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Amendments to the objective analysis procedure for the ten level model (Rectangle - March 1973)

1. The first six sections of this note deal with the modifications to the height analysis scheme described in Technical Note No. 37 and the seventh describes the changes to the relative humidity scheme. The modifications to the height analysis scheme fall into five groups.

The data check procedures at both the surface and U/A levels have been amended so as to reject a smaller amount of correct information. The surface analysis is now performed simultaneously with those of the 300 and 500mb contour heights and 500-1000 thickness, so that surface detail may be incorporated into the upper air pattern without using the adjustment procedure described in Technical Note No. 37 (whereby differences between the 1000mb U/A and surface analyses are added proportionately into the upper levels of the model). The details of the surface analysis process have been changed so that small systems can be more accurately represented. The interpolation formulae for the 600, 800, 900mb heights, which have proved unsatisfactory on several occasions, have been replaced by a polynomial fitting procedure, and finally there is a group of small modifications needed either to produce improved thermal structure or smoother relative geostrophic vorticity fields.

These changes in the analysis leave the overall system as follows. The eight standard levels are analysed in the following three groups:-

(i) 100mb

(ii) 1000, 500, 300,mb, T/T

(iii) 200, 400, 700, 850mb

(The T/T analysis is not kept after being performed since it is equal to the difference between the 500 and 1000mb levels). There are four scans at each of the first two groups of levels and three at the third. The curvature correction to the data is made after the end of the second scan at all groups of levels as before. Similarly, the procedure for treating the differences between the observations and the final analysis at 100mb as the random errors of the radio sondes, has been maintained. When the three groups of levels have been completed, the 600, 800, 900mb are interpolated by fitting polynomials to the bottom six standard levels and the results of this process are the ten analysed levels of contour height.

2. Data check modifications

2.1. Upper Air

The data check at the U/A levels has been reformulated so that where there is an observation of both wind speed (v) and contour height (h) the height observation is rejected if

$$|h_{observed} - h_{calculated}| > x \text{ metres}$$

where $x = x^1 + bv - c$

In cases where there is no observation of wind, the height observation is rejected if

$$|h_{observed} - h_{calculated}| > x^1 \text{ metres}$$

x^1 varies from 60 to 240 metres, depending on the level and the type of observation and is tabulated in the Appendix; b,c are fixed at

2.0 secs and 20 metres respectively. This modification is designed to take account of two sources of error in $h_{calculated}$:

(a) The first scan of the analysis is designed to define the large scale features of the situation rather than the detail, and the differences between the results of this scan and the final analysis could be substantial in the region of a jet stream, which will be rather smoothed out on the first scan.

(b) The analysis process gives considerable weight to the background field (12hr forecast field on the first scan); therefore a small error in the forecast movement of a jet stream could result in an error of the order of x^1 in the value of $h_{calculated}$ even fairly close to an observation. The observed wind is taken as the estimator of the height gradient in the region of the observation, because, although it is subject to observational errors, these are no worse than the errors in the forecast field (an alternative estimator), and gross errors in the wind observations will have already been trapped by the quality control procedures.

2.2. Surface

At the surface, a similar allowance for errors or smoothing in the first scan of the analysis has been made, except that the relevant observations in this case are those near the centres of depressions. The nearness to a depression is expressed in terms of ϕ^1, which is derived by linear interpolation from the ϕ's worked out at the four grid points surrounding the observation. At a grid point (x,y) we define $\phi(x,y)$ as a function of the background field heights at the nine points used as background points in the quadric fitting procedure. Numbering the points (1-9), and the corresponding background heights (h_1 - h_9) (suffix j refers to the grid point (x,y)) then

$$\phi(x,y) = \frac{\sum_{i=1}^{9} (h_i - h_j)}{\sum_{i=1}^{9} |h_i - h_j|}$$

If $\phi(x,y) \leq 0$, then $\phi(x,y)$ is set equal to 0. Therefore $\phi(x,y)$ has a range of values from 0 to 1, where any value of $\phi(x,y) > 0$ implies that the geostrophic vorticity at (x,y) tends to be cyclonic. An observation of height is rejected if

$$|h_{observed} - h_{calculated}| > x \text{ metres}$$

when $\phi^1 > 0$ and $h_{observed} < h_{calculated}$ then

$$x = x^1 + a\phi - b \;;$$

and when $\phi^1 = 0$ or $h_{calculated} < h_{observed}$ then

$$x = x^1$$

where a = 100.0 metres, b = 2.0 metres.

This relaxes the rejection criterion when the 12 hr forecast field implies a depression near to an observation, whose height is lower than that obtained from the first scan of the analysis. It is in the areas of depressions that the first scan, which is designed to define the large scale pattern, differ by the largest amount from the actual field.

