

FORMULAE AND TABLES OF CONSTANTS
FOR USE IN THE DISCUSSION OF
THE RESULTS OF SOUNDINGS WITH
REGISTERING BALLOONS AND PILOT
BALLOONS

PREPARED FOR THE COMPUTERS' HANDBOOK OF
THE METEOROLOGICAL OFFICE

1913

CALCULUS OF THE UPPER AIR

FORMULAE AND TABLES OF CONSTANTS FOR USE IN
THE DISCUSSION OF THE RESULTS OF SOUND-
INGS WITH REGISTERING BALLOONS AND PILOT
BALLOONS.

*Prepared for the Computers' Handbook of the
Meteorological Office.*

1. Characteristic equation.

$$p = R\rho\theta$$

for pressure in millibars and temperature in degrees absolute:—

For dry air $R = 2.870 \times 10^3.$

For air containing 6.11 mb. of water vapour (i.e. saturated
at 273°)

$$R = 2.876 \times 10^3.$$

For air containing 12.24 mb. of water vapour (i.e. saturated
at 283°)

$$R = 2.883 \times 10^3.$$

For air containing 35.41 mb. of water vapour (i.e. saturated
at 300°)

$$R = 2.905 \times 10^3.$$

These values of R are computed from the formula

$$\frac{R_w}{R_o} = \frac{p_o}{p_o - \frac{3}{8} \frac{\theta_o}{\theta} p_w},$$

where R_o is the value of R for dry air; R_w is the value of R
for a mixture of air and water vapour which is saturated at
the temperature θ and which has a partial pressure of dry air
of 1000 mb. at the freezing point.

Other values of R computed on the same plan are given in
Table I, p. 7.

2. Change of pressure-difference between two verticals at different levels. Rate of increase of pressure-difference (in millibars per metre increase of height)

$$= 0.342 \frac{p}{\theta} \left(\frac{\Delta\theta}{\theta} - \frac{\Delta p}{p} \right).$$

3. Wind Velocity at different levels. Calculation of the gradient wind at any level at which the pressure is p millibars and the absolute temperature θ , in terms of the pressure-difference Δp millibars per 100 kilometres.

$$V = \frac{R}{2\omega \sin \lambda} \cdot \frac{\theta}{p} \cdot \frac{\Delta p}{L}$$

$$= K \cdot \frac{\theta}{p} \cdot \frac{\Delta p}{L}.$$

V = velocity in metres per second,

R = constant of characteristic equation,

λ = latitude,

ω = angular velocity of earth's rotation,

Δp = pressure-difference between two points on same level,

L = their distance apart in hundred-kilometres.

[The curvature of the path is neglected.]

For the values of $R/(2\omega \sin \lambda)$ see Table II.

For *average values* of p/θ see Table III.

4. Calculation of the pressure-differences per 100 kilometres from the wind components, W . to E . and S . to N . at different levels.

If the pressure gradient northward be $\Delta_N p$ per 100 kilometres, then

$$\Delta_N p = \frac{1}{K} \cdot \frac{p}{\theta} (W. \text{ to } E.).$$

If the pressure gradient westward be $\Delta_W p$ per 100 kilometres, then

$$\Delta_W p = \frac{1}{K} \cdot \frac{p}{\theta} (S. \text{ to } N.).$$

For values of K see Table II.

For *average values* of p/θ see Table III.

For the factors of wind components see Table IV.

5. Calculation of the pressure-difference and temperature-difference for 100 kilometres from the components of wind velocity.

First compute the slope of pressure per 100 kilometres to North from the West-East component by the formula

$$\Delta_N p = \frac{1}{K} \cdot \frac{p}{\theta} (W. \text{ to } E.),$$

and slope of pressure per 100 kilometres to East from the North-South component by the formula

$$\Delta_W p = \frac{1}{K} \cdot \frac{p}{\theta} (S. \text{ to } N.).$$

Thence compute by subtraction of successive lines the change of pressure-difference per kilometre of elevation.

Then for the slope of temperature to the *North*

$$\Delta_N \theta = \frac{\theta}{p} \left(\frac{\text{increase per kilometre of pressure slope to North}}{34.2} \times \theta + \Delta_N p \right),$$

and for the slope of temperature to the *West*

$$\Delta_W \theta = \frac{\theta}{p} \left(\frac{\text{increase per kilometre of pressure slope to West}}{34.2} \times \theta + \Delta_W p \right).$$

If the values of θ and θ/p are not known for the special occasion an estimate must be used which may be taken from the tables of *average values* of θ from Table V and of θ/p from Table III.

In these equations Δp and $\Delta \theta$ signify a *slope* of pressure and temperature, that is the value of Δ is positive to the North when the quantity is smaller Northward. This is unfortunately in contradiction with the usual interpretation of Δ , and such a contradiction ought to be avoided. Another symbol that would avoid the ambiguity has not yet been found.

An example of applying the foregoing formulae is appended. The data are published in C. J. P. Cave's *Structure of the Atmosphere*, for a pilot balloon ascent at Ditcham Park on April 29th, 1908, and are represented in his model of a N.W. current aloft crossing a S.W. current at the surface.

6. Calculation of the amount of outflow from a South to North current and of inflow towards a North to South current which is required in order to maintain the system unchanged.

If H is the vertical depth of the current, λ the latitude, then

$$\frac{1}{H} \frac{dH}{d\lambda} = 0.175 \cot \lambda.$$

Whence it follows that for the maintenance of the thickness of a current of breadth l degrees of longitude, flowing to the North, the amount of air to be removed is equivalent to a crossflow of

$$0.175 \frac{\cos^2 \lambda}{\sin \lambda} lV,$$

where V is the velocity of the current.

For a current flowing southward with the same velocity a similar amount of air must be supplied in order to maintain the thickness of the current as it flows on.

The values of vapour pressure for Table I are taken from Tables 57 and 58 of Landolt and Börnstein's *Physikalisch-chemische Tabellen*.

The modification of the characteristic equation on account of water vapour, keeping R constant, is dealt with by V. Bjerknes by means of an alteration from the actual to the "virtual temperature." See *Dynamic Meteorology and Hydrography*, p. 26, Carnegie Institution, Washington, 1910.

TABLES FOR FACILITATING COMPUTATION

TABLE I. Saturation pressures in millibars of aqueous vapour over ice and water at different temperatures and the corresponding values of R in the characteristic equation for air.

(i) Temperatures below the freezing point.

Temperature in degrees Absolute	ICE VAPOUR		WATER VAPOUR	
	Saturation pressure in millibars	R	Saturation pressure in millibars	R
223°	0.04	2870		
225°	0.06	2870		
230°	0.10	2870		
235°	0.17	2870		
240°	0.29	2870		
245°	0.48	2870		
250°	0.79	2871		
255°	1.28	2871	1.52	2872
260°	2.01	2872	2.34	2872
265°	3.13	2873	3.36	2874
270°	4.79	2875	4.90	2875
273°	6.11	2876	6.11	2876

(ii) Temperatures above the freezing point.

Temperature in degrees Absolute	Saturation pressure in millibars	R	Temperature in degrees Absolute	Saturation pressure in millibars	R
273°	6.11	2876	308°	55.85	2924
275°	7.05	2877	309°	59.03	2927
280°	9.99	2880	310°	62.27	2930
285°	13.96	2884	311°	65.75	2933
290°	19.30	2890	312°	69.39	2937
295°	26.16	2895	313°	73.27	2940
300°	35.41	2905	314°	77.28	2944
301°	37.41	2907	315°	81.47	2948
302°	39.77	2909	316°	85.87	2952
303°	42.06	2911	317°	90.49	2956
304°	44.53	2913	318°	95.40	2961
305°	47.17	2915	319°	100.38	2965
306°	49.85	2918	320°	105.63	2970
307°	52.79	2921			

CALCULUS OF THE UPPER AIR

Height in kilo-metres	Velocity (metres per sec.)	Direction	W to E comp.	$\frac{W}{K}$ to $\frac{E}{K}$ $K=25.4$	$\frac{p}{\theta}$	$\frac{W \text{ to } E}{K} \times \frac{p}{\theta} = \Delta_p p$	Change per k. (increase)	Incr. per k. $\frac{34.2}{34.2}$	θ	Incr. per k. $\frac{34.2}{34.2} \times \theta$	Incr. per k. $\frac{34.2}{34.2} \times \theta + \Delta_p p$	$\left(\frac{\text{Incr. per k.}}{34.2} \times \theta + \Delta_p p \right) \div p/\theta$	$\Delta_p \theta$
6	20.5	300	+17.75	+70	1.88	+1.32	+26	+0073	248	+1.81	+3.13	+1.66	+1.66
5	15.0	300	+12.99	+51	2.08	+1.06	+29	+0085	254	+2.16	+3.22	+1.55	+1.55
4	8.5	280	+8.37	+33	2.33	+77	+12	+0035	261	+91	+1.68	+74	+74
3	6.5	265	+6.48	+25	2.58	+65	+22	+0064	267	+1.71	+1.06	+41	+41
2	8.0	250	+7.52	+30	2.90	+87	+33	+0091	272	+2.48	+3.35	+1.08	+1.08
1	5.0	240	+4.33	+17	3.18	+54	+08	+0023	278	+64	+1.18	+37	+37
0	5.0	220	+3.21	+13	3.50	+46							
Height in kilo-metres	Velocity (metres per sec.)	Direction	S to N comp.	$\frac{S \text{ to } N}{K}$ $K=25.4$	$\frac{p}{\theta}$	$\frac{S \text{ to } N}{K} \times \frac{p}{\theta} = \Delta_p p$	Change per k. (increase)	Incr. per k. $\frac{34.2}{34.2}$	θ	Incr. per k. $\frac{34.2}{34.2} \times \theta$	Incr. per k. $\frac{34.2}{34.2} \times \theta + \Delta_p p$	$\left(\frac{\text{Incr. per k.}}{34.2} \times \theta + \Delta_p p \right) \div p/\theta$	$\Delta_p \theta$
6	20.5	300	-10.25	-41	1.88	-77	-15	-0044	248	-1.09	-1.86	-98	-98
5	15.0	300	-7.50	-30	2.08	-62	-48	-0140	254	-3.56	-4.18	-2.01	-2.01
4	8.5	280	-1.47	-06	2.33	-14	-19	-0056	261	-1.47	-1.61	-69	-69
3	6.5	265	+0.57	+02	2.58	+05	-27	-0079	267	-2.11	-2.06	-80	-80
2	8.0	250	+2.74	+11	2.90	+32	± 0	0	272	0	+32	+11	+11
1	5.0	240	+2.50	+10	3.18	+32	-21	-0061	278	-1.70	-1.38	+45	+45
0	5.0	220	+3.84	+15	3.50	+53							

TABLE II. *K*. Values of $R/2\omega \sin \lambda$ for different latitudes. DRY AIR.

Latitude	$R/2\omega \sin \lambda$	Latitude	$R/2\omega \sin \lambda$	Latitude	$R/2\omega \sin \lambda$
1	1127.1	31	38.3	61	22.6
2	565.2	32	37.2	62	22.3
3	376.9	33	36.2	63	22.1
4	282.8	34	35.3	64	22.0
5	226.3	35	34.4	65	21.8
6	188.7	36	33.6	66	21.6
7	161.9	37	32.8	67	21.4
8	141.7	38	32.0	68	21.3
9	126.1	39	31.3	69	21.1
10	113.8	40	30.7	70	21.0
11	103.3	41	30.1	71	20.9
12	94.9	42	29.5	72	20.7
13	87.7	43	28.9	73	20.6
14	81.5	44	28.4	74	20.5
15	76.2	45	27.9	75	20.4
16	71.6	46	27.4	76	20.3
17	67.5	47	27.0	77	20.2
18	63.8	48	26.5	78	20.2
19	60.6	49	26.1	79	20.1
20	57.7	50	25.8	80	20.0
21	55.0	51	25.4	81	20.0
22	52.7	52	25.0	82	19.9
23	50.5	53	24.7	83	19.9
24	48.5	54	24.4	84	19.8
25	46.7	55	24.1	85	19.8
26	45.0	56	23.8	86	19.8
27	43.4	57	23.5	87	19.7
28	42.0	58	23.3	88	19.7
29	40.7	59	23.0	89	19.7
30	39.4	60	22.8	90	19.7

For the correction for humidity see Table I.

TABLE III. Table for values of p/θ at different levels—Average of results in *Geophysical Journal*, 1912.

Height kilo-metres	p/θ	Height kilo-metres	p/θ	Height kilo-metres	p/θ	Height kilo-metres	p/θ
20	.26	15	.53	10	1.18	5	2.11
19	.28	14	.64	9	1.35	4	2.35
18	.32	13	.75	8	1.52	3	2.61
17	.39	12	.87	7	1.70	2	2.91
16	.46	11	1.02	6	1.90	1	3.24
						Gd.	3.55

TABLE IV. Table of factors for components of wind in **W** to **E** and **S** to **N** directions.

W to $E+$ S to $N-$	W to $E+$ S to $N+$	W to $E-$ S to $N+$	W to $E-$ S to $N-$	W to E	S to N	Directions in degrees from N			
						90°	90°	270°	270°
360°	180°	180°	0°	0	1-0000	89°	91°	269°	271°
359°	181°	179°	1°	-0175	-9998	88°	92°	268°	272°
358°	182°	178°	2°	-0349	-9994	87°	93°	267°	273°
357°	183°	177°	3°	-0523	-9986	86°	94°	266°	274°
356°	184°	176°	4°	-0698	-9976	85°	95°	265°	275°
355°	185°	175°	5°	-0872	-9962	84°	96°	264°	276°
354°	186°	174°	6°	-1046	-9946	83°	97°	263°	277°
353°	187°	173°	7°	-1219	-9925	82°	98°	262°	278°
352°	188°	172°	8°	-1392	-9903	81°	99°	261°	279°
351°	189°	171°	9°	-1564	-9877	80°	100°	260°	280°
350°	190°	170°	10°	-1736	-9848	79°	101°	259°	281°
349°	191°	169°	11°	-1908	-9816	78°	102°	258°	282°
348°	192°	168°	12°	-2079	-9781	77°	103°	257°	283°
347°	193°	167°	13°	-2250	-9744	76°	104°	256°	284°
346°	194°	166°	14°	-2419	-9703	75°	105°	255°	285°
345°	195°	165°	15°	-2588	-9659	74°	106°	254°	286°
344°	196°	164°	16°	-2756	-9613	73°	107°	253°	287°
343°	197°	163°	17°	-2924	-9563	72°	108°	252°	288°
342°	198°	162°	18°	-3090	-9511	71°	109°	251°	289°
341°	199°	161°	19°	-3256	-9455	70°	110°	250°	290°
340°	200°	160°	20°	-3420	-9397	69°	111°	249°	291°
339°	201°	159°	21°	-3584	-9336	68°	112°	248°	292°
338°	202°	158°	22°	-3750	-9272	67°	113°	247°	293°
337°	203°	157°	23°	-3907	-9205				

Directions in degrees from N				S to N	W to E	W to $E-$ S to $N-$	W to $E+$ S to $N+$	W to $E+$ S to $N-$
336°	204°	156°	24°	-4067	-9135	66°	114°	246°
335°	205°	155°	25°	-4226	-9063	65°	115°	245°
334°	206°	154°	26°	-4384	-8988	64°	116°	244°
333°	207°	153°	27°	-4540	-8910	63°	117°	243°
332°	208°	152°	28°	-4695	-8829	62°	118°	242°
331°	209°	151°	29°	-4848	-8746	61°	119°	241°
330°	210°	150°	30°	-5000	-8660	60°	120°	240°
329°	211°	149°	31°	-5150	-8572	59°	121°	239°
328°	212°	148°	32°	-5299	-8480	58°	122°	238°
327°	213°	147°	33°	-5446	-8387	57°	123°	237°
326°	214°	146°	34°	-5592	-8290	56°	124°	236°
325°	215°	145°	35°	-5736	-8192	55°	125°	235°
324°	216°	144°	36°	-5878	-8090	54°	126°	234°
323°	217°	143°	37°	-6018	-7986	53°	127°	233°
322°	218°	142°	38°	-6157	-7880	52°	128°	232°
321°	219°	141°	39°	-6293	-7771	51°	129°	231°
320°	220°	140°	40°	-6428	-7660	50°	130°	230°
319°	221°	139°	41°	-6561	-7547	49°	131°	229°
318°	222°	138°	42°	-6691	-7431	48°	132°	228°
317°	223°	137°	43°	-6820	-7314	47°	133°	227°
316°	224°	136°	44°	-6947	-7193	46°	134°	226°
315°	225°	135°	45°	-7071	-7071	45°	135°	225°

TABLE V. *Average temperature at different levels for months.*1. *For British Isles. Taken from Geophysical Memoirs, No. 2 (W. H. Dines).*

Height kilo- metres	January	February	March	April	May	June	July	August	September	October	November	December
14	216	217	219	221	222	223	222	221	219	217	216	215
13	216	217	219	221	222	223	223	221	219	218	217	216
12	217	218	219	220	221	222	222	221	221	219	218	217
11	217	217	217	219	220	221	222	222	221	220	219	218
10	220	220	220	222	224	225	226	226	226	224	223	221
9	224	223	224	226	229	231	234	233	233	231	228	225
8	230	229	230	232	236	238	241	241	241	238	235	232
7	237	236	237	239	242	245	247	248	247	245	241	238
6	243	243	244	246	249	252	255	255	254	251	249	245
5	250	249	250	252	256	259	261	262	261	258	255	252
4	257	256	257	259	262	265	267	268	267	264	261	258
3	263	262	263	265	268	271	273	274	273	270	267	264
2	267	266	267	270	273	276	278	279	278	275	272	269
1	271	271	273	276	279	282	283	283	281	279	275	272
Gd.	276	276	277	282	285	288	289	289	286	283	280	277

2. *For England. Taken from Geophysical Memoirs, No. 5 (Ernest Gold).*

Height kilo- metres	January	February	March	April	May	June	July	August	September	October	November	December
16	218.0	217.0	222.3	—	222.7	222.3	225.0	224.1	222.1	210.5	220.3	218.3
15	216.4	217.0	225.8	—	230.1	222.3	225.0	223.7	221.1	211.2	220.4	216.8
14	214.9	212.8	223.5	220.6	218.6	220.3	226.7	222.1	221.4	211.7	220.4	218.8
13	214.7	213.0	221.9	222.6	218.4	219.4	226.2	221.3	220.9	213.4	223.5	219.0
12	215.3	214.6	219.3	223.1	218.1	217.2	226.0	219.9	221.2	217.0	224.0	218.4
11	217.3	215.5	217.7	220.8	218.4	217.1	225.7	221.5	221.6	221.2	224.9	219.3
10	221.1	219.8	220.5	221.0	220.9	221.8	227.6	227.5	225.4	227.6	226.5	220.8
9	227.0	225.3	224.9	222.5	228.2	229.1	231.7	235.0	231.4	235.4	229.3	222.7
8	233.5	231.5	229.7	227.6	235.5	237.4	238.4	242.4	238.3	242.5	231.9	225.4
7	240.0	238.8	236.2	233.1	243.4	245.5	245.9	249.5	246.0	249.8	236.7	230.9
6	246.5	245.8	242.7	239.7	251.8	253.2	252.6	256.6	253.1	256.9	244.2	238.4
5	252.5	252.6	249.6	246.8	258.2	260.1	259.7	263.4	259.7	263.7	252.6	245.9
4	259.0	259.4	255.8	253.8	265.1	266.6	265.6	269.5	265.8	269.8	259.3	253.6
3	265.1	265.4	262.2	260.2	270.6	272.3	271.3	274.4	271.8	275.5	265.2	260.7
2	270.3	270.5	267.8	266.1	276.1	277.6	277.1	278.9	277.1	281.3	270.7	266.4
1	273.9	275.0	271.5	271.4	281.2	282.7	281.7	283.8	281.8	285.6	275.1	270.8
Gd.	276.4	275.0	278.3	279.4	284.9	286.8	287.3	288.7	285.8	288.1	280.1	275.7

TABLE VI. *Average values of pressure and temperature at different levels over high pressure (1033 mb.) and low pressure (982 mb.) at the surface; with pressure-differences and temperature-differences at each level. Compiled from the diagram and tables of W. H. Dines, F.R.S., in M. O. 210 b.*

Pressure			Diff.	Diff.	Temperature	
k.	mb.	mb.	Δp mb.	$\Delta \theta$ °A	°A	°A
15	116	123	7		"Low"	"High"
14	139	146	7	- 9	224	215
13	162	170	8	- 11	226	215
12	186	194	8	- 8	225	217
11	215	237	22	- 4	225	221
10	255	279	24	+ 1	225	226
9	293	323	30	+ 7	226	223
8	341	372	31	+ 13	227	240
7	391	421	30	+ 15	232	247
6	450	482	32	+ 14	240	254
5	518	550	32	+ 13	248	261
4	591	626	35	+ 12	255	267
3	644	712	38	+ 9	263	272
2	762	813	51	+ 8	269	277
1	864	915	51	+ 4	275	279
0	982	1033	51	+ 3	279 984 mb.	282 1031 mb.

Standard deviation of P_9 13·8 mb.

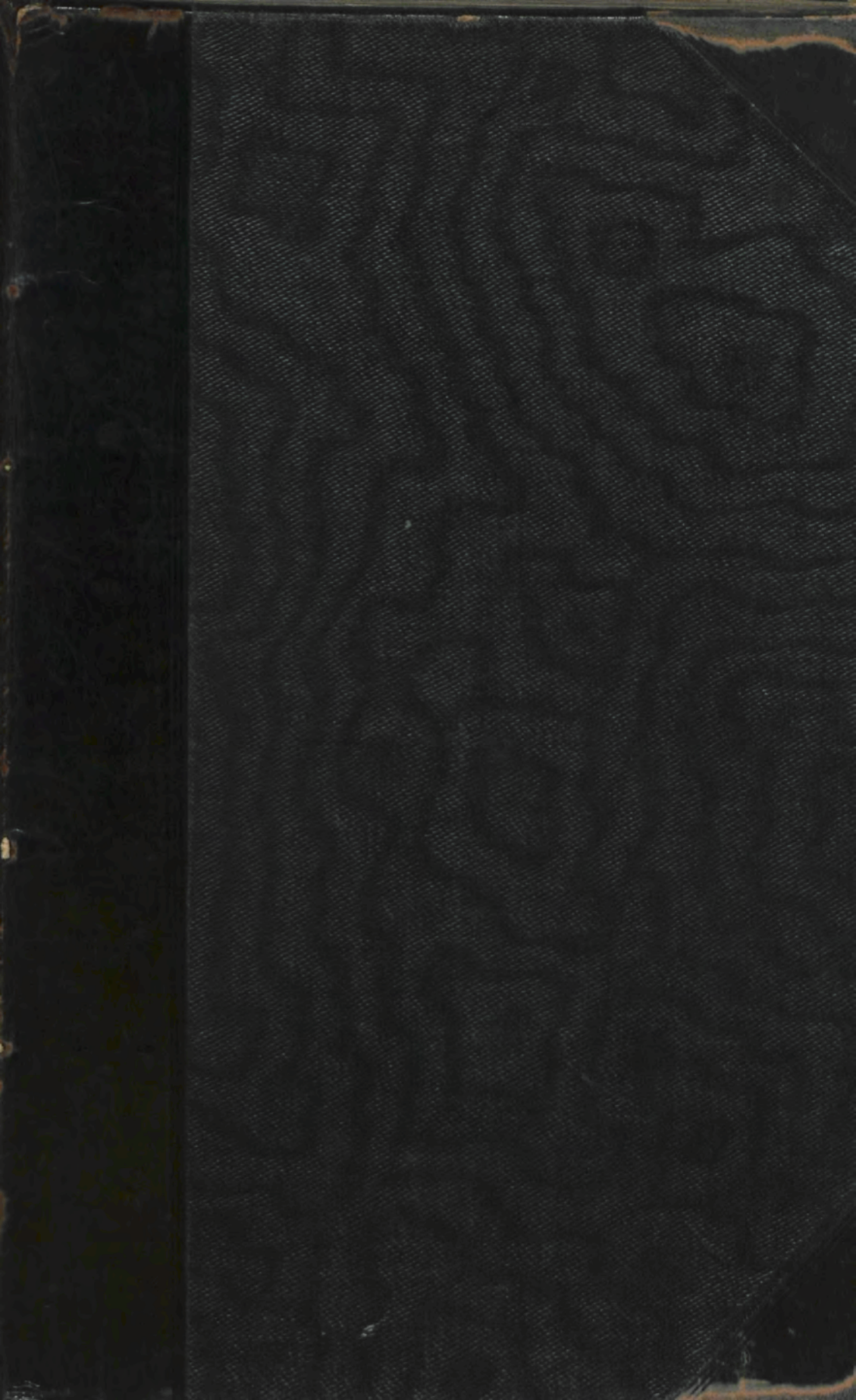
Standard deviation of P_8 14·1.

Correlation coefficient between the variations of P_9 and P_8 from the means for the month (English ascents) ·80.

W. N. S.

METEOROLOGICAL OFFICE,
Nov. 22, 1913.

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TABLE OF CONTENTS.

UNITS OF MEASUREMENT IN METEOROLOGY.

International Meteorological Tables.

UNITS OF MEASUREMENT WITH THEIR ABBREVIATIONS AND THEIR EQUIVALENTS	PAGE
Measures of length	3
— area	5
— volume	6
— mass	6
— density	7
— angle and time	7
Geodetic Measures	8
Duration of sunshine in various latitudes	9
Measures of velocity	9
— acceleration	9
— force	9
— work and energy	10
— rate of working	10
— geopotential	10
— solar radiation	10
Measures of temperature	10
Reduction of temperature to sea level	11
Measures of quantity of heat	12
Measures of pressure	13
Reduction of barometric readings to the freezing point of water	13
— — — — — to sea-level	14
Hygrometric Measures	14
Measures of wind	15
Resultant wind : Lambert's formula	15
Wind Roses	16
TABLES FOR COMPUTATION	...
Numerical Computation : Numbers 1 to 99 with their reciprocals, quarter-squares, squares, and square roots ; cubes and cubic roots	17
Length of 1° of longitude in different latitudes in metres, nautical miles, and statute miles	19
Logarithms of numbers to four figures	20
Trigonometric computation	22
Trigonometric ratios	22
Traverse tables of components	23-32
CONVERSION TABLES	33-52

NOTE.—The Director of the Meteorological Office will be much obliged if any numerical errors detected in the tables may be pointed out to him, as well as any omissions of data necessary for Meteorological Computation.

May 13th, 1916,

THE COMPUTER'S HANDBOOK.

INTRODUCTION.

UNITS OF MEASUREMENT IN METEOROLOGY.

It has come to be recognised at the Meteorological Office, for reasons which are set out in the preface to the Seaman's Handbook of Meteorology and elsewhere, that the most suitable units for the expression of the Meteorological elements are those based upon the C.G.S. (centimetre-gramme-second) system with temperatures expressed in degrees upon the scale to which the name of Kelvin is often given, which takes for its zero the datum-point of the absolute thermodynamic scale and which uses the centigrade degree as the interval of successive steps of temperature. For practical purposes the scale is the same as that of the hydrogen thermometer starting from a point 273 degrees centigrade below the freezing point of water.

In order to give effect to this conclusion it is necessary to change the ordinary practice of British meteorologists and to express pressures in C.G.S. units of pressure, dynes per square centimetre or in some unit derived therefrom, as the *thousand dynes per square centimetre* known as a millibar (or by some meteorologists and chemists as a kilobar) instead of the inch, temperatures in the Kelvin scale instead of that of Fahrenheit, wind velocities in metres per second instead of miles per hour or feet per second, the depth of rainfall in millimetres instead of inches or "points," heights above sea level in metres and so on. In this way the out-door practice of meteorologists will conform, with certain modifications on scientific lines, to the conventions which are prescribed but not always practised for the measurement of physical quantities in the laboratory.

This is by no means the first change of practice that has come within the experience of meteorologists, and those of us who endeavour to bring the older records into line with modern observations must be prepared to interpret figures which *prima facie* convey no meaning to the modern observer. The study of weather is older than the millimetre, and it would be a considerable loss to the science to write off as unintelligible what can be interpreted with some knowledge of the practice of the past.

The basis of numerical computation for students of meteorology has been set out authoritatively by the International Meteorological Committee in a publication in French, English, and German, dated at Paris and St. Petersburg on 1st June, 1890. It is entitled *International Meteorological Tables published in Conformity with a Resolution of the Congress of Rome, 1879*. The work was carried out mainly at the Bureau Central Météorologique of Paris under the superintendence of Monsieur E. Mascart, the Director of the Bureau, and Monsieur Chauveau, *Météorologiste Adjoint*, and published by Gauthier-Villars et Fils, Quai des Grands-Augustins 55, Paris.

This publication includes a number of meteorological computations and tables:—for the variation of gravity with latitude and altitude, the length of 1° of meridian in various latitudes and of 1° of longitude in various parallels, the duration of sunshine in various latitudes, the reduction of barometer readings to 32° and to sea level, including the influence of gravity upon barometrical measurements, the pressure of aqueous vapour, weight of aqueous vapour in a cubic metre of saturated air, relative humidity computed from the dew point, the mean direction of the wind by Lambert's formula, and the conversion of measures of magnetic force from the British units previously employed to C.G.S. units and *vice-versâ*, and it is prefaced by and amplified by tables of conversion between the fundamental standards and divers British units and metric units; and also between some of the measures which had been in practical use in the past but which had become obsolete or were becoming so with the gradual extension of the metric system.

It has been the object of the meteorological authorities in all countries that the numerical values given in this publication should form the basis of all tables of conversion and computation which are employed by observers and students throughout the world, so that the meaning of any small differences in results should be freed from ambiguity on account of the process of computation.

Meteorological questions nearly always depend for their solution on the comparison of results from different parts of the world and comparability is often more important than extreme numerical precision, so that the values used in the computation of the International Tables are sufficiently accurate for all the computations of meteorological practice and will remain so for many years to come, but in the meantime alterations in the accepted comparisons

of fundamental standards of the various countries may receive, and some have received, the sanction of law, and it is hardly permissible to an official to display, or affect to use, tables which are ostensibly based on equivalents which are not lawful.

The suggestion is sometimes lightly made that a new edition of the tables is required to bring them up to date, but the recomputation of tables is a work of great labour without any justification in the results to be achieved.

Still, the progress in metrological exactitude which naturally follows the establishment of such institutions as the Bureau International des Poids et Mesures, the Reichsanstalt, the National Physical Laboratory, and the Bureau of Standards must not be disregarded; and the horizon of meteorological computation has been much extended by the development of the study of the upper air which requires the tables for computation to be similarly extended over ranges which were outside the meteorological practice of 1890. The present work affords a means of reviewing our position with regard to such matters without undertaking the burden of re-editing the International Tables. We shall therefore quote here from the introduction to that publication the bases of computation therein adopted, and where revised *fundamental* numerical relations have been formally adopted by the legislature a note of the changes will be made, and any tables which are affected by the alteration and which are set out in detail will take account of the alteration.

The quotations from the International Tables referring to units of measurement, now obsolete, will be supplemented by information of the same kind derived from other sources.

The values taken from the International Tables are marked with an asterisk.*

UNITS OF MEASUREMENT WITH THEIR ABBREVIATIONS AND THEIR EQUIVALENTS.

Measures of Length. (International Tables, Chapter I., Section I.)

Metric Units.

	at 0° C.	at 62° F.
metre :	1 m.	= *39·37079 in. (Kater 1818).
		= *3·28089917 ft.
kilometre :	1 k.	= *0·6213824 mi.
C.G.S. centimetre :	1 cm.	= *0·3937079 in.
millimetre :	1 mm.	= *0·03937079 in.

British Units.

	(at 62° F.)	(at 0° C.)
mile : 1 mi.	=	*1609·3149 m.
yard : 1 yd.	=	*0·91438348 m.
foot : 1 ft.	=	*30·479449 cm.
inch : 1 in.	=	*25·39954 mm.

By Order in Council of May 19, 1898, the statutory equivalent between the standard metre and the British standard yard now gives

metre : 1 m.	=	1·093614 yard.
1 m.	=	39·3701 in.
yard : 1 yd.	=	·914399 m.
inch : 1 in.	=	2·54000 cm.

Ancient French Units.

1 toise = 6 ft.	=	*1·9490366 m.	=	*2·1315305 yd.
1 foot = 12 inches	=	*0·3248394 m.	=	*1·0657653 ft.
1 inch = 12 Paris lines	=	*27·069953 mm.	=	*1·0657653 in.
1 line	=	*2·255829 mm.	=	*0·0888138 in.

Russian Measures.

1 verst = 106678 cm. (E.)

(For barometric heights.) 1 demiligne = *05 in. = *1·269977 mm. at 13 $\frac{1}{2}$ ° R. or 62° F.

NOTE : For the measurement of pressure in Russia the unit of a demiligne ($\frac{1}{10}$ in.) of mercury at 62° F. was employed.

Special Tables for the conversion of the old Russian barometric tables into English and French measures will be found in International Meteorological Tables, Section I., Chapter IV.

Measures of Area.**Metric Units.**

C.G.S. Square centimetre (sq. cm. or cm ²).	
1 cm. ²	= ·1550 in. ²
	= ·001076 ft. ²
	= ·0001196 yard ² .

British Units.

1 in. ²	= 6·4516 cm. ²
1 ft. ²	= 929·03 cm. ²
1 yard ²	= 8361·3 cm. ²

Measures of Volume.**Metric Units.**

C.G.S. Cubic centimetre (cc. or cm. ³).	
1 cc.	= ·0610 c. in.
1 litre = 1000 cc.	= ·03531 c. ft.
	= 1·7598 pint.
	= ·2200 gallon.

British Units.

1 c. in.	= 16·387 cc.
1 c. ft.	= 28·317 litre = 28317 cc.
1 pint	= ·5682 „
1 gallon	= 4·5460 „

Measures of Mass. (International Tables, Chapter I., Section II.)**Metric Units.**

Kilogramme : 1 kg. = *15432·350 gr. Troy. (W. H. Miller, 1844-46, and Broch, 1883).
= 2·204621 lb.

C.G.S. Gramme : 1 g. = 15·432350 gr.

Metric Tonne : 1 t. = 1000 kg. = 2204·621 lb.

British Units.

Pound : 1 lb. = 453·59 g. (Everett, C.G.S. Units and Constants.)
Do. do.
Ounce (Avoir.) : 1 oz. = 28·3495 g.
Ounce (Troy and Apothecary) : 1 $\bar{5}$ = 31·1034912 g.
Grain : 1 gr. = *0·06479894 g.
Ton : 1 ton = 1·01605 $\times 10^6$ g. (Everett.)

By Order in Council, May 19, 1898, the statutory equivalent between the pound and the kilogramme is 1 lb. = ·45359243 kg. giving 1 kg. = 2·2046223 lbs.

Measures of Density.

C.G.S. Gramme per cubic centimetre. (g/cc.)

1 g/cc. = 62·43 lb/c ft.

British Units.

1 lb/c ft. = ·01602 g/cc.

The density of mercury at the normal freezing point of water is 13·5955 g/cc.

The density of dry air varies with the pressure and the temperature in accordance with following formula—

$$\Delta = \frac{p}{p_0} \times \frac{T_0}{T} \times \Delta_0.$$

From the observations of Regnault, at 760 mm. and 273a. the value of Δ_0 is *1292·78 g/m³, from which we compute the following table:—

Pressure.	Temperature.	Density.
mb.	a.	g/m ³ .
1,000	100	3483·21
"	200	1741·60
"	250	1393·28
"	275	1266·62
"	278·66	1250·00
"	284	1226·48
"	284·11	1226·00
"	290·27	1200·00
"	300	1161·07

For the purpose of meteorological computation we may take the standard density as 1201 g/m³ at 1000 mb. and 290a.

Measures of Angle and Time. (International Tables, Chapter I., Section III.)**Angles, &c.**

Length of circumference of circle \div diameter, $\pi = 3·14159265$
log $\pi = \cdot 49715$
 $1/\pi = \cdot 318309886$
log (1/ π) = 1·50285

Radian.

$\pi r = 180^\circ$
1 r = 57·29578°
1° = ·017453 r.

Time.

Year : 1 year = *365·2422166 mean solar days.
 1 mean solar day = *0·002737909 year.
 1 sidereal day = ·99727 mean solar day.
 If 1 year = 360°, 1 mean solar day = 0° 59' 8·33".
 1 week = 6° 53' 58".
 30 days = 29° 34' 10".
 Hour : 1 hr. = *0·0001140795 year.
 = *0° 2' 27·847".
 Minute : 1 min. = *0·00000190132 year.
 = 2·464".
 Second : 1 s. = *3·168866 × 10⁻⁸ year.
 1 s. = ·041066".

Length of the seconds pendulum at London = 39·13929 in.

NOTE.—The symbols ' and " should be reserved for minutes and seconds of angle exclusively and not used for minutes and seconds of time nor for feet and inches. It is also desirable that the symbol ° should be reserved for degrees of angle; the practice of using it for degrees of temperature is archaic but quite undesirable, because no statement of temperature is complete without a specification of the scale. For this purpose the abbreviation *a* will be regarded as sufficient in the case of absolute temperature without a degree sign.

Rotation of the Earth: relative to a star $\omega = \cdot 00007292$ r/s.
 relative to the sun, 1 hr. = 15°.
 1° = 4 min.

Geodetic Measures. (International Tables, Chapter II.)

Equatorial semi-axis = *6377397 m. (Bessel)
 = *6378253 m. ± 75 m. (Clarke)
 = *6378393 m. ± 79 m. (Faye)
 = *6378238 m. (Fischer)
 Polar semi-axis = *6356079 m. (Bessel)
 = *6356521 m. ± 111 m. (Clarke)
 = *6356549 m. ± 109 m. (Faye)
 = *6356230 m. (Fischer)
 Three axes. $a = *6378294$ m. (Clarke's Ellipsoid)
 $a^1 = *6376350$ m.
 $b = *6356068$ m.
 Nautical mile = (1) mean length of arc of one minute of latitude
 which varies from 1842·7 m. at the Equator
 to 1861·3 m. at the poles.
 = 1852 m. (Annuaire du Bureau des Longitudes)
 or = (2) mean length of arc of a minute of the great
 circle of a sphere with surface equal to that
 of Clarke's Ellipsoid.
 = *1853·152 m. (Admiralty).
 1 kilometre = 0·5396212 nautical mile.
 90 degrees of latitude = 10,007,011 m.

Gravity. $g = *980·617 (1 - \cdot 00259 \cos 2 \lambda) \left(1 - \frac{5}{4} \frac{h}{E}\right)$

where λ is the latitude, h the height above sea level, E the earth's radius.

This formula takes into account the additional attraction of the high ground and supposes the mean density of the elevated area to be equal to one half of the mean density of the earth.

Value of g at sea level in London = 981·19 cm/s².
 Mean density of earth = about 5·5 g/cc.
 Mean density of surface of earth = 2·65 g/cc.
 Volume of earth = 1·082 × 10²¹ m³ (K. and L.).
 Mass of earth = 5·98 × 10²⁷ g.
 = 5·87 × 10²¹ tons.
 Area of land = 1·45 × 10¹⁸ cm².
 Area of ocean = 3·67 × 10¹⁸ cm².
 Mean depth of ocean (Murray) = 3·85 × 10³ cm.
 Volume of ocean = 1·41 × 10²⁴ cc.
 Mass of ocean = 1·45 × 10²⁴ g.
 Mass of the atmosphere = 5·34 × 10²¹ g.

Duration of Sunshine in Various Latitudes.

Definition of sunrise: The time at which the centre of the sun, owing to refraction, would just appear on the horizon.

$\sin \frac{1}{2}$ (semi-course of the sun) =

$$\sqrt{\frac{\sin\left(\frac{\pi}{4} + \frac{\lambda - D + r}{2}\right) \sin\left(\frac{\pi}{4} - \frac{\lambda - D - r}{2}\right)}{\cos \lambda \cos D}}$$

where λ is the latitude, D the northerly declination of the sun, r the apparent elevation due to refraction.

The mean value of r (corresponding to $t = 283a$. and $H = 1013$ mb) is 33' 47" and may be taken as 34'.

The sun's declination may be taken from the Meteorological Calendar.

Diagrams showing the position of the sun at each hour of the day at stated times of the year for latitudes 50°, 55° and 60°, but taking no account of refraction are given in the Observer's Handbook.

Measures of Velocity. (Everett: 1 metre = 39·370 in.)**Metric Units.**

C.G.S. centimetre per second : 1 cm/s = ·032809 ft/s.
 = ·022369 mi/hr. = ·019435 knot.
 metre per second : 1 m/s = 3·2809 ft/s.
 = 2·2369 mi/hr. = 1·9435 knot.

British Units.

foot per second : 1 ft/s. = 3·04797 m/s.
 mile per hour 1 mi/hr. = ·44704 m/s.
 knot 1 knot = ·51453 m/s.

Measures of Acceleration.

C.G.S. centimetre per second per second : 1 cm/s² = ·032809 ft/s².
 decametre per second per second : 1 leo. (= 1000 cm/sec.²)
 = 32·809 ft/s².

Measures of Force. (Everett.)**Metric Units.**

C.G.S. gramme centimetre per second per second :
 1 dyne = weight of ·00102 g.
 kilogramme weight : 1 kg. = ·981 × 10⁵ dynes.

British Units.

1 pound weight = 4·45 × 10⁵ dynes.
 1 poundal = 13825 dynes.

Measures of Work and Energy. Everett ($g=981$).**British Units.**

Footpound : 1 ft. lb. = 13825 g.cm. = 1.3562×10^7 ergs.
 Foot poundal = 4.2139×10^5 ergs.
 Foot ton = 3.097×10^7 g.cm. = 3.0380×10^{10} ergs.

Metric Units.

C.G.S. gramme (centimetre per second)².
 1 erg = 2.3731×10^{-6} foot poundals.
 1 joule = 10^7 ergs.
 1 kilogrammetre = 10^5 g.cm. = 9.81×10^7 ergs.

NOTE.—The equivalent of the foot pound in C.G.S. units is dependent upon the geographical position at which the comparison is made because the value of gravity which enters into the computation varies with latitude.

Measures of Rate of Working.

C.G.S. Erg per second, i.e. gramme (centimetre per second)²/second.
 1 watt = 10^7 ergs per second.
 1 horse power = 7.46×10^9 ergs per second.
 1 force de cheval = 7.36×10^9 ergs per second.

Measures of Geopotential.

C.G.S. 1 (c/s)². 1 leometre = 10^5 (c/s)².

Measures of Solar Radiation.

The intensity of solar radiation may also be measured in these units by indicating the energy received in one second upon a square centimetre exposed to the radiation.

Thus at the Meteorological Office the amount of vertical radiation from sun and sky is measured in milliwatts per square centimetre. The relation to the usual unit is—

1 milliwatt per square centimetre = .01435 gramme calories per square centimetre per minute.

1 gramme calorie per square centimetre per minute = 69.7 milliwatts per square centimetre.

1 British Thermal unit = 252.00 gramme calories.

The "solar constant" is the strength of the solar heat stream at the outer boundary of the earth's atmosphere.

Mean value of solar constant = 1.93 g. cal. per cm² per min. (Abbot). = 135 milliwatts per cm². = 32.4 kilowatt hours per m² per diem. = 11673 joules per cm² per diem.

Measures of Temperature. (International Tables, Chapter III., Section I.)**Absolute Scale.**

Equivalent intervals 1 a = 1° C. = 1.8° F. = 1.6° R.

The most recent computations of the position of the absolute zero in the centigrade scale of the hydrogen gas-thermometer place it at -273.02°. Taking the starting point of the absolute scale as -273° C. we get the following equivalents between the expression of the same temperature on the absolute scale A, the Fahrenheit scale F, the centigrade scale C, and the Réaumur scale R.

$$A = \frac{5}{9} (459.4 + F) = 273 + C = \frac{5}{4} (218.4 + R).$$

Fahrenheit Scale.

Freezing point of water under standard conditions 32° F.

Boiling point of water under standard conditions (1013.231 mb) 212° F.

$$F = \frac{9}{5} A - 459.4 = 32 + \frac{9}{5} C = 32 + \frac{9}{4} R.$$

The Fahrenheit scale as originally devised ran from 0° the lowest temperature then reached (the temperature of a mixture of ice and salt) to 8° the temperature of the human body. Subsequently each degree was divided into twelve parts. Then the boiling point of water under a pressure 29.922 (760 mm.) inches of mercury, 212°, and the freezing point of water, 32°, were taken as defining the scale.

Centigrade Scale.

Freezing point of water under standard conditions 0° C.

Boiling point of water under standard conditions 1013.231 mb.

(760 mm. mercury at 0° lat. 45°) 100° C.

$$C = A - 273 = \frac{5}{9} (F - 32) = \frac{5}{4} R.$$

9 (C - 10) = 5 (F - 50) is convenient for ordinary air temperatures.

Réaumur Scale.

Freezing point of water under standard conditions, 0° R.

Boiling point of water under standard conditions 1013.231 mb.

(760 mm. mercury at 0° lat. 45°) 80° R.

$$R = \frac{4}{5} (A - 273) = \frac{4}{9} (F - 32) = \frac{4}{5} C.$$

This scale was until recently the customary scale on the continent for domestic and economic purposes, but it is now generally replaced by the centigrade scale. The editors of the International Tables regarded it as so little in use that it was not necessary to give conversion tables.

Other Scales.—The older books and tracts on meteorological subjects often give temperatures in figures without specifying the method of graduation, and in consequence it is difficult to interpret the readings in terms of the scales with which we are familiar. Accordingly it may be well to mention here some of the various scales whose scheme of graduation is known. Those which have come to the notice of the Office in manuscript records of meteorological observations are De l'Isle's Scale, in which the boiling point of water is zero and the freezing point 150°, and a scale in which temperatures were measured above and below "temperate" (60° F.) in a scale which had apparently two degrees for each degree Fahrenheit. The temperate mark which is common to many thermometers is apparently 13° C. (55.4° F.).

Sir Isaac Newton's Scale (Phil. Trans. 1701) the freezing point of water is 0° and the temperature of the human body is 12°.

Reduction of Temperature to Sea Level. (International Tables, Chapter III., Section II.)

The tables on this subject included in the International Meteorological tables are empirical tables based upon the assumption that on the average the rate of fall of temperature with height is proportional to the height with a coefficient which must be determined by observation. The coefficients provided for in the tables range from 1° C. in 500 m. to 1° C. in 100 m. and from 1° F. in 1,000 ft. to 1° F. in 200 ft., and therefore include the values appropriate for France—viz., in Spring and Autumn :—

A fall of temperature of 1° C. for 180 m.
 1° C. for 200 m.
 In Winter 1° C. for 160 m.
 In Summer 1° F. for 300 ft.

and the value commonly used in England—viz., 1° F. for 300 ft.

Other coefficients have been recently introduced in the preparation of the climatic tables of the Meteorological Office, of which the metric equivalents are given in the following table :—

TABLE showing ELEVATION in metres corresponding with the REDUCTION of the normal maximum and minimum temperatures of each month and of the year by one tenth of a degree of the centigrade scale.

	England, Wales and Ireland.		Scotland.	
	Max.	Min.	Max.	Min.
January	14 m.	14 m.	22 m.	16 m.
February	11 "	14 "	22 "	16 "
March	11 "	14 "	16 "	16 "
April	11 "	11 "	16 "	14 "
May	11 "	11 "	19 "	14 "
June	11 "	14 "	25 "	16 "
July	11 "	14 "	25 "	16 "
August	11 "	14 "	22 "	16 "
September	11 "	16 "	19 "	16 "
October	11 "	16 "	16 "	19 "
November	11 "	16 "	16 "	19 "
December	14 "	14 "	19 "	16 "
Year	12 "	14 "	20 "	16 "

Measures of Quantity of Heat.

