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Numerical Modelling of Discontinuous Atmospheric Flows

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DISCONTINUOUS ATMOSPHERIC FLOW

1. Introduction

Numerical modelling of large scale atmospheric flow is very successful. Useful forecasts of the flow pattern are obtained out to 3 days in nearly every case and to 5 days in about half the cases. The problems occur in converting these forecasts into useful statements about actual weather. In order to do this the detailed vertical structure of the atmosphere must be accurately predicted so that different air masses can be clearly identified.

A large scale weather map, such as those published in the newspapers, appears smooth. The dynamical theory of vertically averaged atmospheric motion shows that the behaviour is essentially the same as two dimensional incompressible flow. That system of equations, given smooth initial data, has smooth solutions indefinitely (Kato, 1965). A correct numerical solution can be obtained by any stable consistent finite difference or Galerkin scheme. The most effective way of obtaining nonlinear stability in this case is to use the conservative schemes of Arakawa (1966), or a Galerkin method. The concept of smooth solutions extends to three dimensional flow provided that the scale remains large and the stratification strong. However, detailed observations of the vertical structure of the atmosphere indicate that the solutions are far from smooth. The structure is often like a series of layers of air with markedly different properties and sharp interfaces between them (Danielson, 1959). These interfaces support waves and turbulence. The amount of moisture and dust in the different layers is usually very different, leading to marked changes in weather when different layers reach the surface.

A system of equations whose scaling allows for the presence of discontinuities has been developed (Hoskins, 1982) and the existence of solutions for piecewise constant initial data shown (Cullen and Purser, 1984). There is a need to solve the complete equations of atmospheric motion in a way which will generate these piecewise constant solutions and model the waves and turbulence on the interfaces. As in other branches of fluid dynamics when discontinuities develop, it is essential to choose the correct conservation law form of the equations if numerical solutions are not to converge to unphysical solutions. It is also necessary to refine the solution procedure to help it to capture the discontinuities.

In this paper this is illustrated for three situations. In forecasting atmospheric fronts, a convergence to the correct solution can be obtained by absorbing certain acceleration terms into the turbulence model. In the case of flow over large scale mountains, the difficulty is to confine the influence of the mountains to a shallow layer above its top when the stratification is strong. In hemispheric scale forecasting, absorption of extra terms into the turbulence model can lend to a substantial reduction in error through eliminating spurious solutions. However, some useful detail is lost at the same time.

2. Governing equations

2.1 Primitive equations

The normal form of equations used in atmospheric modelling are the equations of compressible gas flow with the hydrostatic approximation. For present purposes we also make the Boussinesq approximation and treat the effect of the Earth's rotation as constant in space. The resulting system of equations, as used by Gent and McWilliams (1983), is also appropriate for the ocean:

$$\frac{Du}{Dt} + \frac{\partial\phi}{\partial x} - fv = 0 \qquad (2.1.1)$$

$$\frac{Dv}{Dt} + \frac{\partial\phi}{\partial y} + fu = 0 \qquad (2.1.2)$$

$$\frac{D\theta}{Dt} = 0 \qquad (2.1.3)$$

$$\frac{\partial\phi}{\partial z} + g\theta/\theta_0 = 0 \qquad (2.1.4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (2.1.5)$$

$$w = 0 \quad \text{at} \quad z = 0, H \qquad (2.1.6)$$

The vertical coordinate z is a function of pressure, and the lower boundary condition is simplified so that it is applied at a constant pressure surface near the ground, rather than the Earth's surface. ϕ represents the height of a constant pressure surface, θ is the potential temperature and f the Coriolis parameter. The rest of the notation is standard. The behaviour of this system has been analysed by Norbury and Cullen (1985). Standard methods (Courant and Hilbert (1962), section VI. 3) show that it is a symmetric hyperbolic system with five real characteristic directions; three parallel to particle paths corresponding to three advected quantities, and two parallel to the z axis. Thus it is natural to look for quantities conserved or varying in a simple way along trajectories, and for discontinuities propagating along them.