3. Simultaneous analysis of surface and U/A levels

Let an estimate of the error at level (n)

$$E^{(n)} = \sum_{l} \left\{ p_l \left(H_{Al}^{(n)} - H_{Ol}^{(n)} \right)^2 \right\} + S^2 \sum_{m} \left\{ p_m \left(\underline{V}_{Om}^{(n)} - \underline{V}_{Am}^{(n)} \right)^2 \right\} + q \sum_{j} \left(H_{Aj}^{(n)} - H_{Bj}^{(n)} \right)^2$$

where $H_{Aj}^{(n)} = a^{(n)} x_j^2 + b^{(n)} y_j^2 + 2h^{(n)} x_j y_j + 2g^{(n)} x_j + 2f^{(n)} y_j + c_j^{(n)}$

$\underline{V}_{Am}^{(n)}$ is geostrophic wind derived as $\beta g / af \left(-\dfrac{\partial H_{Aj}^{(n)}}{\partial y}, \dfrac{\partial H_{Aj}^{(n)}}{\partial x} \right)$

$H_{Ol}^{(n)}$ is observation of height

$\underline{V}_{Om}^{(n)}$ is observation of wind

p, q, S^2 are weighting factors as defined in Technical Note No. 37. The subscripts j, l, m refer to background points, height observations and wind observations respectively; (x,y) are coordinates relative to the grid point under consideration in units of grid length.

Let $\underline{Q}^{(n)} = (a^{(n)}, b^{(n)}, h^{(n)}, g^{(n)}, j^{(n)}, c^{(n)})$; then if we wish to minimise $E^{(n)}$ subject to variation in $a^{(n)}$, $b^{(n)}$ etc. we obtain the set of equations

$$\frac{\partial E^{(n)}}{\partial a^{(n)}} = \frac{\partial E^{(n)}}{\partial b^{(n)}} = \frac{\partial E^{(n)}}{\partial c^{(n)}} = \frac{\partial E^{(n)}}{\partial h^{(n)}} = \frac{\partial E^{(n)}}{\partial f^{(n)}} = \frac{\partial E^{(n)}}{\partial g^{(n)}} = 0$$

which may be written in matrix form:

$$\underline{\underline{A}}^{(n)} \cdot \underline{Q}^{(n)} = \underline{d}^{(n)} \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

where $\underline{\underline{A}}^{(n)}$ is a function only of p, q, S^2 and the position of observations, and $\underline{d}^{(n)}$ is a function also of the background heights, observed heights and observed winds. These equations have solution

$$\underline{Q}^{(n)} = \underline{\underline{A}}^{(n)^{-1}} \cdot \underline{d}^{(n)}$$

(assuming that the matrix $\underline{\underline{A}}$ is non-singular)

Therefore the analysed height $= C^{(n)} = \underline{\Delta} \cdot \underline{\underline{A}}^{(n)^{-1}} \cdot \underline{d}^{(n)}$ ......(2)

where $\underline{\Delta} = (0, 0, 0, 0, 0, 1)$

Equation (2) is linear, and therefore if we analyse 500-1000 thickness based on upper air observations (thickness and thermal wind) we may expect the same answer as we would obtain from the difference between the 500mb and 1000mb analyses performed independently on the upper air ascents. (A difference can occur if the data check rejects an observation at either 500 or 1000mb but not in the 500-1000 thickness analysis or vice versa.) The matrix $\underline{\underline{A}}$ should be the same for 500, 1000 and thickness since it merely depends on the positions and weights of the observations. The situation is not the same when the 1000mb field has been analysed using surface reports, since they tend to be far more numerous than upper air reports, and not necessarily in the same position, so that the matrix $\underline{\underline{A}}^{(1000)}$ is not identical to $\underline{\underline{A}}^{(500)}$ or $\underline{\underline{A}}^{(\text{thickness})}$, and our two values for the thickness may be different. The theory which follows shows how it is possible to take these two independent estimates of the thickness and adjust the 1000, 500 and thickness analyses, so that the thickness analysis is equal to the difference between the 1000 and 500. At the same time, since the 300mb analysis is performed with the 500, 1000, and thickness (as one of the primary levels) the same argument applies, in that we can obtain estimates of the 300-500 thickness both by direct analysis and from the difference between the 300 and 500mb contour heights analysed independently. However, apart from aireps data, the observations at 300mb, 500mb and thickness will be the same. Therefore in the adjustments to the 300mb level, we assume that the same data as was present in 500mb analysis would be present in the 300-500mb thickness analysis and our answer would be that obtained from the difference between the 300 and 500mb height analyses. On these assumptions we can obtain adjustments to the 300, 300-500 thickness analysis, so that the 300, 500, 1000mb levels are consistent with the analyses of the 300-500 and 500-1000 thickness.

