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AN INVESTIGATION OF THE TURBULENCE BALANCE EQUATIONS IN THE ATMOSPHERIC BOUNDARY LAYER

by

R. Rayment & S.J. Caughey

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Abstract

Three Cardington turbulence probes were operated at heights of 61, 91 and 152 m over a five hour period in an unstable boundary layer. The results have been used to estimate (at the middle level) and assess the relative importance of the terms in the turbulence balance equations for kinetic energy, temperature variance, stress and the vertical and total horizontal heat fluxes.

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An Investigation of the turbulence balance equations

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Introduction

During the past decade the relative importance (and dependence on stability) of some of the terms in the turbulence balance equations has been assessed in the first 20 m of the atmosphere (see e.g. Wyngaard and Coté, 1971, Wyngaard et al 1971, etc). In this respect the 1968 Kansas experiment carried out by the Boundary Layer Branch of the Meteorology Laboratory of AFCRL was unique in providing direct measurements of turbulent production, transport and dissipation rates over a range of stability conditions. This permitted a detailed investigation of the equations to be carried out, however a general specification of the balance equations in the boundary layer as a whole has not yet been achieved due to the sparseness of data from above the surface layer. Furthermore at these higher levels the averaging time required to obtain physically meaningful estimates of the turbulent statistics is still in doubt (Wyngaard et al 1974).

This paper adds some further information on the region between 50 m and 150 m using data from an experiment carried out at Earles Croome ($52^{o}05'$N, $02^{o}15'$W) on the 24th August 1972 as part of an investigation of synoptic inversions (Readings et al, 1973, Caughey et al 1975). The terrain in the vicinity of the site is of typically rural character, however the Malvern hills are only a few miles to the west and this implies that terms in the balance equations arising from horizontal inhomogeneity could perhaps on occasions be significant and that deductions about the nature of imbalance quantities must be tentative.

On this day three Cardington turbulence probes (see Readings and Butler, 1972) were located at heights of 61, 91 and 152 metres. The total run duration was 5 hours [beginning at 1031Z] and over this period there was a complete cover of thin stratus/stratocumulus and a temperature inversion persisted throughout the day. Though a comparison with a similar study in other weather conditions cannot yet be made, the lack of rapid changes on this day, both in insolation and advection made it likely to yield a useful data set. Results from two consecutive uninterrupted runs of three hours and two hours duration will be described.

2. Derivation of the mean and gradient quantities

Shown in Figure 1 are the mean hourly temperatures ($\overline{T}$) and wind speeds ($\overline{u}$) (small corrections have been applied from a previous comparison run at 3 m height). These profiles indicate that, over the total interval, $\overline{T}$ increased by about 1°C whilst the variation in $\overline{u}$ was rather erratic. Of particular note is the establishment of a reverse wind gradient during hours four and five.

Hourly mean potential temperature profiles are shown in Figure 2 with some radiosonde ascents (at Defford, Worcestershire) for comparison. These indicate that the probes were situated near a turning point in the lapse rate (the reliability of the radiosonde profiles is difficult to assess, they could easily have overestimated the depth of the superadiabatic region). Thus there is some uncertainty on how best to determine the value of $\partial\overline{\theta}/\partial Z$ to use in the balance equations. Several regression lines were tried (i.e. ln-ln, linear, etc) but although these produced somewhat different gradients (up to 100%) it was decided to use a logarithmic line through the three heights since this represented, to some extent, an average line. Of course the only validity for this procedure lies in the physical interpretability of the results and as such it seems justified.

For temperature therefore,

$$\frac{\partial\overline{\theta}}{\partial Z} = .0000977 + C_1/Z \ {}^{\circ}\text{C cm}^{-1}$$

where

$$\text{T} = C_1 \ln Z + C_2$$

Tables 1 and 2 give the vertical gradients of $\overline{\theta}$ and $\overline{u}$ at 91 m. The potential temperature gradient remained fairly constant but the wind speed gradient decreased and eventually reversed in the fourth and fifth hours. Gradient Richardson numbers (see Tables 1 and 2) showed a marked trend towards greater instability across the period so that although the day was overcast it was quite unstable. After applying comparison corrections Readings and Butler (1972) quote residual errors for $\overline{T}$ and $\overline{u}$ of $\pm$ .02°C and $\pm$ 3 cms sec^{-1}. For the log linear regression the maximum error in the gradient is (for the heights used here)

$$E_{max} = \pm \left| \xi \right| x.00025$$

where $\left| \xi \right|$ is the error in the mean, thus the maximum errors in $\partial\overline{\theta}/\partial Z$ and $\partial\overline{u}/\partial Z$, due to errors in the mean quantities, are about .000005°C cm^{-1} and .00075 s^{-1} respectively.

3. Derivation of the variances, covariances, triple products and their vertical gradients

The spectral analysis of this data revealed that low frequency oscillations could, on occasions, contribute significantly to the turbulent statistics. This was particularly true for the products involving temperature and made the decision on whether to use values calculated from the mean or a linear regression difficult. Both sets were computed and compared and since the conclusions were not essentially different only the set calculated about the mean will be given here. Thus the quantities u', v', w' and T' represent the departures of the longitudinal, lateral and vertical wind components and temperature from their respective means. In addition, as pointed out in section 1, the averaging periods required to produce meaningful values of the fluxes (particularly the stress $\overline{u'w'}$) and triple products are uncertain but are expected to be of the order of hours (Wyngaard et al 1974). Hence all terms in the balance equations were calculated over both one and two hours.

Vertical profiles of the mean statistics showed a general and unexpected tendency for a persistent maximum or minimum at the middle level. This may correspond to the similar tendency in the profiles of $\overline{\theta}$ and $\overline{u}$ and might have been due to some upwind terrain effect or this level could have been situated in a transition region in the organisation of convective elements see e.g. Kaimal and Haugen (1967). The problem still remains of how best to determine the terms to be entered in the balance equations, if this is possible at all with such profiles. In analogy with the method chosen for the potential temperature and wind speed a logarithmic regression was fitted to the data at the three levels and then used to determine a value and gradient (where necessary) at 91 m. The results for the one and two hourly averages are included in Tables 1 and 2 respectively.

