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LAGRANGIAN PLUME DEVELOPMENT WITH IMPOSED MEAN VERTICAL MOTION

by

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1. Introduction

Most of the extensive theory on the transport and dispersion of pollutants in the atmospheric boundary layer is specific to uniform conditions over level terrain. A glance at any Ordnance Survey map reveals that there are extensive areas in the UK in which the effects of the relief upon the mean air flow and turbulence cannot realistically be ignored, and surprisingly few areas of even a few square miles in which topographic irregularities may not have distinct, if sometimes subtle, effects upon atmospheric dispersion.

This paper is a companion to Maryon, Whitlock and Jenkins (1984, referred to henceforth as MWJ) which described short-range dispersion experiments carried out on the hill Blashaval, North Uist in 1982. It deals with the initial stages of plume dispersion in the vertical for elevated releases, in neutral conditions, into the surface stress layer. In particular it is concerned with the displacement of the centroid of a puff according to Lagrangian similarity theory, an attempt being made to adapt formulae developed for conditions of level, uniform terrain to the situation where the plume is embedded in a profile of mean vertical velocity, as occurred in many of the experiments carried out on the upwind slope at Blashaval. Throughout this paper the mean vertical velocity $\overline{w}_\varepsilon(z)$ refers to the motion imposed, via mass continuity, by distortion of the mean flow due to the presence of the hill, and is always regarded as perpendicular to the local surface.

The work may, however, also prove to be of wider interest, in that vertical motions which are broad-scale in relation to surface layer turbulence may occur widely in convective situations; as yet little theory has been applied specifically to the detail of plume development in such conditions.

2. Lagrangian similarity theory

Although the standard deviation of particle displacements σ_y is necessarily used to represent horizontal spread, due to the presence of a solid boundary σ_z is a less satisfactory measure of vertical dispersion from an elevated source in the surface layer, except in the immediate vicinity of the point of release. Similarity theory can be usefully applied to the rate of displacement of the mean height of an ensemble of particles $\frac{d\bar{z}}{dt}$, and in particular for surface releases. $\bar{z}$ is the height above the surface of the centroid of an instantaneous puff released at a height z_s.

In neutral conditions, and subject to a vertical forcing as a result of topographically induced divergence, $\frac{d\bar{z}}{dt}$ in the near field depends upon the time since release, t, the turbulent momentum flux represented by the friction velocity u_*, the height of release, and upon the gradient of induced vertical velocity $\frac{d\bar{w}_\varepsilon}{\partial z}$, which is presumed steady. With a slight, acceptable loss of accuracy the profile of vertical velocity may be assumed linear, ie $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ = constant, so that dimensional considerations yield

$$\frac{dz}{dt} = u_* f\left(\frac{z_s}{u_* t}, \frac{z_s}{u_*}\frac{\partial \bar{w}_\varepsilon}{\partial z}\right) \qquad (1)$$

where the second argument of f is a dimensionless vertical velocity or profile. Turbulence measurements made over level terrain have established that there is little dependence upon height in the surface stress layer, and for $t \gg z_s/u_*$, there is little dependence of $\frac{d\bar{z}}{dt}$ upon $z_s/u_* t$ in conditions of zero mean vertical velocity. Writing $\frac{\partial \bar{w}_\varepsilon}{\partial z} = 0$ in (1) we immediately obtain the standard formulation

$$\left.\frac{d\bar{z}}{dt}\right|_{\bar{w}_\varepsilon = 0} = k u_* \qquad (2)$$

where k is a constant now generally taken as von Karman's constant. Batchelor's similarity theory assumes that (2) may be applied to an elevated release provided that sufficient time has passed for the particles to 'forget' their origin. Integrating (2) with respect to time,

$$\bar{z} = k u_* (t + t') \qquad (3)$$

where $t' \propto z_s/u_*$ is the time required for a notional surface release to produce a plume displacement coincident with that from an elevated release: it follows (Fig 1) that the centroid of the notional plume would pass a little below z_s. The figure is also used to illustrate (diagrammatically) the contrast in Eulerian and Lagrangian plume displacement. In the early stages of plume development, although the mean displacement of a discrete puff rises, as (2) implies (pecked line), the Eulerian displacement is at first slight (F approximates the centroid of a profile at a fixed downwind distance). This can be seen from Hunt's (1985) analysis of $\frac{d\bar{z}}{dx}$ and indeed from random walk simulations. The difference is due to the relative effects of the profiles of eddy diffusivity and wind strength in the Eulerian and Lagrangian situations. The line of maximum concentration descends (Smith, 1957; Fig. 1, dotted line). It should be mentioned at this point that none of the formulations in this paper include the relatively slight effects of the interactions involving horizontal turbulence upon the vertical displacement.

3. Raupach's formula and its extension

Raupach (1983) addressed the question of the Lagrangian mean displacement following release at z_s. Starting with the standard Langevin equation which forms the basis of the conventional random walk dispersion models he derived the linear ordinary differential equation

$$\frac{d\bar{w}}{dt} = - \frac{\bar{w}(t)}{T_s} + \frac{\sigma_w^2}{z_s}\left(1 - e^{-t/T_s}\right) \tag{4}$$

where $\bar{w}$ is the ensemble mean vertical velocity of the particles and T_s the Lagrangian time scale at $z = z_s$. To obtain (4) Raupach assumed an exponential autocorrelation function for $w(t)$ after expanding the dimensionless displacement of the plume binomially under an assumption of small t and hence small displacement, and truncating to one term. Using $\sigma_w^2 \frac{T_s}{z_s} = ku_*$, (from standard diffusivity theory) the solution to (4) is

$$\frac{d\bar{z}}{dt} = ku_*\left[1 - e^{-t/T_s}(1 + t/T_s)\right] \tag{5}$$

and hence

$$\frac{\bar{z}}{z_s} = 1 + kc\left[(t/T_s - 2) + (t/T_s + 2)e^{-t/T_s}\right] \tag{6}$$

In (6) C is a constant used in relating the Lagrangian time scale to the height:

$$T_L(z) = \frac{Cz}{u_*}, \quad \text{or} \quad T_s = \frac{Cz_s}{u_*} \tag{7}$$

which Raupach estimates at 0.26, corresponding to $\sigma_w = 1.24\, u_*$. Expression (5) tends to zero as $t \to 0$ and, despite the assumptions, tends to ku_* as $t \to \infty$. Raupach compared (5) and (6) with random walk simulations for t/T_s up to 10 and found the formulae largely validated.

So far only displacements in conditions of zero mean vertical motion have been reviewed. We now consider the introduction of a uniform Eulerian gradient of vertical velocity, $\frac{\partial \bar{w}_E}{\partial z}$. The basic random walk in neutral conditions may be written

$$w_{i+1} = \left(1 - \frac{\Delta t}{T_L(z_i)}\right) w_i + \sqrt{\frac{2\Delta t}{T_L(z_i)}} \cdot \sigma_w \mu_{i+1} \tag{8}$$

where w_i, w_{i+1} are instantaneous vertical velocities of a particle at timesteps i, $i+1$ separated by a time interval Δt, μ a $N(0,1)$ random variable, $T_L(z)$ the Lagrangian timescale at height z estimated from ku_*z/σ_w^2, and σ_w approximated by $1.24\, u_*$ (see Ley, 1982, Thomson, 1984). The most straightforward adaptation of (8) for conditions of non-zero mean vertical velocity is obtained by substituting $w_i - \bar{w}_E(z_i)$ for w_i (etc):

$$w_{i+1} = \left(1 - \frac{\Delta t}{T_L(z_i)}\right) w_i + \sqrt{\frac{2\Delta t}{T_L(z_i)}} \cdot \sigma_w \mu_{i+1} + \Delta t\, w_i \frac{\partial \bar{w}_E}{\partial z} + \bar{w}_E(z_i) \frac{\Delta t}{T_L(z_i)} \tag{9}$$

where $\bar{w}_\varepsilon(z)$ is the Eulerian mean vertical wind velocity at height z.