Define $S = \chi^{(3)}E^{(3)} + \chi^{(5)}E^{(5)} + \chi^{(10)}E^{(10)} + \chi^{(5/10)}E^{(5/10)} + \chi^{(3/5)}E^{(3/5)}$

where $E^{(3)} = E^{(n)}$ for 300mb

$E^{(5)} = E^{(n)}$ for 500mb

$E^{(10)} = E^{(n)}$ for 1000mb

$E^{(5/10)} = E^{(n)}$ for 500-1000 thickness

$E^{(3/5)} = E^{(n)}$ for 300-500 thickness

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$\chi^{(n)}$ are the weights of the $E^{(n)}$ in S and may be arbitrarily specified.

We wish to minimise S subject to variations in $a^{(3)}$, $a^{(5)}$, $a^{(10)}$, $a^{(3/5)}$, $a^{(5/10)}$, $b^{(3)}$ .... etc. under the constraints

$$c^{(5)} = c^{(10)} + c^{(5/10)} \qquad \text{.........(5)}$$

$$c^{(3)} = c^{(5)} + c^{(3/5)} \qquad \text{.........(6)}$$

Similar constraints have not been put on the other coefficients in the quadric fit analysis, hence the quadric surface obtained directly for the thickness is only equal to the difference between the surfaces fitted at the 500 and 1000mb levels actually at the grid point under consideration (and therefore approximately in a small neighbourhood around the grid point). In practice it is probably unnecessary to impose a more stringent constraint than this, and it is also very much simpler to perform the analysis for only two constraints.

This leads to a set of 30 equations

$$\frac{\partial S}{\partial a^{(3)}} = \frac{\partial S}{\partial b^{(3)}} = \frac{\partial S}{\partial h^{(3)}} = \frac{\partial S}{\partial g^{(3)}} = \frac{\partial S}{\partial f^{(3)}} = 0 \; ; \; \frac{\partial S}{\partial c^{(3)}} = \lambda \qquad (7)$$

$$\frac{\partial S}{\partial a^{(5)}} = \frac{\partial S}{\partial b^{(5)}} = \frac{\partial S}{\partial h^{(5)}} = \frac{\partial S}{\partial g^{(5)}} = \frac{\partial S}{\partial f^{(5)}} = 0 \; ; \; \frac{\partial S}{\partial c^{(5)}} = \mu - \lambda \qquad (8)$$

$$\frac{\partial S}{\partial a^{(10)}} = \frac{\partial S}{\partial b^{(10)}} = \frac{\partial S}{\partial h^{(10)}} = \frac{\partial S}{\partial g^{(10)}} = \frac{\partial S}{\partial f^{(10)}} = 0 \; ; \; \frac{\partial S}{\partial c^{(10)}} = -\mu \qquad (9)$$

$$\frac{\partial S}{\partial a^{(3/5)}} = \frac{\partial S}{\partial b^{(3/5)}} = \frac{\partial S}{\partial h^{(3/5)}} = \frac{\partial S}{\partial g^{(3/5)}} = \frac{\partial S}{\partial f^{(3/5)}} = 0 \; ; \; \frac{\partial S}{\partial c^{(3/5)}} = -\lambda \qquad (10)$$

$$\frac{\partial S}{\partial a^{(5/10)}} = \frac{\partial S}{\partial b^{(5/10)}} = \frac{\partial S}{\partial h^{(5/10)}} = \frac{\partial S}{\partial g^{(5/10)}} = \frac{\partial S}{\partial f^{(5/10)}} = 0 \; ; \; \frac{\partial S}{\partial c^{(5/10)}} = -\mu \qquad (11)$$

λ, μ are Lagrange undertermined multipliers of equations (5) and (6) respectively. Equations (7) - (11) may be rewritten as

$$\chi^{(3)} \underline{\underline{A}}^{(3)} \cdot \underline{Q}^{(3)} = \chi^{(3)} \underline{d}^{(3)} + \lambda \underline{\Lambda}^{T}$$

$$\chi^{(5)} \underline{\underline{A}}^{(5)} \cdot \underline{Q}^{(5)} = \chi^{(5)} \underline{d}^{(5)} + (\mu - \lambda) \underline{\Lambda}^{T}$$

$$\chi^{(10)} \underline{\underline{A}}^{(10)} \cdot \underline{Q}^{(10)} = \chi^{(10)} \underline{d}^{(10)} - \mu \underline{\Lambda}^{T}$$

$$\chi^{(3/5)} \underline{\underline{A}}^{(3/5)} \cdot \underline{Q}^{(3/5)} = \chi^{(3/5)} \underline{d}^{(3/5)} - \lambda \underline{\Lambda}^{T}$$

$$\chi^{(5/10)} \underline{\underline{A}}^{(5/10)} \cdot \underline{Q}^{(5/10)} = \chi^{(5/10)} \underline{d}^{(5/10)} - \mu \underline{\Lambda}^{T}$$