2.2 The geostrophic momentum approximation

A scale analysis appropriate to large-scale atmospheric flow suggests that the velocities in (2.2.2) to (2.1.6) can be approximated by their geostrophic values.

$$u_g = -\partial\phi/\partial y \qquad (2.2.1)$$

$$v_g = \partial\phi/\partial x \qquad (2.2.2)$$

In accordance with the prescription of Lighthill (1961), this approximation is only made in the advected quantities, not in the trajectories themselves. This allows uniform validity of the approximation for large times. Hoskins (1982) shows that the resulting system is still valid in the presence of discontinuities provided the scale parallel to the discontinuity is large. Therefore we replace $\left(\frac{Du}{Dt}, \frac{Dv}{Dt}\right)$ in (2.2.2) and (2.1.2) by $\left(\frac{Du_g}{Dt}, \frac{Dv_g}{Dt}\right)$.

2.3 Lagrangian conservation form

Equations (2.2.2) to (2.2.6) subject to the geostrophic momentum approximation can be written in the following Lagrangian form :

$$\frac{DM}{Dt} = -fu_g \qquad (2.3.1)$$

$$\frac{DN}{Dt} = -fv_g \qquad (2.3.2)$$

$$\frac{D\theta}{Dt} = 0 \qquad (2.3.3)$$

$$\frac{D\tau}{Dt} = 0 \qquad (2.3.4)$$

where $(M,N,\theta) = \nabla(\phi + \tfrac{1}{2}f^2(x^2+y^2))$ (2.3.5)

and τ is the specific volume. The natural boundary condition is that no fluid crosses the boundary of the integration domain. The work of Hoskins shows that this system of equations can generate a discontinuity in a finite time from smooth initial data. After this time we assume that the Lagrangian form remains valid while the Eulerian form does not. The correctness of this can only be tested by actual experiment.

2.4 Implications for finite difference solutions

If it is assumed that (2.3.1) to (2.3.5) hold for almost all the fluid volume, then a finite difference solution of the original equations (2.1.1) to (2.1.6) must be made consistent with these conservation laws. The requirement of conservation of quantities along trajectories cannot only be met in an Eulerian scheme by upwinding, however this violates the requirement that the volume of fluid with a particular value of a conserved quantity is also conserved. The latter requirement cannot be met exactly in a conventional scheme. The best practical procedure is to use a quadratic conservative scheme (Arakawa (1966)) and to use an artificial viscosity which acts as little as possible on the vertical component of the vorticity, essentially the geostrophic part of the flow.

3. Results

3.1 Fronts

Fig. 1 shows a solution of (2.3.1) to (2.3.5) obtained by representing the initial data as piecewise constant, in the manner of Glimm (1965) for hyperbolic conservation laws, integrating the ordinary differential equations for each segment, and fitting the results together in the way described by Cullen and Purser (1984). This method is like a moving finite element method except that the volumes rather than the shapes of elements are specified.

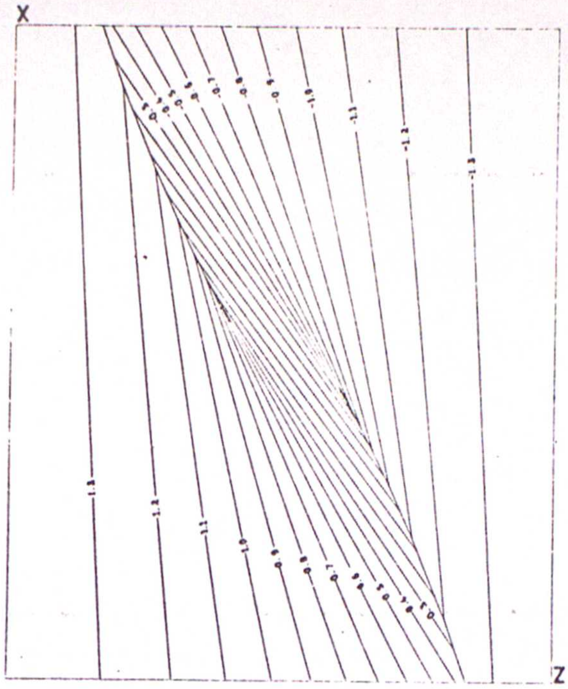

Fig. 1 Cross section of a front obtained by a Lagrangian method, contours of potential temperature.