Following Raupach's approach, (9) may be expressed, at small times,

$$\frac{d\bar{w}}{dt} = -\bar{w}\left(\frac{1}{T_s} - \frac{\partial \bar{w}_\varepsilon}{\partial z}\right) + \frac{\sigma_w^2}{z_s}\left(1 - e^{-t/T_s}\right) + \frac{\bar{w}_\varepsilon(z)}{T_L(z)} \quad (10)$$

Substituting for $T_L(z)$ the last term becomes $\frac{u_*}{c}\frac{\partial \bar{w}_\varepsilon}{\partial z}$ which is assumed constant in the surface stress layer. (10) may then be solved to give

$$\frac{d\bar{z}}{dt} = \frac{1}{1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}}\left\{ z_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\left[1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\cdot e^{-t/T_s\left(1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)}\right] + ku_*\left[1 - e^{-t/T_s}\left\{1 - \frac{1}{T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}}\left(1 - e^{t\frac{\partial \bar{w}_\varepsilon}{\partial z}}\right)\right\}\right]\right\} \quad (11)$$

provided $T_s \neq \left(\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)^{-1}$. A simpler solution exists for that particular but unimportant situation. In (11), $T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}$ is in direct proportion to $\frac{\bar{w}_\varepsilon(z)}{ku_*}$, the ratio of the Eulerian and turbulence vertical velocity components at source height, and in effect represents the corresponding term in (1). The other term in (1), z_s/u_*t, is present in t/T_s (via (7)).

Integrating with respect to time,

$$\bar{z} - z_s = \frac{1}{1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}}\left\{ z_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\left[t + T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\left(e^{-t/T_s\left(1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)} - 1\right)\right] + ku_*\left[t + \frac{\left(1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)\left(1 - e^{-t/T_s}\right)}{\frac{\partial \bar{w}_\varepsilon}{\partial z}} + \frac{e^{-t/T_s\left(1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)} - 1}{\frac{\partial \bar{w}_\varepsilon}{\partial z}\left(1 - T_s\frac{\partial \bar{w}_\varepsilon}{\partial z}\right)}\right]\right\} \quad (12)$$

Unfortunately, although these formulae hold for small t they are not applicable as $t \rightarrow \infty$ as an expression involving T_s or z_s remains in the limit. In fact, in (11), as $t \rightarrow \infty$

$$\bar{w} \longrightarrow \frac{z_s \frac{\partial \bar{w}_\varepsilon}{\partial z} + k u_*}{1 - T_s \frac{\partial \bar{w}_\varepsilon}{\partial z}} \qquad (13)$$

when it should tend to zero provided $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$. As a result, (12) accurately reproduces the mean displacement of the random walk (9) only for a brief initial period and often starts to diverge at about $t/T_s = 2$. The random walk and computed displacement curves are shown in Figs 2-4. These figures correspond to the Blashaval tracer dispersion experiments 3, 4 and 8 (ie one for each of the three release sites on the upwind slope of the hill). u_* and $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ were estimated using the observed wind velocity and computed divergence as described in MWJ. Each random walk computation used 3000 particle trajectories.

Figs 2-4 show that as t/T_s increases, the downward displacement according to (12) comes to exceed greatly that of the random walk simulations. Expression (13) draws attention to the presence of $z_s \frac{\partial \bar{w}_\varepsilon}{\partial z}$, the mean vertical velocity at source height, in that part of (12) modelling the descending motions, even in the limit as $t \rightarrow \infty$. Clearly, as a puff is depressed more and more particles collect near the surface in a region of small $\bar{w}_\varepsilon$, so that the mean displacement should gradually approach a position where its turbulence and mean flow 'components' are in approximate equilibrium (if conditions are uniform). The situation is that of case (a) in Fig. 5, which illustrates possible puff centroid trajectories for different relative magnitudes of ku_* and the gradient of mean vertical velocity.

Ideally, then, the numerator of (13) should tend to zero for downward mean displacements and this can, in effect, be achieved by replacing z_s by $z_s + bz_d$ in (11), where z_d is the mean displacement $\bar{\bar{z}} - z_s$ and b a coefficient. Unfortunately the resultant integral corresponding to (12) would be very complicated. A time-stepping technique to obtain $\bar{\bar{z}}$ can be used, but for practical purposes it is more straightforward to consider a simple empirical adjustment to (12). If (12) is summarized

$$z_d = A(t)z_s + B(t)$$

where $A(t)z_s$, $B(t)$, are respectively the mean flow and turbulence terms in the mean displacement, substituting $z_s + bz_d$ for z_s, where b is a coefficient < 1 should have the effect of linking the downward motion to the mean level of the cloud of particles in a reasonably consistent way. This adjustment gives the empirical formula

$$z_d = \frac{A(t)z_s + B(t)}{1 - bA(t)} \qquad (14)$$

An arbitrary choice of 0.5 for b yielded the curves plotted in Figs 2-4, which compare well with the random walk simulations up to $t = 10T_s$, for all three sites, although for the weaker $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ Expt. 4, $b \simeq 0.3$ would give a better fit. The results seem encouraging for prospects of modelling mean puff displacements over those distances likely to be of practical interest, in these conditions.

In the following sections some alternative approaches to the computation of puff displacement are discussed; standard Lagrangian formulae relating the horizontal mean puff displacements to the vertical are extended to elevated releases and much simplified conditions of mean vertical motion in an Appendix.

4. An alternative approach to puff displacement

Expression (11) draws attention to the relation and interactions between $z_s \frac{\partial \bar{w}_\varepsilon}{\partial z}$ and ku_*, the background and turbulent components of vertical plume motion. Returning to the similarity formulae for a surface release, if the assumption is made that $\frac{d\bar{z}}{dt}$ is linearly dependent upon the superimposed vertical velocity, in the near field, it follows from (1) and (2) that

$$\frac{d\bar{z}}{dt} = f_2(ku_*, \bar{w}_\varepsilon(z_s)) = ku_* + r_1 z_s \frac{\partial \bar{w}_\varepsilon}{\partial z}$$

where r_1 is a coefficient. This formula has obvious shortcomings, and the work of the previous section suggests that a more flexible formula might be expressed.

$$\frac{d\bar{z}}{dt} = R(t) + (z_s + r_2 z_d) \frac{\partial \bar{w}_\varepsilon}{\partial z} \tag{15}$$

where $R(t)$ is Raupach's solution (5). The simple superimposition (15) with r_2 taken as 1, might be expected to give a reasonable estimate of the net vertical displacement close to the source, where a narrow plume is embedded in an Eulerian vertical motion. As t increases, however, the particles will be spread over greater vertical depths, and the assumption of linearity may become less valid.

