LONDON, METEOROLOGICAL OFFICE.

Met.O.15 Internal Report No. 002.

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London, Met.Off., Met.O.15 Intern.Rep.No.002, 1976, 31cm.
Pp. [9], 5 pls. 16 Refs.

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METEOROLOGICAL OFFICE
London Road, Bracknell, Berks.

MET.O.15 INTERNAL REPORT

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A BRIEF SURVEY OF SOME CHARACTERISTICS OF SNOWFLAKES

By

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DATE: NOVEMBER 1976

Cloud Physics Branch (Met.O.15)

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A Brief Survey of Some Characteristics of Snowflakes.

1. Introduction

Snowflakes are aggregations of ice crystals. The latter are individual ice particles whose shape and size depend upon conditions inside the cloud in which they are initially formed and grow. As the crystals become heavier they start to fall downwards through the cloud and sweep up other, smaller, crystals and cloud drops. In this way an aggregate of crystals is formed which may contain only a few crystals or may contain up to a hundred or more.

It has been found that the maximum fall speeds of snowflakes are $\sim$ 2 m sec^{-1} and consequently snow is found to fall only from layer clouds and from weak or decaying convective clouds which contain weak updraughts.

The size of the largest snowflake in a given storm shows a dependence upon ground temperature. The maximum surface temperature at which snow has been observed to fall is about + 4°C but this is a rare event; at + 1°C precipitation has a 50% chance (approx) of being in the form of snow.

2. Snowflake Fall Speeds and Densities

Detailed measurements of snowflake fall speeds have been made by Magono (1953), Langleben (1954), Magono and Nakamura (1965) and Jiusto and Bosworth (1971). The latter give a useful summary of past work.

Langleben's measurements were all made in terms of the size of the snowflake when melted. His empirical expression for velocity was $V = k\, d^{0\cdot 31}$

where d (cm) is the melted flake diameter. The value of k varies with the type of crystal forming the snowflake

crystal type	k
Prismatic columns and plates	234
Rimed dendrites	221
Dendrites (T = 33°F)	203
Dendrites (T < 32°F)	100

When the crystal type is uncertain k is often set at 200 (see fig. 1)

Expressions for fall velocity in terms of the dimensions of unmelted snowflakes have been given by Magono (1953) :

$$V = 132\left(\frac{r}{0\cdot 40 + 0\cdot 63r}\right)^{1/2} \text{ cm sec}^{-1} \quad \text{for unrimed flakes}$$

$$V = 194\left(\frac{r}{0\cdot 45 + 0\cdot 60r}\right)^{1/2} \text{ cm sec}^{-1} \quad \text{for rimed flakes.}$$

where Υ (cm) is the flake 'radius'. Because of the irregular shape of the snowflakes they were assigned a diameter given by the arithmetic mean of the long and short axes. The results are shown in fig. 1

Jiusto and Bosworth (1970) consider data from a number of sources; they do not distinguish between rimed and unrimed flakes but consider instead whether they are mainly dendritic or composed of plates and columns (compact flakes). The respective expressions are

$$V = 123\, \Upsilon^{0.2} \quad \text{cm sec}^{-1}$$
$$V = 178\, \Upsilon^{0.2} \quad \text{cm sec}^{-1}$$

they are plotted in fig. 1. For flakes composed of both dendritic and compact crystals, or where the composition is unknown, the mean of the two expressions is suggested :

$$V = 150\, \Upsilon^{0.2} \quad \text{cm sec}^{-1}$$

It can be seen that for a particular type of flake there is little variation of speed with diameter especially beyond about 2 cm diameter. Magono (1953) has explained this by considering the air flow not only around a falling flake but also through it. Neglecting the latter leads to a velocity dependence upon (radius of flake)$^{1/2}$. Air flowing through the flake introduces extra drag proportional to the volume of the flake; the terminal velocity shows a progressively weaker dependence upon (flake radius)$^{1/2}$ as the size increases.

Jiusto and Bosworth (1971) also quote an expression (due to Holroyd (1971)) for the density of dry snowflakes (i.e. those showing no sign of melting) :

$$\rho \Upsilon' = 0.0085 \quad \text{gm cm}^{-2}$$

where ρ = snowflake density (gms cm^{-3}) and

$\Upsilon' = d'/2$ and d' = the diameter of the flake defined as the harmonic mean of the long and short axes.