C.G.S. 1 gramme-calorie, the heat required to raise the temperature of 1 gramme of water by 1°C from t°C. The calorie for 20° (T=293a)=4.180 × 10⁷ ergs: The calorie for 15° (T=288a)=4.184 × 10⁷ ergs.

The mean calorie, one hundredth of the heat required to raise one gramme of water from 273a to 373a,=4.184 × 10⁷ ergs. British thermal unit (lb. F) = 252.00 calories.

Capacity for Heat or *Thermal capacity* of a material, the number of Calories which will raise the unit mass of the material through one degree of temperature.

The capacity for heat of one gramme of water at various temperatures, interpolated from Kaye and Laby.

Temperature.	Capacity for heat.	Dynamical Equivalent.	Temperature.	Capacity for heat.	Dynamical Equivalent.
a.	Calories.	Joules.	a.	Calories.	Joules.
270	1.0130	4.234	290	1.0007	4.183
273	1.0094	4.219	293	1.0000	4.180
275	1.0076	4.212	295	.9997	4.179
280	1.0042	4.198	300	.9990	4.176
285	1.0021	4.189	305	.9985	4.174

Specific Heat of a substance is the ratio of the capacity for heat of the substance to the capacity for heat of water at a standard temperature and therefore numerically the same as the capacity for heat in Calories.

Some specific heats.

Water, numerically the same as the capacity for heat (see Table above).

Sea Water at 290a	...	0.94.
Ice at 260a	...	0.502.
Dry Air at constant pressure	...	0.2417.
" at constant volume	...	0.1715.
Water Vapour at constant pressure	...	0.4652.
" at constant volume	...	0.340.

Latent Heat of one gramme of water, 79.77.
steam, 597 at 273a, 539 at 373a.

Measures of Pressure. (International Tables, Chapter IV., Section I.) Observer's Handbook: 1 in. = 2.54000 cm.

C.G.S. dyne per square centimetre = 1 microbar = 2.95306 × 10⁻⁵ mercury inches = 7.50076 × 10⁻⁴ mercury millimetres.
millibar: 1 mb. = 0.0295306 mercury inches = 0.750076 mercury millimetres.

Mercury inch: 1 mercury inch at 32° F. in latitude 45° = 33.8632 mb.

Mercury millimetre: 1 mm. at °C. in latitude 45° = 1.333200 mb.
760 mm. " " = 1013.231 mb

Russian half-lines (normal at 62° F.).
600 half-lines = 759.68 mm. = 1012.804 mb.
1 half-line = 1.668801 mb.

British pound per square inch in London.
1000 mb. = 14.496 lb./in.² in London.

Reduction of Barometric Readings to the Freezing Point of Water. (International Tables, Chapter IV., Section II.)

Coefficient of cubical expansion of mercury per degree centigrade.
Mean value °0.0001818.

Coefficient of expansion of mercury per degree Fahrenheit.
Mean value °0.0001010.

Coefficient of linear expansion of brass per degree centigrade.
Mean value °0.0000184.

Coefficient of expansion of brass per degree Fahrenheit.
Mean value °0.0000102.

Formulae for Correction.*

$$\text{Metric measures } C = \frac{0.0001634}{1 + 0.0001818 t} \times t \times H.$$

$$\text{British measures } C = \frac{0.0000908 (t - 32) + 0.000306}{1 + 0.0001010 (t - 32)} H.$$

These formulae are applicable in the case of barometers of the Fortin or Newman pattern in which the reading is made directly upon a linear scale without any adjustment of the graduation on account of the capacity of the cistern. In barometers of the Kew pattern in which the graduation is adjusted to allow for the capacity of the cistern a modification is necessary on account of the expansion of the cistern. The modification will depend on the dimensions of the tube and cistern. With the sizes usual in the practice of the Meteorological Office the computation is modified by taking .0001050 as the coefficient of expansion of mercury relative to glass in the barometer.

In consequence of this modification the correction for temperature of a Fortin barometer must be increased by one-twentieth of its own magnitude to give a correction appropriate for a Kew pattern barometer. For this reason it is desirable that the Kew pattern barometer should be so adjusted as to give true readings in millibars at the mean of the ordinary temperatures at which it is read. When the graduation is accurate at 28° F. as in the case of the Kew barometer graduated in inches, one-twentieth of the correction is generally appreciable.

Reduction of Barometric Readings to Sea Level.*

(International Tables, Chapter IV., Section II.)

Laplace (*Mécanique Céleste* 2^e Partie Livre IX., chap. IV.) gives the following formula—

$$Z = K (1 + a\theta) \frac{1}{1 - k \cos 2\lambda} \left(1 + \frac{Z}{R}\right) \log \frac{p_0}{p},$$

in which—

Z is the altitude of the station above the sea level.

λ its latitude.

R the mean terrestrial radius.

a the coefficient of expansion of air.

θ the equivalent mean temperature of the air between the station and a place supposed to be situated in the same vertical at sea level.

p and p_0 the atmospheric pressures at the two points.

K a constant called the barometric constant.

k is the constant of variation of gravity with latitude (see p. 8) and is numerically 0.00259.

K is given by the formula—

$$K = \Delta \times H / (a \log e),$$

where H is the normal barometric height 76 cm.

Δ is the density of mercury at 273a.

a the density of air at 273a and normal pressure.

$\log e$ the modulus of common logarithms.

whence $K = 18,400$ metres or 60,370 feet.

For the details of the application of these formulæ to the preparation of a table for the reduction of barometric readings to sea level reference should be made to the International Meteorological Tables or to Section I., § 2, of this handbook or to the Observer's Handbook, p. 121.

The practical application of the formulæ to the computation of heights from the records of balloon ascents is given in this Handbook, Section II., § 2.

Hygrometric Measures. (International Tables, Chapter V.)

Pressure of Aqueous Vapour.

Observations of dew point give the temperature of saturation of the air. The corresponding vapour pressure is given in the International Meteorological Tables by a table calculated by Dr. Broch from the classical observations of Regnault. A brief table is given in this Handbook, Section I., § 1.

Percentage Relative Humidity.

The relative humidity is defined as the ratio of the weight of vapour contained in a cubic metre of the air under observation to the weight which it would contain if saturated with water vapour without changing its temperature.

To obtain this ratio we require the saturation pressure of aqueous vapour at the temperature of the dew point and at the temperature of the air; both these can be obtained from the table of saturation pressures and obtaining their ratio is a simple numerical operation. We may represent it by π_{dp} / π_t . The percentage relative humidity is then obtained by multiplying by 100.

Thus Relative Humidity = $100 \pi_{dp} / \pi_t$.

Wet and Dry Bulbs.

Tables for obtaining the relative humidity and the vapour pressure from readings of the wet and dry bulbs are given in this Handbook, Section I., § 1.

the value of Δ is 1292.78 g/m^3 . Generally quoted in physical tables as 1293.052 g/m^3 .

At 1000 mb. and temperature 269.43a the figure is the same. See Section I., § 3.

Measures of Wind. (International Tables, Chapter VI.)

Wind may be measured by noting the direction in which the current of air is moving, defining the direction by the orientation, and by measuring the velocity, or the force on a measured area, by means of an anemometer or by assigning a number on the Beaufort scale by estimation. (See Observer's Handbook, p. 39.) If the observations of wind are to be subjected to numerical computation they should all be expressed in terms of velocity, preferably in metres per second, by the conversion tables which are given in the Handbook referred to.

Resultant Wind.

Lambert's formula:—Calculation of mean direction and force from a group of observations to eight points.

Lambert's formula (*Nouveaux Mémoires de l'Académie Royale des Sciences et Belles-Lettres*: Berlin, 1777) expresses the angle a which the direction of a wind equal to the resultant or geometrical sum of all the observed winds make with the N.S. line.

For that purpose the sum of the components directed towards the West and towards the South must be obtained and their ratio taken.

If M_N is the sum of the measures of wind from the North.

North-East, and so on.

Then the sum of the "components" directed towards the West is

$$\Sigma_E = M_E - M_W + (M_{NE} + M_{SE} - M_{NW} - M_{SW}) \cos 45^\circ.$$

And the sum of the components directed towards the South is

$$\Sigma_N = M_N - M_S + (M_{NE} + M_{NW} - M_{SE} - M_{SW}) \sin 45^\circ.$$

And the angle a is given by

$$\tan a = \Sigma_E / \Sigma_N.$$

The quadrant to which the resultant is to be referred will depend upon the signs of the numerator and denominator. Thus if both Σ_E and Σ_N are positive the resultant wind is from some point between North and East, if Σ_E is positive and Σ_N negative the resultant wind is between East and South. If both components are negative it is between South and West, and if Σ_E is negative and Σ_N positive it is between North and West.

The resultant wind is obtained, as in all cases of geometrical summation, by taking the square root of the sum of the squares of the components:—Thus the resultant wind

$$W = \sqrt{(\Sigma_E^2 + \Sigma_N^2)}.$$

These formulæ are only strictly applicable when the wind is expressed in terms of velocity, that is in metres per second or in some other unit of velocity. When the wind measures are in Beaufort numbers they ought first to be converted into velocity

Erratum.

page 15, line 3. The formula should read

$$w = \Delta \delta \cdot \frac{\pi}{P_0} \cdot \frac{T_0}{T}$$

where P_0, T_0 are the standard pressure and temperature.

R the mean terrestrial radius.

α the coefficient of expansion of air.

θ the equivalent mean temperature of the air between the station and a place supposed to be situated in the same vertical at sea level.

p and p_0 the atmospheric pressures at the two points.

K a constant called the barometric constant.

k is the constant of variation of gravity with latitude (see p. 8) and is numerically 0.00259.

K is given by the formula—

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To obtain this ratio we require the saturation pressure of aqueous vapour at the temperature of the dew point and at the temperature of the air; both these can be obtained from the table of saturation pressures and obtaining their ratio is a simple numerical operation. We may represent it by π_{dp} / π_t . The percentage relative humidity is then obtained by multiplying by 100.

Thus Relative Humidity = $100 \pi_{dp} / \pi_t$.

Wet and Dry Bulbs.

Tables for obtaining the relative humidity and the vapour pressure from readings of the wet and dry bulbs are given in this Handbook, Section I., § 1.

Weight of Vapour in the Air.—The weight w of vapour in a cubic metre of air may be computed from the formula

$$w = \Delta \delta / p \times \pi / T$$

where Δ is the weight of 1 cubic metre of dry air under standard conditions of pressure and temperature, and δ is the ratio of the density of aqueous vapour to air at the same temperature. This may be taken as equal to .622. π is the pressure of aqueous vapour in the air and p is the pressure of the atmosphere and t its temperature. T is the temperature on the absolute scale.

For the standard conditions 1013.231 mb. (760 mm.) and 273a, the value of Δ is .1292.78 g/m³. Generally quoted in physical tables as 1293.052 g/m³.

At 1000 mb. and temperature 269.43a the figure is the same. See Section I., § 3.

Measures of Wind. (International Tables, Chapter VI.)

Wind may be measured by noting the direction in which the current of air is moving, defining the direction by the orientation, and by measuring the velocity, or the force on a measured area, by means of an anemometer or by assigning a number on the Beaufort scale by estimation. (See Observer's Handbook, p. 39.) If the observations of wind are to be subjected to numerical computation they should all be expressed in terms of velocity, preferably in metres per second, by the conversion tables which are given in the Handbook referred to.

Resultant Wind.

Lambert's formula:—Calculation of mean direction and force from a group of observations to eight points.

Lambert's formula (Nouveaux Mémoires de l'Académie Royale des Sciences et Belles-Lettres: Berlin, 1777) expresses the angle α which the direction of a wind equal to the resultant or geometrical sum of all the observed winds make with the N.S. line.

For that purpose the sum of the components directed towards the West and towards the South must be obtained and their ratio taken.

If M_N is the sum of the measures of wind from the North, North-East, and so on.

Then the sum of the components directed towards the West is

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And the sum of the components directed towards the South is

$$\Sigma_N = M_N - M_S + (M_{NE} + M_{NW} - M_{SE} - M_{SW}) \sin 45^\circ.$$

And the angle α is given by

$$\tan \alpha = \Sigma_E / \Sigma_N.$$

The quadrant to which the resultant is to be referred will depend upon the signs of the numerator and denominator. Thus if both Σ_E and Σ_N are positive the resultant wind is from some point between North and East, if Σ_E is positive and Σ_N negative the resultant wind is between East and South. If both components are negative it is between South and West, and if Σ_E is negative and Σ_N positive it is between North and West.

The resultant wind is obtained, as in all cases of geometrical summation, by taking the square root of the sum of the squares of the components:—Thus the resultant wind

$$W = \sqrt{(\Sigma_E^2 + \Sigma_N^2)}.$$

These formulæ are only strictly applicable when the wind is expressed in terms of velocity, that is in metres per second or in some other unit of velocity. When the wind measures are in Beaufort numbers they ought first to be converted into velocity

measures by a scale of equivalents, but this is not always done, the resultant being expressed as "mean force," which may be sufficiently intelligible for ordinary use.

Sometimes the formula is used when only the direction of winds is given, in which case the application of the formula implies the quite improbable assumption that all the observed winds are of equal force or velocity.

Such cases are better dealt with by means of a wind rose.

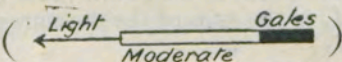
Wind-Roses.

When it is desired to express the velocity or force of wind on a chart it is better to use a diagrammatic representation instead of a numerical one. The almost universal practice is to represent the direction of the wind by a line drawn on the chart *up to* the point for which the observation is noted. The convention with regard to direction is easily understood, lines drawn from left to right across the paper represent a line from **East** to **West**, as is usual with geographical charts, and lines drawn from top to bottom represent the line from North to South.

The conventions with regard to the representation of force or velocity are various. For the charts of the Daily Weather Service the force is represented by the number of feathers on the tail of the direction line in accordance with a practice that is gradually becoming general. But the length of the line is still often used, and for the purpose of representing frequency of occurrence of winds of selected forces it is very convenient.

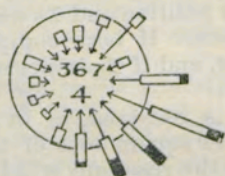
After many trials a form of wind rose has been adopted by the Meteorological Office for expressing average frequencies of calms, light winds, moderate winds and gales, which is thus described. (Report of the Meteorological Council, 1891-2, p. 7.)

New Wind-Rose.—In the new form of Wind Rose which is given below, the arrows which fly with the wind show by their length the frequency of winds of various directions and by their thickness the various forces; light winds, forces 1 to 3: moderate

winds 4 to 7: and gales 8 to 12 (). The

circle supplies a scale for estimating the frequency of winds in any direction. The length from the heads of the arrows to the circumference represents 5 per cent. of the whole number of observed winds (100 per cent. = 4 inches). The upper figures in the centre of the wind rose are the total number of observations, the percentage of calms being given underneath.

Example of Wind Rose.



This form of wind-rose is convenient because it enables one to add the variations of some other variable such as temperature, rainfall without disturbing the characteristics of the wind-rose itself.

TABLES.

NUMBERS FOR APPROXIMATE NUMERICAL COMPUTATIONS.

GENERAL INSTRUCTIONS FOR NUMERICAL COMPUTATION.

Carry out all numerical computations to one figure, or in cases where precision is necessary, &c., two figures beyond the decimal place which is to be given as correct in the result, in order to avoid the accumulation of errors from a succession of operations.

When the final result is obtained, cast up the last figure retained if the working figure which immediately follows it is 5 or more.

It is a useful practice in the computation of physical quantities, if the measurement is carried to such a number of places that the value of the last figure given in the result is uncertain, to write the uncertain figure a little below the line. Thus, the quoting of a temperature as 25.2_3 would mean that the computation has been carried to tenths of a degree but that the decimal fraction is uncertain on account of possible errors of measurement.

When a large amount of computation of one kind has to be done various calculating machines can be employed. In the Meteorological Office the Comptometer and Burroughs adding machine are employed for adding, but in dealing with the ordinary work of climatological stations, Crelle's tables are used.

$$\pi = 3.14159265 : \frac{1}{\pi} = 0.3183098862.$$

NUMBERS 1 TO 99, THEIR RECIPROCAL, QUARTER SQUARES, SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS TO FOUR FIGURES.

n	$1/n$	$\frac{1}{4}n^2$	n^2	\sqrt{n}	n^3	$\sqrt[3]{n}$
1	1	0	1	1	1	1
2	5000	1	4	1.414	8	1.260
3	3333	2	9	1.732	27	1.442
4	2500	4	16	2.000	64	1.587
5	2000	6	25	2.236	125	1.710
6	1667	9	36	2.449	216	1.817
7	1429	12	49	2.646	343	1.913
8	1250	16	64	2.828	512	2.000
9	1111	20	81	3.000	729	2.080
10	1000	25	100	3.162	1000	2.154
11	9091	30	121	3.317	1331	2.224
12	8333	36	144	3.464	1728	2.289
13	7692	42	169	3.606	2197	2.351
14	7143	49	196	3.742	2744	2.410
15	6667	56	225	3.873	3375	2.466
16	6250	64	256	4.000	4096	2.520
17	5882	72	289	4.123	4913	2.571
18	5556	81	324	4.243	5832	2.621
19	5263	90	361	4.359	6859	2.668
20	5000	100	400	4.472	8000	2.714
21	4762	110	441	4.583	9261	2.759
22	4546	121	484	4.690	10665	2.802
23	4348	132	529	4.796	12167	2.844
24	4167	144	576	4.899	13824	2.884

n	$1/n$	$\frac{1}{4}n^2$	n^2	\sqrt{n}	n^3	$\sqrt[3]{n}$
25	4000	156	625	5.000	1563	2.924
26	3846	169	676	5.099	1758	2.962
27	3704	182	729	5.196	1968	3.000
28	3571	196	784	5.291	2195	3.037
29	3448	210	841	5.385	2439	3.072
30	3333	225	900	5.477	2700	3.107
31	3226	240	961	5.568	2979	3.141
32	3125	256	1024	5.657	3277	3.175
33	3030	272	1089	5.745	3594	3.208
34	2941	289	1156	5.831	3930	3.240
35	2857	306	1225	5.916	4288	3.271
36	2778	324	1296	6.000	4666	3.302
37	2703	342	1369	6.083	5065	3.332
38	2632	361	1444	6.164	5487	3.362
39	2564	380	1521	6.245	5932	3.391
40	2500	400	1600	6.325	6400	3.420
41	2439	420	1681	6.403	6892	3.448
42	2381	441	1764	6.481	7409	3.476
43	2326	462	1849	6.557	7951	3.503
44	2273	484	1936	6.633	8518	3.530
45	2222	506	2025	6.708	9113	3.557
46	2174	529	2116	6.782	9734	3.583
47	2128	552	2209	6.856	1038	3.609
48	2083	576	2304	6.928	1106	3.634
49	2041	600	2401	7.000	1176	3.659
50	2000	625	2500	7.071	1250	3.684
51	1961	650	2601	7.141	1327	3.708
52	1923	676	2704	7.211	1406	3.733
53	1887	702	2809	7.280	1489	3.756
54	1852	729	2916	7.348	1575	3.780
55	1818	756	3025	7.416	1664	3.803
56	1786	784	3136	7.483	1756	3.826
57	1754	812	3249	7.550	1852	3.849
58	1724	841	3364	7.616	1951	3.871
59	1695	870	3481	7.681	2054	3.893
60	1667	900	3600	7.746	2160	3.915
61	1639	930	3721	7.810	2270	3.936
62	1613	961	3844	7.874	2383	3.958
63	1587	992	3969	7.937	2500	3.979
64	1563	1024	4096	8.000	2621	4.000
65	1539	1056	4225	8.062	2746	4.021
66	1515	1089	4356	8.124	2875	4.041
67	1493	1122	4489	8.185	3008	4.062
68	1471	1156	4624	8.246	3144	4.082
69	1449	1190	4761	8.307	3285	4.102
70	1429	1225	4900	8.367	3430	4.121
71	1408	1260	5041	8.426	3579	4.141
72	1389	1296	5184	8.485	3732	4.160
73	1370	1332	5329	8.544	3890	4.179
74	1351	1369	5476	8.602	4052	4.198
75	1333	1406	5625	8.660	4219	4.217
76	1316	1444	5776	8.718	4390	4.236
77	1299	1482	5929	8.775	4565	4.254
78	1282	1521	6084	8.832	4744	4.273
79	1266	1560	6241	8.888	4930	4.291

n	$1/n$	$\frac{1}{4}n^2$	n^2	\sqrt{n}	n^3	$\sqrt[3]{n}$
80	1250	1600	6400	8.944	5120	4.309
81	1235	1640	6561	9.000	5314	4.327
82	1220	1681	6724	9.055	5514	4.344
83	1205	1722	6889	9.110	5718	4.362
84	1191	1764	7056	9.165	5927	4.380
85	1177	1806	7225	9.220	6141	4.397
86	1163	1849	7396	9.274	6361	4.414
87	1149	1892	7569	9.327	6585	4.431
88	1136	1936	7744	9.381	6815	4.448
89	1124	1980	7921	9.434	7050	4.465
90	1111	2025	8100	9.487	7290	4.481
91	1099	2070	8281	9.539	7536	4.498
92	1087	2116	8464	9.592	7787	4.514
93	1075	2162	8649	9.644	8044	4.531
94	1064	2209	8836	9.695	8306	4.547
95	1053	2256	9025	9.747	8574	4.563
96	1042	2304	9216	9.798	8847	4.579
97	1031	2352	9409	9.849	9127	4.595
98	1020	2401	9604	9.899	9412	4.610
99	1010	2450	9801	9.950	9703	4.626

LENGTH OF 1° OF LONGITUDE IN DIFFERENT LATITUDES IN METRES, NAUTICAL MILES AND STATUTE MILES.

(International Meteorological Tables.)

Degrees of Latitude.	Metres.	Nautical Miles.	Statute Miles.	Degrees of Latitude.	Metres.	Nautical Miles.	Statute Miles.
0	111,307	60,064	69,164	50	71,687	38,684	44,545
2	111,239	60,027	69,122	52	68,670	37,056	42,670
4	111,037	59,918	68,996	54	65,568	35,382	40,743
6	110,701	59,737	68,788	56	62,385	33,664	38,765
8	110,230	59,482	68,495	58	59,126	31,906	36,740
10	109,627	59,157	68,120	60	55,793	30,107	34,669
12	108,890	58,759	67,662	62	52,392	28,272	32,555
14	108,021	58,290	67,122	64	48,926	26,402	30,402
16	107,022	57,751	66,502	66	45,399	24,498	28,210
18	105,893	57,142	65,800	68	41,816	22,565	25,984
20	104,635	56,463	65,018	70	38,182	20,604	23,726
22	103,250	55,716	64,158	72	34,500	18,617	21,438
24	101,740	54,901	63,219	74	30,775	16,607	19,123
26	100,106	54,019	62,204	76	27,012	14,576	16,785
28	98,350	53,072	61,113	78	23,216	12,528	14,426
30	96,475	52,060	59,948	80	19,391	10,464	12,049
32	94,482	50,984	58,709	82	15,542	8,387	9,658
34	92,374	49,847	57,400	84	11,673	6,299	7,253
36	90,153	48,648	56,019	86	7,790	4,204	4,841
38	87,822	47,391	54,571	88	3,897	2,103	2,422
40	85,384	46,075	53,056	90	0	0.000	0.000
42	82,841	44,703	51,476				
44	80,196	43,275	49,832				
46	77,454	41,796	48,129				
48	74,616	40,264	46,365				

4129

insert decimal
point in place of
comma.

B 2

insert decimal point
in place of comma.

LOGARITHMS OF NUM-

Base of "natural" or Napierian

Modulus or factor of conversion from natural logarithms

Reciprocal or factor of

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

BERS TO FOUR FIGURES.

logarithms $e = 2.718281828$.(base e) to common logarithms (base 10) $\log_{10} e = .43429$.conversion, $\log_e 10 = 2.3026$.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	5	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TRIGONOMETRICAL RATIOS.

De- grees.	Sin.	Cos.	Tan.	Cot.	Sec.	Cosec.	De- grees.
0°	0	1	0	∞	1	∞	90°
1	·0175	·9999	·0175	57·29	1·000	57·30	89
2	·0349	·9994	·0349	28·64	1·001	28·65	88
3	·0523	·9986	·0524	19·08	1·001	19·11	87
4	·0698	·9976	·0699	14·30	1·002	14·34	86
5	·0872	·9962	·0875	11·43	1·004	11·47	85
6	·1045	·9945	·1051	9·514	1·006	9·567	84
7	·1219	·9926	·1228	8·144	1·008	8·206	83
8	·1392	·9903	·1405	7·115	1·010	7·185	82
9	·1564	·9877	·1584	6·314	1·012	6·392	81
10	·1737	·9848	·1763	5·671	1·015	5·759	80
11	·1908	·9816	·1944	5·145	1·019	5·241	79
12	·2079	·9782	·2126	4·705	1·022	4·810	78
13	·2250	·9744	·2309	4·331	1·026	4·445	77
14	·2419	·9703	·2493	4·011	1·031	4·134	76
15	·2588	·9659	·2680	3·732	1·035	3·864	75
16	·2756	·9613	·2868	3·487	1·040	3·623	74
17	·2924	·9563	·3057	3·271	1·046	3·420	73
18	·3090	·9511	·3249	3·078	1·051	3·236	72
19	·3256	·9455	·3443	2·904	1·058	3·072	71
20	·3420	·9397	·3640	2·747	1·064	2·924	70
21	·3584	·9336	·3839	2·605	1·071	2·790	69
22	·3746	·9272	·4040	2·475	1·079	2·669	68
23	·3907	·9205	·4245	2·356	1·086	2·559	67
24	·4067	·9136	·4452	2·246	1·095	2·459	66
25	·4226	·9063	·4663	2·145	1·103	2·366	65
26	·4384	·8988	·4877	2·050	1·113	2·281	64
27	·4540	·8910	·5095	1·963	1·122	2·203	63
28	·4695	·8830	·5317	1·881	1·133	2·130	62
29	·4848	·8746	·5543	1·804	1·143	2·063	61
30	·5000	·8660	·5774	1·732	1·155	2·000	60
31	·5150	·8572	·6009	1·664	1·167	1·942	59
32	·5299	·8481	·6249	1·600	1·179	1·887	58
33	·5446	·8387	·6494	1·540	1·192	1·836	57
34	·5592	·8290	·6745	1·483	1·206	1·788	56
35	·5736	·8192	·7002	1·428	1·221	1·743	55
36	·5878	·8090	·7265	1·376	1·236	1·701	54
37	·6018	·7986	·7536	1·327	1·252	1·662	53
38	·6157	·7880	·7813	1·280	1·269	1·624	52
39	·6293	·7772	·8098	1·235	1·287	1·589	51
40	·6428	·7660	·8391	1·192	1·305	1·556	50
41	·6561	·7547	·8693	1·150	1·325	1·524	49
42	·6691	·7431	·9004	1·111	1·346	1·494	48
43	·6820	·7314	·9325	1·072	1·367	1·466	47
44	·6947	·7193	·9657	1·036	1·390	1·440	46
45°	·7071	·7071	1·000	1·000	1·414	1·414	45°
De- grees.	Cos.	Sin.	Cot.	Tan.	Cosec.	Sec.	De- grees.

TRAVERSE TABLES.

The components of geometrical resolution for obtaining the "Latitude" and "Departure" from "Course" and "Distance" or the components in the S. to N. and W. to E. direction of a vector whose magnitude and direction measured from North are specified. The vector is in all cases directed towards the origin.

The following table is a common one in books on Navigation and is used to compute the change in the geographical position of a ship from the course and distance run. The effect of the curvature of the earth is disregarded, hence the process is equivalent, with the same limitation, to "resolving" a length drawn in a specified direction into two components along and perpendicular to the meridian. The table is therefore applicable for finding the components of wind velocity from the observed velocity and direction and *vice-versâ*.

Some care is necessary in determining the proper signs to be given to the components indicated for the different angles of direction in the head and foot of the tables. From the point of view of a ship computing its new position, the distance run terminates at the new position—in geometrical language, the vector is drawn *to the origin*. In forming wind components the signs indicated in the table will be correct if the angle assigned to direction gives the point *from* which the wind comes. Thus an East wind has direction 90°.

Direction from 1° to 5° from a cardinal point.

Direction from North measured through East.											
Lat. —	359°	358°	357°	356°	355°	Dep. +					
Lat. +	181°	182°	183°	184°	185°	Dep. +					
Lat. +	179°	178°	177°	176°	175°	Dep. —					
Lat. —	1°	2°	3°	4°	5°	Dep. —					
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	1°0	0°0	1°0	0°0	1°0	0°1	1°0	0°1	1°0	0°1	1
2	2°0	0°0	2°0	0°1	2°0	0°1	2°0	0°1	2°0	0°2	2
3	3°0	0°1	3°0	0°1	3°0	0°2	3°0	0°2	3°0	0°3	3
4	4°0	0°1	4°0	0°1	4°0	0°2	4°0	0°3	4°0	0°3	4
5	5°0	0°1	5°0	0°2	5°0	0°3	5°0	0°3	5°0	0°4	5
6	6°0	0°1	6°0	0°2	6°0	0°3	6°0	0°4	6°0	0°5	6
7	7°0	0°1	7°0	0°2	7°0	0°4	7°0	0°5	7°0	0°6	7
8	8°0	0°1	8°0	0°3	8°0	0°4	8°0	0°6	8°0	0°7	8
9	9°0	0°2	9°0	0°3	9°0	0°5	9°0	0°6	9°0	0°8	9
10	10°0	0°2	10°0	0°3	10°0	0°5	10°0	0°7	10°0	0°9	10
11	11°0	0°2	11°0	0°4	11°0	0°6	11°0	0°8	11°0	1°0	11
12	12°0	0°2	12°0	0°4	12°0	0°6	12°0	0°8	12°0	1°0	12
13	13°0	0°2	13°0	0°5	13°0	0°7	13°0	0°9	13°0	1°1	13
14	14°0	0°2	14°0	0°5	14°0	0°7	14°0	1°0	13°9	1°2	14
15	15°0	0°3	15°0	0°5	15°0	0°8	15°0	1°0	14°9	1°3	15
16	16°0	0°3	16°0	0°6	16°0	0°8	16°0	1°1	15°9	1°4	16
17	17°0	0°3	17°0	0°6	17°0	0°9	17°0	1°2	16°9	1°5	17
18	18°0	0°3	18°0	0°6	18°0	0°9	18°0	1°3	17°9	1°6	18
19	19°0	0°3	19°0	0°7	19°0	1°0	19°0	1°3	18°9	1°7	19
20	20°0	0°3	20°0	0°7	20°0	1°0	20°0	1°4	19°9	1°7	20
21	21°0	0°4	21°0	0°7	21°0	1°1	20°9	1°5	20°9	1°8	21
22	22°0	0°4	22°0	0°8	22°0	1°2	21°9	1°5	21°9	1°9	22
23	23°0	0°4	23°0	0°8	23°0	1°2	22°9	1°6	22°9	2°0	23
24	24°0	0°4	24°0	0°8	24°0	1°3	23°9	1°7	23°9	2°1	24
25	25°0	0°4	25°0	0°9	25°0	1°3	24°9	1°7	24°9	2°2	25
26	26°0	0°5	26°0	0°9	26°0	1°4	25°9	1°8	25°9	2°3	26
27	27°0	0°5	27°0	0°9	27°0	1°4	26°9	1°9	26°9	2°4	27
28	28°0	0°5	28°0	1°0	28°0	1°5	27°9	2°0	27°9	2°4	28
29	29°0	0°5	29°0	1°0	29°0	1°5	28°9	2°0	28°9	2°5	29
30	30°0	0°5	30°0	1°0	30°0	1°6	29°9	2°1	29°9	2°6	30
31	31°0	0°5	31°0	1°1	31°0	1°6	30°9	2°2	30°9	2°7	31
32	32°0	0°6	32°0	1°1	32°0	1°7	31°9	2°2	31°9	2°8	32
33	33°0	0°6	33°0	1°2	33°0	1°7	32°9	2°3	32°9	2°9	33
34	34°0	0°6	34°0	1°2	34°0	1°8	33°9	2°4	33°9	3°0	34
35	35°0	0°6	35°0	1°2	35°0	1°8	34°9	2°4	34°9	3°1	35
36	36°0	0°6	36°0	1°3	36°0	1°9	35°9	2°5	35°9	3°1	36
37	37°0	0°6	37°0	1°3	36°9	1°9	36°9	2°6	36°9	3°2	37
38	38°0	0°7	38°0	1°3	37°9	2°0	37°9	2°7	37°9	3°3	38
39	39°0	0°7	39°0	1°4	38°9	2°0	38°9	2°7	38°9	3°4	39
40	40°0	0°7	40°0	1°4	39°9	2°1	39°9	2°8	39°9	3°5	40
41	41°0	0°7	41°0	1°4	40°9	2°1	40°9	2°9	40°8	3°6	41
42	42°0	0°7	42°0	1°5	41°9	2°2	41°9	2°9	41°8	3°7	42
43	43°0	0°8	43°0	1°5	42°9	2°3	42°9	3°0	42°8	3°7	43
44	44°0	0°8	44°0	1°5	43°9	2°3	43°9	3°1	43°8	3°8	44
45	45°0	0°8	45°0	1°6	44°9	2°4	44°9	3°1	44°8	3°9	45
46	46°0	0°8	46°0	1°6	45°9	2°4	45°9	3°2	45°8	4°0	46
47	47°0	0°8	47°0	1°6	46°9	2°5	46°9	3°3	46°8	4°1	47
48	48°0	0°8	48°0	1°7	47°9	2°5	47°9	3°3	47°8	4°2	48
49	49°0	0°9	49°0	1°7	48°9	2°6	48°9	3°4	48°8	4°3	49
50	50°0	0°9	50°0	1°7	49°9	2°6	49°9	3°5	49°8	4°4	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	89°		88°		87°		86°		85°		Lat. —
Dep. —	91°		92°		93°		94°		95°		Lat. +
Dep. +	269°		268°		267°		266°		265°		Lat. +
Dep. +	271°		272°		273°		274°		275°		Lat. —
Direction measured from North through East.											

Vector directed towards origin. "Latitude" is component in S. and N. direction.
 "Departure" is component in W. and E. direction.

Direction from 6° to 10° from a cardinal point.

Direction measured from North through East.											
Lat. —	354°		353°		352°		351°		350°		Dep. +
Lat. +	186°		187°		188°		189°		190°		Dep. +
Lat. —	174°		173°		172°		171°		170°		Dep. —
Lat. —	6°		7°		8°		9°		10°		Dep. —
Distance	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	1°0	0°1	1°0	0°1	1°0	0°1	1°0	0°2	1°0	0°2	1
2	2°0	0°2	2°0	0°2	2°0	0°3	2°0	0°3	2°0	0°3	2
3	3°0	0°3	3°0	0°4	3°0	0°4	3°0	0°5	3°0	0°5	3
4	4°0	0°4	4°0	0°5	4°0	0°6	4°0	0°6	3°9	0°7	4
5	5°0	0°5	5°0	0°6	5°0	0°7	4°9	0°8	4°9	0°9	5
6	6°0	0°6	6°0	0°7	5°9	0°8	5°9	0°9	5°9	1°0	6
7	7°0	0°7	6°9	0°9	6°9	1°0	6°9	1°1	6°9	1°2	7
8	8°0	0°8	7°9	1°0	7°9	1°1	7°9	1°3	7°9	1°4	8
9	9°0	0°9	8°9	1°1	8°9	1°3	8°9	1°4	8°9	1°6	9
10	9°9	1°0	9°9	1°2	9°9	1°4	9°9	1°6	9°8	1°7	10
11	10°9	1°1	10°9	1°3	10°9	1°5	10°9	1°7	10°8	1°9	11
12	11°9	1°3	11°9	1°5	11°9	1°7	11°9	1°9	11°8	2°1	12
13	12°9	1°4	12°9	1°6	12°9	1°8	12°8	2°0	12°8	2°3	13
14	13°9	1°5	13°9	1°7	13°9	1°9	13°8	2°2	13°8	2°4	14
15	14°9	1°6	14°9	1°8	14°9	2°1	14°8	2°3	14°8	2°6	15
16	15°9	1°7	15°9	1°9	15°8	2°2	15°8	2°5	15°8	2°8	16
17	16°9	1°8	16°9	2°1	16°8	2°4	16°8	2°7	16°7	3°0	17
18	17°9	1°9	17°9	2°2	17°8	2°5	17°8	2°8	17°7	3°1	18
19	18°9	2°0	18°9	2°3	18°8	2°6	18°8	3°0	18°7	3°3	19
20	19°9	2°1	19°9	2°4	19°8	2°8	19°8	3°1	19°7	3°5	20
21	20°9	2°2	20°8	2°6	20°8	2°9	20°7	3°3	20°7	3°6	21
22	21°9	2°3	21°8	2°7	21°8	3°1	21°7	3°4	21°7	3°8	22
23	22°9	2°4	22°8	2°8	22°8	3°2	22°7	3°6	22°7	4°0	23
24	23°9	2°5	23°8	2°9	23°8	3°3	23°7	3°8	23°6	4°2	24
25	24°9	2°6	24°8	3°0	24°8	3°5	24°7	3°9	24°6	4°3	25
26	25°9	2°7	25°8	3°2	25°7	3°6	25°7	4°1	25°6	4°5	26
27	26°9	2°8	26°8	3°3	26°7	3°8	26°7	4°2	26°6	4°7	27
28	27°8	2°9	27°8	3°4	27°7	3°9	27°7	4°4	27°6	4°9	28
29	28°8	3°0	28°8	3°5	28°7	4°0	28°6	4°5	28°6	5°0	29
30	29°8	3°1	29°8	3°7	29°7	4°2	29°6	4°7	29°5	5°2	30
31	30°8	3°2	30°8	3°8	30°7	4°3	30°6	4°8	30°5	5°4	31
32	31°8	3°3	31°8	3°9	31°7	4°5	31°6	5°0	31°5	5°6	32
33	32°8	3°4	32°8	4°0	32°7	4°6	32°6	5°2	32°5	5°7	33
34	33°8	3°6	33°7	4°1	33°7	4°7	33°6	5°3	33°5	5°9	34
35	34°8	3°7	34°7	4°3	34°7	4°9	34°6	5°5	34°5	6°1	35
36	35°8	3°8	35°7	4°4	35°6	5°0	35°6	5°6	35°5	6°3	36
37	36°8	3°9	36°7	4°5	36°6	5°1	36°5	5°8	36°4	6°4	37
38	37°8	4°0	37°7	4°6	37°6	5°3	37°5	5°9	37°4	6°6	38
39	38°8	4°1	38°7	4°8	38°6	5°4	38°5	6°1	38°4	6°8	39
40	39°8	4°2	39°7	4°9	39°6	5°6	39°5	6°3	39°4	6°9	40
41	40°8	4°3	40°7	5°0	40°6	5°7	40°5	6°4	40°4	7°1	41
42	41°8	4°4	41°7	5°1	41°6	5°8	41°5	6°6	41°4	7°3	42
43	42°8	4°5	42°7	5°2	42°6	6°0	42°5	6°7	42°3	7°5	43
44	43°8	4°6	43°7	5°4	43°6	6°1	43°5	6°9	43°3	7°6	44
45	44°8	4°7	44°7	5°5	44°6	6°3	44°4	7°0	44°3	7°8	45
46	45°7	4°8	45°7	5°6	45°6	6°4	45°4	7°2	45°3	8°0	46
47	46°7	4°9	46°6	5°7	46°5	6°5	46°4	7°4	46°3	8°2	47
48	47°7	5°0	47°6	5°8	47°5	6°7	47°4	7°5	47°3	8°3	48
49	48°7	5°1	48°6	6°0	48°5	6°8	48°4	7°7	48°3	8°5	49
50	49°7	5°2	49°6	6°1	49°5	7°0	49°4	7°8	49°2	8°7	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	84°		83°		82°		81°		80°		Lat. —
Dep. —	96°		97°		98°		99°		100°		Lat. +
Dep. +	264°		263°		262°		261°		260°		Lat. +
Dep. +	276°		277°		278°		279°		280°		Lat. —
Direction measured from North through East.											

Direction from 11° to 15° from a cardinal point.

Direction measured from North through East.											
Lat. —	349°	348°	347°	346°	345°	Dep. +					
Lat. +	191°	192°	193°	194°	195°	Dep. +					
Lat. +	169°	168°	167°	166°	165°	Dep. —					
Lat. —	11°	12°	13°	14°	15°	Dep. —					
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	1'0	0'2	1'0	0'2	1'0	0'2	1'0	0'2	1'0	0'3	1
2	2'0	0'4	2'0	0'4	1'9	0'4	1'9	0'5	1'9	0'5	2
3	2'9	0'6	2'9	0'6	2'9	0'7	2'9	0'7	2'9	0'8	3
4	3'9	0'8	3'9	0'8	3'9	0'9	3'9	1'0	3'9	1'0	4
5	4'9	1'0	4'9	1'0	4'9	1'1	4'9	1'2	4'8	1'3	5
6	5'9	1'1	5'9	1'2	5'8	1'3	5'8	1'5	5'8	1'6	6
7	6'9	1'3	6'8	1'5	6'8	1'6	6'8	1'7	6'8	1'8	7
8	7'9	1'5	7'8	1'7	7'8	1'8	7'8	1'9	7'7	2'1	8
9	8'8	1'7	8'8	1'9	8'8	2'0	8'7	2'2	8'7	2'3	9
10	9'8	1'9	9'8	2'1	9'7	2'2	9'7	2'4	9'7	2'6	10
11	10'8	2'1	10'8	2'3	10'7	2'5	10'7	2'7	10'6	2'8	11
12	11'8	2'3	11'7	2'5	11'7	2'7	11'6	2'9	11'6	3'1	12
13	12'8	2'5	12'7	2'7	12'7	2'9	12'6	3'1	12'6	3'4	13
14	13'7	2'7	13'7	2'9	13'6	3'1	13'6	3'4	13'5	3'6	14
15	14'7	2'9	14'7	3'1	14'6	3'4	14'6	3'6	14'5	3'9	15
16	15'7	3'1	15'7	3'3	15'6	3'6	15'5	3'9	15'5	4'1	16
17	16'7	3'2	16'6	3'5	16'6	3'8	16'5	4'1	16'4	4'4	17
18	17'7	3'4	17'6	3'7	17'5	4'0	17'5	4'4	17'4	4'7	18
19	18'7	3'6	18'6	4'0	18'5	4'3	18'4	4'6	18'4	4'9	19
20	19'6	3'8	19'6	4'2	19'5	4'5	19'4	4'8	19'3	5'2	20
21	20'6	4'0	20'5	4'4	20'5	4'7	20'4	5'1	20'3	5'4	21
22	21'6	4'2	21'5	4'6	21'4	4'9	21'3	5'3	21'3	5'7	22
23	22'6	4'4	22'5	4'8	22'4	5'2	22'3	5'6	22'2	6'0	23
24	23'6	4'6	23'5	5'0	23'4	5'4	23'3	5'8	23'2	6'2	24
25	24'5	4'8	24'5	5'2	24'4	5'6	24'3	6'0	24'1	6'5	25
26	25'5	5'0	25'4	5'4	25'3	5'8	25'2	6'3	25'1	6'7	26
27	26'5	5'2	26'4	5'6	26'3	6'1	26'2	6'5	26'1	7'0	27
28	27'5	5'3	27'4	5'8	27'3	6'3	27'2	6'8	27'0	7'2	28
29	28'5	5'5	28'4	6'0	28'3	6'5	28'1	7'0	28'0	7'5	29
30	29'4	5'7	29'3	6'2	29'2	6'7	29'1	7'3	29'0	7'8	30
31	30'4	5'9	30'3	6'4	30'2	7'0	30'1	7'5	29'9	8'0	31
32	31'4	6'1	31'3	6'7	31'2	7'2	31'0	7'7	30'9	8'3	32
33	32'4	6'3	32'3	6'9	32'2	7'4	32'0	8'0	31'9	8'5	33
34	33'4	6'5	33'3	7'1	33'1	7'6	33'0	8'2	32'8	8'8	34
35	34'4	6'7	34'2	7'3	34'1	7'9	34'0	8'5	33'8	9'1	35
36	35'3	6'9	35'2	7'5	35'1	8'1	34'9	8'7	34'8	9'3	36
37	36'3	7'1	36'2	7'7	36'1	8'3	35'9	9'0	35'7	9'6	37
38	37'3	7'3	37'2	7'9	37'0	8'5	36'9	9'2	36'7	9'8	38
39	38'3	7'4	38'1	8'1	38'0	8'8	37'8	9'4	37'7	10'1	39
40	39'3	7'6	39'1	8'3	39'0	9'0	38'8	9'7	38'6	10'4	40
41	40'2	7'8	40'1	8'5	39'9	9'2	39'8	9'9	39'6	10'6	41
42	41'2	8'0	41'1	8'7	40'9	9'4	40'8	10'2	40'6	10'9	42
43	42'2	8'2	42'1	8'9	41'9	9'7	41'7	10'4	41'5	11'1	43
44	43'2	8'4	43'0	9'1	42'9	9'9	42'7	10'6	42'5	11'4	44
45	44'2	8'6	44'0	9'4	43'8	10'1	43'7	10'9	43'5	11'6	45
46	45'2	8'8	45'0	9'6	44'8	10'3	44'6	11'1	44'4	11'9	46
47	46'1	9'0	46'0	9'8	45'8	10'6	45'6	11'4	45'4	12'2	47
48	47'1	9'2	47'0	10'0	46'8	10'8	46'6	11'6	46'4	12'4	48
49	48'1	9'3	47'9	10'2	47'7	11'0	47'5	11'9	47'3	12'7	49
50	49'1	9'5	48'9	10'4	48'7	11'2	48'5	12'1	48'3	12'9	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	79°		78°		77°		76°		75°		Lat. —
Dep. —	101°		102°		103°		104°		105°		Lat. +
Dep. +	259°		258°		257°		256°		255°		Lat. +
Dep. +	281°		282°		283°		284°		285°		Lat. —
Direction measured from North through East.											

Vector directed towards origin. "Latitude" is component in S. and N. direction.
"Departure" is component in W. and E. direction.

Direction from 16° to 20° from a cardinal point.

