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by

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SUMMARY

The angular momentum of the atmosphere is the sum of the wind term, $\underline{W}$, due to the winds relative to the Earth's surface, and the matter term $\underline{M}$. We show that if the Earth were an oblate spheroid with an equipotential surface, the atmospheric torque on the Earth would be $-\underline{\Omega} \wedge \underline{M}$, $\underline{\Omega}$ being the rotation rate of the solid Earth. As a result, the equatorial components of $\underline{M}$ for a wave propagating at azimuthal angular velocity σ without changing shape are $(\Omega-\sigma)/\Omega$ times the equatorial components of $\underline{W}$. Laplace's equation for tidal motions "on a sphere" strictly applies to motions on such an oblate spheroid and its solutions apply a torque on the Earth equal to $-\underline{\Omega} \wedge \underline{M}$. We show that the resulting relationship between $\underline{M}$ and $\underline{W}$ also implies that in any separable wave solution of the tidal equations the surface wind is simpled related to the vertical integral of the wind and the equivalent depth.

Analyses of the equatorial components of the matter term by ECMWF and UKMO weather forecast systems are dominated by chaotic oscillations with periods between 8 and 10 days which are well forecast out to 5 days ahead. We argue that these are essentially free solutions of the tidal equations which exert considerable torques on the Earth. The main feature of the equatorial components of the wind term is a seasonally modulated diurnal oscillation. Analyses and 2 day forecasts of this phenomenon are in less good agreement. We argue that it is thermally forced and depends on the compressibility of the atmosphere.

1. INTRODUCTION

On a planet which is rotating with (nearly) constant angular velocity, $\underline{\Omega}$, the angular momentum of the atmosphere

$$\underline{A} = \int_{atmos} \rho \underline{r} \wedge (\underline{u} + \underline{\Omega} \wedge \underline{r}) \, d\tau \ , \qquad (1)$$

where $\underline{r}$ is the position vector from the planet's centre of mass of the volume elements $d\tau$ which have densities ρ and velocities $\underline{u}$ relative to the planet's surface. The contribution to $\underline{A}$ from the winds relative to the underlying planet is usually called the wind term, $\underline{W}$, and the contribution from the rotation of the atmosphere with the angular velocity of the planet the matter term, $\underline{M}$:

$$\underline{W} = \int_{atmos} \rho \underline{r} \wedge \underline{u} \, d\tau \qquad \underline{M} = \int_{atmos} \rho \underline{r} \wedge (\underline{\Omega} \wedge \underline{r}) \, d\tau \qquad (2)$$

$\underline{A}$, $\underline{W}$ and $\underline{M}$ are often expressed as components (e.g. $\underline{A} = (A_1 , A_2, A_3)$) relative to a body fixed frame with the 3rd component aligned with $\underline{\Omega}$ and hence referred to as the axial component. On the Earth the two other components lie in the equatorial plane, the first "equatorial component" along the Greenwich meridian and the second along 90^o E.

The total torque on the atmosphere at the Earth's surface produces fluctuations in the atmosphere's angular momentum (AAM) and equal and opposite changes in the angular momentum of the underlying planet. Fluctuations in A_3, together with tidally induced changes in the Earth's moment of inertia, have been shown (see Barnes et al. 1983) to account for most of the variations in the rotation rate of the solid Earth and hence of the length of the day over timescales of between a few days and a few years. Most of the fluctuations in A_3 occur in W_3 despite the fact that M_3 (which is nearly constant) is about 60 times larger than W_3.

Evidence for the hypothesis that the equatorial components of AAM play a similarly dominant role in exciting the wobble of the Earth's pole of rotation about the principal axis of its moment of inertia for similar timescales is, as yet, much less clear-cut.

Brzezinski (1987) has shown that M_1 and M_2 exhibit strong oscillations with periods of between 8 and 12 days and chaotically varying amplitude. Figure 1 presents a timeseries of M_2 for 1990. The fluctuations in figure 1 are entirely typical of those in both M_1 and M_2, their amplitude varies little with season and they are forecast quite accurately out to 5 days by both the UKMO and ECMWF forecast systems (Bell et al. 1991). Similar fluctuations are not apparent in W_1 and W_2 - see for example figure 2. Indeed the data appear to be very noisy and for some time there seemed to be no discernible agreement between evaluations of W_1 and W_2 based on analyses from several major forcast systems. Eventually T. M. Eubanks inferred that W_2 has a diurnal variation with a seasonal modulation. This is clearly apparent in figure 3 which presents evaluations of W_2 at 00Z (full line) and 12Z (dashed line) from (a) ECMWF data for 1988 and (b) UKMO data for 1990. The values in this and later figures have been smoothed by the application of an 11 day running mean filter. Similar evaluations of W_1 are presented in figures 4. A small diurnal variation is apparent in the UKMO data (figure 4(b)) and an even smaller one in the ECMWF data (figure 4(a)).

The fluctuations in $\underline{M}$ and $\underline{W}$ just described are so rapid that, unless they are aliased into longer periods, they do not greatly excite the wobble of the pole (whose resonant period is of the order of 430 days). The understanding of their nature is nevertheless the subject of this paper.

Eubanks et al. (1988) suggested that the fluctuations in M_1 and M_2 could be interpreted as free travelling wave solutions of Laplace's tidal equations, since the period of the wave with largest projection on these components of AAM is approximately 8 to 10 days. Whilst this seemed plausible (and we will argue that their interpretation is correct) we were unclear how free (unforced) solutions of Laplace's equations on a sphere could either provide the torque to the underlying Earth which must accompany the observed fluctuation or maintain themselves whilst doing so.

The seasonal modulation of the diurnal variation in W_2 strongly suggests that it is thermally forced. Since the torques accompanying an oscillation of the observed amplitude in W_2 alone would be large (section 6) it seemed likely that the diurnal fluctuation consists of a travelling wave with diurnal period which is stationary in inertial space. Originally our examination of solutions of the tidal equations was motivated by desires to support this interpretation and to explain the sharp difference between the periods of fluctuations dominating the wind and matter terms. Figure 5, however, practically settles the first of these two issues, presenting evaluations of (a) W_1 and (b) W_2 at 06Z and 18Z from ECMWF data for 1988 which show that W_1 has a seasonally modulated diurnal oscillation. Figures 3(a), 4(a) and 5 together show that the diurnal oscillation moves westward and is stationary in inertial space.

The evaluations of W_1 at 00Z and 12Z from UKMO data for 1988 presented in figures 14 and 15 of Bell et al. (1991) (note that the scaling on the ordinate of these figures should be marked as 10^{-7} not 10^{-6}) show a diurnal oscillation in W_1 between 00Z and 12Z comparable with that in W_2. Of the W_1 data calculated from UKMO, ECMWF, NMC and JMA analyses between 1979 and 1988, only UKMO data had such a large seasonal cycle in the difference between 00Z and 12Z. The UKMO data for 1989 (not shown) and 1990 (figure 4b) display a markedly smaller oscillation. The only major change to the UKMO system during this period was the introduction of a new analysis system on 30 November 1988 (Lorenc et al. 1991). Whilst this improvement in agreement between centres is encouraging it is by no means certain that the analyses are now accurate. The amplitude of the seasonal cycle in the change in W_2 in ECMWF 60 hour forecasts started at 12Z in 1988 was almost twice that of the change between 00Z and 12Z analyses (see Bell et al. (1991) figure 12). Understanding of the nature of the diurnal oscillation should facilitate the assessment of the accuracy of the analyses and forecasts.

The Earth's bulge, being about 20 km high, is the most significant topography on the planet. The mountain torque on the atmosphere due to the bulge is calculated in section 2 and shown to equal $-\underline{\Omega} \wedge \underline{M}$. This result leads to a simple expression (20) for the ratio of the equatorial matter and wind terms for a travelling wave disturbance of a given period, which explains in large part the sharp difference between the significant periods in the equatorial wind and matter terms. Section 3 shows that, as first established by Lamb (1932), Laplace's tidal equations on a sphere strictly apply to motions on a spheroid of constant geopotential (e.g. the Earth with its bulge). Their solutions thus contain implicit mountain torques equal to $\underline{\Omega} \wedge \underline{M}$. Section 4 summarizes results on free and forced linear solutions of the tidal equations in preparation for sections 5 and 6. Section 5 discusses the interpretation of the matter term oscillations, the magnitude of the associated torques and the accuracy of the approximations implicit in the linear solutions. Lindzen's (1965) solution for diurnal oscillations is presented in section 6 and the wind term oscillation associated with it calculated and compared with figures 3 to 5. Conclusions are drawn in section 7.