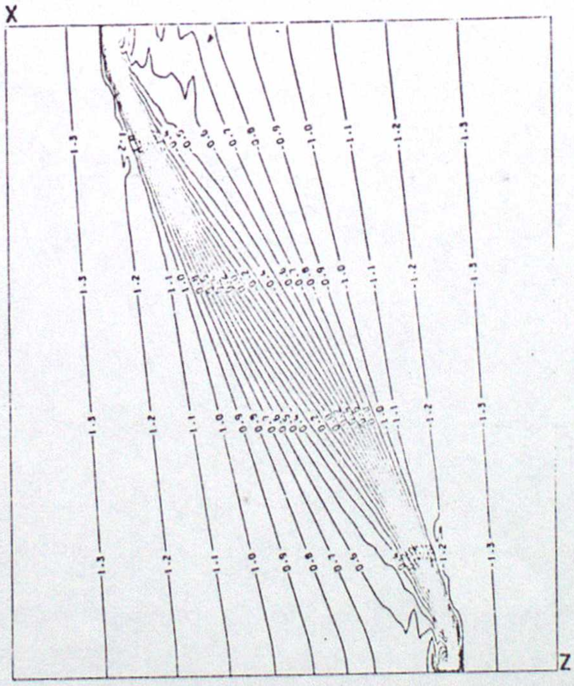

Fig. 2 Cross section of a front obtained by a finite difference method using a 200 x 20 grid, contours of potential temperature.

Fig. 2 shows a solution of the Eulerian equations (2.1.1) to (2.1.6) obtained by adding artificial viscosity of the form

$$\frac{D(u-u_g)}{Dt} + K\nabla^2 u \qquad (3.1.1)$$

to equation (2.1.1), an equivalent term to equation (2.1.2) and $K\nabla^2\theta$ to equation (2.1.3). This has the effect of enforcing the geostrophic momentum approximation on the scale of the discontinuity and allows a value of K to be used which is 100 times smaller than that required if the extra acceleration term is not included in the viscosity. The solution does not capture the change in slope with height of the front very well. As the resolution is increased to 200 x 40, there is very slow convergence towards the solution shown in Fig. 1. The adaptive method has a clear advantage here.

3.2 Mountain flow

Fig. 3 shows a cross section of the flow over the Alps predicted by a limited area forecast model which uses equations (2.1.1) to (2.1.6) and also includes the effect of moisture. The wind component across the Alps is about 15 ms^{-1}. A large standing wave is generated, also transient waves.

This structure is only observed in the atmosphere on much smaller horizontal scales. On the scale of the Alps, the constraint of the Earth's rotation makes the flow tend to go round, rather than over the mountain. Even when flow crosses the mountain, observations of clouds suggest that the rapid flow is confined to a thin layer near the ridge crest.

