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SOME CHARACTERISTICS OF EDDY MOTION IN THE ATMOSPHERE

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SOME CHARACTERISTICS OF EDDY MOTION IN THE ATMOSPHERE

§ 1—Introduction

Most of our practical knowledge of the diffusing power of atmospheric turbulence is based on measurements of the coefficient of eddy diffusion, usually denoted by K. The magnitude of the coefficient has been estimated in a variety of ways, for example, by observing the average effect of turbulence on the distribution of temperature (1, 2, 3, 4),* wind velocity (1, 3, 5, 6, 7, 8) or water vapour (4, 9) in the atmosphere. These methods give mean values of K appertaining to the phenomenon acting on a comparatively large scale. In a few instances the effects of atmospheric turbulence acting on a smaller scale have been investigated by measuring the dispersion of smoke clouds (8, 17). The results obtained by these methods, however, do not throw much light on the mechanism of the processes at work, although in the case of smoke experiments the diffusion is rendered visible.

We have, of course, a general idea of the nature of the mechanism. When air moves over surfaces of discontinuity eddies are continually being formed and they, in turn, give rise to other eddies when they come into contact with other masses of air. Usually the ground forms the surface at which the eddies originate. Under certain conditions of density stratification, however, the generation of eddies may be prevented near the ground and it may then occur at some higher level where stratification ceases. It is certain that the diffusing power of turbulent air is in some way dependent on the characteristics of the eddies. G. I. Taylor (2) considered the coefficient K roughly to be equal to $\frac{1}{2}\bar{w}d$ where $\bar{w}$ is the average vertical component of velocity due to the eddies and d is the average vertical distance through which any portion of the atmosphere is raised or lowered while it forms part of an eddy. An examination of the details of eddy motion, therefore, may be expected to yield some information on the mechanism of turbulence in the atmosphere.

In the Symons Memorial Lecture given before the Royal Meteorological Society in 1927, G. I. Taylor (10) described some methods of studying the characteristics of eddy motion and outlined some of the results which he had obtained. The work described in the present paper has been carried out on somewhat similar lines, the main purpose of the experiments and analyses being to obtain some information concerning the magnitudes of the components of eddy velocity and the frequency of occurrence of eddies in the layer of air immediately above the ground. The investigation was made on Salisbury Plain in 1926.

§ 2—Methods of Observation

Eddies formed in air travelling over an irregular surface may have axes of revolution in any direction. The effect of such eddies at a particular point is to cause instantaneous changes in the velocity and direction of the wind. The ribbon-like appearance of the records of wind velocity and direction obtained with a pressure-tube anemometer is due to this effect. If the difference between the instantaneous wind and the mean wind be called the eddy wind, it is convenient to regard the eddy wind as resolvable into components in three perpendicular directions. Let the axis of x be along the mean direction of the wind, let y be along the horizontal direction perpendicular to the mean wind and let z be in the vertical direction. Denote the components of eddy velocity (by this is meant the velocity of the eddy wind) in these three directions by u, v and w respectively and let the average wind velocity be denoted by U.

The ordinary type of wind-direction recording instrument (such as the Baxendell) provides a way of obtaining mean values of the v component. For the sine of the angle θ between the instantaneous direction of the vane and the

* The numbers in brackets refer to the bibliography on p. 16.

mean direction of the wind is the ratio of v/U. The record of direction is usually in the form of a more or less irregular band, the mean semi-width of which taken over an hour say, gives a mean value of θ and from this we can get a mean value for the ratio v/U.

In a somewhat similar way a velocity recorder, such as the Dines pressure-tube anemometer, gives a way of obtaining mean values of the u component of eddy velocity. In this case, however, the semi-width of the trace is a measure of the actual component u and the ratio u/U is obtained by dividing the half width by the mean velocity or by the expression

$$u/U = (U \text{ max.} - U \text{ min.})/(U \text{ max.} + U \text{ min.})$$

This method of dealing with wind records is more fully explained in a paper by G. I. Taylor (11).

A large number of anemometer records have been analysed in this way and in extracting the data the following routine was adopted. The mean velocity or direction entered to any hour of the day was the hourly mean centring on the hour. The width of the trace was measured by taking horizontal lines along the upper and lower edges of the trace such that approximately equal areas were cut off on each side of the lines.

The head of the Dines anemometer was 13·5 metres above the ground and the connecting pipes were of 1-inch " Simplex " tubing (about $\frac{3}{4}$-inch internal diameter). The direction vane was of the R.A.E. type.

Some data have also been obtained with a vane recording changes in direction in the vertical plane. This vane was somewhat smaller than the R.A.E. type and it was fitted, together with its recording drum, on the top of the vane of a second pressure-tube anemometer, at a height of 3·5 metres. A photograph of the apparatus is shown in Fig. 2. Mean values of w/U were obtained from the record.

Owing to inertia and friction, the standard instruments do not register the effects of small eddies. For this purpose, a small bi-directional vane, kindly lent by Professor G. I. Taylor, has been found of considerable use. The vane, which is similar to that shown diagrammatically in Fig. 4, has a gimbal mounting and records movements in horizontal and vertical planes by means of a single pen. There is no provision for a time scale, however, and the trace therefore consists of an irregular mass of continuous lines which lie roughly within an ellipse or circle. It was found that a run of one minute gave a record, the breadth and width of which could be measured with some degree of consistency. From measurements of the breadth and width of the records the ratios v/U and w/U have been found. The mean velocity U was measured by a small dial airmeter mounted to one side of the vane. Some photographs of typical records are shown in Fig. 1. The dimensions of the vane are given in Fig. 4. and are the same as those of a lighter vane described in the following paragraph. The weight of the vane was 86 grams.

In order to introduce a time scale into the records given by the type of vane just described, use has been made of the kinematograph camera. In these experiments, a very light vane (A, Fig. 3) weighing 19 grams, was used, the metal portions being made of piano wire and the fins of stiff paper. Kinematograph pictures were taken from behind so that an end-on view of the vane was obtained, and from the pictures the instantaneous angles of the vane from the mean direction could subsequently be obtained. The mean wind velocity was recorded by a small dial airmeter (B) included in the pictures, a mirror (C) being arranged to reflect the dial to the required direction. In order to obtain some information about the x component of eddy velocity, a small swinging-plate anemometer (D) was also included, while to obviate any doubt as to the regularity of the rate of exposure by the kinematograph operator, a timing device was included in the pictures. This consisted of a synchronous motor (E) controlled by an electrically maintained tuning fork. A stop watch (F) gave a check on the duration of records. A photograph of the apparatus, fitted up for an experiment, is shown in Fig. 3, while dimensions of the vane and swinging plate are given in Fig. 4.

For the analysis of the records, the pictures were projected optically on to squared paper. The co-ordinates of the cross of the fins of the vane could then

To face page 4.