Apart from the difficulty associated with estimating the 'representativeness' of the measurements and the method of deriving gradients the intrinsic instrumental errors associated with the measurements must be considered. This has been examined in detail for the Cardington instrumentation by Readings and Butler (1971 and 1972) and Rayment (1975). In general the likely errors in the variances/covariances are about 5% whilst those for the triple products are around 10%.

4. High frequency fluctuation measurements

The estimate of the rates of dissipation of turbulent kinetic energy (ε) and temperature variance (N) were obtained by the method previously described by Caughey and Rayment (1974). Briefly, the estimate of ε and N are obtained from the outputs of band pass filter units chosen to lie in the inertial subrange band of frequencies. The filter used in this study had a centre frequency of 1.30 Hz and half-power values of 3.19 Hz and 0.53 Hz. In line with the other quantities the hourly profiles of these variables also revealed a maximum or minimum at the middle level, so for consistency values of ε and N at 91 m were derived from logarithmic regression lines. The likely error associated with these measurements is of order 5%.

5. The energy balance equation

This equation is usually written in the form (under the assumptions of horizontal homogeneity and zero mean vertical velocity),

$$\frac{\partial}{\partial t}(\bar{E}) = \underset{(M)}{-\overline{u'w'}\frac{\partial \bar{u}}{\partial Z}} + \underset{(B)}{\frac{g}{T}\overline{w'T'}} - \left[\underset{(P)}{\frac{\partial}{\partial Z}\frac{1}{\rho}\overline{w'p'}} + \underset{(T)}{\frac{\partial}{\partial Z}\overline{w'E}}\right] - \varepsilon$$

where $E = \frac{1}{2}(u'^2 + v'^2 + w'^2)$

M - represents the mechanical production term.

B - " " buoyant " ".

ε - rate of dissipation of turbulent kinetic energy (T.K.E.). (E)

P - pressure transport term.

T - divergence of the flux of T.K.E. (E)

It is worth emphasizing that during this experiment the turbulence probe resolved only the total horizontal wind speed and thus in the turbulent kinetic energy expression $\overline{u'^2 + v'^2}$ is replaced by $\overline{u_h'^2}$ (under the plausible assumption that $|u'| \simeq |v'| \approx \bar{u}/10$ the error introduced is about 10%). The estimated values of the above terms with respect to the time mean over one and two hours are illustrated in Figure 3 from which several points of interest emerge, (the time derivative terms on the left hand side of this and the other equations to be discussed are negligible).

i. The balance proposed by Busch and Panofsky (1968) namely that ε = M+B seems appropriate on this occasion, especially for the two hour averaged data.

ii. In the later periods where the instability increased M becomes small and ε could then be used to determine B and hence the heat flux to within about 20%.

iii. The sign of the transport term, ie negative, (apart from a small positive value in hour four), representing a net export of energy to higher levels, is the same as that previously found by Lumley and Panofsky (1964) at 91 m and by Wyngaard and Cote (1971) below about 20 m.

In so far as the residual represents the pressure term (P) the results then indicate that $P \approx T$ and this should be compared with the result deduced from surface layer data that $T \approx B$ and could imply that a region close to the surface in which $T \approx B$ (and hence $\mathcal{E} \approx M + P$) is overlain by one in which $T \approx P$ (and thus $\mathcal{E} \approx M + B$). Further more comprehensive measurements are required to test the realism of this 'two-layer' idea.

iv. The other terms that have been omitted from the balance equation (under the assumption of horizontal homogeneity and zero mean vertical velocity) may be roughly estimated from the data. These are

$$(\overline{u'^2} - \overline{w'^2})\, \partial \bar{u}/\partial x + \bar{u}\, \frac{\partial \bar{E}}{\partial x} + \bar{w}\, \frac{\partial \bar{E}}{\partial z}$$

– the final term is about unity (c.g.s. units). The derivatives in the x direction are not known but insofar as Taylors hypothesis ($dx = \bar{u}\, dt$) gives some idea of their magnitude then all these terms add up to a value near unity. Thus it seems reasonable to conclude that the major contributor to R_E is the pressure term. Consideration of the typical errors in the quantities (see earlier) does not yield different conclusions.

6. <u>The temperature variance balance equation</u>

The approximate form of this equation is

$$\frac{\partial}{\partial t}\left(\tfrac{1}{2}\overline{T'^2}\right) = -\overline{w'T'}\,\frac{\partial \bar{\theta}}{\partial Z} - N + \frac{K}{\rho c_p}\frac{\partial^2}{\partial Z^2}\left(\tfrac{1}{2}\overline{T'^2}\right) - \frac{\partial}{\partial z}\overline{w'\left(\tfrac{1}{2}T'^2\right)}$$

The third term on the R.H.S. may be written approximately as,

$$\left[\frac{\overline{T'^2_{Z3}} - \overline{T'^2_{Z2}}}{2} - \overline{T'^2_{Z2}} + \overline{T'^2_{Z1}}\right]\frac{0.2}{1350 \times 10^4}$$

(where $\frac{K}{\rho c_p} \approx 0.2$). Thus this term is several orders of magnitude smaller than the others and will be neglected. The remaining terms are plotted in Figure 4 and it appears that apart from the first period there is a large positive residual. Since there are no other significant terms it would appear that an error in the measurements is likely. Alternative values for the inertial sub-range constant (for temperature) do not affect N significantly although the much larger values quoted by Gibson et al (1970) would make N smaller and increase the imbalance. It is felt that $\partial\bar{\theta}/\partial Z$ is the quantity most likely to be in error although applying the maximum error correction to $\partial\bar{\theta}/\partial Z$ still leaves a positive residual (though the dissipation and production terms do then become nearly equal for hours 1 and 2 and 2 and 3).