Holroyd's (1971) expression was derived from a re-analysis of some data of Magono and Nakamura (1965). Rogers (1974) has re-analysed the results on wet snowflakes to give

$$\rho \Upsilon' = 0.362 \quad \text{gms cm}^{-2}$$

but the fit is not good.

The original authors, Magono and Nakamura (1965) did not consider wet and dry flakes separately and found, for the mixture :

$$\rho d'^2 = 0.02 \quad \text{gms cm}^{-1}$$

3. Maximum Snowflake Sizes and Precipitation Rates

There is evidence to suggest that snowflakes reach their maximum sizes at temperatures close to $0^{\circ}C$. Quantitative evidence has come from Magono (1953), Hobbs (1973) and Rogers (1974) - figures 2a and 2b. The maximum dimensions are found to decrease rather rapidly at temperatures below $0^{\circ}C$ and there is a secondary maximum between $-10^{\circ}C$ and $-15^{\circ}C$.

The secondary maximum is thought to be due to the aggregation of dendritic crystals which form at temperatures between $-12^{\circ}C$ and $-16^{\circ}C$. The maximum at around $0^{\circ}C$ is thought to be due to preferential aggregation arising from increased adhesion near the melting point of ice (Mason (1971), Hobbs (1974)).

Precipitation rates are obviously related to snowflake sizes. Rogers (1974) has indicated that the greatest precipitation rates ($\gtrsim 2\frac{1}{2}$ mm. of water per hour) are associated with surface temperatures near $0^{\circ}C$.

4. Snowflake Size Distributions and Concentrations

The distribution of snowflake sizes has been studied by only a few workers and in almost all of these cases it is the melted snowflake diameter (i.e. the diameter of the spherical drop of water resulting from the melting of the flake) that has been measured, rather than the true dimension of the flake.

Whether melted or true sizes are measured the basic method is to collect the flakes on a horizontal surface of area A for a known time t. The surface distribution of the particles $N_h(D)$ is the number of particles of sizes between D and $D + \Delta D$, ie $n(D)$ falling onto unit area per unit time :

$$N_h(D) = \frac{n(D)}{A t \Delta D}$$

To convert this to the more useful volume or spatial concentration $N(D)$ requires knowledge of the fall speed $V(D)$ then

$$N(D) = \frac{n(D)}{A t \Delta D} \cdot \frac{1}{V(D)}$$

The units are usually n° of flakes/m^3/mm size interval.

Melted Flakes Quantitative measurements on melted flakes have been reported by Imai et al (1955), Magono (1957), Gunn and Marshall (1958) and Ohtake (1969, 1970).

The most useful are those of Gunn and Marshall (1958). They have obtained the size distribution of melted diameters (for diameters > 1 mm) for various intensities of precipitation - figure 3. Following the work of Marshall and Palmer (1948) on raindrop sizes Gunn and Marshall (1958) have fitted to their distribution data on equation of the form $N(D) = N_o \exp(-\Lambda D)$ $m^3 mm^{-1}$

where $D =$ water drop diameter (cm)

Λ and N_o were found to depend upon precipitation intensity (R, $mm\ hr^{-1}$):

$$N_o = 3.8 \times 10^3 \times R^{-0.87} \qquad m^{-3}\ mm^{-1}$$

$$\Lambda = 25.5 \times R^{-0.48} \qquad cm^{-1}$$

These authors used the results of Langleben (1954) to derive fall velocities.

In a survey of the available data Sekhon and Srivastava (1970) have found a more useful fit to be

$$N_o = 2.5 \times 10^3 \times R^{-0.94} \qquad m^{-3}\ mm^{-1}$$

$$\Lambda = 22.9 \times R^{-0.45} \qquad cm^{-1}$$

They also give equations for the precipitation content (W) and the median volume diameter of the precipitation (D_o):

$$W = 0.25\ R^{0.86} \qquad gm\ m^{-3}$$

$$D_o = 0.14\ R^{0.45} \qquad cm$$

The total concentration of snowflakes of all sizes (> 1 mm melted diameter) can be obtained by summing under the curves of fig. 3

Unmelted Flakes Measurements of unmelted flakes seem to have been made by only one worker - Rogers (1974).