We can obtain expressions for $c^{(3)}, c^{(5)}, c^{(10)}, c^{(3/5)}, c^{(5/10)}$ as follows

$$c^{(3)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(3)^{-1}} \cdot \underline{d}^{(3)} + \lambda/\chi^{(3)}\, \underline{\Lambda} \cdot \underline{\underline{A}}^{(3)^{-1}} \cdot \underline{\Lambda}^{T} \quad \ldots(7)\text{a}$$

$$c^{(5)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(5)^{-1}} \cdot \underline{d}^{(5)} + \frac{(\mu-\lambda)}{\chi^{(5)}}\, \underline{\Lambda} \cdot \underline{\underline{A}}^{(5)^{-1}} \cdot \underline{\Lambda}^{T} \quad \ldots(8)\text{a}$$

$$c^{(10)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(10)^{-1}} \cdot \underline{d}^{(10)} - \mu/\chi^{(10)}\, \underline{\Lambda} \cdot \underline{\underline{A}}^{(10)^{-1}} \cdot \underline{\Lambda}^{T} \quad \ldots(9)\text{a}$$

$$c^{(3/5)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(3/5)^{-1}} \cdot \underline{d}^{(3/5)} - \lambda/\chi^{(3/5)}\, \underline{\Lambda} \cdot \underline{\underline{A}}^{(3/5)^{-1}} \cdot \underline{\Lambda}^{T} \quad \ldots(10)\text{a}$$

$$c^{(5/10)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(5/10)^{-1}} \cdot \underline{d}^{(5/10)} - \mu/\chi^{(5/10)}\, \underline{\Lambda} \cdot \underline{\underline{A}}^{(5/10)^{-1}} \cdot \underline{\Lambda}^{T} \quad \ldots(11)\text{a}$$

Writing $w^{(i)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(i)^{-1}} \cdot \underline{\Lambda}^{T}$ and $c_N^{(i)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(i)^{-1}} \cdot \underline{d}^{(i)}$ the equations may be rewritten as

$$c^{(3)} = c_N^{(3)} + \lambda/\chi^{(3)}\ \omega^{(3)} \quad \ldots\ldots(12)$$

$$c^{(5)} = c_N^{(5)} + \frac{(\mu-\lambda)}{\chi^{(5)}}\ \omega^{(5)} \quad \ldots\ldots(13)$$

$$c^{(10)} = c_N^{(10)} - \mu/\chi^{(10)}\ \omega^{(10)} \quad \ldots\ldots(14)$$

$$c^{(3/5)} = c_N^{(3/5)} - \lambda/\chi^{(3/5)}\ \omega^{(3/5)} \quad \ldots\ldots(15)$$

$$c^{(5/10)} = c_N^{(5/10)} - \mu/\chi^{(5/10)}\ \omega^{(5/10)} \quad \ldots\ldots(16)$$

$c_N^{(i)}$ is the value which is obtained for C at level i before the analysis constraints are applied and the term dependent on $w^{(i)}$ is a correction required by the constraints and can be added on afterwards.

$w^{(i)} = \underline{\Lambda} \cdot \underline{\underline{A}}^{(i)^{-1}} \cdot \underline{\Lambda}^{T}$ represents the weight of level (i) relative to the other levels being analysed at the same time and is a function of the positions and weights of the observations at level (i). We can solve the set of equations (5), (6), (12) - (16) for

$\lambda, \mu, c^{(3)}, c^{(5)}, c^{(10)}, c^{(3/5)}, c^{(5/10)}$ in terms of $C_N^{(i)}$ and $\omega^{(i)}$, which can be calculated direct from the data. We need not perform the 300-500 thickness analysis if we assume that

$C_N^{(3/5)} = C_N^{(3)} - C_N^{(5)}$ and that $\omega^{(3/5)} = \omega^{(5)}$; equation (15) then reads

$$c^{(3/5)} = C_N^{(3)} - C_N^{(5)} - \lambda/\chi_{(3/5)}\, \omega^{(5)} \qquad .....(17)$$

and we can solve the new set substituting equation (17) for equation (15). (See appendix for details of solution.)
This analysis has proceeded on the assumption that we are fitting a quadric surface in the neighbourhood of the grid point at all five levels. There are two occasions on which this assumption breaks down.

(a) When the "plane-fit" technique, outlined in Technical Note No. 37, is being used in the surface analysis

(b) when there is no data available at the grid point under consideration and the equation $h_A = h_B$ is in use.

The plane fit takes a weighted mean of several estimates of the contour height at the grid point under consideration.

viz.

$$h_A = \frac{\sum_i p_i h_{iE}}{\sum_i p_i}$$

where h_{iE} are estimates of contour height obtained from height and wind observations (i) and p_i are their associated weights.