7. The stress equation

In the usual nomenclature this equation is,

$$\frac{\partial}{\partial t}\left(\overline{u'w'}\right) = -\overline{w'^2}\frac{\partial \bar{u}}{\partial Z} + \frac{g}{T}\overline{u'T'} - \frac{\partial}{\partial Z}\left(\overline{w'u'w'}\right) - \frac{1}{\rho}\left(\overline{w'\frac{\partial p}{\partial x}} + \overline{u'\frac{\partial p}{\partial Z}}\right)$$

It is possible to estimate the first three terms on the right hand side from the present data and as with the energy balance equation the large residual indicates that the remaining terms involving pressure fluctuations are probably significant (see Figure 5). The implied sign of these terms would agree with that deduced for unstable conditions in the surface layer by Wyngaard et al (1971). In contrast however Wyngaard et al found the transport term negligible which is the opposite of the indications here. The influence of negative wind shear is clearly reflected by the change in magnitude and sign of the terms for hours 4 and 5, all the terms becoming small.

8. The vertical heat flux equation

Under the usual assumptions this equation may be written,

$$\frac{\partial}{\partial t}\left(\overline{w'T'}\right) = -\overline{w'^2}\frac{\partial \bar{\theta}}{\partial Z} + \frac{g}{T}\overline{T'^2} - \frac{\partial}{\partial Z}\overline{w'^2T'} - \frac{1}{\rho}\overline{T'\frac{\partial p}{\partial Z}}$$

The variation of the first three terms is shown in Figure 6. Both the production terms are positive and of nearly equal magnitude whilst the transport term is small, so that if the residual represents the pressure gradient interaction term then all the results broadly agree with those of Wyngaard et al (1971). It is worth noting that the stratification production of vertical heat flux term ($-\overline{w'^2}\frac{\partial\bar{\theta}}{\partial Z}$) seems almost independent of stability whilst the temperature variance term decreases substantially.

9. The horizontal heat flux equation

Ignoring terms usually taken as negligible,

$$\frac{\partial}{\partial t}(\overline{u'T'}) = -\overline{w'T'}\frac{\partial \overline{u}}{\partial Z} - \overline{u'w'}\frac{\partial \overline{\theta}}{\partial Z} - \frac{\partial}{\partial Z}(\overline{w'w'T'}) - \frac{1}{\rho}\left(\overline{T'\frac{\partial p}{\partial x}}\right)$$

and the estimates of the terms available from this study are illustrated in Figure 7. The most notable feature is the correspondence between the shear production term and the transport term - especially in the more unstable periods. This is in contrast to Wyngaard's surface layer results which indicated that the transport term was negligible. It follows that if the residual is due chiefly to the pressure term then it must be in balance with the stratification production term. This conclusion is of course tentative because of the uncertainty regarding $\partial\overline{\theta}/\partial Z$.

10. Concluding Comments:-

One possible implication from this study is the importance of the pressure terms in the balance equations, which would be in agreement with earlier conclusions from surface layer work. However this conclusion must remain tentative in view of the inhomogeneous nature of the site and the possible influence of advective effects.

Other methods for deriving the terms in the equations were considered, e.g. the gradients derived from the logarithmic regression line were combined with the actual flux measurements at 91m. In this instance large residuals were again obtained. Other methods of deriving the vertical gradients and flux values e.g. by linear or power law regressions do not lead to any markedly different conclusions. As mentioned previously the log-regression is a compromise between these two and implies that all the vertical gradients can be represented by c/Z where c is a constant. Thus some curvature in the vertical profile is allowed.

It is clear that before any generalisations can be expected to emerge many more multilevel runs, covering periods of several hours, will be required. Furthermore the representivity of the higher order moments, particularly at greater heights, will require careful consideration.

Acknowledgement

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References

BUSCH, N.E. and PANOFSKY, H.A. 1968 "Recent spectra of atmospheric turbulence". Quart. J. R. Met. Soc., 94, 132-148.

CAUGHEY, S.J. and RAYMENT, R. 1974 "High frequency temperature fluctuations in the atmospheric boundary layer". Boundary Layer Met., 5, 489-503.

CAUGHEY, S.J. MOSS, S.H., and RAYMENT, R. 1975 "Some spectral characteristics of atmospheric turbulence above 50 m". Met Office Turbulence and Diffusion Note No. 66.

GIBSON, C.H., STEGEN, G.R., and WILLIAMS, R.B. 1970 "Statistics of the fine structure of turbulent velocity and temperature fields measured at high Reynolds number". J. Fluid Mech. 41, Part 1, 153-167.

KAIMAL, J.C. and HAUGEN, D.A. 1967 "Characteristics of the vertical velocity fluctuations on a 430 m tower". Quart. J.R. Met. Soc., 93, 305-317.

LUMLEY, J.L. and PANOFSKY, H.A. 1964 "The structure of atmospheric turbulence" Interscience publishers, New York, 239 pp.

RAYMENT, R. 1970 "Introduction to the fast Fourier Transform (FFT) in the production of spectra". Met. Mag., 99, 261-270.

RAYMENT, R. 1975 "Turbulence studies from a tethered balloon". Ph.D. Thesis, Reading University.

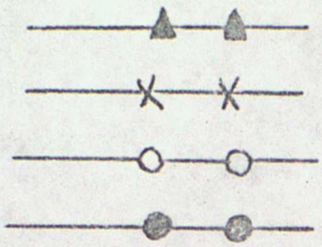
READINGS, C.J. and BUTLER, H.E. 1971 "The 1969 comparison of AFCRL and Cardington turbulence sensors". Met Office Turbulence and Diffusion Note No. 8.

READINGS, C.J., GOLTON, E. and BROWNING, K.A. 1973 "Fine scale structure and mixing within an inversion". Boundary Layer Met., 4, 275-287.