Defining the snowflake diameter as the arithmetic mean of the greatest length and perpendicular width and considering sizes > 1 mm, Rogers (1974) has found that an equation of the form

$$N(D) = N_o \exp(-\lambda D)$$ fits the distribution

of flake sizes to a reasonable degree. The mean values of N_0 and λ are given as

$$N_0 = 130.0 \quad m^{-3}\,mm^{-1}$$

$$\lambda = 0.296\ mm^{-1} \quad (\text{st. dev. } 0.145\ mm^{-1})$$

He gives the relation between λ and R as

$$\lambda = 0.293\ R^{-1}$$

but does not relate N_0 and R (Using Rogers' (1974) data a relationship of the form $N_0 = aR^b$, where a and b are constants has been derived by the least squares method. The equation is $N_0 = 95R^{0.37}$ the data do, however, show considerable scatter around this line)

In deriving these results fall velocities were estimated from the results of Jiusto and Bosworth (1971). A summary of the characteristic parameters of the snowflake size distributions of Rogers (1974) is enclosed - table 1.

Since maximum snowflake dimensions are temperature dependent then λ could be expected to depend on temperature. Rogers (1974) gives graphs relating λ and temperature, also λ and maximum snowflake diameter - figures 4a and 4b. Rogers indicates that the exponential form of his equation is rather inadequate in describing the distribution of relatively large snowflakes in small sample volumes.

As before, the total numbers of flakes of all sizes ($>$ 1 mm) can be found by summing under the distribution curve.

The results of Sekhon and Srivastava (1970) and of Rogers (1974) for melted and unmelted snowflakes respectively are compared in Figure 5. The curves refer to a precipitation rate of 1 mm hr^{-1} since this is close to the mean of the data of Rogers (1974). Bearing in mind the distributions apply only for sizes $>$ 1 mm the total concentrations of melted and unmelted flakes are 110 m^{-3} and 330 m^{-3} respectively (obtained by integrating from 1 mm to ∞). There is a degree of consistency between the two curves. The median unmelted flake size is about 3 mm and using Magono and Nakamura's (1965) relation for a mixture of wet and dry flakes ($\rho d^2 = 0.02\ gm\ cm^{-1}$) yields a median density of about 0.2 gms cm^{-3}. This is reasonably comparable with a value of very roughly 0.3 gms cm^{-3} which could be expected from the equality of the precipitation rates.

In his table of results (table 1) Rogers (1974) gives the total concentrations for sizes $>$ 1 mm. It can be seen that this figure varies over about two orders of magnitude (20 - 1400 flakes/m^3) as does the precipitation rate (0.02 — 2.6 mm hr^{-1}). It should be noted that the maximum snowflake concentration does not necessarily lead to the maximum precipitation rate (Rogers' results are an example) since snowflake size is important and as already mentioned this is related to surface temperature.

5. Conclusions

Snowflake fall speeds have been measured by a number of workers and useful empirical equations found. There is good agreement between the results of Magono (1953) and Jiusto and Bosworth (1970).

Maximum snowflake sizes and maximum snowfall rates seem to be found at around $0^{\circ}C$ with lower values occurring at colder temperatures.

Experimental results on snowflake size distributions is sparse. What data there is shows the distribution to be an exponentially decreasing function of size. The parameters of the distribution (amplitude and slope) can be related to precipitation intensity.