The solution of (15) is

$$\bar{z} = \frac{z_s}{r_2}\left(e^{r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z} t} + r_2 - 1\right)$$

$$+ k u_* \left[\frac{e^{r_2\frac{\partial \bar{w}_\varepsilon}{\partial z}t} - 1}{r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z}} + \frac{\left(2 + r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z} T_s\right)\left(e^{-t/T_s} - e^{r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z} t}\right) + t e^{-t/T_s}\left(r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z} + 1/T_s\right)}{T_s \left(r_2 \frac{\partial \bar{w}_\varepsilon}{\partial z} + 1/T_s\right)^2}\right] \quad \ldots (16)$$

which is still rather complicated. In the first term $z_s e^{\frac{\partial \bar{w}_\varepsilon}{\partial z} t}$ is from the motion of a puff along a streamline of the mean flow; the second term incorporates the turbulence component of the displacement. If $r_2 \equiv 1$ the somewhat simplified solution corresponds to the equation

$$\frac{d\bar{z}}{dt} = R(t) + \bar{z}\frac{\partial \bar{w}_\varepsilon}{\partial z} \qquad (17)$$

a super-position of simple components. Reasonable fits to the random walk simulation out to $t = 10T_s$ can be obtained, but the optimal magnitude of r_2 is near unity for the depressed flows of Exp. 3 and 8, and about 0.7 for the near-equilibrium case 4. Results with an accuracy comparable to (14) cannot be obtained using a single choice of r_2 . Figs 2-4 illustrate the curves for which $r_2 = 0.9$; the solutions (17) are close to those illustrated, except in the case of Exp. 3, from which they diverge towards $t = 8T_s$. The weaker consistency (on the whole) of (16) in comparison with (14) may be due to the physics of the dispersion process in a descending mean flow being less well represented, the mutual interaction of the turbulence and the mean vertical velocity being absent.

5. A simplified approach

It would be helpful if a simpler formula of useful range could be derived from (17). Various empirical ways of combining the mean vertical flow and turbulent components of the centroid displacement were considered: Raupach's formula (6) expresses $\bar{z}$ as the sum of the source height z_s and the displacement due to turbulent spread. One suggestion is to replace z_s with the height the centroid of a puff would assume if it travelled along a streamline resulting from the mean horizontal and vertical wind profiles:

$$\bar{z} = z_s e^{t\frac{\partial \bar{w}_\varepsilon}{\partial z}} \left\{ 1 + kc\left[\left(t/T_s - 2\right) + \left(t/T_s + 2\right) e^{-t/T_s} \right] \right\} \quad (18)$$

Here, $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$, and $\bar{z}$ is the sum of the height of the streamline above the surface plus a displacement term. This formula gives no consistent improvement over (12): the second term is too small as insufficient account is taken of the synchronous vertical spread of the material, and $\bar{z}$ is eventually depressed far too strongly in comparison with random walk solutions. The obvious adjustment

$$\bar{z} = z_s e^{t\frac{\partial \bar{w}_\varepsilon}{\partial z}} + z_s kc\left[\left(t/T_s - 2\right) + \left(t/T_s + 2\right) e^{-t/T_s} \right] \quad (19)$$

suffers from the opposite effect - the turbulence term is now too large (as no account is taken of the synchronous depression of the plume) and $\bar{z}$ is not depressed sufficiently. Replacing z_s in the second term with the intermediate level $\bar{z}$ yields

$$\bar{z} = z_s e^{t\frac{\partial \bar{w}_\varepsilon}{\partial z}} \left\{ 1 - kc\left[\left(t/T_s - 2\right) + \left(t/T_s + 2\right) e^{-t/T_s} \right] \right\}^{-1} \quad (20)$$

This implicit formula is superior to (18) and (19) near the source giving an acceptable solution to $t = 5T_s$ or more (Figs 2-4). It is prone to instability in the far field, however, as there are no constraints on the magnitude of $\bar{z}$, with its contribution to the turbulence term.

One way forward lies in the application of an empirical weighting coefficient $r(t)$ to the turbulence term (for any of these formulae), taking a value of unity for $t = 0$ and approaching some appropriate asymptotic value at large t: for example a hyperbolic curve as described in section 5. The random walk simulations have been reproduced fairly successfully in this way, but it introduces additional complexity, and again there is some difficulty in the optimization of the free parameters, which tend to vary with situation.

5. Time-stepping techniques

Although not so simple to use, time-stepping methods can be applied to formulae which are not integrable analytically, and have the advantage of greater potential flexibility. A programmable calculator would be quite adequate for their application.

It was noted in section 3 that (11) can be improved by substituting $z_s + bz_d$ for z_s, where b is a weighting coefficient; ie

$$\frac{d\bar{z}}{dt} = (z_s + bz_d)\frac{\partial \bar{w}_s}{\partial z} E(t) + ku_* F(t) \qquad (21)$$

where $E(t)$, $F(t)$ are the complicated terms representing the mean flow and turbulence interaction. Reference to the relatively simple equations (11) and (15) suggests that any analytical solution would be very complicated. A forward time-stepping solution was made, which showed that for $b = 1$, ie $z_s + bz_d = \bar{z}$, the divergence of (21) from the random walk simulation is in the opposite sense to (11) – the downward displacement is insufficient. Evidently an empirical weighting is needed which can be applied to the vertical velocity to give greater depression in the far field. The best results were obtained by using a hyperbolic function $b(t)$ ranging from 1 at $t = 0$ to an arbitrary value < 1 for large t : the parameter defining the asymptotic value of $b(t)$ as $t \to \infty$ was taken as proportional to the relative magnitudes of the mean flow and turbulence components of the vertical motion, $T_s \frac{\partial \bar{w}_\varepsilon}{\partial z}$. This ratio constituted the only variation in an otherwise fixed hyperbolic formulation of $b(t)$. The forward stepping solutions compared very well with all the random walk simulations: the fits were as good as, and occasionally much improved upon, (14).

This interesting result is presented as encouraging for prospects of developing formulae of wider practical application; no general validity is claimed, of course, for this specific formulation. It is possible that empirical time-stepping formulae such as those discussed here may be adapted to more realistic slope conditions if changes in parameters such as $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ can be sufficiently well estimated, and incremented in the course of the integration.

6. Upward mean vertical motion

For elevated releases with ascending mean motion surface layer analysis is of less importance. Lateral convergence and upward motion may occur on the flanks of a hill, viewed from upwind, as apparently happened in the Blashaval exp. 9 (see MWJ). The Section 3 displacement formulae (11) and (15) both tend to ku_* as t increases, for very small vertical velocity gradients so that (12), (16) and (21) all yield fairly accurate results when compared with the random walk. The optimal weighting r_2 for (16) was about 0.2.

Of the Section 4 formulae (not illustrated) the best result is given by (19) in which the turbulence term is of appropriate magnitude for an ascending puff, and the displacement is virtually identical to the random walk. Both (18) and (20) exaggerate the displacement, as is inherent in the formulae. (20) in particular, becomes unstable.

In conclusion, for a small positive $\frac{\partial \bar{w}_\varepsilon}{\partial z}$, either (19) or the basic analytical formula (12) is adequate. Curves (12) and (16) are shown in Fig. 6.

7. Discussion

The presence of a conical hill has a profound influence on the mean air flow and turbulence and accordingly upon plume evolution (MWJ). Briefly, the main speed-up occurs close to the surface, and in this zone the turbulence adjusts rapidly to the increased shear (Jackson and Hunt, 1975). At higher levels the turbulence time scales are comparable to the time of transit over the hill, and the eddies respond to streamline convergence in approximate accordance with rapid distortion theory (Britter et al, 1981). In this region, the theory suggests, for a conical hill σ_w is increased, σ_u decreased. Recent work described in MK85 and Mason 1985 suggests that a transitional zone exists, which lies between about 3 m and 20 m above the surface at Blashaval. Little is known about the effects of the mean flow distortion and changes in the character of the turbulence upon the magnitudes of the concentration flux $\overline{w'c'}$.