WYNGAARD, J.C. and COTÉ, O.R.	1971	"The budgets of turbulent kinetic energy and temperature variance in the atmospheric surface layer". J. Atmos. Sci. 28, 190-201.
WYNGAARD, J.C., COTÉ, O.R. and IZUMI, Y.	1971	"Local free convection, similarity and the budgets of shear stress and heat flux". J. Atmos. Sci. 28, 1171-1182.
WYNGAARD, J.C., ARYA, S.P.S. and COTÉ, O.R.	1974	"Some aspects of the structure of convective planetary boundary layers". J. Atmos. Sci. 31, 747-754.

List of Figures

Figure 7 Terms in the horizontal heat flux balance equation,



$- \partial/\partial Z \, (\overline{w'u'T'})$

$- \overline{w'T'} \, \partial \bar{u}/\partial Z$

$- \overline{u'w'} \, \partial \bar{\theta}/\partial Z$

residual

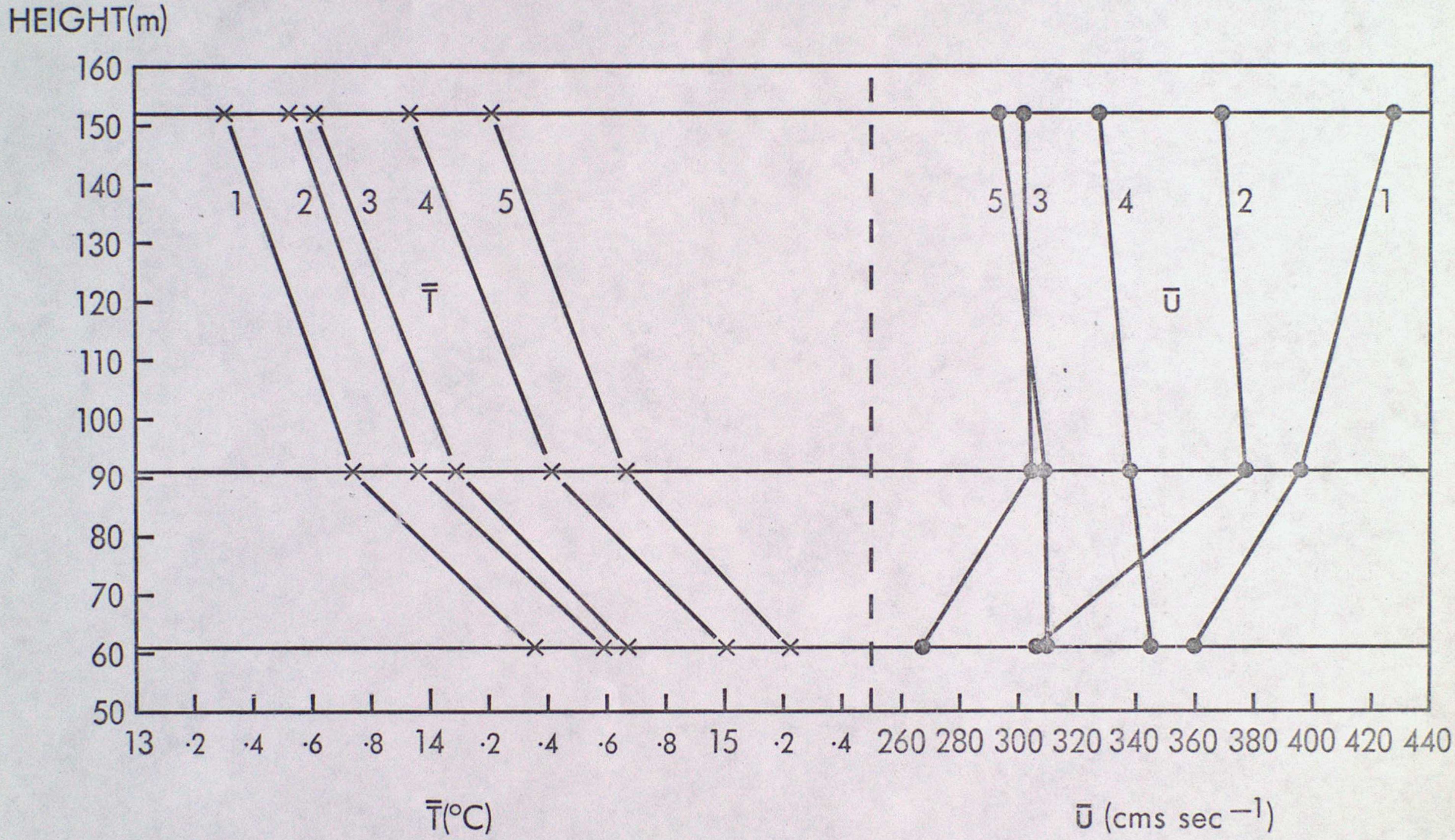

HEIGHT(m)
160
150
140
130
120
110
100
90
80
70
60
50
1
2
3
4
5
$\overline{T}$
5
3
4
2
1
$\overline{U}$
13 ·2 ·4 ·6 ·8 14 ·2 ·4 ·6 ·8 15 ·2 ·4
260 280 300 320 340 360 380 400 420 440
$\overline{T}$(°C)
$\overline{U}$ (cms sec $^{-1}$)

FIGURE 1.