Direction measured from North through East.											
Lat. —	344°		343°		342°		341°		340°		Dep. +
Lat. +	196°		197°		198°		199°		200°		Dep. +
Lat. +	164°		163°		162°		161°		160°		Dep. —
Lat. —	16°		17°		18°		19°		20°		Dep. —
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	1'0	0'3	1'0	0'3	1'0	0'3	0'9	0'3	0'9	0'3	1
2	1'9	0'6	1'9	0'6	1'9	0'6	1'9	0'7	1'9	0'7	2
3	2'9	0'8	2'9	0'9	2'9	0'9	2'8	1'0	2'8	1'0	3
4	3'8	1'1	3'8	1'2	3'8	1'2	3'8	1'3	3'8	1'4	4
5	4'8	1'4	4'8	1'5	4'8	1'5	4'7	1'6	4'7	1'7	5
6	5'8	1'7	5'7	1'8	5'7	1'9	5'7	2'0	5'6	2'1	6
7	6'7	1'9	6'7	2'0	6'7	2'2	6'6	2'3	6'6	2'4	7
8	7'7	2'2	7'7	2'3	7'6	2'5	7'6	2'6	7'5	2'7	8
9	8'7	2'5	8'6	2'6	8'6	2'8	8'5	2'9	8'5	3'1	9
10	9'6	2'8	9'6	2'9	9'5	3'1	9'5	3'3	9'4	3'4	10
11	10'6	3'0*	10'5	3'2	10'5	3'4	10'4	3'6	10'3	3'8	11
12	11'5	3'3	11'5	3'5	11'4	3'7	11'3	3'9	11'3	4'1	12
13	12'5	3'6	12'4	3'8	12'4	4'0	12'3	4'2	12'2	4'4	13
14	13'5	3'9	13'4	4'1	13'3	4'3	13'2	4'6	13'2	4'8	14
15	14'4	4'1	14'3	4'4	14'3	4'6	14'2	4'9	14'1	5'1	15
16	15'4	4'4	15'3	4'7	15'2	4'9	15'1	5'2	15'0	5'5	16
17	16'3	4'7	16'3	5'0	16'2	5'3	16'1	5'5	16'0	5'8	17
18	17'3	5'0	17'2	5'3	17'1	5'6	17'0	5'9	16'9	6'2	18
19	18'3	5'2	18'2	5'6	18'1	5'9	18'0	6'2	17'9	6'5	19
20	19'2	5'5	19'1	5'8	19'0	6'2	18'9	6'5	18'8	6'8	20
21	20'2	5'8	20'1	6'1	20'0	6'5	19'9	6'8	19'7	7'2	21
22	21'1	6'1	21'0	6'4	20'9	6'8	20'8	7'2	20'7	7'5	22
23	22'1	6'3	22'0	6'7	21'9	7'1	21'7	7'5	21'6	7'9	23
24	23'1	6'6	23'0	7'0	22'8	7'4	22'7	7'8	22'6	8'2	24
25	24'0	6'9	23'9	7'3	23'8	7'7	23'6	8'1	23'5	8'6	25
26	25'0	7'2	24'9	7'6	24'7	8'0	24'6	8'5	24'4	8'9	26
27	26'0	7'4	25'8	7'9	25'7	8'3	25'5	8'8	25'4	9'2	27
28	26'9	7'7	26'8	8'2	26'6	8'7	26'5	9'1	26'3	9'6	28
29	27'9	8'0	27'7	8'5	27'6	9'0	27'4	9'4	27'3	9'9	29
30	28'8	8'3	28'7	8'8	28'5	9'3	28'4	9'8	28'2	10'3	30
31	29'8	8'5	29'6	9'1	29'5	9'6	29'3	10'1	29'1	10'6	31
32	30'8	8'8	30'6	9'4	30'4	9'9	30'3	10'4	30'1	10'9	32
33	31'7	9'1	31'6	9'6	31'4	10'2	31'2	10'7	31'0	11'3	33
34	32'7	9'4	32'5	9'9	32'3	10'5	32'1	11'1	31'9	11'6	34
35	33'6	9'6	33'5	10'2	33'3	10'8	33'1	11'4	32'9	12'0	35
36	34'6	9'9	34'4	10'5	34'2	11'1	34'0	11'7	33'8	12'3	36
37	35'6	10'2	35'4	10'8	35'2	11'4	35'0	12'0	34'8	12'7	37
38	36'5	10'5	36'3	11'1	36'1	11'7	35'9	12'4	35'7	13'0	38
39	37'5	10'7	37'3	11'4	37'1	12'1	36'9	12'7	36'6	13'3	39
40	38'5	11'0	38'3	11'7	38'0	12'4	37'8	13'0	37'6	13'7	40
41	39'4	11'3	39'2	12'0	39'0	12'7	38'8	13'3	38'5	14'0	41
42	40'4	11'6	40'2	12'3	39'9	13'0	39'7	13'7	39'5	14'4	42
43	41'3	11'9	41'1	12'6	40'9	13'3	40'7	14'0	40'4	14'7	43
44	42'3	12'1	42'1	12'9	41'8	13'6	41'6	14'3	41'3	15'0	44
45	43'3	12'4	43'0	13'2	42'8	13'9	42'5	14'7	42'3	15'4	45
46	44'2	12'7	44'0	13'4	43'7	14'2	43'5	15'0	43'2	15'7	46
47	45'2	13'0	44'9	13'7	44'7	14'5	44'4	15'3	44'2	16'1	47
48	46'1	13'2	45'9	14'0	45'7	14'8	45'4	15'6	45'1	16'4	48
49	47'1	13'5	46'9	14'3	46'6	15'1	46'3	16'0	46'0	16'8	49
50	48'1	13'8	47'8	14'6	47'6	15'5	47'3	16'3	47'0	17'1	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	74°		73°		72°		71°		70°		Lat. —
Dep. —	106°		107°		108°		109°		110°		Lat. +
Dep. +	254°		253°		252°		251°		250°		Lat. +
Dep. +	286°		287°		288°		289°		290°		Lat. —
Direction measured from North through East.											

Direction from 21° to 25° from a cardinal point.

Direction measured from East through North.											
Lat. -	339°	338°	337°	336°	335°	Dep. +					
Lat. +	201°	202°	203°	204°	205°	Dep. -					
Lat. +	159°	158°	157°	156°	155°	Dep. -					
Lat. -	21°	22°	23°	24°	25°	Dep. -					
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	0.9	0.4	0.9	0.4	0.9	0.4	0.9	0.4	0.9	0.4	1
2	1.9	0.7	1.9	0.7	1.8	0.8	1.8	0.8	1.8	0.8	2
3	2.8	1.1	2.8	1.1	2.8	1.2	2.7	1.2	2.7	1.3	3
4	3.7	1.4	3.7	1.5	3.7	1.6	3.7	1.6	3.6	1.7	4
5	4.7	1.8	4.6	1.9	4.6	2.0	4.6	2.0	4.5	2.1	5
6	5.6	2.2	5.6	2.2	5.5	2.3	5.5	2.4	5.4	2.5	6
7	6.5	2.5	6.5	2.6	6.4	2.7	6.4	2.8	6.3	3.0	7
8	7.5	2.9	7.4	3.0	7.4	3.1	7.3	3.3	7.3	3.4	8
9	8.4	3.2	8.3	3.4	8.3	3.5	8.2	3.7	8.2	3.8	9
10	9.3	3.6	9.3	3.7	9.2	3.9	9.1	4.1	9.1	4.2	10
11	10.3	3.9	10.2	4.1	10.1	4.3	10.0	4.5	10.0	4.6	11
12	11.2	4.3	11.1	4.5	11.0	4.7	10.9	4.9	10.9	5.1	12
13	12.1	4.7	12.1	4.9	12.0	5.1	11.9	5.3	11.8	5.5	13
14	13.1	5.0	13.0	5.2	12.9	5.5	12.8	5.7	12.7	5.9	14
15	14.0	5.4	13.9	5.6	13.8	5.9	13.7	6.1	13.6	6.3	15
16	14.9	5.7	14.8	6.0	14.7	6.3	14.6	6.5	14.5	6.8	16
17	15.9	6.1	15.8	6.4	15.6	6.6	15.5	6.9	15.4	7.2	17
18	16.8	6.5	16.7	6.7	16.6	7.0	16.4	7.3	16.3	7.6	18
19	17.7	6.8	17.6	7.1	17.5	7.4	17.4	7.7	17.2	8.0	19
20	18.7	7.2	18.5	7.5	18.4	7.8	18.3	8.1	18.1	8.5	20
21	19.6	7.5	19.5	7.9	19.3	8.2	19.2	8.5	19.0	8.9	21
22	20.5	7.9	20.4	8.2	20.3	8.6	20.1	8.9	19.9	9.3	22
23	21.5	8.2	21.3	8.6	21.2	9.0	21.0	9.4	20.8	9.7	23
24	22.4	8.6	22.3	9.0	22.1	9.4	21.9	9.8	21.8	10.1	24
25	23.3	9.0	23.2	9.4	23.0	9.8	22.8	10.2	22.7	10.6	25
26	24.3	9.3	24.1	9.7	23.9	10.2	23.8	10.6	23.6	11.0	26
27	25.2	9.7	25.0	10.1	24.9	10.5	24.7	11.0	24.5	11.4	27
28	26.1	10.0	26.0	10.5	25.8	10.9	25.6	11.4	25.4	11.8	28
29	27.1	10.4	26.9	10.9	26.7	11.3	26.5	11.8	26.3	12.3	29
30	28.0	10.8	27.8	11.2	27.6	11.7	27.4	12.2	27.2	12.7	30
31	28.9	11.1	28.7	11.6	28.5	12.1	28.3	12.6	28.1	13.1	31
32	29.9	11.5	29.7	12.0	29.5	12.5	29.2	13.0	29.0	13.5	32
33	30.8	11.8	30.6	12.4	30.4	12.9	30.1	13.4	29.9	13.9	33
34	31.7	12.2	31.5	12.7	31.3	13.3	31.1	13.8	30.8	14.4	34
35	32.7	12.5	32.5	13.1	32.2	13.7	32.0	14.2	31.7	14.8	35
36	33.6	12.9	33.4	13.5	33.1	14.1	32.9	14.6	32.6	15.2	36
37	34.5	13.3	34.3	13.9	34.1	14.5	33.8	15.0	33.5	15.6	37
38	35.5	13.6	35.2	14.2	35.0	14.8	34.7	15.5	34.4	16.1	38
39	36.4	14.0	36.2	14.6	35.9	15.2	35.6	15.9	35.3	16.5	39
40	37.3	14.3	37.1	15.0	36.8	15.6	36.5	16.3	36.3	16.9	40
41	38.3	14.7	38.0	15.4	37.7	16.0	37.5	16.7	37.2	17.3	41
42	39.2	15.1	38.9	15.7	38.7	16.4	38.4	17.1	38.1	17.7	42
43	40.1	15.4	39.9	16.1	39.6	16.8	39.3	17.5	39.0	18.2	43
44	41.1	15.8	40.8	16.5	40.5	17.2	40.2	17.9	39.9	18.6	44
45	42.0	16.1	41.7	16.9	41.4	17.6	41.1	18.3	40.8	19.0	45
46	42.9	16.5	42.7	17.2	42.3	18.0	42.0	18.7	41.7	19.4	46
47	43.9	16.8	43.6	17.6	43.3	18.4	42.9	19.1	42.6	19.9	47
48	44.8	17.2	44.5	18.0	44.2	18.8	43.9	19.5	43.5	20.3	48
49	45.7	17.6	45.4	18.4	45.1	19.1	44.8	19.9	44.4	20.7	49
50	46.7	17.9	46.4	18.7	46.0	19.5	45.7	20.3	45.3	21.1	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. -	69°		68°		67°		66°		65°		Lat. -
Dep. -	111°		112°		113°		114°		115°		Lat. +
Dep. +	249°		248°		247°		246°		245°		Lat. +
Dep. +	291°		292°		293°		294°		295°		Lat. -
Direction measured from North through East.											

Vector directed towards origin. "Latitude" is component in S. and N. direction.
 "Departure" is component in W. and E. direction.

Direction from 26° to 30° from a cardinal point.

Direction measured from North through East.											
Lat. -	334°		333°		332°		331°		330°		Dep. +
Lat. +	206°		207°		208°		209°		210°		Dep. +
Lat. +	154°		153°		152°		151°		150°		Dep. -
Lat. -	26°		27°		28°		29°		30°		Dep. -
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	0.9	0.4	0.9	0.5	0.9	0.5	0.9	0.5	0.9	0.5	1
2	1.8	0.9	1.8	0.9	1.8	0.9	1.7	1.0	1.7	1.0	2
3	2.7	1.3	2.7	1.4	2.6	1.4	2.6	1.5	2.6	1.5	3
4	3.6	1.8	3.6	1.8	3.5	1.9	3.5	1.9	3.5	2.0	4
5	4.5	2.2	4.5	2.3	4.4	2.3	4.4	2.4	4.3	2.5	5
6	5.4	2.6	5.3	2.7	5.3	2.8	5.2	2.9	5.2	3.0	6
7	6.3	3.1	6.2	3.2	6.2	3.3	6.1	3.4	6.1	3.5	7
8	7.2	3.5	7.1	3.6	7.1	3.8	7.0	3.9	6.9	4.0	8
9	8.1	3.9	8.0	4.1	7.9	4.2	7.9	4.4	7.8	4.5	9
10	9.0	4.4	8.9	4.5	8.8	4.7	8.7	4.8	8.7	5.0	10
11	9.9	4.8	9.8	5.0	9.7	5.2	9.6	5.3	9.5	5.5	11
12	10.8	5.3	10.7	5.4	10.6	5.6	10.5	5.8	10.4	6.0	12
13	11.7	5.7	11.6	5.9	11.5	6.1	11.4	6.3	11.3	6.5	13
14	12.6	6.1	12.5	6.4	12.4	6.6	12.2	6.8	12.1	7.0	14
15	13.5	6.6	13.4	6.8	13.2	7.0	13.1	7.3	13.0	7.5	15
16	14.4	7.0	14.3	7.3	14.1	7.5	14.0	7.8	13.9	8.0	16
17	15.3	7.5	15.1	7.7	15.0	8.0	14.9	8.2	14.7	8.5	17
18	16.2	7.9	16.0	8.2	15.9	8.5	15.7	8.7	15.6	9.0	18
19	17.1	8.3	16.9	8.6	16.8	8.9	16.6	9.2	16.5	9.5	19
20	18.0	8.8	17.8	9.1	17.7	9.4	17.5	9.7	17.3	10.0	20
21	18.9	9.2	18.7	9.5	18.5	9.9	18.4	10.2	18.2	10.5	21
22	19.8	9.6	19.6	10.0	19.4	10.3	19.2	10.7	19.1	11.0	22
23	20.7	10.1	20.5	10.4	20.3	10.8	20.1	11.2	19.9	11.5	23
24	21.6	10.5	21.4	10.9	21.2	11.3	21.0	11.6	20.8	12.0	24
25	22.5	11.0	22.3	11.3	22.1	11.7	21.9	12.1	21.7	12.5	25
26	23.4	11.4	23.2	11.8	23.0	12.2	22.7	12.6	22.5	13.0	26
27	24.3	11.8	24.1	12.3	23.8	12.7	23.6	13.1	23.4	13.5	27
28	25.2	12.3	24.9	12.7	24.7	13.1	24.5	13.6	24.2	14.0	28
29	26.1	12.7	25.8	13.2	25.6	13.6	25.4	14.1	25.1	14.5	29
30	27.0	13.2	25.7	13.6	26.5	14.1	26.2	14.5	26.0	15.0	30
31	27.9	13.6	27.6	14.1	27.4	14.6	27.1	15.0	26.8	15.5	31
32	28.8	14.0	28.5	14.5	28.3	15.0	28.0	15.5	27.7	16.0	32
33	29.7	14.5	29.4	15.0	29.1	15.5	28.9	16.0	28.6	16.5	33
34	30.6	14.9	30.3	15.4	30.0	16.0	29.7	16.5	29.4	17.0	34
35	31.5	15.3	31.2	15.9	30.9	16.4	30.6	17.0	30.3	17.5	35
36	32.4	15.8	32.1	16.3	31.8	16.9	31.5	17.5	31.2	18.0	36
37	33.3	16.2	33.0	16.8	32.7	17.4	32.4	17.9	32.0	18.5	37
38	34.2	16.7	33.9	17.3	33.6	17.8	33.2	18.4	32.9	19.0	38
39	35.1	17.1	34.7	17.7	34.4	18.3	34.1	18.9	33.8	19.5	39
40	36.0	17.5	35.6	18.2	35.3	18.8	35.0	19.4	34.6	20.0	40
41	36.9	18.0	36.5	18.6	36.2	19.2	35.9	19.9	35.5	20.5	41
42	37.7	18.4	37.4	19.1	37.1	19.7	36.7	20.4	36.4	21.0	42
43	38.6	18.8	38.3	19.5	38.0	20.2	37.6	20.8	37.2	21.5	43
44	39.5	19.3	39.2	20.0	38.8	20.7	38.5	21.3	38.1	22.0	44
45	40.4	19.7	40.1	20.4	39.7	21.1	39.4	21.8	39.0	22.5	45
46	41.3	20.2	41.0	20.9	40.6	21.6	40.2	22.3	39.8	23.0	46
47	42.2	20.6	41.9	21.3	41.5	22.1	41.1	22.8	40.7	23.5	47
48	43.1	21.0	42.8	21.8	42.4	22.5	42.0	23.3	41.6	24.0	48
49	44.0	21.5	43.7	22.2	43.3	23.0	42.9	23.8	42.4	24.5	49
50	44.9	21.9	44.6	22.7	44.1	23.5	43.7	24.2	43.3	25.0	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. -	64°		63°		62°		61°		60°		Lat. -
Dep. -	116°		117°		118°		119°		120°		Lat. +
Dep. +	244°		243°		242°		241°		240°		Lat. +
Dep. +	296°		297°		298°		299°		300°		Lat. -
Direction measured from North through East.											

Direction from 31° to 35° from a cardinal point.

Direction measured from North through East.											
Lat. —	329°	328°	327°	326°	325°	Dep. +					
Lat. +	211°	212°	213°	214°	215°	Dep. +					
Lat. +	149°	148°	147°	146°	145°	Dep. —					
Lat. —	31°	32°	33°	34°	35°	Dep. —					
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	0.9	0.5	0.8	0.5	0.8	0.5	0.8	0.6	0.8	0.6	1
2	1.7	1.0	1.7	1.1	1.7	1.1	1.7	1.1	1.6	1.1	2
3	2.6	1.5	2.5	1.6	2.5	1.6	2.5	1.7	2.5	1.7	3
4	3.4	2.1	3.4	2.1	3.4	2.2	3.3	2.2	3.3	2.3	4
5	4.3	2.6	4.2	2.6	4.2	2.7	4.1	2.8	4.1	2.9	5
6	5.1	3.1	5.1	3.2	5.0	3.3	5.0	3.4	4.9	3.4	6
7	6.0	3.6	5.9	3.7	5.9	3.8	5.8	3.9	5.7	4.0	7
8	6.9	4.1	6.8	4.2	6.7	4.4	6.6	4.5	6.6	4.6	8
9	7.7	4.6	7.6	4.8	7.5	4.9	7.5	5.0	7.4	5.2	9
10	8.6	5.2	8.5	5.3	8.4	5.4	8.3	5.6	8.2	5.7	10
11	9.4	5.7	9.3	5.8	9.2	6.0	9.1	6.2	9.0	6.3	11
12	10.3	6.2	10.2	6.4	10.1	6.5	9.9	6.7	9.8	6.9	12
13	11.1	6.7	11.0	6.9	10.9	7.1	10.8	7.3	10.6	7.5	13
14	12.0	7.2	11.9	7.4	11.7	7.6	11.6	7.8	11.5	8.0	14
15	12.9	7.7	12.7	7.9	12.6	8.2	12.4	8.4	12.3	8.6	15
16	13.7	8.2	13.6	8.5	13.4	8.7	13.3	8.9	13.1	9.2	16
17	14.6	8.8	14.4	9.0	14.3	9.3	14.1	9.5	13.9	9.8	17
18	15.4	9.3	15.3	9.5	15.1	9.8	14.9	10.1	14.7	10.3	18
19	16.3	9.8	16.1	10.1	15.9	10.3	15.8	10.6	15.6	10.9	19
20	17.1	10.3	17.0	10.6	16.8	10.9	16.6	11.2	16.4	11.5	20
21	18.0	10.8	17.8	11.1	17.6	11.4	17.4	11.7	17.2	12.0	21
22	18.9	11.3	18.7	11.7	18.5	12.0	18.2	12.3	18.0	12.6	22
23	19.7	11.8	19.5	12.2	19.3	12.5	19.1	12.9	18.8	13.2	23
24	20.6	12.4	20.4	12.7	20.1	13.1	19.9	13.4	19.7	13.8	24
25	21.4	12.9	21.2	13.2	21.0	13.6	20.7	14.0	20.5	14.3	25
26	22.3	13.4	22.0	13.8	21.8	14.2	21.6	14.5	21.3	14.9	26
27	23.1	13.9	22.9	14.3	22.6	14.7	22.4	15.1	22.1	15.5	27
28	24.0	14.4	23.7	14.8	23.5	15.2	23.2	15.7	22.9	16.1	28
29	24.9	14.9	24.6	15.4	24.3	15.8	24.0	16.2	23.8	16.6	29
30	25.7	15.5	25.4	15.9	25.2	16.3	24.9	16.8	24.6	17.2	30
31	26.6	16.0	26.3	16.4	26.0	16.9	25.7	17.3	25.4	17.8	31
32	27.4	16.5	27.1	17.0	26.8	17.4	26.5	17.9	26.2	18.4	32
33	28.3	17.0	28.0	17.5	27.7	18.0	27.4	18.5	27.0	18.9	33
34	29.1	17.5	28.8	18.0	28.5	18.5	28.2	19.0	27.9	19.5	34
35	30.0	18.0	29.7	18.5	29.4	19.1	29.0	19.6	28.7	20.1	35
36	30.9	18.5	30.5	19.1	30.2	19.6	29.8	20.1	29.5	20.6	36
37	31.7	19.1	31.4	19.6	31.0	20.2	30.7	20.7	30.3	21.2	37
38	32.6	19.6	32.2	20.1	31.9	20.7	31.5	21.2	31.1	21.8	38
39	33.4	20.1	33.1	20.7	32.7	21.2	32.3	21.8	31.9	22.4	39
40	34.3	20.6	33.9	21.2	33.5	21.8	33.2	22.4	32.8	22.9	40
41	35.1	21.1	34.8	21.7	34.4	22.3	34.0	22.9	33.6	23.5	41
42	36.0	21.6	35.6	22.3	35.2	22.9	34.8	23.5	34.4	24.1	42
43	36.9	22.1	36.5	22.8	36.1	23.4	35.6	24.0	35.2	24.7	43
44	37.7	22.7	37.3	23.3	36.9	24.0	36.5	24.6	36.0	25.2	44
45	38.6	23.2	38.2	23.8	37.7	24.5	37.3	25.2	36.9	25.8	45
46	39.4	23.7	39.0	24.4	38.6	25.1	38.1	25.7	37.7	26.4	46
47	40.3	24.2	39.9	24.9	39.4	25.6	39.0	26.3	38.5	27.0	47
48	41.1	24.7	40.7	25.4	40.3	26.1	39.8	26.8	39.3	27.5	48
49	42.0	25.2	41.6	26.0	41.1	26.7	40.6	27.4	40.1	28.1	49
50	42.9	25.8	42.4	26.5	41.9	27.2	41.5	28.0	41.0	28.7	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	59°		58°		57°		56°		55°		Lat. —
Dep. —	121°		122°		123°		124°		125°		Lat. +
Dep. +	239°		238°		237°		236°		235°		Lat. +
Dep. +	301°		302°		303°		304°		305°		Lat. +

Direction measured from North through East.

Vector directed towards origin. "Latitude" is component in S. and N. direction.
 "Departure" is component in W. and E. direction.

Direction from 36° to 40° from a cardinal point.

Direction measured from North through East.											
Lat. —	324°		323°		322°		321°		320°		Dep. +
Lat. +	216°		217°		218°		219°		220°		Dep. +
Lat. —	144°		143°		142°		141°		140°		Dep. —
Lat. —	36°		37°		38°		39°		40°		Dep. —
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	0.8	0.6	0.8	0.6	0.8	0.6	0.8	0.6	0.8	0.6	1
2	1.6	1.2	1.6	1.2	1.6	1.2	1.6	1.3	1.5	1.3	2
3	2.4	1.8	2.4	1.8	2.4	1.8	2.3	1.9	2.3	1.9	3
4	3.2	2.4	3.2	2.4	3.2	2.5	3.1	2.5	3.1	2.6	4
5	4.0	2.9	4.0	3.0	3.9	3.1	3.9	3.1	3.8	3.2	5
6	4.9	3.5	4.8	3.6	4.7	3.7	4.7	3.8	4.6	3.9	6
7	5.7	4.1	5.6	4.2	5.5	4.3	5.4	4.4	5.4	4.5	7
8	6.5	4.7	6.4	4.8	6.3	4.9	6.2	5.0	6.1	5.1	8
9	7.3	5.3	7.2	5.4	7.1	5.5	7.0	5.7	6.9	5.8	9
10	8.1	5.9	8.0	6.0	7.9	6.2	7.8	6.3	7.7	6.4	10
11	8.9	6.5	8.8	6.6	8.7	6.8	8.5	6.9	8.4	7.1	11
12	9.7	7.1	9.6	7.2	9.5	7.4	9.3	7.6	9.2	7.7	12
13	10.5	7.6	10.4	7.8	10.2	8.0	10.1	8.2	10.0	8.4	13
14	11.3	8.2	11.2	8.4	11.0	8.6	10.9	8.8	10.7	9.0	14
15	12.1	8.8	12.0	9.0	11.8	9.2	11.7	9.4	11.5	9.6	15
16	12.9	9.4	12.8	9.6	12.6	9.9	12.4	10.1	12.3	10.3	16
17	13.8	10.0	13.6	10.2	13.4	10.5	13.2	10.7	13.0	10.9	17
18	14.6	10.6	14.4	10.8	14.2	11.1	14.0	11.3	13.8	11.6	18
19	15.4	11.2	15.2	11.4	15.0	11.7	14.8	12.0	14.6	12.2	19
20	16.2	11.8	16.0	12.0	15.8	12.3	15.5	12.6	15.3	12.9	20
21	17.0	12.3	16.8	12.6	16.5	12.9	16.3	13.2	16.1	13.5	21
22	17.8	12.9	17.6	13.2	17.3	13.5	17.1	13.8	16.9	14.1	22
23	18.6	13.5	18.4	13.8	18.1	14.2	17.9	14.5	17.6	14.8	23
24	19.4	14.1	19.2	14.4	18.9	14.8	18.7	15.1	18.4	15.4	24
25	20.2	14.7	20.0	15.0	19.7	15.4	19.4	15.7	19.2	16.1	25
26	21.0	15.3	20.8	15.6	20.5	16.0	20.2	16.4	19.9	16.7	26
27	21.8	15.9	21.6	16.2	21.3	16.6	21.0	17.0	20.7	17.4	27
28	22.7	16.5	22.4	16.9	22.1	17.2	21.8	17.6	21.4	18.0	28
29	23.5	17.0	23.2	17.5	22.9	17.9	22.5	18.3	22.2	18.6	29
30	24.3	17.6	24.0	18.1	23.6	18.5	23.3	18.9	23.0	19.3	30
31	25.1	18.2	24.8	18.7	24.4	19.1	24.1	19.5	23.7	19.9	31
32	25.9	18.8	25.6	19.3	25.2	19.7	24.9	20.1	24.5	20.6	32
33	26.7	19.4	26.4	19.9	26.0	20.3	25.6	20.8	25.3	21.2	33
34	27.5	20.0	27.2	20.5	26.8	20.9	26.4	21.4	26.0	21.9	34
35	28.3	20.6	28.0	21.1	27.6	21.5	27.2	22.0	26.8	22.5	35
36	29.1	21.2	28.8	21.7	28.4	22.2	28.0	22.7	27.6	23.1	36
37	29.9	21.7	29.5	22.3	29.2	22.8	28.8	23.3	28.3	23.8	37
38	30.7	22.3	30.3	22.9	29.9	23.4	29.5	23.9	29.1	24.4	38
39	31.6	22.9	31.1	23.5	30.7	24.0	30.3	24.5	29.9	25.1	39
40	32.4	23.5	31.9	24.1	31.5	24.6	31.1	25.2	30.6	25.7	40
41	33.2	24.1	32.7	24.7	32.3	25.2	31.9	25.8	31.4	26.4	41
42	34.0	24.7	33.5	25.3	33.1	25.9	32.6	26.4	32.2	27.0	42
43	34.8	25.3	34.3	25.9	33.9	26.5	33.4	27.1	32.9	27.6	43
44	35.6	25.9	35.1	26.5	34.7	27.1	34.1	27.7	33.7	28.3	44
45	36.4	26.5	35.9	27.1	35.5	27.7	35.0	28.3	34.5	28.9	45
46	37.2	27.0	36.7	27.7	36.2	28.3	35.7	28.9	35.2	29.6	46
47	38.0	27.6	37.5	28.3	37.0	28.9	36.5	29.6	36.0	30.2	47
48	38.8	28.2	38.3	28.9	37.8	29.6	37.3	30.2	36.8	30.9	48
49	39.6	28.8	39.1	29.5	38.6	30.2	38.1	30.8	37.5	31.5	49
50	40.5	29.4	39.9	30.1	39.4	30.8	38.9	31.5	38.3	32.1	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	54°		53°		52°		51°		50°		Lat. —
Dep. —	126°		127°		128°		129°		130°		Lat. +
Dep. +	234°		233°		232°		231°		230°		Lat. +
Dep. +	306°		307°		308°		309°		310°		Lat. —
Direction measured from North through East.											

Direction from 41° to 45° from a cardinal point.

Direction measured from North through East.											
Lat. —	319°	318°	317°	316°	315°	Dep. +					
Lat. +	221°	222°	223°	224°	225°	Dep. +					
Lat. +	139°	138°	137°	136°	135°	Dep. —					
Lat. —	41°	42°	43°	44°	45°	Dep. —					
Distance.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Distance.
1	0.8	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7	1
2	1.5	1.3	1.5	1.3	1.5	1.4	1.4	1.4	1.4	1.4	2
3	2.3	2.0	2.2	2.0	2.2	2.0	2.2	2.1	2.1	2.1	3
4	3.0	2.6	3.0	2.7	2.9	2.7	2.9	2.8	2.8	2.8	4
5	3.8	3.3	3.7	3.3	3.7	3.4	3.6	3.5	3.5	3.5	5
6	4.5	3.9	4.5	4.0	4.4	4.1	4.3	4.2	4.2	4.2	6
7	5.3	4.6	5.2	4.7	5.1	4.8	5.0	4.9	4.9	4.9	7
8	6.0	5.2	5.9	5.4	5.9	5.5	5.8	5.6	5.7	5.7	8
9	6.8	5.9	6.7	6.0	6.6	6.1	6.5	6.3	6.4	6.4	9
10	7.5	6.6	7.4	6.7	7.3	6.8	7.2	6.9	7.1	7.1	10
11	8.3	7.2	8.2	7.4	8.0	7.5	7.9	7.6	7.8	7.8	11
12	9.1	7.9	8.9	8.0	8.8	8.2	8.6	8.3	8.5	8.5	12
13	9.8	8.5	9.7	8.7	9.5	8.9	9.4	9.0	9.2	9.2	13
14	10.6	9.2	10.4	9.4	10.2	9.5	10.1	9.7	9.9	9.9	14
15	11.3	9.8	11.1	10.0	11.0	10.2	10.8	10.4	10.6	10.6	15
16	12.1	10.5	11.9	10.7	11.7	10.9	11.5	11.1	11.3	11.3	16
17	12.8	11.2	12.6	11.4	12.4	11.6	12.2	11.8	12.0	12.0	17
18	13.6	11.8	13.4	12.0	13.2	12.3	12.9	12.5	12.7	12.7	18
19	14.3	12.5	14.1	12.7	13.9	13.0	13.7	13.2	13.4	13.4	19
20	15.1	13.1	14.9	13.4	14.6	13.6	14.4	13.9	14.1	14.1	20
21	15.8	13.8	15.6	14.1	15.4	14.3	15.1	14.6	14.8	14.8	21
22	16.6	14.4	16.3	14.7	16.1	15.0	15.8	15.3	15.6	15.6	22
23	17.4	15.1	17.1	15.4	16.8	15.7	16.5	16.0	16.3	16.3	23
24	18.1	15.7	17.8	16.1	17.6	16.4	17.3	16.7	17.0	17.0	24
25	18.9	16.4	18.6	16.7	18.3	17.0	18.0	17.4	17.7	17.7	25
26	19.6	17.1	19.3	17.4	19.0	17.7	18.7	18.1	18.4	18.4	26
27	20.4	17.7	20.1	18.1	19.7	18.4	19.4	18.8	19.1	19.1	27
28	21.1	18.4	20.8	18.7	20.5	19.1	20.1	19.5	19.8	19.8	28
29	21.9	19.0	21.6	19.4	21.2	19.8	20.9	20.1	20.5	20.5	29
30	22.6	19.7	22.3	20.1	21.9	20.5	21.6	20.8	21.2	21.2	30
31	23.4	20.3	23.0	20.7	22.7	21.1	22.3	21.5	21.9	21.9	31
32	24.2	21.0	23.8	21.4	23.4	21.8	23.0	22.2	22.6	22.6	32
33	24.9	21.6	24.5	22.1	24.1	22.5	23.7	22.9	23.3	23.3	33
34	25.7	22.3	25.3	22.8	24.9	23.2	24.5	23.6	24.0	24.0	34
35	26.4	23.0	26.0	23.4	25.6	23.9	25.2	24.3	24.7	24.7	35
36	27.2	23.6	26.8	24.1	26.3	24.6	25.9	25.0	25.5	25.5	36
37	27.9	24.3	27.5	24.8	27.1	25.2	26.6	25.7	26.2	26.2	37
38	28.7	24.9	28.2	25.4	27.8	25.9	27.3	26.4	26.9	26.9	38
39	29.4	25.6	29.0	26.1	28.5	26.6	28.1	27.1	27.6	27.6	39
40	30.2	26.2	29.7	26.8	29.2	27.3	28.8	27.8	28.3	28.3	40
41	30.9	26.9	30.5	27.4	30.0	28.0	29.5	28.5	29.0	29.0	41
42	31.7	27.6	31.2	28.1	30.7	28.6	30.2	29.2	29.7	29.7	42
43	32.5	28.2	32.0	28.8	31.4	29.3	30.9	29.9	30.4	30.4	43
44	33.2	28.9	32.7	29.4	32.2	30.0	31.7	30.6	31.1	31.1	44
45	34.0	29.5	33.4	30.1	32.9	30.7	32.4	31.3	31.8	31.8	45
46	34.7	30.2	34.2	30.8	33.6	31.4	33.1	32.0	32.5	32.5	46
47	35.5	30.8	34.9	31.4	34.4	32.1	33.8	32.6	33.2	33.2	47
48	36.2	31.5	35.7	32.1	35.1	32.7	34.5	33.3	33.9	33.9	48
49	37.0	32.1	36.4	32.8	35.8	33.4	35.2	34.0	34.6	34.6	49
50	37.7	32.8	37.2	33.5	36.6	34.1	36.0	34.7	35.4	35.4	50
Distance.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Distance.
Dep. —	49°		48°		47°		46°		45°		Lat. —
Dep. —	131°		132°		133°		134°		135°		Lat. +
Dep. +	229°		228°		227°		226°		225°		Lat. +
Dep. +	311°		312°		313°		314°		315°		Lat. —

Direction measured from North through East.

Vector directed towards origin. "Latitude" is component in S. and N. direction.
 "Departure" is component in W. and E. direction.

TABLES OF CONVERSION FROM BRITISH TO METRIC UNITS AND VICE VERSA.

List.

	PAGE.
Inches (0.00 to 10.09) with tenths and hundredths to whole millimetres for rainfall	34, 35
Inches (0.00 to 4.99) with tenths and hundredths to millimetres and tenths for rainfall	36
Millimetres (0 to 50) and tenths to inches correct to hundredths ...	37
Millimetres (50 to 100) and tenths to inches correct to hundredths ...	38
Temperature. Degrees Fahrenheit (−99 to +150) to Degrees Absolute	39–43
Pressure. Inches of Mercury at 32°F. and lat. 45° with tenths and hundredths to millibars	44
Pressure. Millimetres of Mercury 0° C. in latitude 45° to millibars	45
Pressure. Millibars to millimetres of mercury at 0° C. in latitude 45°	45
Pressure. Millimetres of mercury (680.0 to 779.9) with tenths at 0° C. in latitude 45° to millibars and tenths	46, 47
Pressure. Inches (27 to 31.99) with tenths and hundredths to millimetres with tenths and hundredths	48
Pressure. Millimetres (705 to 799) with tenths to inches and thousandths	49, 50
Height Table. Conversion of metres to feet	51
Wind velocity. { Miles per hour to metres per second } { Metres per second to miles per hour }	52

INCHES (0.00 TO 4.99) TO MILLIMETRES. RAINFALL.
(From Crelle's Tables, Sheet 254. Values rounded off.)

Hundredths.	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
Inches.	Millimetres.									
0.0	0	0	1	1	1	1	2	2	2	2
0.1	3	3	3	3	4	4	4	4	5	5
0.2	5	5	6	6	6	6	7	7	7	7
0.3	8	8	8	8	9	9	9	9	10	10
0.4	10	10	11	11	11	11	12	12	12	12
0.5	13	13	13	13	14	14	14	14	15	15
0.6	15	15	16	16	16	17	17	17	17	18
0.7	18	18	18	19	19	19	19	20	20	20
0.8	20	21	21	21	21	22	22	22	22	23
0.9	23	23	23	24	24	24	24	25	25	25
1.0	25	26	26	26	26	27	27	27	27	28
1.1	28	28	28	29	29	29	29	30	30	30
1.2	30	31	31	31	31	32	32	32	33	33
1.3	33	33	34	34	34	34	35	35	35	35
1.4	36	36	36	36	37	37	37	37	38	38
1.5	38	38	39	39	39	39	40	40	40	40
1.6	41	41	41	41	42	42	42	42	43	43
1.7	43	43	44	44	44	44	45	45	45	45
1.8	46	46	46	46	47	47	47	47	48	48
1.9	48	49	49	49	49	50	50	50	50	51
2.0	51	51	51	52	52	52	52	53	53	53
2.1	53	54	54	54	54	55	55	55	55	56
2.2	56	56	56	57	57	57	57	58	58	58
2.3	58	59	59	59	59	60	60	60	60	61
2.4	61	61	61	62	62	62	62	63	63	63
2.5	64	64	64	64	65	65	65	65	66	66
2.6	66	66	67	67	67	67	68	68	68	68
2.7	69	69	69	69	70	70	70	70	71	71
2.8	71	71	72	72	72	72	73	73	73	73
2.9	74	74	74	74	75	75	75	75	76	76
3.0	76	76	77	77	77	77	78	78	78	78
3.1	79	79	79	80	80	80	80	81	81	81
3.2	81	82	82	82	82	83	83	83	83	84
3.3	84	84	84	85	85	85	85	86	86	86
3.4	86	87	87	87	87	88	88	88	88	89
3.5	89	89	89	90	90	90	90	91	91	91
3.6	91	92	92	92	92	93	93	93	93	94
3.7	94	94	94	95	95	95	96	96	96	96
3.8	97	97	97	97	98	98	98	98	99	99
3.9	99	99	100	100	100	100	101	101	101	101
4.0	102	102	102	102	103	103	103	103	104	104
4.1	104	104	105	105	105	105	106	106	106	106
4.2	107	107	107	107	108	108	108	108	109	109
4.3	109	109	110	110	110	110	111	111	111	112
4.4	112	112	112	113	113	113	113	114	114	114
4.5	114	115	115	115	115	116	116	116	116	117
4.6	117	117	117	118	118	118	118	119	119	119
4.7	119	120	120	120	120	121	121	121	121	122
4.8	122	122	122	123	123	123	123	124	124	124
4.9	124	125	125	125	125	126	126	126	126	127

For amounts between 10 and 20 inches, add 254 to the number expressing the units and decimals in millimetres.

Thus: $13.74 = 10 + 3.74 \text{ in.} = 254 + 95 = 349 \text{ mm.}$

For amounts between 20 inches and 30 inches add 508.

" " " 30 " " 40 " " 762.

" " " 40 " " 50 " " 1,016.

INCHES (5 TO 10.09) TO MILLIMETRES. RAINFALL.
(From Crelle's Tables, Sheet 254. Values rounded off.)

Hundredths.	·00	·01	·02	·03	·04	·05	·06	·07	·08	·09
Inches.	Millimetres.									
5.0	127	127	128	128	128	128	129	129	129	129
5.1	130	130	130	130	131	131	131	131	132	132
5.2	132	132	133	133	133	133	134	134	134	134
5.3	135	135	135	135	136	136	136	136	137	137
5.4	137	137	138	138	138	138	139	139	139	139
5.5	140	140	140	140	141	141	141	141	142	142
5.6	142	142	143	143	143	144	144	144	144	145
5.7	145	145	145	146	146	146	146	147	147	147
5.8	147	148	148	148	148	149	149	149	149	150
5.9	150	150	150	151	151	151	151	152	152	152
6.0	152	153	153	153	153	154	154	154	154	155
6.1	155	155	155	156	156	156	156	157	157	157
6.2	157	158	158	158	158	159	159	159	160	160
6.3	160	160	161	161	161	161	162	162	162	162
6.4	163	163	163	163	164	164	164	164	165	165
6.5	165	165	166	166	166	166	167	167	167	167
6.6	168	168	168	168	169	169	169	169	170	170
6.7	170	170	171	171	171	171	172	172	172	172
6.8	173	173	173	173	174	174	174	174	175	175
6.9	175	176	176	176	176	177	177	177	177	178
7.0	178	178	178	179	179	179	179	180	180	180
7.1	180	181	181	181	181	182	182	182	182	183
7.2	183	183	183	184	184	184	184	185	185	185
7.3	185	186	186	186	186	187	187	187	187	188
7.4	188	188	188	189	189	189	189	190	190	190
7.5	191	191	191	191	192	192	192	192	193	193
7.6	193	193	194	194	194	194	195	195	195	195
7.7	196	196	196	196	197	197	197	197	198	198
7.8	198	198	199	199	199	199	200	200	200	200
7.9	201	201	201	201	202	202	202	202	203	203
8.0	203	203	204	204	204	204	205	205	205	205
8.1	206	206	206	207	207	207	207	208	208	208
8.2	208	209	209	209	209	210	210	210	210	211
8.3	211	211	211	212	212	212	212	213	213	213
8.4	213	214	214	214	214	215	215	215	215	216
8.5	216	216	216	217	217	217	217	218	218	218
8.6	218	219	219	219	219	220	220	220	220	221
8.7	221	221	221	222	222	222	223	223	223	223
8.8	224	224	224	224	225	225	225	225	226	226
8.9	226	226	227	227	227	227	228	228	228	228
9.0	229	229	229	229	230	230	230	230	231	231
9.1	231	231	232	232	232	232	233	233	233	233
9.2	234	234	234	234	235	235	235	235	236	236
9.3	236	236	237	237	237	237	238	238	238	239
9.4	239	239	239	240	240	240	240	241	241	241
9.5	241	242	242	242	242	243	243	243	243	244
9.6	244	244	244	245	245	245	246	246	246	246
9.7	246	247	247	247	247	248	248	248	248	249
9.8	249	249	249	250	250	250	250	251	251	251
9.9	251	252	252	252	252	253	253	253	253	254
10.0	254	254	254	255	255	255	256	256	256	256

For amounts of rainfall between 50 inches and 100 inches add 254 for each ten inches.

Thus: between 50 and 60 ins. add 1,270.

" 60 and 70 " " 1,524.

" 70 and 80 " " 1,778.

INCHES (0.00 TO 4.99) TO MILLIMETRES AND TENTHS. RAINFALL.
From Crelle's Tables, Sheet 254. Values rounded off.

Hundredths.	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
Inches.	Millimetres.									
0.0	0.0	0.3	0.5	0.8	1.0	1.3	1.5	1.8	2.0	2.3
0.1	2.5	2.8	3.0	3.3	3.6	3.8	4.1	4.3	4.6	4.8
0.2	5.1	5.3	5.6	5.8	6.1	6.4	6.6	6.9	7.1	7.4
0.3	7.6	7.9	8.1	8.4	8.6	8.9	9.1	9.4	9.7	9.9
0.4	10.2	10.4	10.7	10.9	11.2	11.4	11.7	11.9	12.2	12.4
0.5	12.7	13.0	13.2	13.5	13.7	14.0	14.2	14.5	14.7	15.0
0.6	15.2	15.5	15.7	16.0	16.3	16.5	16.8	17.0	17.3	17.5
0.7	17.8	18.0	18.3	18.5	18.8	19.1	19.3	19.6	19.8	20.1
0.8	20.3	20.6	20.8	21.1	21.3	21.6	21.8	22.1	22.4	22.6
0.9	22.9	23.1	23.4	23.6	23.9	24.1	24.4	24.6	24.9	25.1
1.0	25.4	25.7	25.9	26.2	26.4	26.7	26.9	27.2	27.4	27.7
1.1	27.9	28.2	28.4	28.7	29.0	29.2	29.5	29.7	30.0	30.2
1.2	30.5	30.7	31.0	31.2	31.5	31.8	32.0	32.3	32.5	32.8
1.3	33.0	33.3	33.5	33.8	34.0	34.3	34.5	34.8	35.1	35.3
1.4	35.6	35.8	36.1	36.3	36.6	36.8	37.1	37.3	37.6	37.8
1.5	38.1	38.4	38.6	38.9	39.1	39.4	39.6	39.9	40.1	40.4
1.6	40.6	40.9	41.1	41.4	41.7	41.9	42.2	42.4	42.7	42.9
1.7	43.2	43.4	43.7	43.9	44.2	44.5	44.7	45.0	45.2	45.5
1.8	45.7	46.0	46.2	46.5	46.7	47.0	47.2	47.5	47.8	48.0
1.9	48.3	48.5	48.8	49.0	49.3	49.5	49.8	50.0	50.3	50.5
2.0	50.8	51.1	51.3	51.6	51.8	52.1	52.3	52.6	52.8	53.1
2.1	53.3	53.6	53.8	54.1	54.4	54.6	54.9	55.1	55.4	55.6
2.2	55.9	56.1	56.4	56.6	56.9	57.2	57.4	57.7	57.9	58.2
2.3	58.4	58.7	58.9	59.2	59.4	59.7	59.9	60.2	60.5	60.7
2.4	61.0	61.2	61.5	61.7	62.0	62.2	62.5	62.7	63.0	63.2
2.5	63.5	63.8	64.0	64.3	64.5	64.8	65.0	65.3	65.5	65.8
2.6	66.0	66.3	66.5	66.8	67.1	67.3	67.6	67.8	68.1	68.3
2.7	68.6	68.8	69.1	69.3	69.6	69.9	70.1	70.4	70.6	70.9
2.8	71.1	71.4	71.6	71.9	72.1	72.4	72.6	72.9	73.2	73.4
2.9	73.7	73.9	74.2	74.4	74.7	74.9	75.2	75.4	75.7	75.9
3.0	76.2	76.5	76.7	77.0	77.2	77.5	77.7	78.0	78.2	78.5
3.1	78.7	79.0	79.2	79.5	79.8	80.0	80.3	80.5	80.8	81.0
3.2	81.3	81.5	81.8	82.0	82.3	82.6	82.8	83.1	83.3	83.6
3.3	83.8	84.1	84.3	84.6	84.8	85.1	85.3	85.6	85.9	86.1
3.4	86.4	86.6	86.9	87.1	87.4	87.6	87.9	88.1	88.4	88.6
3.5	88.9	89.2	89.4	89.7	89.9	90.2	90.4	90.7	90.9	91.2
3.6	91.4	91.7	91.9	92.2	92.5	92.7	93.0	93.2	93.5	93.7
3.7	94.0	94.2	94.5	94.7	95.0	95.3	95.5	95.8	96.0	96.3
3.8	96.5	96.8	97.0	97.3	97.5	97.8	98.0	98.3	98.6	98.8
3.9	99.1	99.3	99.6	99.8	100.1	100.3	100.6	100.8	101.1	101.3
4.0	101.6	101.9	102.1	102.4	102.6	102.9	103.1	103.4	103.6	103.9
4.1	104.1	104.4	104.6	104.9	105.2	105.4	105.7	105.9	106.2	106.4
4.2	106.7	106.9	107.2	107.4	107.7	108.0	108.2	108.5	108.7	109.0
4.3	109.2	109.5	109.7	110.0	110.2	110.5	110.7	111.0	111.3	111.5
4.4	111.8	112.0	112.3	112.5	112.8	113.0	113.3	113.5	113.8	114.0
4.5	114.3	114.6	114.8	115.1	115.3	115.6	115.8	116.1	116.3	116.6
4.6	116.8	117.1	117.3	117.6	117.9	118.1	118.4	118.6	118.9	119.1
4.7	119.4	119.6	119.9	120.1	120.4	120.7	120.9	121.2	121.4	121.7
4.8	121.9	122.2	122.4	122.7	122.9	123.2	123.4	123.7	124.0	124.2
4.9	124.5	124.7	125.0	125.2	125.5	125.7	126.0	126.2	126.5	126.7

For amounts above 5 inches and below 10 inches add 127 to the numbers above.
Thus 8.63 inches = $5 + 3.63 = 127 + 92.2 = 219.2$ mm.

