	–	–	1.1804	–	–
$\dot{s}_V$	0.560	3.644	0.0578	0.322	3.23
$\dot{V}$	0.463	3.477	0.1464	0.0330	19.62
$\dot{s}_H$	3.311	11.402	0.0242	0.0093	6.74
$\lvert\nabla w\rvert$	3.009	2.842	0.1738	0.0384	20.53

Note: The variable should be removed from the regression equation if *F* is not significant at the required level. The degrees of freedom of *F* are $\nu_1 = 4$, $\nu_2 = 2092$.

Multiple linear regression can produce only one predictive equation but when the predictand is divided into three groups (NIL, SLIGHT, and MODERATE or SEVERE, bumps in our case) it is theoretically possible to discriminate between the groups along two orthogonal axes (Cooley and Lohnes, 1971). To investigate this possibility we carried out a multiple discriminant analysis using the indices that had been selected by regression. Table 5.5a shows the number of reports of each turbulence intensity. The mean values of the indices in each group and the overall means are given in Table 5.5b while Table 5.5c shows the discriminant functions.

Table 5.5b shows that the indices provide no reliable basis for discriminating between slight and moderate bumpiness since even the difference between the means of V for these two groups is not significant at the 5 per cent level. The differences between the means in the nil group and the means in the other two groups are, however, significant for all the predictors.

TABLE 5.5. THE RESULTS OF DISCRIMINANT ANALYSIS

5.5a. NUMBER OF REPORTS IN EACH GROUP

GROUP	NUMBER
MODERATE+SEVERE	78
SLIGHT	236
NIL	1783

5.5b. GROUP MEANS AND OVERALL MEANS OF THE INDICES

INDEX	$\dot{S}_V$	$\dot{V}$	$\dot{S}_H$	$\|\nabla w\|$
GROUP				
MODERATE + SEVERE	1.77	2.27	5.70	3.92
SLIGHT	1.58	1.44	5.80	3.91
NIL	0.37	0.25	2.88	2.85
ALL	0.56	0.46	3.31	3.01

5.5c. DISCRIMINANT FUNCTIONS

EIGENVALUES	0.04037	0.00123
INDEX		
$\dot{S}_V$	0.27789	−0.36439
$\dot{V}$	0.63691	0.83433
$\dot{S}_H$	0.09049	−0.10878
$\|\nabla w\|$	0.71280	−0.39911

The significance of the discriminant functions was tested using the associated eigenvalue in Bartlett's chi-square approximation (Cooley and Lohnes, 1971). Only the first of the two functions showed significant discrimination. The weighting given by this function to the predictors is shown by the components in the eigenvector. Comparison of these weights with the coefficients given in Table 5.4 shows that both methods of analysis gave very similar relative weights to the indices. We concluded, therefore, that E was as good as any other combination of the indices that we could produce.

To allow comparison of the performance of the combined index E with that of the individual indices and pilots' reports, Figure 5.12 has been produced in a similar format to Figure 5.1 to 5.11.

Only one curve has been plotted in Figure 5.12 to represent both R_M and R_B because the discriminant analysis has shown that any differences between R_M and R_B are not statistically significant.

The index E is clearly better than any individual index and is also better than a pilot's report for indicating bumpiness of all intensities (R_B). (It should be noted that R_M and R_B are significantly different in Figure 5.1 where discrimination was based on pilots' reports.) The difference between E and a pilot's report for predicting moderate or severe bumps is not so clear; a pilot's report appears to be slightly better but the difference is not significant at the 5 per cent level.

5.3 *The effects of topography and flight level*

In the evaluation of E we have ignored topography and flight level (except in so far as some individual indices were calculated in ways that reduced their dependence on pressure). Using E, flight level and topography (coded SEA=1, COASTS=2 and LAND=3, see Figure 3.1) as predictors in the multiple regression program we found that, after regression on E no significant partial correlation remained between bumpiness and flight level but the correlation between bumpiness and topography remained significant (see Table 5.6).

