MET O 11 TECHNICAL NOTE NO. 90

REPORT ON CENTRED OCTAGON FORECASTS

DAVID A FORRESTER

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1. INTRODUCTION

Two versions of the 10-level octagon model have been written using a centred (leapfrog) time-step. In one model (referred to as TS) the grids are staggered in time whilst in the other model (referred to as TU) the grids are unstaggered in time. In both models the grids are staggered in space.

The models are semi-implicit and incorporate the same physics, surface exchanges and surface friction as the split semi-implicit Lax Wendroff 10-level model (Burridge and Gadd 1975). The diffusion scheme used in the TU version is a non-linear $K|\nabla^2|\nabla^2$ type, and time smoothing of the form

$$\overline{\varphi}^{n} = \varphi^{n} + \alpha\left(\varphi^{n+1} - 2\varphi^{n} + \overline{\varphi}^{n-1}\right)$$

with $\alpha = 0.005$ is incorporated. In the TS version, a modified non-linear Dufort - Frankel type diffusion (which incorporates some time smoothing) is used in the form

$$K\,|\tilde{\nabla}^2|\,\tilde{\nabla}^2$$

where $$\left(\tilde{\nabla}^2\varphi\right)^{n} = \frac{1}{\Delta x^2}\left[\left(\overline{\varphi}^{xy}\right)^{n} - \left(\varphi\right)^{n-1}\right]$$

Apart from the diffusion and time smoothing, the models were made as similar to each other as possible.

For a mathematical discussion of the models see Forrester (1976).

The general results are described in Section 5. Detailed results of 7 case studi including charts are in a separate report (Forrester 1977a).

2. GRIDS AND FINITE DIFFERENCE EQUATIONS

The grids used are shown in Fig 1. Grid A is used at each time level in the TU model. Grids A, B are used at alternate time levels in the TS model.

The differential equations are described in Forrester (1976). The finite difference forms of the equations are given below, using the standard notation

$$u_x \equiv \frac{1}{\delta x}\left[u(x+\tfrac{1}{2}\delta x) - u(x-\tfrac{1}{2}\delta x)\right]$$

$$\overline{u}^x \equiv \tfrac{1}{2}\left[u(x+\tfrac{1}{2}\delta x) + u(x-\tfrac{1}{2}\delta x)\right]$$

TU model

$$u_{2t} + \mu\left(u\, u_{2x} + \overline{v}^{xy} u_{2y}\right) + \overline{\omega}^{px} u_{2p} + g h_x + \tfrac{1}{2}\left[(u)^2 + (\overline{v}^{xy})^2\right]\mu_x - f\,\overline{v}^{xy} = 0$$

$$v_{2t} + \mu\left(\overline{u}^{xy} v_{2x} + v\, v_{2y}\right) + \overline{\omega}^{py} v_{2p} + g h_y + \tfrac{1}{2}\left[(\overline{u}^{xy})^2 + (v)^2\right]\mu_y + f\,\overline{u}^{xy} = 0$$

$$h'_{2t} + \mu\left(\overline{u}^{px} h'_{2x} + \overline{v}^{py} h'_{2y}\right) + \beta\omega = 0$$

$$h^{10}_{2t} + \mu\left[\overline{u}^{px}(h^{10}-H)_{2x} + \overline{v}^{py}(h^{10}-H)_{2y}\right] + \beta\omega = 0$$

$$r_{2t} + \mu\left[\overline{u}^{px} r_{2x} + \overline{v}^{py} r_{2y}\right] + \omega\, r_{2p} = 0$$

TS model

$$u_{2t} + \mu\left(\bar{u}^{xy}\bar{u}^{y}_{x} + v\,\bar{u}^{x}_{y}\right) + \bar{\omega}^{py}\,\bar{u}^{xy}_{2p} + g\,\bar{h}^{y}_{2x}$$

$$+ \tfrac{1}{2}\left[(\bar{u}^{xy})^{2} + (v)^{2}\right]\mu_{x} - f v = 0$$

$$v_{2t} + \mu\left(u\,\bar{v}^{y}_{x} + \bar{v}^{xy}\bar{v}^{x}_{y}\right) + \bar{\omega}^{px}\,\bar{v}^{xy}_{2p} + g\,\bar{h}^{x}_{2y}$$

$$+ \tfrac{1}{2}\left[(u)^{2} + (\bar{v}^{xy})^{2}\right]\mu_{y} + f u = 0$$

$$h'_{2t} + \mu\left[\bar{u}^{py}\,\overline{h'}^{y}_{x} + \bar{v}^{px}\,\overline{h'}^{x}_{y}\right] + \bar{\beta}^{xy}\,\bar{\omega}^{xy} = 0$$

$$h^{10}_{2t} + \mu\left[\bar{u}^{py}\,\overline{(h^{10}-H)}^{y}_{x} + \bar{v}^{px}\,\overline{(h^{10}-H)}^{x}_{y}\right] + \bar{\beta}^{xy}\,\bar{\omega}^{xy} = 0$$

$$r_{2t} + \mu\left[\bar{u}^{py}\,\bar{r}^{y}_{x} + \bar{v}^{px}\,\bar{r}^{x}_{y}\right] + \bar{\omega}^{xy}\,\bar{r}^{xy}_{2p} = 0$$