2 . THE ATMOSPHERE'S ANGULAR MOMENTUM AND THE EARTH'S BULGE

Expressions for the equatorial components of $\underline{M}$ and $\underline{W}$ in the body fixed Cartesian co-ordinate system introduced in section 1 are easily found from (2). In spherical polar co-ordinates with r denoting radial distance, ϕ latitude, λ longitude (from the Greenwich meridian) and u the eastward and v the northward components of velocity, the wind term,

$$\underline{W} = \int_{atmos} (\rho ru\underline{\hat{\phi}} - \rho rv\underline{\hat{\lambda}})\, d\tau \ , \tag{3}$$

and the matter term,

$$\underline{M} = \int_{atmos} \rho\Omega r^2 \cos\phi\ \underline{\hat{\phi}}\ d\tau \ . \tag{4}$$

Using the following expression for the horizontal coordinate of any vector $\underline{V}$ with $V_r = 0$,

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} -\cos\lambda\ \sin\phi & -\sin\lambda \\ -\sin\lambda\ \sin\phi & \cos\lambda \end{pmatrix} \begin{pmatrix} V_\phi \\ V_\lambda \end{pmatrix} \tag{5}$$

and the hydrostatic relation,

$$\partial p\ /\ \partial r = -\rho g \ , \tag{6}$$

in (3) one finds that W_1 and W_2 are given by

$$(W_1, W_2) = -a^3\ /\ g \int_0^{p_s} \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \{u \sin\phi(\cos\lambda, \sin\lambda) + v(-\sin\lambda, \cos\lambda)\} \cos\phi\ d\phi\ d\lambda\ dp \ . \tag{7}$$

Using (4), (5) and (6) M_1 and M_2 can similarly be shown to be given by

$$(M_1, M_2) = -\Omega a^4\ /\ g \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} p_s \cos\phi \sin\phi(\cos\lambda, \sin\lambda) \cos\phi\ d\phi\ d\lambda \ . \tag{8}$$

The total torque on the atmosphere due to pressure gradient forces,

$$\underline{\Gamma} = -\int_{surf} \underline{r} \wedge p\underline{n}\ dS = -\int_{atmos} \underline{r} \wedge \underline{\nabla} p\ d\tau \ , \tag{9}$$

where $\underline{n}$ is the unit normal pointing into the Earth. Using (5) once more, the equatorial components of the surface pressure torque may be shown to be expressible as

$$-\left((\underline{r}\wedge\underline{\nabla}p)_1,(\underline{r}\wedge\underline{\nabla}p)_2\right) = \partial p/\partial\phi(-\sin\lambda,\cos\lambda) + \partial p/\partial\lambda\tan\phi(\cos\lambda,\sin\lambda) \ . \tag{10}$$

Assuming that the Earth is of uniform density and that its oblate surface coincides with a surface of constant geopotential, its radius $R(\phi)$ is given by

$$-GM/R - \tfrac{1}{2}\Omega^2R^2\cos^2\phi \simeq -GM/a - \Omega^2a^2/4$$

where a is the radius of the Earth at 45°N. Hence

$$R(\phi) - a \simeq \tfrac{1}{2}\frac{\Omega^2a^4}{GM}(\cos^2\phi - \tfrac{1}{2}) \ . \tag{11}$$

The pressure torque on the Earth's bulge may be calculated by transforming the expression for the pressure torque from spherical coordinates (r, λ, ϕ) to spheroidal co-ordinates (r', λ', ϕ') following the Earth's surface;

$$r = r'[1 + \frac{\Omega^2a}{2g}(\cos^2\phi' - \tfrac{1}{2})] \ ; \ \phi = \phi' \ ; \ \lambda = \lambda' \ . \tag{12}$$

In these co-ordinates the latitudinal pressure gradient

$$\begin{aligned}\partial p/\partial\phi &= \partial r'/\partial\phi\,\partial p/\partial r' + \partial p/\partial\phi' \\ &= \frac{\Omega^2r^2}{g}\cos\phi\sin\phi\,\partial p/\partial r' + \partial p/\partial\phi' \ .\end{aligned} \tag{13}$$

The volume integrals of the horizontal pressure gradient terms cancel after integration by parts (the Earth's surface being parallel to surfaces of constant r') leaving

$$(\Gamma_1, \Gamma_2) = \Omega^2a^4/g\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} p_s\cos\phi\sin\phi(\sin\lambda,-\cos\lambda)\cos\phi\,d\phi\,d\lambda \ . \tag{14}$$

This expression may alternatively be derived (without introducing spheroidal co-ordinates) by calculating the integral (9) of (10) directly. Integration by parts of the first term on the r.h.s. of (10) with respect to ϕ yields boundary contributions which may be written as

$$(\Gamma_1, \Gamma_2) = -\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} p_s R^2\cos\phi\,dR/d\phi\,d\phi\,(\sin\lambda,-\cos\lambda)\,d\lambda \ . \tag{15}$$

(14) is easily re-derived from (15) by using (11) and the shallow atmosphere approximation.

This result, (14), also holds to a similarly good approximation (i.e. of the order of the ratio of the maximum to the minimum radii of the Earth) when the density of the Earth is variable and its gravity field Π non-radial, provided its surface follows the geopotential. This may be shown by again using co-ordinates which follow the Earth's surface

$$g\,r' = \Pi(r,\lambda,\phi) - \frac{\Omega^2 a^2}{2}(\cos^2\phi - ½) .$$

In these coordinates, taking $\partial p/\partial\phi$ as an example,

$$\partial p / \partial\phi = -\rho(\partial\Pi / \partial\phi + \Omega^2 a^2 \cos\phi \sin\phi) + \partial p / \partial\phi'$$

and the total force $-a^{-1}$ ($\partial p/\partial\phi + \partial\Pi/\partial\phi$) involves only the pressure gradient $\partial p/\partial\phi'$ and the centripetal acceleration term (c.f. (13)).

Comparison of (14) with (8) reveals that

$$\Gamma_1 = -\Omega M_2 \ , \ \Gamma_2 = \Omega M_1 \ . \tag{16}$$

So the pressure torque on the atmosphere due to the Earth's bulge is directly proportional to the matter term's contribution to $\underline{A}$ and indeed equals $\underline{\Omega} \wedge \underline{M}$.

In the absence of viscous stresses, the pressure torque equals the rate of change of $\underline{A}$ in the inertial frame;

$$\underline{\Gamma} = (d\underline{A} / dt)_I = (d\underline{A} / dt)_R + \underline{\Omega} \wedge \underline{A} \tag{17}$$

where R and I denote derivatives in the planet's rotating frame and any inertial frame respectively. The result of combining (16) with (17) is the following important equation of motion for the equatorial components of the angular momentum;

$$(dA_1 / dt)_R - \Omega W_2 = 0 \ , \ (dA_2 / dt)_R + \Omega W_1 = 0 \ . \tag{18}$$

Much of the theory of the motion of atmospheric tides is concerned with propagating wave solutions of the form

$$\alpha = Re\{\hat{\alpha}(\phi, z) \exp i(s\lambda + \sigma t)\} \ . \tag{19}$$

in which α is any property of the wave (e.g. its temperature), $\hat{\alpha}(\phi,\ z)$ is a complex function of latitude ϕ and height z, σ is the angular velocity of the wave and s its longitudinal wave-number. The dependence of the relative amplitudes of the wind and matter terms of such wave solutions on their frequency σ follows immediately from (18).

$$(M_1, M_2) = \frac{(\Omega - \sigma)}{\sigma}(W_1, W_2) \ . \tag{20}$$

Hence diurnal oscillations, which are stationary in an inertial space and for which $\sigma = \Omega$, have null matter terms. Waves with periods of N days have $(M_1, M_2) = (N - 1)(W_1, W_2)$. So waves with periods of many days have much larger matter terms than wind terms. The period and structure of any wave with a non-zero wind or matter term is evidently intimately linked with the pressure torque on the equatorial bulge and (18). Since the standard equations for tidal motion are formulated for a spherical Earth, it might be thought that their representation of waves containing angular momentum would be incorrect. In the next section we show that the full tidal equations do satisfy (18) and hence can simulate angular momentum variations faithfully.

3. ANGULAR MOMENTUM AND THE TIDAL EQUATIONS

The standard equations for tidal motions on a sphere are often derived on the assumptions that the ellipticity of the Earth and the centripetal acceleration of fluid parcels are negligible. Here we derive the momentum equations for tidal motions by transforming the full equations of motion into spheroidal co-ordinates. The resulting equations may be viewed as being defined in spherical co-ordinates for a spherical Earth with negligible error. We also demonstrate that the angular momentum of motions governed by these equations satisfies (18).

The latitudinal and longitudinal components of the momentum equation for an inviscid fluid are

$$\rho(dv/dt + vw/r + u^2\tan\phi/r + 2\Omega u\sin\phi + \Omega^2 r\sin\phi\cos\phi) = -1/r\,\partial p/\partial\phi\ , \qquad (21)$$

$$\rho(du/dt + uw/r - uv\tan\phi/r + 2\Omega w\cos\phi - 2\Omega v\sin\phi) = -\frac{1}{r\cos\phi}\,\partial p/\partial\lambda\ . \qquad (22)$$

On transforming the latitudinal component of the momentum equation into spheroidal co-ordinates the centripetal acceleration term is directly balanced by the transformation term of (14). Hence

$$\rho(dv/dt + vw/r + u^2\tan\phi'/r + 2\Omega u\sin\phi') = -1/r\,\partial p/\partial\phi'\ , \qquad (23)$$

The longitudinal component is unaltered. (22) and (23) are exact in spheroidal coordinates. The geometric differences between the spherical and spheroidal surfaces are of order $\Omega^2 a/g$ and can safely be neglected for motions on the Earth. (That the Earth's bulge can nevertheless be dynamically significant is due to the fact that vertical pressure gradients are much larger than horizontal ones.) (22) & (23) may then be viewed as defined in spherical co-ordinates and appropriate for a spherical Earth. The familiar momentum equations for small amplitude tidal motions follow from them by linearising about a state of rest, neglecting vertical velocities and making the shallow atmosphere approximation (setting r=a);

$$\begin{aligned} &\partial u/\partial t - 2\Omega v\sin\phi = -1/(\rho_0 a\cos\phi)\,\partial p/\partial\lambda \\ &\partial v/\partial t + 2\Omega u\sin\phi = -1/(\rho_0 a)\,\partial p/\partial\phi\ . \end{aligned} \qquad (24)$$

That (22) & (23) for spherical co-ordinates on a spherical Earth together satisfy (18) can be shown directly as follows. The angular momentum of a fluid parcel (see (3) and (4)) is

$$\underline{a} = \rho d\tau\{(\Omega r^2\cos\phi + ru)\hat{\underline{\phi}} - rv\hat{\underline{\lambda}}\}\ . \qquad (25)$$

