M.O. 623d

AIR MINISTRY

METEOROLOGICAL OFFICE

PROFESSIONAL NOTES NO. 124

(Fourth Number of Volume VIII)

THE PREPARATION OF STATISTICAL WIND FORECASTS AND AN ASSESSMENT OF THEIR ACCURACY IN COMPARISON WITH FORECASTS MADE BY SYNOPTIC TECHNIQUES

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LONDON
HER MAJESTY'S STATIONERY OFFICE
1959

PRICE 1s. 9d. NET

U.D.C.
551·509·314:551·509·322.7

Prof. Notes Met. Off.
London, 8, No. 124, 1959

FOREWORD

By R. C. SUTCLIFFE, O.B.E., Ph.D., F.R.S.

For many years statistical methods have had an extensive if disappointing application to long-range forecasting, but in short-range forecasting, for periods from a few hours to one or two days, the subjective method depending on the estimating of changes in the geographical distribution of the weather elements—" the conventional synoptic method "—has held the field practically unchallenged throughout the history of organized weather forecasting. A challenge has however come in recent years from two directions: from the mathematical hydrodynamicists who would predict the future weather situation by solving the appropriate physical equations using electronic machines; and from the statisticians who, by using statistical relations derived from past occurrences, will predict a most probable value or a most probable field distribution of some factor or phenomenon, and may also demand electronic computing assistance. A great deal of work has been done in recent years on both these lines, both attractive by virtue of their quantitative and objective procedures but with their intellectual appeal varying according to one's feeling for physics or statistics.

The present paper presents a limited application of statistical methods to the prediction of upper winds. Basically the method rests on the serial correlation of the time-series of wind observations and so provides a prediction, by a regression equation, which lies somewhere between " present conditions " and the " climatic average ". As such it has the feature, physically unacceptable, of invariably predicting that the winds will become lighter or stronger according as they are abnormally strong or abnormally light. Obviously if one has no worthwhile knowledge of the trend in the unique case of interest the procedure is a sensible one, " a good bet ", although the result may hardly justify the word " forecast ".

The true forecast performed by conventional synoptic or dynamical methods, whereby the evolution of the circulation is estimated, is, in the present comparison, somewhat better than the statistical method of " playing the odds ". In a sense this is encouraging to the forecaster as implying a real measure of skill in his judgment of trends in individual cases, although the challenge to achieve greater quantitative accuracy is a telling one.

While, on present evidence, there is no case for abandoning the synoptic-dynamical approach, it is only fair to admit that statistical methods are a serious competitor and may find valid application in some problems, especially where synoptic studies and physical analyses have so far yielded little of prediction value, perhaps at high levels or low latitudes or anywhere where the fewness of the data renders the physical problem intractable. It is well that the authors have put this study forward for no forecaster should be unaware of the potentialities of statistical techniques.

THE PREPARATION OF STATISTICAL WIND FORECASTS AND AN ASSESSMENT OF THEIR ACCURACY IN COMPARISON WITH FORECASTS MADE BY SYNOPTIC TECHNIQUES

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Summary.—The principles and practice of forecasting upper winds by linear regression equations are discussed. An account is given of the accuracy achieved by the statistical technique and by the current synoptic method during trials held for the first four months of 1955. The use of the statistical approach in deciding the effect on forecast accuracy of changing the distribution of observing stations is illustrated.

Introduction.—Over a number of years C. S. Durst has developed a statistical technique for forecasting upper winds which has considerable value in treating a number of the problems arising in forecasting and flight-planning for aviation. For wind forecasting in temperate latitudes, the accuracy of the statistical predictions has proved to fall short (but not far short) of the accuracy currently attained in routine forecasting by skilled forecasters using the more orthodox synoptic method. For this and other reasons, when adequate observations are available, the statistical technique cannot compete with the synoptic approach, but since it makes judicious use of climatology the statistical method can provide useful forecasts in circumstances when the synoptic approach is inapplicable: for example, when observations are lacking over most of a route or region for which forecasts are required, or when forecasters of sufficient experience are not available. In the tropics, where observations are generally sparse and the geostrophic approximation, on which the temperate latitude synoptic method of wind forecasting rests, does not apply, statistical predictions may be difficult to better. It is in the favour of the statistical technique that, when a fresh observation or set of observations becomes available, a fresh forecast can be made in a few minutes by a simple calculation; synoptic forecasts generally take considerably longer to prepare.

The level of accuracy of statistical predictions varies from time to time and from place to place in fairly close correlation with similar variations in the accuracy of synoptic forecasts. For this reason, the expected success of statistical forecasts, which is readily calculated from climatological data, provides a useful gauge of the likely accuracy of synoptic forecasts. This gives a convenient way of estimating the effect on routine upper-wind forecasts of varying the number of observations available and also is a method by which the probable success of forecasts to be made for a hitherto untried air route may be predicted.

In this paper the principles of the method are stated and data are given or data sources are quoted, from which it is possible to set up statistical forecasts of wind in many parts of the world at heights up to 100 mb. (53,000 ft.). A section is included which gives the results of practical tests of the method and compares the accuracies of statistical and synoptic forecasts. As an example of the use of the basic data of statistical forecasting in deciding the decrease in accuracy of synoptic forecasts which would follow if certain facilities were not available, the loss of forecast accuracy to be expected from the withdrawal of Atlantic weather ships is assessed.

The method in principle.—The statistical technique relies simply on the principle of linear regression. It is shown by Brooks and Carruthers,[1,*] or by any standard statistical textbook, that if two scalar variables P and Q are correlated with correlation coefficient r, then the best linear equation which can be used to predict P, knowing Q, is

$$F(P)-\bar{P} = \frac{r\sigma_P}{\sigma_Q}(Q-\bar{Q}), \qquad \ldots\ldots(1)$$

where $F(P)$ is the predicted value of P, $\bar{P}$ and $\bar{Q}$ are the mean values and σ_P and σ_Q are the standard deviations of P and Q respectively.

Equation (1) may be applied to wind components but alternatively a vector regression equation can be formed. Durst[2] has discussed the concept of the stretch vector correlation coefficient defined by the relation

$$r_{st} = \frac{\frac{1}{n}\sum(\mathbf{U}-\bar{\mathbf{U}}).(\mathbf{V}-\bar{\mathbf{V}})}{\sigma_u\,\sigma_v},$$

where $\bar{\mathbf{U}}$ and $\bar{\mathbf{V}}$ are the vector mean values and σ_u and σ_v are the standard vector deviations of the vectors **U** and **V**, the summation being made over n scalar products. Knowledge of the stretch vector correlation enables a vector regression equation to be formed for use when one vector departure $(\mathbf{U}-\bar{\mathbf{U}})$ tends to be proportional in magnitude to, and tends to lie parallel to, a second vector departure $(\mathbf{V}-\bar{\mathbf{V}})$. The equation is

$$F(\mathbf{U})-\bar{\mathbf{U}} = \frac{r_{st}\,\sigma_u}{\sigma_v}(\mathbf{V}-\bar{\mathbf{V}}), \qquad \ldots\ldots(2)$$

where $F(\mathbf{U})$ is the predicted value of **U**. Provided that the correlation between the components of **U** and **V** is the same for all directions, there will be no loss of accuracy in using the vector equation (2) in preference to scalar regression equations relating the components.

When $(\mathbf{U}-\bar{\mathbf{U}})$ tends to have a direction rotated by some angle from $(\mathbf{V}-\bar{\mathbf{V}})$, equation (2) becomes less effective and should be replaced by an analogous equation,

$$\mathbf{K} = \frac{r_{sr}\,\sigma_u}{\sigma_v}(\mathbf{V}-\bar{\mathbf{V}}), \qquad \ldots\ldots(3)$$

where r_{sr} is a more general vector correlation coefficient given by

$$r_{sr} = \frac{\sqrt{[\{\frac{1}{n}\sum|\mathbf{U}-\bar{\mathbf{U}}|\,|\mathbf{V}-\bar{\mathbf{V}}|\cos\theta\}^2 + \{\frac{1}{n}\sum|\mathbf{U}-\bar{\mathbf{U}}|\,|\mathbf{V}-\bar{\mathbf{V}}|\sin\theta\}^2]}}{\sigma_u\,\sigma_v},$$

θ being the angle between $(\mathbf{U}-\bar{\mathbf{U}})$ and $(\mathbf{V}-\bar{\mathbf{V}})$.

The predicted value for $(\mathbf{U}-\bar{\mathbf{U}})$ is then got by rotating **K** through an angle α where,

$$\alpha = \tan^{-1}\frac{\Sigma|\mathbf{U}-\bar{\mathbf{U}}|\,|\mathbf{V}-\bar{\mathbf{V}}|\sin\theta}{\Sigma|\mathbf{U}-\bar{\mathbf{U}}|\,|\mathbf{V}-\bar{\mathbf{V}}|\cos\theta}.$$

Experience suggests that for most problems concerning upper winds, use of equation (3) rather than equation (2) does not improve the accuracy of the predictions materially, although Durst[2] cites a case involving the

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* The index numbers refer to the bibliography on p. 27.

variation of wind with height in the friction layer, when the correlation between the surface and 1,500-ft. winds was increased from 0·73 to 0·79 by the inclusion of the second term on the right-hand side of equation (3).

The vector correlations and regressions discussed in the following sections are of the kind implied by equation (2); for stretch vector correlations the concepts and formulae of partial correlation as developed for scalar variables (Brooks and Carruthers,[1] Ch. 13) can be extended by strict analogy to apply to vectors.

Prediction equations for wind at a point.—The simplest case arises when the wind, $\mathbf{U}_0$, observed at some point at time $t = 0$ is used to predict the wind, $F(\mathbf{U}_t)$, expected at a subsequent time t. The mean values and standard deviations of $\mathbf{U}_0$ and $\mathbf{U}_t$ will be nearly identical. Hence if r_t is the correlation coefficient between $\mathbf{U}_0$ and $\mathbf{U}_t$, equation (2) simplifies to

$$F(\mathbf{U}_t)-\bar{\mathbf{U}} = r_t(\mathbf{U}_0-\bar{\mathbf{U}}). \qquad \ldots\ldots(4)$$

In forecasting, values of $\bar{\mathbf{U}}$ and r_t must be selected before the event and it is customary to choose the climatic normals, $\bar{\mathbf{U}}^*$ and r_t^*, so the practical prediction equation is

$$F(\mathbf{U}_t) = (1-r_t^*)\,\bar{\mathbf{U}}^* + r_t^*\,\mathbf{U}_0. \qquad \ldots\ldots(5)$$

Interpreted physically, equation (5) demonstrates the extreme simplicity of the statistical approach; the statistical forecast is an adjustment of the observed wind towards the seasonal normal; a light wind is usually forecast to freshen a little and a strong wind to abate.