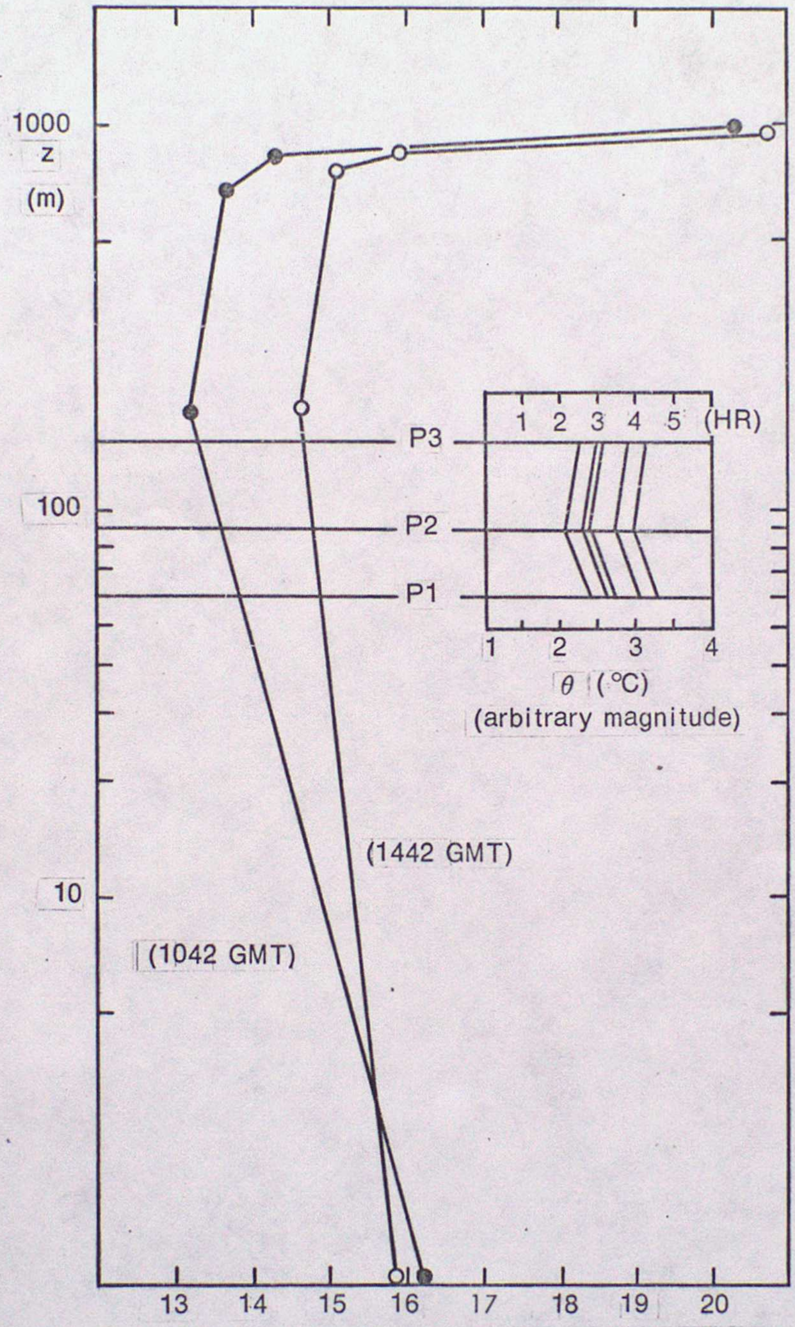
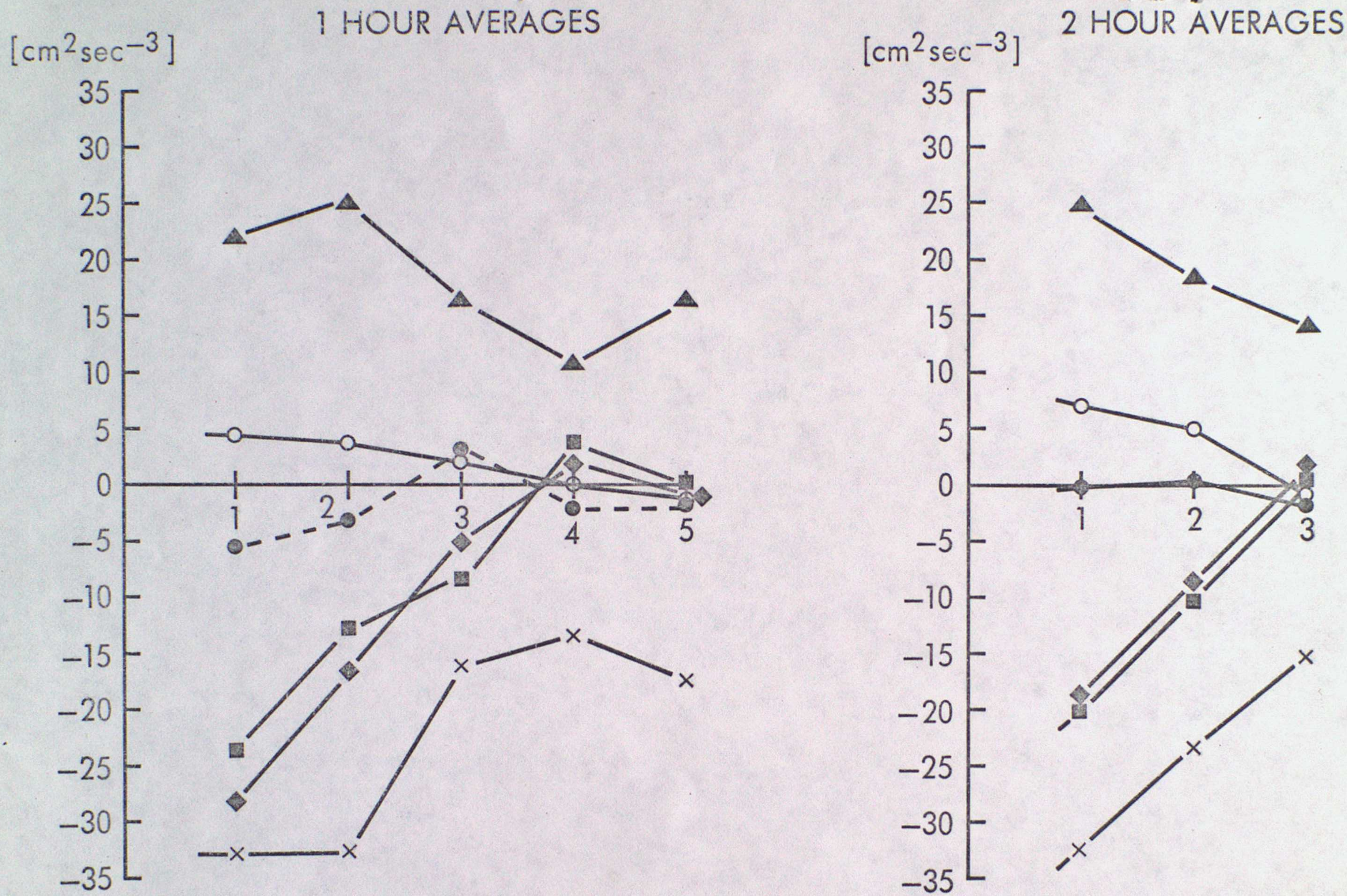

