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DISCUSSION ON THE USE OF ELECTRICAL BAND PASS FILTERS AT CARDINGTON

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FH

CONTENTS

NOTATION

A	Linear gain of electrical filter units
a, b	Time constant of the low pass sections $(= (2\pi C_1 R_1)^2)$ and high pass sections $(= 1/(2\pi C_2 R_2)^2)$ respectively of the band pass filters
c	A constant
c_w	Kolmogorov constant for "transverse" wind fluctuations
c_T	Inertial sub-range constant for temperature fluctuations
E_o	Output peak to peak voltage magnitude from a filter unit of linear gain β (N.B. E_o is a function of frequency)
E_o'	As E_o but from the actual filter units of linear gain A
G	Peak frequency gain of the actual filter units.
$E_{o_{max}}, E'_{o_{max}}$	Maximum peak to peak voltages (in fact the output at the peak frequencies) of the β - filter and A- filter respectively
K_x, K_y	$E'_{o_{max}}/E_{o_{n_x}}$ and $E'_{o_{max}}/E_{o_{n_y}}$ respectively
L_1, L_2	n_x^2 and n_y^2 respectively
n	Frequency (Hz)
n_1, n_2	Half-power cut off frequencies
n_x, n_y	Any two frequencies
n_{peak}	Peak frequency
R	"Equivalent square - filter factor"
$S(n)$	Variance spectrum function
T	Temperature
$\bar{u}$	Mean horizontal wind speed ($cm\ sec^{-1}$)
β	Linear gain of theoretical filter
$\varepsilon_1, \varepsilon_2$	Rates of dissipation ($cm^2\ sec^{-3}$) estimated from the output of band pass filter unit 1 (BPFUI) and band pass filter unit 2 (BPFU2) respectively
χ_1, χ_2	Rate of dissipation of temperature fluctuations ($\frac{1}{2}$ $°C^2 sec^{-1}$) (NB corresponding to the rate of change with time of half the variance of temperature)

τ, s Sampling and averaging times respectively

ϕ Inclination of the air flow to the horizontal

σ^2_{BP} Variance of the output from the filters units

$f(-)^2$ Represents the calculation of the variance of the quantity inside the brackets (eg $f(x)^2 = \sum (x-\bar{x})^2 / (N-1)$ where $N =$ no. of points)

S_ϕ, S_T Scaling factors in radians per volt and $^{\circ}C$ per volt respectively

INTRODUCTION

For a number of years the Jones' band pass filter units (BPFU's) (JONES 1963) have been used to examine turbulent fluctuations within limited frequency bands. In particular two BPFU's, lying within the inertial sub-range of frequencies, have been used to measure wind and temperature fluctuations; thereby enabling estimates of ε and χ to be made using the inertial sub-range similarity relations (eg KOLMOGOROV, 1941 PASQUILL & TYLDSLEY, 1967).

This short note shows that for the 3 years 1968, 1969 and 1970 ε and χ had been over-estimated by about 25% due to an improper use of the experimental constants describing the BPFU's. Other errors had also been introduced into the calculation of variances (see later). All these errors can however be exactly accounted for. Before 1968 a different system of filter unit calibration was employed which did not have the errors mentioned here.

DESCRIPTION OF THE ERRORS

A resistance - capacitance band pass electrical filter can be closely described by an equation of the following form,

$$E_o^2 = E_{in}^2 \beta^2 \Big/ (1+an^2)^2 (1+b/n^2)^2 \qquad (1)$$

(see fig 1)

For given constants a and b the numerical values of the half-power cut-off frequencies n_1 and n_2 depend on the value of β. To clarify the situation here it can be pointed out that it is perfectly possible to pick a and b with $\beta = 1$ such that the expression passes through the previously picked n_1 and n_2. Such a filter is shown by the dashed-dotted line in Fig 1. This would be quite satisfactory for the purposes of determining variances and estimating ε and χ. However, the previous (before 1968) requirements that the peak frequency power gain also be specified (actually 85.6%, see Appendix III), though theoretically unnnecessary for the calculation of variances etc, has been continued to maintain experimental continuity. This means that the filters are designed to be reduceable to that shown by the solid line in Fig 1. The peak frequency power gain restriction arose from fitting the filter curves, to the mathematical expression describing the affect on the variance spectrum of sampling and averaging times (see Appendix II). Further the choice of n_1 and n_2 does not matter for the calculation of variances etc, but n_1 and n_2 have again been fixed from considerations of sampling and averaging times (see Appendix II). With $\beta = 1$, preserving n_1, and n_2, it is impossible to find a's and b's which describe the solid line in Fig 1. Alternatively, preserving the a's and b's as for the solid line in Fig 1, $\beta = 1$ gives the dashed-double-dotted line, but n_1 and n_2 is not then preserved. This filter could be used with the present experimental arrangement but historically n_1 and n_2 have been fixed so that the output from the actual filter units must be reduced to that of the solid line in Fig 1. The filter units used in practice (already built to the pre-1968 pattern) have a linear gain of say A, instead of β, with the same shape and a's and b's as the solid line in Fig 1. So that the following relation applies,

$$E_o'^2 = E_{in}^2 A/(1+an^2)^2(1+b/n^2)^2 \qquad (2)$$

(see pecked line in Fig 1)