The r.m.s. (root-mean-square) error, ε, of forecasts made by equation (5) is given by

$$\varepsilon = [\sigma^2\,(1-2\,r_t^*\,r_t + r_t^{*2}) + (\bar{\mathbf{U}}^*-\bar{\mathbf{U}})^2\,(1-r_t^*)^2]^{\frac{1}{2}}, \qquad \ldots\ldots(6)$$

σ being the standard vector deviation of the winds, $\mathbf{U}$. The second group of terms on the right-hand side of equation (6) represents the square of the bias in the statistical forecasts. Over a single season, this term can make a significant contribution to the error, but when averaged over several seasons its effect will be negligible. In the long run, a good approximation to the r.m.s. error given by equation (6) is

$$\varepsilon = \sigma^*\,(1-r_t^{*2})^{\frac{1}{2}}. \qquad \ldots\ldots(7)$$

The corresponding r.m.s. error, σ_t^*, of persistence forecasts is given by

$$\sigma_t^* = \sigma^*\,(2-2r_t^*)^{\frac{1}{2}}. \qquad \ldots\ldots(8)$$

It is possible to improve upon the predictions of equation (5) by forming a multiple regression equation which takes into account the winds at points additional to that for which the forecast is made. Experiments, reported by Durst,[2] show that the standard error of 24-hr. statistical forecasts can be reduced by 5 to 10 per cent and that of 12-hr. predictions by 10 to 20 per cent, when observations from more than one station are taken into account.

Data required in forecasting wind at a point.—Use of equation (5) requires knowledge of the climatic normal mean winds and temporal stretch vector correlation coefficients. Assessment of the expected errors from equation (7) requires, in addition, knowledge of the standard vector deviation of wind. The climatic mean wind should refer to the time of year and time of day for which the forecast is to be made and therefore should take account of diurnal, seasonal and secular variations. When $\bar{\mathbf{U}}^*$ and σ^* are not known for a particular point and they cannot be calculated from observations, seasonal geostrophic values may be inferred from the climatic maps published in *Upper winds over the world*,[3] or they may be deduced from the seasonal charts of streamlines and isotachs[4] which are available for the levels 700 mb., 500 mb., 300 mb., 200 mb., 150 mb. and 100 mb. Values of $r_t{}^*$ for heights in the troposphere over extra-tropical latitudes are given in Table I. In the extra-tropical stratosphere the appropriate values are not known accurately, but they are certainly higher than those given in Table I; $r_{24}{}^*$ is probably as high as 0·75 at 100 mb.

TABLE I—STRETCH VECTOR CORRELATION COEFFICIENTS BETWEEN WINDS OBSERVED AT VARIOUS TIME INTERVALS

Time interval	6-hr.	12-hr.	18-hr.	24-hr.	48-hr.
Stretch vector correlation coefficient .. .. $r_t{}^*$	0·88	0·75	0·65	0·55	0·35

These values apply to all heights in the troposphere over extra-tropical latitudes.

Prediction equations for wind over a route.—By analogy with equation (5), the prediction equation for the vector mean wind over a route can be written

$$F(\mathbf{V}_t) = (1-R_t{}^*)\bar{\mathbf{V}}^* + R_t{}^*\mathbf{V}_0, \qquad \ldots\ldots(9)$$

where $\mathbf{V}_t$ is the vector mean wind over the route at time t, $\mathbf{V}_0$ is the vector mean wind over the route at time $t = 0$, $\bar{\mathbf{V}}^*$ is the climatic mean value of $\mathbf{V}_t$ and $R_t{}^*$ is the stretch vector correlation coefficient between $\mathbf{V}_t$ and $\mathbf{V}_0$.

$\mathbf{V}_0$ may be computed from measurements made on an appropriate contour chart or streamline/isotach chart, but time will be saved and in some circumstances a more accurate estimate may be got, if $\mathbf{V}_0$ is derived directly by means of a multiple regression equation from wind observations for points along the route. If there are n suitable wind observations such an equation would take the form

$$E(\mathbf{V}_0)-\bar{\mathbf{V}}^* = \sum_{i=1}^{n} A_i(\mathbf{U}_0-\bar{\mathbf{U}}_0{}^*)_i, \qquad \ldots\ldots(10)$$

where $(\mathbf{U}_0)_i$ is the wind observed at the ith station at time $t = 0$, $(\bar{\mathbf{U}}_0{}^*)_i$ is the climatic mean of $(\mathbf{U}_0)_i$, $E(\mathbf{V}_0)$ is the best estimate of $\mathbf{V}_0$ and the A_i are partial regression coefficients. Brooks and Carruthers[1] discuss the computation of partial regression coefficients for scalars; as mentioned above, the same formulae are valid for vectors provided that stretch vector correlations and standard vector deviations are used. To compute the A_i, it is necessary to know: the standard vector deviations, σ_i, of the point winds; the standard vector deviation, σ_L, of the mean wind over the route; the correlations between the winds observed at different places along the route; and the correlations, r_i, between the $(\mathbf{U}_0)_i$ and $\mathbf{V}_0$.

The standard vector deviation of the vector mean wind over a route is given by

$$\sigma_L{}^2 = \frac{2\sigma^2}{L^2}\left\{\int_0^L \int_0^s r_x \, dx \, ds\right\}, \qquad \ldots\ldots(11)$$

where L is the length of the route, r_x is the stretch vector correlation between winds at points distance x apart and σ is the mean value of the standard vector deviations of wind at all points along the route. Formula (11) is similar to the expression derived by Sawyer[5] for equivalent headwinds. The correlation coefficient, r_i, between the wind at some point distance S along a route and the mean wind over the route has been shown by Durst[2] to be given by

$$r_t = \frac{1}{L}\frac{\sigma_t}{\sigma_L}\left\{ \int_0^S r_x dx + \int_0^{L-S} r_x dx \right\}. \qquad \ldots\ldots(12)$$

Cases for which the integrals in equations (11) and (12) have already been evaluated are cited in the section below.

Observations from stations which are not on the route may be used in equation (10), but in computing the correlation with the mean wind over the route using equation (12), it must be remembered that although the integration is taken along the route, the correlations r_x are not, in this case, correlations between two points on the route.

The prediction of route winds by the method just described proceeds, effectively, in two stages represented by equations (10) and (9), and for computing regression coefficients initially this method is often expedient. An alternative approach, which theoretically is a little more accurate, is to relate the forecast route wind directly to observations made at an earlier time by an equation of the form

$$F(\mathbf{V}_t) - \bar{\mathbf{V}}^* = \sum_{i=1}^{n} B_i(\mathbf{U}_0 - \bar{\mathbf{U}}_0^*)_i. \qquad \ldots\ldots(13)$$

In calculating the regression coefficients B_i, the correlations r_x in equation (12) must be replaced by correlations which may be denoted by $r_{x,t}$. $r_{x,t}$ is the correlation coefficient between pairs of winds distance x apart in space and interval t apart in time. Equation (13) can take into account observations made at differing times, provided that appropriate adjustments are made to the regression coefficients.

For aviation, the forecast wind required is likely to be a mean route wind which takes account of the arrival of the aircraft at different points of the route at different times. To meet this circumstance the regression coefficient in equation (13) can be modified further; the integration in equation (12) must be made following the aircraft, in space and time, along the scheduled track, taking at each stage the proper value for $r_{x,t}$.

Equations (9), (10) and (13) have been written in vector notation, but they can be applied equally to the mean equivalent headwind over a route or to the mean-wind components, the regression coefficients being devised from appropriate values of the standard deviations and scalar correlations. For equivalent headwinds, the right-hand side of equation (11) requires reduction by a factor of two (*vide* Sawyer[5]).

The calculation of the regression coefficients may require considerable labour but, once formed, the prediction equations enable forecasts to be made in a matter of minutes by relatively unskilled operators.

Data required in forecasting wind over a route.—*Vector winds.*—the climatic vector mean winds for any route, and also the mean winds for individual stations if these are not otherwise available, may be derived from maps given in *Upper winds over the world*[3] or better, from the normal streamline and isotach charts.[4] The standard vector deviation of wind at individual points can be had from the same sources.

The stretch vector correlation of wind with distance, r_x, has been explored by Durst[2] for both tropical and temperate latitudes. Figure 1 gives values appropriate to the troposphere, which are reproduced from *Geophysica Memoir* No. 93[2] for convenience of reference. Figures 2 and 3 contain corresponding graphs for 200 mb. and 100 mb. The plotted values on which the smoothed curves of Figures 2 and 3 are based were found for winter (December to February) and summer (June to August) by