1000
z
(m)
100
10
P3
P2
P1
1 2 3 4 5 (HR)
1 2 3 4
θ (°C)
(arbitrary magnitude)
(1442 GMT)
(1042 GMT)
13 14 15 16 17 18 19 20

Figure 2



1 HOUR AVERAGES
[cm²sec⁻³]
35
30
25
20
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
1
2
3
4
5
2 HOUR AVERAGES
[cm²sec⁻³]
35
30
25
20
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
1
2
3

FIGURE 3.

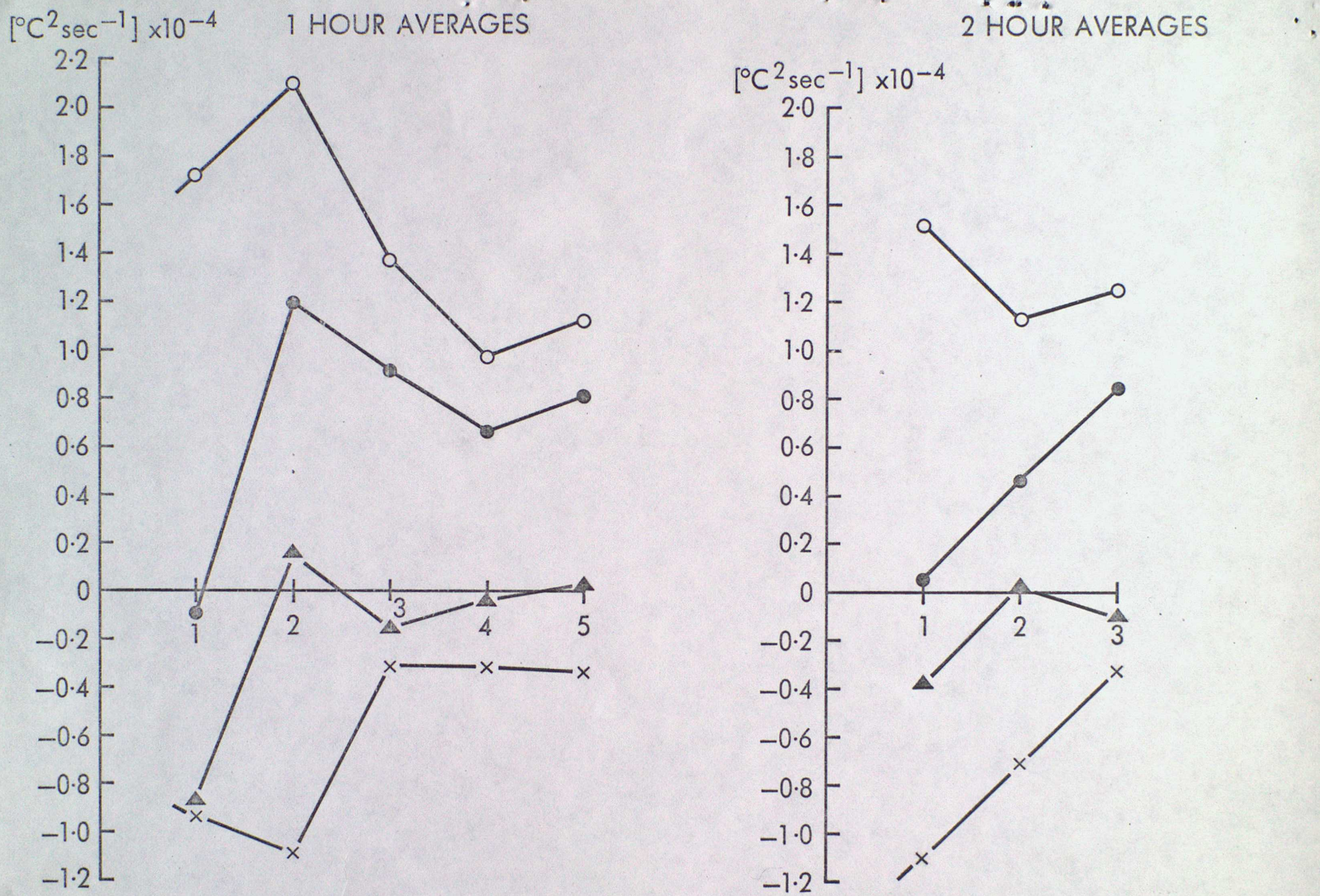

[°C²sec⁻¹] x10⁻⁴
1 HOUR AVERAGES
2·2
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
−0·2
−0·4
−0·6
−0·8
−1·0
−1·2
1
2
3
4
5
2 HOUR AVERAGES
[°C²sec⁻¹] x10⁻⁴
2·0
1·8
1·6
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
−0·2
−0·4
−0·6
−0·8
−1·0
−1·2
1
2
3

FIGURE 4.