For amounts above 10 inches add 254. Thus 13.63 inches = $254 + 92.2 = 346.2$ mm.

MILLIMETRES (0 TO 50) AND TENTHS TO INCHES CORRECT TO HUNDREDTHS,
ON THE BASIS OF 100 MM. = 3.93701 IN.

Tenths.	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
mm.	Inches.									
0	.00	.00	.01	.01	.02	.02	.03	.03	.04	
1	.04	.04	.05	.05	.06	.06	.07	.07	.07	
2	.08	.08	.09	.09	.09	.10	.10	.11	.11	
3	.12	.12	.13	.13	.13	.14	.14	.15	.15	
4	.16	.16	.17	.17	.17	.18	.18	.19	.19	
5	.20	.20	.20	.21	.21	.22	.22	.22	.23	
6	.24	.24	.24	.25	.25	.26	.26	.26	.27	
7	.28	.28	.28	.29	.29	.30	.30	.30	.31	
8	.31	.32	.32	.33	.33	.33	.34	.34	.35	
9	.35	.36	.36	.37	.37	.37	.38	.38	.39	
10	.39	.40	.40	.41	.41	.41	.42	.42	.43	
11	.43	.44	.44	.44	.45	.45	.46	.46	.47	
12	.47	.48	.48	.48	.49	.49	.50	.50	.51	
13	.51	.52	.52	.52	.53	.53	.54	.54	.55	
14	.55	.56	.56	.56	.57	.57	.57	.58	.59	
15	.59	.59	.60	.60	.61	.61	.61	.62	.62	
16	.63	.63	.64	.64	.65	.65	.65	.66	.67	
17	.67	.67	.68	.68	.69	.69	.70	.70	.70	
18	.71	.71	.72	.72	.72	.73	.73	.74	.74	
19	.75	.75	.76	.76	.76	.77	.77	.78	.78	
20	.79	.79	.80	.80	.80	.81	.81	.81	.82	
21	.83	.83	.83	.84	.84	.85	.85	.85	.86	
22	.87	.87	.87	.88	.88	.89	.89	.89	.90	
23	.91	.91	.91	.92	.92	.93	.93	.93	.94	
24	.95	.95	.95	.96	.96	.96	.97	.97	.98	
25	.98	.99	.99	1.00	1.00	1.00	1.01	1.01	1.02	
26	1.02	1.03	1.03	1.04	1.04	1.04	1.05	1.05	1.06	
27	1.06	1.07	1.07	1.07	1.08	1.08	1.09	1.09	1.10	
28	1.10	1.10	1.11	1.11	1.12	1.12	1.13	1.13	1.14	
29	1.14	1.15	1.15	1.15	1.16	1.16	1.17	1.17	1.18	
30	1.18	1.19	1.19	1.19	1.20	1.20	1.21	1.21	1.22	
31	1.22	1.22	1.23	1.23	1.24	1.24	1.24	1.25	1.26	
32	1.26	1.26	1.27	1.27	1.28	1.28	1.28	1.29	1.30	
33	1.30	1.30	1.31	1.31	1.31	1.32	1.32	1.33	1.33	
34	1.34	1.34	1.35	1.35	1.35	1.36	1.36	1.37	1.37	
35	1.38	1.38	1.39	1.39	1.39	1.40	1.40	1.41	1.41	
36	1.42	1.42	1.43	1.43	1.43	1.44	1.44	1.44	1.45	
37	1.46	1.46	1.46	1.47	1.47	1.48	1.48	1.48	1.49	
38	1.50	1.50	1.50	1.51	1.51	1.52	1.52	1.52	1.53	
39	1.54	1.54	1.54	1.55	1.55	1.56	1.56	1.56	1.57	
40	1.57	1.58	1.58	1.59	1.59	1.59	1.60	1.60	1.61	
41	1.61	1.62	1.62	1.63	1.63	1.63	1.64	1.64	1.65	
42	1.65	1.66	1.66	1.67	1.67	1.67	1.68	1.68	1.69	
43	1.69	1.70	1.70	1.71	1.71	1.71	1.72	1.72	1.73	
44	1.73	1.74	1.74	1.74	1.75	1.75	1.76	1.76	1.77	
45	1.77	1.78	1.78	1.78	1.79	1.79	1.80	1.80	1.81	
46	1.81	1.81	1.82	1.82	1.83	1.83	1.83	1.84	1.85	
47	1.85	1.85	1.86	1.86	1.87	1.87	1.87	1.88	1.89	
48	1.89	1.89	1.90	1.90	1.91	1.91	1.91	1.92	1.93	
49	1.93	1.93	1.94	1.94	1.94	1.95	1.95	1.96	1.96	

The table also serves for converting whole millimetres, from 0 to 500, to inches and tenths.

MILLIMETRES (50 TO 100) AND TENTHS TO INCHES, CORRECT TO HUNDREDTHS,
ON THE BASIS OF 100 MM. = 3.93701 IN.

Tenths	0	1	2	3	4	5	6	7	8	9
mm.	Inches.									
50	1'97	1'97	1'98	1'98	1'98	1'99	2'00	2'00	2'00	2'00
51	2'01	2'01	2'02	2'02	2'02	2'03	2'03	2'04	2'04	2'04
52	2'05	2'05	2'06	2'06	2'06	2'07	2'07	2'07	2'08	2'08
53	2'09	2'09	2'09	2'10	2'10	2'11	2'11	2'12	2'12	2'12
54	2'13	2'13	2'13	2'14	2'14	2'15	2'15	2'15	2'16	2'16
55	2'17	2'17	2'17	2'18	2'18	2'19	2'19	2'19	2'20	2'20
56	2'20	2'21	2'21	2'22	2'22	2'22	2'23	2'23	2'24	2'24
57	2'24	2'25	2'25	2'26	2'26	2'26	2'27	2'27	2'28	2'28
58	2'28	2'29	2'29	2'30	2'30	2'30	2'31	2'31	2'32	2'32
59	2'32	2'33	2'33	2'33	2'34	2'34	2'35	2'35	2'35	2'36
60	2'36	2'37	2'37	2'37	2'38	2'38	2'39	2'39	2'39	2'40
61	2'40	2'41	2'41	2'41	2'42	2'42	2'43	2'43	2'43	2'44
62	2'44	2'44	2'45	2'45	2'46	2'46	2'47	2'47	2'48	2'48
63	2'48	2'48	2'49	2'49	2'50	2'50	2'50	2'51	2'51	2'52
64	2'52	2'52	2'53	2'53	2'54	2'54	2'55	2'55	2'55	2'56
65	2'56	2'56	2'57	2'57	2'57	2'58	2'58	2'59	2'59	2'59
66	2'60	2'60	2'61	2'61	2'61	2'62	2'62	2'63	2'63	2'63
67	2'64	2'64	2'65	2'65	2'65	2'66	2'66	2'67	2'67	2'67
68	2'68	2'68	2'69	2'69	2'69	2'70	2'70	2'71	2'71	2'71
69	2'72	2'72	2'72	2'73	2'73	2'74	2'74	2'74	2'75	2'75
70	2'76	2'76	2'76	2'77	2'77	2'78	2'78	2'78	2'79	2'79
71	2'80	2'80	2'80	2'81	2'81	2'82	2'82	2'83	2'83	2'83
72	2'83	2'84	2'84	2'85	2'85	2'86	2'86	2'87	2'87	2'87
73	2'87	2'88	2'88	2'89	2'89	2'89	2'90	2'90	2'91	2'91
74	2'91	2'92	2'92	2'93	2'93	2'93	2'94	2'94	2'94	2'95
75	2'95	2'96	2'96	2'96	2'97	2'97	2'98	2'98	2'98	2'99
76	2'99	3'00	3'00	3'00	3'01	3'01	3'02	3'02	3'02	3'03
77	3'03	3'04	3'04	3'04	3'05	3'05	3'06	3'06	3'06	3'07
78	3'07	3'07	3'08	3'08	3'09	3'09	3'09	3'10	3'10	3'11
79	3'11	3'11	3'12	3'12	3'13	3'13	3'13	3'14	3'14	3'15
80	3'15	3'15	3'16	3'16	3'17	3'17	3'17	3'18	3'18	3'19
81	3'19	3'19	3'20	3'20	3'21	3'21	3'22	3'22	3'22	3'23
82	3'23	3'23	3'24	3'24	3'24	3'25	3'25	3'26	3'26	3'26
83	3'27	3'27	3'28	3'28	3'28	3'29	3'29	3'30	3'30	3'30
84	3'31	3'31	3'31	3'32	3'32	3'33	3'33	3'33	3'34	3'34
85	3'35	3'35	3'35	3'36	3'36	3'37	3'37	3'37	3'38	3'38
86	3'39	3'39	3'39	3'40	3'40	3'41	3'41	3'42	3'42	3'42
87	3'43	3'43	3'43	3'44	3'44	3'44	3'45	3'45	3'46	3'46
88	3'46	3'47	3'47	3'48	3'48	3'48	3'49	3'49	3'50	3'50
89	3'50	3'51	3'51	3'52	3'52	3'53	3'53	3'54	3'54	3'54
90	3'54	3'55	3'55	3'56	3'56	3'57	3'57	3'57	3'58	3'58
91	3'58	3'59	3'59	3'59	3'60	3'60	3'61	3'61	3'62	3'62
92	3'62	3'63	3'63	3'63	3'64	3'64	3'65	3'65	3'66	3'66
93	3'66	3'67	3'67	3'67	3'68	3'68	3'69	3'69	3'70	3'70
94	3'70	3'70	3'71	3'71	3'72	3'72	3'72	3'73	3'73	3'74
95	3'74	3'74	3'75	3'75	3'76	3'76	3'76	3'77	3'77	3'78
96	3'78	3'78	3'79	3'79	3'80	3'80	3'81	3'81	3'81	3'82
97	3'82	3'82	3'83	3'83	3'83	3'84	3'84	3'85	3'85	3'85
98	3'86	3'86	3'87	3'87	3'88	3'88	3'89	3'89	3'89	3'90
99	3'90	3'90	3'91	3'91	3'92	3'92	3'93	3'93	3'93	3'93

The table also serves for converting whole millimetres, from 500 to 1,000, to inches and tenths.

TEMPERATURE. DEGREES FAHRENHEIT TO DEGREES ABSOLUTE.

Tenths.	0	1	2	3	4	5	6	7	8	9
Degrees Fah.	Degrees Absolute, 200 +									
—99	2	2	1	1	0	—	—	—	—	—
—98	8	7	7	6	6	5	4	4	3	3
—97	13	13	12	12	11	11	10	9	9	8
—96	19	18	18	17	17	16	16	15	14	14
—95	24	24	23	23	22	22	21	21	20	19
—94	30	29	29	28	28	27	27	26	26	25
—93	36	35	34	34	33	33	32	32	31	31
—92	41	41	40	39	39	38	38	37	37	36
—91	47	46	46	45	44	44	43	43	42	42
—90	52	52	51	51	50	49	49	48	48	47
—89	58	57	57	56	56	55	54	54	53	53
—88	63	63	62	62	61	61	60	59	59	58
—87	69	68	68	67	67	66	66	65	64	64
—86	74	74	73	73	72	72	71	71	70	69
—85	80	79	79	78	78	77	77	76	76	75
—84	86	85	84	84	83	83	82	82	81	81
—83	91	91	90	89	89	88	88	87	87	86
—82	97	96	96	95	94	94	93	93	92	92
—81	102	102	101	101	100	99	99	98	98	97
—80	108	107	107	106	106	105	104	104	103	103
—79	113	113	112	112	111	111	110	109	109	108
—78	119	118	118	117	117	116	116	115	114	114
—77	124	124	123	123	122	122	121	121	120	119
—76	130	129	129	128	128	127	127	126	126	125
—75	136	135	134	134	133	133	132	132	131	131
—74	141	141	140	139	139	138	138	137	137	136
—73	147	146	146	145	144	144	143	143	142	142
—72	152	152	151	151	150	149	149	148	148	147
—71	158	157	157	156	156	155	154	154	153	153
—70	163	163	162	162	161	161	160	159	159	158
—69	169	168	168	167	167	166	166	165	164	164
—68	174	174	173	173	172	172	171	171	170	169
—67	180	179	179	178	178	177	177	176	176	175
—66	186	185	184	184	183	183	182	182	181	181
—65	191	191	190	189	189	188	188	187	187	186
—64	197	196	196	195	194	194	193	193	192	192
—63	202	202	201	201	200	199	199	198	198	197
—62	208	207	207	206	206	205	204	204	203	203
—61	213	213	212	212	211	211	210	209	209	208
—60	219	218	218	217	217	216	216	215	214	214
—59	224	224	223	223	222	222	221	221	220	219
—58	230	229	229	228	228	227	227	226	226	225
—57	236	235	234	234	233	233	232	232	231	231
—56	241	241	240	239	239	238	238	237	237	236
—55	247	246	246	245	244	244	243	243	242	242
—54	252	252	251	251	250	249	249	248	248	247
—53	258	257	257	256	256	255	254	254	253	253
—52	263	263	262	262	261	261	260	259	259	258
—51	269	268	268	267	267	266	266	265	264	264
—50	274	274	273	273	272	272	271	271	270	269

TEMPERATURE. DEGREES FAHRENHEIT TO DEGREES ABSOLUTE.

Tenths.	0	1	2	3	4	5	6	7	8	9
Degrees Fah.	Degrees Absolute, 200 +									
-49	28.0	27.9	27.9	27.8	27.8	27.7	27.7	27.6	27.6	27.5
-48	28.6	28.5	28.4	28.4	28.3	28.3	28.2	28.1	28.1	28.0
-47	29.1	29.1	29.0	28.9	28.9	28.8	28.8	28.7	28.7	28.6
-46	29.7	29.6	29.6	29.5	29.4	29.4	29.3	29.3	29.2	29.2
-45	30.2	30.2	30.1	30.1	30.0	29.9	29.9	29.8	29.8	29.7
-44	30.8	30.7	30.7	30.6	30.6	30.5	30.4	30.4	30.3	30.3
-43	31.3	31.3	31.2	31.2	31.1	31.1	31.0	30.9	30.9	30.8
-42	31.9	31.8	31.8	31.7	31.7	31.6	31.5	31.4	31.4	31.3
-41	32.4	32.4	32.3	32.3	32.2	32.2	32.1	32.1	32.0	31.9
-40	33.0	32.9	32.9	32.8	32.8	32.7	32.7	32.6	32.6	32.5
-39	33.6	33.5	33.4	33.4	33.3	33.3	33.2	33.2	33.1	33.1
-38	34.1	34.1	34.0	33.9	33.9	33.8	33.8	33.7	33.7	33.6
-37	34.7	34.6	34.6	34.5	34.4	34.4	34.3	34.3	34.2	34.2
-36	35.2	35.2	35.1	35.1	35.0	34.9	34.9	34.8	34.8	34.7
-35	35.8	35.7	35.7	35.6	35.6	35.5	35.4	35.4	35.3	35.3
-34	36.3	36.3	36.2	36.2	36.1	36.1	36.0	35.9	35.9	35.8
-33	36.9	36.8	36.8	36.7	36.7	36.6	36.6	36.5	36.4	36.4
-32	37.4	37.4	37.3	37.3	37.2	37.2	37.1	37.1	37.0	36.9
-31	38.0	37.9	37.9	37.8	37.8	37.7	37.7	37.6	37.6	37.5
-30	38.6	38.5	38.4	38.4	38.3	38.3	38.2	38.2	38.1	38.1
-29	39.1	39.1	39.0	38.9	38.9	38.8	38.8	38.7	38.7	38.6
-28	39.7	39.6	39.6	39.5	39.4	39.4	39.3	39.3	39.2	39.2
-27	40.2	40.2	40.1	40.1	40.0	39.9	39.9	39.8	39.8	39.7
-26	40.8	40.7	40.7	40.6	40.6	40.5	40.4	40.4	40.3	40.3
-25	41.3	41.3	41.2	41.2	41.1	41.1	41.0	40.9	40.9	40.8
-24	41.9	41.8	41.8	41.7	41.7	41.6	41.6	41.5	41.4	41.4
-23	42.4	42.4	42.3	42.3	42.2	42.2	42.1	42.1	42.0	41.9
-22	43.0	42.9	42.9	42.8	42.8	42.7	42.7	42.6	42.6	42.5
-21	43.6	43.5	43.4	43.4	43.3	43.3	43.2	43.2	43.1	43.1
-20	44.1	44.1	44.0	43.9	43.9	43.8	43.8	43.7	43.7	43.6
-19	44.7	44.6	44.6	44.5	44.4	44.4	44.3	44.3	44.2	44.2
-18	45.2	45.2	45.1	45.1	45.0	44.9	44.9	44.8	44.8	44.7
-17	45.8	45.7	45.7	45.6	45.6	45.5	45.4	45.4	45.3	45.3
-16	46.3	46.3	46.2	46.2	46.1	46.1	46.0	45.9	45.9	45.8
-15	46.9	46.8	46.8	46.7	46.7	46.6	46.6	46.5	46.4	46.4
-14	47.4	47.4	47.3	47.3	47.2	47.2	47.1	47.1	47.0	46.9
-13	48.0	47.9	47.9	47.8	47.8	47.7	47.7	47.6	47.6	47.5
-12	48.6	48.5	48.4	48.4	48.3	48.3	48.2	48.2	48.1	48.1
-11	49.1	49.1	49.0	48.9	48.9	48.8	48.8	48.7	48.7	48.6
-10	49.7	49.6	49.6	49.5	49.4	49.4	49.3	49.3	49.2	49.2
-9	50.2	50.2	50.1	50.1	50.0	49.9	49.9	49.8	49.8	49.7
-8	50.8	50.7	50.7	50.6	50.6	50.5	50.4	50.4	50.3	50.3
-7	51.3	51.3	51.2	51.2	51.1	51.1	51.0	50.9	50.9	50.8
-6	51.9	51.8	51.8	51.7	51.7	51.6	51.6	51.5	51.4	51.4
-5	52.4	52.4	52.3	52.3	52.2	52.2	52.1	52.1	52.0	51.9
-4	53.0	52.9	52.9	52.8	52.8	52.7	52.7	52.6	52.6	52.5
-3	53.6	53.5	53.4	53.4	53.3	53.3	53.2	53.2	53.1	53.1
-2	54.1	54.1	54.0	53.9	53.9	53.8	53.8	53.7	53.7	53.6
-1	54.7	54.6	54.6	54.5	54.4	54.4	54.3	54.3	54.2	54.2
0	55.2	55.2	55.1	55.1	55.0	54.9	54.9	54.8	54.8	54.7

TEMPERATURE. DEGREES FAHRENHEIT TO DEGREES ABSOLUTE.

Tenths.	0	1	2	3	4	5	6	7	8	9
Degrees Fah.	Degrees Absolute, 200 +									
0	55.2	55.3	55.3	55.4	55.4	55.5	55.6	55.6	55.7	55.7
1	55.8	55.8	55.9	55.9	56.0	56.1	56.1	56.2	56.2	56.3
2	56.3	56.4	56.4	56.5	56.6	56.6	56.7	56.7	56.8	56.8
3	56.9	56.9	57.0	57.1	57.1	57.2	57.2	57.3	57.3	57.4
4	57.4	57.5	57.6	57.6	57.7	57.7	57.8	57.8	57.9	57.9
5	58.0	58.1	58.1	58.2	58.2	58.3	58.3	58.4	58.4	58.5
6	58.6	58.6	58.7	58.7	58.8	58.8	58.9	58.9	59.0	59.1
7	59.1	59.2	59.2	59.3	59.3	59.4	59.4	59.5	59.6	59.6
8	59.7	59.7	59.8	59.8	59.9	59.9	60.0	60.1	60.1	60.2
9	60.2	60.3	60.3	60.4	60.4	60.5	60.6	60.6	60.7	60.7
10	60.8	60.8	60.9	60.9	61.0	61.1	61.1	61.2	61.2	61.3
11	61.3	61.4	61.4	61.5	61.6	61.6	61.7	61.7	61.8	61.8
12	61.9	61.9	62.0	62.1	62.1	62.2	62.2	62.3	62.3	62.4
13	62.4	62.5	62.6	62.6	62.7	62.7	62.8	62.8	62.9	62.9
14	63.0	63.1	63.1	63.2	63.2	63.3	63.3	63.4	63.4	63.5
15	63.6	63.6	63.7	63.7	63.8	63.8	63.9	63.9	64.0	64.1
16	64.1	64.2	64.2	64.3	64.3	64.4	64.4	64.5	64.6	64.6
17	64.7	64.7	64.8	64.8	64.9	64.9	65.0	65.1	65.1	65.2
18	65.2	65.3	65.3	65.4	65.4	65.5	65.6	65.6	65.7	65.7
19	65.8	65.8	65.9	65.9	66.0	66.1	66.1	66.2	66.2	66.3
20	66.3	66.4	66.4	66.5	66.6	66.6	66.7	66.7	66.8	66.8
21	66.9	66.9	67.0	67.1	67.1	67.2	67.2	67.3	67.3	67.4
22	67.4	67.5	67.6	67.6	67.7	67.7	67.8	67.8	67.9	67.9
23	68.0	68.1	68.1	68.2	68.2	68.3	68.3	68.4	68.4	68.5
24	68.6	68.6	68.7	68.7	68.8	68.8	68.9	68.9	69.0	69.1
25	69.1	69.2	69.2	69.3	69.3	69.4	69.4	69.5	69.6	69.6
26	69.7	69.7	69.8	69.8	69.9	69.9	70.0	70.1	70.1	70.2
27	70.2	70.3	70.3	70.4	70.4	70.5	70.6	70.7	70.7	70.8
28	70.8	70.8	70.9	70.9	71.0	71.1	71.1	71.2	71.2	71.3
29	71.3	71.4	71.4	71.5	71.6	71.6	71.7	71.7	71.8	71.8
30	71.9	71.9	72.0	72.1	72.1	72.2	72.2	72.3	72.3	72.4
31	72.4	72.5	72.6	72.6	72.7	72.7	72.8	72.8	72.9	72.9
32	73.0	73.1	73.1	73.2	73.2	73.3	73.3	73.4	73.4	73.5
33	73.6	73.6	73.7	73.7	73.8	73.8	73.9	73.9	74.0	74.1
34	74.1	74.2	74.2	74.3	74.3	74.4	74.4	74.5	74.6	74.6
35	74.7	74.7	74.8	74.8	74.9	74.9	75.0	75.1	75.1	75.2
36	75.2	75.3	75.3	75.4	75.4	75.5	75.6	75.7	75.7	75.8
37	75.8	75.8	75.9	75.9	76.0	76.1	76.1	76.2	76.2	76.3
38	76.3	76.4	76.4	76.5	76.6	76.6	76.7	76.7	76.8	76.8
39	76.9	76.9	77.0	77.1	77.1	77.2	77.2	77.3	77.3	77.4
40	77.4	77.5	77.6	77.6	77.7	77.7	77.8	77.8	77.9	77.9
41	78.0	78.1	78.1	78.2	78.2	78.3	78.3	78.4	78.4	78.5
42	78.6	78.6	78.7	78.7	78.8	78.8	78.9	79.0	79.0	79.1
43	79.1	79.2	79.2	79.3	79.3	79.4	79.4	79.5	79.6	79.6
44	79.7	79.7	79.8	79.8	79.9	79.9	80.0	80.1	80.1	80.2
45	80.2	80.3	80.3	80.4	80.4	80.5	80.6	80.6	80.7	80.7
46	80.8	80.8	80.9	80.9	81.0	81.1	81.1	81.2	81.2	81.3
47	81.3	81.4	81.4	81.5	81.6	81.6	81.7	81.7	81.8	81.8
48	81.9	81.9	82.0	82.1	82.1	82.2	82.2	82.3	82.3	82.4
49	82.4	82.5	82.6	82.6	82.7	82.7	82.8	82.8	82.9	82.9
50	83.0	83.1	83.1	83.2	83.2	83.3	83.3	83.4	83.4	83.5

TEMPERATURE. DEGREES FAHRENHEIT TO DEGREES ABSOLUTE.

Tenths.	0	1	2	3	4	5	6	7	8	9
Degrees Fah.	Degrees Absolute, 200 +									
50	83.0	83.1	83.2	83.3	83.4	83.5	83.6	83.7	83.8	83.9
51	84.0	84.1	84.2	84.3	84.4	84.5	84.6	84.7	84.8	84.9
52	85.0	85.1	85.2	85.3	85.4	85.5	85.6	85.7	85.8	85.9
53	86.0	86.1	86.2	86.3	86.4	86.5	86.6	86.7	86.8	86.9
54	87.0	87.1	87.2	87.3	87.4	87.5	87.6	87.7	87.8	87.9
55	88.0	88.1	88.2	88.3	88.4	88.5	88.6	88.7	88.8	88.9
56	89.0	89.1	89.2	89.3	89.4	89.5	89.6	89.7	89.8	89.9
57	90.0	90.1	90.2	90.3	90.4	90.5	90.6	90.7	90.8	90.9
58	91.0	91.1	91.2	91.3	91.4	91.5	91.6	91.7	91.8	91.9
59	92.0	92.1	92.2	92.3	92.4	92.5	92.6	92.7	92.8	92.9
60	93.0	93.1	93.2	93.3	93.4	93.5	93.6	93.7	93.8	93.9
61	94.0	94.1	94.2	94.3	94.4	94.5	94.6	94.7	94.8	94.9
62	95.0	95.1	95.2	95.3	95.4	95.5	95.6	95.7	95.8	95.9
63	96.0	96.1	96.2	96.3	96.4	96.5	96.6	96.7	96.8	96.9
64	97.0	97.1	97.2	97.3	97.4	97.5	97.6	97.7	97.8	97.9
65	98.0	98.1	98.2	98.3	98.4	98.5	98.6	98.7	98.8	98.9
66	99.0	99.1	99.2	99.3	99.4	99.5	99.6	99.7	99.8	99.9
67	100.0	100.1	100.2	100.3	100.4	100.5	100.6	100.7	100.8	100.9
68	101.0	101.1	101.2	101.3	101.4	101.5	101.6	101.7	101.8	101.9
69	102.0	102.1	102.2	102.3	102.4	102.5	102.6	102.7	102.8	102.9
70	103.0	103.1	103.2	103.3	103.4	103.5	103.6	103.7	103.8	103.9
71	104.0	104.1	104.2	104.3	104.4	104.5	104.6	104.7	104.8	104.9
72	105.0	105.1	105.2	105.3	105.4	105.5	105.6	105.7	105.8	105.9
73	106.0	106.1	106.2	106.3	106.4	106.5	106.6	106.7	106.8	106.9
74	107.0	107.1	107.2	107.3	107.4	107.5	107.6	107.7	107.8	107.9
75	108.0	108.1	108.2	108.3	108.4	108.5	108.6	108.7	108.8	108.9
76	109.0	109.1	109.2	109.3	109.4	109.5	109.6	109.7	109.8	109.9
77	110.0	110.1	110.2	110.3	110.4	110.5	110.6	110.7	110.8	110.9
78	111.0	111.1	111.2	111.3	111.4	111.5	111.6	111.7	111.8	111.9
79	112.0	112.1	112.2	112.3	112.4	112.5	112.6	112.7	112.8	112.9
80	113.0	113.1	113.2	113.3	113.4	113.5	113.6	113.7	113.8	113.9
81	114.0	114.1	114.2	114.3	114.4	114.5	114.6	114.7	114.8	114.9
82	115.0	115.1	115.2	115.3	115.4	115.5	115.6	115.7	115.8	115.9
83	116.0	116.1	116.2	116.3	116.4	116.5	116.6	116.7	116.8	116.9
84	117.0	117.1	117.2	117.3	117.4	117.5	117.6	117.7	117.8	117.9
85	118.0	118.1	118.2	118.3	118.4	118.5	118.6	118.7	118.8	118.9
86	119.0	119.1	119.2	119.3	119.4	119.5	119.6	119.7	119.8	119.9
87	120.0	120.1	120.2	120.3	120.4	120.5	120.6	120.7	120.8	120.9
88	121.0	121.1	121.2	121.3	121.4	121.5	121.6	121.7	121.8	121.9
89	122.0	122.1	122.2	122.3	122.4	122.5	122.6	122.7	122.8	122.9
90	123.0	123.1	123.2	123.3	123.4	123.5	123.6	123.7	123.8	123.9
91	124.0	124.1	124.2	124.3	124.4	124.5	124.6	124.7	124.8	124.9
92	125.0	125.1	125.2	125.3	125.4	125.5	125.6	125.7	125.8	125.9
93	126.0	126.1	126.2	126.3	126.4	126.5	126.6	126.7	126.8	126.9
94	127.0	127.1	127.2	127.3	127.4	127.5	127.6	127.7	127.8	127.9
95	128.0	128.1	128.2	128.3	128.4	128.5	128.6	128.7	128.8	128.9
96	129.0	129.1	129.2	129.3	129.4	129.5	129.6	129.7	129.8	129.9
97	130.0	130.1	130.2	130.3	130.4	130.5	130.6	130.7	130.8	130.9
98	131.0	131.1	131.2	131.3	131.4	131.5	131.6	131.7	131.8	131.9
99	132.0	132.1	132.2	132.3	132.4	132.5	132.6	132.7	132.8	132.9
100	133.0	133.1	133.2	133.3	133.4	133.5	133.6	133.7	133.8	133.9

TEMPERATURE. DEGREES FAHRENHEIT TO DEGREES ABSOLUTE.

Tenths.	0	1	2	3	4	5	6	7	8	9
Degrees Fah.	Degrees Absolute, 200 +									
100	110.8	110.9	111.0	111.1	111.2	111.3	111.4	111.5	111.6	111.7
101	111.8	111.9	112.0	112.1	112.2	112.3	112.4	112.5	112.6	112.7
102	112.8	112.9	113.0	113.1	113.2	113.3	113.4	113.5	113.6	113.7
103	113.8	113.9	114.0	114.1	114.2	114.3	114.4	114.5	114.6	114.7
104	114.8	114.9	115.0	115.1	115.2	115.3	115.4	115.5	115.6	115.7
105	115.8	115.9	116.0	116.1	116.2	116.3	116.4	116.5	116.6	116.7
106	116.8	116.9	117.0	117.1	117.2	117.3	117.4	117.5	117.6	117.7
107	117.8	117.9	118.0	118.1	118.2	118.3	118.4	118.5	118.6	118.7
108	118.8	118.9	119.0	119.1	119.2	119.3	119.4	119.5	119.6	119.7
109	119.8	119.9	120.0	120.1	120.2	120.3	120.4	120.5	120.6	120.7
110	120.8	120.9	121.0	121.1	121.2	121.3	121.4	121.5	121.6	121.7
111	121.8	121.9	122.0	122.1	122.2	122.3	122.4	122.5	122.6	122.7
112	122.8	122.9	123.0	123.1	123.2	123.3	123.4	123.5	123.6	123.7
113	123.8	123.9	124.0	124.1	124.2	124.3	124.4	124.5	124.6	124.7
114	124.8	124.9	125.0	125.1	125.2	125.3	125.4	125.5	125.6	125.7
115	125.8	125.9	126.0	126.1	126.2	126.3	126.4	126.5	126.6	126.7
116	126.8	126.9	127.0	127.1	127.2	127.3	127.4	127.5	127.6	127.7
117	127.8	127.9	128.0	128.1	128.2	128.3	128.4	128.5	128.6	128.7
118	128.8	128.9	129.0	129.1	129.2	129.3	129.4	129.5	129.6	129.7
119	129.8	129.9	130.0	130.1	130.2	130.3	130.4	130.5	130.6	130.7
120	130.8	130.9	131.0	131.1	131.2	131.3	131.4	131.5	131.6	131.7
121	131.8	131.9	132.0	132.1	132.2	132.3	132.4	132.5	132.6	132.7
122	132.8	132.9	133.0	133.1	133.2	133.3	133.4	133.5	133.6	133.7
123	133.8	133.9	134.0	134.1	134.2	134.3	134.4	134.5	134.6	134.7
124	134.8	134.9	135.0	135.1	135.2	135.3	135.4	135.5	135.6	135.7
125	135.8	135.9	136.0	136.1	136.2	136.3	136.4	136.5	136.6	136.7
126	136.8	136.9	137.0	137.1	137.2	137.3	137.4	137.5	137.6	137.7
127	137.8	137.9	138.0	138.1	138.2	138.3	138.4	138.5	138.6	138.7
128	138.8	138.9	139.0	139.1	139.2	139.3	139.4	139.5	139.6	139.7
129	139.8	139.9	140.0	140.1	140.2	140.3	140.4	140.5	140.6	140.7
130	140.8	140.9	141.0	141.1	141.2	141.3	141.4	141.5	141.6	141.7
131	141.8	141.9	142.0	142.1	142.2	142.3	142.4	142.5	142.6	142.7
132	142.8	142.9	143.0	143.1	143.2	143.3	143.4	143.5	143.6	143.7
133	143.8	143.9	144.0	144.1	144.2	144.3	144.4	144.5	144.6	144.7
134	144.8	144.9	145.0	145.1	145.2	145.3	145.4	145.5	145.6	145.7
135	145.8	145.9	146.0	146.1	146.2	146.3	146.4	146.5	146.6	146.7
136	146.8	146.9	147.0	147.1	147.2	147.3	147.4	147.5	147.6	147.7
137	147.8	147.9	148.0	148.1	148.2	148.3	148.4	148.5	148.6	148.7
138	148.8	148.9	149.0	149.1	149.2	149.3	149.4	149.5	149.6	149.7
139	149.8	149.9	150.0	150.1	150.2	150.3	150.4	150.5	150.6	150.7
140	150.8	150.9	151.0	151.1	151.2	151.3	151.4	151.5	151.6	151.7
141	151.8	151.9	152.0	152.1	152.2	152.3	152.4	152.5	152.6	152.7
142	152.8	152.9	153.0	153.1	153.2	153.3	153.4	153.5	153.6	153.7
143	153.8	153.9	154.0	154.1	154.2	154.3	154.4	154.5	154.6	154.7
144	154.8	154.9	155.0	155.1	155.2	155.3	155.4	155.5	155.6	155.7
145	155.8	155.9	156.0	156.1	156.2	156.3	156.4	156.5	156.6	156.7
146	156.8	156.9	157.0	157.1	157.2	157.3	157.4	157.5	157.6	157.7
147	157.8	157.9	158.0	158.1	158.2	158.3	158.4	158.5	158.6	158.7
148	158.8	158.9	159.0	159.1	159.2	159.3	159.4	159.5	159.6	159.7
149	159.8	159.9	160.0	160.1	160.2	160.3	160.4	160.5	160.6	160.7
150	160.8	160.9	161.0	161.1	161.2	161.3	161.4	161.5	161.6	161.7

PRESSURE.

EQUIVALENTS IN MILLIBARS OF INCHES OF MERCURY

AT 32° F. AND LATITUDE 45°.

Inches of Mercury.	Hundredths of Inches.									
	0	1	2	3	4	5	6	7	8	9
	Millibars.									
27.0	914.3	914.6	915.0	915.3	915.7	916.0	916.3	916.7	917.0	917.4
27.1	917.7	918.0	918.4	918.7	919.0	919.4	919.7	920.1	920.4	920.7
27.2	921.1	921.4	921.8	922.1	922.4	922.8	923.1	923.4	923.8	924.1
27.3	924.5	924.8	925.1	925.5	925.8	926.2	926.5	926.8	927.2	927.5
27.4	927.9	928.2	928.5	928.9	929.2	929.5	929.9	930.2	930.6	930.9
27.5	931.2	931.6	931.9	932.3	932.6	932.9	933.3	933.6	933.9	934.3
27.6	934.6	935.0	935.3	935.6	936.0	936.3	936.7	937.0	937.3	937.7
27.7	938.0	938.3	938.7	939.0	939.4	939.7	940.0	940.4	940.7	941.1
27.8	941.4	941.7	942.1	942.4	942.8	943.1	943.4	943.8	944.1	944.4
27.9	944.8	945.1	945.5	945.8	946.1	946.5	946.8	947.2	947.5	947.8
28.0	948.2	948.5	948.8	949.2	949.5	949.9	950.2	950.5	950.9	951.2
28.1	951.6	951.9	952.2	952.6	952.9	953.2	953.6	953.9	954.3	954.6
28.2	954.9	955.3	955.6	956.0	956.3	956.6	957.0	957.3	957.7	958.0
28.3	958.3	958.7	959.0	959.3	959.7	960.0	960.3	960.7	961.0	961.4
28.4	961.7	962.1	962.4	962.7	963.1	963.4	963.7	964.1	964.4	964.8
28.5	965.1	965.4	965.8	966.1	966.5	966.8	967.1	967.5	967.8	968.1
28.6	968.5	968.8	969.2	969.5	969.8	970.2	970.5	970.9	971.2	971.5
28.7	971.9	972.2	972.6	972.9	973.2	973.6	973.9	974.2	974.6	974.9
28.8	975.3	975.6	975.9	976.3	976.6	976.9	977.3	977.6	978.0	978.3
28.9	978.6	979.0	979.3	979.7	980.0	980.3	980.7	981.0	981.4	981.7
29.0	982.0	982.4	982.7	983.0	983.4	983.7	984.1	984.4	984.7	985.1
29.1	985.4	985.8	986.1	986.4	986.8	987.1	987.5	987.8	988.1	988.5
29.2	988.8	989.1	989.5	989.8	990.2	990.5	990.8	991.2	991.5	991.9
29.3	992.2	992.5	992.9	993.2	993.5	993.9	994.2	994.6	994.9	995.2
29.4	995.6	995.9	996.3	996.6	996.9	997.3	997.6	997.9	998.3	998.6
29.5	999.0	999.3	999.6	1000.0	1000.3	1000.7	1001.0	1001.3	1001.7	1002.0
29.6	1002.4	1002.7	1003.0	1003.4	1003.7	1004.0	1004.4	1004.7	1005.1	1005.4
29.7	1005.7	1006.1	1006.4	1006.8	1007.1	1007.4	1007.8	1008.1	1008.4	1008.8
29.8	1009.1	1009.5	1009.8	1010.1	1010.5	1010.8	1011.2	1011.5	1011.8	1012.2
29.9	1012.5	1012.8	1013.2	1013.5	1013.9	1014.2	1014.5	1014.9	1015.2	1015.6
30.0	1015.9	1016.2	1016.6	1016.9	1017.3	1017.6	1017.9	1018.3	1018.6	1018.9
30.1	1019.3	1019.6	1020.0	1020.3	1020.6	1021.0	1021.3	1021.7	1022.0	1022.3
30.2	1022.7	1023.0	1023.3	1023.7	1024.0	1024.4	1024.7	1025.0	1025.4	1025.7
30.3	1026.1	1026.4	1026.7	1027.1	1027.4	1027.7	1028.1	1028.4	1028.8	1029.1
30.4	1029.4	1029.8	1030.1	1030.5	1030.8	1031.1	1031.5	1031.8	1032.2	1032.5
30.5	1032.8	1033.2	1033.5	1033.8	1034.2	1034.5	1034.9	1035.2	1035.5	1035.9
30.6	1036.2	1036.6	1036.9	1037.2	1037.6	1037.9	1038.3	1038.6	1038.9	1039.3
30.7	1039.6	1039.9	1040.3	1040.6	1041.0	1041.3	1041.6	1042.0	1042.3	1042.6
30.8	1043.0	1043.3	1043.7	1044.0	1044.3	1044.7	1045.0	1045.4	1045.7	1046.0
30.9	1046.4	1046.7	1047.1	1047.4	1047.7	1048.1	1048.4	1048.7	1049.1	1049.4

PRESSURE IN MILLIMETRES OF MERCURY AT 0° C. IN LATITUDE 45° TO
MILLIBARS.

1000 mm. = 1333.20 mb.

Tens.	0	10	20	30	40	50	60	70	80	90
Milli- metres.	Millibars.									
100	133	147	160	173	187	200	213	227	240	253
200	267	280	293	307	320	333	347	360	373	387
300	400	413	427	440	453	467	480	493	507	520
400	533	547	560	573	587	600	613	627	640	653
500	667	680	693	707	720	733	747	760	773	787
600	800	813	827	840	853	867	880	893	907	920
700	933	947	960	973	987	1000	1013	1027	1040	1053
800	1067	1080	1093	1107	1120	1133	1147	1160	1173	1187
900	1200	1213	1227	1240	1253	1267	1280	1293	1307	1320
1000	1333	1347	1360	1373	1387	1400	1413	1427	1440	1453

Increments for changes by millimetre intervals.

mm.	1	2	3	4	5	6	7	8	9
mb.	1.3	2.7	4.0	5.3	6.7	8.0	9.3	10.7	12.0

PRESSURE IN MILLIBARS TO MILLIMETRES OF MERCURY AT 0° C. IN LATITUDE 45°.

1000 mb. = 750.076 mm.

Tens.	0	10	20	30	40	50	60	70	80	90
Millibars.	Millimetres at 0° C. and latitude 45°.									
100	75.0	82.5	90.0	97.5	105.0	112.5	120.0	127.5	135.0	142.5
200	150.0	157.5	165.0	172.5	180.0	187.5	195.0	202.5	210.0	217.5
300	225.0	232.5	240.0	247.5	255.0	262.5	270.0	277.5	285.0	292.5
400	300.0	307.5	315.0	322.5	330.0	337.5	345.0	352.5	360.0	367.5
500	375.0	382.5	390.0	397.5	405.0	412.5	420.0	427.5	435.0	442.5
600	450.0	457.5	465.0	472.5	480.0	487.5	495.0	502.5	510.0	517.5
700	525.0	532.5	540.0	547.5	555.0	562.5	570.0	577.5	585.0	592.5
800	600.0	607.5	615.0	622.5	630.0	637.5	645.0	652.5	660.0	667.5
900	675.0	682.5	690.0	697.5	705.0	712.5	720.0	727.5	735.0	742.5
1000	750.0	757.5	765.0	772.5	780.0	787.5	795.0	802.5	810.0	817.5
1100	825.0	832.5	840.0	847.5	855.0	862.5	870.0	877.5	885.0	892.5
1200	900.0	907.5	915.0	922.5	930.0	937.5	945.0	952.5	960.0	967.5

Increments for changes by millibar intervals.

mb.	1	2	3	4	5	6	7	8	9
mm.	.8	1.5	2.3	3.0	3.8	4.5	5.3	6.0	6.8

MILLIMETRES OF MERCURY (680.0 TO 779.9) WITH TENTHS AT 0° C. IN
LATITUDE 45° TO MILLIBARS AND TENTHS.

Tenths.	0	1	2	3	4	5	6	7	8	9
mm.	Millibars.									
680	9	06.6	06.7	06.8	07.0	07.1	07.2	07.4	07.5	07.6
681	9	07.9	08.0	08.2	08.3	08.4	08.6	08.7	08.8	09.0
682	9	09.2	09.4	09.5	09.6	09.8	09.9	10.0	10.2	10.3
683	9	10.6	10.7	10.8	11.0	11.1	11.2	11.4	11.5	11.6
684	9	11.9	12.0	12.2	12.3	12.4	12.6	12.7	12.8	13.0
685	9	13.2	13.4	13.5	13.6	13.8	13.9	14.0	14.2	14.3
686	9	14.6	14.7	14.8	15.0	15.1	15.2	15.4	15.5	15.6
687	9	15.9	16.0	16.2	16.3	16.4	16.6	16.7	16.8	17.0
688	9	17.2	17.4	17.5	17.6	17.8	17.9	18.0	18.2	18.3
689	9	18.6	18.7	18.8	19.0	19.1	19.2	19.4	19.5	19.6
690	9	19.9	20.0	20.2	20.3	20.4	20.6	20.7	20.8	21.0
691	9	21.2	21.4	21.5	21.6	21.8	21.9	22.0	22.2	22.3
692	9	22.6	22.7	22.8	23.0	23.1	23.2	23.4	23.5	23.6
693	9	23.9	24.0	24.2	24.3	24.4	24.6	24.7	24.8	25.0
694	9	25.2	25.4	25.5	25.6	25.8	25.9	26.0	26.2	26.3
695	9	26.6	26.7	26.8	27.0	27.1	27.2	27.4	27.5	27.6
696	9	27.9	28.0	28.2	28.3	28.4	28.6	28.7	28.8	29.0
697	9	29.2	29.4	29.5	29.6	29.8	29.9	30.0	30.2	30.3
698	9	30.6	30.7	30.8	31.0	31.1	31.2	31.4	31.5	31.6
699	9	31.9	32.0	32.2	32.3	32.4	32.6	32.7	32.8	33.0
700	9	33.2	33.4	33.5	33.6	33.8	33.9	34.0	34.2	34.3
701	9	34.6	34.7	34.8	35.0	35.1	35.2	35.4	35.5	35.6
702	9	35.9	36.0	36.2	36.3	36.4	36.6	36.7	36.8	37.0
703	9	37.2	37.4	37.5	37.6	37.8	37.9	38.0	38.2	38.3
704	9	38.6	38.7	38.8	39.0	39.1	39.2	39.4	39.5	39.6
705	9	39.9	40.0	40.2	40.3	40.4	40.6	40.7	40.8	41.0
706	9	41.2	41.4	41.5	41.6	41.8	41.9	42.0	42.2	42.3
707	9	42.6	42.7	42.8	43.0	43.1	43.2	43.4	43.5	43.6
708	9	43.9	44.0	44.2	44.3	44.4	44.6	44.7	44.8	45.0
709	9	45.2	45.4	45.5	45.6	45.8	45.9	46.0	46.2	46.3
710	9	46.6	46.7	46.8	47.0	47.1	47.2	47.4	47.5	47.6
711	9	47.9	48.0	48.2	48.3	48.4	48.6	48.7	48.8	49.0
712	9	49.2	49.4	49.5	49.6	49.8	49.9	50.0	50.2	50.3
713	9	50.6	50.7	50.8	51.0	51.1	51.2	51.4	51.5	51.6
714	9	51.9	52.0	52.2	52.3	52.4	52.6	52.7	52.8	53.0
715	9	53.2	53.4	53.5	53.6	53.8	53.9	54.0	54.2	54.3
716	9	54.6	54.7	54.8	55.0	55.1	55.2	55.4	55.5	55.6
717	9	55.9	56.0	56.2	56.3	56.4	56.6	56.7	56.8	57.0
718	9	57.2	57.4	57.5	57.6	57.8	57.9	58.0	58.2	58.3
719	9	58.6	58.7	58.8	59.0	59.1	59.2	59.4	59.5	59.6
720	9	59.9	60.0	60.2	60.3	60.4	60.6	60.7	60.8	61.0
721	9	61.2	61.4	61.5	61.6	61.8	61.9	62.0	62.2	62.3
722	9	62.6	62.7	62.8	63.0	63.1	63.2	63.4	63.5	63.6
723	9	63.9	64.0	64.2	64.3	64.4	64.6	64.7	64.8	65.0
724	9	65.2	65.4	65.5	65.6	65.8	65.9	66.0	66.2	66.3
725	9	66.6	66.7	66.8	67.0	67.1	67.2	67.4	67.5	67.6
726	9	67.9	68.0	68.2	68.3	68.4	68.6	68.7	68.8	69.0
727	9	69.2	69.4	69.5	69.6	69.8	69.9	70.0	70.2	70.3
728	9	70.6	70.7	70.8	71.0	71.1	71.2	71.4	71.5	71.6
729	9	71.9	72.0	72.2	72.3	72.4	72.6	72.7	72.8	73.0

MILLIMETRES OF MERCURY (680.0 TO 779.9) WITH TENTHS AT 0° C. IN
LATITUDE 45° TO MILLIBARS AND TENTHS.