Returning to particle motions in slope conditions, it is not difficult to obtain an expression representing the influence of the gradient of mean vertical wind velocity in which a plume disperses. The vertical component A_w of the acceleration of a particle with respect to the ambient mean flow (a non-inertial frame) may be expressed as the difference between the mean vertical acceleration of the particle with respect to Eulerian space and the incremental change in vertical velocity of the non-inertial frame along the path of the particle. If the particle is displaced from a level z with a perturbation velocity w' (perpendicular to the surface) then, in the simple case of the particle maintaining a uniform Eulerian-space vertical velocity, and allowing the time increment to tend to zero,

$$A_w = -\frac{\partial \bar{w}_\varepsilon}{\partial t} = -\frac{\partial \bar{w}_\varepsilon}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$= -\frac{\partial \bar{w}_\varepsilon}{\partial z}\left[\bar{w}_\varepsilon(z) + w'\right] \qquad (22)$$

Elementary kinematic formulae such as (22) and the corresponding version for horizontal accelerations yield the correct sign for the changes in σ_u and σ_w when streamlines are compressed over a hill summit, that is, in accordance with rapid distortion indications. This reflects the importance of the production terms such as $-\overline{w'^2}\frac{\partial \bar{w}_\varepsilon}{\partial z}$ in second order closure and pressure-strain modelling - Deardorff (1973) contains a discussion.

The random walk (9) can be re-arranged

$$\frac{w_{i+1} - w_i}{\Delta t} = \underset{A}{-\frac{w'_i}{T_L}} + \underset{B}{\sigma_w \mu_{i+1}\sqrt{\frac{2}{\Delta t\, T_L}}} + \underset{C}{w'_i\frac{\partial \bar{w}_\varepsilon}{\partial z}} + \underset{D}{\bar{w}_\varepsilon(z_i)\frac{\partial \bar{w}_\varepsilon}{\partial z}} \qquad (23)$$

Terms A and B correspond to the right hand side of the standard random walk (8) while C and D are identical to (22). The sign is changed: (22) reflects the acceleration of an unimpeded particle with respect to the Eulerian-time reference frame, (23) the influence of the background acceleration on the particle - the random walk is embedded in the profile of mean vertical velocity.

If, for $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$, (17) is regarded as giving a reasonable approximation to the near-source mean displacement (where the plume is narrow and the turbulence/mean vertical velocity interaction less complicated) then the positive accelerations experienced by descending particles by virtue of the last two terms of (23) may account for the slightly slower initial descent of the random walk for significant $\left|\frac{\partial \bar{w}_\varepsilon}{\partial z}\right|$ (Figs 2, 4, where solution (16) is close to (17) initially). In the simulation, as time passes, many particles collect near the solid surface, where $\bar{w}_\varepsilon$ is small, while some attain considerable height, where $\bar{w}_\varepsilon$ may be large. In the former case term D is very small and a sign inspection shows that both ascending and descending motions are retarded by $w_i' \frac{\partial \bar{w}_\varepsilon}{\partial z}$ in (23). The descending particles are in any event constrained by the surface, so that the important influence is upon the ascending particles - the plume is depressed. For the elevated particles, when terms C and D are both significant the positive acceleration will have a stronger effect although the number affected is relatively small (unless the vertical motion, and hence acceleration, is small). The net result, it is suggested, is that the random walk simulations, after a somewhat slow initial descent, are eventually depressed below the level indicated by (17).

In reality there is a turbulence/mean flow interaction provided the dynamically-induced distortion is significant, which may have consequences for plume development. A 3-dimensional equilibrium must be attained between the horizontal divergence due to orographic deflection of the mean wind, and the turbulent motions, leading to a site-dependent profile of mean vertical velocity. For descending mean motion $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ increases downwards as far as the top of the equilibrium zone. See Fig 7 for instance, which shows a profile of $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ computed using a 3-dimensional Jackson and Hunt (1975) type analytical model of the airflow over Blashaval.

It will be noted that the source release height at Blashaval (8m) and the level to which the plume centroid sinks lie in the region where a transition from equilibrium to rapid distortion is taking place. The effect of a vertical velocity profile of Fig. 7 type is thus to force the particles down towards the zone of strong surface stress a little more strongly than would be expected from a constant gradient. With weaker profiles many particles will attain higher levels, perhaps to be spread more effectively by the high σ_w associated with rapid distortion conditions - at Blashaval the recorded at 14 m was well in excess of the values measured at lower heights (MK85) - always assuming that a high σ_w is associated with an increased $\overline{w'c'}$. These effects may explain the results of the small $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ dispersion experiments, 2 and 3, at Blashaval (MWJ) where a rapid vertical spread was combined with maximum concentrations close to the surface. In the random walk simulations σ_w is estimated from prescribed values of u_* and accordingly will only roughly approximate conditions over a hill slope and summit where the σ_w profile is complex. In addition, prescribed $\frac{\partial \bar{w}_\varepsilon}{\partial z}$ can only approximate the equilibrium Eulerian-space profile of a real situation.

9. Summary and Conclusions

This paper approaches the problem of the short-range dispersion of a pollutant released from an elevated source in the atmospheric surface layer over complex terrain by studying the effect of a non-zero mean vertical velocity (vertical in the sense perpendicular to the local terrain) imposed in the simple case of mean flow distortion around a roughly conical hill of moderate slope. Analytical formulae for mean plume displacement (see Sections 3, 4 and the Appendix) rapidly increase in complexity in these circumstances, and are applicable only in steady, uniform conditions. Although the region of applicability may be increased by introducing time-stepping methods of solution (section 5) the most hopeful prospects, encompassing the whole concentration field, must lie in random walk techniques.

In conclusion,

(i) Of the various mean displacement formulae derived from the basic random walk equation the analytical solutions (11) and (12) following the method of Raupach are applicable only to about $t = 2T_s$ as a result of the simplifying assumptions. The justifiable empirical adjustment (14) provides a very good fit to the random walk simulation over the whole range to $t = 10T_s$, a weighting $b = 0.5$ serving for all the situations tested. This is perhaps the most useful of the formulae, and although at first sight rather complicated, it is easily programmable.

(ii) Integrating the elementary super-position (15) yields reasonable comparisons with the random walk out to $t = 6T_s$ for $r_2 \equiv 1$ (i.e. formula (17)), and further improvements can be obtained by varying r_2. No single value of r_2 was appropriate for all the experiments, however, rendering the refinement difficult to apply a priori. This may be connected with the poor representation of the physics in the formula. The simplified formulae of section 5 suffer from the same disadvantage, if weighting functions are used, although (20) as it stands gave acceptable results to about $t = 5T_s$, at least, before tending to become unstable.

(iii) Simple time-stepping techniques allowed the integration of basic formulae such as (21), and are potentially of greater flexibility. Very close fits to the random walks out to $t = 10T_s$ were obtained from the numerical integration of (21), but again only through the introduction of a time-dependent coefficient $b(t)$, whose asymptotic magnitude at large times depends upon the ratio of the mean and turbulent components of vertical puff displacement. This hyperbolic form contained two free parameters, for which a single choice of values sufficed for all the cases investigated. These results are interpreted as offering some prospect of progress in modelling puff displacements in realistic (non-uniform) flows.

(iv) Changes in positive $\frac{\partial \bar{w}_E}{\partial z}$ may be small, in reality, for a plume ascending from an elevated source, so that the presence of z_s away from the near field in (11) and (19) is acceptable. For the one case with (weak) ascending mean motion which was modelled both formulae gave good results, (19) in fact being almost identical to the random walk mean displacement.

(v) The random walk described is embedded in a prescribed gradient of mean vertical motion which it is difficult to estimate from field observations. Numerical models may (in principle) provide reasonable approximations for situations which can be simulated with sufficient accuracy. The parameter u_* must also be prescribed. The disadvantages of the simple formulation are not felt to be too serious, but for estimating mean displacements the random walk appears to offer few advantages over the basic offset (17) close to the source. In section 8 the respective behaviour of the two curves is discussed.

Acknowledgement is due to Mr D J Thomson who provided and adapted his random walk model of dispersion and made some helpful suggestions, to Miss M L Macari for graphical assistance, and to other members of the Boundary Layer Branch for discussion.