Using (17) and the identities

$$\partial\hat{\underline{\phi}} / \partial\phi = -\hat{\underline{r}}\ ,\ \ \partial\hat{\underline{\lambda}} / \partial\phi = 0\ ,\ \ \partial\hat{\underline{\phi}} / \partial\lambda = -\sin\phi\,\hat{\underline{\lambda}}\ ,$$

$$\partial\hat{\underline{\lambda}} / \partial\lambda = -\cos\phi\,\hat{\underline{r}} + \sin\phi\,\hat{\underline{\phi}}\ ,$$

$$u = r\cos\phi\ d\lambda/dt\ ,\ \ v = r\ d\phi/dt\ ,$$

in (25), $(d\underline{a}/dt)_I$ may be calculated to be

$$(d\underline{a}/dt)_I = r\rho d\tau\ (\,du/dt + uw\,/\,r - uv\tan\phi\,/\,r\ + 2\Omega w\cos\phi - 2\Omega v\sin\phi)\,\hat{\underline{\phi}} \qquad (26)$$
$$-\ r\rho d\tau\ (\,dv/dt + vw\,/\,r + u^2\tan\phi\,/\,r\ + 2\Omega u\sin\phi + \Omega^2 r\sin\phi\cos\phi)\,\hat{\underline{\lambda}}\ .$$

(22) & (23) may evidently be interpreted as statements concerning the angular momentum of the fluid parcels. Substituting them into (26) shows that

$$-\left(\frac{d\underline{a}}{dt}\right)_I = r\rho d\tau\left\{\frac{\partial p/\partial\lambda\ \hat{\underline{\phi}}}{\rho r\cos\phi} + \left(\frac{-1}{\rho r}\,\partial p/\partial\phi + \Omega^2 a\sin\phi\cos\phi\right)\hat{\underline{\lambda}}\right\}\ . \qquad (27)$$

That the implied torque (11) is related to the matter term as in (17) and hence that (18) holds is easily established by integrating (27) over the whole (spherical) "atmospheric" shell, using (6) and then integrating the pressure torque terms by parts.

4: A SUMMARY OF SOLUTIONS OF THE TIDAL EQUATIONS

The following account of thermally forced atmospheric tides and solutions of Laplace's tidal equation on a sphere summarises results given in Chapman & Lindzen (1970) and Longuet-Higgins (1968). These works can be consulted for further algebraic details.

The tides are assumed to be describable as small wave-like motions in a thin layer of an inviscid perfect gas covering the rotating Earth. Investigations are limited to propagating linear wave solutions of the form of (19). The basic state of the atmosphere on which the tidal motions are superposed is taken to be a vertically stratified state of rest with no horizontal thermal variation and the tidal motions and rest state of the atmosphere are taken to be in hydrostatic balance. Thus, denoting the density and pressure of the rest state by $\rho_0(z)$ and $p_0(z)$ respectively and those of the motions by $\delta\rho$ and δp,

$$\partial p_0 / \partial z = -\rho_0 g \quad and \quad \partial \delta p / \partial z = -\delta\rho\, g \ . \tag{28}$$

The horizontal motions are governed by (24). For motions of the form (19), the velocities u and v are easily expressed in terms of the pressure field δp. The horizontal divergence of the velocities,

$$\underline{\nabla}_h \cdot \underline{u}_h = \frac{i\sigma}{4a^2\Omega^2} F(\delta p / \rho_0) \ , \tag{29}$$

where F is the differential operator given by

$$F \equiv \frac{1}{\cos\phi}\frac{d}{d\phi}\left(\frac{\cos\phi}{\eta^2 - \sin^2\phi}\frac{d}{d\phi}\right) - \frac{1}{\eta^2-\cos^2\phi}\left(\frac{s}{\eta}\frac{\eta^2+\sin^2\phi}{\eta^2-\sin^2\phi} + \frac{s^2}{\cos^2\phi}\right) \tag{30}$$

with $\eta = \sigma / 2\,\Omega$. The 3D divergence of the 3D velocity field is related to the material time derivative of the density by the continuity equation appropriate for a compressible fluid;

$$D\rho / Dt = -\rho_0(\underline{\nabla}_h \cdot \underline{u}_h + \partial w / \partial z) \ , \tag{31}$$

and the system is closed by the thermodynamic equation appropriate for a compressible perfect gas;

$$Dp / Dt = \gamma g H\, D\rho / Dt + (\gamma - 1)\rho_0 J \ . \tag{32}$$

In these equations $\gamma = c_p / c_v = 1.4$, $H(z) = p_0(z) \, / \, (\rho_0(z)\, g)$, w is the vertical velocity,

$$D/Dt\,(p, \rho) = \partial/\partial t\,(\delta p, \delta\rho) + w\, d/dz\,(p_0, \rho_0) \ , \tag{33}$$

and J is the distribution of diabatic heating.

Equations (28) - (33), for modes of the form (19) may be reduced to a single equation for

$$G = -\frac{1}{\gamma p_0} Dp/Dt \ , \tag{34}$$

which is essentially the pressure tendency following the motion: from (29), (31), (33) and (34) one can show that

$$\frac{\gamma}{g p_0} \frac{\partial}{\partial z}(p_0 G) = -\underline{\nabla}_h . \underline{u}_h = \frac{-1}{4a^2\Omega^2} F(i\sigma\, \delta p \,/\, \rho_0) \tag{35}$$

and a second expression for $i\sigma\, \partial\, (\delta p \,/\, \rho_0) \,/\, \partial z$ in terms of G and J is easily found from (33) using (31), (32) and (35); combining the vertical derivative of the first expression with the second gives

$$H\partial^2 G/\partial z^2 + (dH/dz - 1)\partial G \,/\, \partial z = \frac{g}{4a^2\Omega^2} F\{(dH/dz + \kappa)G - \frac{\kappa J}{\gamma g H}\} \ . \tag{36}$$

where $\kappa = (\gamma - 1)/\gamma$.

This equation is clearly suited to solution by the method of separation of variables. The heating function, J, and G are expanded as series,

$$(J,\, G) = \sum_m (\hat{J}_m(z),\, \hat{G}_m(z))\, \Theta_m(\phi) \exp i(s\lambda + \sigma t) \ , \tag{37}$$

where $\Theta_m(\phi)$ are eigenfunctions of the operator F;

$$F\{\Theta_m\} = -\epsilon\, \Theta_m \ ; \ \epsilon \equiv \frac{4a^2\Omega^2}{g h_m} \ . \tag{38}$$

The horizontal structure of the w and δp fields is identical to that of J and G. The vertical structure equation is

$$H\, d^2\hat{G}_m/dz^2 + (dH/dz - 1)\, d\hat{G}_m/dz + (\frac{dH/dz + \kappa}{h_m})\, \hat{G}_m = \frac{\kappa \hat{J}_m}{\gamma g H h_m} \ . \tag{39}$$

For unforced (resonant) solutions of (38) and (39) with $J_m = 0$, the separation constant h_m is an eigenvalue of (39) and the frequency of the oscillation is determined by (38). For solutions forced at a given frequency (by J_m), h_m is determined by (38).

Love's method for the solution of (38) is particularly relevant to the discussion of angular momentum fluctuations. Love re-expressed the velocities u and v in terms of a streamfunction Ψ and velocity potential Φ ;

$$au = \frac{1}{\cos\phi}\, \partial\Phi \,/\, \partial\lambda + \partial\Psi \,/\, \partial\phi \ ; \ av = \partial\Phi \,/\, \partial\phi - \frac{1}{\cos\phi}\, \partial\Psi \,/\, \partial\lambda \ . \tag{40}$$

He used the curl of the momentum equations (24) to obtain one relation between Φ and Ψ and its divergence to obtain a second relation between Φ , Ψ and $\delta p \,/\, \rho_0$. For pressure fields, δp, which are eigenfunctions of F,

$$\nabla^2 \Phi = \underline{\nabla}_h \cdot \underline{u}_h = \frac{i\sigma}{4a^2\Omega^2} F(\delta p / \rho_0) = -\frac{i\sigma}{g h_m} \delta p / \rho_0 \tag{41}$$

(see (29) and (38)), so $\delta p / \rho_0$ can be eliminated from the second relation. On expanding Φ and Ψ as series of spherical harmonics,

$$(\Phi, \Psi) = \sum_{n=s} (A_n^s,\ iB_n^s)\ \hat{V}(z)\ P_n^s(\sin\phi) \exp i(s\lambda + \sigma t) \ , \tag{42}$$

the relations reduce to recurrence relations between the A_{ns} and B_{ns} (note that spherical harmonics and their coefficients are denoted using only subscripts in the body of the text). The solutions fall into two sets distinguished by the symmetry of their motions about the equator. The set which contains the angular momentum satisfies

$$\begin{pmatrix} M_s & p_{s+1} & 0 & 0 & \cdot \\ q_s & K_{s+1} & p_{s+2} & 0 & \cdot \\ 0 & q_{s+1} & M_{s+2} & p_{s+3} & \cdot \\ \cdot & 0 & q_{s+2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} \begin{pmatrix} B_s^s \\ A_{s+1}^s \\ B_{s+2}^s \\ A_{s+3}^s \\ \cdot \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cdot \end{pmatrix} \tag{43}$$

Here K_n M_n, p_n and q_n are numerical constants which depend only on n, s and ϵ. For future reference (in section 5) we note that the first of the relations in (43) is

$$A_2^1 = \frac{5}{9} \frac{(\Omega - \sigma)}{\Omega} B_1^1 \ . \tag{44}$$

It is obvious from (8) that only the $P_{21}(\sin\phi) \exp i\lambda$ spherical harmonic of the surface pressure field contributes to the matter terms. (41) shows that the Laplacian of the velocity potential is directly proportional to the pressure field, δp , if δp is an eigenfunction of (38). Hence A_{21} is directly proportional to the matter term. Furthermore only the $P_{11}(\sin\phi) \exp i\lambda$ component of the streamfunction, with coefficient $i\,B_{11}$, contributes to the wind terms as may be shown by integrating (7) by parts:

$$(W_1, W_2) = \frac{2a^2}{g} \iiint \Psi \cos\phi (\cos\lambda, \sin\lambda) \cos\phi \ d\phi\ d\lambda\ dp \ . \tag{45}$$

Analogues of (45) can be found for equations using pressure and sigma coordinates.