Tenths.	0	1	2	3	4	5	6	7	8	9
mm.	Millibars.									
730	9	73.2	73.4	73.5	73.6	73.8	73.9	74.0	74.2	74.3
731	9	74.6	74.7	74.8	75.0	75.1	75.2	75.4	75.5	75.6
732	9	75.9	76.0	76.2	76.3	76.4	76.6	76.7	76.8	77.0
733	9	77.2	77.4	77.5	77.6	77.8	77.9	78.0	78.2	78.3
734	9	78.6	78.7	78.8	79.0	79.1	79.2	79.4	79.5	79.6
735	9	79.9	80.0	80.2	80.3	80.4	80.6	80.7	80.8	81.0
736	9	81.2	81.4	81.5	81.6	81.8	81.9	82.0	82.2	82.3
737	9	82.6	82.7	82.8	83.0	83.1	83.2	83.4	83.5	83.6
738	9	83.9	84.0	84.2	84.3	84.4	84.6	84.7	84.8	85.0
739	9	85.2	85.4	85.5	85.6	85.8	85.9	86.0	86.2	86.3
740	9	86.6	86.7	86.8	87.0	87.1	87.2	87.4	87.5	87.6
741	9	87.9	88.0	88.2	88.3	88.4	88.6	88.7	88.8	89.0
742	9	89.2	89.4	89.5	89.6	89.8	89.9	90.0	90.2	90.3
743	9	90.6	90.7	90.8	91.0	91.1	91.2	91.4	91.5	91.6
744	9	91.9	92.0	92.2	92.3	92.4	92.6	92.7	92.8	93.0
745	9	93.2	93.4	93.5	93.6	93.8	93.9	94.0	94.2	94.3
746	9	94.6	94.7	94.8	95.0	95.1	95.2	95.4	95.5	95.6
747	9	95.9	96.0	96.2	96.3	96.4	96.6	96.7	96.8	97.0
748	9	97.2	97.4	97.5	97.6	97.8	97.9	98.0	98.2	98.3
749	9	98.6	98.7	98.8	99.0	99.1	99.2	99.4	99.5	99.6
750	9	99.9	00.0	00.2	00.3	00.4	00.6	00.7	00.8	01.0
751	10	01.2	01.4	01.5	01.6	01.8	01.9	02.0	02.2	02.3
752	10	02.6	02.7	02.8	03.0	03.1	03.2	03.4	03.5	03.6
753	10	03.9	04.0	04.2	04.3	04.4	04.6	04.7	04.8	05.0
754	10	05.2	05.4	05.5	05.6	05.8	05.9	06.0	06.2	06.3
755	10	06.6	06.7	06.8	07.0	07.1	07.2	07.4	07.5	07.6
756	10	07.9	08.0	08.2	08.3	08.4	08.6	08.7	08.8	09.0
757	10	09.2	09.4	09.5	09.6	09.8	09.9	10.0	10.2	10.3
758	10	10.6	10.7	10.8	11.0	11.1	11.2	11.4	11.5	11.6
759	10	11.9	12.0	12.2	12.3	12.4	12.6	12.7	12.8	13.0
760	10	13.2	13.4	13.5	13.6	13.8	13.9	14.0	14.2	14.3
761	10	14.6	14.7	14.8	15.0	15.1	15.2	15.4	15.5	15.6
762	10	15.9	16.0	16.2	16.3	16.4	16.6	16.7	16.8	17.0
763	10	17.2	17.4	17.5	17.6	17.8	17.9	18.0	18.2	18.3
764	10	18.6	18.7	18.8	19.0	19.1	19.2	19.4	19.5	19.6
765	10	19.9	20.0	20.2	20.3	20.4	20.6	20.7	20.8	21.0
766	10	21.2	21.4	21.5	21.6	21.8	21.9	22.0	22.2	22.3
767	10	22.6	22.7	22.8	23.0	23.1	23.2	23.4	23.5	23.6
768	10	23.9	24.0	24.2	24.3	24.4	24.6	24.7	24.8	25.0
769	10	25.2	25.4	25.5	25.6	25.8	25.9	26.0	26.2	26.3
770	10	26.6	26.7	26.8	27.0	27.1	27.2	27.4	27.5	27.6
771	10	27.9	28.0	28.2	28.3	28.4	28.6	28.7	28.8	29.0
772	10	29.2	29.4	29.5	29.6	29.8	29.9	30.0	30.2	30.3
773	10	30.6	30.7	30.8	31.0	31.1	31.2	31.4	31.5	31.6
774	10	31.9	32.0	32.2	32.3	32.4	32.6	32.7	32.8	33.0
775	10	33.2	33.4	33.5	33.6	33.8	33.9	34.0	34.2	34.3
776	10	34.6	34.7	34.8	35.0	35.1	35.2	35.4	35.5	35.6
777	10	35.9	36.0	36.2	36.3	36.4	36.6	36.7	36.8	37.0
778	10	37.2	37.4	37.5	37.6	37.8	37.9	38.0	38.2	38.3
779	10	38.6	38.7	38.8	39.0	39.1	39.2	39.4	39.5	39.6

INCHES (27.00 TO 31.99) WITH TENTHS AND HUNDREDTHS TO MILLIMETRES
WITH TENTHS AND HUNDREDTHS.

1 inch = 25.4000 mm.

Hun- dredths	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
Inches and Tenths.	Millimetres.									
27.0	6	85.80	86.05	86.31	86.56	86.82	87.07	87.32	87.58	87.83
27.1	6	88.34	88.59	88.85	89.10	89.36	89.61	89.86	90.12	90.37
27.2	6	90.88	91.13	91.39	91.64	91.90	92.15	92.40	92.66	92.91
27.3	6	93.42	93.76	93.93	94.18	94.44	94.69	94.94	95.20	95.45
27.4	6	95.96	96.21	96.47	96.72	96.98	97.23	97.48	97.74	97.99
27.5	6	98.50	98.75	99.01	99.26	99.52	99.77	100.02	100.28	100.53
27.6	7	01.04	01.29	01.55	01.80	02.06	02.31	02.56	02.82	03.07
27.7	7	03.58	03.83	04.09	04.34	04.60	04.85	05.10	05.36	05.61
27.8	7	06.12	06.37	06.63	06.88	07.14	07.39	07.64	07.90	08.15
27.9	7	08.66	08.91	09.17	09.42	09.68	09.93	10.18	10.44	10.69
28.0	7	11.20	11.45	11.71	11.96	12.22	12.47	12.72	12.98	13.23
28.1	7	13.74	13.99	14.25	14.50	14.76	15.01	15.26	15.52	15.77
28.2	7	16.28	16.53	16.79	17.04	17.30	17.55	17.80	18.06	18.31
28.3	7	18.82	19.07	19.33	19.58	19.84	20.09	20.34	20.60	20.85
28.4	7	21.36	21.61	21.87	22.12	22.38	22.63	22.88	23.14	23.39
28.5	7	23.90	24.15	24.41	24.66	24.92	25.17	25.42	25.68	25.93
28.6	7	26.44	26.69	26.95	27.20	27.46	27.71	27.96	28.22	28.47
28.7	7	28.98	29.23	29.49	29.74	30.00	30.25	30.50	30.76	31.01
28.8	7	31.52	31.77	32.03	32.28	32.54	32.79	33.04	33.30	33.55
28.9	7	34.06	34.31	34.57	34.82	35.08	35.33	35.58	35.84	36.09
29.0	7	36.60	36.85	37.11	37.36	37.62	37.87	38.12	38.38	38.63
29.1	7	39.14	39.39	39.65	39.90	40.16	40.41	40.66	40.92	41.17
29.2	7	41.68	41.93	42.19	42.44	42.70	42.95	43.20	43.46	43.71
29.3	7	44.22	44.47	44.73	44.98	45.24	45.49	45.74	46.00	46.25
29.4	7	46.76	47.01	47.27	47.52	47.78	48.03	48.28	48.54	48.79
29.5	7	49.30	49.55	49.81	50.06	50.32	50.57	50.82	51.08	51.33
29.6	7	51.84	52.09	52.35	52.60	52.86	53.11	53.36	53.62	53.87
29.7	7	54.38	54.63	54.89	55.14	55.40	55.65	55.90	56.16	56.41
29.8	7	56.92	57.17	57.43	57.68	57.94	58.19	58.44	58.70	58.95
29.9	7	59.46	59.71	59.97	60.22	60.48	60.73	60.98	61.24	61.49
30.0	7	62.00	62.25	62.51	62.76	63.02	63.27	63.52	63.78	64.03
30.1	7	64.54	64.79	65.05	65.30	65.56	65.81	66.06	66.32	66.57
30.2	7	67.08	67.33	67.59	67.84	68.10	68.35	68.60	68.86	69.11
30.3	7	69.62	69.87	70.13	70.39	70.64	70.89	71.14	71.40	71.65
30.4	7	72.16	72.41	72.67	72.92	73.18	73.43	73.68	73.94	74.19
30.5	7	74.70	74.95	75.21	75.46	75.72	75.97	76.22	76.48	76.73
30.6	7	77.24	77.49	77.75	78.00	78.26	78.51	78.76	79.02	79.27
30.7	7	79.78	80.03	80.29	80.54	80.80	81.05	81.30	81.56	81.81
30.8	7	82.32	82.57	82.83	83.08	83.34	83.59	83.84	84.10	84.35
30.9	7	84.86	85.11	85.37	85.62	85.88	86.13	86.38	86.64	86.89
31.0	7	87.40	87.65	87.91	88.16	88.42	88.67	88.92	89.18	89.43
31.1	7	89.94	90.19	90.45	90.70	90.96	91.21	91.46	91.72	91.97
31.2	7	92.48	92.73	92.99	93.24	93.50	93.75	94.00	94.26	94.51
31.3	7	95.02	95.27	95.53	95.78	96.04	96.29	96.54	96.80	97.05
31.4	7	97.56	97.81	98.07	98.32	98.58	98.83	99.08	99.34	99.59
31.5	8	00.10	00.35	00.61	00.86	01.12	01.37	01.62	01.88	02.13
31.6	8	02.64	02.89	03.15	03.40	03.66	03.91	04.16	04.42	04.67
31.7	8	05.18	05.43	05.69	05.94	06.20	06.45	06.70	06.96	07.21
31.8	8	07.72	07.97	08.23	08.48	08.74	08.99	09.24	09.50	09.75
31.9	8	10.26	10.51	10.77	11.02	11.28	11.53	11.78	12.04	12.29

*7.

MILLIMETRES (705-799) WITH TENTHS TO INCHES AND THOUSANDTHS.
(1 metre = 39.3701 inches.)

		Tenths of a Millimetre.									
		0	1	2	3	4	5	6	7	8	9
		English Inches.									
705	2	7.756	7.760	7.764	7.768	7.772	7.776	7.780	7.783	7.787	7.791
706	2	7.795	7.799	7.803	7.807	7.811	7.815	7.819	7.823	7.827	7.831
707	2	7.835	7.839	7.843	7.846	7.850	7.854	7.858	7.862	7.866	7.870
708	2	7.874	7.878	7.882	7.886	7.890	7.894	7.898	7.902	7.906	7.909
709	2	7.913	7.917	7.921	7.925	7.929	7.933	7.937	7.941	7.945	7.949
710	2	7.953	7.957	7.961	7.965	7.969	7.972	7.976	7.980	7.984	7.988
711	2	7.992	7.996	8.000	8.004	8.008	8.012	8.016	8.020	8.024	8.028
712	2	8.032	8.035	8.039	8.043	8.047	8.051	8.055	8.059	8.063	8.067
713	2	8.071	8.075	8.079	8.083	8.087	8.091	8.094	8.098	8.102	8.106
714	2	8.110	8.114	8.118	8.122	8.126	8.130	8.134	8.138	8.142	8.146
715	2	8.150	8.154	8.157	8.161	8.165	8.169	8.173	8.177	8.181	8.185
716	2	8.189	8.193	8.197	8.201	8.205	8.209	8.213	8.217	8.220	8.224
717	2	8.228	8.232	8.236	8.240	8.244	8.248	8.252	8.256	8.260	8.264
718	2	8.268	8.272	8.276	8.280	8.283	8.287	8.291	8.295	8.299	8.303
719	2	8.307	8.311	8.315	8.319	8.323	8.327	8.331	8.335	8.339	8.343
720	2	8.346	8.350	8.354	8.358	8.362	8.366	8.370	8.374	8.378	8.382
721	2	8.386	8.390	8.394	8.398	8.402	8.406	8.409	8.413	8.417	8.421
722	2	8.425	8.429	8.433	8.437	8.441	8.445	8.449	8.453	8.457	8.461
723	2	8.465	8.469	8.472	8.476	8.480	8.484	8.488	8.492	8.496	8.500
724	2	8.504	8.508	8.512	8.516	8.520	8.524	8.528	8.531	8.535	8.539
725	2	8.543	8.547	8.551	8.555	8.559	8.563	8.567	8.571	8.575	8.579
726	2	8.583	8.587	8.591	8.594	8.598	8.602	8.606	8.610	8.614	8.618
727	2	8.622	8.626	8.630	8.634	8.638	8.642	8.646	8.650	8.654	8.657
728	2	8.661	8.665	8.669	8.673	8.677	8.681	8.685	8.689	8.693	8.697
729	2	8.701	8.705	8.709	8.713	8.717	8.720	8.724	8.728	8.732	8.736
730	2	8.740	8.744	8.748	8.752	8.756	8.760	8.764	8.768	8.772	8.776
731	2	8.780	8.783	8.787	8.791	8.795	8.799	8.803	8.807	8.811	8.815
732	2	8.819	8.823	8.827	8.831	8.835	8.839	8.843	8.846	8.850	8.854
733	2	8.858	8.862	8.866	8.870	8.874	8.878	8.882	8.886	8.890	8.894
734	2	8.898	8.902	8.906	8.909	8.913	8.917	8.921	8.925	8.929	8.933
735	2	8.937	8.941	8.945	8.949	8.953	8.957	8.961	8.965	8.969	8.972
736	2	8.976	8.980	8.984	8.988	8.992	8.996	9.000	9.004	9.008	9.012
737	2	9.016	9.020	9.024	9.028	9.031	9.035	9.039	9.043	9.047	9.051
738	2	9.055	9.059	9.063	9.067	9.071	9.075	9.079	9.083	9.087	9.091
739	2	9.094	9.098	9.102	9.106	9.110	9.114	9.118	9.122	9.126	9.130
740	2	9.134	9.138	9.142	9.146	9.150	9.154	9.157	9.161	9.165	9.169
741	2	9.173	9.177	9.181	9.185	9.189	9.193	9.197	9.201	9.205	9.209
742	2	9.213	9.217	9.220	9.224	9.228	9.232	9.236	9.240	9.244	9.248
743	2	9.252	9.256	9.260	9.264	9.268	9.272	9.276	9.280	9.283	9.287
744	2	9.291	9.295	9.299	9.303	9.307	9.311	9.315	9.319	9.323	9.327
745	2	9.331	9.335	9.339	9.343	9.346	9.350	9.354	9.358	9.362	9.366
746	2	9.370	9.374	9.378	9.382	9.386	9.390	9.394	9.398	9.402	9.406
747	2	9.409	9.413	9.417	9.421	9.425	9.429	9.433	9.437	9.441	9.445
748	2	9.449	9.453	9.457	9.461	9.465	9.469	9.472	9.476	9.480	9.484
749	2	9.488	9.492	9.496	9.500	9.504	9.508	9.512	9.516	9.520	9.524
750	2	9.528	9.531	9.535	9.539	9.543	9.547	9.551	9.555	9.559	9.563
751	2	9.567	9.571	9.575	9.579	9.583	9.587	9.591	9.594	9.598	9.602
752	2	9.606	9.610	9.614	9.618	9.622	9.626	9.630	9.634	9.638	9.642
753	2	9.646	9.650	9.654	9.657	9.661	9.665	9.669	9.673	9.677	9.681
754	2	9.685	9.689	9.693	9.697	9.701	9.705	9.709	9.713	9.717	9.720

MILLIMETRES (755-799) WITH TENTHS TO INCHES AND THOUSANDTHS.
(1 metre = 39.3701 inches.)

Milli- metres.	Tenths of a Millimetre.										
	0	1	2	3	4	5	6	7	8	9	
English Inches.											
755	2	9.724	9.728	9.732	9.736	9.740	9.744	9.748	9.752	9.756	9.760
756	2	9.764	9.768	9.772	9.776	9.780	9.783	9.787	9.791	9.795	9.799
757	2	9.803	9.807	9.811	9.815	9.819	9.823	9.827	9.831	9.835	9.839
758	2	9.843	9.846	9.850	9.854	9.858	9.862	9.866	9.870	9.874	9.878
759	2	9.882	9.886	9.890	9.894	9.898	9.902	9.906	9.909	9.913	9.917
760	2	9.921	9.925	9.929	9.933	9.937	9.941	9.945	9.949	9.953	9.957
761	2	9.961	9.965	9.968	9.972	9.976	9.980	9.984	9.988	9.992	9.996
762	3	0.000	0.004	0.008	0.012	0.016	0.020	0.024	0.028	0.031	0.035
763	3	0.039	0.043	0.047	0.051	0.055	0.059	0.063	0.067	0.071	0.075
764	3	0.079	0.083	0.087	0.091	0.094	0.098	0.102	0.106	0.110	0.114
765	3	0.118	0.122	0.126	0.130	0.134	0.138	0.142	0.146	0.150	0.154
766	3	0.157	0.161	0.165	0.169	0.173	0.177	0.181	0.185	0.189	0.193
767	3	0.197	0.201	0.205	0.209	0.213	0.217	0.220	0.224	0.228	0.232
768	3	0.236	0.240	0.244	0.248	0.252	0.256	0.260	0.264	0.268	0.272
769	3	0.276	0.280	0.283	0.287	0.291	0.295	0.299	0.303	0.307	0.311
770	3	0.315	0.319	0.323	0.327	0.331	0.335	0.339	0.343	0.346	0.350
771	3	0.354	0.358	0.362	0.366	0.370	0.374	0.378	0.382	0.386	0.390
772	3	0.394	0.398	0.402	0.406	0.409	0.413	0.417	0.421	0.425	0.429
773	3	0.433	0.437	0.441	0.445	0.449	0.453	0.457	0.461	0.465	0.468
774	3	0.472	0.476	0.480	0.484	0.488	0.492	0.496	0.500	0.504	0.508
775	3	0.512	0.516	0.520	0.524	0.528	0.531	0.535	0.539	0.543	0.547
776	3	0.551	0.555	0.559	0.563	0.567	0.571	0.575	0.579	0.583	0.587
777	3	0.591	0.594	0.598	0.602	0.606	0.610	0.614	0.618	0.622	0.626
778	3	0.630	0.634	0.638	0.642	0.646	0.650	0.654	0.657	0.661	0.665
779	3	0.669	0.673	0.677	0.681	0.685	0.689	0.693	0.697	0.701	0.705
780	3	0.709	0.713	0.717	0.720	0.724	0.728	0.732	0.736	0.740	0.744
781	3	0.748	0.752	0.756	0.760	0.764	0.768	0.772	0.776	0.780	0.783
782	3	0.787	0.791	0.795	0.799	0.803	0.807	0.811	0.815	0.819	0.823
783	3	0.827	0.831	0.835	0.839	0.843	0.846	0.850	0.854	0.858	0.862
784	3	0.866	0.870	0.874	0.878	0.882	0.886	0.890	0.894	0.898	0.902
785	3	0.905	0.909	0.913	0.917	0.921	0.925	0.929	0.933	0.937	0.941
786	3	0.945	0.949	0.953	0.957	0.961	0.965	0.968	0.972	0.976	0.980
787	3	0.984	0.988	0.992	0.996	1.000	1.004	1.008	1.012	1.016	1.020
788	3	1.024	1.028	1.031	1.035	1.039	1.043	1.047	1.051	1.055	1.059
789	3	1.063	1.067	1.071	1.075	1.079	1.083	1.087	1.091	1.094	1.098
790	3	1.102	1.106	1.110	1.114	1.118	1.122	1.126	1.130	1.134	1.138
791	3	1.142	1.146	1.150	1.154	1.157	1.161	1.165	1.169	1.173	1.177
792	3	1.181	1.185	1.189	1.193	1.197	1.201	1.205	1.209	1.213	1.217
793	3	1.220	1.224	1.228	1.232	1.236	1.240	1.244	1.248	1.252	1.256
794	3	1.260	1.264	1.268	1.272	1.276	1.280	1.283	1.287	1.291	1.295
795	3	1.299	1.303	1.307	1.311	1.315	1.319	1.323	1.327	1.331	1.335
796	3	1.339	1.343	1.346	1.350	1.354	1.358	1.362	1.366	1.370	1.374
797	3	1.378	1.382	1.386	1.390	1.394	1.398	1.402	1.405	1.409	1.413
798	3	1.417	1.421	1.425	1.429	1.433	1.437	1.441	1.445	1.449	1.453
799	3	1.457	1.461	1.465	1.468	1.472	1.476	1.480	1.484	1.488	1.492

HEIGHT TABLE. METRES TO FEET.

1 metre = 3.28084 feet.

Metres.		Feet.									
		0	1	2	3	4	5	6	7	8	9
0		0.00	3.28	6.56	9.84	13.12	16.40	19.68	22.97	26.25	29.53
10		32.81	36.09	39.37	42.65	45.93	49.21	52.49	55.77	59.06	62.34
20		65.62	68.90	72.18	75.46	78.74	82.02	85.30	88.58	91.86	95.14
30		98.43	101.71	104.99	108.27	111.55	114.83	118.11	121.39	124.67	127.95
40		131.23	134.51	137.80	141.08	144.36	147.64	150.92	154.20	157.48	160.76
50		164.04	167.32	170.60	173.88	177.17	180.45	183.73	187.01	190.29	193.57
60		196.85	200.13	203.41	206.69	209.97	213.25	216.54	219.82	223.10	226.38
70		229.66	232.94	236.22	239.50	242.78	246.06	249.34	252.62	255.91	259.19
80		262.47	265.75	269.03	272.31	275.59	278.87	282.15	285.43	288.71	291.99
90		295.28	298.56	301.84	305.12	308.40	311.68	315.96	319.24	322.52	325.80
100		328.08	331.36	334.65	337.93	341.21	344.49	347.77	351.05	354.33	357.61
110		360.89	364.17	367.45	370.73	374.02	377.30	380.58	383.86	387.14	390.42
120		393.70	396.98	400.26	403.54	406.82	410.11	413.39	416.67	419.95	423.23
130		426.51	429.79	433.07	436.35	439.63	442.91	446.19	449.48	452.76	456.04
140		459.32	462.60	465.88	469.16	472.44	475.72	479.00	482.28	485.56	488.85
150		492.13	495.41	498.69	501.97	505.25	508.53	511.81	515.09	518.37	521.65
160		524.93	528.22	531.50	534.78	538.06	541.34	544.62	547.90	551.18	554.46
170		557.74	561.02	564.30	567.59	570.87	574.15	577.43	580.71	583.99	587.27
180		590.55	593.83	597.11	600.39	603.67	606.96	610.24	613.52	616.80	620.08
190		623.36	626.64	629.92	633.20	636.48	639.76	643.04	646.33	649.61	652.89
Metres.		Feet									
		0	10	20	30	40	50	60	70	80	90
200		656.17	688.98	721.79	754.59	787.40	820.21	853.02	885.83	918.64	951.44
300		984.25	1017.06	1049.87	1082.68	1115.49	1148.29	1181.10	1213.91	1246.72	1279.53
400		1312.34	1345.14	1377.95	1410.76	1443.57	1476.38	1509.19	1541.99	1574.80	1607.61
500		1640.42	1673.23	1706.04	1738.85	1771.65	1804.46	1837.27	1870.08	1902.89	1935.70
600		1968.50	2001.31	2034.12	2066.93	2099.74	2132.55	2165.35	2198.16	2230.97	2263.78
700		2296.59	2329.40	2362.20	2395.01	2427.82	2460.63	2493.44	2526.25	2559.06	2591.86
800		2624.67	2657.48	2690.29	2723.10	2755.91	2788.71	2821.52	2854.33	2887.14	2919.95
900		2952.76	2985.56	3018.37	3051.18	3083.99	3116.80	3149.61	3182.41	3215.22	3248.03
1000		3280.84	3313.65	3346.46	3379.27	3412.07	3444.88	3477.69	3510.50	3543.31	3576.12

WIND VELOCITY.

A.—MILES PER HOUR TO METRES PER SECOND.

1 Mile per hour = 0.44704 Metres per second.

Miles per hour.	0	1	2	3	4	5	6	7	8	9
	Metres per second.									
0	0.0	0.5	0.9	1.3	1.8	2.2	2.7	3.1	3.6	4.0
10	4.5	4.9	5.4	5.8	6.3	6.7	7.2	7.6	8.1	8.5
20	8.9	9.4	9.8	10.3	10.7	11.2	11.6	12.1	12.5	13.0
30	13.4	13.9	14.3	14.8	15.2	15.7	16.1	16.5	17.0	17.4
40	17.9	18.3	18.8	19.2	19.7	20.1	20.6	21.0	21.5	21.9
50	22.4	22.8	23.3	23.7	24.1	24.6	25.0	25.5	26.0	26.4
60	26.8	27.3	27.7	28.2	28.6	29.1	29.5	30.0	30.4	30.9
70	31.3	31.7	32.2	32.6	33.1	33.5	34.0	34.4	34.9	35.3
80	35.8	36.2	36.7	37.1	37.6	38.0	38.4	38.9	39.3	39.8
90	40.2	40.7	41.1	41.6	42.0	42.5	42.9	43.4	43.8	44.3
100	44.7	45.2	45.6	46.0	46.5	46.9	47.4	47.8	48.3	48.7
110	49.2	49.6	50.1	50.5	51.0	51.4	51.9	52.3	52.8	53.2
120	53.6	54.1	54.5	55.0	55.4	55.9	56.3	56.8	57.2	57.7
130	58.1	58.6	59.1	59.5	59.9	60.4	60.8	61.2	61.7	62.1
140	62.5	63.0	63.5	63.9	64.4	64.8	65.3	65.7	66.2	66.6

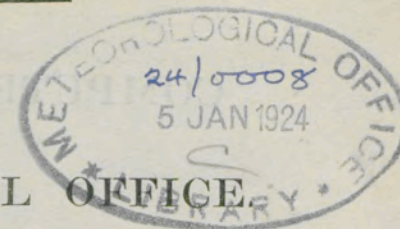
B.—METRES PER SECOND TO MILES PER HOUR.

1 Metre per second = 2.23694 Miles per hour.

Metres per second.	Miles per hour.	Metres per second.	Miles per hour.	Metres per second.	Miles per hour.	Metres per second.	Miles per hour.
1	2.24	16	35.79	31	69.35	46	102.90
2	4.47	17	38.03	32	71.58	47	105.14
3	6.71	18	40.27	33	73.82	48	107.37
4	8.95	19	42.50	34	76.06	49	109.61
5	11.18	20	44.74	35	78.29	50	111.85
6	13.42	21	46.98	36	80.53	51	114.09
7	15.66	22	49.21	37	82.77	52	116.32
8	17.90	23	51.45	38	85.01	53	118.56
9	20.13	24	53.69	39	87.24	54	120.80
10	22.37	25	55.92	40	89.48	55	123.03
11	24.61	26	58.16	41	91.72	56	125.27
12	26.84	27	60.40	42	93.95	57	127.51
13	29.08	28	62.64	43	96.19	58	129.74
14	31.32	29	64.87	44	98.43	59	131.98
15	33.55	30	67.11	45	100.66	60	134.22

FOR OFFICIAL USE.

M.O. 223. Section I.



METEOROLOGICAL OFFICE

THE COMPUTER'S HANDBOOK.

Section I.—Computations based on the physical properties of atmospheric air:—
Humidity and density.

Published by the Authority of the Meteorological Committee.



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COMPUTER'S HANDBOOK.

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COMPUTATIONS BASED ON THE PHYSICAL PROPERTIES OF
ATMOSPHERIC AIR.

CONTENTS.

§ 1.—Tables for the computation of humidity from the readings of the dry-and-wet thermometers on the Kelvin scale of absolute temperature based on Glaisher's factors.

§ 2.—Reduction of barometer readings to sea level.

§ 3.—The relations of pressure, temperature, and density of air under atmospheric conditions.

1879 [47] A full discussion of methods of determining humidity will be found in the Report on Hygrometric Methods, W. N. Shaw, Quarterly Weather Report, 1877, p. [35], or Phil. Trans., vol. 179, p. 73, 1888. The bibliography of the subject is brought up to 1899 in a note by Pernter, Report of the International Meteorological Committee, St. Petersburg, 1899, p. 83. In this note the use of the hair hygrometer at all stations where a ventilated psychrometer is not available is advocated.

COMPUTER'S HANDBOOK.

SECTION I.

Computations based on the physical properties of
atmospheric air.

§ 1. TABLES FOR THE COMPUTATION OF HUMIDITY FROM THE READINGS OF THE DRY-AND-WET THERMOMETERS ON THE KELVIN SCALE OF ABSOLUTE TEMPERATURE BASED ON GLAISHER'S FACTORS.

The process of computation is by Glaisher's factors or multipliers, which by operating on the depression of the wet bulb below the dry bulb give the depression of the dew-point below the temperature of the air or dry bulb. Thence by a table of saturation pressures the vapour-pressure and relative humidity are obtained. In a revised edition the values of vapour-pressure will be incorporated with the readings of the wet bulb so that the use of a separate table will be obviated.

For computation with the Fahrenheit scale (tenths of a degree) a separate set of Hygrometrical Tables is provided, price 4s. 6d.

Values of Glaisher's Factors used for Dry Bulb Temperatures from 265a to 314a.

Temperature A.	Glaisher's Factor.	Temperature A.	Glaisher's Factor.
265°	8.55	290°	1.85
66°	8.27	91°	1.83
67°	7.83	92°	1.81
68°	7.28	93°	1.79
69°	6.61	94°	1.77
70°	5.80	95°	1.75
71°	4.92	96°	1.74
72°	4.06	97°	1.72
73°	3.32	98°	1.70
74°	2.81	99°	1.69
75°	2.54	300°	1.68
76°	2.39	01°	1.67
77°	2.31	02°	1.66
78°	2.26	03°	1.65
79°	2.21	04°	1.64
80°	2.17	05°	1.63
81°	2.13	06°	1.62
82°	2.10	07°	1.61
83°	2.06	08°	1.60
84°	2.02	09°	1.59
85°	1.99	10°	1.58
86°	1.95	11°	1.57
87°	1.92	12°	1.56
88°	1.89	13°	1.55
89°	1.87	14°	1.54

Dry Bulb	270°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
99	270.0	271.0	272.0	273.0	274.0	274.9	275.9	276.9	277.9	278.9
98	70.0	71.0	71.9	72.9	73.9	74.9	75.9	76.9	77.9	78.9
97	69.9	70.9	71.9	72.9	73.9	74.8	75.8	76.8	77.8	78.8
96	69.9	70.9	71.9	72.8	73.8	74.8	75.8	76.8	77.7	78.7
95	69.9	70.9	71.8	72.8	73.7	74.7	75.7	76.7	77.7	78.7
94	69.9	70.8	71.8	72.8	73.7	74.7	75.6	76.6	77.6	78.6
93	69.8	70.8	71.8	72.7	73.6	74.6	75.6	76.6	77.5	78.5
92	69.8	70.8	71.7	72.7	73.6	74.6	75.5	76.5	77.5	78.4
91	69.8	70.7	71.7	72.6	73.5	74.5	75.5	76.4	77.4	78.4
90	69.8	70.7	71.6	72.6	73.5	74.4	75.4	76.4	77.3	78.3
89	69.7	70.7	71.6	72.5	73.4	74.4	75.3	76.3	77.3	78.2
88	69.7	70.7	71.6	72.5	73.4	74.3	75.3	76.2	77.2	78.2
87	69.7	70.6	71.5	72.4	73.3	74.2	75.2	76.1	77.1	78.1
86	69.7	70.6	71.5	72.4	73.3	74.2	75.1	76.1	77.0	78.0
85	69.6	70.6	71.5	72.3	73.2	74.1	75.1	76.0	77.0	77.9
84	69.6	70.5	71.4	72.3	73.1	74.0	75.0	75.9	76.9	77.9
83	69.6	70.5	71.4	72.2	73.1	74.0	74.9	75.9	76.8	77.8
82	69.5	70.5	71.3	72.2	73.0	73.9	74.8	75.8	76.8	77.7
81	69.5	70.4	71.3	72.1	73.0	73.8	74.8	75.7	76.7	77.6
80	69.5	70.4	71.3	72.1	72.9	73.8	74.7	75.6	76.6	77.5
79	69.5	70.4	71.2	72.0	72.8	73.7	74.6	75.6	76.5	77.5
78	69.4	70.3	71.2	72.0	72.8	73.6	74.6	75.5	76.4	77.4
77	69.4	70.3	71.1	71.9	72.7	73.6	74.5	75.4	76.4	77.3
76	69.4	70.3	71.1	71.9	72.6	73.5	74.4	75.3	76.3	77.2
75	69.3	70.2	71.1	71.8	72.6	73.4	74.3	75.3	76.2	77.1
74	69.3	70.2	71.0	71.8	72.5	73.4	74.3	75.2	76.1	77.0
73	69.3	70.2	71.0	71.7	72.5	73.3	74.2	75.1	76.0	77.0
72	69.2	70.1	70.9	71.7	72.4	73.2	74.1	75.0	76.0	76.9
71	69.2	70.1	70.9	71.6	72.3	73.1	74.0	74.9	75.9	76.8
70	69.2	70.0	70.8	71.5	72.3	73.1	73.9	74.8	75.8	76.7
69	69.2	70.0	70.8	71.5	72.2	73.0	73.8	74.8	75.7	76.6
68	69.1	70.0	70.7	71.4	72.1	72.9	73.8	74.7	75.6	76.5
67	69.1	69.9	70.7	71.4	72.1	72.8	73.7	74.6	75.5	76.4
66	69.0	69.9	70.6	71.3	72.0	72.8	73.6	74.5	75.4	76.3
65	69.0	69.8	70.6	71.2	71.9	72.7	73.5	74.4	75.3	76.2
64	69.0	69.8	70.5	71.2	71.8	72.6	73.4	74.3	75.2	76.1
63	68.9	69.8	70.5	71.1	71.8	72.5	73.3	74.2	75.1	76.0
62	68.9	69.7	70.4	71.1	71.7	72.4	73.2	74.1	75.0	75.9
61	68.9	69.7	70.4	71.0	71.6	72.3	73.2	74.0	74.9	75.8
60	68.8	69.6	70.3	70.9	71.5	72.2	73.1	73.9	74.8	75.7
Dry Bulb	270°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Vapour Pressure.									
%	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
100	4.89	5.27	5.66	6.09	6.54	7.03	7.55	8.09	8.68	9.29
90	4.40	4.74	5.09	5.48	5.89	6.33	6.80	7.28	7.81	8.36
80	3.91	4.22	4.53	4.87	5.23	5.62	6.04	6.47	6.94	7.43
70	3.42	3.69	3.96	4.26	4.58	4.92	5.29	5.66	6.08	6.50
60	2.93	3.16	3.40	3.65	3.92	4.22	4.53	4.85	5.21	5.57
50	2.45	2.64	2.83	3.05	3.27	3.52	3.78	4.05	4.34	4.65
40	1.96	2.11	2.26	2.44	2.62	2.81	3.02	3.24	3.47	3.72
30	1.47	1.58	1.70	1.83	1.96	2.11	2.27	2.43	2.60	2.79
20	0.98	1.05	1.13	1.22	1.31	1.41	1.51	1.62	1.73	1.86
10	0.49	0.53	0.57	0.61	0.65	0.70	0.76	0.81	0.87	0.93

Dry Bulb	270°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
59	268.8	269.6	270.3	270.9	271.4	272.2	273.0	273.8	274.7	275.6
58	68.8	69.6	70.2	70.8	71.4	72.1	72.9	73.7	74.6	75.5
57	68.7	69.5	70.2	70.7	71.3	72.0	72.8	73.6	74.5	75.4
56	68.7	69.5	70.1	70.7	71.2	71.9	72.7	73.5	74.4	75.3
55	68.6	69.4	70.0	70.6	71.1	71.8	72.6	73.4	74.3	75.2
54	68.6	69.4	70.0	70.5	71.0	71.7	72.5	73.3	74.2	75.1
53	68.6	69.3	69.9	70.4	70.9	71.6	72.4	73.2	74.1	74.9
52	68.5	69.3	69.9	70.4	70.9	71.5	72.3	73.1	74.0	74.8
51	68.5	69.2	69.8	70.3	70.8	71.4	72.2	73.0	73.9	74.7
50	68.4	69.2	69.7	70.2	70.7	71.3	72.1	72.9	73.8	74.6
49	68.4	69.1	69.7	70.1	70.6	71.2	71.9	72.8	73.6	74.5
48	68.4	69.1	69.6	70.1	70.5	71.1	71.8	72.6	73.5	74.3
47	68.3	69.0	69.6	70.0	70.4	71.0	71.7	72.5	73.4	74.2
46	68.3	68.9	69.5	69.9	70.3	70.9	71.6	72.4	73.3	74.1
45	68.2	68.9	69.4	69.8	70.2	70.7	71.5	72.3	73.1	74.0
44	68.2	68.8	69.3	69.7	70.1	70.6	71.4	72.1	73.0	73.8
43	68.1	68.8	69.3	69.6	70.0	70.5	71.2	72.0	72.9	73.7
42	68.1	68.7	69.2	69.6	69.9	70.4	71.1	71.9	72.7	73.5
41	68.0	68.7	69.1	69.5	69.8	70.3	71.0	71.7	72.6	73.4
40	68.0	68.6	69.0	69.4	69.7	70.2	70.8	71.6	72.5	73.3
39	67.9	68.5	69.0	69.3	69.5	70.0	70.7	71.5	72.3	73.1
38	67.9	68.5	68.9	69.2	69.4	69.9	70.6	71.3	72.2	72.9
37	67.8	68.4	68.8	69.1	69.3	69.8	70.4	71.2	72.0	72.8
36	67.7	68.3	68.7	69.0	69.2	69.6	70.3	71.0	71.9	72.6
35	67.7	68.3	68.6	68.9	69.1	69.5	70.1	70.9	71.7	72.5
34	67.6	68.2	68.6	68.8	68.9	69.3	70.0	70.7	71.5	72.3
33	67.6	68.1	68.5	68.7	68.8	69.2	69.8	70.5	71.3	72.1
32	67.5	68.1	68.4	68.5	68.7	69.1	69.7	70.4	71.2	71.9
31	67.4	68.0	68.3	68.4	68.5	68.9	69.5	70.2	71.0	71.7
30	67.4	67.9	68.2	68.3	68.4	68.8	69.3	70.0	70.8	71.6
29	67.3	67.8	68.1	68.2	68.3	68.6	69.2	69.9	70.6	71.4
28	67.2	67.7	68.0	68.1	68.1	68.4	69.0	69.7	70.4	71.2
27	67.1	67.6	67.9	67.9	67.9	68.2	68.8	69.5	70.2	71.0
26	67.1	67.6	67.8	67.8	67.8	68.1	68.6	69.3	70.0	70.8
25	67.0	67.5	67.7	67.6	67.6	67.9	68.4	69.1	69.8	70.5
24	66.9	67.4	67.5	67.5	67.4	67.7	68.2	68.8	69.6	70.3
23	66.8	67.3	67.4	67.4	67.3	67.5	68.0	68.6	69.4	70.1
22	66.7	67.2	67.3	67.2	67.1	67.3	67.7	68.4	69.1	69.8
21	66.6	67.0	67.2	67.0	66.9	67.1	67.5	68.2	68.9	69.5
20	66.6	66.9	67.0	66.9	66.7	66.8	67.3	67.9	68.6	69.3
10	65.3	65.4	65.2	64.5	63.9	63.7	64.0	64.5	65.1	65.6
Dry Bulb	270°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Temperature Difference.	Increase of Vapour Pressure for small differences of Temperature.									
°	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
.1	.04	.04	.04	.05	.05	.05	.05	.06	.06	.07
.2	.08	.08	.09	.09	.10	.10	.11	.12	.12	.13
.3	.11	.12	.13	.14	.15	.16	.16	.18	.18	.20
.4	.15	.16	.17	.18	.20	.21	.22	.24	.24	.27
.5	.19	.20	.22	.23	.24	.26	.27	.29	.30	.33
.6	.23	.23	.26	.27	.29	.31	.32	.35	.37	.40
.7	.27	.27	.30	.32	.34	.36	.38	.41	.43	.47
.8	.30	.31	.34	.36	.39	.42	.43	.47	.49	.54
.9	.34	.35	.39	.41	.44	.47	.49	.53	.55	.60

Dry Bulb	280°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
99	279.9	280.9	281.9	282.9	283.9	284.9	285.9	286.9	287.9	288.9
98	79.9	80.9	81.9	82.9	83.9	84.8	85.8	86.8	87.8	88.8
97	79.8	80.8	81.8	82.8	83.8	84.8	85.8	86.8	87.8	88.7
96	79.7	80.7	81.7	82.7	83.7	84.7	85.7	86.7	87.7	88.7
95	79.7	80.6	81.6	82.6	83.6	84.6	85.6	86.6	87.6	88.6
94	79.6	80.6	81.6	82.6	83.5	84.5	85.5	86.5	87.5	88.5
93	79.5	80.5	81.5	82.5	83.5	84.5	85.4	86.4	87.4	88.4
92	79.4	80.4	81.4	82.4	83.4	84.4	85.4	86.3	87.3	88.3
91	79.4	80.3	81.3	82.3	83.3	84.3	85.3	86.2	87.2	88.2
90	79.3	80.3	81.3	82.2	83.2	84.2	85.2	86.2	87.1	88.1
89	79.2	80.2	81.2	82.2	83.1	84.1	85.1	86.1	87.0	88.0
88	79.1	80.1	81.1	82.1	83.1	84.0	85.0	86.0	87.0	87.9
87	79.1	80.1	81.0	82.0	83.0	83.9	84.9	85.9	86.9	87.8
86	79.0	80.0	80.9	81.9	82.9	83.9	84.8	85.8	86.8	87.7
85	78.9	79.9	80.9	81.8	82.8	83.8	84.7	85.7	86.7	87.7
84	78.8	79.8	80.8	81.7	82.7	83.7	84.6	85.6	86.6	87.6
83	78.8	79.7	80.7	81.7	82.6	83.6	84.6	85.5	86.5	87.5
82	78.7	79.6	80.6	81.6	82.5	83.5	84.5	85.4	86.4	87.4
81	78.6	79.6	80.5	81.5	82.4	83.4	84.4	85.3	86.3	87.3
80	78.5	79.5	80.4	81.4	82.4	83.3	84.3	85.2	86.2	87.2
79	78.4	79.4	80.4	81.3	82.3	83.2	84.2	85.1	86.1	87.0
78	78.4	79.3	80.3	81.2	82.2	83.1	84.1	85.0	86.0	86.9
77	78.3	79.2	80.2	81.1	82.1	83.0	84.0	84.9	85.9	86.8
76	78.2	79.1	80.1	81.0	82.0	82.9	83.9	84.8	85.8	86.7
75	78.1	79.0	80.0	80.9	81.9	82.8	83.8	84.7	85.7	86.6
74	78.0	78.9	79.9	80.9	81.8	82.7	83.7	84.6	85.6	86.5
73	77.9	78.9	79.8	80.8	81.7	82.6	83.6	84.5	85.5	86.4
72	77.8	78.8	79.7	80.7	81.6	82.5	83.5	84.4	85.3	86.3
71	77.7	78.7	79.6	80.6	81.5	82.4	83.4	84.3	85.2	86.2
70	77.6	78.6	79.5	80.5	81.4	82.3	83.3	84.2	85.1	86.1
69	77.6	78.5	79.4	80.4	81.3	82.2	83.1	84.1	85.0	86.0
68	77.5	78.4	79.3	80.3	81.2	82.1	83.0	84.0	84.9	85.8
67	77.4	78.3	79.2	80.2	81.1	82.0	82.9	83.8	84.8	85.7
66	77.3	78.2	79.1	80.1	81.0	81.9	82.8	83.7	84.7	85.6
65	77.2	78.1	79.0	79.9	80.8	81.8	82.7	83.6	84.5	85.5
64	77.1	78.0	78.9	79.8	80.7	81.7	82.6	83.5	84.4	85.4
63	77.0	77.9	78.8	79.7	80.6	81.6	82.5	83.4	84.3	85.2
62	76.9	77.8	78.7	79.6	80.5	81.4	82.3	83.3	84.2	85.1
61	76.8	77.7	78.6	79.5	80.4	81.3	82.2	83.1	84.0	85.0
60	76.7	77.6	78.5	79.4	80.3	81.2	82.1	83.0	83.9	84.8

Dry Bulb	280°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Vapour Pressure.									
%	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
100	9.96	10.65	11.40	12.19	13.02	13.91	14.85	15.84	16.89	18.01
90	8.96	9.59	10.26	10.97	11.72	12.52	13.37	14.26	15.20	16.21
80	7.97	8.52	9.12	9.75	10.42	11.13	11.88	12.67	13.51	14.41
70	6.97	7.46	7.98	8.53	9.11	9.74	10.40	11.09	11.82	12.61
60	5.98	6.39	6.84	7.31	7.81	8.35	8.91	9.50	10.13	10.81
50	4.98	5.33	5.70	6.10	6.51	6.96	7.43	7.92	8.45	9.01
40	3.98	4.26	4.56	4.88	5.21	5.56	5.94	6.34	6.76	7.20
30	2.99	3.20	3.42	3.66	3.91	4.17	4.46	4.75	5.07	5.40
20	1.99	2.13	2.28	2.44	2.60	2.78	2.97	3.17	3.38	3.60
10	1.00	1.07	1.14	1.22	1.30	1.39	1.49	1.58	1.69	1.80