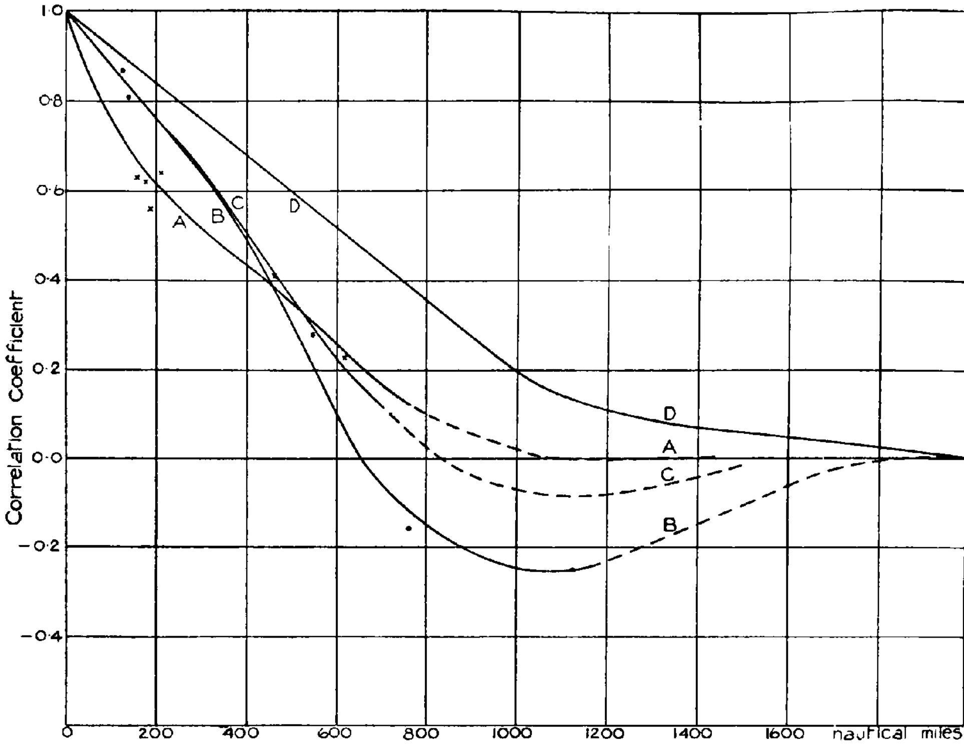

FIGURE 1—VARIATION OF WIND WITH DISTANCE

A: tropics and subtropics
B: temperate latitudes
C: temperate latitudes for an east–west route
D: a generalization of relation of geostrophic winds as given by Sawyer[5]

comparing winds from Langenhagen, Hemsby, Liverpool, Aldergrove, weather ship " J " and weather ship " C ";† taking these observations in pairs, correlation coefficients were calculated for distances from 300 to 1,650 nautical miles. The stations used are aligned roughly from east to west. The correlations between north–south components fall off more sharply with distance than those between east–west components. The 200-mb. curves differ little from the tropospheric curves but at 100 mb., particularly for east–west components in winter, the winds are more closely correlated, distance for distance, than at the lower levels.

The integrals in equations (11) and (12) have been evaluated for the curves shown in Figure 1; Durst[2] gives graphs from which $\int r_x dx$ and σ_L/σ may be inferred for routes up to 2,000 nautical miles long, at any level in the troposphere.

Values of R_t* for use in equation (9) are known for some routes. Durst[2] quotes figures appropriate to the London–Rome route at 40,000 ft. for periods from 6 to 24 hr., plus some data for other European routes. R_t* for a 2,000-mile transatlantic route is believed to be about 0·75 at 10,000 and 20,000 ft. for t = 24 hr. (*vide* Harley[6]).

Data sources quoted above may assist in the calculation of regression

† When the computations and tests described in this paper were made, the weather ships "A", " C " and "J " were stationed at 62° 00′N., 33° 00′W.; 52° 45′N., 35° 30′W.; and 52° 30′N., 20° 00′W. respectively.

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Errata

Page 9, Figure 2; the dash–dot curve is the *east–west component, summer* and not the *south–west component, summer* as stated in the key.

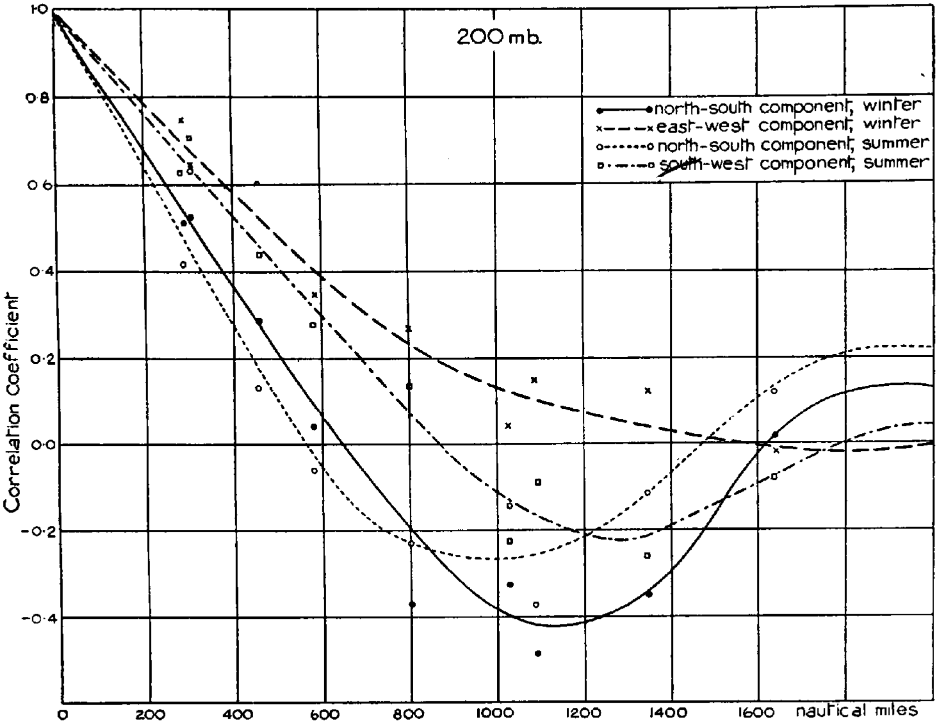

FIGURE 2—VARIATION OF CORRELATION COEFFICIENT OF WIND AT 200 MB. BETWEEN STATIONS AT VARIOUS DISTANCES APART

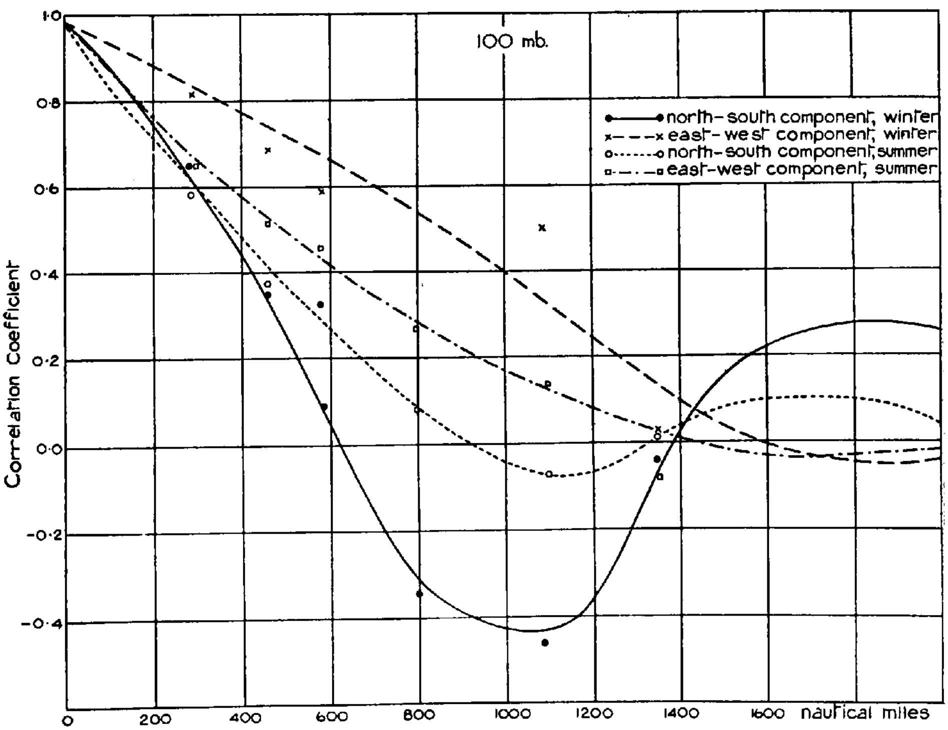

FIGURE 3—VARIATION IN CORRELATION COEFFICIENT OF WIND AT 100 MB. BETWEEN STATIONS AT VARIOUS DISTANCES APART

coefficients when the statistical prediction is made from equations (10) and (9). To use equation (13), $r_{x,t}$ must be known over various ranges of x and t. Values of $r_{x,t}$ have not been published, but it is possible to get approximations, which have given reasonably successful prediction equations, by assuming that

$$r_{x,t} = r_x \, . \, r_t.$$

For particular points this assumption is not very exact; for example, according to Durst,[2] at 300 mb. in the summer of 1948 r_t at Liverpool was 0·63 and r_x, where x is the distance from Aldergrove to Liverpool, was 0·80; the correlation between the wind at Aldergrove and the wind at Liverpool 12 hr. later was not $0{\cdot}80 \times 0{\cdot}63 = 0{\cdot}504$, however, but was 0·72. The difference is due to the tendency for the Aldergrove pressure pattern to be repeated at Liverpool some time later. As a result of such tendencies $r_{x,t}$ may be either greater or less than $r_x \, . \, r_t$, but the net effect of the approximation on the regression coefficients is not very serious, to judge from the practical success of the prediction equations.

Wind components.—In the troposphere the regression coefficients for wind components may be derived from the data quoted for vector winds if it is assumed that r_x and r_t are independent of the direction along which the components are taken and so are equal to the values obtained for vector winds. The standard deviation of a wind component is $2^{-\frac{1}{2}}$ times the standard vector deviation.

These assumptions are not valid for the stratosphere, where the statistics must be developed for each component independently.

Equivalent headwinds.—Tables of mean equivalent headwinds and their standard deviation on some of the principal air routes of the world have been published in two *Meteorological Reports*[7,8].

Theoretically, values of r_x for equivalent headwinds should be very nearly the same as those applicable to headwind components. Sawyer[5] prepared a graph which gives r_x for distances up to 2,500 miles and has computed σ_L/σ for equivalent headwinds over routes up to 4,000 miles long. Sawyer's values of r_x are somewhat higher, distance for distance, than those derived by Durst[2] for vectors; Sawyer worked with geostrophic winds whereas Durst used wind observations, which may explain the discrepancy.

Practical tests of the statistical method.—By autumn 1954, a large volume of wind data had been analysed and standard deviations and correlation coefficients had become available for a large area. A considerable number of pilot experiments in statistical forecasting had been made, but up to that time it had been possible to carry out only small-scale trials. It was decided therefore that a more comprehensive series of tests should be undertaken. A programme was drawn up for providing a comparison between the statistical method and procedures in current use at the Central Forecasting Office and London Airport. The programme was carried out during the first four months of 1955. The trials are reported and discussed at length in a paper of the Meteorological Research Committee, London (Johnson[9]); a summary follows.