5. INTERPRETATION OF MATTER TERM OSCILLATIONS

Since the oscillations in the equatorial matter terms lack a consistent amplitude and long term stability of phase it is natural to suppose that they may be unforced, resonant solutions of the tidal equations. The range of periods of angular momentum oscillations to be expected from the unforced eigensolutions of the tidal equations can be inferred from the work of previous authors such as Madden (1979). For small values of ϵ, the solutions of the tidal equations divide cleanly into gravity waves and planetary waves. The planetary waves have

$$\begin{aligned} \sigma / \Omega &\to 2s / n(n+1) \\ \Phi &\to 0 \qquad\qquad as\, \epsilon \to 0 \qquad (46) \\ \Psi &\to i B_n^s \hat{V}(z) P_n^s(\sin\phi) \exp i(s\lambda + \sigma t) \ . \end{aligned}$$

Figure 3 of Madden (1979) (which is based on fig. 2(b) of Longuet-Higgins (1968)) shows how the eigenfrequencies σ of solutions with $s = 1$ decrease as $\epsilon^{-1/2}$ decreases (i.e. $h^{1/2}$ decreases). The set of solutions which contain angular momentum (see (43)) have even values of $n - s$ so solutions with $n = 1$ and $n = 3$ are likely to contain the most angular momentum.

The equivalent depth, h_m, is an eigenvalue of equation (39) with $J_m = 0$, solved subject to suitable boundary conditions. For an isothermal atmosphere the deepest equivalent depth has

$$h = H / (1 - \kappa) \ , \ H = p_0 / \rho_0 g \ . \qquad (47)$$

According to Madden, calculations of h_m with realistic altitude dependent temperatures differ from 10 km by less than 20%. Within this range (8 km $< h_m <$ 12 km) the period of the $n = 3$ oscillation lies between 8 and 9 days, which is in remarkably good agreement with the observational data. The $n = 1$ oscillation has a period of about 1·2 days and thus, as discussed at the end of section 2, will appear more dominantly in the wind term than the matter term.

The neglect of the atmosphere's mean zonal winds and horizontal thermal variations from these calculations may seem crude but the rapidity of the phase speed of the oscillations (50 m/s at the equator) makes these approximations less significant than one might at first suppose (see Madden (1979) for a summary of numerical calculations concerning this point). The least accurate of the assumptions used in deriving the tidal equations are probably the neglect of thermal variations in heating between continents and oceans and the torques associated with the principal orography other than the equatorial bulge (see Hsu & Hoskins 1989)).

The peak minus trough fluctuation in A_1 over a few days is typically about 8 10^{24} kg m^2 s^{-1}. Such oscillations are due to fluctuations in the P_{21} spherical harmonic of the surface pressure with half amplitude in the fluctuation at 45° N of only 2 mb. The torque on the Earth associated with these oscillations (see (17)) is nevertheless large compared with the net torques which produce the largest 40 - 60 day oscillations in the axial component of the AAM. The largest changes in W_3 over the course of 20 days are of the order of 2.5 10^{25}

kg m^2 s^{-1}. From (16) the equatorial matter term torques are 16 10^{24} kg m^2 s^{-1} per day and hence are consistently 10 times as large as those of the 30 - 60 day oscillation.

It is easy to overlook the fact that much larger torques are required to maintain M_2 at its non-zero mean value (see figure 1). The climatological maps of dynamic height at 1000 mb presented in Hoskins *et al.* (1989) show that the main reason why M_2 is positive is that the Siberian high (centred near 90° W) is not balanced by a correspondingly dominant high pressure system near 90° E. It would be inaccurate to say that the positive value of M_2 is due to the Siberian high alone, because the former is a global quantity and the latter a local one. Indeed the seasonal evolution of M_2 cannot be described simply in terms of the Siberian high. But providing these remarks are borne in mind one can consider the Siberian high to be able to rotate with the Earth because of its torque on the Earth's bulge.

The relationship between (20) and (44) is of some interest. Using (42) in (45) to express (W_1, W_2) as a function of B_{11} and V(z), and (41) with (42) in (8) to express (A_1, A_2) as a function of A_{21} and V(z), one finds that

$$(M_1, M_2) = \frac{24\pi i\Omega a^2}{5\sigma} h_m (\rho_0 \hat{V})_{z=0} A_2^1(1, i) \exp(i\sigma t) ,$$

(48)

$$(W_1, W_2) = \frac{8\pi a^2 i}{3g} \int_0^{p_s} \hat{V}\, dp\, B_1^1(1, i) \exp(i\sigma t) .$$

Combining these with (44) gives

$$(M_1, M_2) = \frac{(\Omega-\sigma)}{\sigma} g h_m (\rho_0 \hat{V})_{z=0} \left(\int_0^{p_s} \hat{V} dp\right)^{-1} (W_1, W_2) . \tag{49}$$

Finally comparing this last equation with (20) one infers that

$$\int_0^{p_s} \hat{V} dp = g h_m (\rho_0 \hat{V})_{z=0} . \tag{50}$$

In a barotropic fluid (50) merely states that ρ_0 g $h_m = p_s$ (i.e. the equivalent depth h_m = H). But (50) is true for all thermal profiles T(z) and heating profiles J(z) for all separable solutions (of the form (37)) whether they contain angular momentum or not. This point may be proved as follows. From (35) and (41), for a single mode,

$$\gamma / g\, \partial / \partial z(p_0 G_m) = \frac{i\sigma}{g h_m} \delta p_m \tag{51}$$

$$\Rightarrow \int_0^{\infty} \delta p_m\, dz = \frac{h_m}{i\sigma} [\gamma p_0 G_m]_0^{\infty} . \tag{52}$$

At the lower boundary z = 0, w = 0 and from (33)

$$-\gamma p_0 G_m = i\sigma\, \delta p_m(0) . \tag{53}$$

So

$$\int_0^\infty \delta p_m \, dz = h_m \, \delta p_m(0) \ . \tag{54}$$

Finally, from (24), $\hat{V}(z)$ is proportional to $\widehat{\delta p} / \rho_0$ and (50) is easily derived.

The description of h_m as the equivalent depth is quite natural in (50), even when h_m is negative, and (50) provides an additional sense to the concept of the equivalent depth. Despite its simple form (50) does not appear to have been noticed before. It can be understood as being based on the properties of angular momentum which lead to (50) for oscillations containing angular momentum and the fact that the vertical structure equation (39) and its boundary conditions (formed from (51) and (53)) are independent of σ. For any solution of the form (37) with a given heating function $J_m(z)$, a solution with the same $J_m(z)$ and vertical structure can be found at some frequency σ (which will usually be different from that for the first solution). Since (50) must hold for this second solution it must hold for the original solution and for all solutions whether they contain angular momentum or not.

6. INTERPRETATION OF WIND TERM OSCILLATIONS

The locking of the main oscillation in the wind terms to the solar day and its seasonal variation makes it clear that the oscillation is principally thermally forced. For this reason we explore here the diurnally forced oscillation rather than the resonant solution whose period is slightly longer than a day. From (20) it is clear that the matter term of the diurnal oscillation is identically zero. The pressure torque on the equatorial bulge is hence zero consistent with the angular momentum of the oscillation being stationary in an inertial frame.

Following the method outlined in section 4, the excitation of the thermal tide may be sought by calculating the projection of the heating function J onto each of the eigenfunctions of the operator F. Heating with horizontal thermal structure $\Theta_m(\phi)\exp(is\lambda)$ excites the same horizontal component in the perturbation pressure δp (see (35) and (36)) and the Laplacian of the velocity potential $\nabla^2\Phi$ (see (41)). Hence, according to (43) the $P_{1\,1}(\sin\phi)$ $\exp i\lambda$ component of the streamfunction (the B_{11} coefficient) is excited by heating functions with amplitude in $P_{2n\,1}(\sin\phi)\exp i\lambda$ $(n \geq 1)$. These heating functions are all antisymmetric about the equator. We anticipate that of these heating functions $P_{2\,1}(\sin\phi)\exp i\lambda$ will have by far the largest amplitude and that its amplitude will have a seasonal cycle.

As Lindzen (1965) pointed out, however, the Hough functions $\Theta_m(\phi)\exp(is\lambda)$ for the diurnal tide are all orthogonal to $P_{2\,1}(\sin\phi)\exp i\lambda$. This point arises from (44) which shows that for modes with $\sigma = \Omega$, $A_{2\,1} = 0$. The Hough functions are consequently not complete for the diurnal tide and must be supplemented by solutions for which the projection of G on $P_{2\,1}(\sin\phi)\exp i\lambda$ is zero.