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1	LANGLEBEN (1954	melted flakes (K = 200)
2	MAGONO (1953)	rimed flakes
3		unrimed flakes
4	JIUTSO AND BOSWORTH (1970)	compact flakes
5		dendritic flakes

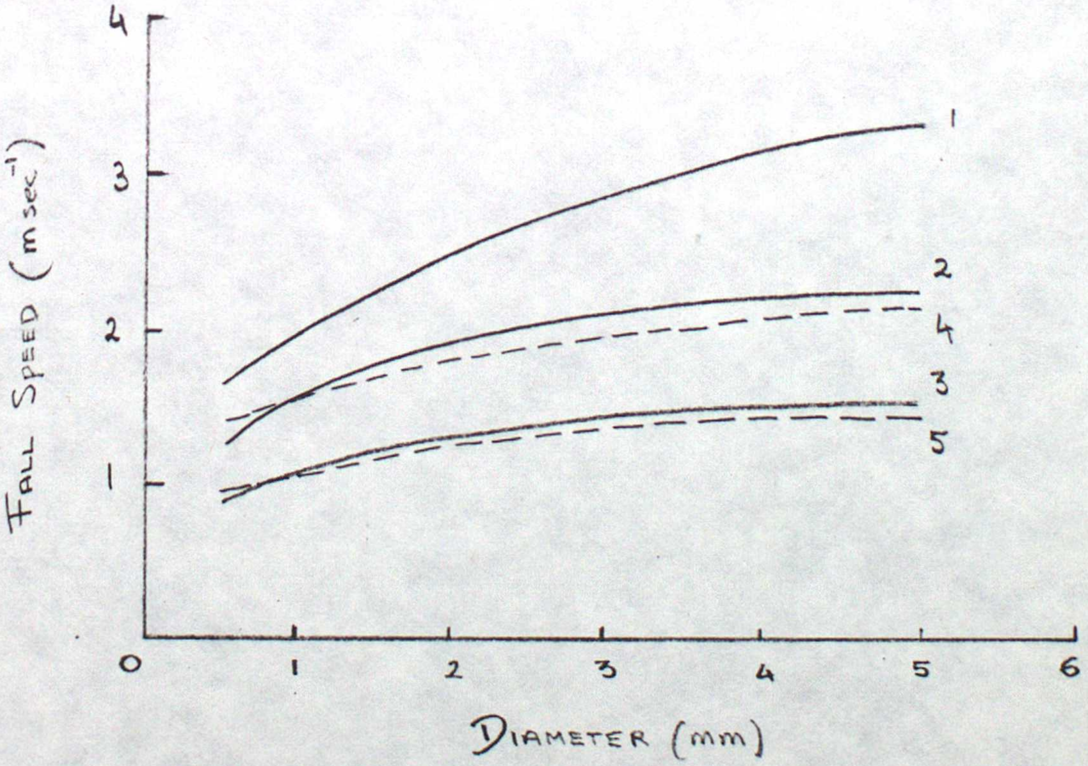

Figure 1. Snowflake fall speeds