Dry Bulb	280°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer									
%	°	°	°	°	°	°	°	°	°	°
59	276.6	277.4	278.4	279.3	280.2	281.1	282.0	282.9	283.8	284.7
58	76.5	77.3	78.3	79.2	80.0	81.0	81.8	82.7	83.6	84.6
57	76.3	77.2	78.2	79.0	79.9	80.8	81.7	82.6	83.5	84.4
56	76.2	77.1	78.0	78.9	79.8	80.7	81.6	82.5	83.4	84.3
55	76.1	77.0	77.9	78.8	79.7	80.6	81.5	82.3	83.2	84.2
54	76.0	76.9	77.8	78.7	79.5	80.4	81.3	82.2	83.1	84.0
53	75.9	76.8	77.7	78.6	79.4	80.3	81.2	82.1	82.9	83.9
52	75.8	76.6	77.5	78.4	79.3	80.2	81.0	81.9	82.8	83.7
51	75.6	76.5	77.4	78.3	79.2	80.0	80.9	81.8	82.7	83.6
50	75.5	76.4	77.3	78.2	79.0	79.9	80.8	81.6	82.5	83.4
49	75.4	76.3	77.2	78.0	78.9	79.8	80.6	81.5	82.4	83.3
48	75.2	76.1	77.0	77.9	78.7	79.6	80.5	81.3	82.2	83.1
47	75.1	76.0	76.9	77.7	78.6	79.5	80.3	81.2	82.1	82.9
46	75.0	75.9	76.7	77.6	78.4	79.3	80.2	81.0	81.9	82.8
45	74.8	75.7	76.6	77.5	78.3	79.2	80.0	80.9	81.7	82.6
44	74.7	75.6	76.5	77.3	78.2	79.0	79.8	80.7	81.5	82.4
43	74.6	75.4	76.3	77.1	78.0	78.9	79.7	80.5	81.4	82.2
42	74.4	75.3	76.1	77.0	77.8	78.7	79.5	80.4	81.2	82.1
41	74.3	75.1	76.0	76.8	77.7	78.5	79.4	80.2	81.0	81.9
40	74.1	75.0	75.8	76.7	77.5	78.3	79.2	80.0	80.8	81.7
39	74.0	74.8	75.7	76.5	77.3	78.2	79.0	79.8	80.6	81.5
38	73.8	74.7	75.5	76.3	77.1	78.0	78.8	79.6	80.5	81.3
37	73.7	74.5	75.3	76.2	77.0	77.8	78.6	79.4	80.3	81.1
36	73.5	74.3	75.2	76.0	76.8	77.6	78.4	79.2	80.1	80.9
35	73.3	74.1	75.0	75.8	76.6	77.4	78.2	79.0	79.9	80.7
34	73.2	74.0	74.8	75.6	76.4	77.2	78.0	78.8	79.6	80.5
33	73.0	73.8	74.6	75.4	76.2	77.0	77.8	78.6	79.4	80.3
32	72.8	73.6	74.4	75.2	76.0	76.8	77.6	78.4	79.2	80.0
31	72.6	73.4	74.2	75.0	75.8	76.6	77.4	78.2	79.0	79.8
30	72.4	73.2	74.0	74.8	75.6	76.4	77.2	77.9	78.7	79.6
29	72.2	73.0	73.8	74.6	75.4	76.2	76.9	77.7	78.5	79.3
28	72.0	72.8	73.6	74.4	75.1	75.9	76.7	77.5	78.2	79.0
27	71.8	72.6	73.4	74.1	74.9	75.7	76.4	77.2	78.0	78.8
26	71.6	72.3	73.2	73.9	74.7	75.5	76.2	77.0	77.7	78.5
25	71.3	72.1	72.9	73.7	74.4	75.2	75.9	76.7	77.4	78.2
24	71.1	71.9	72.7	73.4	74.2	74.9	75.6	76.4	77.2	78.0
23	70.9	71.6	72.4	73.2	73.9	74.6	75.4	76.1	76.9	77.6
22	70.6	71.3	72.1	72.9	73.6	74.4	75.1	75.8	76.6	77.3
21	70.3	71.1	71.9	72.6	73.3	74.1	74.8	75.5	76.2	77.0
20	70.0	70.8	71.6	72.3	73.0	73.7	74.4	75.2	75.9	76.7
10	66.4	67.0	67.6	68.2	68.9	69.5	70.1	70.7	71.3	72.0

Dry Bulb	280°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Temperature Difference.	Increase of Vapour Pressure for small differences of Temperature.									
°	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
1	.07	.07	.08	.08	.09	.09	.10	.11	.11	.12
2	.14	.15	.16	.16	.18	.19	.20	.21	.22	.24
3	.21	.22	.24	.25	.27	.28	.30	.32	.34	.36
4	.28	.30	.32	.33	.36	.38	.40	.42	.45	.48
5	.35	.37	.40	.41	.45	.47	.50	.53	.56	.60
6	.42	.45	.47	.50	.54	.56	.59	.63	.67	.71
7	.49	.52	.55	.58	.63	.66	.69	.74	.78	.83
8	.56	.60	.63	.67	.71	.75	.79	.84	.90	.95
9	.63	.67	.71	.75	.80	.85	.89	.95	1.01	1.07

Dry Bulb	290°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
99	289.9	290.9	291.9	292.9	293.9	294.9	295.9	296.9	297.9	298.9
98	89.8	90.8	91.8	92.8	93.8	94.8	95.8	96.8	97.8	98.8
97	89.7	90.7	91.7	92.7	93.7	94.7	95.7	96.7	97.7	98.7
96	89.7	90.6	91.6	92.6	93.6	94.6	95.6	96.6	97.6	98.6
95	89.6	90.6	91.6	92.5	93.5	94.5	95.5	96.5	97.5	98.5
94	89.5	90.5	91.5	92.4	93.4	94.4	95.4	96.4	97.4	98.4
93	89.4	90.4	91.4	92.3	93.3	94.3	95.3	96.3	97.3	98.3
92	89.3	90.3	91.3	92.2	93.2	94.2	95.2	96.2	97.2	98.2
91	89.2	90.2	91.2	92.2	93.1	94.1	95.1	96.1	97.1	98.1
90	89.1	90.1	91.1	92.1	93.0	94.0	95.0	96.0	97.0	98.0
89	89.0	90.0	91.0	92.0	92.9	93.9	94.9	95.9	96.9	97.8
88	88.9	89.9	90.9	91.9	92.8	93.8	94.8	95.8	96.8	97.7
87	88.8	89.8	90.8	91.8	92.7	93.7	94.7	95.7	96.6	97.6
86	88.7	89.7	90.7	91.6	92.6	93.6	94.6	95.6	96.5	97.5
85	88.6	89.6	90.6	91.5	92.5	93.5	94.5	95.4	96.4	97.4
84	88.5	89.5	90.5	91.4	92.4	93.4	94.4	95.3	96.3	97.3
83	88.4	89.4	90.4	91.3	92.3	93.3	94.3	95.2	96.2	97.2
82	88.3	89.3	90.3	91.2	92.2	93.2	94.1	95.1	96.1	97.0
81	88.2	89.2	90.2	91.1	92.1	93.1	94.0	95.0	95.9	96.9
80	88.1	89.1	90.1	91.0	92.0	92.9	93.9	94.9	95.8	96.8
79	88.0	89.0	90.0	90.9	91.9	92.8	93.8	94.7	95.7	96.7
78	87.9	88.9	89.8	90.8	91.7	92.7	93.7	94.6	95.6	96.6
77	87.8	88.8	89.7	90.7	91.6	92.6	93.6	94.5	95.5	96.4
76	87.7	88.7	89.6	90.6	91.5	92.5	93.4	94.4	95.3	96.3
75	87.6	88.5	89.5	90.5	91.4	92.3	93.3	94.3	95.2	96.2
74	87.5	88.4	89.4	90.3	91.3	92.3	93.2	94.1	95.1	96.1
73	87.4	88.3	89.3	90.2	91.2	92.1	93.1	94.0	95.0	95.9
72	87.2	88.2	89.1	90.1	91.0	92.0	92.9	93.9	94.8	95.8
71	87.1	88.1	89.0	90.0	90.9	91.9	92.8	93.7	94.7	95.7
70	87.0	88.0	88.9	89.8	90.8	91.7	92.7	93.6	94.6	95.5
69	86.9	87.8	88.8	89.7	90.7	91.6	92.6	93.5	94.4	95.4
68	86.8	87.7	88.7	89.6	90.5	91.5	92.4	93.4	94.3	95.2
67	86.7	87.6	88.5	89.5	90.4	91.3	92.3	93.2	94.1	95.1
66	86.5	87.5	88.4	89.3	90.3	91.2	92.2	93.1	94.0	95.0
65	86.4	87.3	88.3	89.2	90.1	91.1	92.0	92.9	93.9	94.8
64	86.3	87.2	88.2	89.1	90.0	90.9	91.9	92.8	93.7	94.7
63	86.2	87.1	88.0	88.9	89.9	90.8	91.7	92.6	93.6	94.5
62	86.0	87.0	87.9	88.8	89.7	90.6	91.6	92.5	93.4	94.4
61	85.9	86.8	87.8	88.7	89.6	90.5	91.4	92.4	93.3	94.2
60	85.8	86.7	87.6	88.5	89.4	90.4	91.3	92.2	93.1	94.1
Dry Bulb	290°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Vapour Pressure.									
%	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
100	19.20	20.44	21.76	23.14	24.62	26.17	27.81	29.53	31.36	33.28
90	17.28	18.40	19.58	20.83	22.16	23.55	25.03	26.58	28.22	29.95
80	15.36	16.35	17.42	18.51	19.70	20.94	22.25	23.62	25.09	26.62
70	13.44	14.31	15.23	16.20	17.23	18.32	19.47	20.67	21.95	23.30
60	11.52	12.26	13.07	13.88	14.77	15.70	16.69	17.72	18.82	19.97
50	9.60	10.22	10.88	11.57	12.31	13.09	13.91	14.77	15.68	16.64
40	7.68	8.18	8.70	9.26	9.85	10.47	11.12	11.81	12.54	13.31
30	5.76	6.13	6.53	6.94	7.39	7.85	8.34	8.86	9.41	9.99
20	3.84	4.09	4.35	4.63	4.92	5.23	5.56	5.91	6.27	6.66
10	1.92	2.04	2.18	2.31	2.46	2.62	2.78	2.95	3.14	3.33

Dry Bulb	290°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
59	285.6	286.5	287.5	288.4	289.3	290.2	291.1	292.1	293.0	293.9
58	85.5	86.4	87.3	88.2	89.1	90.1	91.0	91.9	92.8	93.7
57	85.4	86.3	87.2	88.1	89.0	89.9	90.8	91.7	92.6	93.6
56	85.2	86.1	87.0	87.9	88.8	89.7	90.7	91.6	92.5	93.4
55	85.1	86.0	86.9	87.8	88.7	89.6	90.5	91.4	92.3	93.2
54	84.9	85.8	86.7	87.6	88.5	89.4	90.4	91.3	92.1	93.1
53	84.8	85.7	86.6	87.5	88.4	89.3	90.2	91.1	92.0	92.9
52	84.6	85.5	86.4	87.3	88.2	89.1	90.0	90.9	91.8	92.7
51	84.5	85.4	86.3	87.1	88.0	88.9	89.9	90.7	91.6	92.5
50	84.3	85.2	86.1	87.0	87.9	88.8	89.7	90.6	91.4	92.4
49	84.2	85.0	85.9	86.8	87.7	88.6	89.5	90.4	91.3	92.2
48	84.0	84.9	85.8	86.6	87.5	88.4	89.3	90.2	91.1	92.0
47	83.8	84.7	85.6	86.5	87.4	88.2	89.1	90.0	90.9	91.8
46	83.7	84.5	85.4	86.3	87.2	88.0	88.9	89.8	90.7	91.6
45	83.5	84.4	85.2	86.1	87.0	87.9	88.8	89.6	90.5	91.4
44	83.3	84.2	85.1	85.9	86.8	87.7	88.6	89.4	90.3	91.2
43	83.1	84.0	84.9	85.7	86.6	87.5	88.4	89.2	90.1	91.0
42	83.0	83.8	84.7	85.5	86.4	87.3	88.2	89.0	89.9	90.8
41	82.8	83.6	84.5	85.3	86.2	87.1	87.9	88.8	89.6	90.5
40	82.6	83.4	84.3	85.1	86.0	86.9	87.7	88.6	89.4	90.3
39	82.4	83.2	84.1	84.9	85.8	86.6	87.5	88.4	89.2	90.1
38	82.2	83.0	83.9	84.7	85.6	86.4	87.3	88.1	89.0	89.9
37	81.9	82.8	83.7	84.5	85.4	86.2	87.1	87.9	88.7	89.6
36	81.8	82.6	83.5	84.3	85.1	86.0	86.8	87.7	88.5	89.4
35	81.6	82.4	83.2	84.1	84.9	85.7	86.6	87.4	88.3	89.1
34	81.3	82.2	83.0	83.8	84.7	85.5	86.4	87.2	88.0	88.9
33	81.1	81.9	82.8	83.6	84.4	85.2	86.1	86.9	87.7	88.6
32	80.9	81.7	82.5	83.4	84.2	85.0	85.9	86.7	87.5	88.4
31	80.6	81.5	82.3	83.1	83.9	84.7	85.6	86.4	87.2	88.1
30	80.4	81.2	82.0	82.9	83.7	84.5	85.3	86.1	86.9	87.8
29	80.2	81.0	81.8	82.6	83.4	84.2	85.1	85.8	86.6	87.5
28	79.9	80.7	81.5	82.3	83.1	83.9	84.8	85.5	86.3	87.2
27	79.6	80.4	81.2	82.0	82.8	83.6	84.5	85.2	86.0	86.9
26	79.4	80.1	80.9	81.7	82.5	83.3	84.2	84.9	85.7	86.6
25	79.1	79.8	80.6	81.4	82.2	83.0	83.8	84.6	85.4	86.2
24	78.8	79.5	80.3	81.1	82.0	82.7	83.5	84.3	85.0	85.9
23	78.5	79.2	80.0	80.8	81.6	82.3	83.2	83.9	84.7	85.5
22	78.1	78.9	79.7	80.5	81.2	82.0	82.8	83.6	84.3	85.1
21	77.8	78.6	79.4	80.1	80.9	81.6	82.4	83.2	83.9	84.8
20	77.4	78.2	79.0	79.7	80.5	81.2	82.1	82.8	83.5	84.4
10	72.7	73.4	74.1	74.8	75.4	76.1	76.8	77.4	78.1	78.8
Dry Bulb	296°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Temperature Difference.	Increase of Vapour Pressure for small differences of Temperature.									
°	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
.1	.12	.13	.14	.15	.16	.16	.17	.18	.19	.20
.2	.25	.26	.28	.30	.31	.33	.34	.37	.38	.40
.3	.37	.40	.41	.44	.47	.49	.52	.55	.58	.61
.4	.50	.53	.55	.59	.62	.66	.69	.73	.77	.81
.5	.62	.66	.69	.74	.78	.82	.86	.92	.96	1.01
.6	.74	.79	.83	.89	.93	.98	1.03	1.10	1.15	1.21
.7	.87	.92	.97	1.04	1.09	1.15	1.20	1.28	1.34	1.41
.8	.99	1.06	1.10	1.19	1.24	1.31	1.38	1.46	1.54	1.62
.9	1.12	1.19	1.24	1.33	1.40	1.48	1.55	1.65	1.73	1.82

Dry Bulb	300°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity.	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
99	299.9	300.9	301.9	302.9	303.9	304.9	305.9	306.9	307.9	308.9
98	99.8	00.8	01.8	02.8	03.8	04.8	05.8	06.8	07.8	08.8
97	99.7	00.7	01.7	02.7	03.7	04.7	05.7	06.7	07.7	08.7
96	99.6	00.6	01.6	02.6	03.6	04.6	05.6	06.6	07.6	08.6
95	99.5	00.5	01.5	02.5	03.5	04.5	05.4	06.4	07.4	08.4
94	99.4	00.4	01.4	02.4	03.3	04.3	05.3	06.3	07.3	08.3
93	99.3	00.3	01.3	02.2	03.2	04.2	05.2	06.2	07.2	08.2
92	99.2	00.2	01.1	02.1	03.1	04.1	05.1	06.1	07.1	08.1
91	99.1	00.0	01.0	02.0	03.0	04.0	05.0	06.0	07.0	08.0
90	98.9	299.9	00.9	01.9	02.9	03.9	04.9	05.8	06.8	07.8
89	98.8	99.8	00.8	01.8	02.8	03.8	04.7	05.7	06.7	07.7
88	98.7	99.7	00.7	01.7	02.7	03.6	04.6	05.6	06.6	07.6
87	98.6	99.6	00.6	01.6	02.5	03.5	04.5	05.5	06.4	07.4
86	98.5	99.5	00.4	01.4	02.4	03.4	04.4	05.3	06.3	07.3
85	98.4	99.4	00.3	01.3	02.3	03.3	04.2	05.2	06.2	07.2
84	98.3	99.2	00.2	01.2	02.2	03.1	04.1	05.1	06.1	07.0
83	98.1	99.1	00.1	01.1	02.0	03.0	04.0	05.0	05.9	06.9
82	98.0	99.0	00.0	00.9	01.9	02.9	03.9	04.8	05.8	06.8
81	97.9	98.9	299.8	00.8	01.8	02.8	03.7	04.7	05.7	06.6
80	97.8	98.7	99.7	00.7	01.7	02.6	03.6	04.6	05.5	06.5
79	97.6	98.6	99.6	00.6	01.5	02.5	03.5	04.4	05.4	06.4
78	97.5	98.5	99.5	00.4	01.4	02.4	03.3	04.3	05.2	06.2
77	97.4	98.4	99.3	00.3	01.3	02.2	03.2	04.1	05.1	06.1
76	97.3	98.2	99.2	00.2	01.1	02.1	03.0	04.0	05.0	05.9
75	97.1	98.1	99.1	00.0	01.0	01.9	02.9	03.9	04.8	05.8
74	97.0	98.0	98.9	299.9	00.9	01.8	02.8	03.7	04.7	05.6
73	96.9	97.8	98.8	99.7	00.7	01.7	02.6	03.6	04.5	05.5
72	96.7	97.7	98.6	99.6	00.6	01.5	02.5	03.4	04.4	05.3
71	96.6	97.6	98.5	99.5	00.4	01.4	02.3	03.3	04.2	05.2
70	96.5	97.4	98.4	99.3	00.3	01.2	02.2	03.1	04.1	05.0
69	96.3	97.3	98.2	99.2	00.1	01.1	02.0	03.0	03.9	04.9
68	96.2	97.1	98.1	99.0	00.0	00.9	01.9	02.8	03.8	04.7
67	96.0	97.0	97.9	98.9	299.8	00.8	01.7	02.7	03.6	04.5
66	95.9	96.8	97.8	98.7	99.7	00.6	01.6	02.5	03.4	04.4
65	95.8	96.7	97.6	98.6	99.5	00.5	01.4	02.3	03.3	04.2
64	95.6	96.5	97.5	98.4	99.4	00.3	01.2	02.2	03.1	04.0
63	95.4	96.4	97.3	98.3	99.2	00.1	01.1	02.0	02.9	03.9
62	95.3	96.2	97.2	98.1	99.1	00.0	00.9	01.8	02.8	03.7
61	95.1	96.1	97.0	97.9	98.9	299.8	00.7	01.7	02.6	03.5
60	95.0	95.9	96.8	97.7	98.7	99.6	00.6	01.5	02.4	03.3
Dry Bulb	300°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity.	Vapour Pressure.									
%	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
100	35.29	37.42	39.65	42.01	44.49	47.09	49.83	52.69	55.71	58.88
90	31.76	33.68	35.69	37.81	40.04	42.38	44.85	47.42	50.14	52.99
80	28.23	29.94	31.72	33.61	35.59	37.67	39.86	42.15	44.57	47.10
70	24.70	26.19	27.76	29.41	31.14	32.96	34.88	36.88	39.00	41.22
60	21.17	22.45	23.79	25.21	26.69	28.25	29.90	31.61	33.43	35.33
50	17.65	18.71	19.83	21.01	22.25	23.55	24.92	26.35	27.86	29.44
40	14.12	14.97	15.86	16.80	17.80	18.84	19.93	21.08	22.28	23.55
30	10.59	11.23	11.90	12.60	13.35	14.13	14.95	15.81	16.71	17.66
20	7.06	7.48	7.93	8.40	8.90	9.42	9.97	10.54	11.14	11.78
10	3.53	3.74	3.97	4.20	4.45	4.71	4.98	5.27	5.57	5.89

Dry Bulb	300°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Relative Humidity.	Wet Thermometer.									
%	°	°	°	°	°	°	°	°	°	°
59	294.8	295.8	296.7	297.6	298.5	299.5	300.4	301.3	302.3	303.2
58	94.7	95.6	96.5	97.4	98.4	99.3	00.2	01.2	02.1	03.0
57	94.5	95.4	96.3	97.3	98.2	99.1	00.0	01.0	01.9	02.8
56	94.3	95.3	96.2	97.1	98.0	98.9	299.9	00.8	01.7	02.6
55	94.2	95.1	96.0	96.9	97.8	98.7	99.7	00.6	01.5	02.4
54	94.0	94.9	95.8	96.7	97.7	98.6	99.5	00.4	01.3	02.2
53	93.8	94.7	95.6	96.6	97.5	98.4	99.3	00.2	01.1	02.0
52	93.6	94.5	95.5	96.4	97.3	98.2	99.1	00.0	00.9	01.8
51	93.4	94.4	95.3	96.2	97.1	98.0	98.9	2.9.8	00.7	01.6
50	93.3	94.2	95.1	96.0	96.9	97.8	98.7	99.6	00.5	01.4
49	93.1	94.0	94.9	95.8	96.7	97.6	98.5	99.4	00.3	01.2
48	92.9	93.8	94.7	95.6	96.5	97.4	98.3	99.2	00.1	01.0
47	92.7	93.6	94.5	95.4	96.3	97.2	98.1	99.0	299.9	00.8
46	92.5	93.4	94.3	95.2	96.1	97.0	97.9	98.8	99.6	00.5
45	92.3	93.2	94.1	95.0	95.9	96.8	97.7	98.5	99.4	00.3
44	92.1	93.0	93.9	94.8	95.7	96.5	97.4	98.3	99.2	00.1
43	91.9	92.8	93.7	94.6	95.4	96.3	97.2	98.1	99.0	299.9
42	91.6	92.5	93.4	94.3	95.2	96.1	97.0	97.8	98.7	99.6
41	91.4	92.3	93.2	94.1	95.0	95.9	96.7	97.6	98.5	99.4
40	91.2	92.1	93.0	93.9	94.8	95.6	96.5	97.4	98.2	99.1
39	91.0	91.9	92.7	93.6	94.5	95.4	96.3	97.1	98.0	98.9
38	90.7	91.6	92.5	93.4	94.3	95.1	96.0	96.9	97.7	98.6
37	90.5	91.4	92.3	93.1	94.0	94.9	95.7	96.6	97.5	98.3
36	90.3	91.1	92.0	92.9	93.7	94.6	95.5	96.3	97.2	98.0
35	90.0	90.9	91.7	92.6	93.5	94.3	95.2	96.1	96.9	97.8
34	89.7	90.6	91.5	92.3	93.2	94.1	94.9	95.8	96.6	97.5
33	89.5	90.3	91.2	92.1	92.9	93.8	94.6	95.5	96.3	97.2
32	89.2	90.1	90.9	91.8	92.6	93.5	94.3	95.2	96.0	96.9
31	88.9	89.8	90.6	91.5	92.3	93.2	94.0	94.9	95.7	96.6
30	88.6	89.5	90.3	91.2	92.0	92.9	93.7	94.6	95.4	96.2
29	88.3	89.2	90.0	90.9	91.7	92.6	93.4	94.2	95.1	95.9
28	88.0	88.9	89.7	90.6	91.4	92.2	93.1	93.9	94.7	95.6
27	87.7	88.6	89.4	90.2	91.2	91.9	92.7	93.6	94.4	95.2
26	87.4	88.2	89.1	89.9	90.7	91.5	92.4	93.2	94.0	94.8
25	87.0	87.9	88.7	89.5	90.4	91.2	92.0	92.8	93.6	94.5
24	86.7	87.5	88.4	89.2	90.0	90.8	91.6	92.4	93.2	94.0
23	86.3	87.2	88.0	88.8	89.6	90.4	91.2	92.0	92.8	93.6
22	86.0	86.8	87.6	88.4	89.2	90.0	90.8	91.6	92.4	93.2
21	85.6	86.4	87.2	88.0	88.8	89.6	90.4	91.2	92.0	92.8
20	85.2	86.0	86.8	87.6	88.4	89.2	90.0	90.8	91.5	92.3
10	79.6	80.3	81.0	81.7	82.4	83.2	83.9	84.6	85.4	86.1
Dry Bulb	300°	1°	2°	3°	4°	5°	6°	7°	8°	9°
Temperature Difference.	Increase of Vapour Pressure for small differences of Temperature.									
°	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.	mb.
.1	.21	.22	.24	.25	.26	.27	.29	.30	.32	.33
.2	.43	.45	.47	.50	.52	.55	.57	.60	.63	.66
.3	.64	.67	.71	.74	.78	.82	.86	.91	.95	1.00
.4	.85	.89	.94	.99	1.04	1.10	1.14	1.21	1.27	1.33
.5	1.07	1.11	1.18	1.24	1.30	1.37	1.43	1.51	1.58	1.66
.6	1.28	1.34	1.42	1.49	1.56	1.64	1.72	1.81	1.90	1.99
.7	1.49	1.56	1.65	1.74	1.82	1.92	2.00	2.11	2.22	2.32
.8	1.70	1.78	1.89	1.98	2.08	2.19	2.29	2.42	2.54	2.65
.9	1.92	2.01	2.12	2.23	2.34	2.47	2.57	2.72	2.85	2.99

PSYCHROMETRIC TABLES.—INSTRUCTIONS FOR USE.

1. To obtain the relative humidity from the readings of the dry and wet bulbs.

A series of dry bulb temperatures in steps of 1°C . are set out in a row at the head of the table, and relative humidities (percentages of saturation) are set out in the column on the left-hand side of each page. In the body of the table are inserted the corresponding wet bulb temperatures.

If the temperatures are merely read to whole degrees the procedure is simple. Look out the dry bulb temperature in the heading, and follow down the column until the wet bulb temperature is reached. Then read off the relative humidity at the side.

But when the whole degree is not sufficiently accurate, we require to compute the humidity from readings of the dry bulb and wet bulb to the tenth of a degree. The percentage humidity may, however, be given as a whole number. On account of various uncertainties, partly meteorological and partly physical, the decimal places of the percentage humidity are beyond the working limit of observations with the wet and dry bulb. Even the figure in the units place is uncertain when the temperature of evaporation, or wet bulb, is much below that of the air.

Hence, provisionally, the following process is generally sufficiently accurate for temperatures read to tenths:—Adjust the temperatures by adding to each or subtracting from each the same amount, so that the adjusted dry bulb is an exact degree. Look out the adjusted dry bulb reading in the heading, follow down the column to the adjusted wet bulb reading, and read off the humidity at the left-hand side.

For example:—

Dry bulb	285.6
Wet bulb	279.2

Add .4 to each, run down the 286 column to 279.6. Humidity 42 or 43.

To test this:—Subtract .6 from each, run down the 285 column to 278.6. Humidity 41 or 42.

In the first case the drier ought to be chosen, because the same depression of wet means drier air at lower air temperature; and in the second case, for the converse reason, the moister should be chosen. So in this instance the result is the same either way.

2. To obtain the vapour-pressure in the air.

At the bottom of the left-hand page is a table of vapour-pressures for steps of 10 per cent. in relative humidity, shown in the column on the left, and dry bulb temperatures shown in the row at the top. The numbers in any column are simply proportional to the relative humidity, so that the necessary adjustment for steps of 1 per cent. is got by moving the decimal point.

3. To make the adjustment for the fraction of a degree.

Use the table on the right-hand page which gives the addition to be made to the pressure at saturation for each tenth of a degree at the several temperatures indicated in the heading. The corresponding correction at any degree of humidity is proportional. Thus the correction for $.5^{\circ}$ at 45 per cent. humidity may be taken as the correction for $\frac{45}{100} \times \frac{5^{\circ}}{10}$, or $.2^{\circ}$.

Thus for dry bulb 285.6, humidity 42, we get from p. I 6 and 7—
dry bulb 285, humidity 40, vapour-pressure 5.56 mb.

add for	“	285	“	2	“	.28	“
add for	“	.6	“	40	“	.22	“
	“	285.6	“	42	“	6.1	“

NOTE ON THE COMPUTATION OF HUMIDITY.

Glaisher's hygrometric tables, which are generally used in the British Isles, are based on the determination of the dew point from readings of the dry and wet bulb thermometers by means of the formula

$$t - d = A(t - t')$$

where t is the temperature of the dry bulb

t' is the temperature of the wet bulb

d is the temperature of the dew point

A is a factor which depends on the temperature of the dry bulb.

The values of this factor A were determined from the comparison of many thousand simultaneous observations of the dry and wet bulb thermometers and of the Daniell Hygrometer taken at the Royal Observatory, Greenwich, from the year 1841 to 1854, and from observations taken at high temperature in India and others at low and medium temperatures at Toronto.

Special Tables, abstracted from the detailed tables computed by Mr. Glaisher, were prepared by the Meteorological Office for the use of its observers, for computing relative humidity, vapour pressure and dew point from readings of dry and wet bulb thermometers.

The Hygrometric Tables used in other countries are based on Regnault's formula $e'' = e' - A(t - t')B$, where e' and e'' are the vapour pressures at the temperatures of the wet bulb and the dew point respectively, t and t' the readings of the dry and wet bulbs, b the pressure of the air, and A , a constant. In the last edition (1911) of Jelinek's Table, as revised by the late Professor Pernter, the values of the constant A depend on the ventilation to which the thermometers are exposed. Three cases are distinguished (1) indoor readings without the use of a fan, or outdoor screen readings on occasions of calm; (2) screen readings on occasion of light winds; (3) screen readings on occasions of moderate or strong wind, or readings obtained with Assmann's ventilated psychrometer or with sling psychrometers. When the temperature of the wet bulb is below the freezing point, different values of A are introduced

according as the wet bulb is coated with ice or with super-cooled water. We have thus the following six values of A (e' , e'' and B being measured in millimetres, t and t' in centigrade degrees): (1) wet bulb coated with water, calm 0.001200, light wind 0.000800, strong wind 0.000656; (2) wet bulb coated with ice, calm 0.001060, light wind 0.000706, strong wind 0.000579.

Jelinek's Tables are used at official stations in Austria. The tables used in other countries are in substantial agreement with Jelinek's Tables, for the cases "light wind" or "moderate wind," wet bulb coated with water, according as artificial ventilation is or is not used. Values deduced from Glaisher's Tables differ little from those computed from Jelinek's Tables at ordinary temperatures and humidities, but on occasions of great dryness and at low temperatures the differences are considerable.

e' , e'' and B must all be measured in the same units.

Take as an example:—

Assmann's Psychrometer. Dry Bulb 20.9 C=293.9a.
Wet Bulb 15.3 C=288.3a.
Air pressure, 1000 mb.

$$\begin{aligned} \text{From the bottom of pp. 6 and 7, } e' &= 16.89 \text{ mb.} \\ &+ .34 \text{ } \} = 17.23 \\ AB(t-t') &= 0.00656 \times 10^3 \times 5.6 = 3.67 \\ e'' &= 13.56 \end{aligned}$$

The vapour pressure is 13.56 mb.

The relative humidity is $13.56/24.5 = 55\%$

Note.—The same readings 293.9a., 288.3a. give by Glaisher's Table, Humidity = 53% and Vapour Pressure 13.0mb. At ordinary pressures at moderate heights the factor AB may be taken as $\frac{3}{4}$.

The formulæ used by Pernter when the cover of the wet bulb is not frozen are

Calm (wind velocity in the screen containing the thermometers, 0 to 0.5 metres per second)

$$e'' = e' - .0012 B(t-t')(1+t'/610).$$

Light Wind (wind velocity in the screen, 1 to 1.5 metres per second)

$$e'' = e' - .0008 B(t-t')(1+t'/610).$$

Strong Wind (wind velocity in the screen greater than 2.5 metres per second)

$$e'' = e' - .000656 B(t-t')(1+t'/610).$$

When the water in the cover of the bulb is frozen the formulæ are modified by reducing the numerical co-efficients to .001060, .000706 and .000579 respectively and by writing 689 for 610 in the last factor.

The temperatures are on the centigrade scale; and the vapour pressure is given in the same units as those used for B .

With these may be associated the tables prepared for ASSMANN'S VENTILATED PSYCHROMETER in which the bulbs are exposed to an artificial current of air. For these tables the factor A is taken to be .5/755 and they therefore agree very closely with Pernter's formula for the case of strong wind.

The tables used in INDIA compiled originally by Blanford and recently revised are based upon the formula

$$e'' = e' - .000437 B(T-T') \left(1 + \frac{T'-32}{1098}\right)$$

where T and T' are the temperatures in Fahrenheit degrees.

When converted to the centigrade scale the formula becomes

$$e'' = e' - .00079 B(t-t')(1+t'/610)$$

and is therefore practically identical with Pernter's formula for light wind.

The tables used in the UNITED STATES OF AMERICA are those compiled by Marvin from Ferrel's formula

$$e'' = e' - .000367 B(T-T') \left(1 + \frac{T-T'}{1571}\right)$$

They are intended for use with sling thermometers which are whirled in the air before taking a reading: they are therefore comparable with Pernter's table for strong wind. Numerically, the formulæ are nearly identical so long as the effect of the final factor is negligible, since the dominant factor used by Marvin is .000367, and when allowance is made for the different scale of temperature that used by Pernter for strong winds is .000364.

The difference of form suggests that the final factor becomes important when the difference of the dry and wet thermometers becomes large. Numerically, however, the difference is unimportant in practice on account of the great denominator. But the difference of principle in using a subtractive term in $(T-T')^2$ is important since A. Angot of the Bureau Central Météorologique has shown that such a term is necessary and has proposed the following formula—

$$e'' = e' \{1 - 0.0159(t-t')\} - B(t-t') \{0.000776 - 0.000028(t-t')\}$$

for temperatures of the wet bulb above the freezing point.

The instructions of the Bureau Central Météorologique, however, prescribe a formula—

$$e'' = e' - .00079 B(t-t'),$$

which practically agrees with Pernter's light wind formula and the Indian formula.

The experience of the Egyptian Meteorological Service confirms the view that some modification of the linear form of the equation is necessary for computing humidity when the air is very dry. The linear formula has occasionally been found to give negative values for the vapour pressure, showing that even the constant for strong wind is too large when the humidity is very low.

The operation of the different formulæ may be effectively examined by pushing these conditions to the extreme, and applying the computation to the case of perfectly dry air when vapour pressure and humidity are both zero.

If this be understood to mean that the dew point must be taken as at absolute zero, Glaisher's principle at once leads to most improbable depressions of the wet thermometer, for all air temperatures quoted in the table of p. 13. With other formulæ values for $t-t'$ can be obtained when e'' is taken to be zero.

Ice-covered Bulbs.

The formulæ which have been quoted are held by their authors to be applicable in the cases in which the wet bulb is covered with a coating of water. When a coating of ice is produced other considerations arise which include the latent heat of fusion of ice and the difference of the saturation pressures of water-vapour and ice-vapour at the same temperature.

These considerations have brought in a new set of tables by B. J. Birkeland of the Norwegian Meteorological Institute who has given a series of values for the constant A , arranged according to the degree of ventilation of the wet thermometer.

For a ventilation of 2 m/s Birkeland's tables agree with those of the aspiration psychrometer. In both, for the frozen wet bulb, e' in the formula is taken as the pressure of ice-vapour while the pressure of water-vapour is taken for e in computing the relative humidity.

Neither Glaisher nor Marvin takes account of any difference between ice-covered bulbs and water-covered bulbs. The formulæ of Pernter are affected by the reduction of the constant A in the ratio of 600:680 and by the change of the denominator of the t' fraction from 610 to 690. Tables of vapour-pressure of water are used even when the wet bulb is coated with ice, but the difference $t - t'$ is then augmented by $\cdot 4$ for the purpose of computation.

Angot's extended formula for the wet bulb below the freezing point is—

$$e'' = e' - [1 - 0.059(t - t')] - B(t - t')[0.000682 - 0.000028(t - t')] \text{ when } t' < 0,$$

while the one prescribed in his book of instructions is—

$$e'' = e' - 0.00069 B(t - t') \text{ when } t' < 0.$$

The following are the various psychrometric formulæ adapted for absolute units.

e'' , e' , B , are in millibars and θ and θ' in absolute temperature.

AUSTRIA (Pernter).

$$\theta' > 273a.$$

Calm.

$$e'' = e' - 0.0012 B(\theta - \theta') \left(1 + \frac{\theta' - 273}{610}\right)$$

Light Wind.

$$e'' = e' - 0.0008 B(\theta - \theta') \left(1 + \frac{\theta' - 273}{610}\right)$$

Strong Wind.

$$e'' = e' - 0.000656 B(\theta - \theta') \left(1 + \frac{\theta' - 273}{610}\right)$$

FRANCE (Angot).

$$\theta' > 273a.$$

$$\left\{ \begin{array}{l} e'' = e' - 0.0159(\theta - \theta') \\ - B(\theta - \theta')[0.000776 - 0.000028(\theta - \theta')] \end{array} \right\} \left\{ \begin{array}{l} e'' = e' - 0.059(\theta - \theta') \\ - B(\theta - \theta')[0.000682 - 0.000028(\theta - \theta')] \end{array} \right\}$$

FRANCE (Bureau Central Météorologique).

$$\theta' > 273a.$$

$$e'' = e' - 0.00079 B(\theta - \theta')$$

$$\theta' < 273a.$$

$$\theta' < 273a.$$

$$e'' = e' - 0.00069 B(\theta - \theta')$$

INDIA.

$$e'' = e' - 0.00079 B(\theta - \theta') \left(1 + \frac{\theta' - 273}{610}\right)$$

U.S.A. (Marvin).

$$e'' = e' - 0.000660 B(\theta - \theta') \left(1 + \frac{\theta - \theta'}{870}\right)$$

THE WET BULB EXPOSED TO A CURRENT OF PERFECTLY DRY AIR.

In order to obtain some definite evidence as to the effect of perfectly dry air upon the temperature of the wet bulb an apparatus was arranged by Sir James Dewar, F.R.S., in the laboratory of the Royal Institution, by means of which air was dried by liquefaction and redistillation and then passed over a thermometer bulb surrounded by moistened cotton wool. The temperatures of the dry bulb were at the freezing point of water or not far above it. The curves representing the gradual fall of temperature of the wet bulb after the commencement of the current of dry air show that, if the temperature of the air is not too high, there is a depression of the wet bulb below the freezing point owing to the water being supercooled. After the effect has proceeded for some time the water suddenly freezes and the temperature of the wet bulb at once rises to the freezing point; then the cooling of the ice-covered bulb sets in and proceeds until a steady state is approached. In the experiments there is no suggestion of regularity about the point at which supercooling gives way to freezing. The results of the experiments are set out in the following table:—

Depression of the Wet Bulb in perfectly dry air. (From the laboratory of the Royal Institution.)

No. of Experiment.	5	2	4	6	3	15	16	9	10
Temperature of Dry Bulb.	272.9	273	273	273	273.1	278.2 278.7	278.6 278.1	281.1	281.2
Flow of air in litres/sec.*	.22	.067	.094	.16	.134	.22	.24	.22	.47
Temperature of Water Bulb.†	<272.4		<271.2	<269.3	<269.9	272.4	272.2 272.6	<274.1	274.1
Temperature of Ice Bulb.	269.3	270.2	270.1	270.2					
Temperature Difference. "Depression of Wet."	3.6	2.8	2.9	2.8	>3.2	6.3	5.8	7.0	7.1

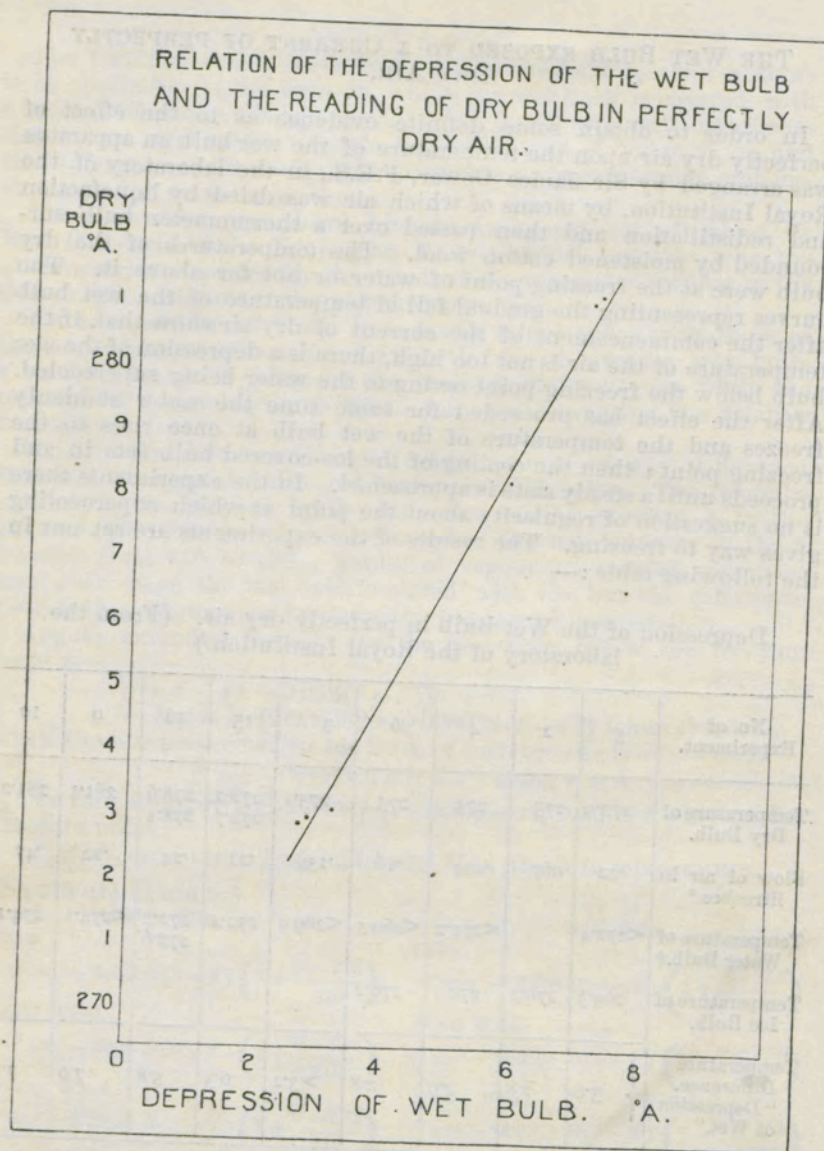
The symbol $<$ is inserted when the shape of the curve suggests that the temperature had not reached a steady state when freezing took place or the experiment terminated.

The two temperatures of the dry bulb in experiments 15 and 16 are the extremes between which the water bath varied during the experiment.

The relation of the depression of the wet bulb to the temperature of the dry bulb in perfectly dry air, according to the results given in the foregoing table, is shown in the following diagram:—

* The cross section of the channel was of the order 1 cm², so that the velocity of the stream of air passing the wet-bulb may be determined very roughly in metres per second by multiplying the numbers given in this line by 10.

† The figures in this row show the extent of the supercooling.



The experiments 9, 10, 15, 16, in which the wet bulb remained above the freezing point, are consistent with Regnault's formula provided that A has the value 0.0009. The depression of the wet bulb was about half a degree less than it would have been according to the Austrian light-wind-formula or the formulæ quoted for other countries. Glaisher's Tables when applied to Dewar's figures give as the humidity of the dry air about 30 per cent.

The experiments with a frozen bulb suggest 0.0016 as the factor in Regnault's formula. The depression of the ice-covered bulb did not exceed 3.6a, whereas, even according to Pernter's formula for a calm, it should have been as much as 4.3a.

COMPUTER'S HANDBOOK.

SECTION I. § 2.

Reduction of Barometer Readings to Sea Level.

Laplace's formula quoted on p. 14, Introduction, when amplified to allow for the aqueous vapour in the atmosphere becomes the following (see International Tables, Chapter IV., Section II., Table VIII., pp. B 32-51, 208-227)—

$$Z = K (1 + a\theta) \left(\frac{1}{1 - 0.378 \frac{\phi}{\eta}} \right) \frac{1}{1 - k \cos 2\lambda} \left(1 + \frac{Z}{R} \right) \log_{10} \frac{H_0}{H},$$

in which—

Z is the altitude of the station above sea level.

λ its latitude.

R the mean terrestrial radius.

a the coefficient of expansion of air.

θ the mean temperature of the air between the station and a place supposed to be situated in the same vertical at sea level.

ϕ = the mean pressure of aqueous vapour in the air column.

$$\eta = \frac{H + H_0}{2}$$

H and H_0 the atmospheric pressures at the two points.

K a constant called the barometric constant.

k is the constant of variation of gravity with latitude (see Introduction, p. 8) and is numerically 0.00259.

K is given by the formula—

$$K = \Delta \times H_n / (a \log_{10} e),$$

where H_n is the normal barometric height 76 cm.

Δ is the density of mercury at 0°C.

a the standard density of air under 76 cm. of mercury in lat. 45 and temperature 0°C.

$\log e$ the modulus of common logarithms.

The term $\left(\frac{1}{1 - 0.378 \frac{\phi}{\eta}} \right)$ is the correction for the presence of aqueous vapour in the atmosphere.

The term $(1 - 0.00259 \cos 2\lambda)$ is the correction for the variation of gravity with latitude.

At a meeting of Directors of Meteorological Institutes and Observatories held at Innsbruck in 1905, it was agreed that the reduction of barometric readings to mean sea level should be carried out in such a manner that the final results should not differ by more than 0.3 millimetre (= 0.012 inch = 0.41 millibar) from those which would have been given by the above formula,

(1) if the humidity at the time of observing had been taken for ϕ the mean humidity of the air column,

(2) if the mean temperature of the air column had been computed from the temperature at the time of observing and a vertical temperature gradient of 0.5°C. per 100 metres.

It can be shown that for altitudes up to 500 metres we obtain results usually within the limits of accuracy laid down in the above resolution

(1) if we omit the term referring to the humidity, and (2) if we adopt the dry bulb reading at the time of observation for the temperature of the air column and neglect the vertical temperature gradient.

$H_0 - H = MH$ is the amount which must be added to the observed reading, corrected for temperature, to reduce it to Mean Sea Level.

The tables published by the International Committee give values of $M \times 1000$, to the nearest 0.1, for increments of altitude of 10 metres and increments of temperature of 2a. By multiplying these values by H , the observed pressure (corrected for temperature), and dividing by 1000, the required increment can be found.

Table II (a) of the Observer's Handbook (*q.v.*) has been deduced in this manner from the values of $1000 \times M$ given in the International Tables, interpolation being resorted to where necessary. It gives the amounts which must be added to readings of 950, 1000 and 1050 millibars respectively for every 10 metres of altitude up to 500 metres and for intervals of temperature of 10a between 250a and 310a. The values for intermediate altitudes, temperatures and pressures must be found by interpolation. Special reduction cards are supplied by the Meteorological Office to its observers in which the amount of interpolation required is reduced to a minimum. These cards are only applicable for the altitudes for which they are made out.

In applying the correction for altitude, the temperature of the dry bulb in the screen and not that of the attached thermometer must always be used.

For the reduction of readings taken at altitudes above 500 metres the tables issued by the International Committee should be consulted.

Determination of depths by barometer readings.

When temperature is measured on the Absolute Scale instead of Centigrade the factors $K(1 + a\theta)$ in Laplace's formula are replaced by 67.4 θ .