These results suggest that E is useful at all levels used in this investigation and that improved predictions of bumpiness would be produced if we could take adequate account of topography. We think, however, that our present treatment of topography is too crude for its inclusion to be worth while.

5.4 *Extension of the forecast period and verification of E*

The evaluation of the individual indices and the selection of E were based on a comparison of pilots' reports from the period 1000 to 1319 GMT with indices forecast for 1200 GMT.

It would be desirable to verify E on completely independent occasions but with the limited data available that was not possible. We did, however, check E against the pilots' reports from observing periods 0840–0959 and 1320–1439 GMT. This enabled us to see whether the forecast period could be extended and how dependent E was on fitting to a particular set of pilots' reports.

TABLE 5.6. THE RELATIONSHIP BETWEEN BUMPINESS, FLIGHT LEVEL AND TOPOGRAPHY AFTER REGRESSION OF BUMPINESS ON *E*

Variables in equation

	Coefficient	*F* to remove
Constant	0.002	–
E	0.994	88.10

Variables not in equation

	Partial Correlation	*F* to enter
Flight level	–0.006	0.08
Topography	0.052	5.71

The degrees of freedom for *F* are 2, 2094

Therefore, $F = 5.71$ is significant at the 1 per cent level

The performance of *E* with the second set of pilots' reports is shown in Figure 5.13. The differences between Figure 5.12 and Figure 5.13 are small but significant. In order to investigate the causes of the slight degradation of the performance of *E* in the two samples of pilots' reports we ran the regression program on the second set of reports, entering as predictors *E*, $\dot{V}$ $\dot{S}_V$ $\dot{S}_H$ and $|\nabla w|$. Having selected *E* as the best predictor the program rejected $\dot{V}$ $\dot{S}_V$ and $|\nabla w|$ showing that *E* already contained all the predictive capacity of those indices. The program then put $\dot{S}_H$ into the regression equation with a negative coefficient, thereby effectively eliminating it from the equation, since it was already in *E* with a positive coefficient. Examination of the correlation matrix showed that the correlation between S_H and bumpiness had fallen from 0.096 in the set of pilots' reports from the period 1000–1319 GMT (2097 reports) to –0.004 in the set of reports from the periods 0840–0959 and 1320–1429 GMT (1586 reports). If the two batches of pilots' reports are regarded as random samples from a normal distribution, which they are not, but we can make no better assumption, the probability of the observed change of correlation coefficient occurring by chance with a single pair of variables is about 0.26 per cent. We had four indices so the probability of such a change in correlation coefficients between the other three variables and bumpiness were all well within the limits of expected random variation. We, therefore, conclude that the value of S_H as a turbulence predictor may be limited to a rather short period near the time of validity of the forecast.