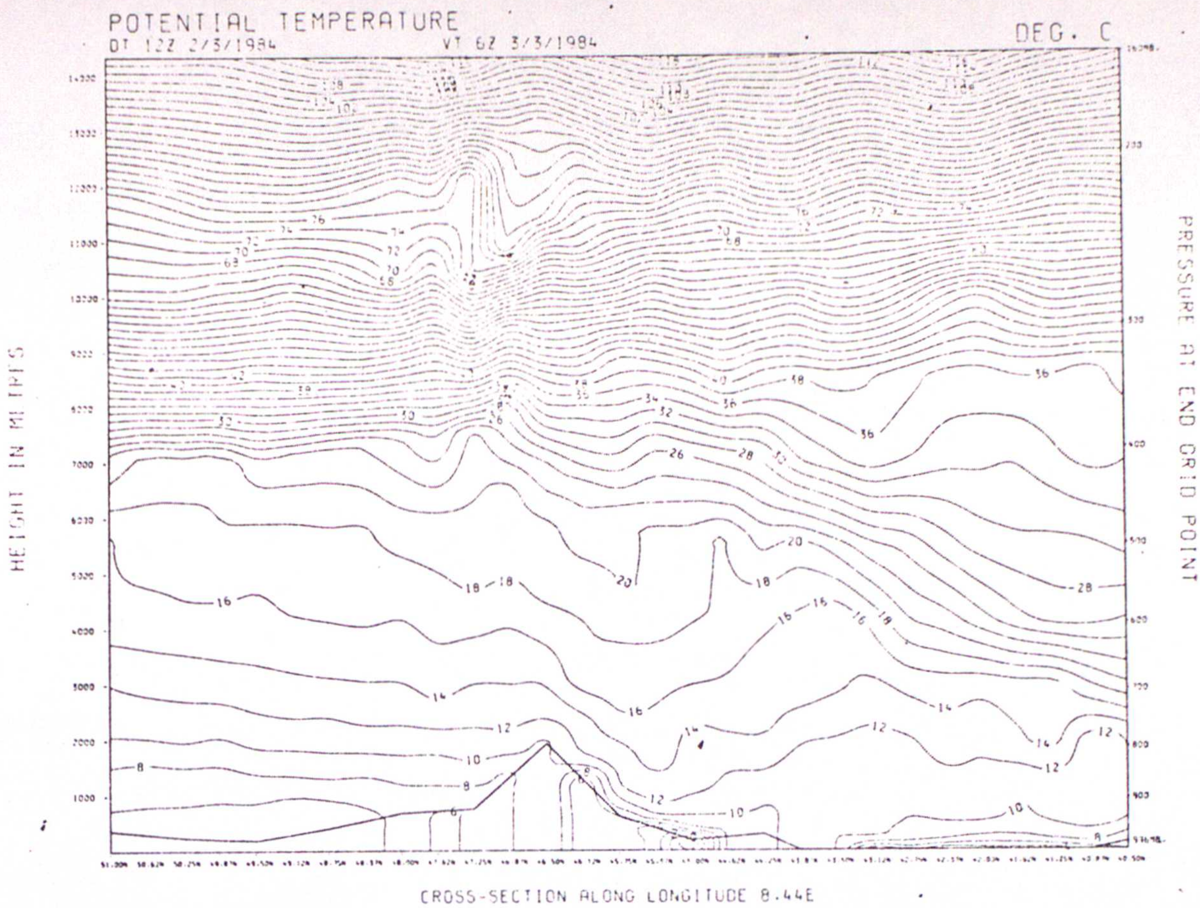

Fig. 3. 18 hour forecast potential temperature cross section along 8°E.

Fig. 4 shows a two dimensional finite difference solution of the equations using a viscosity of the form (3.1.1). This suppresses the waves, but the disturbance due to the mountain extends up to the top of the atmosphere.