3. MAXIMUM TIMESTEP FOR LINEAR COMPUTATIONAL STABILITY

The condition for linear computational stability is

$$\lambda^2 \leq 1 .$$

where
$$\lambda = \alpha_U \lambda_U + \alpha_V \lambda_V + \alpha_c \sqrt{\lambda_{c1}^2 + \lambda_{c2}^2}$$

with
$$\alpha_U = U \frac{\delta t}{\delta x} \qquad \alpha_V = V \frac{\delta t}{\delta x} \qquad \alpha_c = c \frac{\delta t}{\delta x}$$

For the TU model

$$\lambda_U = \sin n\delta x$$

$$\lambda_V = \sin m\delta y$$

$$\lambda_{c1} = 2 \sin \tfrac{1}{2} n\delta x$$

$$\lambda_{c2} = 2 \sin \tfrac{1}{2} m\delta y$$

and for the TS model

$$\lambda_U = 2 \sin \tfrac{1}{2} n\delta x \cos \tfrac{1}{2} m\delta y$$

$$\lambda_V = 2 \sin \tfrac{1}{2} m\delta y \cos \tfrac{1}{2} n\delta x$$

$$\lambda_{c1} = \sin n\delta x \cos \tfrac{1}{2} m\delta y$$

$$\lambda_{c2} = \sin m\delta y \cos \tfrac{1}{2} n\delta x$$

where n, m are the wave numbers in the x, y directions and (U,V) and c are the advection and gravity wave speeds.

Allowing for the variation of the direction of the advection wind to give the most unstable situation for given n, m one finds

$$\lambda = \alpha\sqrt{\lambda_u^2 + \lambda_v^2} + \alpha_c\sqrt{\lambda_{c1}^2 + \lambda_{c2}^2}$$

where

$$\alpha = \sqrt{U^2 + V^2}\,\frac{\delta t}{\delta x}$$

Table 1 gives the maximum values of $\frac{\delta t}{\delta x}$ for various values of c and $\sqrt{U^2+V^2}$ for both the TU and TS models.

With a grid length δx = 300 Km, this indicates a maximum timestep (in the semi-implicit case with 2 implicit modes) of 20 minutes for the TU model and 22½ minutes for the TS model if the maximum wind speed is 100 ms^{-1}, and 27 minutes for the TU model and 38 minutes for the TS model if the maximum wind speed is 50 ms^{-1}.

4. EXPERIMENTS

Integrations to 6 days were carried out at weekends using the 12Z Sunday update analysis as initial fields. Runs of the TU version began on 8/2/76 and were carried on until 6/2/77. The TS version was run successfully to 6 days on the following occasions 7/11, 21/11, 28/11 1976 and 2/1, 9/1, 16/1, 23/1 1977.

The radiation scheme was incorporated into the TU version and run on 5/12 1976 and 2/1, 9/1, 16/1, 23/1, 30/1 1977.

A statistics package written originally for the Fine Mesh Octogan (White, 1976) was run on 10/10, 17/10, 24/10, 31/10, 7/11 1976 and 9/1, 16/1, 23/1 1977 to produce RMS height errors (over the whole octagon and also over a 20 x 20 rectangle over the British Isles) for the TU, TS and split explicit models, and Hovmöller diagrams for the TU, TS and split explicit models and the verifying initialisations.

The models were run using a $7\frac{1}{2}$ minute time-step. A 6 day forecast required 60 minutes CPU. It was later found that with increased time smoothing ($\alpha = 0.05$) the TU model would run successfully with a time-step of 15 minutes thus reducing the CPU time for a 6 day forecast to 30 minutes.