Data are given for four sets of forecasts: London Airport (L.A.P.); Central Forecasting Office, Dunstable (C.F.O.); statistical; and persistence. L.A.P. and C.F.O. forecasts were made by the established synoptic method. Predictions were made of the mean tailwind components over approximately great-circle routes from Shannon to Gander at 500 mb. and from weather ship "A" to London at 200 mb. Winds at 700 mb., 500 mb., 300 mb., 200 mb. and 100 mb. were forecast for Liverpool. All the forecasts were based on information received at C.F.O. before

0300 G.M.T. each day and were valid for the following 1500 G.M.T., so the synoptic forecasts were based upon a 1500 G.M.T. chart and upon the 2100 G.M.T. upper-wind observations; the statistical forecasts were based upon 2100 G.M.T. winds when available, otherwise upon 1500 G.M.T. information. For both route and point, estimates of wind or wind observations at 1500 G.M.T. were taken as persistence forecasts for the following 1500 G.M.T. The regression coefficients required in making the statistical forecasts are tabulated in an appendix to Johnson's paper.[9]

Route wind forecasts.—Estimates of the mean wind over the route, the " actuals " against which the route forecasts were verified, were made by three methods:

(i) A template was made at London Airport to get from the working charts the average geostrophic wind over a band approximately 300 miles wide, centred on the route.

(ii) At C.F.O., winds estimated at a series of points equally spaced along the routes were averaged, both geostrophic and observed winds being taken into account.

(iii) Statistical estimates were made using equation (10) with appropriate coefficients.

Means and standard deviations for each type of estimate are given in Table II. Table III contains details of comparisons between the estimates.

TABLE II—ESTIMATES OF WINDS OVER THE ROUTES FOR THE FOUR-MONTH PERIOD JANUARY–APRIL, 1955

Route	Estimate	Mean (kt.)	Standard deviation (kt.)
Shannon–Gander 500 mb.	L.A.P.	—13·2	18·7
	C.F.O.	—13·5	19·2
	" Best "	—13·4	18·9
	Statistical	—15·1	17·5
O.W.S. "A"–London 200 mb.	L.A.P.	20·3	20·4
	C.F.O.	19·9	20·9
	" Best "	20·1	20·5
	Statistical	19·7	20·1

TABLE III—COMPARISON BETWEEN ESTIMATES OF WINDS OVER THE ROUTES FOR THE FOUR-MONTH PERIOD JANUARY–APRIL, 1955

Route	Estimates compared	Mean difference (kt.)	Standard deviation of difference (kt.)	r.m.s. difference (kt.)	Correlation coefficient
Shannon–Gander 500 mb.	C.F.O.–L.A.P.	0·3	5·9	5·9	0·95
	C.F.O.–Statistical	1·6	7·6	7·7	0·92
	L.A.P.–Statistical	1·9	7·4	7·6	0·92
	" Best "–Statistical	1·7	6·9	7·1	0·93
O.W.S. "A"–London 200 mb.	C.F.O.–L.A.P.	0·4	6·1	6·1	0·96
	C.F.O.–Statistical	0·2	4·0	4·0	0·98
	L.A.P.–Statistical	0·6	6·6	6·6	0·95
	" Best "–Statistical	0·4	4·6	4·6	0·97

The agreement between the C.F.O. and L.A.P. estimates was evidently close: the mean differences were negligible, the correlation coefficients were high and the mean-square differences between the estimates were small in comparison with the variances of the estimates themselves. The C.F.O. and L.A.P. estimates were averaged to give a further value, which has been termed the " best " estimate; this was used in verifying the forecasts. The statistical estimates agree well with the other three, although they have a systematic error of about 2 kt. for the route Shannon–Gander.

In forecasting winds over the routes, both C.F.O. and L.A.P. used templates to get a mean geostrophic wind from their forecast charts. Table IV gives the means and standard deviations of the four types of forecasts, with statistics relating to the forecast errors. An important statistic for assessing the success of wind forecasts is the r.m.s. error; r.m.s. errors are set out in Table IV but for analysis it is convenient to break the r.m.s. error down into its component parts, bias and standard deviation.

TABLE IV—VERIFICATION OF FORECASTS OF WIND OVER THE ROUTES FOR JANUARY–APRIL, 1955

Route		Mean (kt.)	Standard deviation (kt.)	Bias (kt.)	Standard deviation of forecast errors (kt.)	r.m.s. error (kt.)	Correlation coefficient
Shannon–Gander 500 mb.	Actual	—13·4	18·9				
	L.A.P. forecast	—13·4	18·0	0·0	11·1	11·0	0·82
	C.F.O. forecast	—13·7	19·5	0·3	10·9	10·9	0·84
	Statistical forecast	—19·2	12·3	5·8	12·3	13·6	0·77
	Persistence forecast	—13·5	19·2	0·1	13·4	13·4	0·75
O.W.S. "A"–London 200 mb.	Actual	20·1	20·5				
	L.A.P. forecast	18·2	19·9	1·9	12·2	12·3	0·82
	C.F.O. forecast	18·1	19·9	2·0	10·9	11·1	0·85
	Statistical forecast	22·3	16·1	—2·2	12·8	12·9	0·78
	Persistence forecast	19·9	20·9	0·2	14·7	14·7	0·75

Over the route Shannon–Gander the bias in the synoptic forecasts was negligible, but in the statistical forecasts the bias was nearly 6 kt. The reason for the bias can easily be understood by reference to equation (6), where the second term on the right-hand side represents the square of the bias; this is proportional to $(\bar{U}^* - \bar{U})$ which averaged 17 kt. during the four months of the trial. For the route weather ship "A"–London the statistical forecast bias of 2 kt. can be explained similarly. The 2-kt. biases in the synoptic forecasts for the 200-mb. route were statistically significant† at the 5 per cent level, which suggests there was a real tendency for the synoptic forecasters slightly to underestimate the 200-mb. tailwinds. Except for the 500-mb. statistical forecasts, the bias made a negligible contribution to the r.m.s. error.

† The tests of significance, which were complicated by serial correlations in the forecast errors, are discussed in detail by Johnson.[9]

Comparing the standard deviations of the forecast errors, both L.A.P. and C.F.O. improved upon the statistical forecasts by a little over a knot at 500 mb. For the 200-mb. route, C.F.O. were more successful than L.A.P., the 1·3-kt. difference being significant at the 7 per cent level. This result is interesting because, although the two stations used very similar methods of forecasting at 500 mb., at 200 mb. C.F.O. made rather more use of the thickness technique both in analysis and forecasting. The statistical forecast error exceeded the C.F.O. error by almost 2 kt. at 200 mb. (2 per cent significance). The last column of Table IV gives correlations between forecast and actual winds. Synoptic forecasts show the higher correlations.

Table V demonstrates the frequencies of errors greater than specific magnitudes. Figures given in the last two columns indicate that there was little to choose between the two techniques as far as avoidance of extreme errors was concerned; in the remainder of the table, frequencies for statistical forecasts exceed those for synoptic forecasts.

TABLE V—FREQUENCIES OF ERRORS GREATER THAN SPECIFIED MAGNITUDES
(120 forecasts made)

Route	Type of forecast	Error size (kt.)							
		2½	7½	12½	17½	22½	27½	32½	37½
Shannon–Gander 500 mb.	L.A.P.	93	62	32	13	6	1	0	0
	C.F.O.	98	56	33	13	6	1	0	0
	Statistical	102	68	47	26	12	4	1	1
	Persistence	103	72	40	21	12	3	2	1
O.W.S. "A"–London 200 mb.	L.A.P.	101	63	37	20	6	3	3	0
	C.F.O.	98	59	32	14	4	2	1	1
	Statistical	103	66	43	21	8	4	1	0
	Persistence	110	77	44	30	17	8	3	0

Figures 4 and 5 graph the daily values of the forecasts for comparison with estimated actuals. The L.A.P. and C.F.O. graphs suggest that while there was a high correlation between the two forecasts, the forecast errors were not correlated to the same degree; the correlation coefficient between the C.F.O. and L.A.P. errors was 0·61 at 500 mb. and 0·58 at 200 mb. It is thus possible to reduce the standard deviation of the synoptic forecast errors by combining the two sets of forecasts, the expected reduction amounting to 10 per cent. (It is also theoretically possible to improve the statistical forecasts by basing the regression equation on the synoptic forecasts instead of upon the latest actuals.) The statistical forecast graph demonstrates the systematic errors to which the regression equation gave rise over short periods; see, for example, the curves for the first half of January at 500 mb., when the statistical predictions show clearly the effects of consistent regression toward the climatic mean tailwind of −35 kt.

Point wind forecasts.—Forecasts of winds at Liverpool were verified against wind observations or against estimates made from the working charts. One estimate was required at 500 mb. and fourteen at 100 mb.

Means of the south and west wind components for each type of forecast, corresponding standard vector deviations and statistics relating to the forecast errors are listed in Table VI.

TABLE VI—VERIFICATION OF FORECASTS OF WIND AT LIVERPOOL FOR JANUARY–APRIL, 1955

Level (mb.)		Mean component (kt.) S.	Mean component (kt.) W.	Standard vector deviation (kt.)	Bias in component (kt.) S.	Bias in component (kt.) W.	Standard vector deviation of errors (kt.)	r.m.s. error (kt.)	Correlation coefficient
700	Actual	—3·8	7·0	27·0					
	L.A.P.	—3·1	8·2	28·2	—0·7	—1·2	19·0	18·9	0·77
	C.F.O.	—2·9	8·3	26·5	—0·9	—1·3	18·5	18·5	0·76
	Statistical	—2·6	8·7	17·2	—1·2	—1·7	20·1	20·1	0·67
	Persistence	—3·8	6·8	27·0	0	0·2	24·5	24·4	0·59
500	Actual	—7·4	15·3	39·9					
	L.A.P.	—6·7	15·2	39·3	—0·7	0·1	25·5	25·4	0·79
	C.F.O.	—6·3	14·3	35·7	—1·1	1·0	26·6	26·5	0·76
	Statistical	—7·4	16·7	25·7	0	—1·4	28·9	28·8	0·69
	Persistence	—7·5	15·0	39·9	0·1	0·3	35·2	35·1	0·61
300	Actual	—9·9	22·6	56·1					
	L.A.P.	—9·0	25·1	51·1	—0·9	—2·5	35·3	35·2	0·79
	C.F.O.	—9·9	22·0	47·9	0	0·6	36·6	36·5	0·76
	Statistical	—11·2	26·8	36·5	1·3	—4·2	40·3	40·3	0·70
	Persistence	—10·0	22·3	56·0	0·1	0·3	48·3	48·1	0·63
200	Actual	—11·2	21·5	39·1					
	L.A.P.	—11·8	23·7	42·1	0·6	—2·2	25·4	25·4	0·81
	C.F.O.	—10·1	21·8	37·7	—1·1	—0·3	24·1	24·1	0·80
	Statistical	—11·0	26·1	23·5	—0·2	—4·6	26·2	26·5	0·76
	Persistence	—11·4	21·3	39·0	0·2	0·2	30·1	30·0	0·70
100	Actual	—10·4	17·5	23·5					
	C.F.O.	—10·0	18·4	22·0	—0·4	—0·9	13·8	13·7	0·82
	Statistical	—8·9	19·9	15·1	—1·5	—2·4	13·6	13·9	0·84
	Persistence	—10·5	17·4	23·5	0·1	0·1	15·1	15·0	0·79