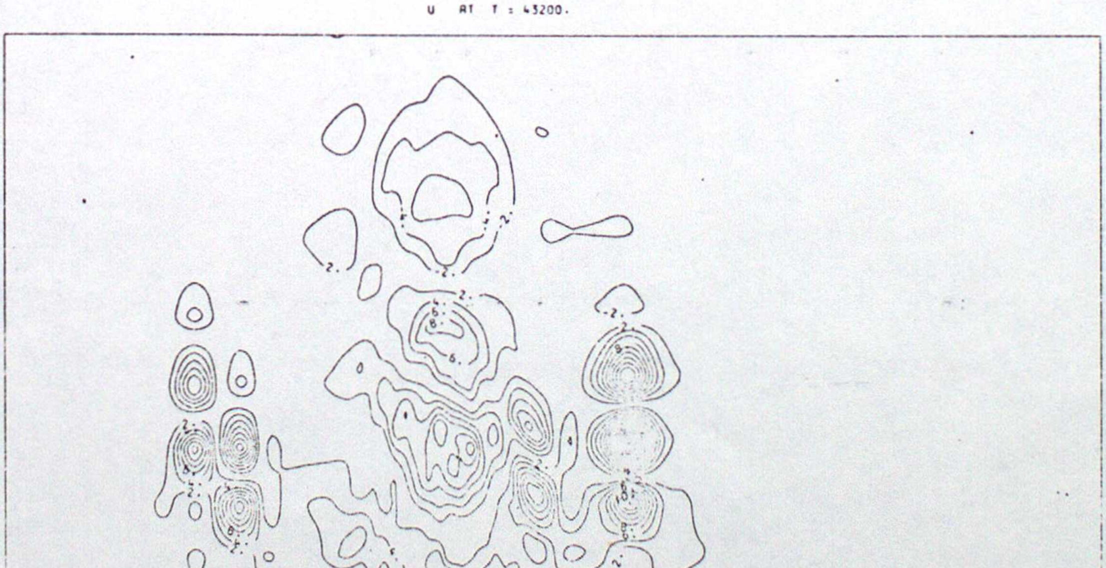

Fig. 4 Cross-mountain wind using finite difference model on 200 x 20 grid.

A solution using an adaptive Lagrangian method being presented by S. Chynoweth (poster presentation) prevents this extensive disturbance which may play a part in generating the spurious waves in the forecast model.

3.3 Short range forecasts

Figs. 5 and 6 show 24 hour surface pressure forecasts made using the U.K. operational hemispheric model, in the first case using (2.1.1) to (2.1.6) with artificial viscous terms added, and in the second case with acceleration terms absorbed into the viscosity as in equation (3.1.1). The verifying analysis is shown in Fig. 7.