Thus to convert the output voltage to that from a filter unit with the previously specified half-power cut-off frequencies n_1 and n_2 it is necessary to multiply the output, $E_o'^2$ by β^2/A^2 ie

$$E_o = \beta E_o'/A \qquad (3)$$

A is not easily measured but it is shown in Appendix I that

$$A = G(1+\sqrt{ab})^2 \qquad (4)$$

where G is the actual peak frequency gain $(E'_{o_{max}}/E_{in})$ which is more easily measured.

Later on it is shown how the output from the filter, shown by the solid line in Fig 1, compares with a unity gain "square filter" passing through n_1 and n_2.

The procedure in the years 1968, 1969 and 1970 was to multiply the outputs by $1/G$ instead of β/A thereby omitting the factor $\beta/(1+\sqrt{ab})^2$. Further in the calculation of variances of the filter unit outputs the computer programmes made no allowance for the fact that the filter units are <u>not</u> "square" filters. This had <u>only</u> been allowed for in the final estimation of ε and χ. Thus the old ε's and χ's have errors due to the omission of $\beta/(1+\sqrt{ab})^2$ and the old variances (or standard deviations) have errors due to this and the omission of the "equivalent square filter" factor.

MAGNITUDE OF THE FILTER CONSTANTS

To assess the magnitude of these errors, numerical values of β, a, b and R (the equivalent square filter factor) need to be determined for the filter units used in practice.

From equation (1)

$$\beta = (1 + a n_{1/2}^2)(1 + b/n_{\frac{1}{2}}^2)/\sqrt{2} \qquad (5)$$

where $n_{1/2}$ is either n_1 or n_2, the half-power cut-off frequencies. Considerations of "averaging" and "sampling" times and the setting of the peak frequency gain, in earlier uses of the filter units, before 1968 (as already mentioned) (JONES 1963), meant that the values used for n_1, n_2, a and b are those shown in Table 1. (See Appendix II for details). Putting the values in Table 1 into equation (5) gives $\beta = 1.094$ for both filter units. Therefore, $\beta/(1+\sqrt{ab})^2 = 0.925$ for both (as given in Jones 1963).

A method of determining the a's and b's of the actual filter units (after they have been built) is shown in Appendix IV. The values are in fact very close to those in Table 1.

The magnitude of R will now be discussed.

THE "EQUIVALENT SQUARE FILTER" FACTOR, R

In general, with the usual approximation throughout, (Pasquill and Tyldesley, 1967), we can write

$$\sigma_\phi^2 = c\int S(n')\,dn' \qquad (6)$$

where σ_ϕ^2 is the variance of the vertical angle of the wind, n' the frequency in $\mathrm{rad\ sec^{-1}}$ and c is a constant. Since any measuring system has a certain frequency response (6) can more accurately be written as

$$\sigma_\phi^2 = c\int S(n')\,w(n')\,dn' \qquad (7)$$

for an electrical filter defined:-

$$\begin{aligned} w(n') &= 0 \quad \text{for} \quad n_2' \leq n \leq n_1' \\ &= 1 \quad \text{for} \quad n_1' \leq n \leq n_2' \end{aligned}$$

(ie a square filter)

using $n' = 2\pi n$ where n is the frequency in Hz, the usual expression for σ_ϕ^2 in the inertial subrange is:-

$$\sigma_\phi^2 = \frac{c_w\,\varepsilon^{2/3}}{(2\pi)^{2/3}\,\bar{u}^{4/3}} \int_{n_1}^{n_2} n^{-5/3}\,dn \qquad (8)$$

Where C_w is the Kolmogorov Constant

$\bar{u}$ the mean horizontal wind speed

ε the turbulent kinetic energy dissipation rate

and n_1 and n_2 are the half-power cut-off frequencies of the filter unit lying within the inertial subrange.