The simplest way to find such a solution is to look for one with

$$\delta p = \hat{p}(z)\cos\phi\sin\phi\exp i(\lambda + \Omega t) \ . \tag{55}$$

Solving (24) for the velocities u and v one obtains

$$u = \frac{\hat{p}}{\rho_0 a\Omega}\sin\phi\exp i(\lambda + \Omega t)$$
$$v = \frac{i\hat{p}}{\rho_0 a\Omega}\exp i(\lambda + \Omega t) \ . \tag{56}$$

This velocity field has zero horizontal divergence

$$\underline{\nabla}_h \cdot \underline{u}_h = 0$$

and is generated by

$$\Psi = -\frac{\hat{p}}{\rho_0\Omega}\cos\phi\exp i(\lambda + \Omega t) \ . \tag{57}$$

Thus from (29) for δp given by (55), $F(\delta p) = 0$ and, by (35), $G = 0$. (33a) then implies that

$$w = i\Omega\, \delta p \,/\, (\rho_0 g) \ . \tag{58}$$

Using (31) and (32) with the results that G and the horizontal divergence are zero one infers that

$$gH\, \partial w \,/\, \partial z = \kappa J \ . \tag{59}$$

Thus pressure perturbations of the form (55) are induced by diabatic heating with the same horizontal and temporal structure

$$J = \hat{J}(z) \cos\phi \sin\phi \exp i(\lambda + \Omega t) \ . \tag{60}$$

Since w = 0 at the surface, (58) implies that no surface pressure (and hence no matter term) tide is excited. The pressure perturbation is related to J through (58) and (59)

$$i\Omega \hat{p}(z) = \rho_0(z) \int_0^z \frac{\kappa \hat{J}(\zeta)}{H(\zeta)} d\zeta \ . \tag{61}$$

In summary, in this mode of motion the diurnal heating causes vertical expansion, which induces pressure fluctuations, but no horizontal divergence. The wave propagates horizontally like the pure barotropic Rossby wave with $\epsilon = 0$ (see (16)). The standard equation (36) fails to describe the motion because $F\{J\} = 0$.

Figures 2 to 5 present χ_{1w} and χ_{2w} which are non-dimensional functions introduced by Barnes et al. (1983). They are related to W_1 and W_2 by

$$(\chi_1^w, \chi_2^w) = \frac{1.43}{\Omega(C-A)} (W_1, W_2) \ . \tag{62}$$

Substituting (45) into (62) and then using (57) and (61) the diurnal wind terms induced by heating in P_{21} are

$$(\chi_1^w, \chi_2^w) = \frac{2.86 a^2 i}{\Omega^3 (C-A)} \iiint \int_0^z \frac{\kappa \hat{J}}{gH} d\zeta \ \cos^2\phi \exp i(\lambda + \Omega t)(\cos\lambda, \sin\lambda) \cos\phi \ d\phi\ d\lambda\ dp \ . \tag{63}$$

Several forms of heating could make significant contributions to J; direct absorption of solar radiation by water vapour, sensible heating in the surface layer, or latent heat release (contributions from absorption by ozone in the stratosphere are probably less important because (61) weights contributions from the lower atmosphere). None of these appears to have been deetermined with much accuracy. The most studied contribution is that of water vapour which Lindzen (1967), following Siebert (1961), estimates to be given by

$$\hat{J}_{water\ vapour} = 0.085\, S \frac{\Omega R_c}{\kappa} \exp(-x'/3) \tag{64}$$

where S varies sinusoidally with the season being 1 in the Northern Hemisphere summer, the phase of J has been chosen to be zero at 12Z local time, $R_c = g\,H / T_0$ is the gas constant, and $p_0(z) = p_0(0)\ \exp\text{-}x'$. Ignoring thermal veriations the vertical integrals are easily evaluated,

$$\int_0^{p_s} \int_0^{z} \exp(-x'/3)\, d\zeta\, dp = ¾\, H\, p_0(0)$$

and χ_{1w} and χ_{2w} are found to be given by

$$(\chi_1^w, \chi_2^w) = \frac{2.86 a^2 i}{\Omega^2 (C-A)} \frac{H p_0(0)}{T_0} 0.085 S \int_0^{2\pi} \exp i(\lambda + \Omega t)(\cos\lambda, \sin\lambda) d\lambda\ . \tag{65}$$

Taking $\Omega = 7.3\ 10^{-5}\ s^{-1}$, $C - A = 7\ 10^{37} / 300\ kg\ m^2$, $a = 6.4\ 10^6$ m, $T_0 = 300$ K, $H = 10^4$ m and $p_0(0) = 10^5\ N\ m^{-2}$ one finds that

$$(\chi_1^w, \chi_2^w) \approx i\, S\, 10^{-7} \exp(i\Omega t)\,(1, i)\ . \tag{66}$$

The seasonal variation in χ_{2w}(00Z) - χ_{2w}(12Z) is hence estimated to be 4 10^{-7} with its maximum during the northern hemisphere summer (July). χ_{1w} lags χ_{2w} by 6 hours; so χ_{1w} has its maximum value at 06Z GMT in July.

The χ_{1w} and χ_{2w} 00Z - 12Z differences presented in Figures 3 to 5 appear to have good phase agreement with (66) but amplitudes between 50 and 100% larger than calculated. The amplitude of the diurnal variations in the ECMWF 48 and 60 hour forecasts are about 3 times as large as suggested by (66). Both UKMO and ECMWF forecast models use equations for compressible fluids and should be able to represent solutions of the form discussed in this section very well. It seems most likely that the differences are due to difficiencies in the representation of diabatic heating.

Fluctuations in the UKMO analyses up to 1988 of W_1 between 00Z and 12Z were of concern because they suggested that the equatorial wind term oscillation was not stationary in an inertial frame. The implied torques could be large if this were the case: if χ_{2w} oscillated as observed whilst χ_{1w} were constant the torque would be almost 10 times larger than that involved in the 30 - 60 day oscillation.The accompanying wind velocity errors could nevertheless be small; a vertically uniform fluctutation in the P_{11} spherical harmonic of the streamfunction would give wind velocities with a half amplitude of only 0.1 m s^{-1}.

7. CONCLUSIONS

As noted by Lamb (1932), fluid motions on a spheroidal surface which follows an equipotential are well described by equations for motions on a spherical surface provided they omit the centripetal acceleration term (i.e. $\Omega^2 r \sin\phi \cos\phi$ in (21)). Thus Laplace's tidal equations and the primitive equations used by most weather forecast systems describe motions which (implicitly) exert a torque on the Earth's bulge equal to $-\underline{\Omega} \wedge \underline{M}$. The phase velocity of solutions of Laplace's tidal equations which contain angular momentum thus depends on the ratio of their wind to their matter terms according to (20). Solutions with null matter terms exert no torque on the Earth and are stationary in an inertial frame whilst solutions with no wind term move with the Earth.

The fluctuations in M_1 and M_2 illustrated by figure 1 can be interpreted as free (unforced) solutions of the tidal equations which are able to rotate rapidly in inertial space by the torque they exert on the Earth's bulge. Their wind terms are small because $\sigma << \Omega$ (see (20)). A large mean value can be maintained in the M_2 component, which rotates with the Earth, because of its torque on the bulge. This component reflects the large asymmetries in the global circulation associated with the Siberian high. The seasonally modulated diurnal fluctuations in the wind terms are thermally forced, virtually stationary in inertial space and hence have null matter terms. Inaccuracies in their evaluation almost certainly derive from errors in the representation of diabatic heating.

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FIGURE CAPTIONS

Figure 1:

Timeseries of χ_{2p} evaluated from UKMO analyses valid at 00Z (full line) and 12Z (dashed line) for 1990. The matter term M_2 is related to (the non-dimensional) χ_{2p} by $M_2 = (C - A)\,\Omega\,\chi_{2p}$.

Figure 2:

Timeseries of χ_{2w} evaluated from UKMO analyses valid at 00Z for 1990. The wind term W_2 is related to (the non-dimensional) χ_{2w} by $W_2 = (C - A)\,\Omega\,\chi_{2w} / 1.43$.

Figure 3:

Timeseries of χ_{2w} evaluated at 00Z (full line) and 12Z (dashed line) from (a) uninitialised ECMWF analyses for 1988 and (b) UKMO analyses for 1990. All timeseries have been smoothed using an 11 day running mean filter.

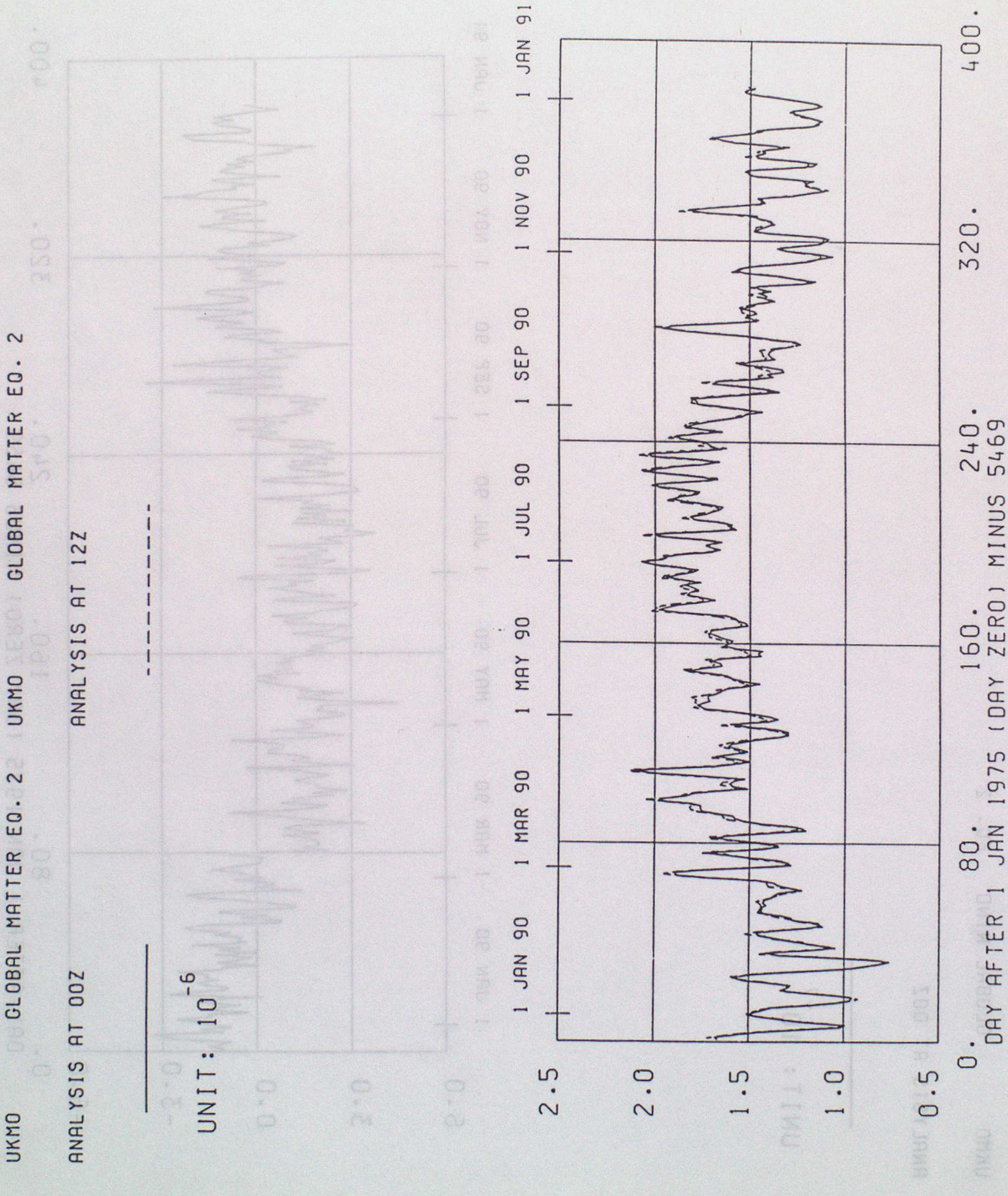
Figure 4:

Timeseries of χ_{1w} evaluated at 00Z (full line) and 12Z (dashed line) from (a) uninitialised ECMWF analyses for 1988 and (b) UKMO analyses for 1990. All timeseries have been smoothed using an 11 day running mean filter.

Figure 5:

Timeseries of (a) χ_{1w} and (b) χ_{2w} evaluated from uninitialised ECMWF analyses at 06Z (full line) and 18Z (dashed line) for 1988. All timeseries have been smoothed using an 11 day running mean filter.

Figure 1



UKMO GLOBAL MATTER EQ.22 (UKMO GLOBAL MATTER EQ. 2
ANALYSIS AT 00Z
ANALYSIS AT 12Z
UNIT: 10^{-6}
2.5
2.0
1.5
1.0
0.5
1 JAN 90
1 MAR 90
1 MAY 90
1 JUL 90
1 SEP 90
1 NOV 90
1 JAN 91
0.
80.
160.
240.
320.
400.
DAY AFTER 1 JAN 1975 (DAY ZERO) MINUS 5469

Figure 2

UKMO GLOBAL WIND EQ. 2

ANALYSIS AT 00Z

UNIT: 10^{-7}

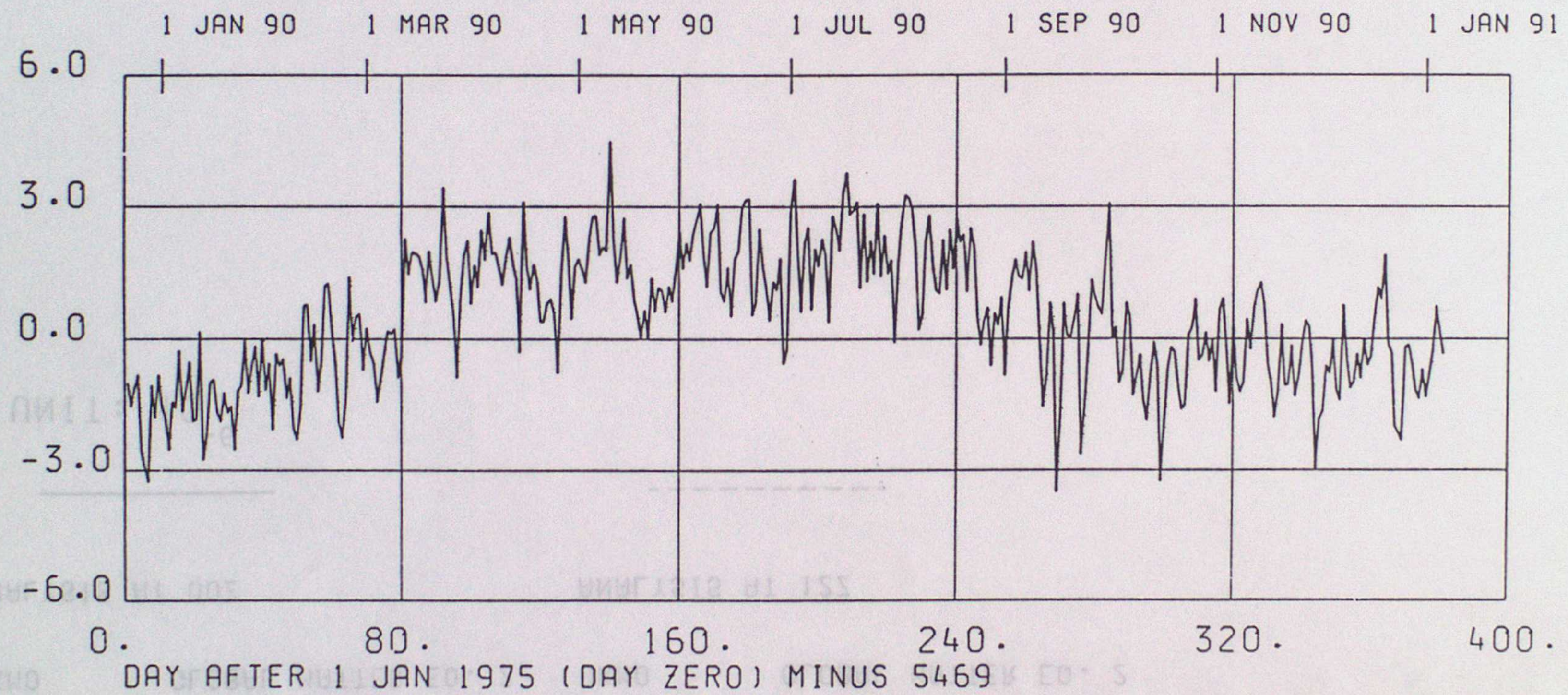


Figure 3a

ECMWF GLOBAL WIND EQ. 2

ANALYSIS AT 00Z

ECMWF GLOBAL WIND EQ. 2

ANALYSIS AT 12Z

UNIT: 10^{-7}

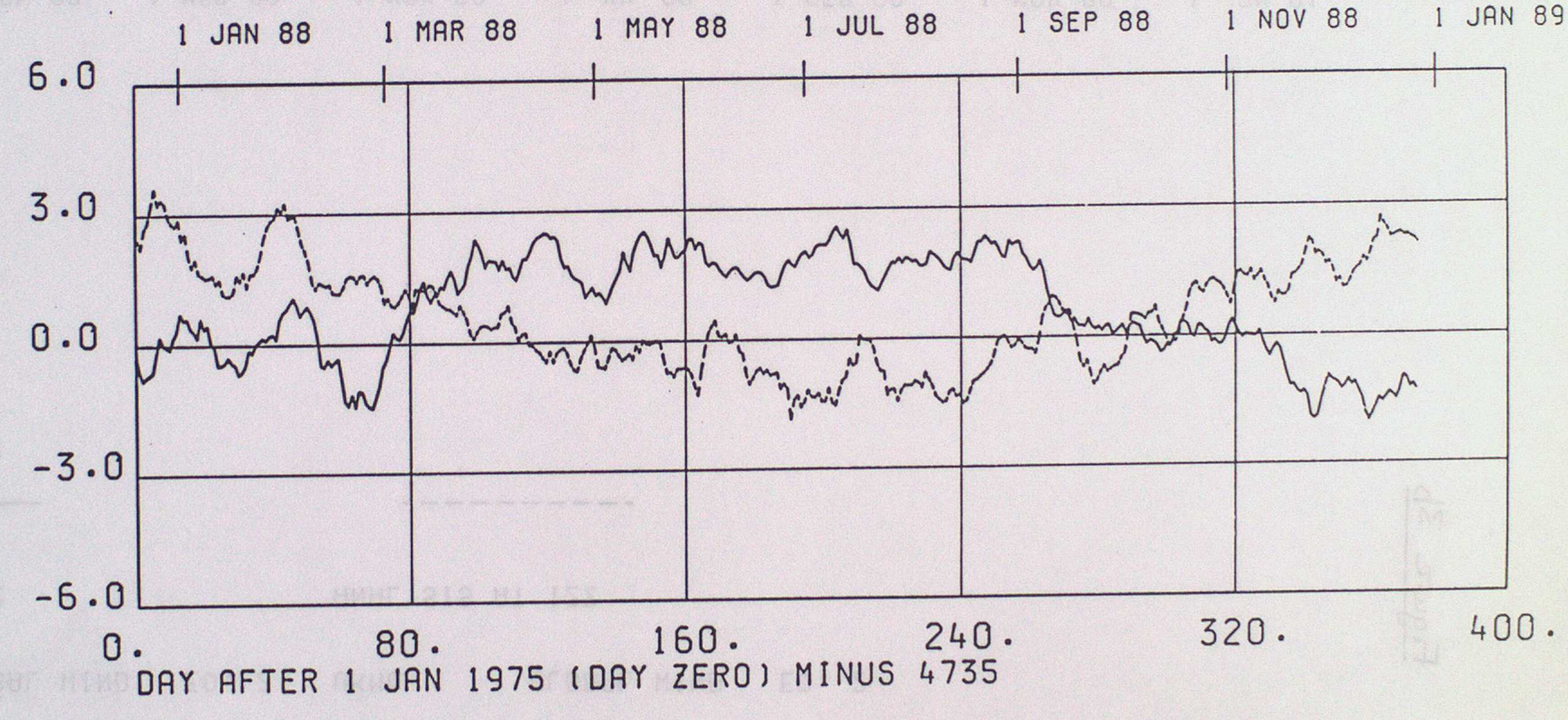


Figure 3b

UKMO GLOBAL WIND EQ. 2 UKMO GLOBAL WIND EQ. 2

ANALYSIS AT 00Z ANALYSIS AT 12Z

UNIT: 10^{-7}

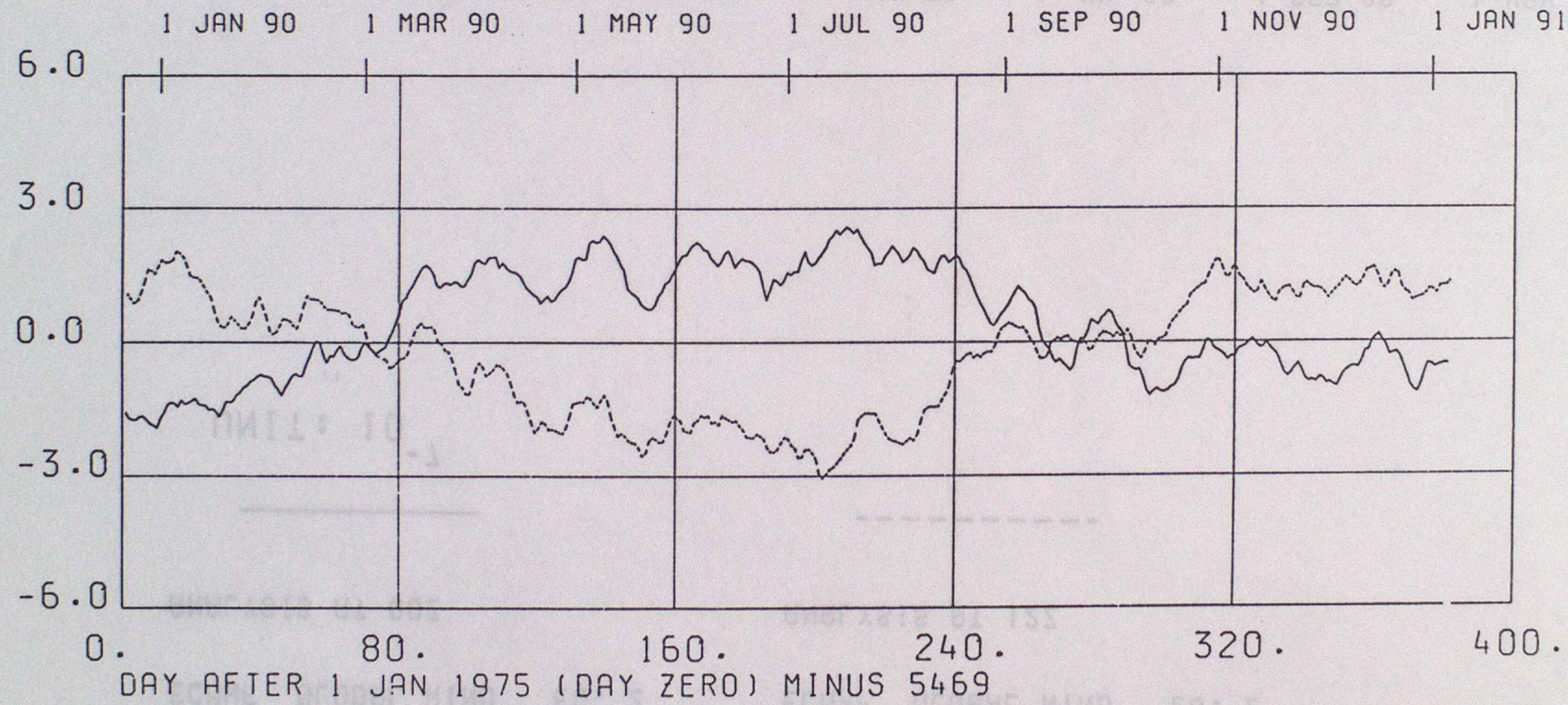


Figure 4a

ECMWF GLOBAL WIND EQ. 1 ECMWF GLOBAL WIND EQ. 1