The following tables are designed primarily for the determination of depths from barometer readings.

On Page I. 22 is a table of 67.4 θ ($\log p - \log 950$).

23 " " 67.4 θ ($\log p - \log 1000$).

24 " " 67.4 θ ($\log p - \log 1050$).

A Table giving the corrections for the variation of gravity with latitude is printed on the opposite page, p. I 21. These corrections are small for depths less than 1000 m.

Example A.

Sea level pressure 990 mb. Station level 953 mb.

Temperature 270a. Latitude 45°.

From I. 17. Depth of sea level below 950 mb = 326 m.

" station " 950 mb = 25 m.

Height of station = 301 m.

Example B.

Sea level pressure 990 mb. Pressure at bottom of mine 1080 mb.

Mean Temperature 284a. Latitude 20°.

Mean vapour pressure 10 mb.

Depth of Mine = Depth of bottom below 1050 mb.

+ Depth of 1050 " 1000

+ Depth of 1000 " 950

- Depth of 990 " 950

= 227 + 408 + 427 - 343 = 719

Corrected 719 $(1 + .0019)$ $(1 + \frac{10}{1035})$ = 719 + 1.4 + 3.7 = 724 m.

CORRECTIONS ON ACCOUNT OF LATITUDE TO BE APPLIED TO DEPTHS IN TABLE: IN METRES PER 1000 METRES.

°	Metres.	°	Metres.	°	Metres.
0	+2.6				
1	2.6	31	+1.2	61	-1.4
2	2.6	32	1.1	62	1.5
3	2.6	33	1.1	63	1.5
4	2.6	34	1.0	64	1.6
5	2.6	35	0.9	65	1.7
6	2.5	36	0.8	66	1.7
7	2.5	37	0.7	67	1.8
8	2.5	38	0.6	68	1.9
9	2.5	39	0.5	69	1.9
10	2.4	40	0.5	70	2.0
11	2.4	41	0.4	71	2.0
12	2.4	42	0.3	72	2.1
13	2.3	43	0.2	73	2.1
14	2.3	44	0.1	74	2.2
15	2.2	45	0.0	75	2.2
16	2.2	46	-0.1	76	2.3
17	2.1	47	0.2	77	2.3
18	2.1	48	0.3	78	2.4
19	2.0	49	0.4	79	2.4
20	2.0	50	0.5	80	2.4
21	1.9	51	0.5	81	2.5
22	1.9	52	0.6	82	2.5
23	1.8	53	0.7	83	2.5
24	1.7	54	0.8	84	2.5
25	1.7	55	0.9	85	2.6
26	1.6	56	1.0	86	2.6
27	1.5	57	1.1	87	2.6
28	1.5	58	1.1	88	2.6
29	1.4	59	1.2	89	2.6
30	1.3	60	1.3	90	2.6

DEPTHS BELOW THE 950 MB. ISOBARIC SURFACE
FOR GIVEN PRESSURES AND TEMPERATURES.

Pressure.	Difference from station.	Mean Temperature of the column ; Dry air in Lat. 45°.						
		250	260	270	280	290	300	310
mb.	mb.	Depth in Metres.						
951	1	7	8	8	8	9	9	9
952	2	15	16	17	17	18	18	19
953	3	23	24	25	26	27	27	29
954	4	31	32	33	34	36	37	38
955	5	39	40	42	43	45	46	48
956	6	46	48	50	51	53	55	57
957	7	54	56	58	60	62	64	67
958	8	61	64	66	69	71	73	76
959	9	69	72	75	77	80	83	85
960	10	76	80	83	86	89	92	95
961	11	84	88	91	94	98	101	104
962	12	92	96	99	103	106	110	114
963	13	99	103	107	111	115	119	124
964	14	107	111	116	120	124	128	133
965	15	115	119	124	128	133	137	142
966	16	122	127	132	137	142	146	152
967	17	130	135	140	145	151	155	161
968	18	137	143	148	154	160	165	170
969	19	145	151	157	162	169	174	180
970	20	153	159	165	171	177	183	189
971	21	160	167	173	180	186	192	199
972	22	168	175	181	188	195	201	208
973	23	175	182	189	196	203	210	218
974	24	183	190	198	205	212	219	227
975	25	190	198	206	213	221	228	236
976	26	198	206	214	222	230	237	245
977	27	205	214	222	230	238	246	255
978	28	212	221	230	238	247	255	264
979	29	220	229	238	247	256	264	273
980	30	227	237	246	255	264	273	283
981	31	235	244	254	263	273	282	292
982	32	242	252	262	272	281	291	301
983	33	250	260	270	280	290	300	310
984	34	258	268	278	288	299	309	319
985	35	265	276	286	297	307	318	328
986	36	272	283	294	305	316	327	338
987	37	280	291	302	313	324	335	347
988	38	287	299	310	322	333	344	356
989	39	295	306	318	330	342	353	365
990	40	302	314	326	338	350	362	375
991	41	310	322	334	346	359	371	384
992	42	317	329	342	355	368	380	393
993	43	324	337	350	363	376	389	402
994	44	331	345	358	371	385	398	411
995	45	339	352	366	380	393	407	420
996	46	346	360	374	388	402	416	429
997	47	353	368	382	396	410	424	438
998	48	361	375	390	404	419	433	447
999	49	369	383	398	413	427	442	457
1000	50	376	391	406	421	436	451	466

DEPTHS BELOW THE 1000 MB. ISOBARIC SURFACE
FOR GIVEN PRESSURES AND TEMPERATURES.

Pressure.	Difference from station.	Mean Temperature of the column ; Dry air in Lat. 45°.						
		250	260	270	280	290	300	310
mb.	mb.	Depth in Metres.						
1001	1	7	7	8	8	8	9	9
1002	2	14	15	16	17	17	18	18
1003	3	22	23	23	24	25	26	27
1004	4	29	30	31	33	34	35	36
1005	5	36	37	39	41	42	44	45
1006	6	44	45	47	49	50	52	54
1007	7	51	53	55	57	59	61	63
1008	8	58	60	62	65	67	70	72
1009	9	65	68	70	73	76	79	81
1010	10	72	75	78	81	84	87	90
1011	11	80	83	86	89	92	96	99
1012	12	87	91	94	98	101	105	108
1013	13	94	98	102	106	109	113	117
1014	14	101	106	110	114	118	122	126
1015	15	109	113	118	122	127	131	135
1016	16	116	121	125	130	135	139	144
1017	17	124	128	133	138	143	148	153
1018	18	131	136	141	146	152	157	162
1019	19	138	143	149	154	160	165	171
1020	20	145	151	157	163	169	174	180
1021	21	152	159	165	171	177	183	189
1022	22	159	166	172	179	185	191	197
1023	23	166	173	180	187	193	200	206
1024	24	173	181	188	195	202	209	215
1025	25	181	188	195	203	210	217	224
1026	26	188	196	203	211	218	226	233
1027	27	195	203	211	219	226	234	242
1028	28	202	210	218	227	235	243	251
1029	29	209	218	226	235	243	251	260
1030	30	216	225	234	243	251	260	269
1031	31	223	232	241	250	259	268	277
1032	32	230	240	249	258	267	276	286
1033	33	237	247	257	266	276	285	295
1034	34	244	254	264	274	284	293	303
1035	35	252	262	272	282	292	302	312
1036	36	259	269	280	290	300	311	321
1037	37	266	276	287	298	308	319	330
1038	38	273	284	295	306	317	327	338
1039	39	280	291	302	314	325	336	347
1040	40	287	298	310	321	333	344	356
1041	41	294	306	318	329	341	353	365
1042	42	301	313	325	337	350	362	374
1043	43	308	321	333	345	358	370	383
1044	44	315	328	341	353	366	378	391
1045	45	322	335	348	361	374	387	400
1046	46	329	342	356	369	382	395	409
1047	47	336	350	363	377	390	404	417
1048	48	343	357	371	385	398	412	426
1049	49	350	365	379	393	407	421	434
1050	50	357	372	386	401	415	429	443

I 24 DEPTHS BELOW THE 1050 MB. ISOBARIC SURFACE
FOR GIVEN PRESSURES AND TEMPERATURES.

Pressure.	Difference from station.	Mean Temperature of the column; Dry air in Lat. 45°.						
		250	260	270	280	290	300	310
mb.	mb.	Depth in Metres.						
1051	1	7	7	7	8	8	8	9
1052	2	14	14	15	15	16	17	18
1053	3	21	21	22	23	24	25	26
1054	4	28	28	30	31	32	33	34
1055	5	35	35	37	39	40	42	43
1056	6	42	43	45	46	48	50	52
1057	7	49	50	52	54	56	58	60
1058	8	56	57	60	62	64	66	69
1059	9	63	64	67	70	72	75	78
1060	10	70	72	75	77	80	83	86
1061	11	77	79	82	85	88	92	94
1062	12	84	86	90	93	96	100	103
1063	13	91	93	97	101	104	108	111
1064	14	98	100	104	108	112	116	120
1065	15	105	108	112	116	120	124	128
1066	16	111	115	119	124	128	133	136
1067	17	118	122	127	131	136	141	145
1068	18	125	129	134	139	144	149	154
1069	19	131	136	141	147	152	157	162
1070	20	138	143	149	154	160	165	171
1071	21	145	150	156	162	168	174	180
1072	22	151	157	163	170	175	182	188
1073	23	158	164	171	177	183	190	197
1074	24	165	172	178	185	192	198	205
1075	25	172	179	186	193	200	207	214
1076	26	179	186	193	201	208	215	222
1077	27	185	193	201	209	216	223	231
1078	28	192	201	208	216	224	231	239
1079	29	199	208	216	224	232	239	247
1080	30	206	215	223	232	240	248	256
1081	31	213	221	230	239	248	256	264
1082	32	220	229	238	247	255	264	273
1083	33	226	235	245	254	263	272	281
1084	34	233	242	252	261	271	280	289
1085	35	240	250	259	269	279	288	298
1086	36	247	257	266	276	286	296	306
1087	37	253	263	274	284	294	304	314
1088	38	260	270	281	291	302	312	323
1089	39	267	277	288	299	310	320	331
1090	40	273	284	295	306	317	328	340
1091	41	280	292	303	314	325	336	348
1092	42	287	299	310	322	333	345	356
1093	43	294	306	318	330	341	353	365
1094	44	301	313	325	337	349	361	373
1095	45	308	320	332	344	356	369	381
1096	46	314	327	339	352	364	377	390
1097	47	321	334	346	359	372	385	398
1098	48	327	340	353	366	380	393	406
1099	49	334	348	361	374	387	401	414
1100	50	341	354	368	382	395	409	422

Note on the Reduction of Pressure Readings to Mean Sea Level by the use of a Slide Rule.

(As used in the reductions for the Réseau Mondial.)

The full statement of Laplace's formula for the reduction of pressure readings to mean sea-level is given in the Introduction p. 14.

For altitudes up to 1000 metres the variation of pressure with height is given with sufficient accuracy by the formula :

$$p_0 = pe^{z/KT}$$

where p_0 is the pressure at M.S.L.

p is the pressure at Station Level.

z is the altitude in metres.

K is a constant approximately equal to 29.3°

T is the temperature on the absolute scale.

From this formula we deduce :

$$p_0 - p = p(e^{z/KT} - 1) \\ = \frac{pz}{KT} \left(1 + \frac{z}{2KT} + \frac{z^2}{6K^2T^2} + \dots \right)$$

Thus for the same values of p and z and two different temperatures T_1 and T_2 , the ratio of the corrections is :

$$\frac{\frac{1}{T_1} \left(1 + \frac{z}{2KT_1} + \dots \right)}{\frac{1}{T_2} \left(1 + \frac{z}{2KT_2} + \dots \right)} = \frac{T_2 + \frac{z}{2K}}{T_1 + \frac{z}{2K}} \cdot \frac{T_2}{T_1} + \dots$$

If T_2 and T_1 are not greatly different, this can be written :

$$\left(T_2 + \frac{z}{2K} \right) \div \left(T_1 + \frac{z}{2K} \right)$$

where z is the height in metres and $2K = 58.6$.

Hence the correction for height z at temperature T may be determined by a formula

$$p_0 - p = \frac{pf}{1000(T + z/2K)} \dots (1).$$

where f , the "factor," depends only on the height.

As the temperatures to be used do not differ greatly from 300a, the factor f has been computed so as to make the formula correct for the temperature T_2 where

$$T_2 + z/2K = 300.$$

Sufficient accuracy is obtained by taking $\frac{1}{2}z/K$ and therefore also T_2 correct to the nearest integer. The values of the "factor" have been calculated from the values of M in the International Meteorological Tables, pp. 182ff. and the results are given on Page I 27 in the columns headed "Factor" for every metre up to 500 metres, and for every 10 metres from 500 to 1000. In the column headed "Correction" is given the approximate value of $z/2K$ for the corresponding heights. The value of the "factor" for any height z is 300 times the correction required to reduce a reading of 1000 mb. from this height to sea-level at a temperature of $300 - z/2K$. If the station pressure is 1000 mb. then to compute the barometer correction for height z with an air temperature T , it is necessary to divide the

* The constant 29.3 is equal to $67.4 \log_{10} e$ (see p. I 20).

Some Particulars of the Gases of the Atmosphere.

Gas.	Specific Gravity.		Δ Density at 1000mb, 290a.	Gas constant R , $p=R\theta$, A.			Boiling Point (1000mb).	Proportional composition in the troposphere.	
	O., 16.	Dry Air, 1.		R .	$1/R$.	γ		By volume or pressure.	By weight.
Dry Air ...	14.48	1.00	kg/m ³ 1.201	*	*	*	a	%	%
Water Vapour ...	9.01	.6221	—	2.870 4.61	.3483 .217	1.4 1.3	— 372.6	100 0 to 4	100 0 to 2.5
Nitrogen ...	14.01	.967	1.162	2.966	.3372	1.4	78	78.03	75.48
Oxygen ...	16.00	1.105	1.327	2.597	.3851	1.4	90	20.99	23.18
Argon ...	19.94	1.377	1.653	2.083	.4801	5/3	87	0.94	1.29
Carbon-dioxide ...	22.15	1.529	1.836	1.878	.5326	1.3	193	0.03	.045
Hydrogen ...	1.008	.0696	.0836	41.22	.0243	1.4	20	0.01?	.0007?
Neon ...	10.1	.697	.837	4.12	.243	5/3	34	.0012	.0008
Helium ...	1.99	.137	.165	20.80	.048	5/3	6	.0004	3×10^{-8}
Krypton ...	41.5	2.87	3.44	1.00	1.00	5/3	121	$\frac{1}{3} \times 10^{-6}$	1.5×10^{-5}
Xenon ...	65	4.5	5.4	.63	1.58	5/3	164	$\frac{1}{3} \times 10^{-6}$	2×10^{-6}
Geocoronium ...	0.2?	.01?	.017?	200?	.005?	?	?	?	?

* The unit for R is chosen so that, when Δ is in kg/m³ and θ is in degrees absolute, p may be in millibars. R is a measure of the work done when the gas expands against constant pressure. The unit in which R is expressed is from this point of view 10 leometres per degree or 0.1 joule per gramme per degree. The ratio of the capacities for heat is denoted by γ . The capacities at constant pressure and at constant volume are $c_p = R\gamma/(\gamma-1)$ and $c_v = R/(\gamma-1)$ respectively.

COMPUTER'S HANDBOOK.

SECTION I. § 3

TABLE OF THE DENSITIES OF AIR IN
GRAMMES PER CUBIC METRE AT PRES-
SURES FROM 100 MILLIBARS TO 1080
MILLIBARS AND TEMPERATURES FROM
200 ABSOLUTE TO 310 ABSOLUTE.

$$\text{Formula ; } \Delta = 348.3 (p - \frac{3}{8}e) / T$$

where Δ is the density in grammes per cubic metre

p is the pressure of the air in millibars

e is the pressure of the water-vapour present in the air
in millibars

T the absolute temperature on the centigrade scale.

DENSITY OF DRY AIR at Pressures from 100 to

Temperature Absolute.	200	205	210	215	220	225	230	235	240	245	250
Pressure mb.	Mass of one cubic										
100	g 174	g 170	g 166	g 162	g 158	g 155	g 151	g 148	g 145	g 142	g 139
120	209	204	199	194	190	186	182	178	174	171	167
140	244	238	232	227	222	217	212	208	203	199	195
160	279	272	265	259	253	248	242	237	232	228	223
180	313	306	299	292	285	279	273	267	261	256	251
200	348	340	332	324	317	310	303	296	290	284	279
220	383	374	365	356	348	340	333	326	319	313	306
240	418	408	398	389	380	371	363	356	348	341	334
260	453	442	431	421	412	402	394	385	377	370	362
280	487	476	464	453	443	433	424	415	406	398	390
300	522	510	498	486	475	464	454	445	435	427	418
320	557	544	531	518	506	495	485	474	464	455	446
340	592	578	564	551	538	526	515	504	493	483	474
360	627	612	597	583	570	557	545	534	522	512	501
380	662	646	630	615	602	588	575	563	551	540	529
400	696	680	663	648	633	619	606	593	580	569	557
420	731	714	697	680	665	650	636	622	609	597	585
440	766	748	730	713	697	681	666	652	638	626	613
460	801	782	763	745	728	712	697	682	667	654	641
480	836	816	796	778	760	743	727	711	696	682	668
500	871	850	829	810	792	774	757	741	725	711	696
520	905	884	863	842	823	805	788	771	754	739	724
540	940	918	896	875	855	836	818	800	783	768	752
560	975	952	929	907	886	867	848	830	812	796	780
580	1009	986	962	940	918	898	878	860	841	825	808
600	1044	1019	995	972	950	929	909	889	870	853	836
620	1079	1053	1028	1004	982	960	939	919	899	882	864
640	1114	1087	1061	1037	1013	991	969	948	928	910	892
660	1149	1121	1094	1069	1044	1022	999	978	957	939	919
680	1183	1155	1128	1101	1076	1053	1030	1008	986	967	947
700	1218	1189	1161	1134	1108	1084	1060	1037	1016	995	975
720	1253	1223	1194	1166	1139	1114	1090	1067	1045	1024	1003
740	1288	1257	1227	1199	1171	1145	1120	1097	1074	1052	1031
760	1323	1291	1260	1231	1202	1176	1151	1126	1103	1081	1059
780	1358	1325	1293	1263	1234	1207	1181	1156	1132	1109	1087
800	1393	1359	1327	1296	1266	1238	1211	1186	1161	1138	1114
820	1428	1393	1360	1328	1297	1269	1241	1215	1190	1166	1142
840	1462	1427	1393	1360	1329	1300	1272	1245	1219	1195	1170
860	1497	1461	1426	1393	1361	1331	1302	1275	1248	1223	1198
880	1532	1495	1459	1425	1392	1362	1332	1304	1277	1252	1226

880 mb. and Temperatures from 200 to 310a.

255	260	265	270	275	280	285	290	295	300	305	310	Temperature Absolute.
metre of Dry air.												Pressure mb.
g 137	g 134	g 131	g 129	g 127	g 124	g 122	g 120	g 118	g 116	g 114	g 112	100
164	161	158	155	152	149	147	144	142	139	137	135	120
191	187	184	181	177	174	171	168	165	162	160	157	140
219	214	210	206	203	199	195	192	189	186	183	180	160
246	241	237	232	228	224	220	216	212	209	206	202	180
273	268	263	258	253	249	244	240	236	232	228	225	200
301	295	289	284	279	274	269	264	260	255	251	247	220
328	321	315	310	304	299	293	288	283	279	274	270	240
355	348	342	335	329	323	318	312	307	302	297	292	260
383	375	368	361	355	348	342	336	330	325	320	315	280
410	402	394	387	380	373	367	360	354	348	342	337	300
437	429	421	413	405	398	391	384	378	371	365	360	320
464	455	447	438	431	423	415	408	401	395	388	382	340
492	482	473	464	456	448	440	432	425	418	411	404	360
519	509	499	490	481	473	464	456	448	441	434	427	380
546	536	526	516	507	497	489	480	472	464	457	449	400
574	563	552	542	532	522	513	504	496	488	480	472	420
601	589	578	567	557	547	538	528	519	511	502	494	440
628	616	605	593	583	572	562	552	543	534	525	517	460
656	643	631	619	608	597	587	576	567	557	548	539	480
683	670	657	645	633	622	611	600	590	580	571	562	500
710	697	683	671	659	647	635	624	614	604	594	584	520
738	723	710	696	684	672	660	648	638	627	617	607	540
765	750	736	722	709	697	684	672	661	650	639	629	560
792	777	762	748	735	721	709	697	685	673	662	652	580
820	804	788	774	760	746	733	721	708	697	685	674	600
847	830	815	800	785	771	758	745	732	720	708	697	620
874	857	841	826	810	796	782	769	756	743	731	719	640
902	884	867	851	836	821	807	793	779	766	754	742	660
929	911	894	877	861	846	831	817	803	789	776	764	680
956	938	920	903	886	871	855	841	826	813	799	787	700
984	964	946	929	912	896	880	865	850	836	822	809	720
1011	991	972	955	937	920	904	889	874	859	845	832	740
1038	1018	999	980	962	945	929	913	897	882	868	854	760
1065	1045	1025	1006	988	970	953	937	921	905	891	876	780
1093	1072	1051	1032	1013	995	978	961	944	929	913	899	800
1120	1099	1077	1058	1038	1020	1002	985	968	952	936	921	820
1148	1125	1104	1083	1064	1045	1026	1009	992	975	959	944	840
1175	1152	1130	1109	1089	1069	1051	1033	1015	998	982	966	860
1202	1179	1156	1135	1114	1094	1075	1057	1039	1021	1005	989	880

DENSITY OF DRY AIR at Pressures from 900 to

Temperature Absolute.	200	205	210	215	220	225	230	235	240	245	250
Pressure mb.	Mass of one cubic										
900	g 1567	g 1529	g 1492	g 1458	g 1424	g 1393	g 1363	g 1334	g 1306	g 1280	g 1254
920	1602	1563	1525	1490	1455	1424	1393	1364	1335	1309	1282
940	1636	1597	1558	1522	1487	1455	1423	1393	1364	1337	1309
960	1671	1631	1592	1555	1519	1485	1454	1423	1393	1365	1337
980	1706	1665	1625	1587	1551	1516	1484	1453	1422	1394	1365
1000	1741	1699	1658	1620	1583	1547	1514	1482	1451	1422	1393
1020	1776	1733	1691	1652	1614	1578	1545	1512	1480	1451	1421
1040	1811	1767	1725	1685	1646	1609	1575	1542	1509	1479	1449
1060	1846	1801	1758	1717	1678	1640	1605	1571	1538	1507	1477
1080	1881	1835	1791	1750	1710	1671	1635	1601	1568	1536	1505
CORRECTIONS FOR											
The figures are to be subtracted from											
Corrections to density for relative humidity of	50 %	g 0	g 0	g 0	g 0	g 0	g 0	g 0	g 0	g 0	g 0
	100 %	0	0	0	0	0	0	0	0	0	1

1080 mb. and Temperatures from 200 to 310a.

255	260	265	270	275	280	285	290	295	300	305	310	Temperature Absolute.
metre of Dry air.												Pressure mb
g 1229	g 1206	g 1182	g 1161	g 1140	g 1119	g 1100	g 1081	g 1062	g 1044	g 1028	g 1011	900
1257	1233	1209	1187	1165	1144	1124	1105	1086	1068	1050	1033	920
1284	1260	1235	1213	1190	1169	1148	1129	1109	1091	1073	1056	940
1311	1286	1261	1238	1215	1193	1173	1153	1133	1114	1096	1078	960
1339	1313	1288	1264	1241	1218	1197	1177	1157	1137	1119	1101	980
1366	1340	1314	1290	1266	1243	1222	1201	1180	1160	1142	1123	1000
1393	1367	1340	1316	1291	1268	1246	1225	1204	1183	1165	1146	1020
1421	1393	1367	1342	1317	1293	1271	1249	1228	1207	1187	1168	1040
1448	1420	1393	1367	1342	1318	1295	1273	1251	1230	1210	1190	1060
1475	1447	1419	1393	1367	1343	1320	1297	1275	1253	1233	1213	1080
WATER VAPOUR.												
the densities given in the main Table.												
g 0	g 1	g 1	g 1	g 2	g 2	g 3	g 4	g 6	g 8	g 10	g 13	50%
1	1	2	2	3	5	6	9	12	15	20	26	100%

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METEOROLOGICAL OFFICE.

THE COMPUTER'S HANDBOOK.

Section II—Sub-section I—The Computation of
Wind Components from observations
of Pilot Balloons.

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COMPUTER'S HANDBOOK.

SECTION II.

DYNAMICAL METEOROLOGY. CALCULUS OF THE UPPER AIR.

Subsection I. The Computation of Wind-Components from Observations of Pilot Balloons.

In the following account of the different methods by which the computations relating to pilot-balloon-work may be carried out, the figures relating to a pilot balloon which was followed by two theodolites at South Farnborough on November 2nd, 1912, have been worked out (1) in the normal manner as a two-theodolite ascent, and (2) as a single-theodolite-ascent (using the observations at the home station only). Working in this second manner it is assumed that the balloon rose at a uniform and known rate. The figures used in the second manner have been dealt with in two separate ways, (*a*) doing all computation by slide rule, and (*b*) using tables of logarithms for the main part of the computation. Some account of a graphical method of working is also given.

The two-theodolite-method of working pilot-balloon-ascents is especially valuable where it is desired to investigate in detail the structure of the lower layers of the atmosphere, and to obtain information upon the vertical movements therein. These layers are especially disturbed by vertical currents, and the necessary assumption of a uniform rate of rising of the balloon may lead to considerable inaccuracies where single-theodolite working is employed. The chief drawback to the employment of two theodolites lies in the length of time involved both in carrying out the observations, and more especially in the later calculations. In cases where information as to the movements of the upper air is required for immediate use, this makes it almost a necessity to employ the single-theodolite-method. In addition, it may be said that this method is probably quite satisfactory in those cases where it is only desired to investigate the general features of the air movements to a great height, and not to enter into the details of the structure.

The Velocity of Lift in Still Air.

In work with either two theodolites or one instrument only, a knowledge of the rate of rising of the balloon

through still air is necessary, in the first case, in order that the vertical air movements may be deduced, and in the second, for filling in the "assumed height" at each time interval. The formula now generally used for determining the rate of rising is "Lifting velocity $q = L / (W + L)^{1/2}$," where q is assumed to be constant for those sizes of balloon used in pilot-balloon-work, although its value may vary somewhat for larger balloons. L is the free lift or carrying capacity of the balloon, and W is the dead weight of rubber and attachments. The units usually employed are metres per minute for velocity and grammes for weight. This formula is based on the assumption that the resistance to rising varies as the square of the rising velocity and directly as the sectional area of the balloon. Numerous experiments were carried out in a lofty shed at Farnborough with different types of pilot balloon, and from these the value 81 was deduced for the constant q , so that the formula becomes

$$\text{Lifting velocity (metres per minute)} = 81 L^{1/2} / (W + L)^{1/2}.$$

For a detailed account of these experiments see a paper by J. S. Dines in the Quarterly Journal of the Royal Meteorological Society, Vol. 39, p. 101. (N.B. " W " is used in the paper for the total lift, or the $W + L$ above).

Computation by Slide Rule.*

The forms issued for use in working the results of pilot-balloon-ascents have been made to a large extent self-explanatory, the letters and equations used being fully explained on the reverse side of the forms. See the specimens reproduced on pp. II 13-16; the following account should be read in conjunction with them. It may be noted that A and B represent the positions of the observing stations and C the point on the earth's surface immediately under the balloon.

With regard to the two-theodolite method, if the two instruments are set up with their azimuth zeros adjusted in the manner recommended on the forms, the readings will in general all lie between 0° and 180° . These angles are more easily dealt with in the subsequent calculations than angles which fall between 180° and 360° . Where a slide rule is

* Special slide rules are now made for Pilot Balloon Work, see below, p. 19, but the following account refers to the type in common use.

used for the computation, the values of AC and BC are obtained from one setting of the slide as follows:

Reverse the slide so that the sine and tangent scales face upwards. Set the cursor at the point on the upper scale corresponding with the length of the base line, 870 metres in the example chosen. Move the slide till the angle C , which is the difference between A and B , on the sine or upper scale falls under the line on the cursor. The distances AC and BC will then read on the upper distance scale against the azimuth angles B and A respectively on the sine scale in accordance with the formulæ

$$AC = (AB / \sin C) \times \sin B, \quad BC = (AB / \sin C) \times \sin A$$

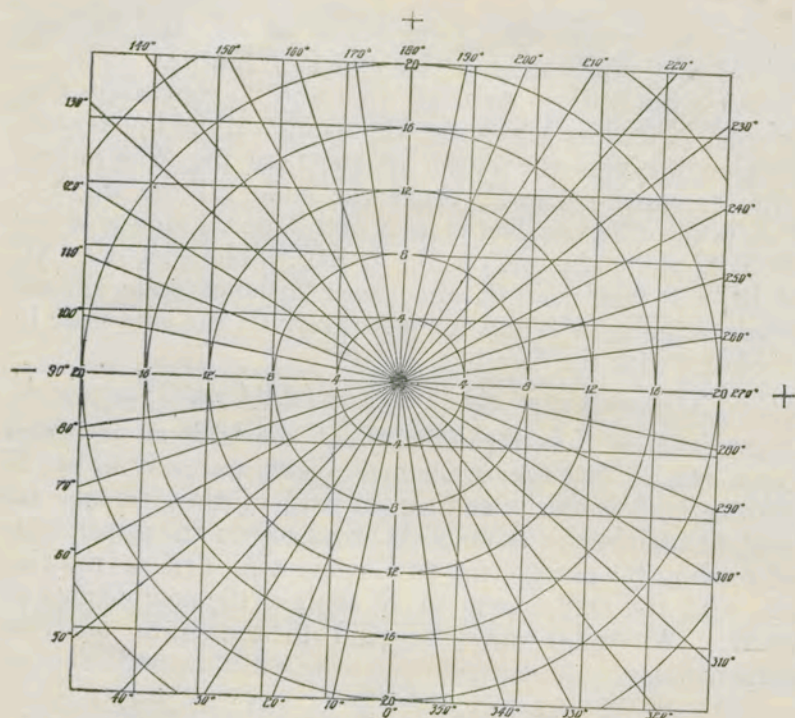
The angle $A + K$ is the azimuth of the balloon measured from north, so that the component distances to E and to N are directly deduced by setting the end of the sine scale on the slide against the value of AC read off on the upper scale and reading the component to E against $A + K$ on the sine scale and the component to N against the complement of $A + K$. Attention must be paid to the signs of these components.

East and North Components of the Velocity.

The velocities to E and N are then deduced by dividing the differences between successive distances to E and N by 60 or 30 according as the readings have been taken at 1 minute or $\frac{1}{2}$ minute intervals. These velocities are thus deduced in metres per second. In the example illustrated readings were taken at the first half-minute from the start and thereafter at succeeding whole minutes, this being done with a view to obtaining more detailed information as to the changes immediately above the surface of the ground than would be possible were whole-minute-readings only taken throughout.

Resultants.

The resultant velocity and direction of the wind may be obtained from the components to E and to N either graphically by means of a specially prepared diagram or by further use of the slide rule. If the graphical method is used, a diagram should be prepared having rectangular ruling for use with the component velocities together with circular and radial ruling for the resultant velocities and directions. For convenience of working it will be found desirable to have the diagram not less than



12 inches (30 cm.) square. The small figure *below* is not meant for actual use in working but is appended to illustrate the method. Thus if the component velocities are $V_{WE} = +8.0$, $V_{SN} = -12.0$, the point represented by $+8.0, -12.0$ is found on the diagram with the aid of the rectangular ruling and the resultant velocity and direction $14.4, 326^\circ$ are read on the circular and radial ruling. It may be noted that the angle ϕ is used to denote the angle between N and the direction *from* which the balloon comes, measured in a clockwise direction; thus movement of the balloon to the S.E. denotes a N.W. wind, and a value for ϕ of 315° .

If the slide rule is used for determining the resultant wind, ϕ is first determined from the equation $\tan \phi = V_{WE}/V_{SN}$.* For this the end of the tangent scale is set against the larger of the two components and the cursor moved to the smaller of the two. The angle read under the cursor on the tangent scale is then the angle the resultant wind makes with one of the four cardinal lines. The actual value of ϕ which corresponds may be filled in by inspection of the magnitude and sign of the two components. Thus taking the interval

* $\tan \phi$ is strictly $-V_{WE}/-V_{SN}$, i.e., when each component is positive ϕ lies between 180° and 270° and not between 0° and 90° .

2-3 minutes from the example shown, the angle read from the slide rule is 10° and as the large component is *from* the west and the small *from* the north ϕ is evidently $270 + 10^\circ = 280^\circ$. To obtain the resultant velocity V , after noting the angle on the tangent scale which is indicated under the cursor the slide is pushed in till the same angle on the sine scale comes under the cursor. The end of the sine scale then indicates the value of V based on the formula $V = V_{WE}/\sin \phi$ or $V = V_{SN}/(\sin \text{ of complement of } \phi)$. The above process is only applicable with those slide rules where the sine and tangent scales are both divided and read against the same logarithm scale on the rule, and both run from *left to right* along the slide.

Computation of the Heights above the Stations.

The next two columns in the sheet give heights of the balloon above stations A and B. It may be noted that these heights should differ by a constant amount which is equal to the difference in level of the two stations. The heights above A and B are determined directly by evaluating $AC \tan E_a$ and $BC \tan E_b$ on the slide rule. The end of the tangent scale is put opposite the value of AC (or BC) and the cursor moved to E_a (or E_b) on the tangent scale, when the height is read under the cursor on the logarithm scale. The rate of ascent is given in m/s and determined from the difference between successive height readings by dividing by 30 or 60 according to whether the readings are made at $\frac{1}{2}$ or 1 minute intervals. From the mean of the two determinations for the rate of ascent the vertical air velocity is deduced by subtraction of the lifting velocity of the balloon (2.4 m/s in the example shown).

It generally happens that either one or both of the angles A and B lies between 90° and 180° . The sine scale on a slide rule does not extend above 90° , so that in working with angles greater than 90° it is necessary to remember that $\sin a = \sin (180 - a)$ and to work with the supplement of the angle instead of with the angle itself. This is facilitated if against each angle on the sine scale its supplement is also noted. Thus beneath the value 30° will be written 150° and so on. On the other hand after some practice the worker becomes so used to dealing with these supplements that the visual guidance afforded by the additional notation becomes quite unnecessary. In the same way the tangent scale only runs to 45° on the slide and in the rather rare cases where altitudes in excess of 45° are experienced it

should be remembered that $\tan a = 1/\tan (90^\circ - a)$ and thus instead of multiplying by $\tan E_a$ the process of division by the tangent of the complement of E_a is used.

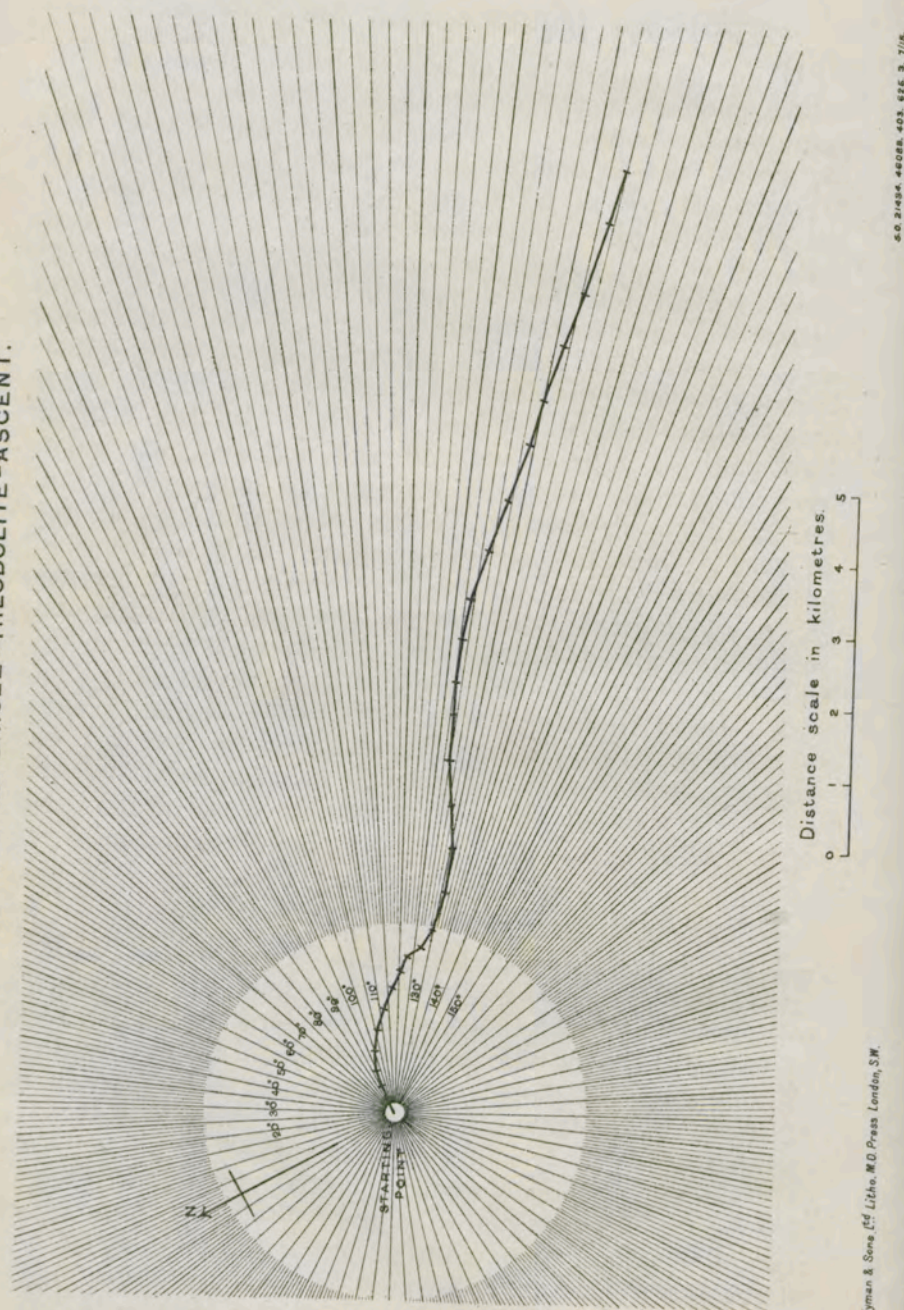
Observations with a single theodolite.

In the sheets headed Method 2a and Method 2b (pp. II 16, 17), the altitude and azimuth readings from the home station have been worked up as a single-theodolite-ascent assuming a uniform rate of ascent of the balloon of 142 metres per minute. The heights " h " are filled in for the time of each observation on this assumption. In sheet 2a the remaining computation has been carried out by the slide rule. The horizontal distance $h \cot E$ and the two component distances D_E and D_N are obtained from one setting of the slide in the following manner. The cursor is set at the height h on the logarithm scale and the slide moved till the altitude angle, E , on the tangent scale comes under the cursor. The distance $h \cot E$ is then read off the logarithm scale against the end of the tangent scale and the components D_E and D_N against the angles A and "*complement of A* " on the sine scale. D_E and D_N may be obtained in this manner without further movement of the slide than that required for the determination of $h \cot E$, provided that the slide rule used has both its sine and tangent scales ruled to read against the upper logarithm scale. The signs of the two components must be carefully noted. V_{WE} and V_{SN} are obtained from D_E and D_N by subtraction of successive readings and division by 30 or 60 as in the two-theodolite method. Either of the two methods previously given may then be followed for determining ϕ and V .

Slide Rule and Logarithms.

It will have been noticed from the above that when choosing a slide rule for pilot-balloon-work it is desirable to obtain one in which the sine and tangent scales are both divided to read against the upper logarithm scale on the rule, and in which both scales read from left to right along the slide. In some rules the sine scale reads against the upper logarithm scale, while the tangents read against the lower. The two types of ruling may be readily distinguished by the fact that in the first type the low angles round about 1° on the sine and tangent scales are very approximately opposite to each other, since the sine and tangent of these low angles are practically equal to one another. In the other type the 1° on the sine scale is roughly opposite to $7^\circ 30'$ on the tangent

TRACK OF BALLOON.
SOUTH FARNBOROUGH PILOT-BALLOON-ASCENT N°28, NOV. 2ND 1912.
PLOTTED AS SINGLE-THEODOLITE-ASCENT.



Wyman & Sons, Ltd. Litho. W.D. Press London, SW.

S.O. 21434. 45088. 403. 625. 3. 1/16.

scale. In the first and most suitable type the tangent readings begin at $0^{\circ}35'$, in the second at $5^{\circ}45'$. Thus a further disadvantage of this second type is that tangents of angles less than $5^{\circ}45'$ cannot be obtained. It is desirable to obtain the largest size of slide rule which is readily procurable, that is, one about 20 inches in length. The accuracy obtainable with this rule is sufficient for the purpose, considering the degree of accuracy which can be obtained from the original readings.

Some workers prefer the use of tables of logarithms to the slide rule for calculating the component distances D_E and D_N from the height h and the altitude and azimuth readings E and A . The sheets marked 2b illustrate the method of working and forms employed. The original readings and assumed heights are entered on the same form and in the same manner as that used for working by slide rule. The additional form supplied for entering the logarithms of the different quantities is almost self explanatory. The reading to which each section on the paper is devoted is shown by the time, in minutes, on the top line, $\frac{1}{2}$, 1, 2, &c. Under this is entered the value of $\log h$ for each time. Where the same type of balloon, weighted to give the same free lift is regularly employed, a table of these values of $\log h$ may be kept and the figures copied out on to the form. The value of $\log h$ is followed by $\log \cot E$ and $\log \cos A$; the addition of these three logarithms giving the logarithm of D_N . Then if $\log \tan A$ is added to the preceding total, the logarithm of D_E is obtained. D_E and D_N when obtained in this manner are entered on the regular form and the remainder of the computation is carried out as in the method where the slide rule is used.

Graphical Method.

There remains the additional method of working pilot-balloon-ascent by purely graphical means or by variations of this method in which the slide rule is employed for some parts of the calculation and the graphical method for the remainder.

In the graphical method the horizontal projection of the path traversed by the balloon is plotted on paper and thus a clear representation of the directions of the different air currents met with is obtained. This forms a distinct advantage of the method. On the other hand, working by this means is not so rapid as working by slide rule, and in the case of a high ascent on a day of strong wind the track when plotted becomes so long as to be somewhat inconvenient to deal with.

The work may be done on plain or squared paper, but paper ruled with radial lines which correspond with the azimuth readings of the theodolite is more convenient for the purpose. Such paper has been published by the Egyptian Meteorological Service, and in the example shown, use has been made of one of these sheets

for working up a single-theodolite ascent. Where the balloon has been followed by two theodolites, the home station is usually taken at the centre of the radiating lines. This "centre" is not at the centre of the rectangular sheet of paper but near to one end, and the direction of the north line should be so chosen that the path of the balloon will be roughly along the longer axis of the paper, and thus provide for as great a length of track as possible with a given size of sheet. Having decided upon the most suitable direction for north, it is necessary to fix upon the most convenient scale to be used in plotting the distances. Usually it is found convenient to have two different scales in regular use, one open scale for those cases where the balloon is not followed to more than 20 km. from the base and a more contracted one suitable for the greater distances obtained on clear days. The two scales may conveniently be in the ratio 1 : 2.

With the N.-S. line and the scale of measurement determined upon, the position of the second observing station can be plotted upon the paper and the positions of the balloon at the times of successive readings are then plotted from the azimuth angles measured at the two stations. The azimuths at the home station are read directly by means of the radial lines, while a protractor is used for the distant station. The point of intersection of the lines given by these two azimuth angles denotes the position of the balloon. Having obtained the points representing the successive positions of the balloon, the velocity and direction of the wind for each interval may be read off with suitable scales. As regards velocity, it is evident that a different scale will be required for each of the distance scales which is employed for plotting the results. For measuring the wind directions it is found most convenient to make use of a drawing board and T square. The chart is mounted upon the board so that the N.-S. line runs parallel to one edge, and wind directions are then read off by means of a transparent celluloid protractor used in conjunction with the T square. The actual distance of the balloon from the home station may be read off, and heights are then determined from these distances and the known altitude angles either by a further use of the graphical method or by means of a slide rule. It will be noticed that in the above method wind velocities and directions are obtained directly, and not by means of the components to East and to North. The component velocities obviously may also be read directly from the diagram if desired.

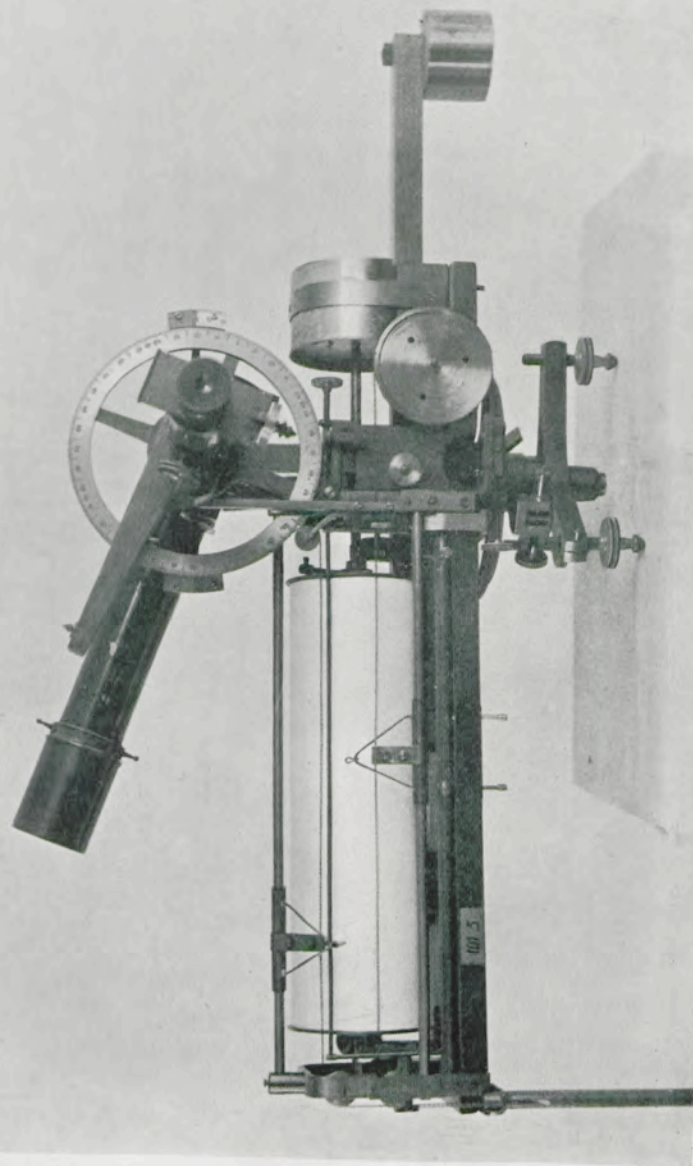
Where single-theodolite observations only are employed, the horizontal distance of the balloon must be determined from the assumed height and known altitude angle, either by means of a diagram or by slide rule. When this is done the position at the time of each reading is plotted directly on the radial paper and the wind velocity and direction for the successive intervals are read off in the same manner as in the case where two theodolites are employed. These graphical methods have been developed by Mr. C. J. P. Cave in his pilot-balloon-work at Ditcham Park.

Observations with the self-recording theodolite.