For the RC band pass filter

$$w(n) = \beta^2 / (1 + an^2)^2 (1 + b/n^2)^2 \qquad (9)$$

Thus for variances calculated from the band pass filters output there is

$$\sigma^2_{\phi_{BP}} = \frac{c_w \varepsilon^{2/3}}{(2\pi)^{2/3} \bar{u}^{4/3}} \int_0^\infty \frac{\beta^2 n^{-5/3}}{(1+an^2)^2(1+b/n^2)^2} dn \tag{10}$$

It is worth noting that an^2 and b/n^2 are dimensionless. The two integrals in equations (8) and (10) have solutions:

$$\int_{n_1}^{n_2} n^{-5/3} dn = \frac{3}{2}\left(n_1^{-2/3} - n_2^{-2/3}\right) \tag{11}$$

and

$$\int_0^\infty \frac{n^{-5/3}}{(1+an^2)^2(1+b/n^2)^2} dn = \frac{1.2092}{(ab-1)^3 b^{1/3}}\left[(ab)^{1/3}(ab+2) - 2ab - 1\right] \tag{12}$$

(Edwards 1960)

Letting

$$R = \sigma^2_{\phi_{BP}} \Big/ \sigma^2_{\phi} \tag{13}$$

gives

$$R = \beta^2 \int_0^\infty \frac{n^{-5/3}\, dn}{(1+an^2)^2(1+b/n^2)^2} \Bigg/ \int_{n_1}^{n_2} n^{-5/3} dn = 1.09 \tag{14}$$

for the current values of the filter constants. This compares with the graphical value used before of 1.05.

MAGNITUDE OF THE ERRORS IN THE CALCULATION OF VARIANCES

The true variance of the input signal ϕ or T in the frequency range n_1 to n_2 can be written:-

$$\sigma_\phi^2 = \frac{\sigma_{\phi\,BP}^2}{R} = \frac{f(E_o S_\phi)^2}{R} = \frac{f(E_o' \beta S_\phi / A)^2}{R} \tag{15}$$

where E_o and E_o' are as defined for equations (3), S_ϕ is the conversion factor in rad volt^{-1} and $f(-)^2$ is a function representing the calculation of variance in the normal way. Under the old system variances were calculated as follows:

$$\sigma_{\phi_{OLD}}^2 = f\left(\frac{E_o'}{G} S_\phi\right)^2 \tag{16}$$

ie, as mentioned before, the term $\beta/(1+\sqrt{ab})^2$ and R were omitted. Therefore, to correct the old variances they need to be multiplied by

$$\frac{\beta^2}{R(1+\sqrt{ab})^4}$$ ie multiplying equation (16)

by this make it equivalent to equation (15). Therefore, the true variance from the old value is:

$$\sigma_\phi^2 = \sigma_{\phi_{OLD}}^2 \frac{\beta^2}{R(1+\sqrt{ab})^4} = \sigma_{\phi_{OLD}}^2 \frac{0.925^2}{1.09} = 0.787\,\sigma_{\phi_{OLD}}^2 \tag{17}$$

and this correction is the same for BPFU1 and BPFU2 for temperature, T, and inclination, ϕ.

MAGNITUDE OF THE ERRORS IN THE CALCULATIONS OF ε AND χ

Equations (8) can be rewritten as follows:

$$\varepsilon = \frac{\sigma_\phi^3 (2\pi)\bar{u}^2}{C_w^{3/2}\left(\int_{n_1}^{n_2} n^{-5/3}\,dn\right)^{3/2}} \tag{18}$$

but under the old system ε was evaluated:-

$$\varepsilon_{OLD} = \frac{\sigma^3_{\phi OLD}\,(2\pi)\,\bar{u}^{-2}}{(1.05)^{3/2}\,C_W^{3/2}\left(\int_{u_1}^{u_2} u^{-5/3}\,du\right)^{3/2}} \tag{19}$$

Thus using equation (17), the correct ε in terms of the old is

$$\varepsilon = \varepsilon_{OLD}\,(1.05 \times 0.787)^{3/2} = \varepsilon_{OLD} \times 0.75 \tag{20}$$

Before <u>March 1970</u> χ was evaluated:-

$$\chi_{OLD} = \text{constant} \times \left(\frac{\sigma^2_T}{\sigma^2_\phi}\,\frac{\varepsilon}{\bar{u}^2}\right)_{OLD} \tag{21}$$

The corrections to σ_T and σ_ϕ cancel out so that the correct χ is

$$\chi = \chi_{OLD} \times 0.75 \tag{22}$$

However, before <u>March 1971</u> χ was evaluated:-

$$\chi_{OLD} = \text{constant} \times \left(\sigma^2_T\,\sigma_\phi\right)_{OLD} \tag{23}$$

Therefore, the correct χ is:-

$$\chi = \chi_{OLD}\,(0.787)^{3/2} = \chi_{OLD} \times 0.70 \tag{24}$$

All the above corrections for variances, ε and χ need to be applied to all computer output before March 1971. Also the ε- profiles published in the Radio Science Journal, 1969 need to be correct for ε .