This solution can be obtained by considering the minimum of

$$E = \sum_i p_i (h_A - h_{iE})^2$$

under variations in h_A. Substituting for $E^{(10)}$ in this way, equation (9)a becomes

$$c^{(10)} = \frac{1}{\sum p_i} \sum p_i h_{iE} - \frac{\mu}{\chi^{(10)}} \cdot \frac{1}{\sum p_i} \cdot \frac{1}{2}$$

$$\left[\underline{\underline{A}}^{(10)^{-1}} = \frac{1}{\sum p_i} \; ; \; \underline{d}^{(10)} = \sum p_i h_{iE} \text{ , now both scalars} \right]$$

In case (b) the equation $h_A = h_B$ may be obtained by minimising $E = (h_A - h_B)^2$ subject to variations in h_A. Substituting for $E^{(n)}$ at any level in this way, we then obtain

$$\underline{\Lambda} \cdot \underline{\underline{A}}^{(n)^{-1}} \cdot \underline{\Lambda}^T = 1, \quad \underline{\Lambda} \cdot \underline{\underline{A}}^{(n)^{-1}} \cdot \underline{d}^{(n)} = h_B^{(n)}$$

At the surface we take $\underline{\Lambda} \cdot \underline{\underline{A}}^{(n)^{-1}} \cdot \underline{\Lambda}^T = \frac{1}{p_B}$ instead of 1 when using the plane-fit, where p_B is the weight of the background point, to ensure continuity of the field $w^{(i)}$ in the horizontal. In both cases (a) and (b) equations (12), (13), (14), (16), (17) can be solved using the values for $C_N^{(n)}$ and $w^{(n)}$ obtained from the above analysis in the ordinary way.

This linking of the levels is only performed at the second group of levels viz:- 1000mb, 500mb, 300mb. The first scan is for checking the horizontal consistency of the data and the levels are not linked, because the effect of the constraints is likely to raise the 1000mb heights slightly near the centres of depressions and thus increase the probability of rejecting a correct observation in the data check.

The values of χ at each of the five levels are fixed for all four scans and have been determined by experiment to produce optimum values for the spot values at the centres of depressions on the surface chart, while at the same time ensuring reasonable values for the partial thicknesses in the bottom 200mb of the model atmosphere. On this basis the values used are

$$\chi^{(3)} = \chi^{(5)} = \chi^{(3/5)} = 1.0$$

$$\chi^{(10)} = 3.33 \qquad \chi^{(5/10)} = 0.67$$

The weight of the thickness analysis in S has been made smaller than those of the 500 or 300mb analyses, because where the 1000mb surface is below the ground, the thermal wind has been derived by the equation

$$\underline{u}_{500} - \underline{u}_{1000} = 1.454 \; (\underline{u}_{500} - \underline{u}_{850}) .$$

$\underline{u}_n$ = wind at pressure level n mb.

4. Modifications to the surface analysis

Since the surface analysis using surface data is performed simultaneously with the 500, 300, 500-1000 thickness analyses, it may use the 1000mb forecast field as first guess rather than obtain its first guess from a previously performed 1000mb U/A analysis. (In practice, this latter procedure seems to have been detrimental; low pressure systems which have been well forecast have been smoothed out by the 1000mb U/A analysis, so that the surface analysis has been presented with a poor quality background field.)

The first two scans of the procedure use quadric fitting and the third and fourth plane fitting everywhere over the field. (See Technical Note No. 37 for details of the plane fit process.) The weighting function on the third and fourth scans has been modified so that at a grid point (x,y) it is now a function of $\phi(x, y)$ as defined in the data check procedure. This is based on the assumption that the results of the second scan will have the positions of all the features correct, but the actual spot values of the centres of the depressions will still be in error. Let $p = 1/(1+p'r^n)$ be the weighting function as defined in Technical Note No. 37. Then the new

$$p = 1/\left(1+p'r^n\left[\frac{1}{(1+\phi)^n}+a\phi r^4\right]\left[1-\tfrac{1}{2}\phi\right]\right)$$

This is designed so that when $\phi = 0$, the weighting function has its original value, but as $\phi \to 1$ the weight of the background field decreases relative to the observations and the distance at which an observation has an effective weight of zero decreases. This is to allow for the fact that the distance at which an observation is a good estimate of the height at the grid point decreases as that grid point moves nearer to the centre of a depression. The actual value of the distance at which the weight of an observation reaches zero is about 500km, when analysing at the presumed centre of a low i.e. $\phi = 1$, so that the smallest such feature is presumed to have a half wave length of 1000km (trough to ridge). The other reason for using a tight weighting function near the centres of depressions is that there can be an error in the estimate of the position of the actual centre due to the distribution of the observations themselves unless their weight is reduced fairly sharply with distance. The background field has a lower weight relative to the observations on the plane fit scans, since it is the product of quadric fitting and the values of the analysed heights close to low centres will be poor estimates of the final values.