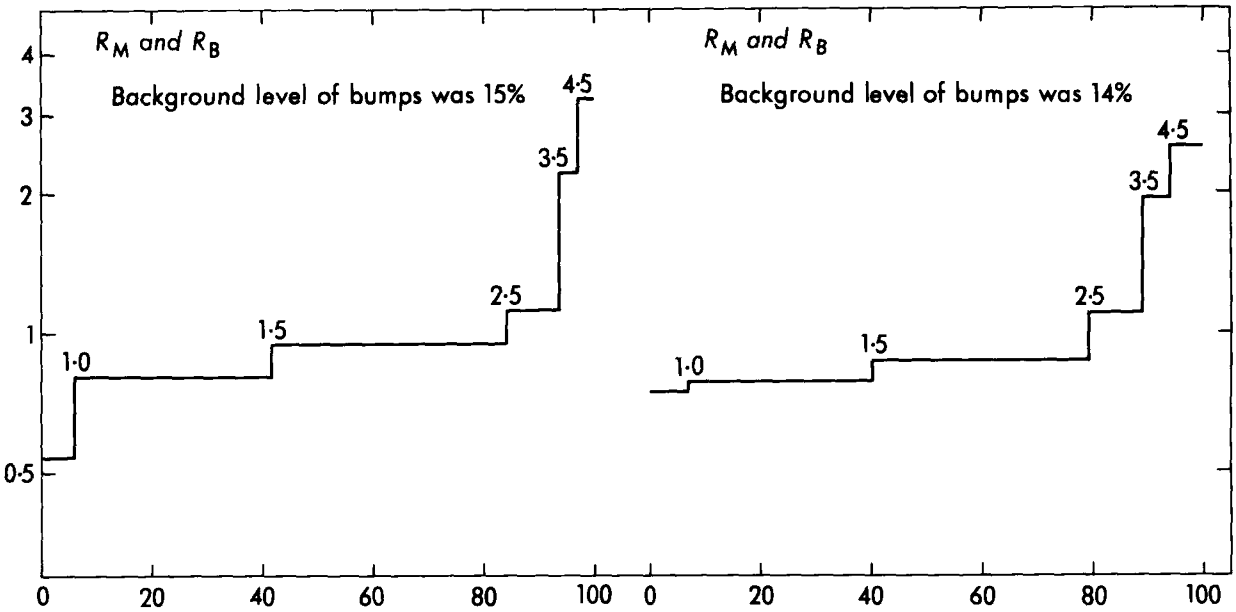


FIGURE 5.12 THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF E AND BUMPINESS IN OBSERVING PERIOD 1000-1319 GMT

Total number of reports 2097

FIGURE 5.13. THE RELATIONSHIP BETWEEN 12-HOUR FORECASTS OF E AND BUMPINESS IN OBSERVING PERIODS 0840-0959 AND 1320-1439 GMT

Total number of reports 1586

Cumulative percentage of observations with index less than or equal to value shown

E was selected using an ensemble of turbulence reports drawn from a wide range of flight levels and several different synoptic situations. It is, therefore, of interest to see what a field of E for a particular hour and a particular height band looks like. Figure 5.14 shows the grid-point values of E for the layer 300–350 mb calculated from the 10-level model forecast valid at 1200 GMT on 1 May 1972. These values of E are a combination of $|\nabla w|$, $\dot{V}$ and $\dot{S}_H$ at 300 mb (taken as representative of the layer 250–350 mb) and $\dot{S}_V$ (for the layer 300-400 mb). The complete index is, therefore, representative of the layer 300–350 mb. Comparison of Figures 5.14 and 3.6 shows that the field of E had a fairly close association with the pilots' reports of bumpiness. It gave no warning of the moderate bumps reported south of 50°N but neither did any other meteorological index examined.

6 - DISCUSSION

Pilots' reports were collected on seven days and the meteorological indices were fully available on only five of those days. We cannot, therefore, consider either the relationships demonstrated between the reports from different pilots or those between the pilots' reports and the meteorological indices as reliably established. We have attempted to test the significance of our results but some of the tests we have had to use are based on the assumption that the variables are normally distributed and all assume random sampling. Our data do not satisfy either requirement so the statistical tests do not have their usual reliability.