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Appendix

Horizontal displacement in the surface stress layer

Using Batchelor's expression for displacement in the x direction

$$\frac{d\bar{x}}{dt} = \bar{u}(c Z_d) \tag{B1}$$

where c is a constant subsequently assumed to be Euler's constant (Chatwin 1968) and Z_d is mean displacement in the vertical, Pasquill and Smith (1983) relate $\bar{x}$ and Z_d for a surface release by assuming a logarithmic profile for $\bar{u}$, substituting in (2) and integrating. They obtain

$$\bar{x} = \frac{Z_d}{k^2}\left[\log_e \frac{cZ_d}{Z_o} - 1 + \frac{Z_o}{Z_d}(1 - \log_e c)\right] \tag{B2}$$

For an elevated release the horizontal translation may plausibly be estimated

$$\frac{d\bar{x}}{dt} = \bar{u}(Z_s + cZ_d) \tag{B3}$$

to give a solution for $t \gg T_s$

$$\bar{x} = \frac{1}{k^2}\left[\frac{Z_s + cZ_d}{c}\left(\log_e \frac{Z_s + cZ_d}{Z_o} - 1\right) - \frac{Z_s}{c}\left(\log_e \frac{Z_s}{Z_o} - 1\right)\right] \tag{B4}$$

It would be useful to incorporate a profile of mean vertical velocity but unfortunately the adoption of even a linear profile leads to integrals which have no simple analytical solution. The simplifying assumption that $\bar{w}_\varepsilon$ is constant yields quite straightforward formulae, but is inappropriate for most real situations. One case where it might apply is where $\bar{w}_\varepsilon > -ku_*$ as, for an ascending plume, particularly from an elevated release, it may be assumed that $\bar{w}_\varepsilon$ is approximately constant without too great an error. Substitution using $\frac{d\bar{z}}{dt} = ku_* + \bar{w}_\varepsilon$, $\bar{w}_\varepsilon$ constant leads to

$$\bar{x} = \frac{Z_d}{k^2}\left[1 - \frac{\bar{w}_\varepsilon}{ku_* + \bar{w}_\varepsilon}\right]\left[\log_e \frac{cZ_d}{Z_o} - 1 + \frac{Z_o}{Z_d}(1 - \log_e c)\right] \qquad \text{(B5)}$$

and

$$\bar{x} = \frac{1}{k^2}\left[1 - \frac{\bar{w}_\varepsilon}{ku_* + \bar{w}_\varepsilon}\right]\left[\frac{Z_s + cZ_d}{c}\left(\log_e \frac{Z_s + cZ_d}{Z_o} - 1\right) - \frac{Z_s}{c}\left(\log_e \frac{Z_s}{Z_o} - 1\right)\right] \qquad \text{(B6)}$$

corresponding to (B2) and (B4) respectively. the term $\bar{w}_\varepsilon(ku_* + \bar{w}_\varepsilon)^{-1}$ is a hyperbola asymptotic to the lines 1 and $\bar{w}_\varepsilon = ku_*$, reflecting the approach of $\bar{x}$ to zero for large $\bar{w}_\varepsilon$ and to infinity should turbulent and negative mean velocity components near balance.

An obvious shortcoming of formulae (B4) and (B6) is that they assume a plume displacement of the similarity form $\bar{w} = ku_*$ or $\bar{w} = ku_* + \bar{w}_\varepsilon$ for elevated releases. Raupach, by comparing the far-field solution of (6) with (3) concluded that a hypothetical plume released from ground level at time $-t'$ would have a trajectory passing under the real elevated source at height $.8Z_s$ (see Fig. 1). This height remains unchanged if ku_* is replaced with $ku_* + \bar{w}_\varepsilon$ so that the simple expedient of substituting $.8Z_s$ for Z_s in both (B4) and (B6) may give reasonable solutions away from the near vicinity of the source. For large $\bar{w}_\varepsilon$, of course, the domain over which (B4), (B6) are applicable is narrow.

List of Illustrations

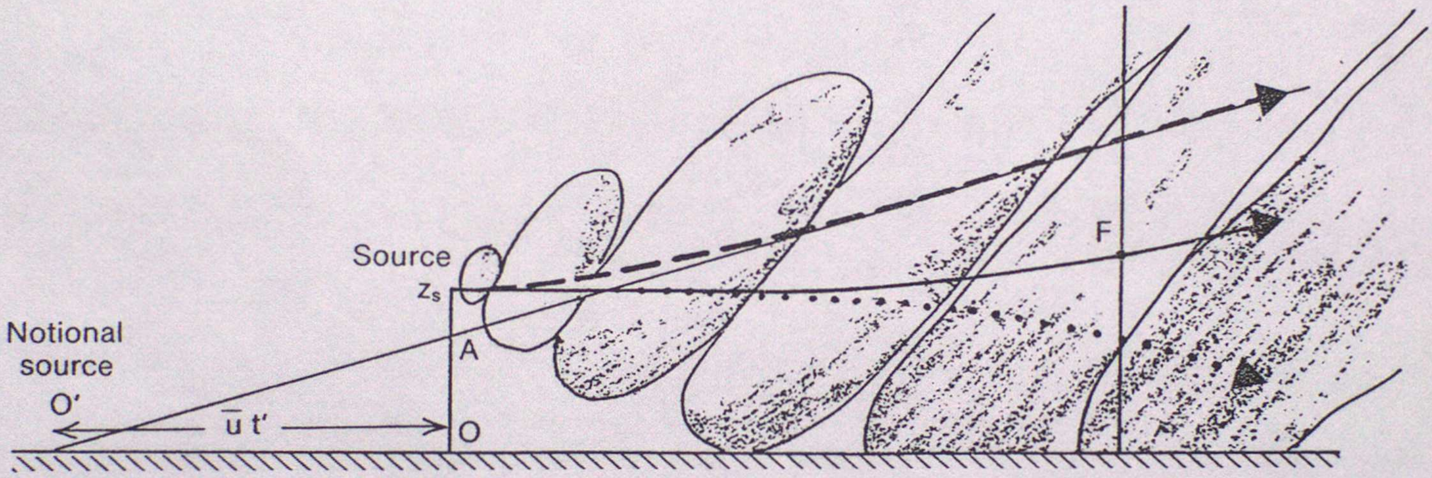
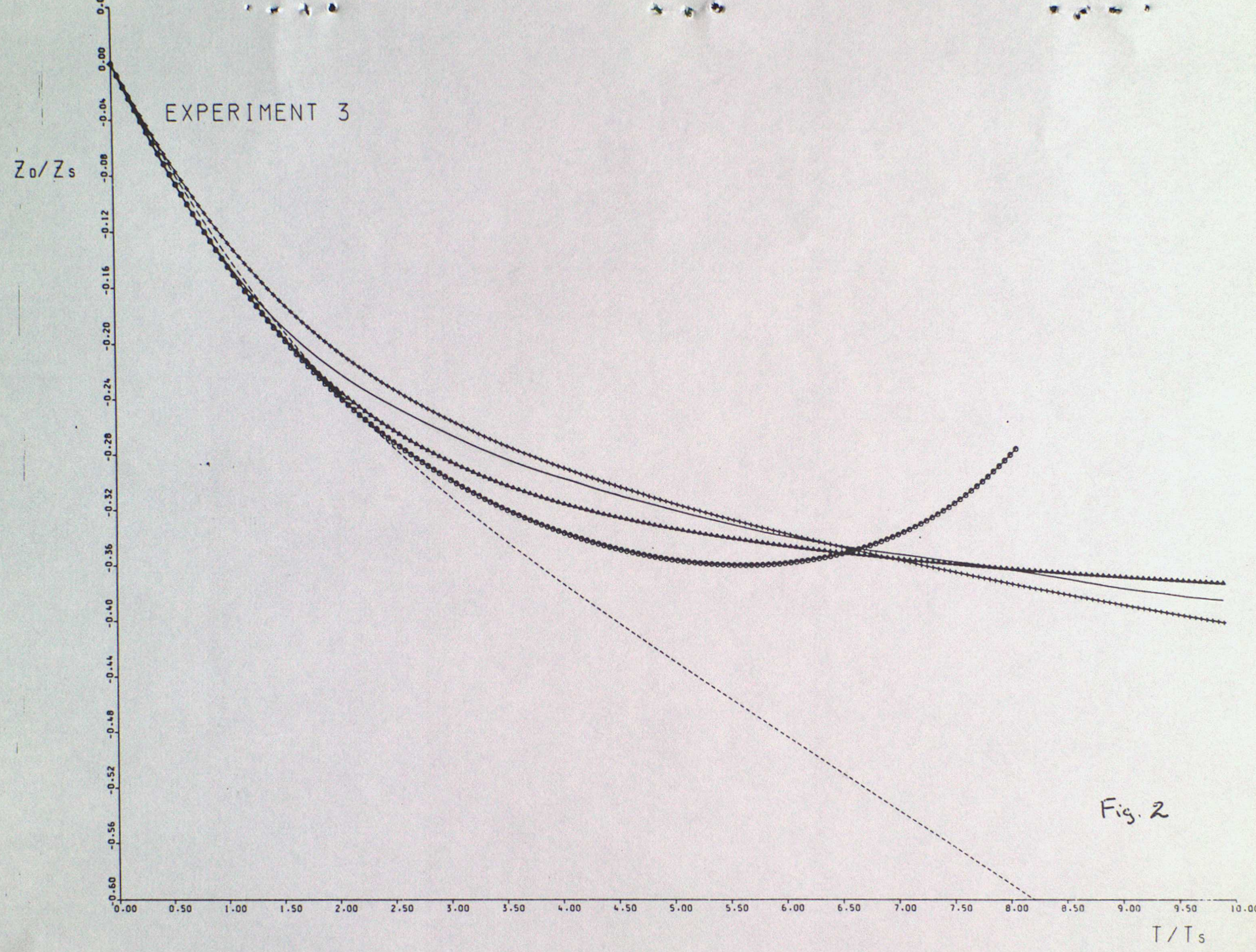