<u>CORRECT PROCEDURES FOR EVALUATING VARIANCES, ε AND χ</u>

From equation (15) the output voltages from the band pass filters need to be multiplied by $\left(\frac{\beta}{A}\,\frac{S_\phi}{R}\right)$ before the variances are calculated.

(NB In practice there is also a digital to voltage conversion since a paper tape data logger is used). If the gain of the filter unit is changed all that needs to be measured is the peak frequency gain G since, as previously mentioned

$$A = G\left(1+\sqrt{ab}\right)^2$$

For the filter units in current use

$$\frac{\beta}{A}\frac{S_\phi}{R} = \frac{0.85\, S_\phi}{G} \tag{25}$$

Where S_ϕ is the sensor calibration in radians per volt or S_ϕ is replaced by S_T in ^{o}C per volt.

In the equation for ε (equations (18)) the constant C_w is taken as $0.47 \times 4/3$ (Pasquill and Tyldesley 1967). The term 2π is included to convert Hz into radians per second for which C_w is known.

The analogous expression for χ is:-

$$\chi = \frac{\sigma_T^2\, \varepsilon^{1/3} (2\pi)^{2/3}}{C_T \bar{u}^{2/3} \left(\int_{n_1}^{n_2} n^{-5/3}\, dn\right)} \tag{26}$$

substituting for ε this becomes

$$\chi = \frac{\sigma_T^2\, \sigma_\phi\, 2\pi}{C_T C_W^{1/2} \left(\int_{n_1}^{n_2} n^{-5/3}\, dn\right)} \tag{27}$$

The value for C_T is taken as 0.70 (Ellison, private communication)

Thus for BPFU1

$$\varepsilon_1 = 6.31\, \sigma_{\phi_1}^3 \bar{u}^2 \quad \& \quad \chi_1 = 5.65\, \sigma_{T_1}^2 \sigma_{\phi_1}$$

while for BPFU2

$$\left.\begin{aligned}\varepsilon_2 = 1.05\, \sigma_{\phi_2}^3 \bar{u}^2 \quad \& \quad \chi_2 = 0.94\, \sigma_{T_2}^2 \sigma_{\phi_2}\end{aligned}\right\} \tag{28}$$

using the values given in Table 1 for the cut-off frequencies. A survey of the solid-state filter units, used in 1970, shows small shifts in the cut-off frequencies, so that

$$\varepsilon_1 = 6.26\, \sigma_{\phi_1}^3 \bar{u}^{-2} \quad \& \quad \varepsilon_2 = 1.04\, \sigma_{\phi_2}^3 \bar{u}^{-2}$$

gives slightly more accurate answers. In any case the set of relations (28) give results accurate to at least 1%,

APPENDIX I DERIVATION OF $A = G(1+\sqrt{ab})^2$ AND n_{peak}

Equation (1) is:

$$\frac{E_o^2}{E_{in}^2} = \beta^2 / (1+an^2)^2(1+b/n^2)^2 \qquad \text{I(i)}$$

Differentiating with respect to frequency gives

$$\frac{\partial(E_o^2/E_{in}^2)}{\partial n} = \beta^2\left[\frac{-2((1+an^2)(-2bn^{-3})+(1+b/n^2)(2an))}{(1+an^2)^3(1+b/n^2)^3}\right]$$

This equals zero when

$$n = (b/a)^{1/4} \qquad \text{I(ii)}$$

which is therefore the peak frequency, n_{peak}.

Substituting (ii) into (i) gives

$$E_{o_{max}}^2 = \frac{E_{in}^2\beta^2}{(1+\sqrt{ab})^4} \qquad \text{I(iii)}$$

For the actual filter units

$$E'^2_{o_{max}} = \frac{E_{in}^2 A^2}{(1+\sqrt{ab})^4} \qquad \text{I(iv)}$$

Letting the peak frequency gain G be

$$G = E'_{o_{max}} / E_{in}$$

gives

$$A = G(1+\sqrt{ab})^2 \qquad \text{I(v)}$$

Experimentally $E'_{o_{max}}$ is the peak to peak output voltage at the peak frequency corresponding to a known input voltage E_{in} at the peak frequency.