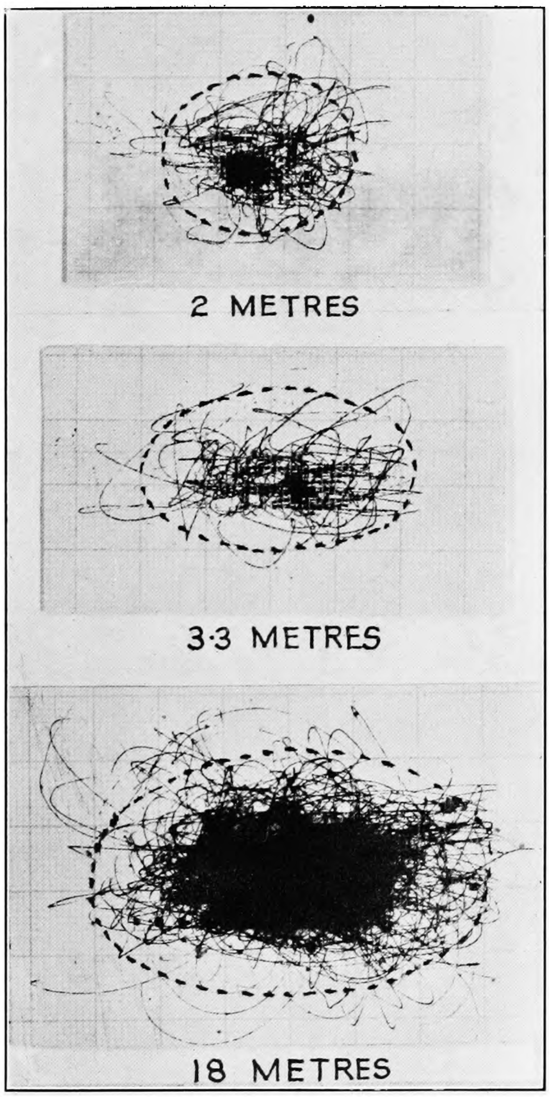


FIG. 1.—TYPICAL DIAGRAMS OBTAINED WITH A SMALL BI-DIRECTIONAL VANE AT DIFFERENT HEIGHTS ABOVE THE GROUND.

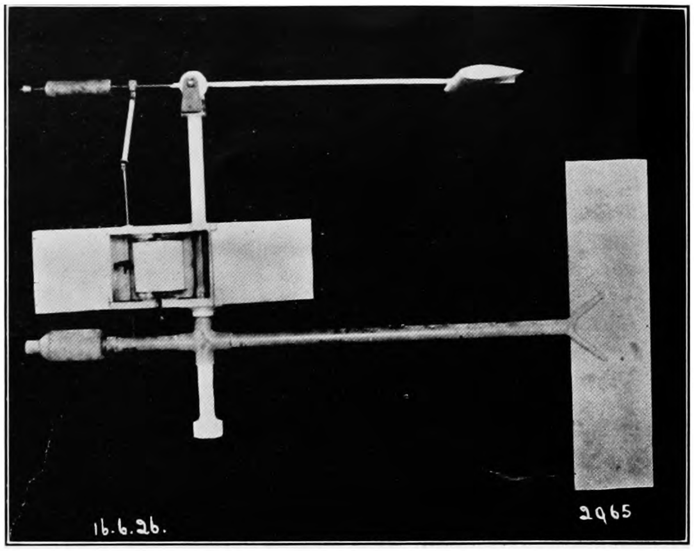


FIG. 2.—VERTICAL GUSTINESS RECORDER.

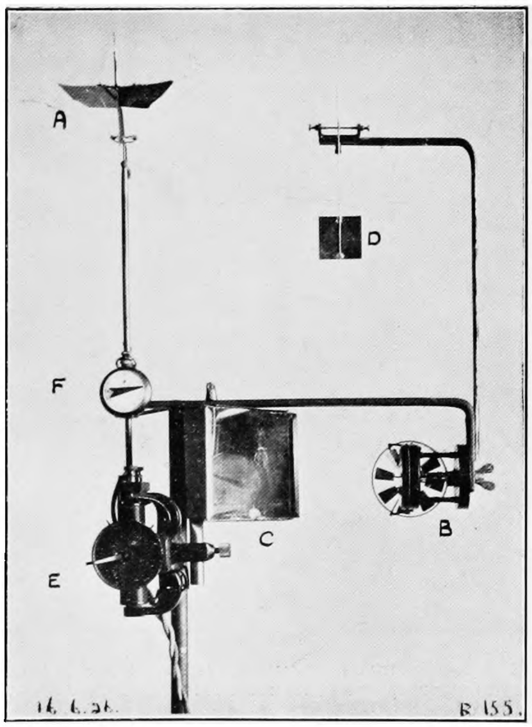


FIG. 3.—LIGHT BI-DIRECTIONAL VANE, SWINGING-PLATE ANEMOMETER AND AUXILIARY APPARATUS USED IN KINEMATOGRAPH EXPERIMENTS.

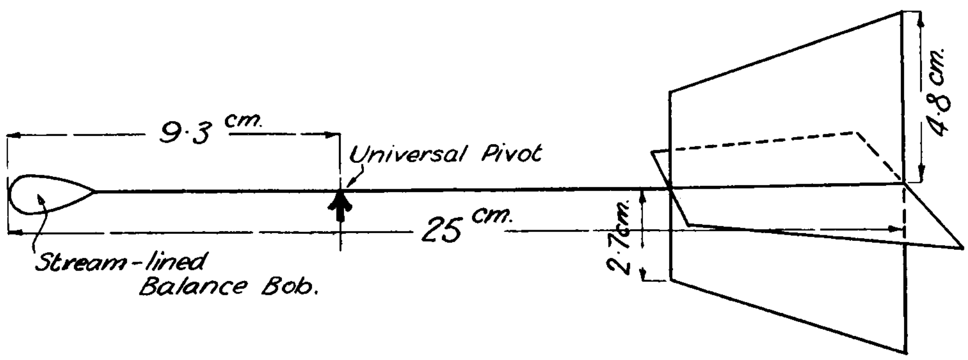
Fig. 4. *To face page 4.*

KINEMATOGRAPH EXPERIMENTS

DIMENSIONAL DRAWING OF INSTRUMENTS USED

BI-DIRECTIONAL VANE.

WEIGHT OF MOVING PART 19 gm.



SWINGING-PLATE ANEMOMETER.

WEIGHT OF MOVING PART (WITHOUT VARIABLE WEIGHT) 2 gm.

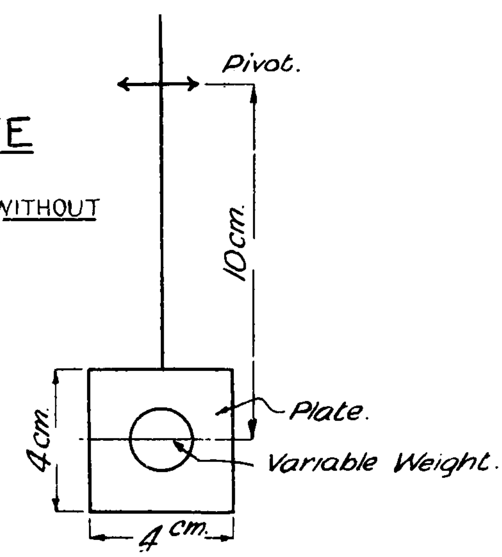


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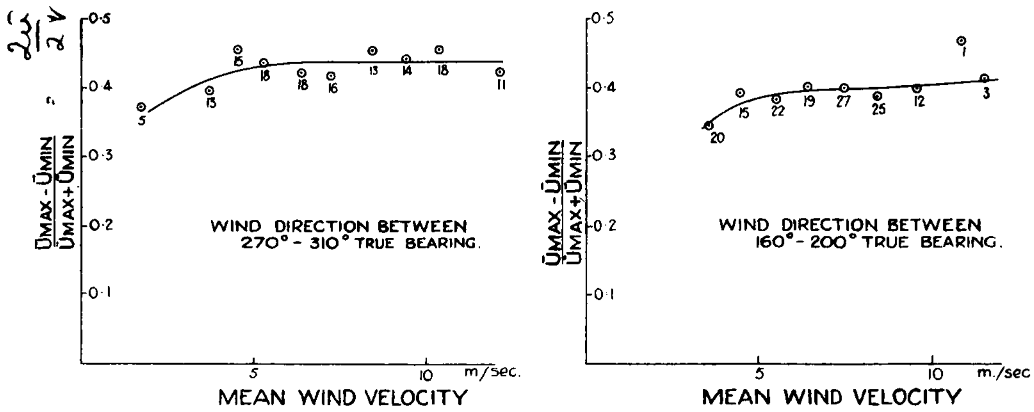
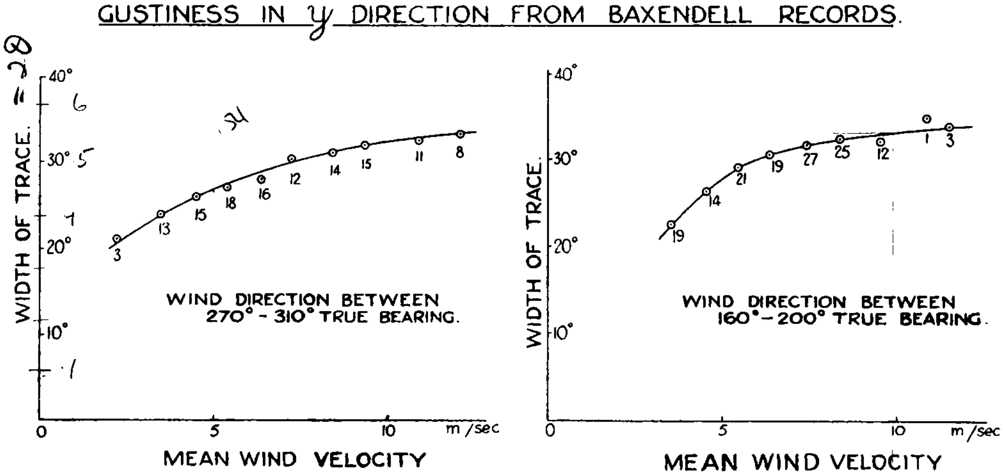
FIG. 5. To face page 5.