5. Vertical Interpolation

The interpolation formulae for the three non-standard levels (600, 800, 900) as described in Technical Note No. 37 are unsatisfactory when the ascent is colder and more stable than the ICAO atmosphere, in that they generate unrealistic static stabilities. To overcome this problem, it was decided to fit at each grid point (independently) a polynomial function of log p to the analysed heights at the bottom six standard levels of the atmosphere viz 1000, 850, 700, 500, 400, 300mb. So that small errors in these values, which might affect the resultant static stabilities, may be smoothed out, a polynomial of fourth order is fitted by least squares, giving the errors one degree of freedom (a fifth degree polynomial itself). The values for the analysed heights are then derived from this polynomial for the required levels viz 1000, 900, 800, 700, 600, 500, 400, 300mb. Since the data points for the least squares fitting are the same at all grid points, a set of orthogonal polynomials is generated over the six standard levels so that the actual fitting process may be as fast as possible. It would be possible to perform the least squares fit with the 300mb level fixed so that the 200-300mb thickness remained unchanged. The magnitude of the normalised error polynomial is sufficiently small, however, for the 300mb level to be effectively fixed in the least squares fitting. The extra time used in solving the least squares problem at each grid point with this constraint, was felt to be wasted.

6. Further modifications

6.1. It was found in practice that the 1000-900 and 900-800 thicknesses had a tendency to become too large near the centres of deep lows using the original constants (as specified in Technical Note No. 37) both for adjusting the 850 and 700mb background fields in the light of the 1000 and 500mb analyses and also in the distance weighting functions used in the analyses of these two levels. The distance weighting functions for the final group of levels (850, 700, 400, 200) have therefore been tightened up so that the data should be fitted more closely. At the same time, the 850 and 700mb background field adjustment constants have been altered to give the 1000mb field change more weight than previously.

6.2. The geostrophic relative vorticity of the analysed height fields on the fine mesh have been unsatisfactory in that they contain a large number of short wave features of significant amplitude. In any analysis system which treats each grid point independently, the continuity of the resultant field, and therefore the smoothness of the derivatives of that field, depends on the continuity of the data between successive grid points. It has been found that by using only the six nearest stations to a grid point, neighbouring grid points in dense data areas tend to be analysed on almost completely different sets of data and, as a result, noise on the data, in the form of random observational errors, has been fitted as real meteorological data, thus producing roughnesses. This problem has been overcome by increasing the number of stations taken at a grid point from six to twelve. To prevent observations being accepted from too far away in sparse data areas, the radius of the area from within which data may be selected at a grid point has been reduced. (See Appendix for new values)

The other modification connected with geostrophic vorticities lies in the way that the nine background points are selected near the edge of the field. The system of choosing the points described in Technical Note No. 37 created substantial roughnesses in the field in the outer three rows of the analysis; this has been replaced by the following system. When within three rectangle grid lengths of any edge, the nine background points are taken to be those that would correspond to the point three grid lengths from the edge on the same row or column, except that the background point nearest to the grid point under consideration is replaced by that grid point. See Fig. 1.

APPENDIX

1. Elimination of λ, μ from equations for $c^{(3)}$, $c^{(5)}$, $c^{(5/10)}$, $c^{(3/5)}$, $c^{(10)}$

$$c^{(3)} = c_N^{(3)} + \lambda\, p^{(3)} \tag{18}$$

$$c^{(5)} = c_N^{(5)} + (\mu - \lambda)\, p^{(5)} \tag{19}$$

$$c^{(10)} = c_N^{(10)} - \mu\, p^{(10)} \tag{20}$$

$$c^{(3/5)} = c_N^{(3)} - c_N^{(5)} - \lambda\, p^{(5)} \tag{21}$$

$$c^{(5/10)} = c_N^{(5/10)} - \mu\, p^{(5/10)} \tag{22}$$

$$c^{(3)} - c^{(5)} = c^{(3/5)} \tag{23}$$

$$c^{(5)} - c^{(10)} = c^{(5/10)} \tag{24}$$

Equations (18) - (24) are equations (5), (6), (12)-(14), (16), (17) rewritten with $p^{(i)} = \omega^{(i)} / \chi^{(i)}$. Substituting (18), (19), (21) in (23) gives

$$c_N^{(3)} - c_N^{(5)} + \lambda\,(p^{(3)} + p^{(5)}) - \mu\, p^{(5)} = c_N^{(3)} - c_N^{(5)} - \lambda\, p^{(5)}$$

i.e. $$\lambda\,(p^{(3)} + 2p^{(5)}) = \mu\, p^{(5)} \tag{25}$$

Substituting (19), (20), (22) in (24) yields

$$c_N^{(5)} - c_N^{(10)} + \mu\,(p^{(5)} + p^{(10)}) - \lambda\, p^{(5)} = c_N^{(5/10)} - \mu\, p^{(5/10)}$$

i.e. $$c_N^{(5)} - c_N^{(10)} - c_N^{(5/10)} = \lambda\, p^{(5)} - \mu\,(p^{(5)} + p^{(10)} + p^{(5/10)}) \tag{26}$$