From equation (5) in the text we have

$$\left(1+an_1^2\right)\left(1+b/n_1^2\right) = \left(1+an_2^2\right)\left(1+b/n_2^2\right) \qquad \text{I(vi)}$$

So that

$$\frac{b}{a} = n_1^2 n_2^2 \qquad \text{I(vii)}$$

Therefore the peak frequency is given by

$$n_{peak} = \sqrt{n_1 n_2} \qquad \text{I(viii)}$$

which gives the values in Table 1.

	n_1 (Hz)	n_2 (Hz)	a	b	n_{peak} (Hz)
BPFU1	0.5328	3.1968	0.0513	0.1489	1.305
BPFU2	0.0888	0.5328	1.8485	0.0041(4)	0.217(5)

TABLE 1. The filter unit's constants.

APPENDIX II THEORETICAL NUMERICAL VALUES OF THE FILTER CONSTANTS AND THE RELATION WITH SAMPLING AND AVERAGING TIME

The affect on the spectrum of a sampling time τ and an averaging time s has been shown to be equivalent to a multiplication of the spectrum by:-

$$\left(1 - \frac{\sin^2 \pi n \tau}{(\pi n \tau)^2}\right)\left(\frac{\sin^2 \pi n s}{(\pi n s)^2}\right) \qquad \text{II(i)}$$

(eg JONES, 1963)

If we let τ/s = constant and $n\tau$ and ns = constants we find, by setting the expression II(i) = 0.5, for $\tau/s = 6$ that the half-power cut-off frequencies are

$$n_1 = 0.4485/\tau \quad \& \quad n_2 = 0.4393/s$$

which is a little different to the values quoted in JONES' paper. A common value of $n_1\tau = n_2 s = 0.444$ will be taken.

For BPFU2 $\tau = 5$ and $s = 5/6$ and for BPFU1 $\tau = 5/6$ and $s = 5/36$ which immediately gives the cut-off frequencies given in Table 1. Setting equation I(i) to 0.5, using equation I(vii) and the fact that $n_2 = 6n_1$ gives a quadratic which has a solution:

$$a = \frac{1}{n_1^2}\left(-\frac{37}{72} + \sqrt{\left(\frac{37}{72}\right)^2 + \frac{\beta\sqrt{2}-1}{36}}\right) \qquad \text{II(ii)}$$

also $b = 36 n_1^4 a$

Thus the values of a and b depend on β. Letting, as Jones does, the voltage output to input ratio from the filters be 0.925 (ie 0.856 for power) ie

$$\frac{E_{o_{max}}}{E_{in}} = \frac{\beta}{(1+\sqrt{ab})^2} = 0.925 = L \text{ (say)} \qquad \text{II(iii)}$$

The solution for a then becomes

$$a = \frac{1}{n_1^2}\left(\frac{-(37-12L\sqrt{2}) \pm \sqrt{(37-12L\sqrt{2})^2 - 4\times 36(1-L\sqrt{2})^2}}{72(1-L\sqrt{2})}\right) \qquad \text{II(iv)}$$

which for L = 0.925 gives the a's and b's in Table 1. Also from equation II(iii) $\beta = 1{\cdot}094$. The value chosen for L arose from the attempt to fit the filter curves to the expression II(i). Actually this can only be done roughly because the expression II(i) has ripples along its whole path, even at the peak, and is a non-symmetric function, as can be discovered by plotting it out. Thus L = 0.925 is somewhat arbitrary.

APPENDIX III

1. Setting up the Filter units

The ratio of $E'_{o_{max}}$ to the output at the half-power frequencies, $E'_{o_{n/2}}$, is

$$\frac{E'_{o_{max}}}{E'_{o_{n/2}}} = \frac{(1+an_1^2)(1+b/n_1^2)}{(1+\sqrt{ab})^2} = \frac{(1+an_2^2)(1+b/n_2^2)}{(1+\sqrt{ab})^2} \qquad \text{III(i)}$$

which for the values given in Table 1 is equal to 1.308. Thus in setting up the filter units the resistance - capacitance components are adjusted until the above ratio is satisfied at both half-power cut-off frequencies. Note that in setting up these filters it is not necessary to know the linear gain, A. The linear gain, $\beta = 1.094$, though never set as such directly in the setting up determines a and b and hence the 1.308 for given n_1's and n_2's. If we took $\beta = 1$ (ie the dashed - dot line in Fig 1) the a's and b's would be different and the ratio smaller. The gain A, or more correctly G, only needs to be known for converting the output. Fig 2 shows a comparison between the theoretical curves (solid line) and the experimental points (dots). The ordinate scale is arbitrary but has been taken for the peak value to be

$$\frac{E^2_{o_{max}}}{E^2_{in}} \times 100 = \frac{\beta^2 \times 100}{(1+\sqrt{ab})^4} = 85.6\%$$

An alternative way of looking at this ratio is seen by noting that $\sqrt{\frac{85.6}{50}} = 1.308$. Clearly the measure points fit the theoretical curves very closely.