Figure 1



EXPERIMENT 3
Zo/Zs
T/Ts
0.0
0.00
-0.04
-0.08
-0.12
-0.16
-0.20
-0.24
-0.28
-0.32
-0.36
-0.40
-0.44
-0.48
-0.52
-0.56
-0.60
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
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9.00
9.50
10.00

Fig. 2

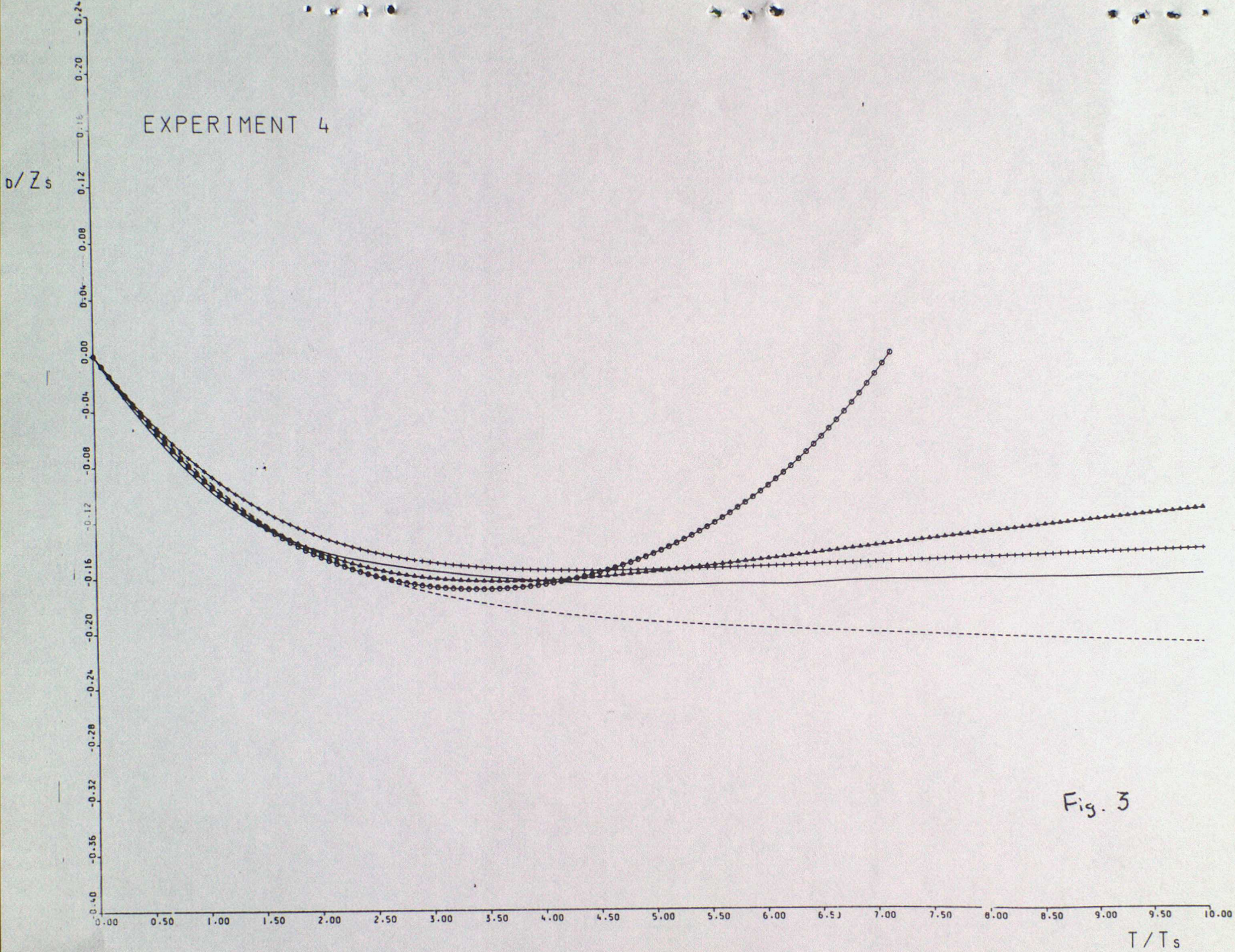
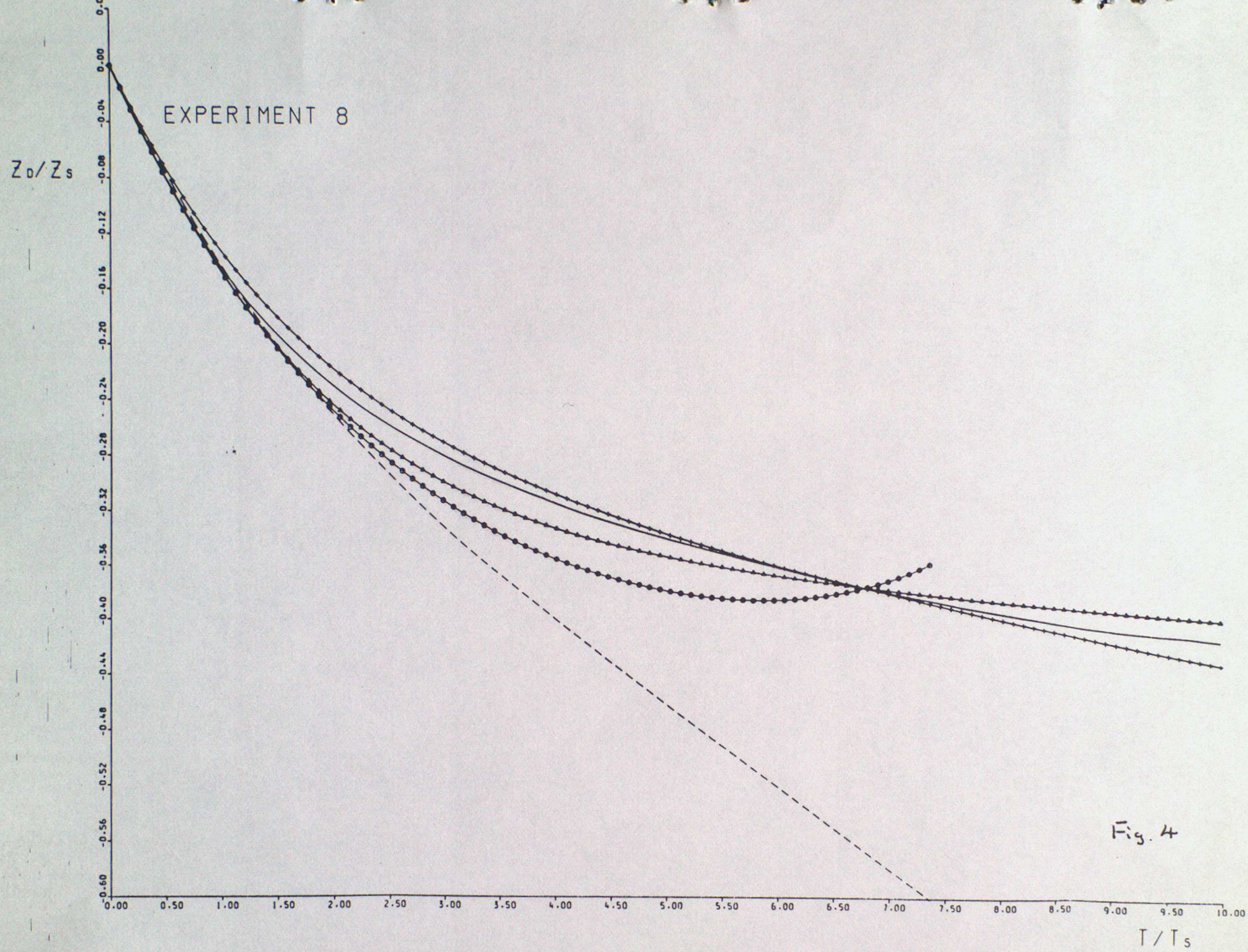