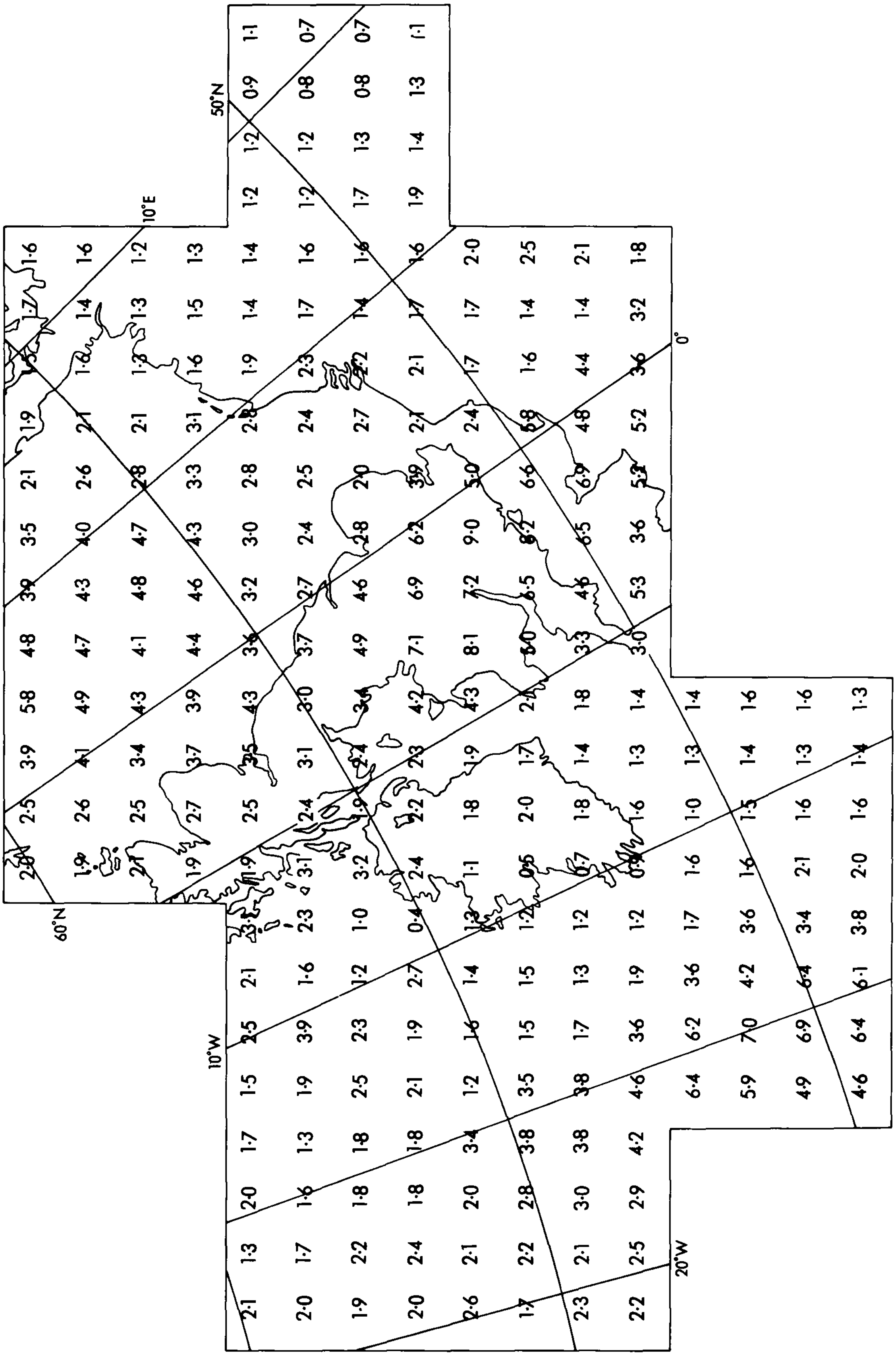


FIGURE 5.14. 12-HOUR FORECAST OF *E* FOR 1200 GMT 1972—LAYER 300-400 mb

The main causes of the difficulty of assessing the significance of our results are, paradoxically the main reasons that we may hope for success. Namely that fields of meteorological parameters are spatially coherent and that turbulent patches often occur in conglomerates that have synoptic scale.

Figures 1.3 to 1.7 show that, in the five days which we could analyse fully, we had only three independent synoptic situations. These are too few to permit reliable positive conclusions.

The proportion of the total distance flow, during our investigation, which was reported as bumpy, was not exceptional, but the proportion of bumps reported well below the tropopause over land and coasts was unusually high. This suggests that a significant proportion of the turbulence was topographic in origin and this possibility is being investigated. It also implies that our sample may have been unusual in the depth of the atmosphere that was simultaneously affected by turbulence so that Figure 3.10, which shows the relationship between pilots' reports and their vertical separation, may not have general validity. Nevertheless, we think that our sample gives a useful guide to the value of one pilot's report of bumps as a warning to another pilot.