Before the experiment with increased time smoothing was done, it was thought that the form of the finite differencing used for the non-linear terms might be the cause of the instability which prevented runs with large time-steps. An experiment was carried out to modify the horizontal advection scheme in the TU model by making the following replacements.

$$u\,u_{2x} \rightarrow \overline{\bar{u}^{x} u_{x}}^{x} \qquad \bar{v}^{xy} u_{2y} \rightarrow \overline{\bar{v}^{x} u_{y}}^{y}$$

$$\bar{u}^{xy} v_{2x} \rightarrow \overline{\bar{u}^{y} v_{x}}^{x} \qquad v\,v_{2y} \rightarrow \overline{\bar{v}^{y} v_{y}}^{y}$$

$$\bar{u}^{x} h'_{2x} \rightarrow \overline{u\,h'_{x}}^{x} \qquad \bar{v}^{y} h'_{2y} \rightarrow \overline{v\,h'_{y}}^{y}$$

and similarly for h_{1000} and r.

The resulting model (still semi-implicit) was run on one particular occasion with the modifications applied only to u,v and again with the modifications applied to h', h_{1000} and r also. There was no improvement in the maximum time-step that could be used. Furthermore, there was very little difference in the forecasts produced.

5. GENERAL RESULTS

During the period 8/2/76 to 7/3/76 the TU model was compared with the semi-implicit operational model and also with the Fine Mesh version of the octagon (with a 150 Km grid length). In general the Centred Octagon forecasts were similar to the Fine Mesh forecasts which differed slightly from, and were generally slightly better than the Operational forecasts. On at least one occasion the Centred Octagon was slightly better than the Fine Mesh over part of the chart.

On 14/3/76 the Split Explicit model (referred to as SE) became operational and was thenceforth used as a comparison.

The differences between the TU model forecasts and the SE forecasts were on most occasions small. On some occasions, the TU model was noticeably better over part of the octagon than the SE model, but noticeably worse over other parts.

It has proved extremely difficult to draw general conclusions as to the circumstances under which the TU model is better or worse than the SE model. There is no decisive case for either model being consistently better on amplitudes or phase speeds.

The TS model forecasts were rather disappointing. In general, they were worse than the TU model forecasts. To what extent this is attributable to the differing diffusion and time-smoothing is uncertain, but it is believed that this effect is small. The theoretical advantages of the time staggered grid (see Forrester, 1977b) were not at all apparent in these experiments.

Considering the 7 case studies described elsewhere it is concluded that the TU model is slightly better than the SE model overall on four occasions, slightly worse on two occasions and that both models are about equal on the other occasion. The TS model is worse than the TU model on most occasions.

The inclusion of the radiation scheme into the TU model was generally beneficial, raising both low and high pressure areas by 3 or 4 mb.

On one occasion (7/11/76) the TU model was also run using diffusion coefficients equal to half the usual value. This improved the surface forecast at 3 days by decreasing the low pressures by about 4 mb, with no noticeable change in the position of the features.

500 mb RMS height errors were computed on several occasions. These were not found particularly helpful, and were frequently contradictory to the subjective assessment of the forecasts. The TU model usually had a larger RMS error at day 3 than the other models. This is due at least partly to an overall mean error in the 500 mb height field of the TU model forecast, the cause of which is unknown. The TS model RMS error was about the same as that of the SE model. The RMS errors at day 3 are included in the separate report.

Hovmöller diagrams for wave numbers 1 to 3, 4 to 5, 6 to 10 and 11 to 16 were obtained on certain occasions for 500 mb heights at 50°N for the models and the verifying initialisations. These diagrams show that all 3 models are very similar at predicting the phase speeds and amplitudes of features, and that all models make the same basic errors.

6. CPU EFFICIENCY

Counting the number of multiplications (M) and additions (A) for the (explcit) dynamical equations for the TU, TS and SE models gives an estimate of the relative speeds of the different models. Weighting M as 3 cycles and A as 2 cycles one finds (approximately)

TU 373M + 560A = 2239 cycles per time-step

TS 393M + 759A = 2697 cycles per time-step

SE 676M + 1657A = 5342 cycles per time-step

Thus the explicit part of the computation in the TU model using a 15 minute time-step is faster than the SE model using a 30 minute time-step.

The implicit part of the calculation adds about 10% more CPU time for every gravity mode.

The optimum number of modes to treat implicitly (to minimise CPU time) is probably 3 or 4.

Thus a semi-implicit TU model will be at least as efficient as a SE model.

7. PROGRAMMING NOTES

The following object and load modules were used:

(1) ALOAD(NTUCOCT)

TU centred octagon (with non-linear diffusion)

(2) ABOJ(NCEVENU2, NCEVENV2, NCEVNTH2, NCEVNHØ2, NCEVENR2)

ALOAD(NTUCOCT)

modified advection (can include u,v,h,r or just u,v)

(3) AOBJ(NROLDMN, NROCTSF, NRXPADJ2, NRLWRAD, NROCTPH2)

ALOAD(NTUCOCT)

TU with radiation (requires LATLONG in column)

(4) ALOAD(NTUCOCTX)

explicit TU version

(5) AOBJ(NTODUF6, NTEDUF6)

ALOAD(NTSCOCT)

TS version (with non-linear Dufort-Frankel type diffusion)

Also ASRCE(NXODDFLD) interpolates onto the odd grid. (Note that the title is moved from words 16-21 to words 59-64 of column 43 row 1).