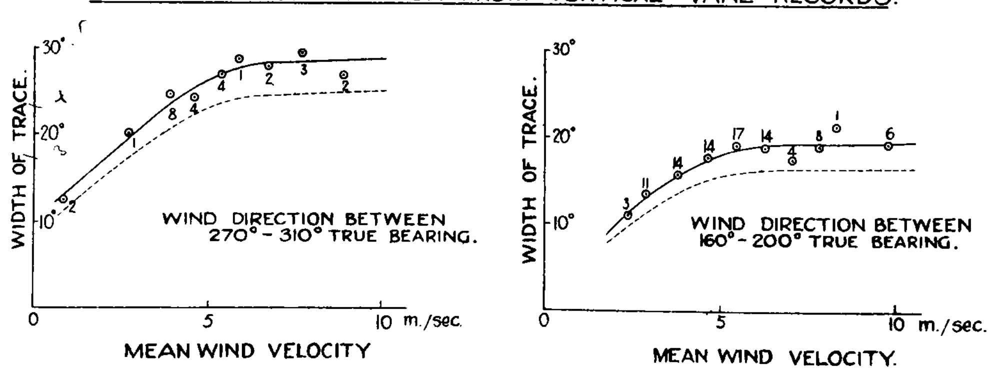
GUSTINESS IN x DIRECTION FROM RECORDS OF DINES ANEMOMETER.



GUSTINESS IN y DIRECTION FROM BAXENDELL RECORDS.



GUSTINESS IN Z DIRECTION FROM VERTICAL - VANE RECORDS.



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be read off and subsequently converted into angles to obtain each of the components v and w. The angle of the swinging plate was measured in a similar manner—actual velocities being obtained by using the readings of the dial airmeter for calibration. Finally from the position of a mark on the disc of the synchronous motor, times were read off with an accuracy of about one fiftieth of a second.

A standard speed for the kinematograph was adopted giving sixteen pictures per second.

§ 3—Analysis of Gustiness as Shown by Records from Dines and Baxendell Instruments and Vertical-Direction Vane

The first analysis was carried out in order to ascertain how the eddy velocity (measured by gustiness) varies with the mean velocity of the wind.

It was also hoped to obtain some information on the effect of contour upon gustiness, and further to find out whether there is any marked difference in the magnitude of the turbulence associated with equatorial and polar air respectively.

The data were selected from occasions during the years 1924, 1925 and 1926 on which the following conditions were fulfilled.

(i) Temperature lapse-rate of approximately zero (between plus 0·5° F. and minus 0·1° F. per 17 metres). This condition eliminates any effect on turbulence due to large temperature lapse or inversion.

(ii) Wind direction between 270° and 310° true bearing (i.e., the bearing reckoned in degrees from true N. through E. up to 360°) for polar air.

(iii) Wind direction between 160° and 200° true bearing for equatorial air.

The times of occurrence of the required condition of the temperature lapse-rate were obtained from the autographic records of an apparatus measuring differences of temperature between heights of one, seven and seventeen metres. A description of this apparatus has been given by N. K. Johnson (18).

With regard to conditions (ii) and (iii) it will be realised that these ranges of wind direction do not provide an infallible differentiation between equatorial and polar air.

For the directions between 270° and 310° the anemometer exposure is excellent. The first kilometre consists of flat grass land with open country beyond. On the other hand, at a distance of 1½ kilometres to the south of the anemometer, a ridge of hills rises to a height of 50 metres above the anemometer site. A discussion of the effect of different exposures to the south and west is given later.

The results are set out in the form of graphs in Fig. 5, and the points in most cases represent the means of several measurements, the number of observations being placed against each point. For the records of the direction vanes the width of the trace has been plotted against mean wind velocity. For the Dines velocity records the gustiness factor u/U obtained from the expression—

$$u/U=(U\text{ max.}-U\text{ min.})/(U\text{ max.}+U\text{ min.})$$

has been plotted. The curves should therefore give an approximate idea as to how the ratio of the component eddy velocity to the mean velocity varies with the latter. Actually the ratio is given by the sines of the semi-angular widths of the direction records, but as the method of measurement of the width is an approximate one only, it was thought that no great error could be introduced by taking actual widths instead of the sines. The Dines and Baxendell records refer to a height of 13·5 metres. The vertical vane, however, was fixed at a height of 3·5 metres, but a result obtained in a later section of the work enables a correction for height to be applied. The dotted curves for the vertical component refer to a height of 13·5 metres and are therefore comparable with the Dines and Baxendell figures.

The first point to be observed from the curves is that the gustiness, i.e., the ratio of the component of eddy velocity to the mean velocity, remains practically constant at velocities greater than six metres per second but that there is a tendency for it to fall off at lower velocities. The change is not so abrupt in the case of the lateral direction records. There is some evidence for suggesting that this apparent decrease of gustiness at low velocities is an instrumental effect due to inertia and friction.

A detailed investigation of the accuracy with which fluctuations of wind velocity and direction are recorded by the Dines anemometer has been made recently at the National Physical Laboratory (12). It is found that fluctuations of velocity about means varying from three to 40 metres per second are recorded accurately when the period of the fluctuations exceeds ten seconds but for shorter periods there is a tendency for the ratio of the recorded amplitude to the true amplitude to diminish, the effect becoming more marked at lower mean velocities. The direction vane gives an accurate representation of oscillations about a mean direction so long as the period of the oscillation exceeds five seconds but the effect of inertia becomes important when the variations take place more rapidly. The over-run of the vane becomes increasingly marked as the period of fluctuation falls from five seconds to one second, strong resonance occurring at about the latter period. Apart from the effects of " micro-turbulence " most of the irregular fluctuations which occur in a natural wind have periods greater than one second and as a rule high winds show more rapid variations than low winds. We should therefore expect the over-run to become more marked at high velocities. The experimental results obtained at the National Physical Laboratory appear to support the idea that the apparent diminution in gustiness of both velocity and direction at low velocities is partly, if not wholly, an instrumental effect. One would expect the effect to be considerably reduced in the case of an instrument possessing very little inertia and friction. Some results given in a later section appear to confirm this. It will be seen that the gustiness shown by a very light vane is practically constant down to velocities as low as 2·5 metres per second.