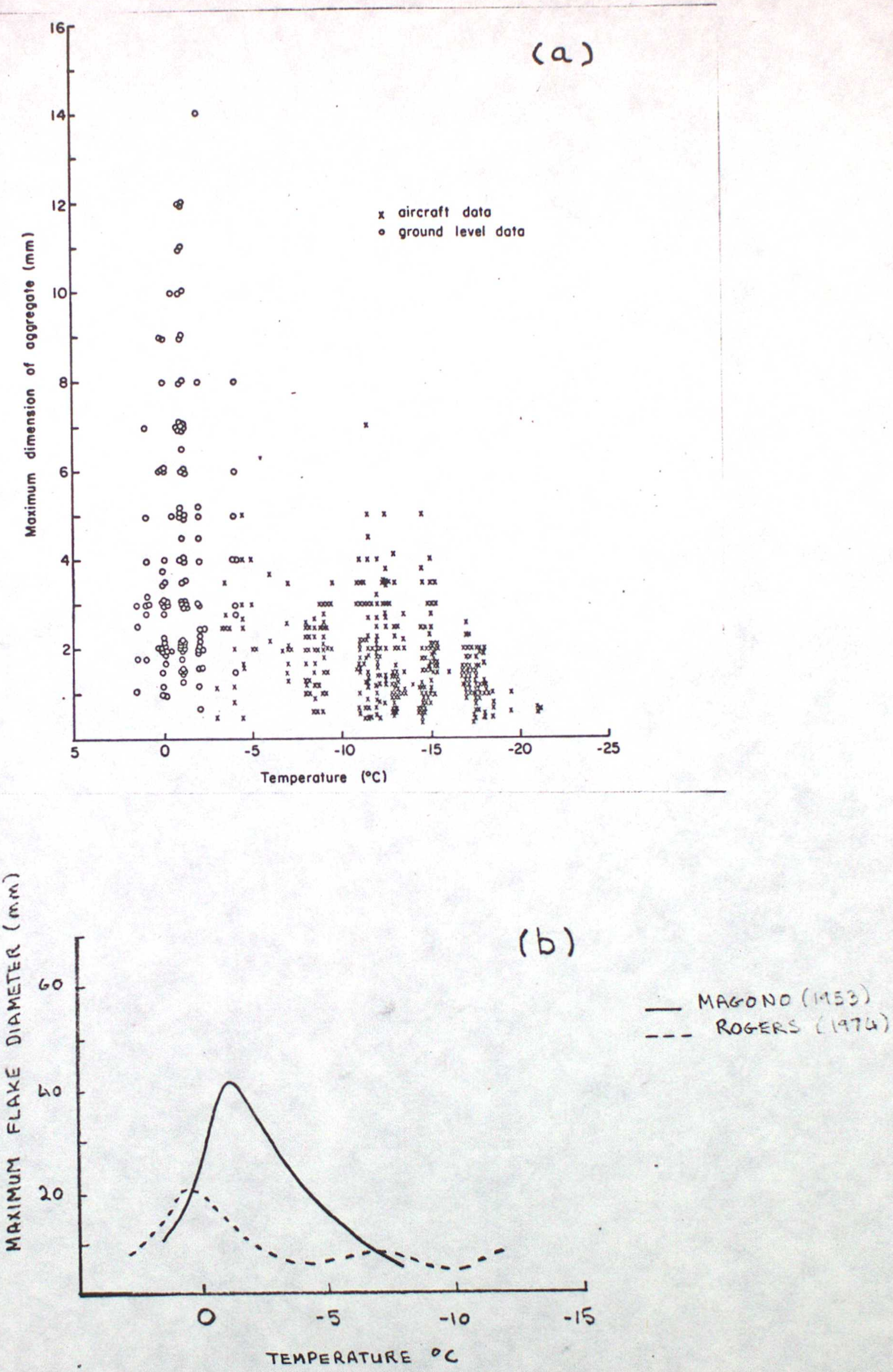

Figure 2. Maximum dimensions of snowflakes as a function of temperature.

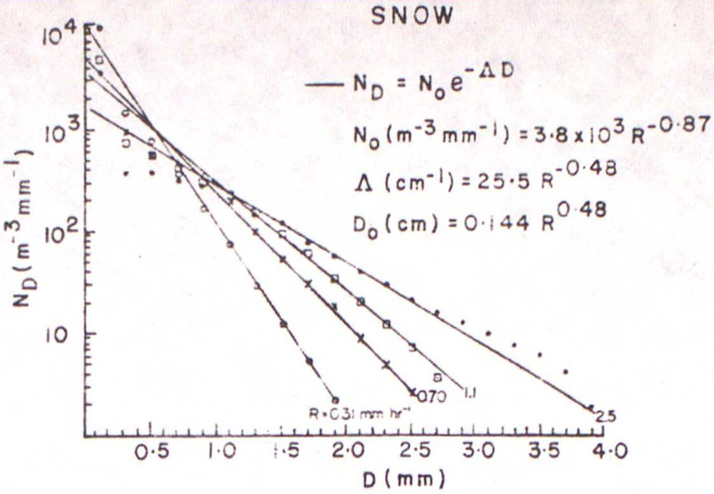

Figure 3. Size distributions of flakes in terms of melted diameter (Gunn & Marshall, 1958)