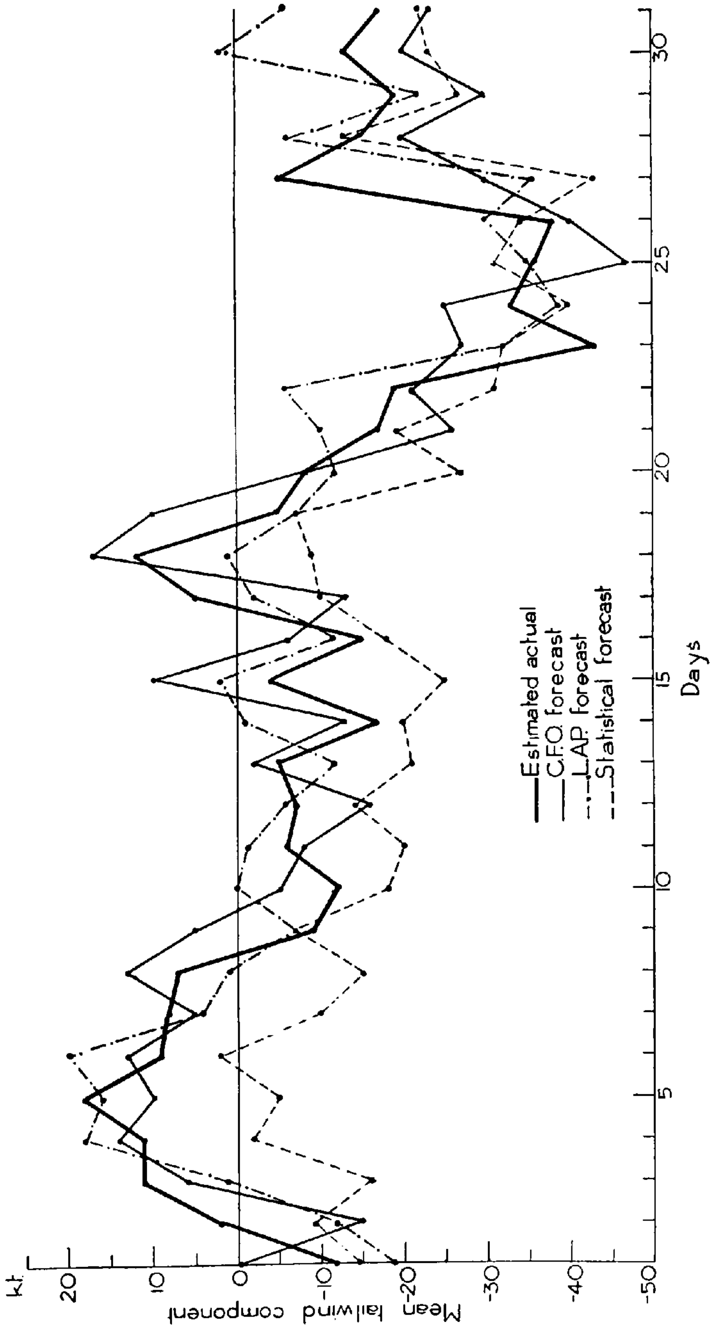

FIGURE 4(*a*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 500 MB. FOR THE ROUTE SHANNON–GANDER, JANUARY 1955

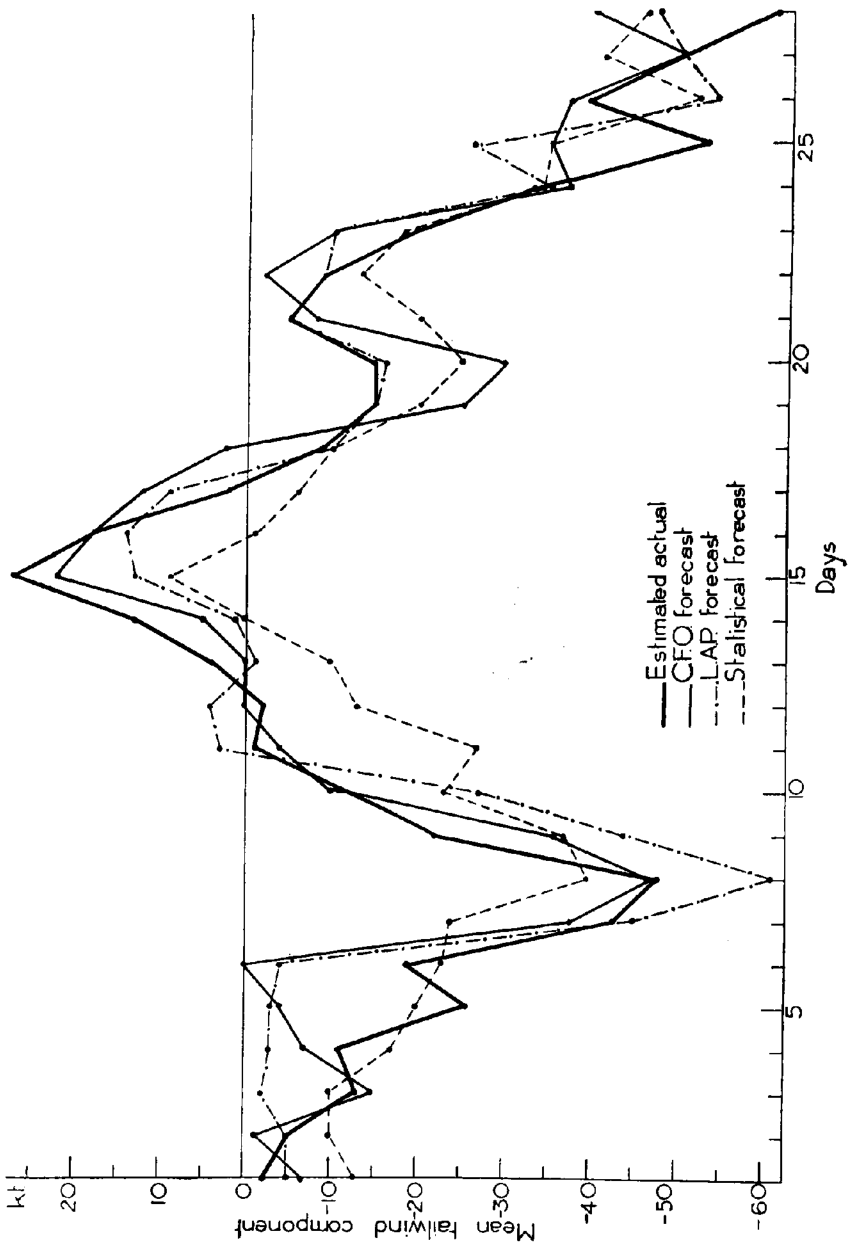

FIGURE 4(*b*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 500 MB. FOR THE ROUTE SHANNON–GANDER, FEBRUARY 1955

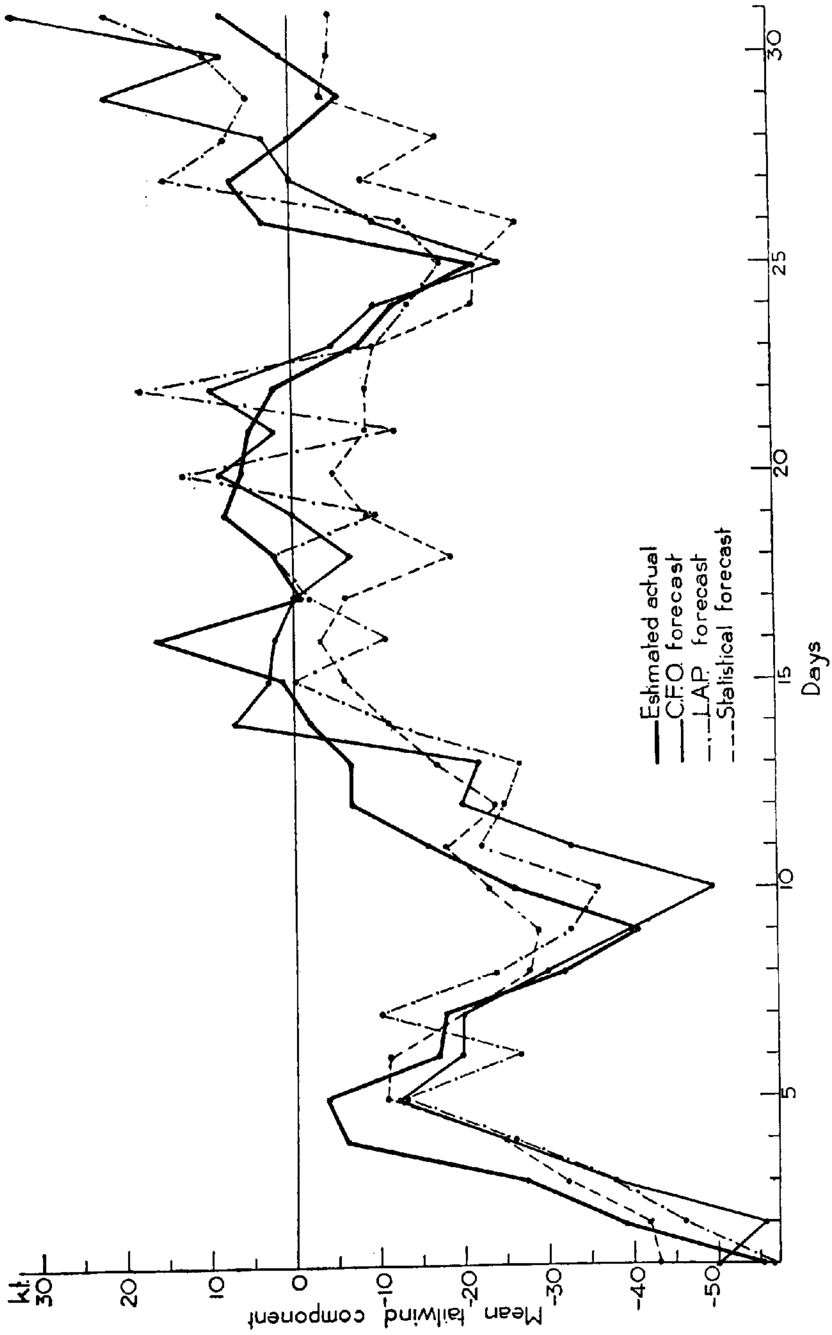

FIGURE 4(*c*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 500 MB. FOR THE ROUTE SHANNON–GANDER, MARCH 1955