2. Description of the filter unit circuits (H E Butler)

The circuit diagram for the solid state filter units is shown in Fig 3. The underlined component values are for BPFU1 (centre frequency 1.305Hz) and the others for BPFU2 (centre frequency 0.217Hz). Each band pass filter was designed as an independent module so that one or more could be plugged into a rack for processing any required sensor output. It is essentially a 2-stage

R-C filter using a linear integrated circuit (IC) amplifier as an interstage buffer and amplifier, with the first stage feeding into the non-inverting, high impedance ($\sim 20\ M\Omega$) input of the IC amplifier, thereby minimising the loading of R_2. The minimum gain is set approximately by $(R_3 + R_4)/R_4$ but can be increased up to 6 times by means of the pre-set feed-back loop on the output without affecting the filter values. The output, which has a very low impedance ($< 1\ \Omega$) is arranged to give a maximum of $\pm 5V$ to feed direct into a data logger. A second output is provided, through an attenuator, to feed a chart recorder.

3. A quick check of the module is provided by short circuiting C_2 and applying a DC test voltage (CAL) and measuring the output. An additional resistor is added in series with the input at this time to preserve the same impedance presented to the IC amplifier.

APPENDIX IV

1. Evaluation of a and b. If the filter units are set up exactly as suggested in Appendix II the a's and b's will be those given in Table 1. However, this may not be the case, particularly after a long time when circuit components may have 'drifted'. Provided the general shape still follows equation (1), the following analysis gives a technique for evaluating the new a's and b's thereby saving time spent in readjusting the R-C components.

The measurements required are: -

$$\frac{E'_{o_{max}}}{E'_{n_x}} = K_x \quad \& \quad \frac{E'_{o_{max}}}{E'_{n_y}} = K_y \qquad \text{IV(i)}$$

where the frequencies n_x and n_y would be close to the half-power cut-off frequencies but not necessarily equal to them.

From equations (2) and I(iv) we have

$$(1 + an_x^2)(1 + b/n_x^2) = K_x(1 + \sqrt{ab})^2 \qquad \text{IV(ii)}$$

and

$$(1 + an_y^2)(1 + b/n_y^2) = K_y(1 + \sqrt{ab})^2 \qquad \text{IV(iii)}$$

Letting $b = c^2a$ gives

$$a^2c^2(K_x - 1) - a(L_1 + c^2/L_1 - 2cK_x) - (1 - K_x) = 0 \qquad \text{IV(iv)}$$

and

$$a^2c^2(K_y - 1) - a(L_2 + c^2/L_2 - 2cK_y) - (1 - K_y) = 0 \qquad \text{IV(v)}$$

where

$$L_1 = n_x^2 \quad \& \quad L_2 = n_y^2$$

Equations IV(iv) and IV(v) give a quadratic for c : -

$$c^2\left[\frac{1 - K_y}{L_1} - \frac{1 - K_x}{L_2}\right] + 2c(K_y - K_x) + L_1(1 - K_y) - L_2(1 - K_x) = 0 \qquad \text{IV(vi)}$$

Solving for c gives the peak frequency of the 'best fit' mathematical expression ie $\sqrt{c} = n_{peak}$. Substituting for c into IV(iv) and IV(v) gives a as a solution to another quadratic and hence also b. Given n_{peak}, a and b, preserving the relations $n_{peak} = \sqrt{n_1 n_2}$ and $n_2 = 6n_1$, the new n_1 and n_2, then β (equation (5)) and R can be found. Thus finally the new constant $\beta S_\phi / AR$ is derived. The new n_1 and n_2 may also have to be inserted in the ε and χ formulae. For the solid state filters, in current use, such variations in the constants have been negligible. It should be noted here that the a's and b's so calculated are not exactly equivalent to the R-C component values because of stray resistances and capacitances.