Writing $K = c_N^{(5)} - c_N^{(10)} - c_N^{(5/10)}$ and substituting from equation (25) in equation (26) for λ in terms of μ

$$K = \mu \left\{ \frac{p^{(5)^2}}{p^{(3)} + 2p^{(5)}} - (p^{(5)} + p^{(10)} + p^{(5/10)}) \right\}$$

i.e.

$$\mu = \frac{-\; K\,(\,p^{(3)} + 2p^{(5)}\,)}{(p^{(3)}p^{(5)} + p^{(3)}p^{(10)} + p^{(3)}p^{(5/10)} + 2p^{(5)}p^{(10)} + 2p^{(5)}p^{(5/10)} + p^{(5)^2}\,)}$$

Writing $D = p^{(3)}p^{(5)} + p^{(3)}p^{(10)} + p^{(3)}p^{(5/10)} + 2p^{(5)}p^{(10)} + 2p^{(5)}p^{(5/10)} + p^{(5)^2}$

$$\mu = -K\,(p^{(3)} + 2p^{(5)})/D \tag{27}$$

$$\lambda = -K\,p^{(5)}/D$$

Substituting from (27) in (18), (19), (20), (22)

$$c^{(3)} = c_N^{(3)} - Kp^{(3)}p^{(5)}\,/\,D$$

$$c^{(5)} = c_N^{(5)} - K\,(\,p^{(3)} + p^{(5)})\,p^{(5)}\,/\,D$$

$$c^{(10)} = c_N^{(10)} + K\,p^{(10)}\,(p^{(3)} + 2p^{(5)}\,)\,/\,D$$

$$c^{(5/10)} = c_N^{(5/10)} + K\,p^{(5/10)}\,(p^{(3)} + 2p^{(5)}\,)\,/\,D$$

Hence we have obtained $c^{(3)}, c^{(5)}, c^{(10)}, c^{(5/10)}$ in terms of

$c_N^{(3)}, c_N^{(5)}, c_N^{(10)}, c_N^{(5/10)}$, and $p^{(3)}, p^{(5)}, p^{(10)}, p^{(5/10)}$

$$p^{(i)} = \frac{1}{\chi^{(i)}}\,\underline{\Lambda}\cdot\underline{\underline{A}}^{(i)^{-1}}\cdot\underline{\Lambda}^{T}\,, \quad c_N^{(i)} = \underline{\Lambda}\cdot\underline{\underline{A}}^{(i)^{-1}}\cdot\underline{d}^{(i)}$$

The operator $\underline{\Lambda}\,.\,\underline{\underline{A}}^{(i)^{-1}}$ represents the solution of the set of equations $\underline{\underline{A}}^{(i)}\cdot\underline{Q}^{(i)} = \underline{d}^{(i)}$ for one unknown. $p^{(i)}$ and $c_N^{(i)}$ can be obtained simultaneously since they are the results of the same operator applied to different vectors.

500mb, 300mb, T/T

Scan	1	2	3	4
p'	1.524158×10^{-7}	1.524158×10^{-7}	1.524158×10^{-5}	1.524158×10^{-5}
q	0.0625	0.0625	0.0625	0.0625
T^2	16	16	64	64
n	8	8	8	8
G	36	36	18	18
Background	12hrF/C	12hrF/C	2nd Scan	3rd Scan

200mb, 400mb, 700mb, 850mb

Scan	1	2	3
p'	1.524158×10^{-5}	1.524158×10^{-5}	1.524158×10^{-3}
q	0.0625	0.0625	0.0625
T^2	16	16	64
n	8	8	8
G	18	18	18
Background†	Adjusted 12hrF/C	Adjusted 12hrF/C	2nd Scan

* The central grid point at 500mb, 300mb, T/T has a weight of $8q$ and at 200mb, 400mb, 700mb, 850mb it has a weight of $16q$.

† The derivation of the background field at 700mb and 850mb from the 12hr forecast field has been changed as follows.