EXPERIMENT 4
D/Zs
T/Ts
0.24
0.20
0.16
0.12
0.08
0.04
0.00
-0.04
-0.08
-0.12
-0.16
-0.20
-0.24
-0.28
-0.32
-0.36
-0.40
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.5)
7.00
7.50
8.00
8.50
9.00
9.50
10.00

Fig. 3



EXPERIMENT 8
ZD/ZS
T/TS
0.04
0.00
-0.04
-0.08
-0.12
-0.16
-0.20
-0.24
-0.28
-0.32
-0.36
-0.40
-0.44
-0.48
-0.52
-0.56
-0.60
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
7.50
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8.50
9.00
9.50
10.00

Fig. 4

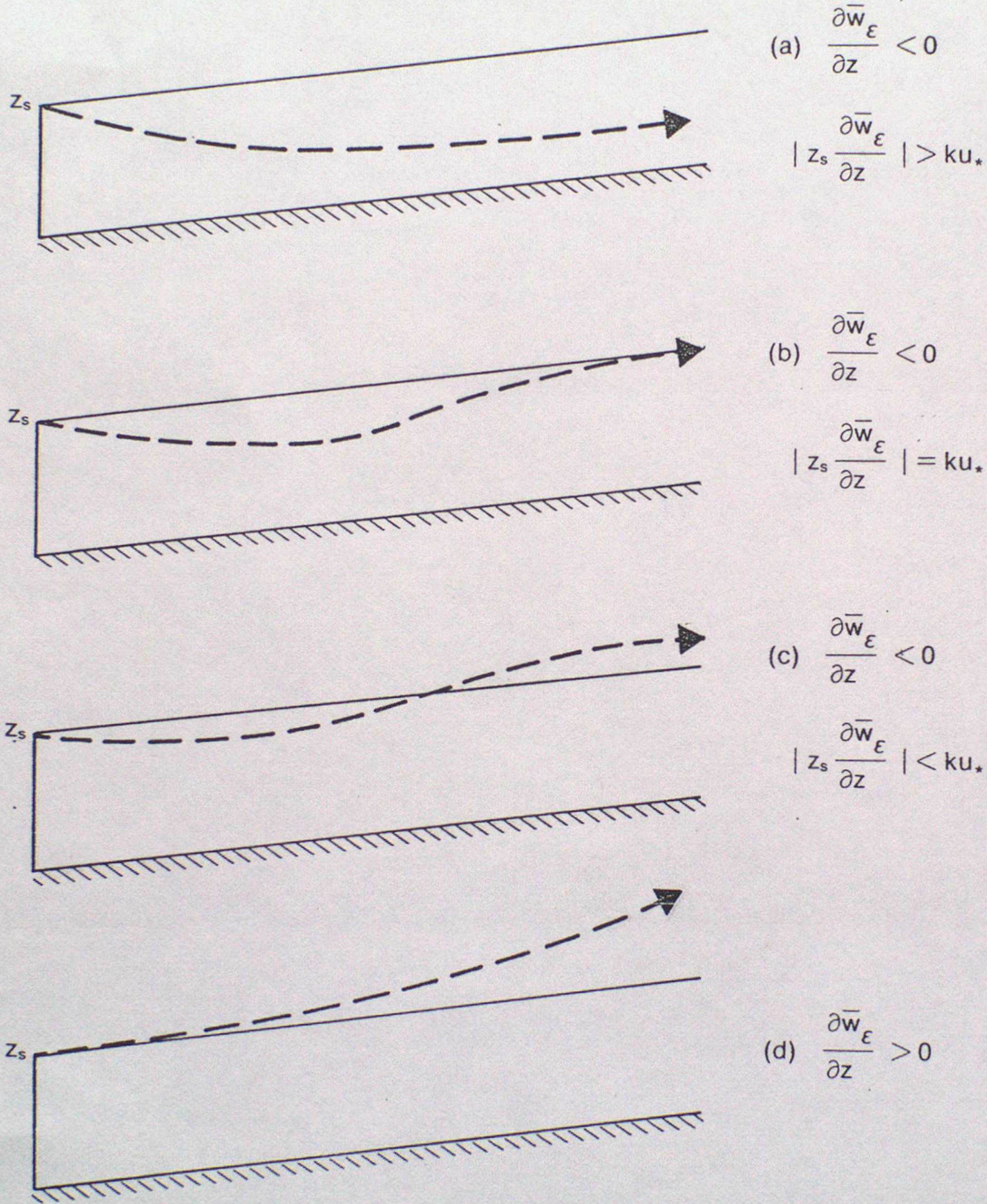
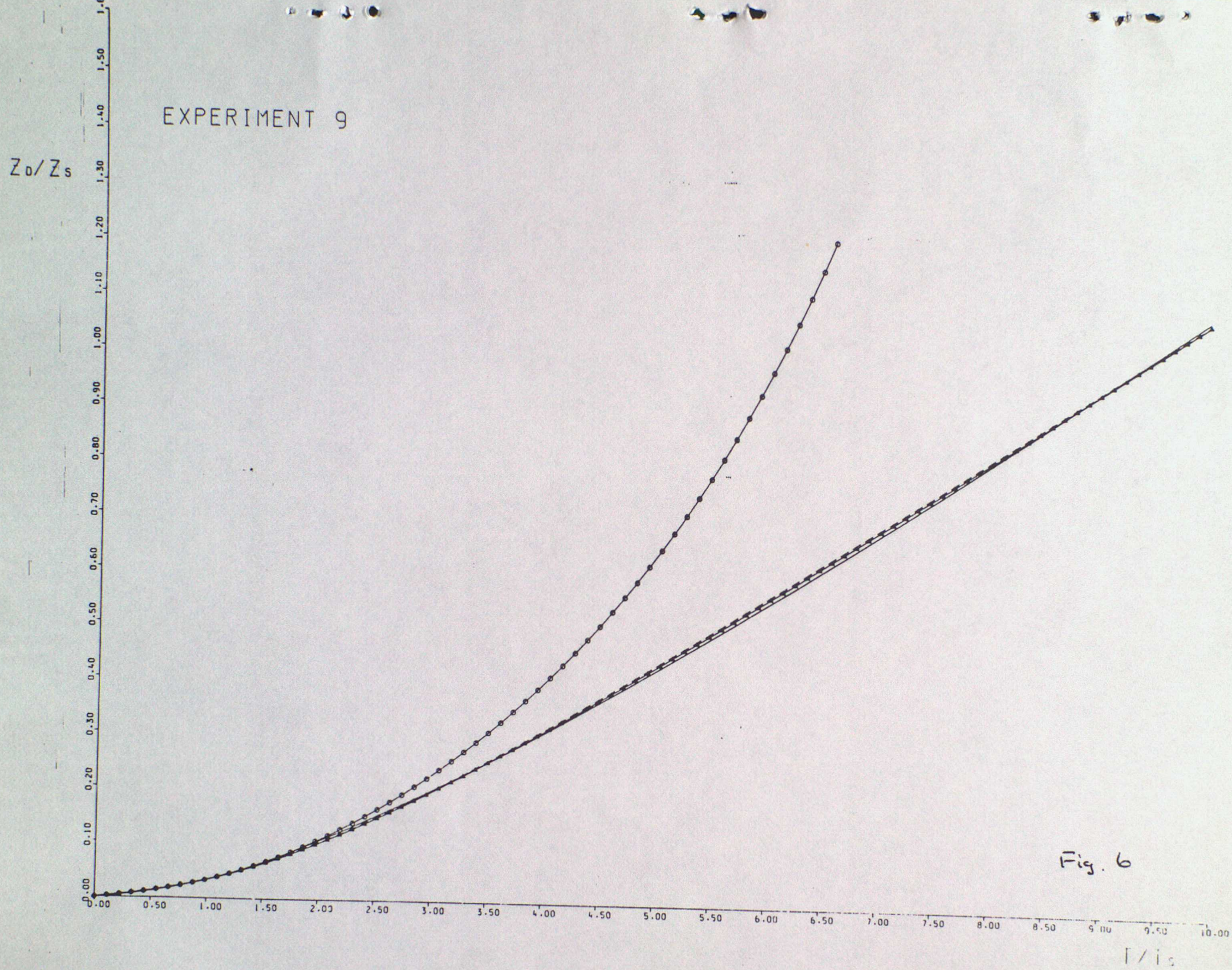