Four other similar investigations of the associations between synoptic-scale meteorological indices and bumpiness are known to the authors. Endlich and Mancuso (1965) in the USA used reports from 4 to 9 February 1963 and the same authors (1967) analysed reports collected during the four ICAO five-day observing periods in 1964 and 1965. Colquhoun (1967) analysed reports over the South Pacific region from the same ICAO observing periods and Bortnikov and Vasil'yev (1974), in the USSR, used reports from the period 11 to 20 March 1969.

All the investigations have shown that vertical wind shear* performed fairly well as a locator of bumpiness but whereas we found deformation to be of little or no value both the Americans and the Russians found it to be a very useful predictor. We think this difference is mainly a result of our sample of synoptic situations. Endlich and Mancuso in comment on the apparent variation in the ability of the Richardson number to locate turbulence in two data sets they had examined say 'During March 1962 (12th to 24th) .... the regions of turbulence were concentrated in certain portions of upper fronts and jet streams. By contrast, in February 1963 winds over the United States were relatively light .... turbulent regions tended to be associated with the troughs rather than with fronts or jet streams.' Our own investigation was dominated by turbulence reported in a jet stream on 1 May 1972 and hence wind speed is a good locator of bumpiness in our data set. We think that any future investigation must cover a wide range of synoptic situations.

The relative failure of the theoretically based indices such as Ri, Φ and ϵ when compared with empirical combinations of indices is disappointing but the evidence is not, in our view, conclusive. There are two reasons for this view. Firstly, when the empiricism is based on only a few distinct synoptic situations over-fitting is almost inevitable and it is highly probable that an empirical combination of indices will be found which outperforms, on that data set, a

* Bortnikov and Vasil'yev actually used the horizontal gradient of temperature which, on the synoptic scale, is closely related to vertical wind shear.

theoretical combination. We cannot avoid this by taking large samples from the few situations. Secondly, the performance of a theoretical index is limited by the resolution of the forecast model and the approximations that are made in its calculation. Roach's original formulation of Φ required greater detail, particularly of the vertical wind-shear vector, than can be obtained from the 10-level model. Brown's approximations, it seems from our results, are not precise enough. Oard (1974) has suggested another formulation based on the thermal wind equation but that, like Roach's original, is highly dependent on vertical gradients of wind which are difficult to evaluate accurately, near the level of maximum wind, when data are available from only 10 standard levels. Considerable work may be needed to find the best way of calculating theoretically based indices from information available in forecast models.

One indication of bumpiness that a pilot can be given is a report from another pilot who has been exposed to bumps in the same region. The value of such a report decreases as the size of the region increases and as the interval between the two exposures increases. Our results suggest that for regions 100 km square and 100 mb deep a pilot's report is a more reliable indicator of bumpiness than a 12-hour forecast of E if the interval between exposures is less than 1 hour but the index is probably better if the interval is more than 3 hours.

A comprehensive service for warning pilots of bumpiness must use both pilots' reports and meteorological indices. How the two types of information should be combined is not obvious. A method has been suggested by Endlich and Mancuso (1967) and we hope that the data we have collected, and any collected in future turbulence campaigns, will be used to verify their method.

Ideally there would be a continuous exchange of information between the aviator and the meteorologist so that the latter could improve his forecasting techniques. Such a learning program would require considerable effort from both meteorologists and pilots. It could be justified only if the improved forecasts can be expected to be of real value to the aviator.

This investigation has shown that we have good reason to expect that meteorologists will eventually be able to give a pilot advance warning of turbulence that is as reliable as a report from another pilot who recently passed through the same region. Thus, in the more turbulent regions meteorologists should be able to tell him that he has a probability of about 0.2 of encountering moderate or severe bumps in each 100 km of flight. This may seem, at first sight, a very low risk, but if he flew more than 310 km in such a region he would be more likely to experience moderate bumps than escape them, and if he flew 1000 km the probability that he would encounter bumps of at least moderate intensity would be about 0.9. This illustrates why it is undesirable to mark an area on a chart as simply turbulent or non-turbulent: the risk of encountering bumps depends not only on the state of the atmosphere but also on the length of exposure.

7 - CONCLUSIONS

We can draw only two firm conclusions from this and similar turbulence investigations. They are as follows.