The question of gustiness as shown by a Dines pressure-tube anemometer has been treated by a number of other observers. Their results are in agreement so far as the higher wind velocities are concerned, but the falling off of the gustiness ratio at lower velocities has not always been evident. Thus, Dr. G. C. Simpson, in his report on the "Beaufort Scale of Wind Force" (13) shows that the range of variation of velocity is proportional to the mean velocity of the wind at all speeds. A similar result was obtained by J. S. Dines (14). On the other hand, some data obtained by A. H. R. Goldie (15) from records taken at Lerwick show that the gustiness ratio does not become appreciably constant until a velocity of about six metres per second is exceeded. This effect is also indicated by some figures given by G. I. Taylor (11).

It seems quite certain, therefore, that eddy velocity is proportional to the mean wind velocity when the latter is greater than about six metres per second. The proportionality probably holds down to much lower wind speeds, but with the standard instruments it is masked by effects of friction and inertia.

Let us now examine the effect of direction on the gustiness of the wind. A. H. R. Goldie (15) has pointed out that polar and equatorial air currents are characterised by different types of gustiness which are explicable in terms of the vertical distribution of temperature normally found in these winds ; here Goldie refers to temperature distribution over considerable heights. The condition of a zero lapse-rate of temperature observed during the present investigation is almost certainly a surface effect and cannot therefore be expected to eliminate the inherent difference in gustiness in equatorial and polar winds. Any difference between the two types of wind is therefore likely to remain superimposed on differences in gustiness due to changes in the contour in the two directions.

In Table I are given the data extracted from Fig. 5. The gustiness corresponding to a mean velocity of 10 metres per second has been taken in each case.

TABLE I

Exposure		Under lee of hills.	Open.
Limits of wind direction		160° to 200°	270° to 310°
Air		Equatorial.	Polar.
Gustiness	x	0·38 (1·0)	0·40 (1·0)
	y	0·29 (0·76)	0·28 (0·70)
	z	0·15 (0·39)	0·21 (0·52)

For the direction gustiness, if 2θ is the angular width of the trace taken from the curve, then sine θ gives the y gustiness in terms which are comparable with the velocity gustiness. The figures in brackets express the y and z components as decimals of the x component of gustiness. Since the mean velocity is the same in each case, it follows that the figures in the table are relative values of the component eddy velocities, but an intercomparison of the three components as represented by the figures in brackets is not strictly justifiable, since it is not certain that the instruments have the same relative sensitivity. However, it is probable that the differences between the sensitivities are not sufficiently great to account for the marked differences in the gustiness in the three directions. We may infer that the three components of eddy velocity are not equal at a height of about 13 metres above the ground.

It will be seen that in the case of the x and y components there is little difference in the gustiness obtained with the two different exposures. It will even be observed that the open exposure (or polar air) gives an appreciably larger vertical component of gustiness than the obstructed exposure (equatorial air). This result is rather surprising, and indicates that the turbulence generated by a ridge of downs fifty metres high does not extend to $1\frac{1}{2}$ kilometres to leeward.

In the paper referred to above, Goldie finds that for equatorial winds coming off the sea, the gustiness (x component) above 6 metres per second is nearly independent of velocity and the value of the gustiness ratio is about 0·3. On the other hand, Table I shows it to be somewhat higher (about 0·4) over land.

These results regarding the effect of contour are put forward tentatively since effects due to the origin of the air have not been eliminated. All that can be said, perhaps, is that the effect of this particular contour is considerably less than had been anticipated.

§ 4—Experiments with the Small Bi-Directional Vane (Taylor)

This vane, already described in a previous section, does not record direction against time, but simply traces out on a fixed chart the actual movements of the vane for any desired interval of time. In measuring up the traces the method adopted was to draw an oval round the mass of lines so that it included practically all the loops except perhaps one or two irregular extremes. The lateral diameter of the oval was then taken to be a relative measure of the ratio of the maximum y component of eddy velocity to the mean wind velocity, and the vertical diameter was taken to represent the corresponding quantity for the z component. Thus the ratio of the horizontal diameter to the vertical diameter gives the ratio of the y and z components. Knowing the mean wind velocity in each case, approximate values of the maximum eddy components can be worked out.

The experiments, all of which were made with temperature gradient approximately zero, may be considered under three headings :—

(*a*) *Height above ground constant* (2 *metres*) ; *wind velocity and direction approximately constant* ; *duration of record variable.*—The mean figures for two series of observations are given in Table II which is self-explanatory. The first three columns are also plotted in Fig. 6.

Table II

Height of vane, 2 metres ; wind direction, 235° true bearing ; wind velocity at height of 2 metres, 470 cm./sec.

Duration of record	Lateral diameter of trace	Vertical diameter of trace	Ratio lateral/vertical
min.	cm.	cm.	
½	4·15	2·75	1·54
1	5·1	3·15	1·62
2	5·45	3·5	1·56
4	5·75	3·5	1·64
8	6·9	4·4	1·68
			Mean ~~1·51~~ 1·61

It will be seen that the diameters of the records increase rapidly during the first minute, but afterwards only slowly and steadily. It would appear, therefore, that the majority of the eddies recorded by the vane have periods of the order of a few seconds. After one minute, further increase in the diameters of records will occur as occasional large eddies pass the vane.

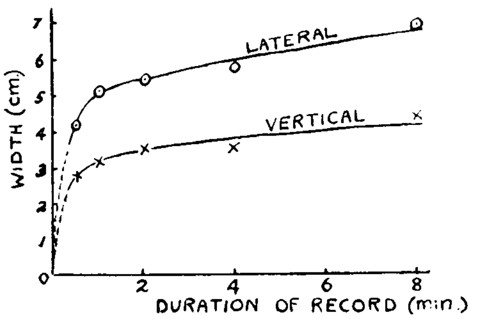

FIG. 6.—BI-DIRECTIONAL VANE: VARIATION OF MEAN RANGES OF LATERAL AND VERTICAL GUSTS WITH DURATION OF RECORD.

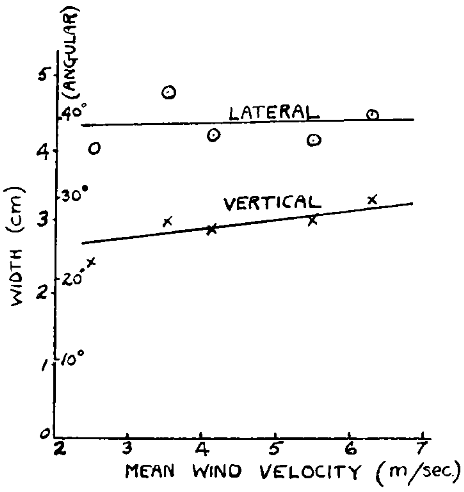

FIG. 7.—BI-DIRECTIONAL VANE : VARIATION OF MEAN RANGES OF LATERAL AND VERTICAL GUSTS WITH WIND VELOCITY.

The ratio of the lateral and the vertical diameter is approximately constant and it will be seen that the lateral is greater by about 50 per cent. This is of great interest in view of the fact that the diameters are relative measures of the y and z components of eddy velocity. It will be discussed in a later section in connexion with some similar results obtained by a somewhat different method. This result may also be compared with the ratio of the eddy velocities shown in Table I for the larger-scale turbulence.

(*b*) *Height of vane constant* (3·3 *metres*) ; *duration of record constant* (1 *minute*) ; *wind velocity variable.*—The results are shown in the form of a graph in Fig. 7. The wind velocity is the velocity measured at the height of the vane (3·3 metres). It will be seen that with this light vane there is very little tendency for the width of the trace to increase with velocity. At any rate, the increase is much less than that shown by the records with the large vanes over a similar range of velocities. This is, of course, to be expected because the effects of friction and inertia must be considerably reduced in the case of the small vane. It certainly seems very probable that with an ideal vane the graphs would be horizontal, that is, the eddy velocity is proportional to mean wind velocity.

The mean ratio of the lateral width to vertical width is 1·48. This agrees very well with the result obtained (at 2 metres) in the preceding section.