z_s
(a) $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$
$\left| z_s \frac{\partial \bar{w}_\varepsilon}{\partial z} \right| > ku_*$
z_s
(b) $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$
$\left| z_s \frac{\partial \bar{w}_\varepsilon}{\partial z} \right| = ku_*$
z_s
(c) $\frac{\partial \bar{w}_\varepsilon}{\partial z} < 0$
$\left| z_s \frac{\partial \bar{w}_\varepsilon}{\partial z} \right| < ku_*$
z_s
(d) $\frac{\partial \bar{w}_\varepsilon}{\partial z} > 0$

Figure 5


EXPERIMENT 9
Zo/Zs
I/is
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
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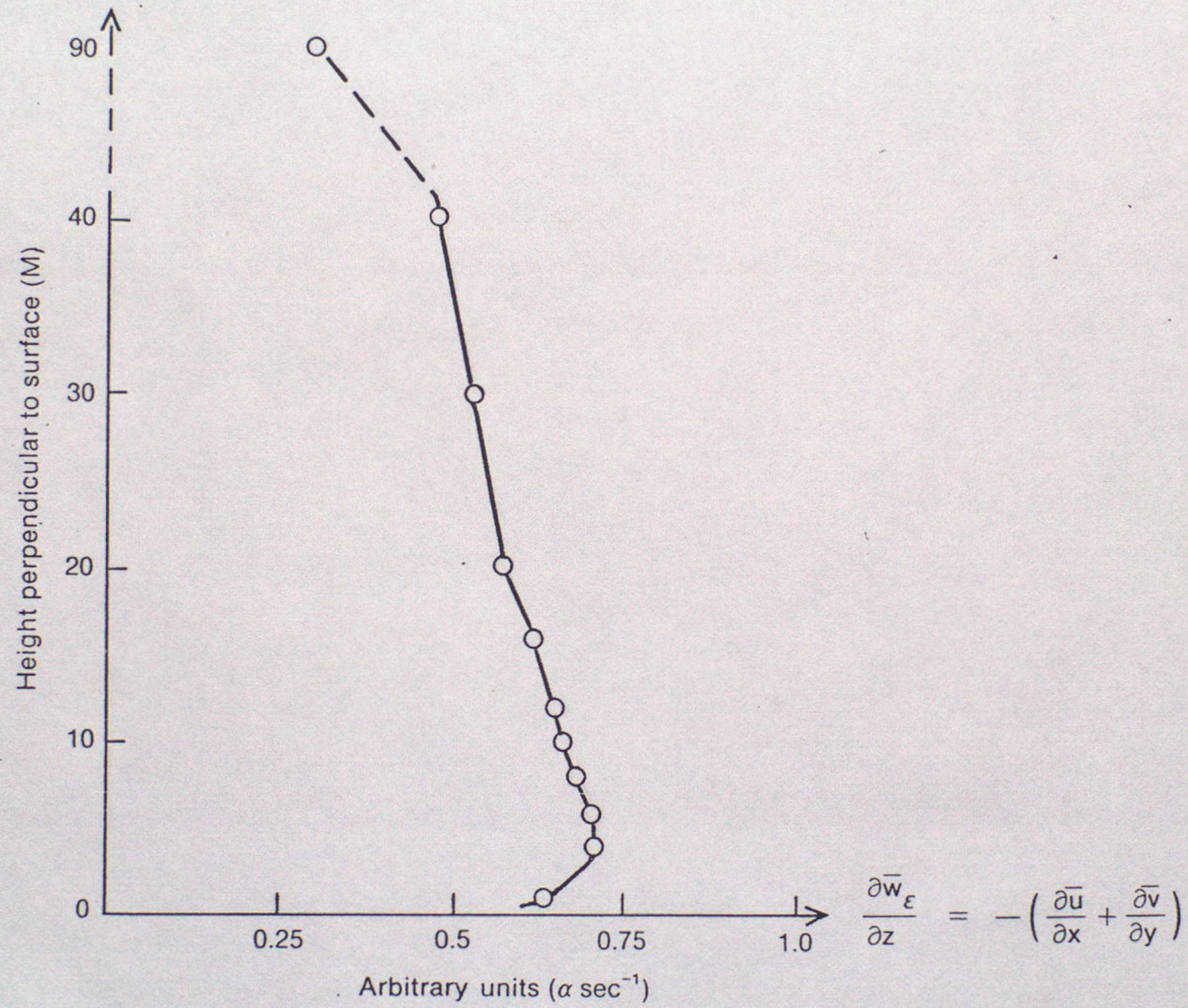
8.50
9.00
9.50
10.00

Fig. 6



90
40
30
20
10
0
Height perpendicular to surface (M)
0.25
0.5
0.75
1.0
Arbitrary units (α sec⁻¹)
$\frac{\partial \bar{w}_{\varepsilon}}{\partial z} = -\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y}\right)$

Figure 7