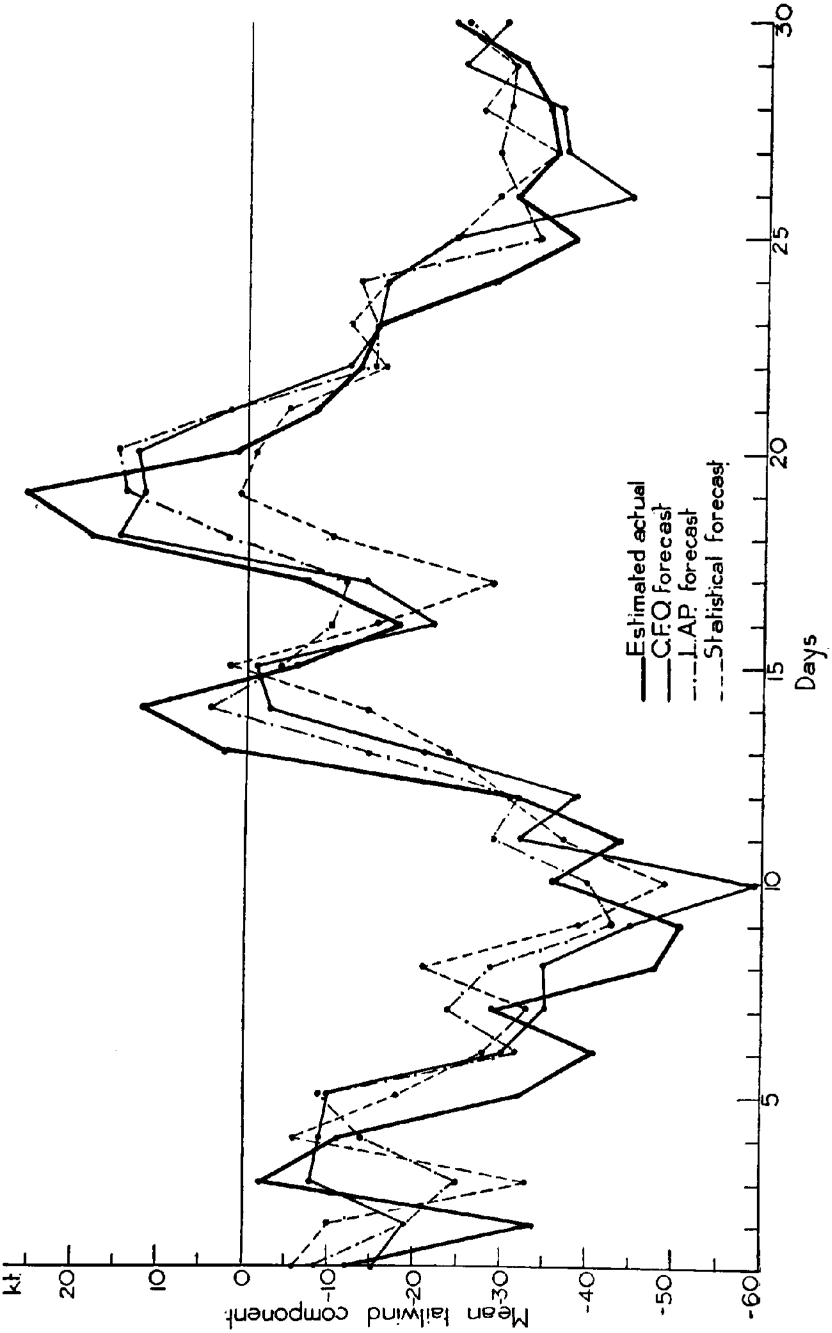

FIGURE 4(*d*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 500 MB. FOR THE ROUTE SHANNON–GANDER, APRIL 1955

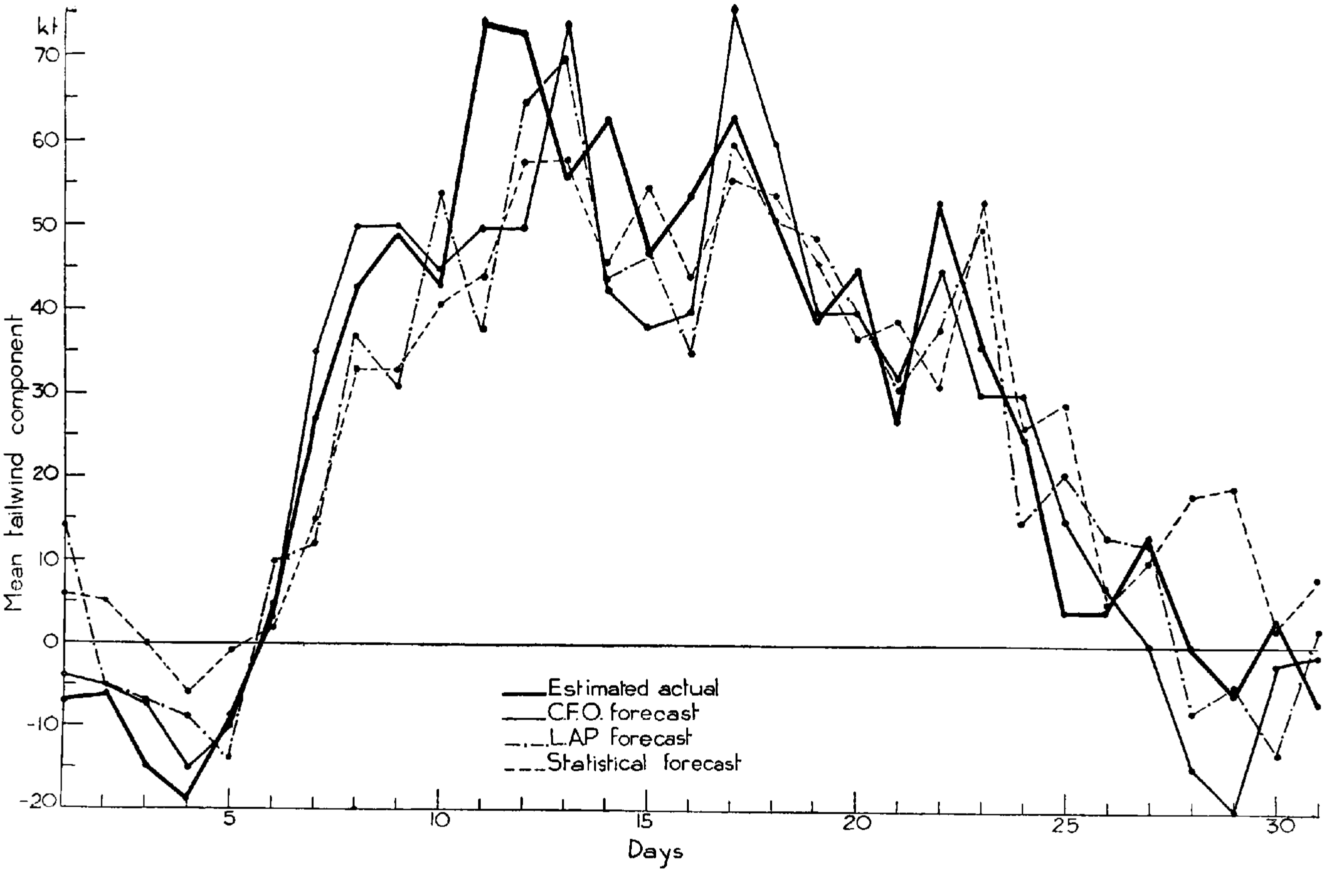

FIGURE 5(*a*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 200 MB. FOR THE ROUTE O.W.S. "A"–LONDON, JANUARY 1955