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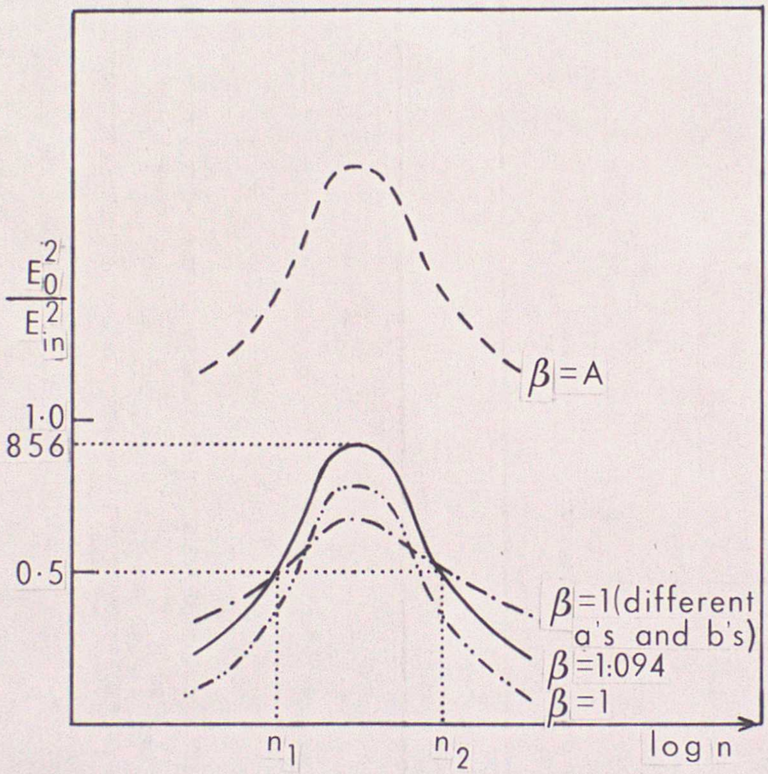


Fig 1 Representation of various band pass filter expressions(see text).