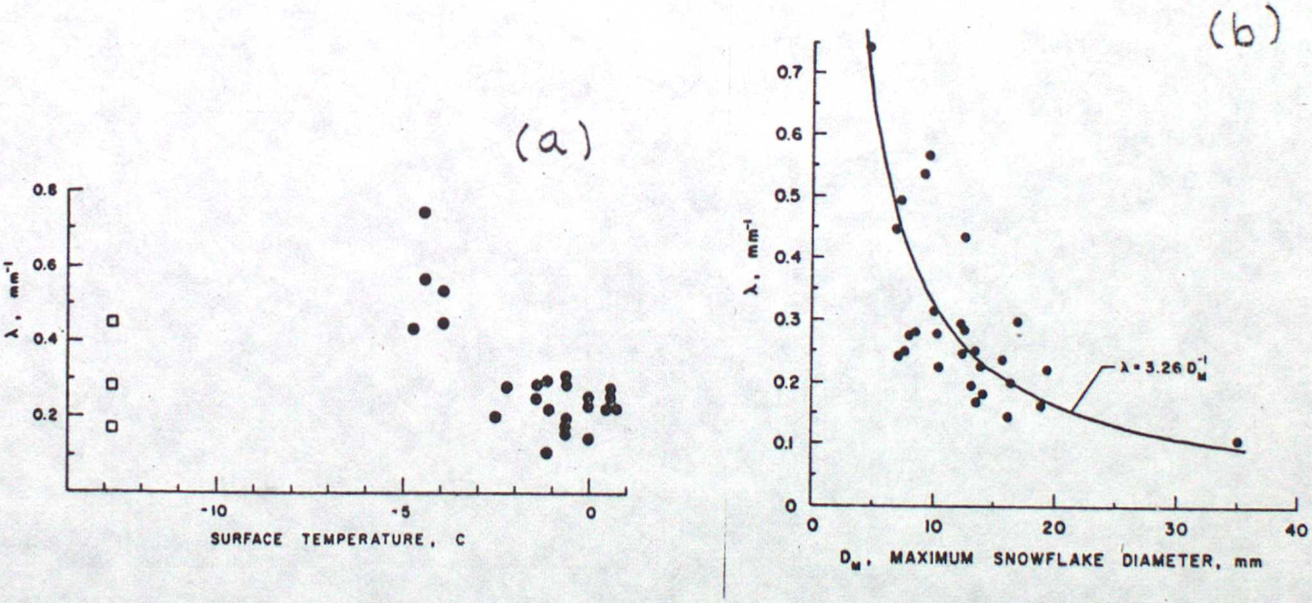

Figure 4. Dependence of λ upon surface temperature and maximum flake diameter

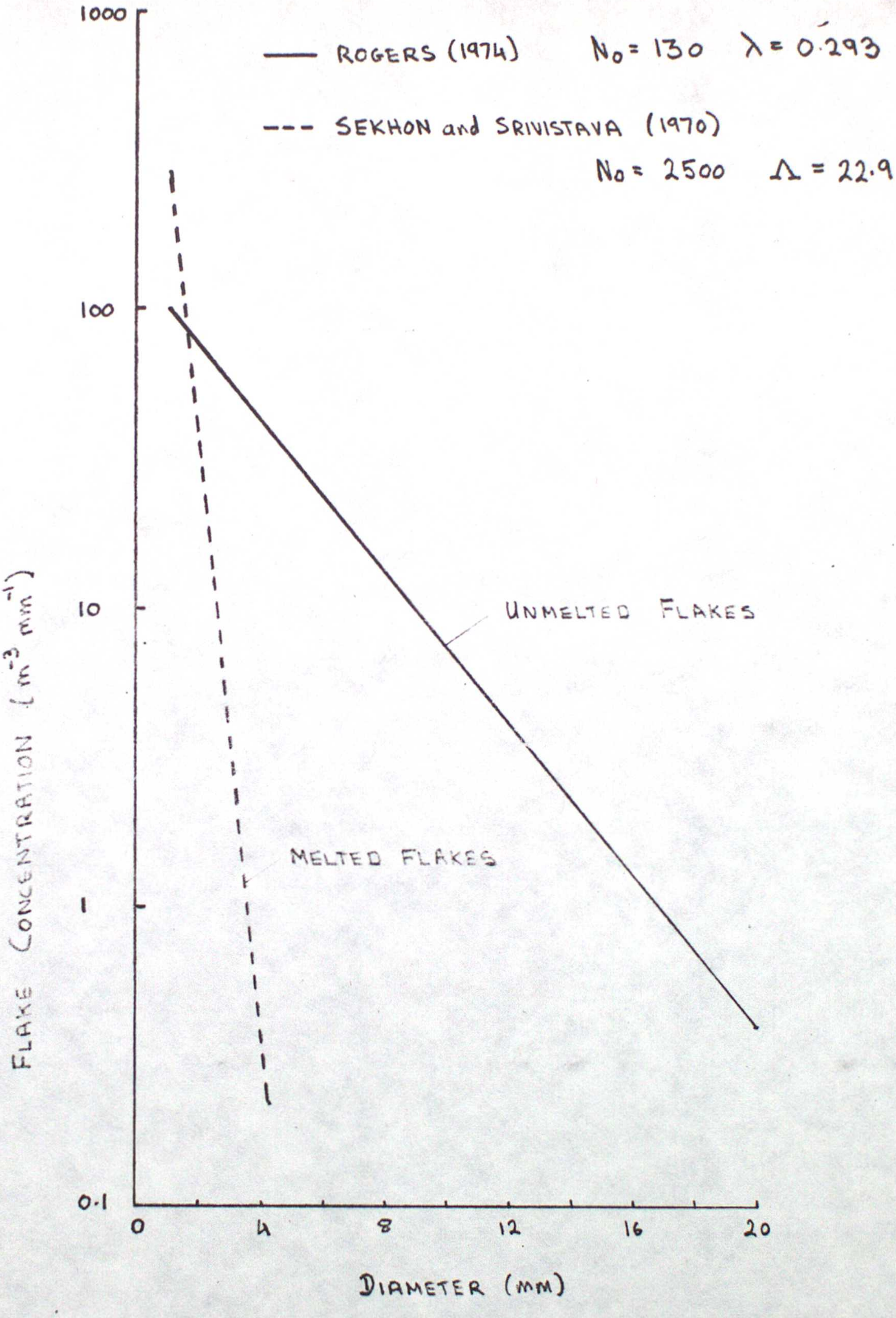

Figure 5. Comparison of melted and unmelted size distributions for a precipitation rate of 1 mm hr^{-1}

[redacted] A summary of the characteristic parameters of the 28 snowflake size distributions which were fitted to the exponential form, $N_D(D) = N_o \exp(-\lambda D)$.

T_{sfc}, C	N_o, m^{-3} mm^{-1}	λ, mm^{-1}	N_T, m^{-3}	D_m, mm	R, mm hr^{-1}
0.6	50.6	0.249	154.7	7.6	1.407
0.6	108.4	0.241	326.2	7.4	1.407
0.6	122.0	0.226	388.1	13.9	0.943
0.6	38.1	0.280	101.9	8.5	0.943
0.6	41.6	0.225	177.3	10.5	0.943
0.0	94.5	0.238	355.5	15.6	1.870
0.0	37.4	0.247	100.3	12.4	0.205