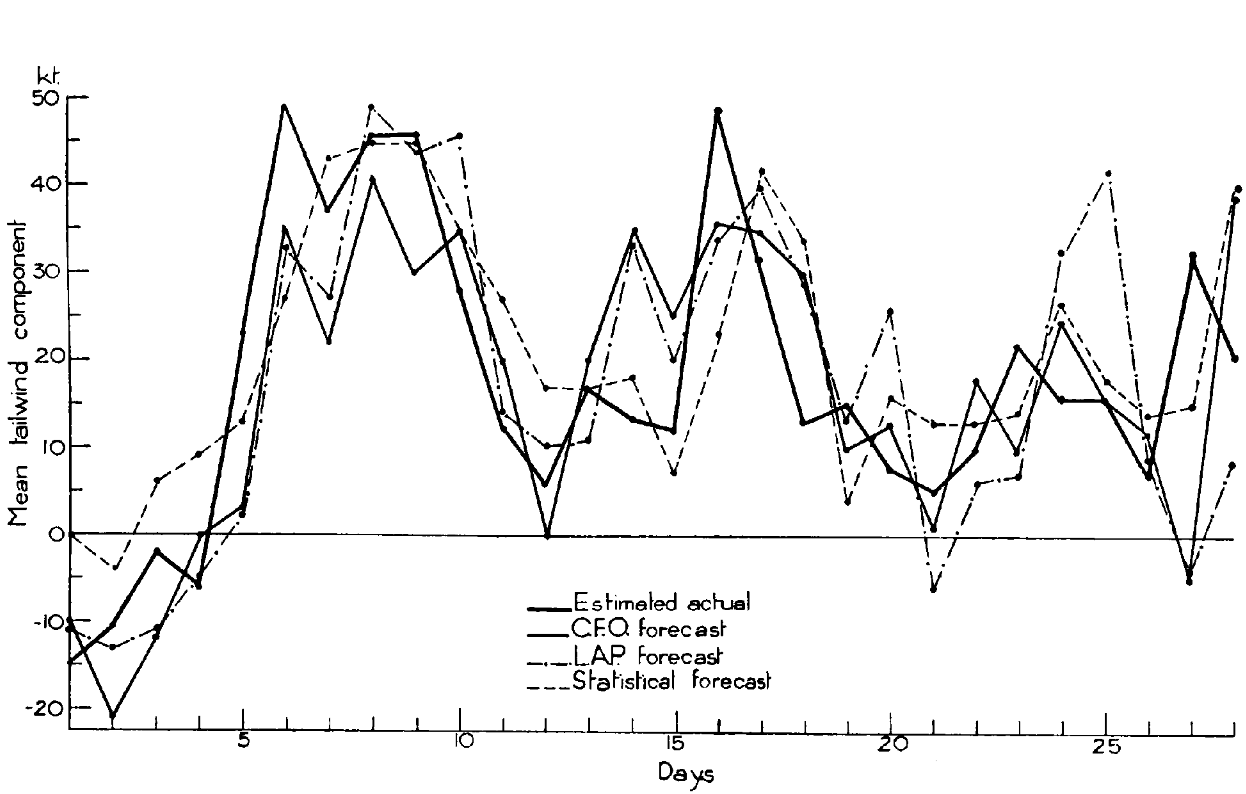

FIGURE 5(*b*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 200 MB. FOR THE ROUTE O.W.S. "A"-LONDON, FEBRUARY 1955

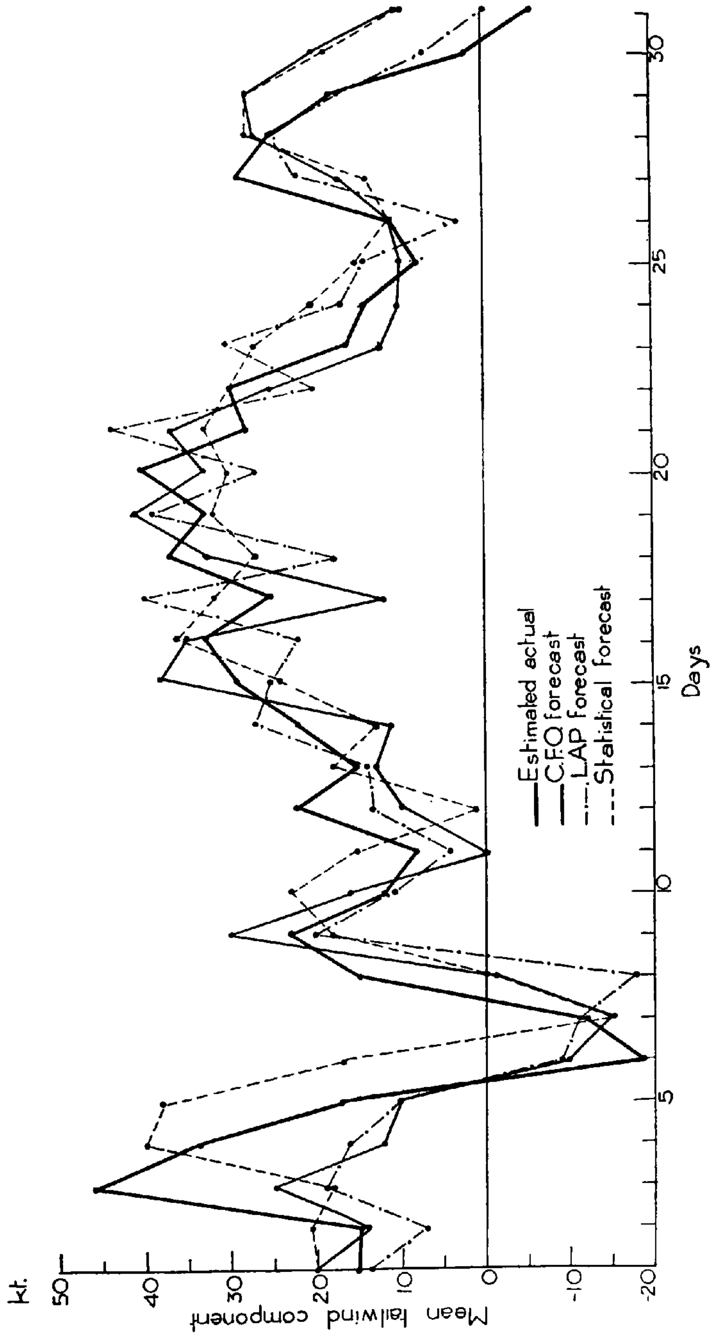

FIGURE 5(*c*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 200 MB. FOR THE ROUTE O.W.S. "A"–LONDON, MARCH 1955

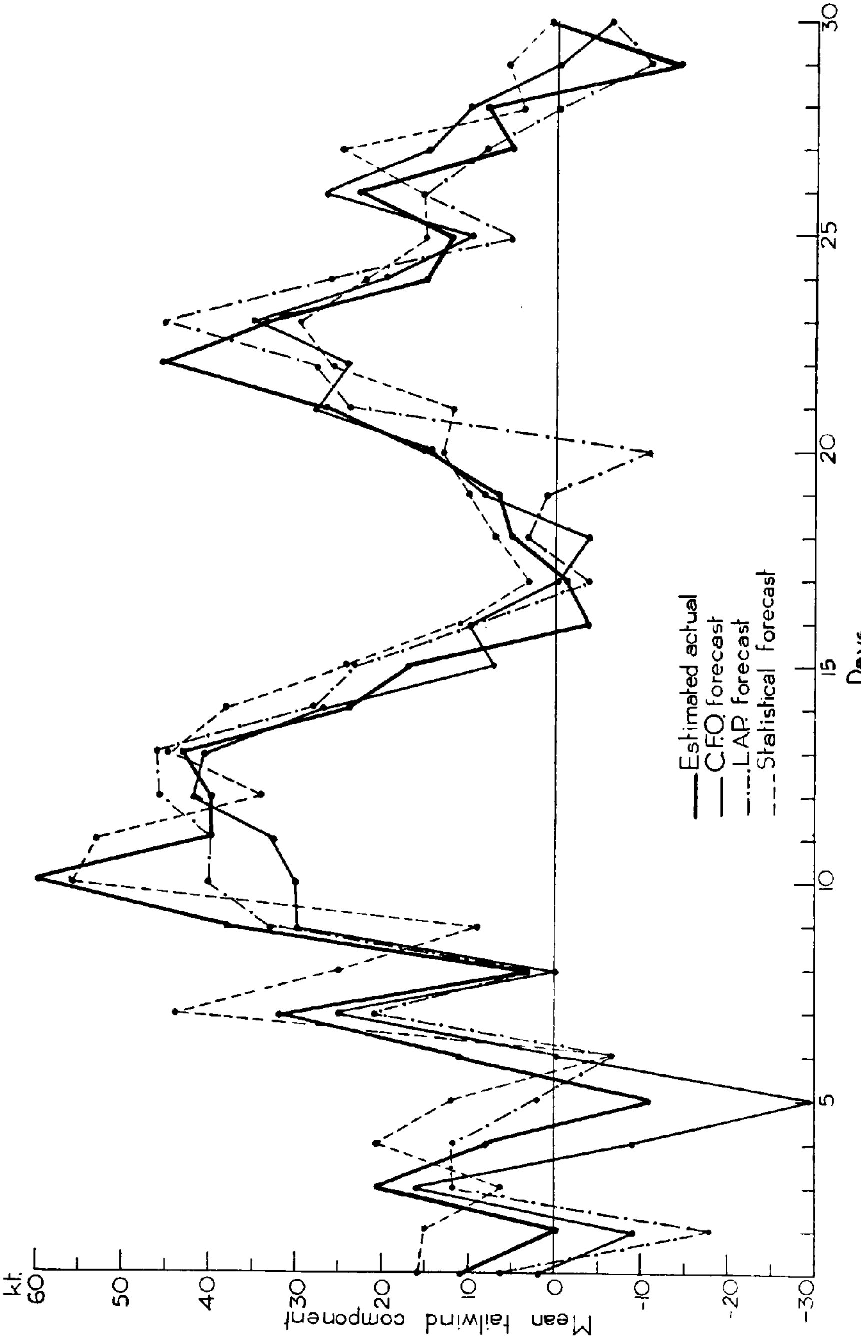

FIGURE 5(*d*)—INDIVIDUAL FORECASTS OF THE MEAN TAILWIND COMPONENT AT 200 MB. FOR THE ROUTE O.W.S. "A"–LONDON, APRIL 1955

For reasons similar to those discussed in the section on route winds, the statistical point forecasts have a small bias. For all types of forecast the contribution of the bias to the r.m.s. error was small.

The synoptic forecasters achieved smaller standard vector deviations of error for all levels up to 200 mb. At 100 mb. there was nothing to choose between the C.F.O. and statistical errors. Table VII gives levels of statistical significance of the difference between pairs of the standard deviations of error.

TABLE VII—LEVELS OF SIGNIFICANCE OF DIFFERENCE BETWEEN STANDARD DEVIATION OF THE FORECAST ERRORS

Location		C.F.O.–Statistical		C.F.O.–L.A.P.	
		difference kt.	significance level %	difference kt.	significance level %
Route Shannon–Gander	500 mb.	—1·43	11	negligible	—
Route O.W.S. "A"–London	200 mb.	—1·85	2	—1·24	7
Liverpool	700 mb.	—1·58	5	negligible	—
	500 mb.	—2·34	4	1·08	27
	300 mb.	—3·64	4	1·36	35
	200 mb.	—2·07	7	—1·27	13
	100 mb.	negligible	—	—	—

In assessing the gain over persistence it must be recalled that the figures quoted in the tables refer to 24-hr. persistence forecasts. The synoptic and statistical forecasts made use of 2100 G.M.T. observations in addition to 1500 G.M.T. data; if 2100 G.M.T. observations had been used for the persistence forecasts when available, the persistence forecast errors would have been reduced by about 10 per cent; persistence would then have attained approximately the same success as the other methods at 100 mb., although at lower levels the synoptic and regression forecasts would still have shown substantial gains.