(i) Turbulence forecasts must be stated in terms of probability if they are to convey the maximum possible information to the recipient.

(ii) Data must be gathered from a wide range of synoptic situations to establish the reliability of turbulence forecasting techniques.

Less certainly we conclude that:

(iii) Forecasts produced by the 10-level model (Benwell *et alii* 1971) contain information which allows positive predictions of bumpiness which are about as good as those based on recent pilots' reports. A prediction by the model that a region will be free from bumps is slightly more reliable than a similar prediction based on a recent report from a single aircraft in that region.

(iv) The inclusion of topographic effects should make a significant improvement to the objective turbulence predictions.

(v) The combination of recent reports from pilots with meteorological indices should improve short-period predictions still further.

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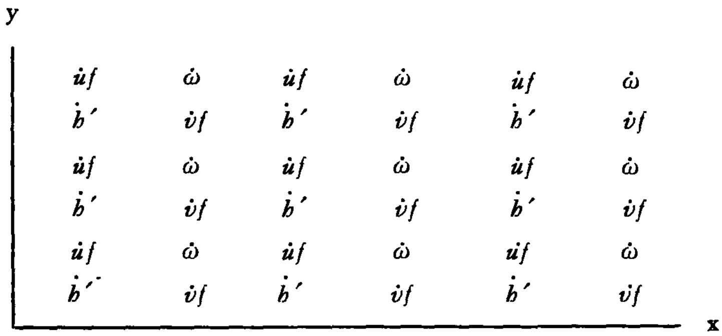
APPENDIX - EVALUATION OF THE INDICES AT GRID POINTS IN THE 10-LEVEL MODEL

In the Bushby-Timpson model, grid-point values of u, v and h' (the components of horizontal velocity in the x and y directions at the point and the vertical thickness of the layer above the point up to the next pressure level in the model) are staggered in both time and space (Benwell *et alii* 1971). However, for the present purpose they may be regarded as arranged at any one pressure level p_r as a grid of points as shown below. The points are approximately 50 km apart. The exact spacing is stored at each point. The Coriolis parameter f is stored at the u and v points

Figure A 1.1



and ω the 'vertical motion' in pressure co-ordinates is stored at the intermediate points. As well as the layer thickness the geopotential of the pressure surface h is stored at an h' point.

All the indices are calculated at h' points which are at the centres of the grid squares to which the pilots are assigned.

It can be seen from Figure A 1.1 that most of the quantities needed to calculate the indices are not directly available at h' points. The required quantities are obtained at h' as follows:

u is the mean of the two nearest values

v is the mean of the two nearest values

V is $(u^2 + v^2)^{1/2}$

w is the mean of the four nearest values

$|\nabla w|$ is obtained from the four nearest values of w

$\frac{\Delta u}{\Delta y}$ is obtained from the two nearest values of u

$\frac{\Delta v}{\Delta x}$ is obtained from the two nearest values of v

$\frac{\Delta u}{\Delta x}$ is the mean of the four $\frac{\Delta u}{\Delta x}$ values calculated at the bearest w points

$\frac{\Delta v}{\Delta y}$ is the mean of the four $\frac{\Delta v}{\Delta y}$ values calculated at the nearest w points

Those indices (see Table 5.1 for a list of symbols) for which all the terms are available at a single pressure level are obtained from the above quantities at each standard pressure level (600, 500 – – – 100 mb) as follows:

$$\zeta_a = \frac{\Delta v}{\Delta x} - \frac{\Delta u}{\Delta y} + f$$

$$D_S = \frac{\Delta v}{\Delta x} + \frac{\Delta u}{\Delta y}, \qquad D_T = \frac{\Delta u}{\Delta x} - \frac{\Delta v}{\Delta y}$$

$$D = (D_S{}^2 + D_T{}^2)^{1/2}$$

$$\Phi_B = (0.3\,\zeta_a{}^2 + D^2)^{1/2}$$

$$H = \frac{1}{V^2}\left(uv\frac{\Delta u}{\Delta x} - u^2\frac{\Delta u}{\Delta y} + v^2\frac{\Delta v}{\Delta x} - uv\frac{\Delta v}{\Delta y}\right)$$

$w = \omega \dfrac{\Delta h}{\Delta p}$ where $\dfrac{\Delta h}{\Delta p}$ is the thickness of a one-mb layer at the pressure level concerned.

$$\left|\nabla w\right| = \left|\nabla \omega\right| \cdot \frac{\Delta h}{\Delta p}.$$

The remaining indices involve vertical gradients and are therefore calculated for the 100-mb layers between standard levels (600–500, 500–400, – – – – – , 200–100 mb).