Let h_n^B = 12 hour forecast field

$h_n^{\prime B}$ = adjusted 12 hour forecast

h_n^A = analysed height field

Then
$$h'^B_{700} = h^B_{700} + \tfrac{1}{2}\left\{ (h^A_{500} - h^B_{500}) + (h^A_{1000} - h^B_{1000}) \right\}$$

$$h'^B_{850} = h^B_{850} + 0.8\,(h^A_{1000} - h^A_{1000}) + 0.2\,(h^A_{500} - h^B_{500})$$

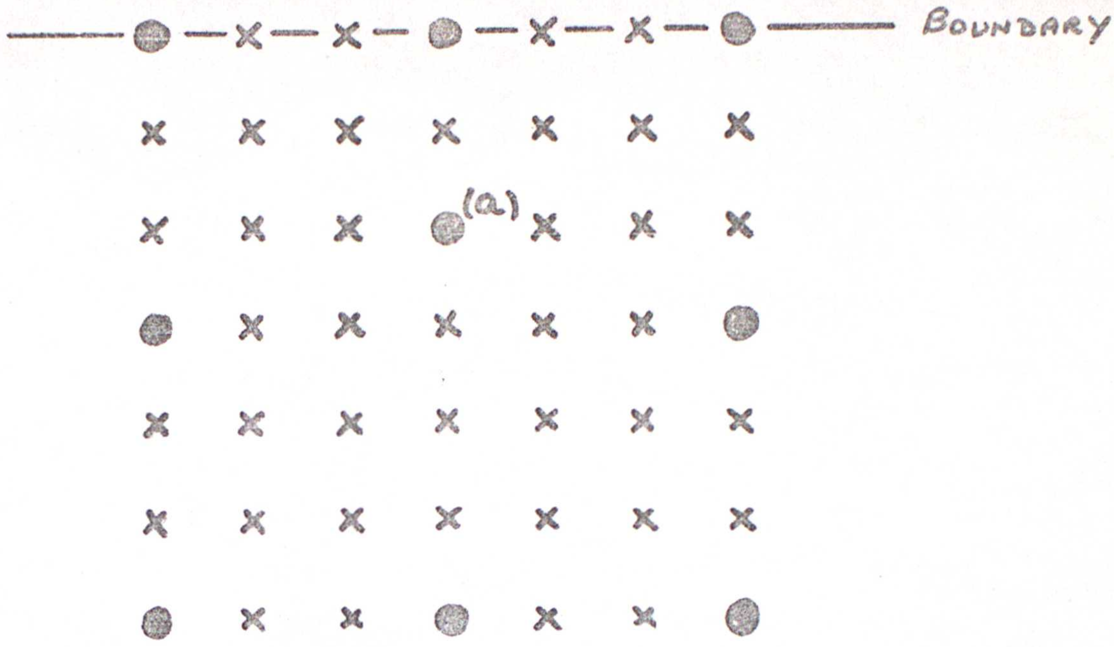
12 stations instead of six are used in the formation of the error estimate E_n at all upper air levels.

Surface (1000mb)

Scan	1	2	3	4
p'	1.524158×10^{-7}	1.524158×10^{-5}	1.524158×10^{-5}	1.524158×10^{-4}
q	0.0625	0.0625	-	-
T^2	16	16	-	-
n	8	8	8	8
G	18	18	12	12
Background	12hrF/C	12hrF/C	2ndScan	3rdScan

18 stations instead of 10 are used in the formation of the error estimate E and also in the plane fit process.

There are no changes to the constants at 100mb.



(a) is the point at which the analysis is being performed.

X Rectangle grid points

● Background points

Fig. 1

In the objective analysis of relative humidity as described in Met O 11 Technical Note No. 37, the assumption is made that the relative humidity for the lowest layer of the model is equal to the relative humidity at the surface. This assumption gave rise to a diurnal variation, in the initial data, of the moisture content of the lowest layer of the model, which in turn led to a diurnal variation in the rainfall forecasts - those based on midnight data giving more rain than those based on midday data. Investigation has shown that over Europe and the North Atlantic there is a considerably better correlation between the humidity mixing ratio at the surface and the mean humidity mixing ratio over the lowest 100mb of the atmosphere than between the corresponding relative humidities.

The analysis of the 950mb layer in the relative humidity analysis has been changed. In the final two scans at 950mb each surface relative humidity observation within the analysis area of the rectangle has been replaced by an estimate of the mean relative humidity over the 950mb layer, at the surface observation, using the following formula.

$$R_{950} = \frac{0.8 \text{ r surface}}{r_s}$$

where

R_{950} is the estimated relative humidity for the 950mb layer.

r surface is the surface humidity mixing ratio in gm kg^{-1} derived from the reported surface temperature, dewpoint and pressure. The formulae used are

$$e = \exp\left(1.80951 + \frac{17.27\, T_d}{T_d + 237.3}\right)$$

$$\text{r surface} = \frac{621.97e}{p-e}$$

where e is the vapour pressure in mb

T_d is the dewpoint in °C

p is the surface pressure in mb

r_s is the interpolated value at the surface observation of the saturated humidity mixing ratio (in gm kg^{-1}) derived from the thickness of the lowest layer obtained from the analysed height fields. The derivation of r_s from the thickness is the same as used in the 10 level model itself. A description may be found in "Changes in the Formulation of the 10 level Model", Burridge, Gadd and White, May 1972.

Outside the analysis area of the rectangle the 950mb relative humidity is assumed equal to the surface relative humidity as it is impossible to calculate r_s.