In Table VIII are tabulated frequencies of errors greater than 50 kt. and 70 kt. respectively. Neglecting persistence, it does not appear that one technique was less liable to give large errors than another.

TABLE VIII—FREQUENCIES OF ERRORS GREATER THAN 50 KT. AND GREATER THAN 70 KT. IN POINT FORECASTS FOR LIVERPOOL AND WEATHER SHIP "J"

Forecast	700 mb.		500 mb.		300 mb.		200 mb.	
	>50 kt.	>70 kt.	>50 kt.	>70 kt.	>50 kt.	>70 kt.	>50 kt.	>70 kt.
L.A.P.	2	1	12	2	35	9	16	4
C.F.O.	2	1	14	0	31	13	12	3
Statistical	1	0	14	0	47	12	17	4
Persistence	7	0	38	6	62	33	25	7

Assessment of the 1955 *trials*.—The representativeness of the weather conditions experienced during the practical tests described above has been discussed elsewhere.[9] It was shown that the biases in the statistical forecasts for the 500 mb. route were abnormally large, but that a true impression of the relative successes of the different techniques could be gained from the comparison of the standard deviations of the forecast errors. Accordingly the trials were taken to have established in practice:

(i) that the synoptic and statistical techniques both give appreciably smaller errors than persistence forecasts at levels up to, and including, 200 mb.; at 100 mb. neither technique improves significantly upon persistence;

(ii) that the synoptic method is capable of giving significantly smaller errors than the statistical technique in temperate latitudes when a good coverage of observations is available.

A test with composite winds.—The route winds forecast in the 1955 trials were instantaneous mean winds over the routes. To show that regression equations are capable in practice of predicting composite route winds, some results are quoted for an experiment in which wind components were forecast for aircraft on a flight of 11-hr. duration on the great-circle track from London to Gander. The forecasts were based on observations from Larkhill (U_1, V_1), weather ship " J " (U_2, V_2), weather ship " C " (U_3, V_3) and Newfoundland (U_4, V_4). Regression equations applied to both wind components and for use at 700 mb. or 500 mb. were:

(i) for an instantaneous route wind at the time of observation,

$$F(U) = 0{\cdot}15\,U_1 + 0{\cdot}31\,U_2 + 0{\cdot}31\,U_3 + 0{\cdot}15\,U_4;$$

(ii) for a route wind referring to 4 hr. after the observation time at London and 15 hr. after the observation time at Gander,

$$F(U) = 0{\cdot}14\,U_1 + 0{\cdot}28\,U_2 + 0{\cdot}26\,U_3 + 0{\cdot}12\,U_4;$$

(iii) for a route wind referring to 16 hr. after the observation time at London and 27 hr. after the observation time at Gander,

$$F(U) = 0{\cdot}12\,U_1 + 0{\cdot}24\,U_2 + 0{\cdot}23\,U_3 + 0{\cdot}10\,U_4.$$

The U_i in these equations are deviations from the appropriate climatic mean values. The equations were devised in 1952 and tested with observations at 500 mb. for the period of 104 days between 22 December 1952 and 4 April 1953 with reasonably satisfactory results as shown in Figure 6.

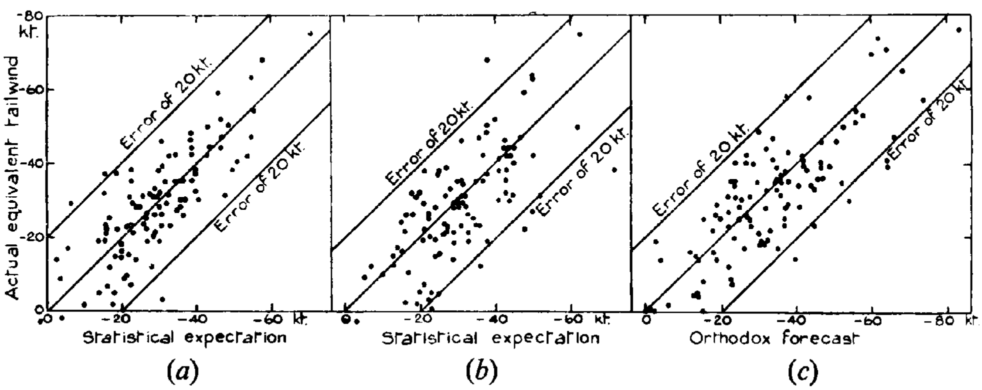

FIGURE 6—COMPARISON OF FORECAST AND ACTUAL TAILWIND AT 500 MB. ON TRANSATLANTIC ROUTES

(*a*) 12 hours ahead, London to Gander
(*b*) 24 hours ahead, London to Gander
(*c*) 24 hours ahead, Shannon to Gander

Forecasts for transatlantic flights in the stratosphere.—There have been no practical tests of statistical forecasts made for stratospheric transatlantic flights; Table IX contains theoretical values of the r.m.s. errors which could be expected.

TABLE IX—ROOT MEAN SQUARE ERRORS (KT.) WHICH WOULD PROBABLY ARISE USING REGRESSION EQUATIONS TO GIVE ESTIMATES OF ROUTE WINDS ACROSS THE ATLANTIC AT 200 MB. AND 100 MB.

Level	Season	Component	Period of forecast			Error with no observation
			0 hr.	12 hr.	24 hr.	
100 mb.	Winter	N	5	6	7	10
		E	3½	8	11	17
	Summer	N	3	4	4½	6½
		E	3½	5½	7	10½
200 mb.	Winter	N	10	13	15	20
		E	7	11½	15	22
	Summer	N	7	8½	9½	12
		E	6	9½	12	17

From earlier discussion it is clear that errors in statistical forecasts at great heights are not very much greater than those made by synoptic methods. Hence it is likely that the errors given in Table IX are not greatly different from those which would be found if a long series of forecasts were made by synoptic methods and compared with true values of the route winds.

Relation between statistical and synoptic forecasting success and its use in estimating the value of weather ships in forecasting for aviation.—Comparison of errors in the statistical and synoptic forecasts set out in Tables IV and VI indicates that the two sets of errors are quite closely related. During analysis of the results of the 1955 trials, monthly values of the errors were examined; the two sets of errors varied from month to month in step with each other, the link being, understandably, through their common dependence on the time-variation of the wind. It seems legitimate to deduce that the standard error in a statistical forecast gives a reasonable indication (though perhaps 10 or 20 per cent too high) of the synoptic forecasting error, and that the statistical method could be used to assess the comparative values of different forecasting arrangements. For instance, it would be possible to compare the accuracy of forecasts made with full information but based on an earlier chart with those made with incomplete data but based on observations received at a later time; alternatively, as described in detail in the following paragraphs, the effect of removing certain stations on the errors of wind forecasts can be assessed.

It was found that the correlation coefficient between the route wind from London to Gander at 700 or 500 mb. and the estimated route wind determined from equation (10) was 0·93. 0·75 represents the correlation between tailwinds over this route at a certain time and 24 hr. later. If we assume that the error in the statistical forecast is not closely related to the error in the estimate of the route wind, so that the correlation coefficient between the route wind 24 hr. later and the observations at the four stations is $0{\cdot}75 \times 0{\cdot}93$, we find that the forecasting error will be $0{\cdot}72\ \sigma_L$, where σ_L is the standard deviation of the route wind in the appropriate season at the appropriate height.

In Table X are set down: estimates of the average of the standard deviations, σ, at points on the route London to Gander for 700 and

500 mb.; the product $0{\cdot}707 \times 0{\cdot}57\ \sigma$ which gives the value of the standard deviation of tailwinds averaged over the route, σ_L; and $0{\cdot}72\ \sigma_L$ (or $0{\cdot}29\ \sigma$) which gives the error for a 24-hr. statistical forecast.

TABLE X—ESTIMATES OF THE ROUTE MEAN SQUARE ERROR FOR 24-HOUR FORECASTS ON THE ROUTE LONDON TO GANDER IN COMPARISON WITH ERRORS MADE BY SYNOPTIC METHODS

	Winter		Spring		Summer		Autumn	
	700 mb.	500 mb.	700 mb.	500 mb.	700 mb.	500 mb.	700 mb.	500 mb.
σ (kt.)	32·5	41·5	28	37	21·5	28	27	35
$0{\cdot}707 \times 0{\cdot}57\sigma$ (kt.)	13	17	11	15	9	11	11	14
$0{\cdot}29\ \sigma$ (kt.)	9·5	12	8	11	6	8	8	10
Harley's errors (kt.)	8·7	12·5	7·8	10·5	6·5	8·5	8·1	10·5

The last line of Table X gives the estimated standard error of 24-hr. forecast winds for the route Shannon to Gander given by Harley's examination[6] of the accuracy of forecasts of tailwind. The agreement between the last two lines is remarkably close and suggests that the greater inaccuracy of the statistical method just balances the greater difficulty of forecasting for the shorter route Shannon to Gander. Nevertheless, with such close agreement it is reasonable to use the statistical errors as a yardstick with which to assess the efficacy of various patterns of observing stations.

Accordingly, in Table XI are given details of the comparative accuracy of tailwinds and of forecasts derived from them when there are: (*a*) no observations; (*b*) land stations only; (*c*) one ship mid-way between London and Gander; (*d*) two ships. By this statistical operation we can form a fairly satisfactory estimate of the actual effect which the presence of weather ships has on the forecasting of winds on the Atlantic air routes.

TABLE XI—ACCURACY OF ESTIMATED ROUTE WINDS AND OF FORECASTS WITH VARIOUS DISTRIBUTIONS OF WEATHER SHIPS ON THE ROUTE LONDON TO GANDER

		Winter		Spring		Summer		Autumn	
		700 mb.	500 mb.	700 mb.	500 mb.	700 mb.	500 mb.	700 mb.	500 mb.
r.m.s. error of estimated route wind	(a) no observations	13	17	11	15	9	11	11	14
	(b) land stations only	12	15	10	13	8	10	10	13
	(c) one ship	7	9	6	8	5	6	6	8
	(d) two ships	5	6	4	6	3	4	4	5
r.m.s. error of 24-hr. forecast	(b) land stations only	12	16	10	14	9	10	10	13
	(c) one ship	10	13	9	12	7	9	9	11
	(d) two ships	9	12	8	11	6	8	8	10

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Printed in England under the authority of Her Majesty's Stationery Office
by The Campfield Press, St. Albans

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