Considering the layer between levels p_r and p_{r-1} we have

$$|\Delta V|^2 = \left(u_r - u_{r-1}\right)^2 + \left(v_r - v_{r-1}\right)^2$$

$\Delta h = h'_p$ i.e. the difference between h (in geopotential metres) at p_r and p_{r-1}

The vertical wind shear S_v is then given by

$$S_V = \frac{|\Delta V|}{\Delta h} \times 1000 \qquad \ldots (1)$$

The calculations of S_V for the layer containing the maximum wind presents some difficulty because there are many occasions when the shear calculated from the winds at the boundaries of that layer gives a very poor estimate of the actual shear within the layer.

An investigation of winds from radiosonde ascents for which maximum winds were reported showed that the level of the maximum wind could be estimated by fitting cubic splines to the winds at standard levels and finding the maximum wind given by the splines.

If the shear $S_{V\,(max)}$ (calculated from equation 1) in the layer containing the maximum wind is greater than the mean of the shears ($S_{V\,(mean)}$) in the layers above and below, no change is made, but otherwise $S_{V\ (max)}$ is recalculated using the formula

$$S_{V\ (max)} = 0.5\ S_{V\ (mean)} + 0.08\ V_{(max)}$$

where $V_{(max)}$ is the highest wind at a standard level.

This formula gives a good estimate of shear in the layer containing the maximum wind for the ascent data and since the correlations between unmodified $S_{V\ (max)}$, $S_{V\ (mean)}$ and $V_{(max)}$ in the ascent data are very similar to those in the 10-level model data it can be applied to the model with reasonable confidence.

The modified $S_{V\ (max)}$ is then used in equation 1 to obtain a new $|\Delta V|$ for that layer and ϵ_{500} and Ri are calculated from the modified $|\Delta V|$ as follows.

$$\epsilon_{500} = \frac{|\Delta V|^2}{24} \cdot \frac{500}{\Delta h} \left(\frac{\Phi_{B(r)} + \Phi_{B(r-1)}}{2} \right)$$

Richardson number is given by

$$Ri = -\frac{1}{\rho\theta} \cdot \frac{\Delta\theta}{\Delta p} \Big/ \left(\frac{\Delta V}{\Delta p}\right)^2 = -\frac{RT}{p\theta} \cdot \frac{\Delta\theta\,\Delta p}{|\Delta V|^2}$$

From the hydrostatic equation temperatures at $p_{r-\frac{1}{2}}$ are given by

$$T_{r-\frac{1}{2}} = \frac{g}{2R} \left(h_{r-1} - h_r\right) \cdot \frac{p_r + p_{r-1}}{p_r - p_{r-1}}$$

$\theta_{r-\frac{1}{2}}$ is found at h' points at $p_{r-\frac{1}{2}}$ from $T_{r-\frac{1}{2}}$. θ is found from θ_r calculated at h' points at p_r and p_{r-1}. This comes from T_r and T_{r-1}. T_r is found from

$$T_r = \tfrac{1}{2}\ T_{r+\frac{1}{2}} + T_{r-\frac{1}{2}}$$

$$\text{Hence } (Ri)_{r-\frac{1}{2}} = -\frac{2RT_{r-\frac{1}{2}}\quad \theta_r - \theta_{r-1}\quad p_r - p_{r-1}}{|\Delta V|^2\quad \theta_{r-\frac{1}{2}}\quad p_r + p_{r-1}}$$

Calculated values are the natural logarithm of this.

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