(*c*) *Duration of record constant* (1 *minute*) ; *wind velocity at fixed height approximately constant, height of vane variable.*—This section of results is the most interesting but is complicated slightly by the fact that mean wind velocity varies with height. The ideal method of carrying out the experiments would have been to arrange three or four vanes at different heights, and take records simultaneously and also measure the wind velocity at these heights at the same time. In this way some definite connexion between gradient of wind velocity and variation of eddy velocity with height might have been established. However, only one vane was available so that simultaneous records were not obtainable, and, owing to the length of time taken over a series of experiments at different heights, it was impossible to ensure that wind velocity at a fixed height did not change during the series of observations. In order to eliminate as far as possible any effect due to variation in wind velocity at a fixed height, the following procedure has been adopted.

Suppose that at a height z metres, the wind as measured was U_z and the lateral and vertical half-widths of the record converted to angles were p and q respectively. Then at the height z the lateral and vertical components of eddy velocity are given approximately by pU_z and qU_z. As all the experiments have been carried out under conditions of approximately zero temperature gradient, the relation of the gradient of wind velocity may be taken as—

$$U_z = \beta z^{0 \cdot 13} \; *$$

where β is the velocity at some standard height.

From this relation the velocity at a constant height of one metre U_1 has been worked out. Then assuming that eddy velocity is proportional to wind velocity at a fixed height, we can find the values of the lateral and vertical components for a standard wind velocity (1 metre/sec.) at 1 metre height, by simple proportion $p\, U_z/U_1$. The series of values thus obtained have height as the only variable—the mean velocity at any fixed height being constant, the velocity being assumed to vary with height according to the power law quoted above.

The results, which in most cases represent the means of several values, are summarised in Table III and are plotted in Fig. 8.

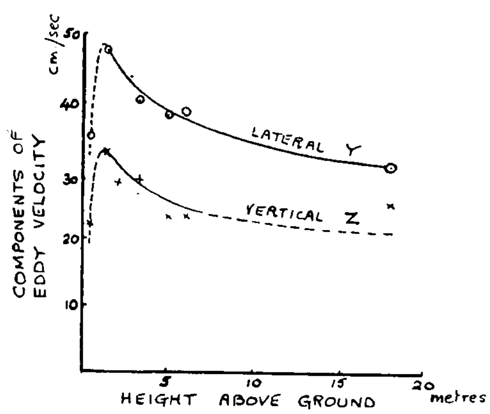

FIG. 8.—BI-DIRECTIONAL VANE: VARIATION OF COMPONENTS OF EDDY VELOCITY WITH HEIGHT ABOVE GROUND.

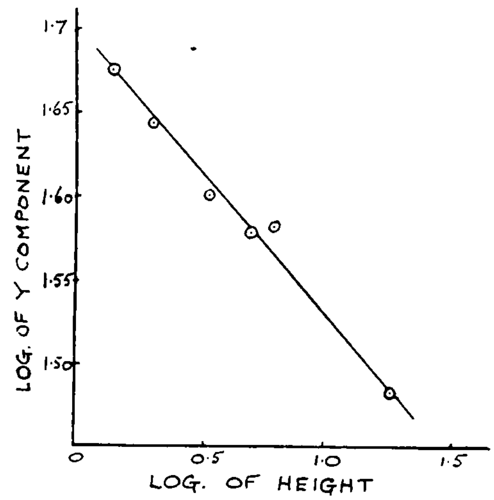

FIG. 9.—BI-DIRECTIONAL VANE: VARIATION OF LATERAL COMPONENT OF EDDY VELOCITY WITH HEIGHT ABOVE GROUND.

TABLE III—y AND z COMPONENTS OF EDDY VELOCITY AT VARIOUS HEIGHTS, FOR WIND VELOCITY OF 1 METRE/SECOND AT 1 METRE HEIGHT

Height	y component	z component	y/z	No. of observations
metres	cm./sec.	cm./sec.		
0·5	34	22	1·63	1
1·4	47·5	32·5	1·45	2
2·0	44	28	1·59	3
3·3	40	28·5	1·40	10
5·0	38	23	1·71	2
6·0	38·5	23	1·67	2
18·0	30·5	25	1·20	6

* This relation was found in a separate investigation and it is considered to be a reliable one for the particular conditions of temperature gradient and local contour.

For that part of the y curve which lies between 1·4 metres and 18 metres, it is found that the points conform to a power law. The logarithms are shown plotted in Fig. 9. This curve leads to the relation

v varies with $z^{-0.17}$
where $v = y$ component of eddy velocity
and z=height above ground.

This is of interest when compared with the relation for wind-velocity gradient under zero temperature gradient.

$$U_z = \beta z^{0.13}$$

It would appear that the y component (above 1 metre from the surface) decreases with height at approximately the same rate as the mean velocity increases with height. The values for the z component do not give a close agreement with such a simple relation, the eddy velocity at 18 metres apparently being higher than that at about 5 metres. Since no great accuracy can be claimed for the method it would be unwise to place much reliance on the change between these two heights. It seems more likely that the variation of the z component with height should be of the same form as that of the y component. We may make a general conclusion that the eddy velocity increases rapidly from the surface up to a height of about one metre, then decreases fairly rapidly, but this decrease becomes less as the height increases.

The ratio of the y and z components does not vary systematically, but has a mean value of about 1·5. This will be referred to in a later section.

§ 5—Analysis of Kinematograph Records

As explained in a previous section, by projecting the kinematograph pictures on to squared paper, the co-ordinates of the positions of the vane and of the swinging plate can be read off at desired short intervals. The figures thus obtained have been plotted against time and the resulting diagrams form records of velocity/time and direction/time with a very open time scale. For each graph, the mean direction or velocity has been obtained and also the mean deviation.

In a preliminary experiment two instruments of different size but identical in shape and form were used. The measurements of the larger pair (used in all the other kinematograph experiments) are shown in Fig. 4. The smaller pair were approximately half size. The weights of the moving parts of the two vanes were 19 grams and 1·4 grams respectively. The instruments were fixed side by side at a height of two metres above the ground. An analysis of a run of six seconds' duration was made, every exposure during this interval being measured. The exposures were taken at a rate of 16 per second so that nearly 100 points were obtained and these, when plotted, showed that most of the eddies recorded by the smaller instruments were also recorded by the larger ones. For further experiments it was decided to employ the larger instruments owing to the very fragile nature of the smaller vane.

(*a*) *Limitations of method as affected by frequency of readings.*—Since the exposures are made at a rate of 16 per second, the work of measuring every exposure for a period of one minute becomes a very laborious process involving the measurement of about 5000 quantities. It was therefore thought advisable to find out how the results would differ if readings were taken from the films at less frequent intervals. With this object in view, a record of 60 seconds' duration was analysed by taking three series of readings of the y component. In the first series the time interval between each reading was $\frac{1}{8}$ second, in the second series it was $\frac{5}{8}$ second and in the third it was five seconds. The three sets are shown graphed in Fig. 10. It is obvious that increasing the interval between the readings tends to flatten out the smaller fluctuations. The curve having an interval of $\frac{1}{8}$ second shows the effect of very small variations; in the curve with an

FIG. 10.

To face page 10.

KINEMATOGRAPH RECORDS OF y COMPONENT.