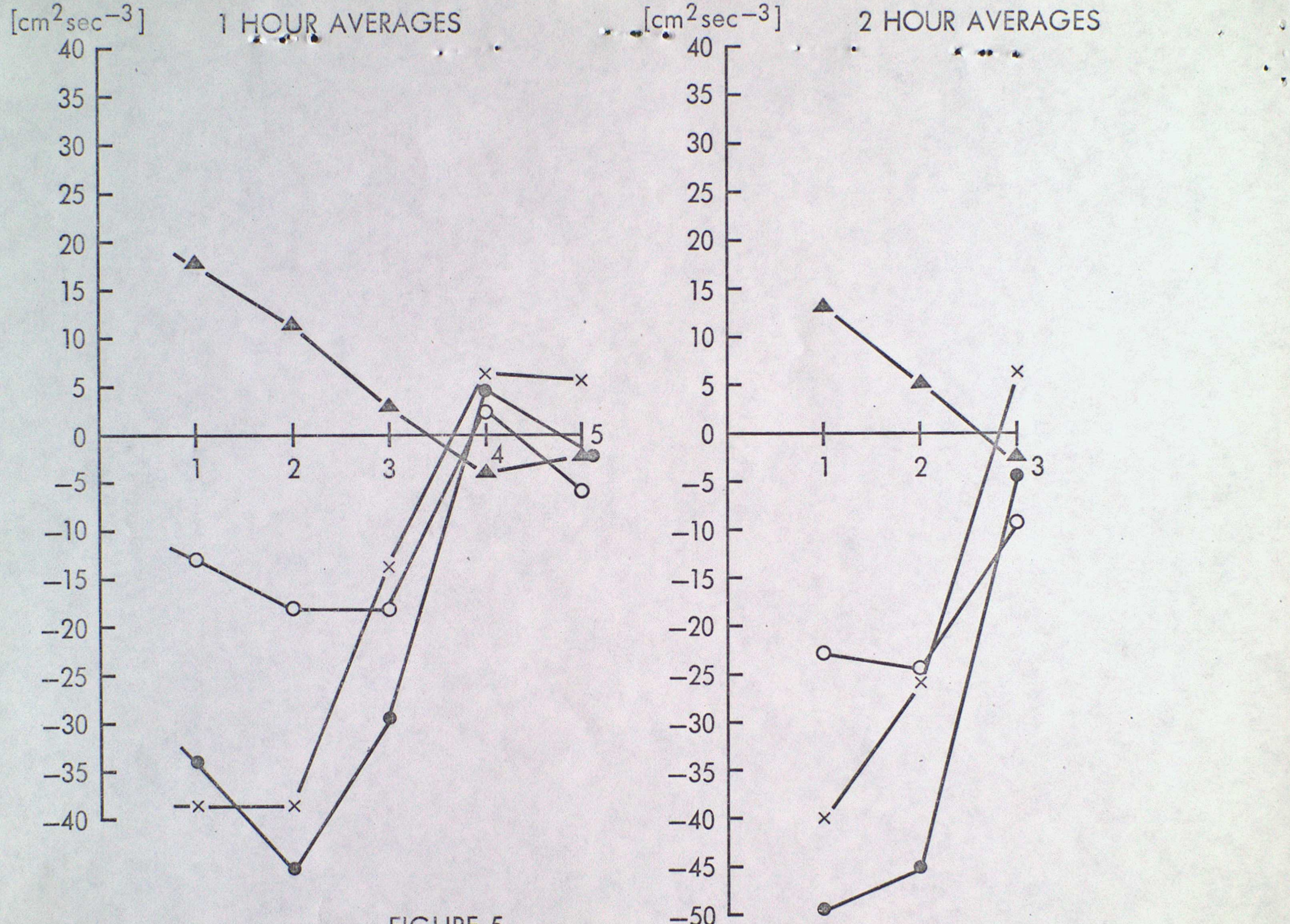

[cm2sec−3]
1 HOUR AVERAGES
[cm2sec−3]
2 HOUR AVERAGES
40
35
30
25
20
15
10
5
0
−5
−10
−15
−20
−25
−30
−35
−40
−45
−50
1
2
3
4
5

FIGURE 5.