REFERENCES

BURRIDGE, D.M. and GADD, A.J. 1975. Met O 11 Technical Note No. 48

FORRESTER, D.A. 1976 Met O 11 Technical Note No. 68

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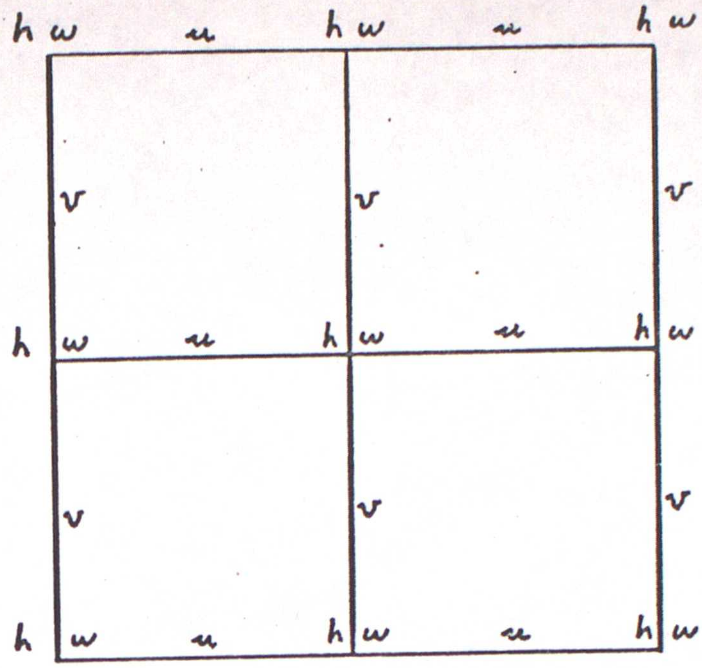
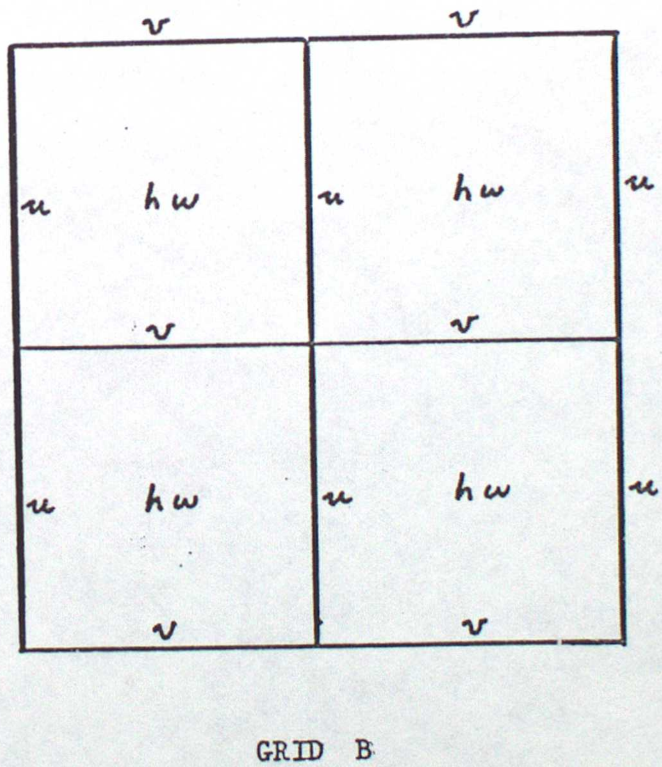
WHITE, P.W. 1976 Met O 11 Technical Note No. 70.



GRID A



GRID B

Fig.(1)

$\sqrt{U^2+V^2}$	c	$\left(\frac{\partial t}{\delta x}\right)_{TU}$	$\left(\frac{\delta t}{\delta x}\right)_{TS}$
50	300	0.0012	0.0025
50	100	0.0032	0.0056
50	50	0.0054	0.0076
100	300	0.0011	0.0022
100	100	0.0027	0.0038
100	50	0.0040	0.0045
150	300	0.0011	0.0019
150	100	0.0023	0.0029
150	50	0.0032	0.0032

TABLE 1

Maximum values of $\frac{\delta t}{\delta x}$ for the TU and TS models.

(Winds are in ms^{-1} and $\frac{\delta t}{\delta x}$ in $s\ m^{-1}$)