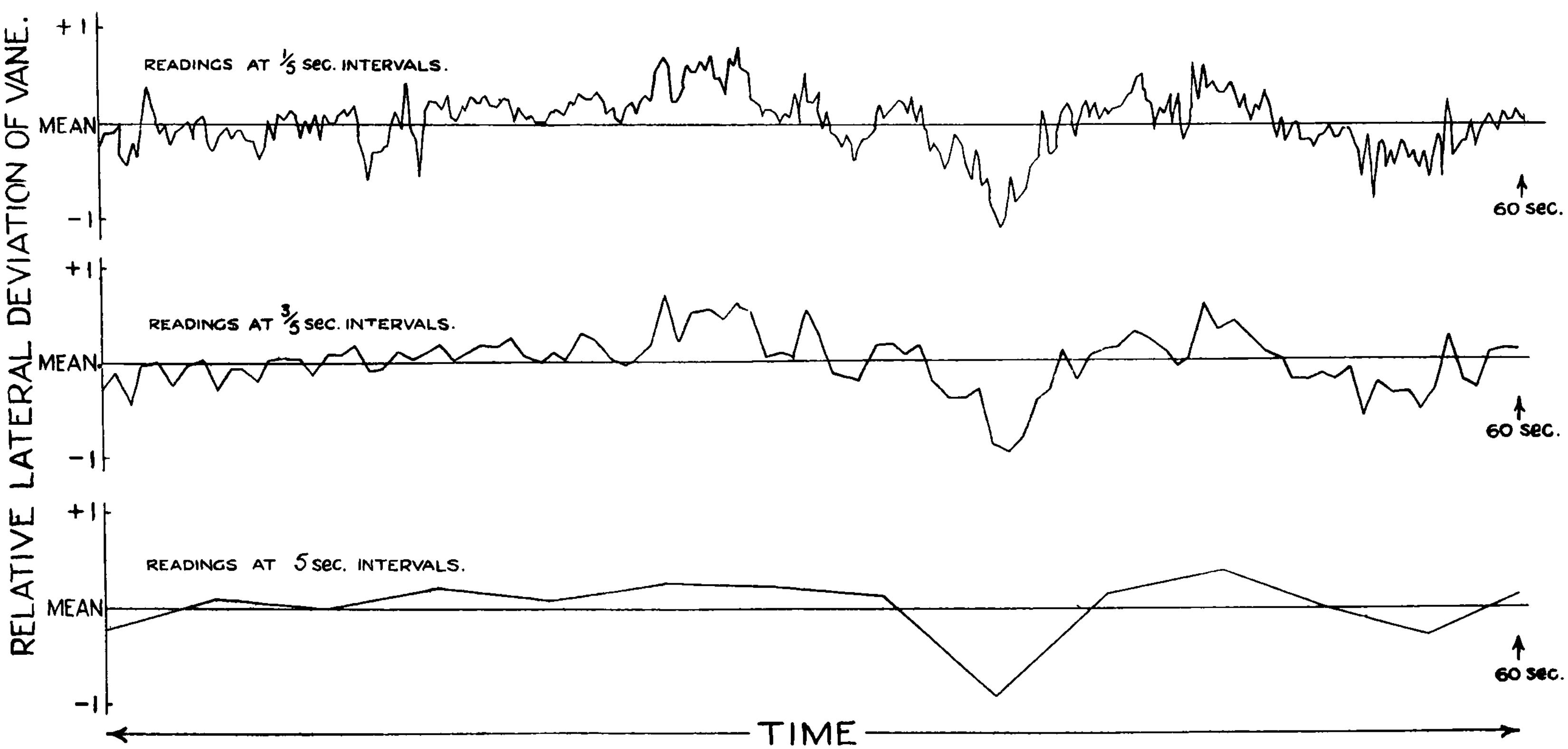

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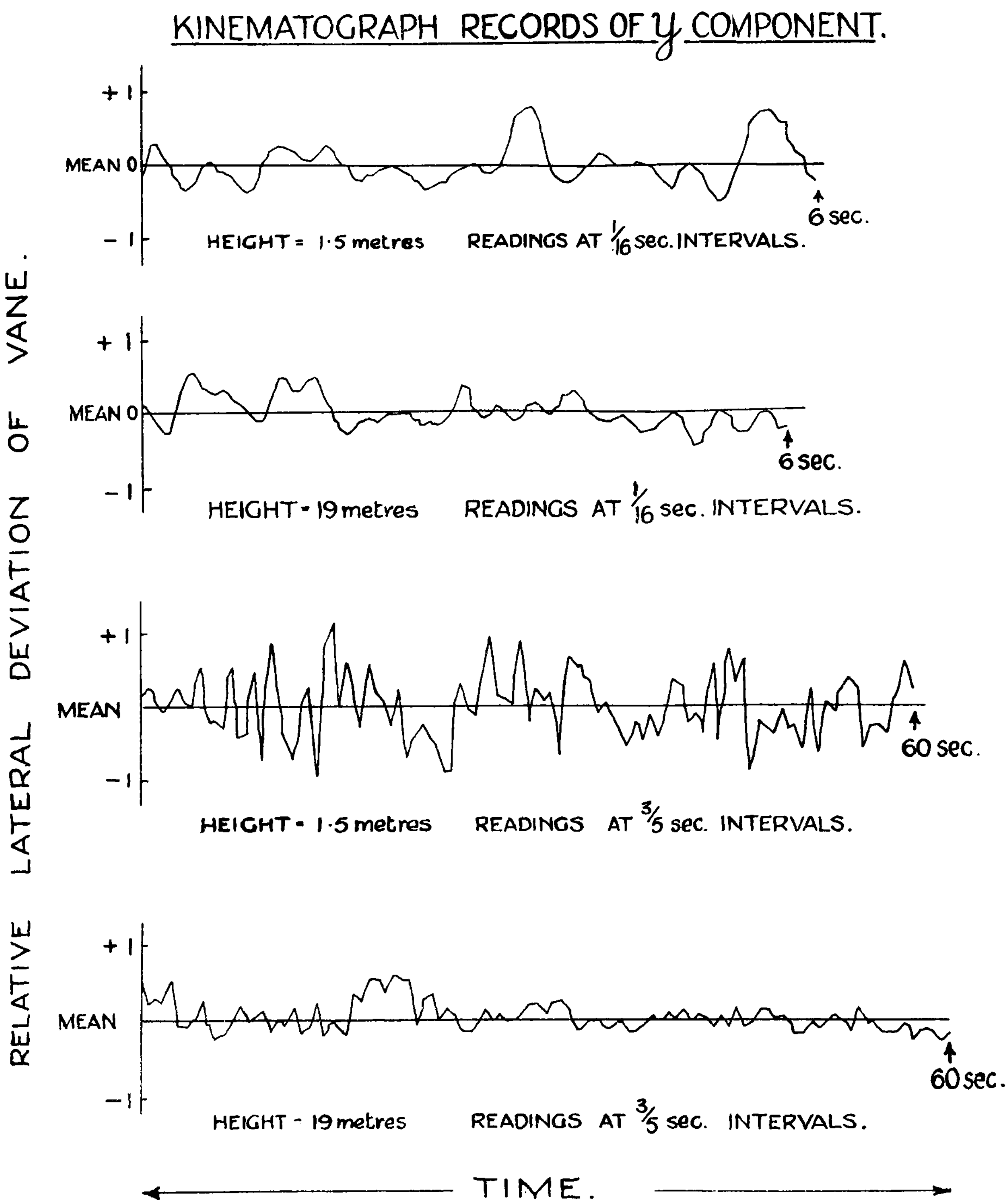
FIG. 11.



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interval of five seconds these are all smoothed out. The figures given in Table IV emphasize the differences between the three series.

TABLE IV

Height of vane = 19 metres ; duration of record = 60 seconds ; mean wind velocity = 900 cm./sec.
y component (v).

Interval between readings (sec.)	$\frac{1}{5}$	$\frac{3}{5}$	5
Mean deviation	0·240	0·223	0·142
Mean component of eddy velocity (cm./sec.)	79	74	47

The mean deviation is not much altered as the interval is changed from $\frac{3}{5}$ second to $\frac{1}{5}$ second. It seems probable that a large number of eddies have periods of the order of a second.