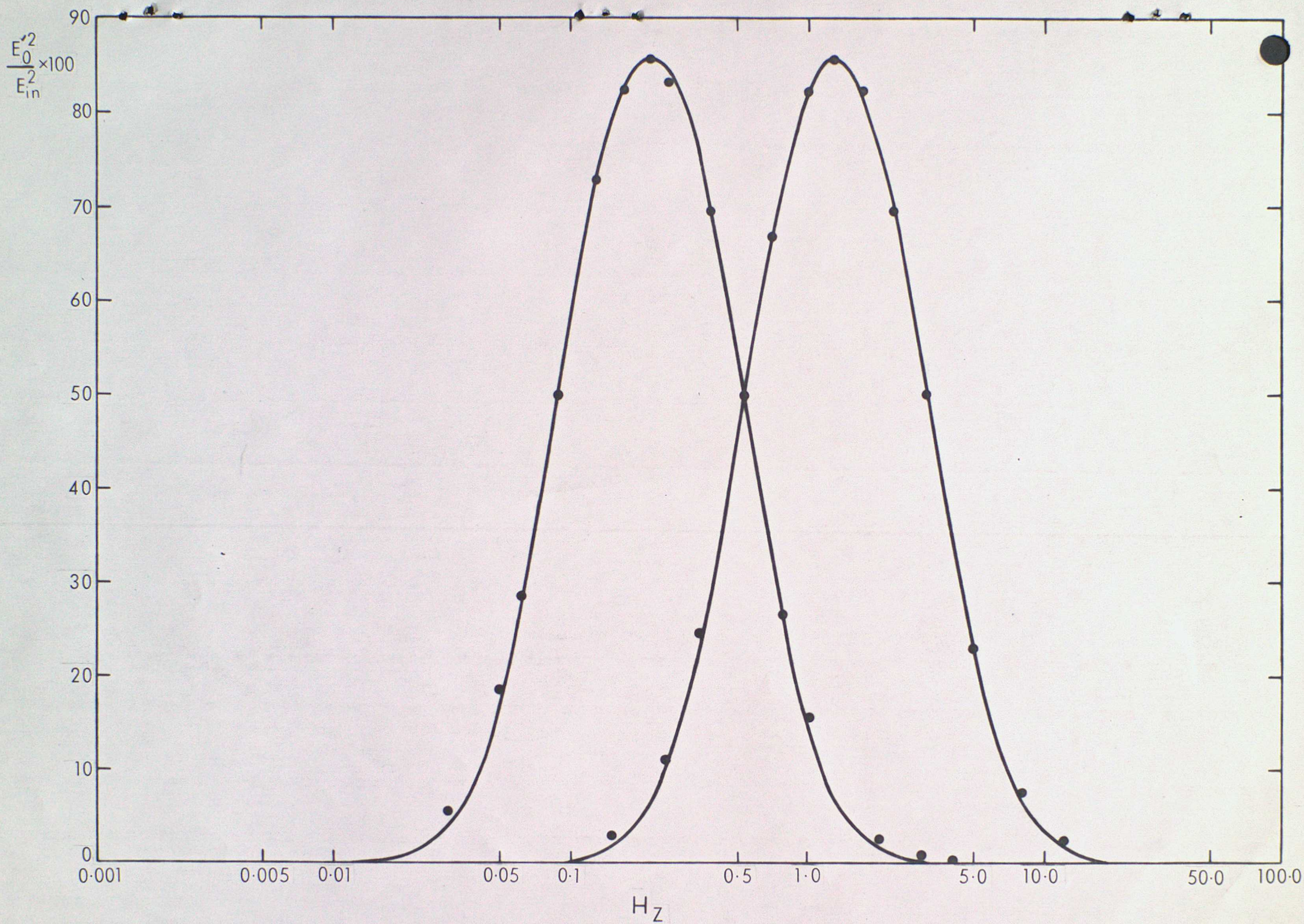
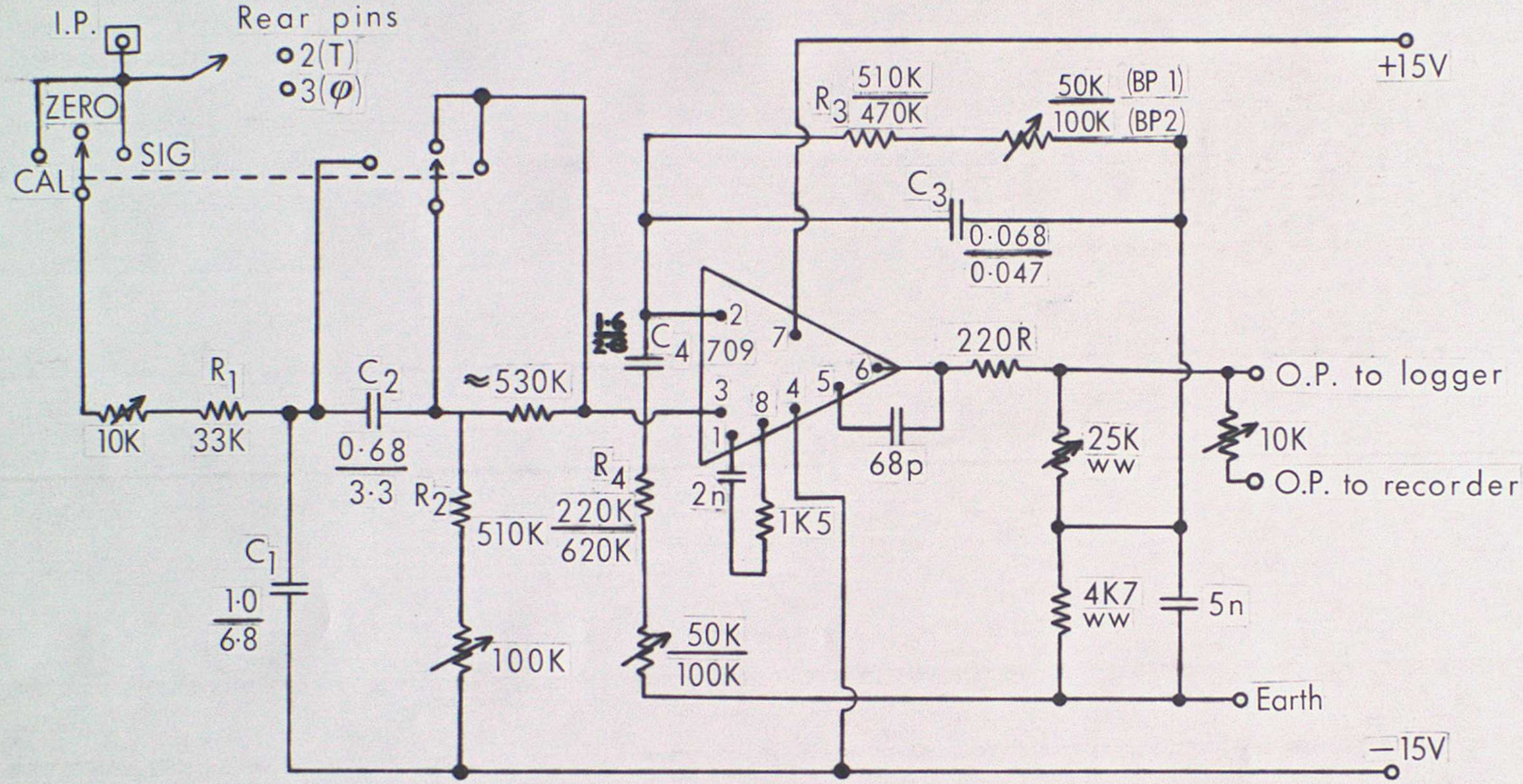


Fig.2 Comparison between experimental(dots) and theoretical(solid line) filter unit curves



Key.

K — Kilohms
R — ohms
ww — wire wound
p — picofarads(10^{-12})
n — nanofarads(10^{-9})
unspecified — microfarads(10^{-6})
I.P. — input
O.P. — output
V — volts

Fig.3 Circuit of the band pass filter units