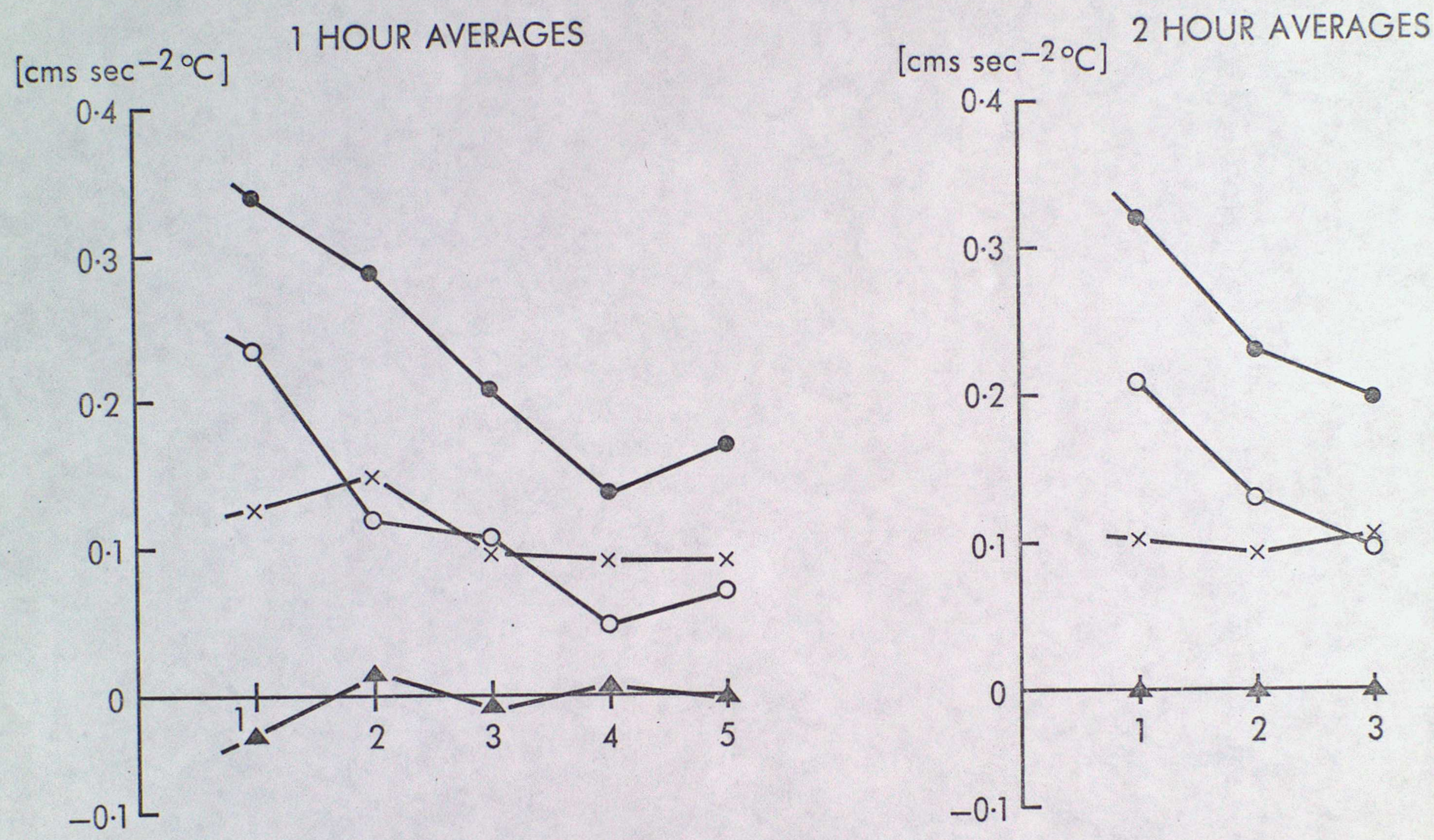
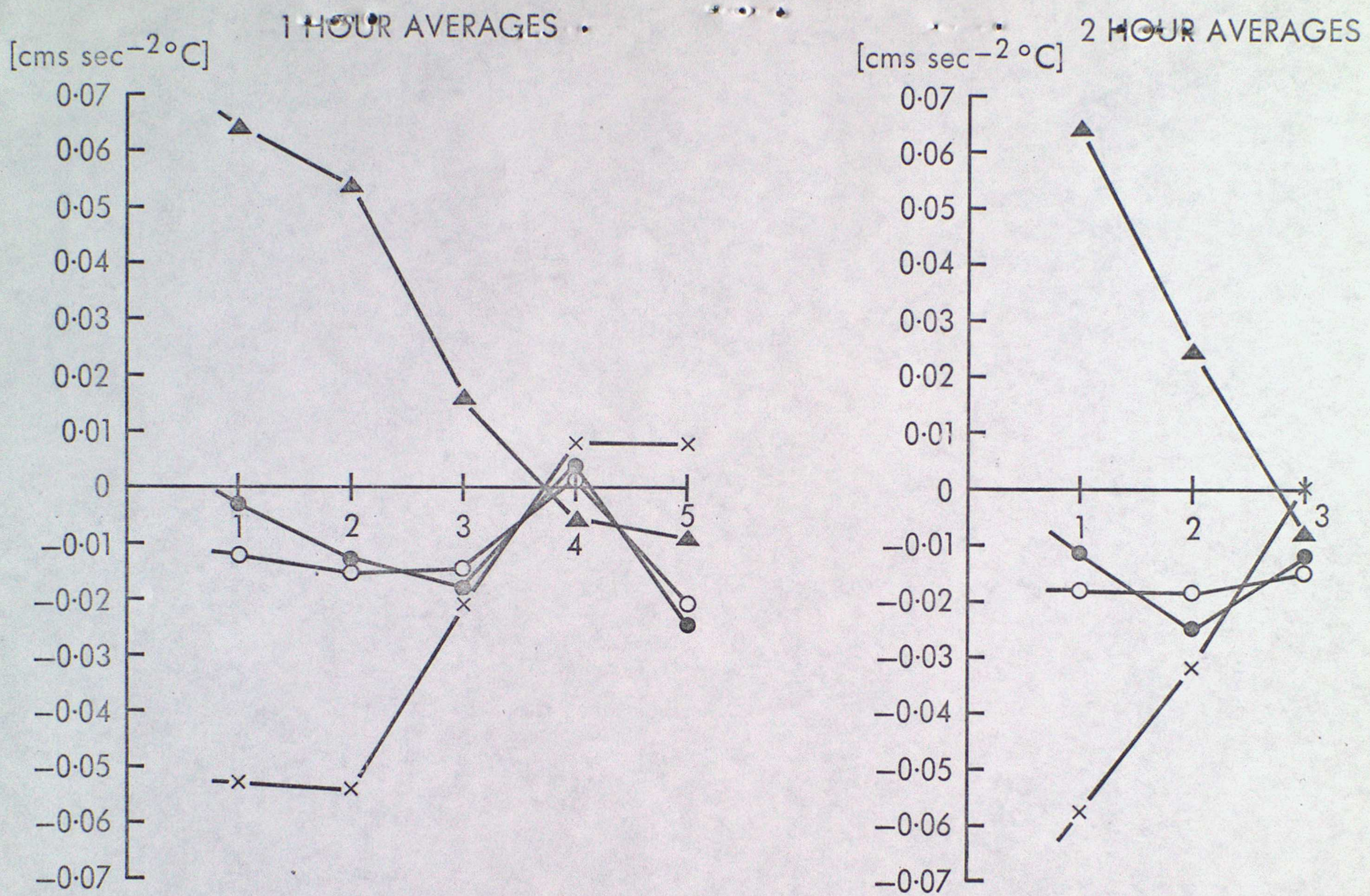

1 HOUR AVERAGES
[cms sec−2 °C]
0·4
0·3
0·2
0·1
0
−0·1
1
2
3
4
5
2 HOUR AVERAGES
[cms sec−2 °C]
0·4
0·3
0·2
0·1
0
−0·1
1
2
3

FIGURE 6.


1 HOUR AVERAGES
[cms sec^{-2} °C]
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0·06
0·05
0·04
0·03
0·02
0·01
0
−0·01
−0·02
−0·03
−0·04
−0·05
−0·06
−0·07
1
2
3
4
5
2 HOUR AVERAGES
[cms sec^{-2} °C]
0·07
0·06
0·05
0·04
0·03
0·02
0·01
0
−0·01
−0·02
−0·03
−0·04
−0·05
−0·06
−0·07
1
2
3

FIGURE 7.