An analysis of a short record showed that reducing the interval from $\frac{1}{5}$ second to $\frac{1}{16}$ second did not make a very great difference to the results. Table V gives the essential data :—

TABLE V

Height of vane = 19 metres ; mean wind velocity = 900 cm./sec. ; duration of record = 7·2 seconds

Interval between readings (sec.)	$\frac{1}{16}$	$\frac{1}{5}$
Mean deviation	0·132	0·115
Mean component of eddy velocity (cm /sec.)	44	38

From these figures it appears that an interval of $\frac{1}{5}$ second is sufficiently short to include most of the small fluctuations, and that for purposes of comparison there is little advantage in taking readings more frequently. The values of the eddy velocity obtained are not very different in the two cases.

(*b*) *Effect of varying the duration of the record*.—It is obvious that unless the duration of a record is sufficiently long to cover the effects of a large number of eddies, the results will be liable to large variations. Some figures obtained from the film dealt with in the preceding section show great inconsistencies on this account. The record of 60 seconds' duration (readings at $\frac{1}{5}$ second intervals) has been considered first by subdividing it into six parts of 10 seconds' duration, then three parts of 20 seconds' duration, next two parts of 30 seconds' duration and, finally, as a complete run of 60 seconds. The results are summarised in Table VI :—

TABLE VI—*y* COMPONENT (v)

Duration	Mean deviation	v
		cm./sec.
10 sec.	0·125	41
	0·151	50
	0·190	63
	0·284	94
	0·170	56
	0·173	57
Mean	0·182	60
20 sec.	0·146	48
	0·357	118
	0·219	72
Mean	0·241	80
30 sec.	0·193	64
	0·304	101
Mean	0·248	82
60 sec.	0·240	79

The mean deviation for any subdivision of the complete run refers to the mean direction during the subdivision and not to the mean direction during the whole run.

In each series where there are two or more sets of readings the differences are very great. A large eddy apparently occurred about the middle of the run, causing relatively large deviations in one of the ten seconds' periods and in one of the twenty seconds' periods. The means for each of the series together with the values for the whole run are representative of the same total number of readings in each case and should be comparable with each other. The mean values of the eddy velocity show a rapid increase at first, but there is little difference between the values for 20, 30 and 60 seconds. A similar analysis of some readings at 5-second intervals of the wind direction shown by the Baxendell recorder showed that the mean values of eddy velocity over longer periods increase slightly with the length of the period. This, of course, is to be expected. since with increase of time the effects of larger and larger eddies are more likely to be recorded.

(c) *Variation of eddy velocity with height above ground.*—The kinematograph method has given some interesting results in this connexion. Two films were taken over periods of a minute. In one case, the instruments were mounted at a height of 1·5 metres above the ground; in the other case, they were mounted on top of a latticed tower at a height of 19 metres. The tower would cause little or no disturbance of the air at the position where the instruments were fixed. The films were analysed in two sections. In one, every exposure was measured over a period of 6 seconds, thus giving readings at intervals of $\frac{1}{16}$ second. This period of 6 seconds, however, is too short to give reliable results, but the curves for the y component are given in Fig. 11, since they show a detailed record of the eddies. In the second analysis, the exposure at intervals of approximately $\frac{3}{8}$ second were measured over the whole period of 60 seconds. Unfortunately, this analysis was carried out before it was decided that an interval of $\frac{1}{8}$ second is more suitable if the effects of most of the smaller eddies are to be included. However, it is considered that the results obtained with the larger interval are sufficiently representative for comparative purposes. The graphs for the y component are given in Fig. 11. The x and z components show variations of a similar type.

The results of the second analysis (60 seconds' run) are set out in Table VII :—

TABLE VII

Duration of record = 60 seconds ; interval between readings $\frac{3}{8}$ second ; wind velocity at 1·5 metres = 470 cm./sec.

Height above ground	1·5 m.			19 m.		
Component	x	y	z	x	y	z
Mean velocity (cm./sec.)	57	66	43	55	40	·31

Owing to the fact that the record at the greater height was made about half-an-hour after the record at 1·5 metres, the wind velocity at constant height was not the same in the two cases. To allow for this, a correction was applied to the eddy velocities at 19 metres on the assumption that eddy velocity is proportional to wind velocity. The corrected values are given in Table VII.

The ratios of the three velocity components at the two heights are as follows :—

1·5 metres :—$x : y : z = 1 : 1{\cdot}16 : 0{\cdot}75$; $y : z = 1{\cdot}54 : 1$
19 ,, :—$x : y : z = 1 : 0{\cdot}73 : 0{\cdot}56$; $y : z = 1{\cdot}29 : 1$

In section 4(c) we found a mean value of about 1·5 for the $y:z$ ratio ; this is in fair agreement with the above values obtained by a more reliable method. It seems fairly certain that under the conditions dealt with in this investigation the three components of eddy velocity are not, in general, equal. In a paper on the nature of turbulent motion G. I. Taylor (11) comes to the conclusion that the components are equal. His deduction, however, is based on observations made at a height of about 60 metres; moreover his conditions were not limited by considering only occasions of zero temperature gradient. It seems reasonable to conclude that if there is an equipartition of eddy energy above a height of about 60 metres it is extinguished at lower heights where the surface of the ground must modify conditions to a great extent.

The figures given in Table VII show that the x component does not alter very much with height whereas the y and z components decrease considerably. The proportional decrease in each case compares very well indeed with the results obtained with the Taylor vane, as the following figures indicate :—

$$\frac{y \text{ component at } 19 \text{ m.}}{y \text{ component at } 1\cdot5 \text{ m.}} = \begin{cases} 0\cdot6 \text{ by Taylor vane.} \\ 0\cdot61 \text{ by kinematograph vane.} \end{cases}$$

$$\frac{z \text{ component at } 19 \text{ m.}}{z \text{ component at } 1\cdot5 \text{ m.}} = \begin{cases} 0\cdot7 \text{ by Taylor vane.} \\ 0\cdot72 \text{ by kinematograph vane.} \end{cases}$$

The values for the Taylor vane are extracted from the smoothed curves in Fig. 8. The close agreement appears to confirm the suggestion which was made in section 4(c), viz. : that the value obtained with the Taylor vane for the z component of eddy velocity at a height of 18 metres (Table III) is too high.

(*d*) *Eddy shearing stresses.*—A point of considerable interest has been observed on the simultaneous records of the three components of eddy velocity.

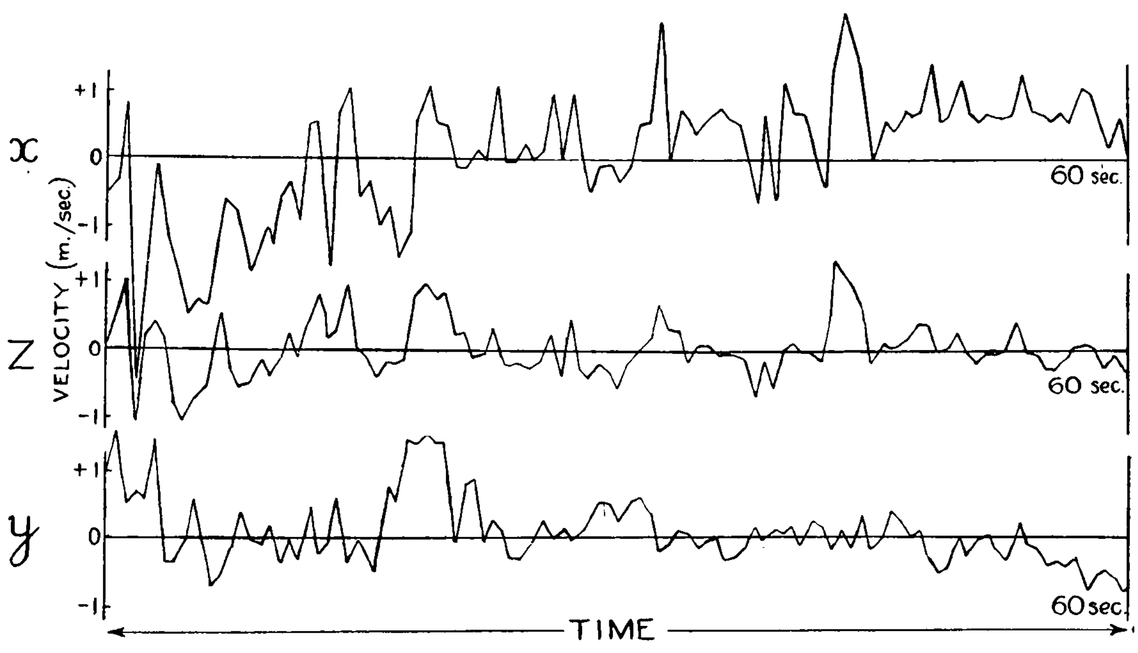

FIG. 12—SIMULTANEOUS RECORDS OF THE THREE COMPONENTS OF EDDY VELOCITY AT 19 METRES ABOVE THE GROUND (MEAN WIND VELOCITY=9 m./sec.)