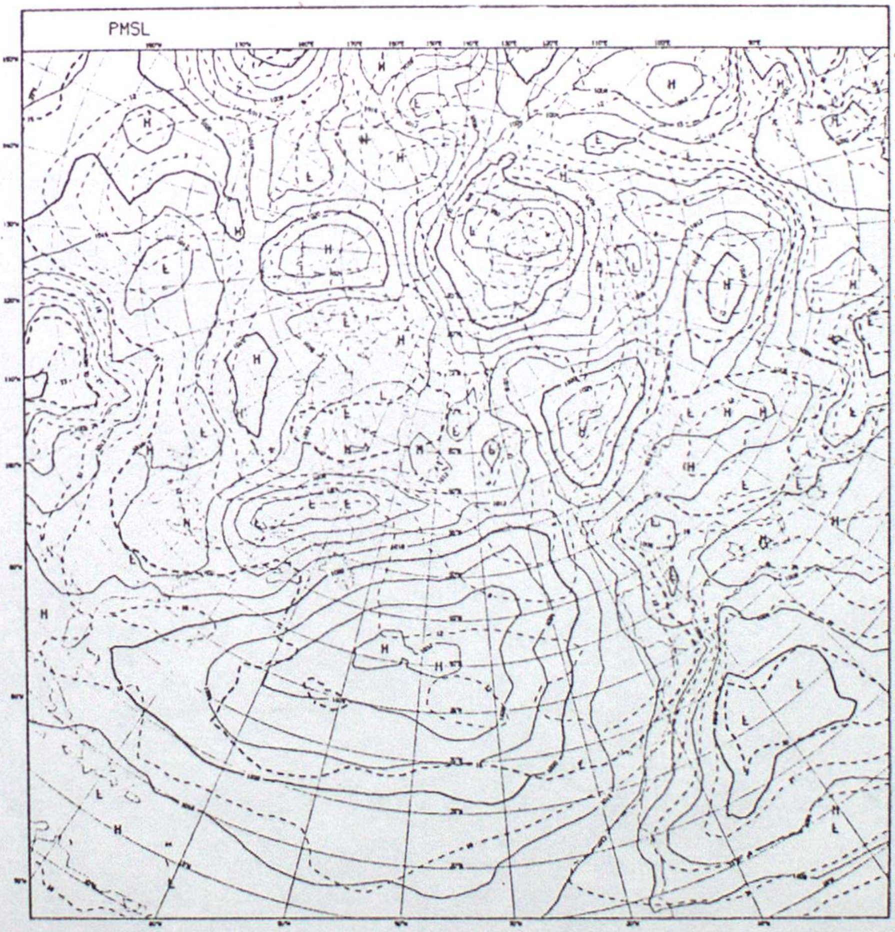

Fig. 5 24 hour forecast using primitive equation model, 192 x 48 x 15 grid. Solid lines-surface pressure (hPa), pecked lines-temperature at 850 hPa.

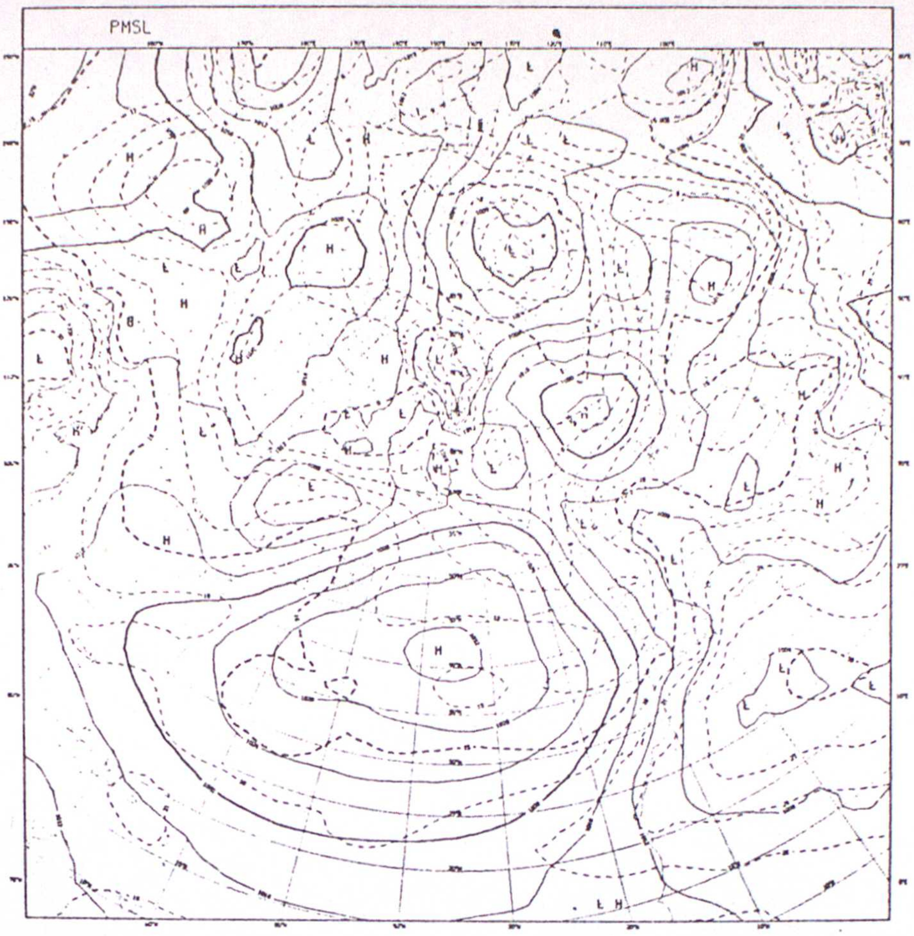

Fig. 6 24 hour forecast using model as Fig. 5 with artificial viscosity (3.1.1).