ANALYSIS AT 00Z ANALYSIS AT 12Z

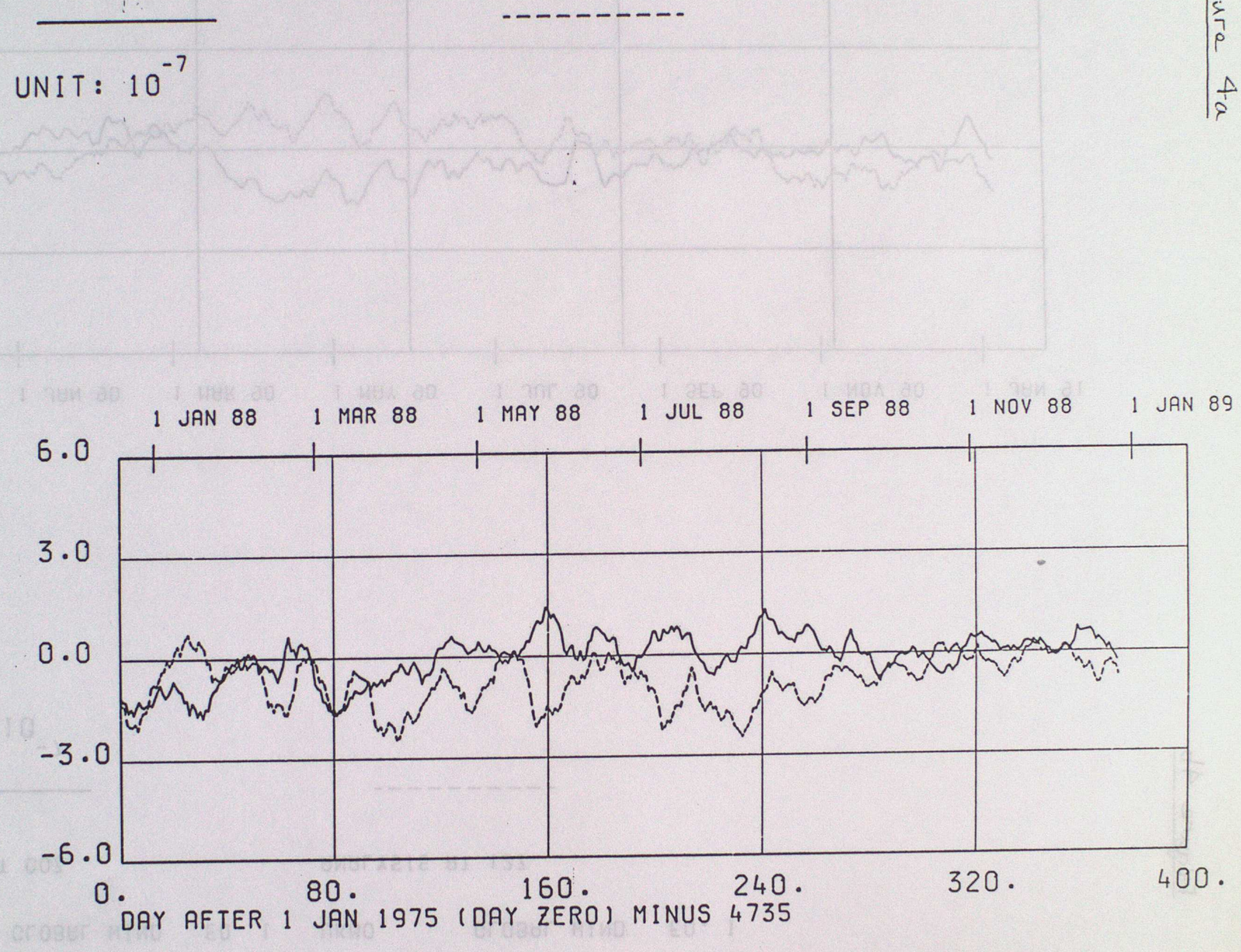


Figure 4b

UKMO GLOBAL WIND EQ. 1 UKMO GLOBAL WIND EQ. 1

ANALYSIS AT 00Z ANALYSIS AT 12Z

UNIT: 10^{-7}

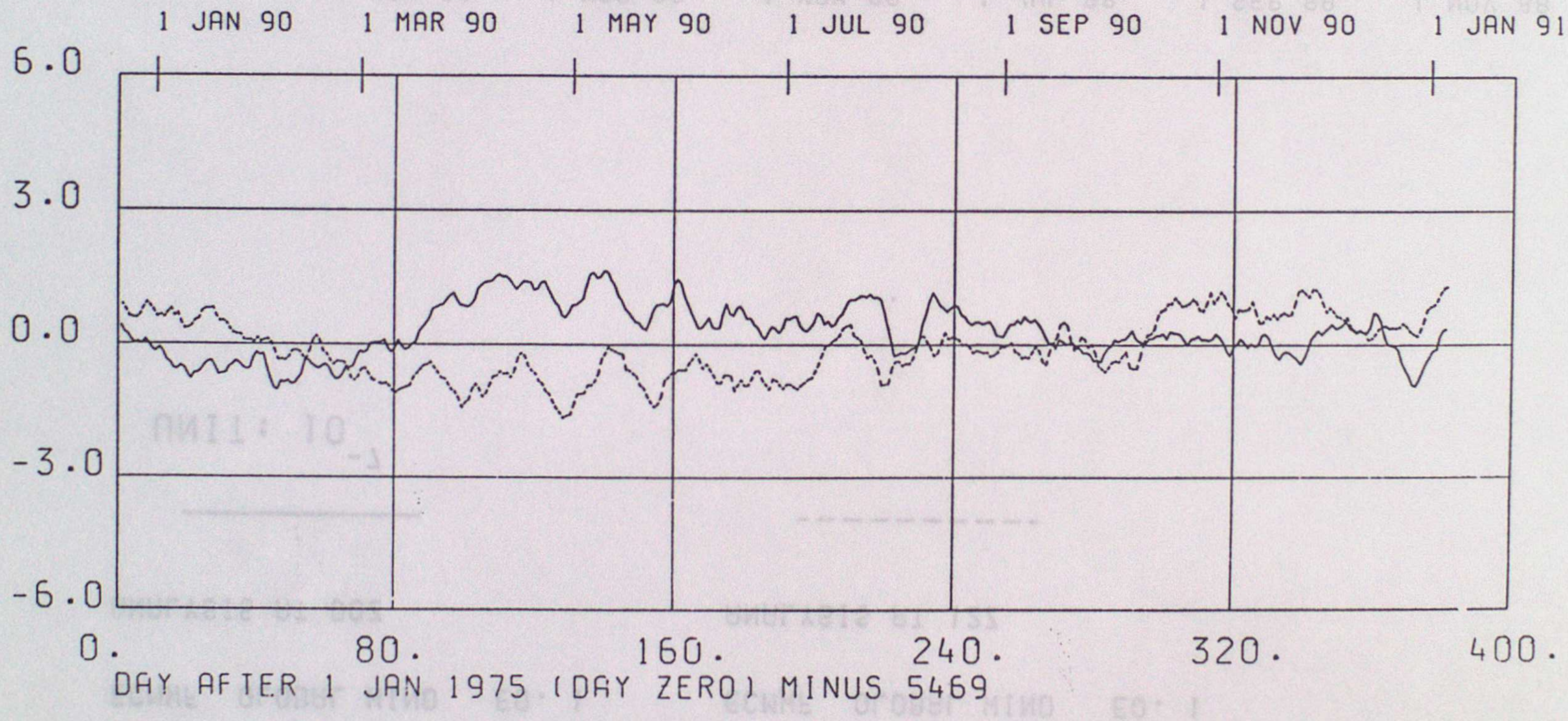


Figure 5a

ECMWF(BHS) GLOBAL WIND EQ. 1 ECMWF(BHS) GLOBAL WIND EQ. 1

ANALYSIS AT 06Z ANALYSIS AT 18Z

UNIT: 10^{-7}

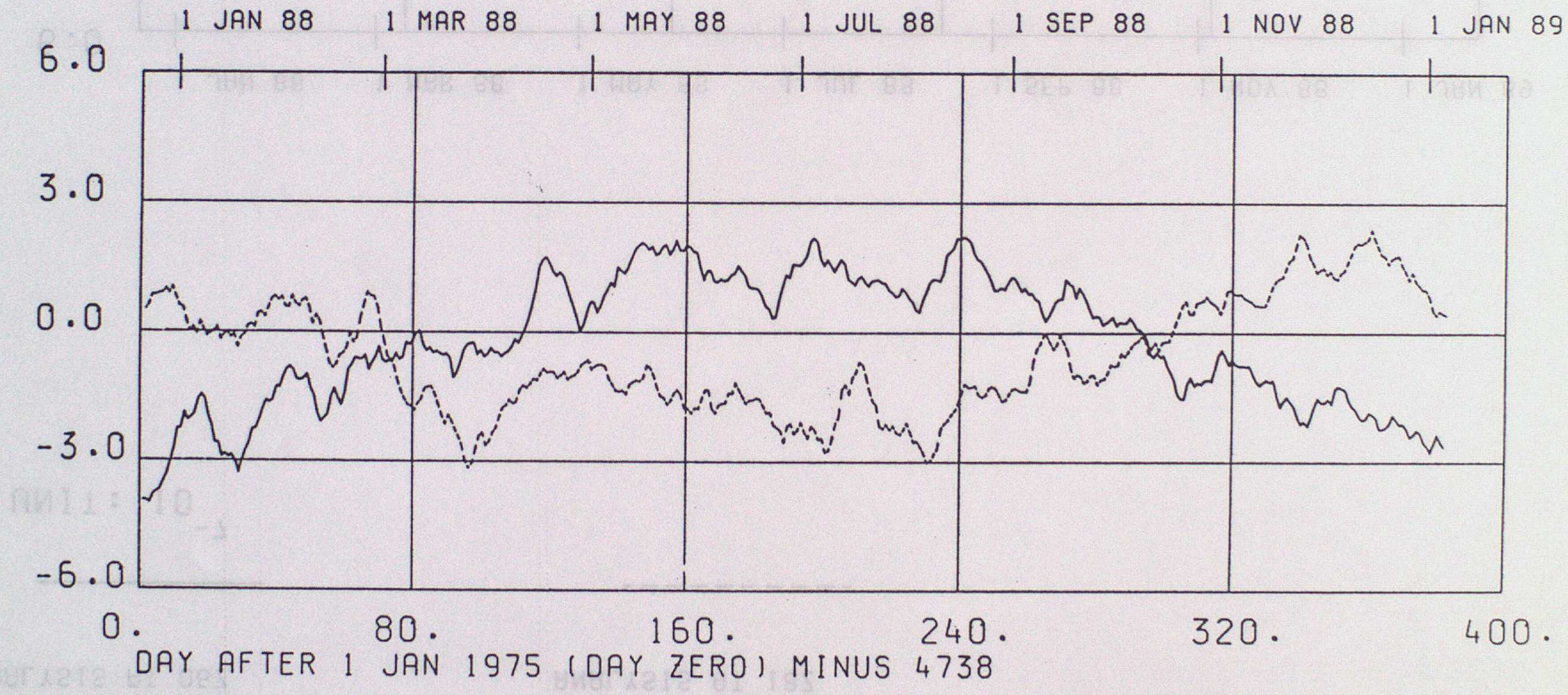
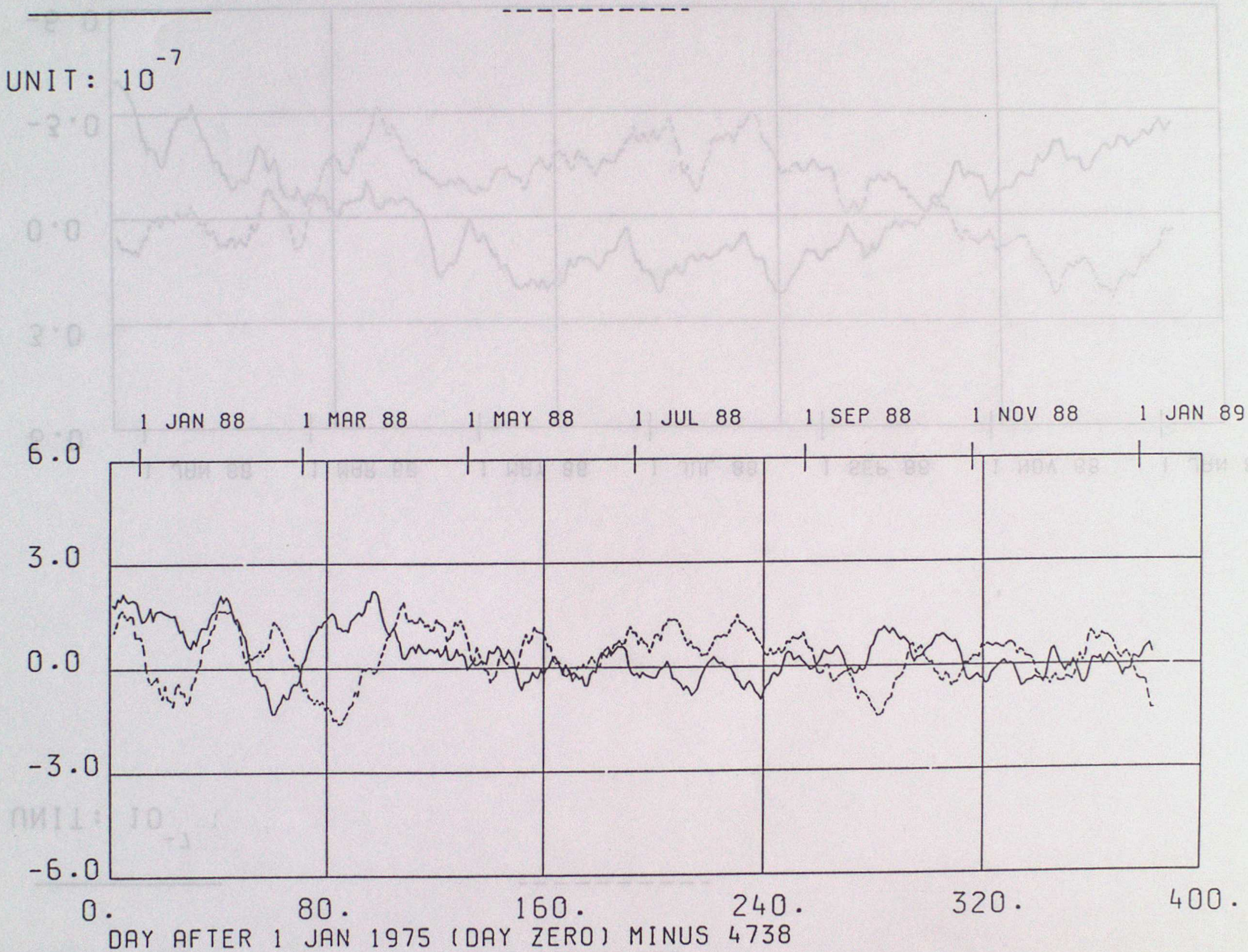


Figure 5b

ECMWF(BHS) GLOBAL WIND EQ. 2 ECMWF(BHS) GLOBAL WIND EQ. 2

ANALYSIS AT 06Z ANALYSIS AT 18Z



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