0.0	88.9	0.145	445.4	16.2	---
-0.3	55.7	0.180	226.6	14.2	2.469
-0.6	70.1	0.161	387.6	18.9	2.240
-0.6	85.1	0.313	223.6	10.1	1.870
-0.6	72.9	0.195	292.2	13.2	0.926
-0.6	131.2	0.299	520.6	16.9	2.038
-1.1	131.4	0.221	621.2	19.4	2.646
-1.1	41.2	0.109	446.9	35.1	2.646
-1.1	96.6	0.294	317.8	12.3	0.613
-1.4	79.5	0.250	262.0	13.5	0.613
-1.4	289.6	0.285	1372.8	12.6	1.263
-2.2	189.1	0.279	538.9	10.3	1.190
-2.5	33.2	0.200	139.8	16.4	1.190
-3.9	486.6	0.447	670.2	6.9	1.885
-3.9	246.3	0.534	327.9	9.2	1.885
-4.4	589.3	0.567	938.9	9.6	0.735
-4.4	246.0	0.742	354.5	4.9	0.735
-4.7	159.4	0.433	290.3	10.3	0.274
-12.8	15.1	0.277	54.1	8.1	0.023
-12.8	12.6	0.169	44.8	13.6	0.023
-12.8	27.8	0.492	20.4	7.4	0.023

Table 1. Data of Rogers (1974)