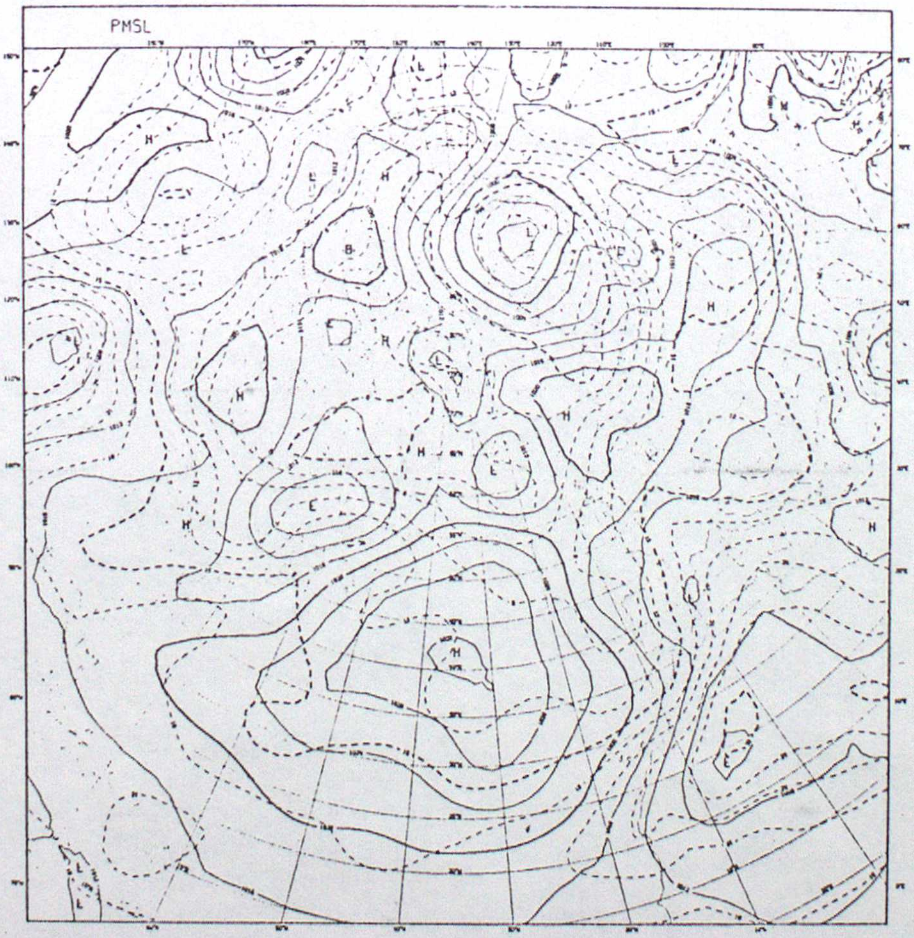

Fig. 7 Surface pressure analysis for 12Z, 13 July 1982.

The forecast with the acceleration absorbed into the artificial viscosity is much smoother, despite a 10-fold reduction in the coefficient. This forecast is more accurate on most of the pressure systems except a deepening depression near the North Pole and another in the Pacific. Some strength has been lost in these. The improvement reflects the removal of much spurious noise, though it has been achieved at the cost of losing some correct detail from the forecast shown in Fig.5. The L_2 error over the region 70°W to 20°E, 30° to 70°N has been reduced from 3.2 to 2.4 hPa.

4. Conclusions

This brief review has shown that numerical procedures for solving the governing equations of atmospheric motion may need careful design to ensure that spurious solutions are not obtained. Turbulence models may have to be designed using knowledge of the structure of approximate systems of equations as well as the complete system. Adaptive methods can achieve far greater accuracy in situations where they are practicable.

5. Acknowledgements

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6.

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