An analysis of the films which were taken at a height of 19 metres showed a fairly close correspondence between the variations of the x and z components ; the variations of the y component showed little connexion with those of the other components. An example is given in Fig. 12 which shows simultaneous records lasting one minute, the readings being taken at intervals of $\frac{3}{8}$ second. In the case of the z component the deviations above the mean line indicate that the vertical air current was directed downwards; the deviations of the x component above the mean line indicate that this component was in the same direction as the mean wind. These two curves give us a clear illustration of the transfer of momentum which is maintained between two layers of air the upper of which possesses the greater mean velocity. It will be noticed that practically all of the peaks on the z component curve, indicating a downward flow of air, correspond with increases of velocity on the x component record; similarly, upward currents are associated with lulls on the x component.

This phenomenon simply demonstrates that the eddy shearing stress for the x, z components is appreciable. The theorem of Osborne Reynolds shows that the stress dragging the lower air in the direction of the velocity U increasing is $-\rho\,\overline{u'\,w'}$, where ρ is the density of the air, u' and w' are simultaneous deviations, at the same point, of components of eddy velocity from the mean; the bar denotes the mean, over a long interval, of the products of the deviations. The numerical value of the eddy shearing stress has been computed from the deviations of the x and z components shown in Fig. 12, thus :—

$$-\rho\,\overline{u'\,w'} = 3{\cdot}62 \text{ gm. cm}^2\text{. sec.}^{-2}$$

If we wish to obtain a value of the eddy viscosity, assuming this to be given by the ratio of the eddy shearing stress to the rate of mean shearing strain, we require to know the gradient of wind velocity at the height (19 metres) at which the experiment was carried out. In the absence of simultaneous measurements of wind velocity at two heights the next best thing is to use the relation already quoted :

$$U = \beta\, z^{0{\cdot}13} \qquad \text{whence } dU/dz = 0{\cdot}13U/z$$

For the case in question U=730 cm./sec., therefore dU/dz=0·05 sec.$^{-1}$. The eddy viscosity, μ, parallel to the wind is therefore

$$\mu = \frac{-\rho\,\overline{u'\,w'}}{dU/dz} = 70 \text{ gm. cm}^{-1}\text{. sec.}^{-1}$$

From the Lindenberg pilot-balloon observations L. F. Richardson (8) obtained a value of 51 for the eddy viscosity parallel to the wind near the ground. We may obtain a rough idea of the value of the coefficient of eddy diffusion K, on the assumption that both this coefficient and the density do not vary with height, by dividing our value of the eddy viscosity by ρ. Taking ρ as $1{\cdot}2 \times 10^{-3}$ gm. cm.$^{-3}$ this gives :

$$K = 6 \times 10^4 \text{ cm}^2\text{. sec.}^{-1}$$

It is apparent from a comparison of the three curves in Fig. 12 that the eddy shearing stresses across the wind are considerably smaller than the shearing stress in the direction of the wind. The actual values are

$$-\rho\,\overline{v'\,w'} = 1{\cdot}50 \text{ gm. cm}^2\text{. sec.}^{-2}$$
$$-\rho\,\overline{u'\,v'} = 0{\cdot}58 \quad ,, \quad ,, \quad ,,$$

Moreover, a record taken at a height of 1·5 metres from the ground shows an apparent diminution in $-\rho\,\overline{u'\,w'}$, the value being roughly less than one quarter of the value 3·62 at 19 metres. Mr. O. F. T. Roberts (17) suggests that the reason for this is probably that since the velocity gradient is so much greater at such a low height, it becomes difficult to decide, when a portion of air comes up from below or down from above, what we can take as the mean wind velocity.

§ 6—Summary of Results

The various results and conclusions arrived at in the course of the present investigation are summarised below. The results all refer to an approximately isothermal atmosphere, moreover those numbered 6 and 10 apply only to exposures similar to that employed in these experiments.

Large-scale turbulence.—Mean values of eddy velocity, etc., over an interval of one hour :—

1 The three components of eddy velocity are proportional to the wind velocity at moderate and high wind velocities. (It is probable that this is also true at low wind velocities, but the records available cannot be relied upon owing to the inertia and friction in the recording instruments.) (Cf. 4 below.)

2. These three components of eddy velocity are not equal in magnitude at a height of about 13 metres.

3. The effect of contour upon turbulence of this scale appears to be less than was anticipated (see Table I).

Intermediate-scale turbulence.—Mean values of eddy velocity, etc., over an interval of the order of a few minutes :—

4. The y and z components of eddy velocity are found to be proportional to the mean wind velocity. (Cf. 1 above.)

5. These two components are also unequal in magnitude.

6. The magnitudes of the components across wind and vertically are in the ratio of 1·0 : 0·7 (see Table II).

7. The magnitudes of these y and z components of eddy velocity increase from the ground up to a height of between one and two metres. Above this height they decrease, the values at 18 m. being about two thirds those at 1½ m. The ratio of these components is constant over this height interval. (See Table III).

8. The mean values of these y and z components of eddy velocity increase with time, the magnitudes for a duration of eight minutes being about fifty per cent. greater than those for one minute (see Fig 6).

Small-scale turbulence.—Mean values of eddy velocity, etc., between a few seconds and one minute :—

9. It seems probable that a large proportion of the small-scale eddies have periods of the order of one second.

10. At a height of 1·5 metres above the ground, the components of eddy velocity along wind, across wind and vertically are in the ratio 1·0 : 1·16 : 0·75.

11. The x component of eddy velocity appears to be nearly constant over the height interval 1½ m. to 19 m.

12. The y and z components, on the other hand, at 19 m. decrease to about two thirds their value at 1½ m. (see Table VII).

13. The eddy shearing stress, $-\rho\overline{u'w'}$, is very noticeable at a height of 19 m., but there is an apparent decrease close to the ground. The two other shearing stresses, $-\rho\overline{u'v'}$ and $-\rho\overline{v'w'}$, are comparatively small even at 19 m. An approximate calculation of the eddy viscosity parallel to the wind and of the coefficient of eddy diffusion gives reasonable values.

§ 7—Conclusion

Some of the conclusions presented above must be regarded as approximate only and accepted with due caution owing to the comparatively small number of observations upon which they are based. Most of the deductions which have been made are regarded as reliable.

The main object of the present investigation was to try out a new method of studying the nature of atmospheric turbulence, and it is thought that the results which have been obtained show that the method possesses great possibilities. The outstanding disadvantage of the method is that it is extremely laborious. To render more trustworthy the various results contained in this report by repeating the experiments would have entailed an enormous amount of work which it was not considered desirable to undertake at the present stage.

Since revising this paper my attention has been drawn to an investigation of a similar character which has been commenced at Vienna by W. Schmidt (16). So far only an account of preliminary experiments is available. The main feature of the method is to obtain what might be called a snapshot of the movements of the air in a small portion of the atmosphere close to the ground. Should the work on the subject be continued, the present investigation may help to decide which is the most profitable line of attack.

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