The self-recording attachment is fitted to the ordinary type of balloon-theodolite and is so arranged that changes in altitude and azimuth are recorded by two separate pens upon a common clock drum. To the carriage on the theodolite

which turns about a vertical axis is attached a horizontal platform upon which the clock and other parts of the recording mechanism are placed. The general arrangement is shown in the photograph. It will be seen that the clock is placed on the right-hand side of the central upright and drives the horizontal drum through a shaft, this drum being on the left hand. The two pen-carriages run horizontally along the front of the drum and are actuated by fine chains which pass over suitable pulleys to grooves cut in the periphery of the vertical and horizontal circles. Movement of the telescope in a vertical or horizontal direction causes the corresponding chain to be wrapped round its circle and in this manner the pen-carriage is drawn along on the guide rod through a distance directly proportional to the angular movement of the telescope. The return movement of the pen-carriage is brought about either by means of a spring mechanism, or in the instrument illustrated by a balance weight which hangs in the tube seen on the extreme left-hand side. Owing to the limitations imposed by the length of the clock drum a horizontal turning movement through $\pm 90^\circ$ only is provided for, this movement corresponding with a movement of the pen from one end of the chart to the other. By setting up the theodolite suitably at the commencement it is generally possible to avoid any trouble through this angular range of movement being exceeded. In those cases where this trouble does occur it is generally possible to a skilled observer to reset the azimuth circle during the flight without losing the balloon from the field of vision.

The clock is arranged to revolve the drum once per hour. No special provision is made for attaching the charts to the clock drum. It is found convenient to fasten the overlapping edges together with gummed paper so that if the duration of the flight exceeds one hour the pens ride over the joint and no record is lost. Provision is made for making time marks either by depressing the actuating chains and so momentarily moving the pens to one side or else by lifting them simultaneously from the paper for a few seconds. The latter is to be preferred. When the instrument is fitted with this device it is customary to make a mark at every minute during the flight, the pens being lifted from the chart exactly at the even minute. The measurements can then be made at the end of each continuous portion of the trace with an accuracy which is certainly equal to that commonly obtained with eye readings. It is recommended that plain charts be used for the record, and the base lines from



SELF-RECORDING THEODOLITE.

which the measurements are made should be ruled while the chart is still in position upon the clock drum. To do this the telescope is clamped in position at each even 10° of altitude and azimuth throughout those parts of the scales which have been traversed during the ascent and the clock drum is rotated under the pens to rule these 10° lines, so that at each part of the trace there is a 10° line of reference from which measurements may be made. Glass scales on which whole and half degree lines are ruled over a range of 10° are used for the tabulations.

A somewhat more detailed description of the instrument may be found in the Second Report on Wind Structure, Report of the Advisory Committee for Aeronautics, 1910-1911. This refers to one of the earliest instruments made and the details have been somewhat modified in later patterns.

Observations of a balloon-tail.

The above descriptions apply to the methods of working up the results of observations on balloons which have been followed by one or two theodolites, in the first case a uniform rate of ascent being assumed. Attempts have been made, and have been attended with some success, to follow the balloon with one theodolite and to determine its distance from the base station at any time by measuring the angle subtended at this point by a tail of known length hanging from the balloon. One of the great difficulties in this method is due to the constant swinging of the tail from the vertical position, and for this reason it has not in the past been much employed. It is possible that in the future the difficulties may be overcome and the method may then come into more general use. It has the advantage of giving the information supplied by the two-theodolite-method at an expenditure of time and trouble not greatly in excess of that required for working in the simple manner by a single theodolite.

In plotting the results of pilot balloon ascents it has become general to represent height by distances measured up the paper (ordinates) and wind velocity whether horizontal or vertical by distances across from left to right (abscissae). The horizontal velocity may be represented either by its two components to E and to N or as the resultant velocity and direction.

Computer's Handbook. Section II., 1. Specimen of form used for computing the results of Pilot Balloon Ascents.
Form 417. METEOROLOGICAL OFFICE, LONDON.

(The particulars below which refer to the individual ascent are printed in italic type.)

PILOT BALLOON ASCENT (TWO THEODOLITES).

Observatory, *South Farnborough.* Date, *November 2nd, 1912.* Time of Start, *12h. 30m.*

Number *28.*

NOTATION AND FORMULÆ.		Position of Theodolites. (Balloon sent up from A). A. <i>By Caretaker's Cottage</i> B. <i>By Golf Course Anemometer</i>	Height above sea level. 75 m. 89 m. 82 m. Mean	Type of Balloon. Hamley's 12 gm. Weight = $W = 12.0$ gm. Free lift = $L = 46.0$ gm. Lifting Velocity = $q \frac{L}{W + L}$ = 142 metres per minute assuming $q = 8\frac{1}{2}$.
t = Time from start in minutes. A, B = Azimuth of Balloon seen from A, B respectively after correction for error in setting. E_a, E_b = Elevation of Balloon at A, B. $C = A \sim B$. $\frac{\sin C}{AB} = \frac{\sin B}{AC} = \frac{\sin A}{BC}$ The angle between AC and north is $A + K$; it may be convenient to enter $A + K$ as $90^\circ +$, $180^\circ +$, $270^\circ +$ or $360^\circ +$. The distance of C to east of A is $AC \sin$ $(A + K)$. The distance of C to north of A is $AC \cos$ $(A + K)$. The resultant velocity makes ϕ with north. $\tan \phi = \frac{V_{ws}}{V_{wn}} \quad V = V_{ws} / \sin$ Resultant horizontal velocity = $V = V_{ws} / \sin$ $\phi = V_{ws} / \cos \phi$. Height above $A = AC \tan E_a$; above $B = BC$ $\tan E_b$. Vertical velocity of air = rate of ascent - lifting velocity.		Mean Distance AB = 870 metres. Bearing of zero reading of A from north = $K = 33^\circ$. Elevation of B at $A + 0^\circ 9'$; Azimuth of B at A 180° at start, — at finish. " A at B 0° ; " A at B 0° — " It is convenient to set up the Theodolites with Azimuths 0° at A, 180° at B or <i>vice versa</i> . The rule is: Look at the other station: if wind blows left to right make Azimuth of other station 0° ; if right to left 180° . Balloon in sight at A, 27 minutes. How lost, in distance. At B, 37 minutes. How lost, in distance.		
Wind by anemometer; Dir. ⁿ . — °; Velf. — $\frac{m}{s}$. Gradient Wind; Dir. ⁿ . — °; Velf. Nil $\frac{m}{s}$.		Atmosphere, clear. Clouds, ci. and ci-st. 6; Fr-cu. 1. Bearing of station from centre of cyclone or other features of the isochronous chart. Belt of high pressure running N.N.W. and S.S.E. over the British Isles.		
NOTES.—All clouds moving from W.N.W.; ci. quickly, fr-cu. slowly.				

Pilot Balloon Ascent Number 28.

L	Home Station.		Our Station.		C.	AC.	BC.	A + K	Balloon E of A.	Balloon N of A.	Velocity.		Resultant.	Height above		Rate of Ascent.		Vert. Vel.
	A.	E _h	B.	E _h							W to E.	Sto N.		A.	B.	A.	B.	
1	56.0	30.1	7.5	4.8	48.5	152	963	89.0	+ 152	m.	m/s	°	m/s	m.	m.	m/s	m/s	m/s
2	57.5	31.8	13.8	9.3	43.7	300	1061	90.5	+ 300	3	+ 5.1	+ 0.1	5.1	81	88	2.9	3.1	+ 0.7
3	60.2	33.6	21.5	16.0	38.7	510	1208	93.2	+ 509	28	+ 4.9	- 0.2	4.9	174	186	3.3	3.1	+ 0.8
4	62.2	34.6	28.8	20.0	33.4	761	1397	95.2	+ 757	69	+ 3.5	- 0.4	3.5	347	365	3.0	2.9	+ 0.5
5	65.0	33.1	37.8	23.0	30.2	1061	1606	101.0	+ 1043	202	+ 4.1	- 0.7	4.2	508	525	2.7	2.7	+ 0.3
6	74.0	31.4	46.7	24.4	27.3	1380	1824	107.0	+ 1320	403	+ 4.6	- 3.4	5.3	682	692	2.8	2.9	+ 0.5
7	79.3	30.5	54.6	25.7	24.7	1697	2048	112.3	+ 1571	644	+ 4.2	- 4.0	5.9	828	843	2.5	2.4	0.0
8	87.95	30.1	66.75	26.9	22.6	1994	2253	117.4	+ 1770	918	+ 3.3	- 4.6	5.8	1000	986	2.6	2.6	+ 0.2
9	90.3	30.45	70.6	28.0	21.2	2210	2402	120.95	+ 1898	1138	+ 2.1	- 3.7	5.7	1145	1152	2.5	2.6	+ 0.2
10	94.05	31.4	74.9	30.5	19.7	2434	2580	123.3	+ 2035	1339	+ 2.3	- 3.4	4.3	1281	1278	2.1	2.2	- 0.3
11	96.5	31.2	78.9	30.75	19.15	2561	2648	127.05	+ 2042	1542	+ 0.1	- 3.4	3.4	1431	1427	2.5	2.5	+ 0.1
12	98.0	28.5	83.5	28.3	14.5	2822	2860	129.5	+ 2178	1795	+ 2.3	- 4.2	3.1	1566	1560	2.3	2.2	- 0.1
13	97.7	26.2	85.4	26.25	12.3	3450	3440	131.0	+ 2267	2287	+ 7.2	- 7.9	4.8	1710	1700	2.4	2.3	- 0.1
14	95.9	24.85	85.1	24.5	10.8	4070	4050	130.7	+ 2607	2650	+ 8.0	- 6.4	10.3	1876	1854	2.8	2.6	+ 0.3
15	94.2	23.4	84.9	23.3	9.3	4630	4625	128.9	+ 3088	2912	+ 8.7	- 4.4	9.8	2000	1999	2.1	2.4	- 0.1
16	93.7	22.2	85.35	22.2	8.35	5370	5370	127.2	+ 3610	2912	+ 11.0	- 5.5	12.3	2148	2108	2.5	1.8	- 0.3
17	93.3	21.8	85.8	21.5	7.5	5970	5975	126.7	+ 4270	3240	+ 8.5	- 5.4	10.1	2323	2314	2.9	3.4	+ 0.8
18	93.3	21.0	86.35	21.05	6.95	6650	6650	126.3	+ 4780	3562	+ 9.7	- 6.3	11.6	2435	2440	1.9	2.1	- 0.4
19	93.7	20.5	87.2	20.5	6.5	7175	7175	126.3	+ 5360	3940	+ 7.2	- 5.3	8.9	2660	2620	3.8	3.0	+ 1.0
						7675	7675	126.7	+ 5790	4255	+ 6.2	- 5.6	8.4	2758	2760	1.9	1.8	- 0.5
									+ 6160	4590				2870	2870			

Computer's Handbook. Section II., 1.

Specimen of form used for Computing the results of Pilot Balloon Ascents.

(The particulars below which refer to the individual ascent are printed in italic type.)

Form 418.

Meteorological Office, London, S.W.

Pilot Balloon Ascent Number 28.

One Theodolite.

Date, *November 2nd, 1912.*

South Farnborough Observatory.

Time of Start, *12h. 30m.*

Position of Theodolite

Height above M.S.L., *75m.*By *Caretaker's Cottage.*Type of Balloon. *Hamley's 12 gm.*Weight = *W = 12.0 gm.*Freelift = *L = 46.0 gm.*Lifting velocity = $q L^{\frac{1}{2}} / (W + L)^{\frac{1}{2}} = 142$ metres per minute.on the assumption $q = 81$ [$q = 81$. J.S.D. 1914]Atmosphere. *Clear.*Clouds. *Ci. and ci.-st. 6; Fr.-cu. 1.*Remarks. *All clouds moving from W.N.W.: ci. quickly, fr.-cu. slowly.*Theodolite set on $O^2 = N.33^{\circ}E.$; known bearing, *Golf Course anemometer pole.* Azimuth reading at start —; at finish —.Balloon kept in sight 27 m. How lost. *In distance.*Wind by Anemometer; Dirⁿ —, Vel^y — m/s.Gradient wind; Dirⁿ —, Vel^y *Nil.* m/s.Bearing of Station from centre of cyclone or other features of the isochronous chart: *A belt of high pressure running N.N.W. and S.S.E. extended over the British Isles.*

t = Time from start.

A = Azimuth of balloon from North. E = Elevation.

h = lifting^g velocity \times time = assumed height.

h cot E = horizontal distance.

 D_E = horiz^l distance to east = (h cot E) sin A. D_N = „ „ north = (h cot E) cos A.The resultant horiz^l vel^y makes ϕ with north; $\tan \phi = V_{WE} / V_{SN}$.The resultant horiz^l vel^y = $V = V_{WE} / \sin \phi = V_{SN} / \cos \phi$.

Method 2a.

Pilot Balloon Ascent Number 28.

t	A	E	h	h cot E	D _E	D _N	V _{WE}	V _{SN}	φ	V
	°	°	m.	m.	m.	m.	m/s	m/s	°	m/s
½	89.0	30.1	71	123	+ 123	+ 2	+ 4.1	+ 0.1	269	4.1
1	90.5	31.8	142	229	+ 229	- 2	+ 3.5	- 0.1	272	3.5
2	93.2	35.6	284	396	+ 396	- 22	+ 2.8	- 0.3	276	2.8
3	95.2	34.6	426	617	+ 615	- 56	+ 3.6	- 0.6	279	3.7
4	101.0	33.1	568	872	+ 855	- 166	+ 4.0	- 1.8	294	4.4
5	107.0	31.4	710	1161	+ 1111	- 340	+ 4.3	- 2.9	304	5.2
6	112.3	30.5	852	1445	+ 1336	- 548	+ 3.7	- 3.5	313	5.1
7	117.4	30.0	994	1720	+ 1528	- 792	+ 3.2	- 4.1	322	5.2
8	120.95	30.1	1136	1960	+ 1681	-1008	+ 2.5	- 3.6	325	4.4
9	123.3	30.45	1278	2172	+ 1817	-1194	+ 2.3	- 3.1	323	3.9
10	127.05	31.4	1420	2322	+ 1854	-1400	+ 0.6	- 3.4	350	3.5
11	129.5	31.2	1562	2578	+ 1990	-1640	+ 2.3	- 4.0	330	4.6
12	131.0	28.5	1704	3136	+ 2368	-2058	+ 6.3	- 7.0	318	9.4
13	130.7	26.2	1846	3752	+ 2847	-2448	+ 8.0	- 6.5	309	10.3
14	128.9	24.85	1988	4295	+ 3342	-2696	+ 8.2	- 4.1	297	9.2
15	127.2	23.4	2130	4925	+ 3920	-2980	+ 9.6	- 4.7	296	10.7
16	126.7	22.2	2272	5565	+ 4470	-3330	+ 9.2	- 5.8	302	10.9
17	126.3	21.8	2414	6025	+ 4860	-3570	+ 6.5	- 4.0	302	7.6
18	126.3	21.0	2556	6650	+ 5365	-3940	+ 8.4	- 6.2	306	10.5
19	126.7	20.5	2698	7220	+ 5780	-4310	+ 6.9	- 6.2	312	9.3
20	127.8	19.7	2840	7925	+ 6260	-4860	+ 8.0	- 9.2	319	12.2
21	128.3	19.0	2982	8660	+ 6750	-5430	+ 8.2	- 9.5	319	12.6
22	129.7	18.3	3124	9450	+ 7270	-6045	+ 8.7	-10.2	319	13.4
23	130.0	18.0	3266	10050	+ 7690	-6455	+ 7.0	- 6.8	314	9.8
24	130.6	17.4	3408	10870	+ 8250	-7075	+ 9.3	-10.3	318	13.9
25	131.1	16.95	3550	11650	+ 8770	-7650	+ 8.7	- 9.6	318	13.0
26	131.7	16.2	3692	12700	+ 9480	-8450	+11.8	-13.3	318	17.8
27	132.0	15.9	3834	13450	+10000	-9010	+ 8.7	- 9.3	317	12.8

Method 2b.

Pilot Balloon Ascent Number 28.

t	A	E	h	h cot E	D _E	D _N	V _{WE}	V _{SN}	φ	V
	°	°	m.		m.	m.				
½	89.0	30.1	71		+ 123	+ 2				
1	90.5	31.8	142		+ 229	- 2				
2	93.2	35.6	284		+ 396	- 22				
3	95.2	34.6	426		+ 615	- 56				
4	101.0	33.1	568		+ 855	- 166				
5	107.0	31.4	710		+ 1113	- 349				
6	112.3	30.5	852		+ 1338	- 549				
7	117.4	30.0	994		+ 1529	- 793				
8	120.95	30.1	1136		+ 1681	-1008				
9	123.3	30.45	1278		+ 1817	-1193				
10	127.05	31.4	1420		+ 1857	-1402				
11	129.5	31.2	1562		+ 1990	-1641				
12	131.0	28.5	1704		+ 2368	-2059				
13	130.7	26.2	1846		+ 2844	-2446				
14	128.9	24.85	1988		+ 3340	-2695				
15	127.2	23.4	2130		+ 3921	-2976				
16	126.7	22.2	2272		+ 4463	-3327				
17	126.3	21.8	2414		+ 4864	-3573				
18	126.3	21.0	2556		+ 5367	-3942				
19	126.7	20.5	2698		+ 5785	-4312				
20	127.8	19.7	2840		+ 6268	-4862				
21	128.8	19.0	2982		+ 6748	-5426				
22	129.7	18.3	3124		+ 7266	-6033				
23	130.0	18.0	3266		+ 7700	-6461				
24	130.6	17.4	3408		+ 8257	-7076				
25	131.1	16.95	3550		+ 8776	-7656				
26	131.7	16.2	3692		+ 9489	-8455				
27	132.0	15.9	3834		+10005	-9007				

Method 2b.

South Farnborough Observatory. Ascent No. 28. Date, November 2nd, 1912.

Observer.....

Computer.....

$W = 12.0$ g.

$L = 46.0$ g.

$V = 142$ m/minute.

$v = \frac{V}{60} = 2.37$ m/s.

t.	$\frac{1}{2}$.	1.	2.	3.	4.	5.
log h ...	1.8513	2.1523	2.4533	2.6294	2.7543	2.8513
log cot E2368	.2076	.1451	.1612	.1858	.2144
log cos A ...	2.2419	3.9408	2.7468	2.9573	1.2806	1.4659
log D_N8300	.3007	1.3452	1.7479	2.2207	2.5316
log tan A ...	1.7581	2.0591	1.2525	1.0409	0.7114	0.5147
log D_E ...	2.0881	2.3598	2.5977	2.7888	2.9321	3.0463
	6.	7.	8.	9.	10.	11.
log h ...	2.9304	2.9974	3.0554	3.1065	3.1523	3.1937
log cot E2299	.2386	.2368	.2307	.2144	.2178
log cos A ...	1.5792	1.6630	1.7112	1.7396	1.7800	1.8035
log D_N ...	2.7395	2.8990	3.0034	3.0768	3.1467	3.2150
log tan A3871	.2854	.2221	.1825	.1221	.0839
log D_E ...	3.1266	3.1844	3.2255	3.2593	3.2688	3.2989
	12.	13.	14.	15.	16.	17.
log h ...	3.2315	3.2662	3.2984	3.3284	3.3564	3.3827
log cot E2652	.3080	.3343	.3638	.3892	.3980
log cos A ...	1.8169	1.8143	1.7979	1.7815	1.7764	1.7723
log D_N ...	3.3136	3.3885	3.4306	3.4737	3.5220	3.5530
log tan A0608	.0654	.0932	.1197	.1276	.1340
log D_E ...	3.3744	3.4539	3.5238	3.5934	3.6496	3.6870
	18.	19.	20.	21.	22.	23.
log h ...	3.4076	3.4310	3.4533	3.4745	3.4947	3.5140
log cot E4158	.4273	.4461	.4630	.4805	.4882
log cos A ...	1.7723	1.7764	1.7874	1.7970	1.8053	1.8081
log D_N ...	3.5957	3.6347	3.6868	3.7345	3.7805	3.8103
log tan A1340	.1276	.1103	.0947	.0808	.0762
log D_E ...	3.7297	3.7623	3.7971	3.8292	3.8613	3.8865
	24.	25.	26.	27.		
log h ...	3.5325	3.5502	3.5673	3.5837		
log cot E5039	.5160	.5368	.5454		
log cos A ...	1.8134	1.8178	1.8230	1.8255		
log D_N ...	3.8498	3.8840	3.9271	3.9546		
log tan A0670	.0593	.0501	.0456		
log D_E ...	3.9168	3.9433	3.9772	4.0002		

The Pilot Balloon Slide Rule.

Special features.

(1.) The tangent, sine and cosine scales are fixed; the scale of natural numbers is on the principal slide and also on an inner slide carried by the principal slide.

In a series of balloon observations the observed angles change slowly, and as the trigonometrical scales are fixed the cursors which indicate the angles are only displaced slightly from one setting to the next.

(2.) The tangent scale runs from 1° to 84° ; the sine scale runs from 1° to 90° , and the part from 10° to 90° is repeated so that the sine and cosine cursors may be set at angles like 44° without interfering with each other.

(3.) The graduations of the sine scale between 70° and 90° are shown on an arc so that they are equally spaced instead of being crowded together.

(4.) Both sine and cosine scales are figured.

(5.) The trigonometrical scales are divided to tenths of a degree, not to sixths.

In the following instructions the trigonometrical scales are referred to as C, S and T. The Principal sliding scale is denoted by D and the inner slide by t.

Single Theodolite.

Displacements.

(1.) Set unity on the inner scale against the rate of ascent on D (142 in the example worked on p. 16).

(2.) Set two cursors on the sine and cosine scales according to the azimuth A and set the third cursor on T according to the elevation E .

(3.) Set the time (in minutes) on t to the T cursor and read off D_E , D_N on the D scale by the S and C cursors.

Note.—With azimuths in the second and fourth quadrants it is convenient to subtract 90° or 270° and then D_E , D_N are given by the C and S cursors respectively.

The horizontal distance of the balloon $h \cot E$ is not usually read off; it is to be found on D opposite 90° on the S scale.

Labour is saved by using 60 metres instead of 1 metre as the unit of displacement; the rate of ascent per second is assumed instead of the rate per minute. See the worked example, p. 21. In this case unity on the t scale is set against 2.37 (rate of ascent in m/s) on D.

Velocity

To find the direction and magnitude of the velocity from its components, the principal slide of this rule should be reversed and the inner slide set to agree with the principal slide.

(1.) Set one cursor at 90° on S and set the slide so that V_{WE} on t corresponds with this cursor. Set the T cursor to agree with V_{SN} on D. The corresponding reading on T is ψ , which is equal to the azimuth ϕ or to one of the angles $180^\circ \sim \phi$ or $360^\circ - \phi$.

(2.) Set the third cursor to the same angle ψ on the S scale. V is given by this cursor on the t scale.

Note.—When ϕ is to fall in the second or fourth quadrant V_{SN} should be set against 90° and the T cursor adjusted to V_{WE} . The angles found on T are *added* to 90° or 270° .

Method 2c.

Pilot Balloon Ascent Number 28.

t	A	E	h	h cot E	D _E	D _N	V _{WE}	V _{SN}	ϕ	V
min.	°	°	m.		$\times 60$ m.	$\times 60$ m.	m/s	m/s	°	m/s
1	90.5	31.8	142	—	3.81	— 0.03	3.8	— 0.03	270	3.8
2	93.2	35.6	284	—	6.60	— 0.37	2.8	— 0.34	277	2.8
3	95.2	34.6	426	—	10.26	— 0.93	3.7	— 0.56	279	3.7
4	101.0	33.1	568	—	14.3	— 2.77	4.1	— 1.84	294	4.5
5	107.0	31.4	710	—	18.5	— 5.67	4.2	— 2.9	305	5.1
6	112.3	30.5	852	—	22.3	— 9.2	3.8	— 3.6	313	5.2
7	117.4	30.0	994	—	25.5	— 13.2	3.2	— 4.0	321	5.1
8	120.95	30.1	1136	—	28.0	— 16.9	2.5	— 3.7	326	4.5
9	123.3	30.45	1278	—	30.3	— 20.0	2.3	— 3.1	323	3.9
10	127.05	31.4	1420	—	30.9	— 23.4	0.6	— 3.4	350	3.5
11	129.5	31.2	1562	—	33.3	— 27.4	2.4	— 4.0	329	4.7
12	131.0	28.5	1704	—	39.5	— 34.3	6.2	— 6.9	318	9.3
13	130.7	26.2	1846	—	47.4	— 40.8	7.9	— 6.5	309	10.2
14	128.9	24.85	1988	—	55.6	— 44.8	8.2	— 4.0	296	9.1
15	127.2	23.4	2130	—	65.3	— 49.5	9.7	— 4.7	296	10.8
16	126.7	22.2	2272	—	74.6	— 55.6	9.3	— 6.1	303	11.1
17	126.3	21.8	2414	—	81.0	— 59.5	6.4	— 3.9	301	7.5
18	126.3	21.0	2556	—	89.3	— 66.0	8.3	— 6.5	308	10.6
19	126.7	20.5	2698	—	96.4	— 71.9	7.1	— 5.9	310	9.3
20	127.8	19.7	2840	—	104.5	— 81.0	8.1	— 9.1	318	12.3
21	128.8	19.0	2982	—	112.2	— 90.3	7.7	— 9.3	320	12.1
22	129.7	18.3	3124	—	120.6	— 100.2	8.4	— 9.9	320	13.1
23	130.0	18.0	3266	—	128.2	— 107.8	7.6	— 7.6	315	10.7
24	130.6	17.4	3408	—	137.6	— 118.0	9.4	— 10.2	317	13.8
25	131.1	16.95	3550	—	146.1	— 128.0	8.5	— 10.0	320	13.1
26	131.7	16.2	3692	—	158.0	141.0	11.9	— 13.0	317	17.6
27	132.0	15.9	3834	—	166.8	150.0	8.8	— 9.0	316	12.6

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M.O. 223 [Sec. II (I) *contd.*]

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II 23

METEOROLOGICAL OFFICE.

COMPUTER'S HANDBOOK.

SECTION II.

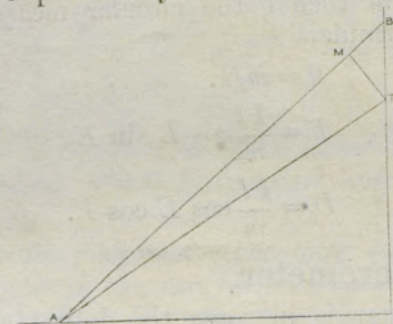
Subsection I.—*continued.*

THE TAIL METHOD OF FINDING THE RATE OF ASCENT OF PILOT BALLOONS.

Description of the Method.

The method consists in observing the apparent length, or distance apart in the eye piece of a telescope, of objects or points forming the top and bottom of a tail attached to the balloon.

It is useful in cases where the wind is fairly strong so that the angle of elevation of the balloon does not exceed 30° or 40°, but it is not much use when the angle is large. It will be best explained by means of a diagram.



Let B represent the centre of the balloon, T some visible object (the "pendant") hanging from the balloon at a known distance BT, A the observer, AC a horizontal line, and TC a vertical one.

Then, the length BT being known as well as the angles BAT and BAC, the height CB can be found.

Let TM be the perpendicular from T on AB and let θ be the circular measure of the angle BAT.

As the angle MAT is small MT cannot be distinguished from part of a circular arc with radius AM and therefore we may write $\theta = MT/AM$ and $AM = MT/\theta$.

Now let l be the length of the tail BT and let E be the elevation BAC of the balloon.

Then $MTB = 90^\circ - MBT = BAC = E$.

so that $MT = l \cos E$.

$$\text{Hence } AM = \frac{l \cos E}{\theta}.$$

The actual distance of the balloon from the observer is AB, but the distinction between AM and AB may be

ignored in practice and we write $AB = \frac{l \cos E}{\theta}$.

Knowing AB we can calculate the height and the horizontal displacement by the formulæ $h = \frac{l \cos E}{\theta} \sin E$

$$\text{and } D = \frac{l \cos E}{\theta} \cos E.$$

The use of the Micrometer.

The angle θ is measured by a micrometer which gives the apparent length of the tail as seen in the telescope. There are various forms of micrometer, but whichever form be used the unit of its scale must correspond to some small angle which is a certain fraction, say, the k th part of a radian. If the apparent length of the tail as given by the micrometer is m then θ the circular measure of BAT is given by the formula,

$$\theta = m/k.$$

$$\text{Accordingly } h = \frac{kl}{m} \cos E \sin E$$

$$D = \frac{kl}{m} \cos E \cos E.$$

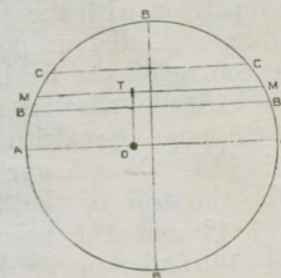
Types of Micrometer.

The micrometer is in some theodolites a glass scale at the focus of the object glass. The scale is divided into millimetres and tenths.

Against this pattern the objection has been urged that specks of dust on the glass may be mistaken for the balloon at a great distance, and the majority of modern instruments have a movable spider's thread in addition to the fixed threads.

The number of units by which the moving thread is displaced from the centre of the field is shown on the milled head of the micrometer. Completed turns of the head are

counted either by reference to a serrated scale at the edge of the field of view or by estimation aided by the provision of additional fixed spider threads.



The figure represents the field of view in one pattern, AA, BB, CC being fixed threads, and MM the thread carried by the micrometer.

Ten units correspond to one turn of the micrometer screw. When MM is at AA the micrometer reading is 0, at BB it is 50, at CC it is 100. In the position shown in the figure, the reading is estimated at about 70, and if the unit figure shown on the head was 9 the observation would be entered as 69.

The scale value of the micrometer should be obtained from the maker: if it is not known, either of the following methods may be adopted.

Estimation of Scale Value.

The scale value is found from the number of divisions on the micrometer scale which correspond to some definite small angle.

With a glass-scale micrometer the most convenient way is to use the diameter of the sun, or if a coloured glass is not available, of the moon. The diameter for any given date can be obtained from a good almanack. Suppose it to be $32'$ and let this diameter correspond to 48 divisions in the micrometer; this is the case with the theodolite used at Benson.

$$\text{The circular measure of } 32' \text{ is } \frac{32}{60} \times \frac{\pi}{180} \text{ or } \frac{1}{107.6} \text{ and}$$

therefore the number of divisions to the radian is 107.6×48 or 5160.

This method is not available with the spider-thread micrometer, as the thread can not be seen except when it

shows against the bright background of the sun, and it can not be adjusted to the tangential position.

With such a micrometer the scale-value may be determined approximately by comparison with the scale of the altitude circle of the theodolite. A distant point is sighted and brought near to the centre of the field. The telescope is adjusted so that the altitude reading comes to an exact tenth of a degree. Suppose the altitude is 2.3° and the micrometer reading is 7.4. Now lower the telescope until the object appears at the top of the field. Suppose the altitude is exactly 1.4° and the micrometer reading is 156.4. In this case 149 micrometer divisions correspond to $.9^\circ$ and the scale value is $149 \times \frac{180}{\pi} \times \frac{10}{9}$ or 9500 units to the radian.

A better method is to use the micrometer to measure the apparent length of a two-foot rule set up at a considerable distance, say, 200 feet away. If the micrometer reading was 94.5 the scale-value would be $94.5 \times 200/2$ or 9450 units to the radian. It should be noted that when precision is aimed at in the use of this method the distance of the object should be measured from the principal focus in front of the object glass. For example, if the focal length were about 18 inches and the distance of the object from the focus were required to be 200 feet, the distance of the object from the object glass would have to be 201 feet 6 inches.

Length and Form of the Tail.

Two points require consideration, the length of the tail and its form.

The longer the tail the greater the proportional accuracy in the micrometer reading, but, on the other hand, when the balloon is quite near, a long tail more than covers the field of view and the method cannot be used. On the whole, for pilot balloons, about 10 metres is a suitable length of tail: in general, this will afford observations from the fourth to the twentieth minute of an ascent. The length of the tail is measured from the centre of the balloon to the middle of the pendant.

The other point is the nature of the pendant. A half-sheet of white foolscap paper gummed together along the shorter edges to form a cylinder and hung from the balloon by cotton will serve. The cylinder should be hung vertically from a cotton loop, and its steadiness is increased if it

is made to spin by cutting the paper upwards for an inch or two from the bottom in a few places and turning the corners outwards. All the corners should be turned out in the same way. Another type of pendant can be made from a circular disc of paper cut along a spiral nearly to the centre. When hung from the centre the outer rings of the spiral drop and it forms a sort of cone which will rotate as it is raised.

The best colour for this paper depends on the background and differs according to the time of day. It has been suggested that in cases when the observations for the first minute or two are important two pendants should be used, one being placed halfway along the tail.

The Observations.

At any rate after the first few minutes of a flight, observations of the positions of the balloon and of apparent length of the tail can be taken at alternate half-minutes. One hand is used for setting the altitude all the time, the other hand is alternately used for setting the azimuth and adjusting the micrometer.

Working up the Observations.

Owing to the swinging of the tail a single observation, however accurate it may be, will not afford a reliable value of the angle BAT, especially if the angle of elevation is large, for if T is not vertically under B the error produced in the angle BAT may be considerable. Oscillations in the vertical plane through the line of sight produce more effect on the angle than oscillations in the perpendicular plane. If the elevation is small the errors will be small, and if the elevation is moderate the positive and negative errors are approximately equal. Hence if the observations be plotted and a smooth curve drawn through them fairly reliable values can be obtained. The smoothing process cuts out the use of a single observation, and it is not possible to say how much of the irregularity of the unsmoothed curve is due to ascending and descending currents and how much is due to the swinging of the tail; there is also a small systematic error because the swinging must take the mean value of BAT too small.

The formula for the height of the balloon is:—

$$h = \frac{kl}{m} \cos E \sin E = \frac{kl}{2m} \sin 2E.$$

k is the scale value, l the length of the tail, m the micrometer reading and E the elevation. It is convenient to

keep to the same length of tail and to make a table of values of $\frac{kl}{2m}$ corresponding to the values of m . When $\frac{kl}{2m}$ is known h is found by a single operation with the slide-rule and from h the displacements can be found in the usual way.

With the pilot-balloon-slide-rule the table of values of $kl/2m$ can be dispensed with if the following instructions are followed.

Lock the t scale so that unity on that scale agrees with $\frac{1}{2}kl$ on the D scale. Suppose that $h = \tan \phi$; then if the rule is set so that m on the D scale is against $2E$ on the sine scale, unity on the t scale is against ϕ on the tangent scale. Keep the tangent-scale cursor at ϕ and move the slide until the t scale is in the position it occupies when the rule is shut up. Then h on that scale comes under the cursor.

Example.

Tail 30 feet long. Scale value 5200 units to radian.
Micrometer reading 24. Elevation 20° . Azimuth 37° .

Here $\frac{lk}{2} = 78000 \text{ ft.}; \frac{lk}{2m} = 3250 \text{ ft.}$

$$h = \frac{lk}{2m} \sin 40^\circ = 2090 \text{ ft.}$$

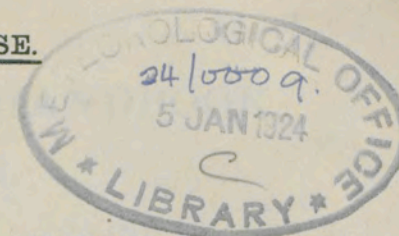
$$D = 2090 / \tan 20^\circ = 5740 \text{ ft.}$$

$$D_N = 5740 \cos 37^\circ = 3450 \text{ ft.}$$

$$D_E = 5740 \sin 37^\circ = 4580 \text{ ft.}$$

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M.O. 223. Section II.



METEOROLOGICAL OFFICE.

THE COMPUTER'S HANDBOOK.

Sub-section II—Computation of Height and Temperature by means of Registering Balloons.

Sub-section III—The Dynamics of the Upper Air.

Sub-section IV—Tables for the Estimation of Geostrophic Winds.

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THE COMPUTER'S HANDBOOK.

SECTION II.

DYNAMICAL METEOROLOGY.—CALCULUS OF THE UPPER AIR.

Sub-Section 2.—Computation of Height and Temperature by means of Registering Balloons.

In the first part of this sub-section the methods devised by Mr. W. H. Dines and adopted by the Meteorological Offices of the United Kingdom, Canada and Australia and with some modifications by that of India for obtaining records of pressure and temperature and deducing the heights are described. In the second part the formula known as Laplace's formula on which such computations are based is discussed and arithmetical methods of obtaining results are exemplified.

Description of the Dines Balloon Meteorograph

The meteorograph for use with registering balloons is light and simple, it weighs 28 grammes, and is shown in Fig. 1. The trace is made by the scratching of a steel point on a piece of thin sheet metal about the size of a postage stamp and the record is read by means of a microscope. The metal is electroplated first with copper and then with silver, and this forms a soft non-corrodible surface free from all scratches, on which the scratch of the actual record is plainly seen.

The trace is the extension-temperature diagram of an aneroid-box, as will be readily seen from the figure. As the balloon rises the aneroid box A opens under the decreased pressure and the two sides of the frame BB which are connected by the spring C move away from each other. One side carries the metal plate and the other the writing point and the result is a scratch on the plate which is practically an arc of a circle with centre near B. Any change of temperature provides a movement perpendicular to this arc and is recorded by the expansion and contraction of a strip of very thin German silver, 125mm. long, 9 broad

and 0.2 thick. The contraction is multiplied some ten times by the lever EFD and the action is as follows:—H is

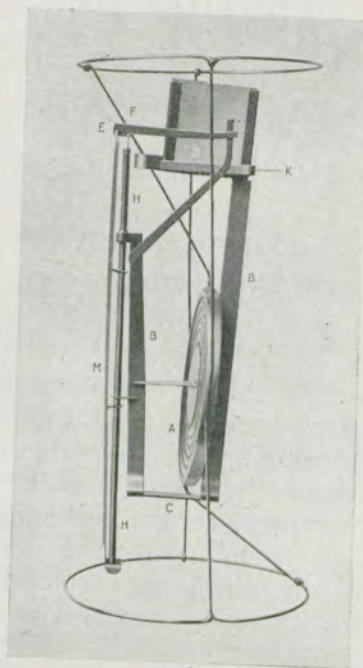


FIG. 1.—General view of the balloon-meteorograph for recording pressure and temperature.

a bar of invar, a nickel steel alloy which does not rust and has no appreciable coefficient of expansion with heat. It ends at E in a short length of flat spring, which is soldered to the lever at F. The strip of German silver, M, is soldered at the one end to the lever at E and at the other to the invar bar. As the German silver contracts with the cold, the end of the lever D, which is turned down and ends in a point, moves outwards giving a scale of about 1mm. to 50°C, which can be easily read by a low power microscope to a sufficient degree of accuracy.

The frame is cut out of one piece of German silver about .8mm. thick, the end C being turned down at right angles so that the frame may open and shut like a pair of spring scissors under the action of the aneroid box; the invar bar is rigidly attached to the frame at one end, and by a sliding joint at the other so as to allow for the expansion of the frame.

The writing point of the lever D is pressed on the plate by the natural spring of the frame and apparatus, but when

the instrument is not in use it is held off by a spring K. When the instrument is to be used the spring is depressed by a small wedge inserted at the back, to which a piece of red string is tied. The finder of the instrument is asked to pull out the string and thus remove the wedge, so that the writing point may be lifted off the trace. The instrument is held by four pieces of stout wire soldered to the edges of the aneroid box as shown, and the small plate that carries the record slides in and out of the plate holder, into which it should fit tightly. In handling the instruments only the wire frame or the edges of the aneroid box should be touched.

The aneroid box is made of very thin metal and some air is left inside it; it is soldered up finally with the opposite faces in contact at the centre. In consequence the temperature correction is large, but this is of no importance, especially as it is impossible to make a box to cover so large a pressure scale that does not require some correction; the advantage is that the extension of the box depends primarily on the elasticity of the included air and but slightly on the metal, and hence there is very little hysteresis.

A second scratching point is employed, rigidly attached to the arm of the frame which does not carry the plate. This gives a fiducial mark on the plate. It is not used in calibration, but it shows whether the plate has moved in the plate-holder between the calibration and the ascent; should such movement occur the scratch made by the fixed pen would be duplicated.

The control exerted by the aneroid box is very powerful so that the friction of the scratching points is quite negligible in comparison. The instrument is protected by a thin cylindrical case of aluminium, and is hung from the balloon so that the axis of the cylinder is vertical; this allows a stream of air to pass over the instrument as it rises or falls and provides sufficient ventilation. Moreover, the thermograph is of bright metal and is protected from the sun by the metal case which is also bright both inside and out; thus it should not be susceptible to radiation, and owing to the thinness of the strip of German silver, which is the basis of the thermograph, it is very sensitive and has very little lag of temperature.

Fig. 2 shows the arrangement of the scratching points on a larger scale.

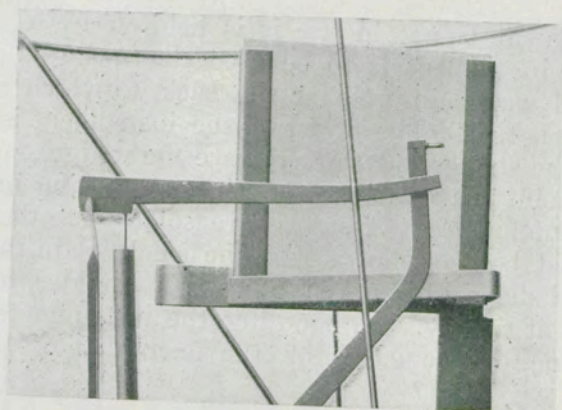


FIG. 2.—Enlarged view of the plate and recording levers.

A hygrograph is easily added if desired; a third writing point is arranged to make a scratch about 4mm. inside the temperature scratch, and the point is held in by a short length of hair against a light spring. (Fig. 3.)

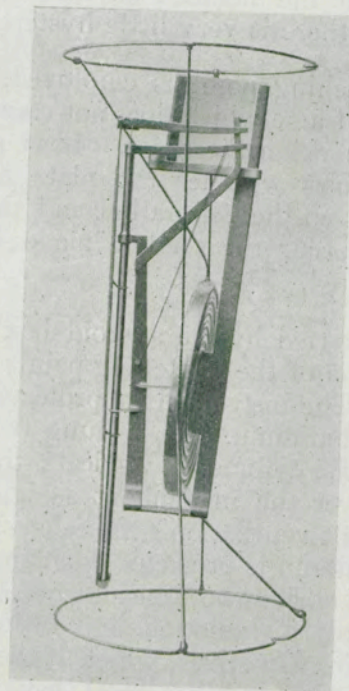


FIG. 3.—General view of a meteorograph arranged to record relative humidity, by means of a third lever, in addition to pressure and temperature.

Calibration of the meteorographs.

New instruments should be calibrated some time in the week before they are used; for old instruments, the zeros of which are not likely to alter, some time in the preceding two or three months will suffice. The calibration is made on the piece of plated metal on which the actual trace is to be recorded.*

The calibration is based upon three marks covering the whole range of pressure likely to be met with, at temperatures of about 280, 250, and 220 on the absolute scale, and cross marks, consisting in reality of small blurs, made at definite pressures on each temperature mark. The calibrating vessel is designed for making these marks. It is a cylindrical brass vessel, into which the meteorograph fits easily. It is fitted with an air-tight cover and well lagged with felt. With it an air-pump and mercury-gauge corrected for temperature are used, so that any desired decrease of pressure

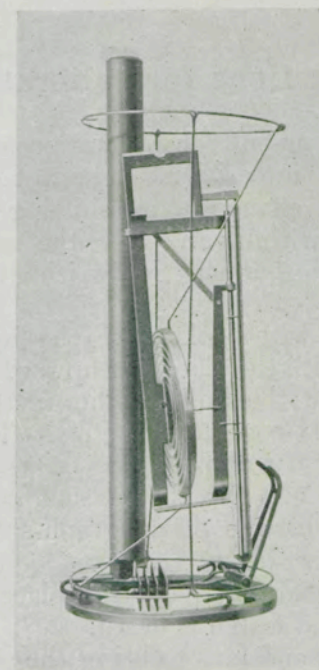


FIG. 4.—Meteorograph in the calibrating tank showing the tube used as an air-thermometer and the electric striker.

can be obtained. It is convenient to have a fair-sized reservoir between the air-pump and the calibrating vessel. By the

* Instructions for plating are given below, page 30.

use of a tap in the communicating pipe and a small tap for letting air in, it is easy to adjust the pressure. Liquid carbonic acid is used for cooling, and can be obtained in steel cylinders. The liquid CO_2 is allowed to run into a coil of copper tube that surrounds and is well soldered to the calibrating vessel. The vessel itself is filled with petrol.

The pressures are marked by striking the strip of the thermograph by a small electric striker (Fig. 4). This striker and its armature are inside the vessel and the electromagnet is outside. A vertical brass tube inside the vessel forms the bulb of an air thermometer which may be fitted up and used for determining the temperature of the petrol. It also serves as a guide for placing the meteorograph in position. A thermometer graduated down to 210a is used for taking the temperature of the petrol. The thermometer should be tested occasionally at the freezing points of water and mercury, 273a and 234a. The electric striker is arranged to work with a four-volt-accumulator or a primary battery.

Instructions for calibrating.

Prepare the instrument for calibration by placing a clean piece of plated metal in position. The metal must fit tightly so that there is no chance of its shifting in the slightest degree until it is withdrawn after the ascent. Insert a small wedge to bring down the writing points upon the plate.

By means of a wire-hook place the instrument in the calibrating vessel with the plate-end upwards and the sloping wire in contact with the bulb of the air-thermometer, and see that it is entirely covered by petrol. The fluid-pressure of this petrol must be allowed for on the pressure scale. The instrument should now be in such a position that the electric striker will hit the thermograph-strip when contact with the battery is made.

Take the temperature of the petrol, place the cover of the vessel in position and fasten it down.

Use the air pump and taps to bring the air-pressure inside to a suitable value p_0 and make a momentary electric contact. Reduce to p_1 and again to p_2 and so on, making a contact each time.

Remove the instrument from the vessel and examine the plate with a magnifying glass to see that it is properly marked. Read and note down the temperature of the petrol; place the instrument carefully on one side and if a batch of

instruments is being calibrated proceed similarly with the next.

Now reduce the temperature in the vessel to about 250a. This is done by making a pressure-tight connection between the copper-coil and a cylinder of carbonic acid. For this a piece of flexible metallic tubing with a metal-union at each end is best; rubber tubing will not do. Note that the carbonic acid cylinder must be so placed that the valve is downward when it is opened. Only a small quantity of the liquid is required.

Take the temperature of the petrol after stirring well and seeing that it has become nearly stationary and then insert the meteorograph as before.

Use the air-pump and make contacts at exactly the same pressures p_0, p_1, p_2 , etc., as before.

Remove the instrument, examine it to see that the marks are clear, take the temperature of the petrol and proceed with the next.

Reduce the temperature to about 220a and repeat the process.

Remove the wedges, place each instrument in its own numbered case and take care that they experience no rough usage before the ascent.

It would be convenient if fixed temperatures and fixed pressures could always be employed but this is not possible, or hardly so, in the case of the low temperatures. By the aid of a little more CO_2 the temperature can be lowered but it is not possible to adjust the supply with sufficient nicety to bring the temperature down exactly to a predetermined value.

The case is different with the pressures p_0, p_1, p_2 , etc., and definite values should be used and kept to. The pressure of the petrol vapour makes it difficult to go below about 120mb. at the higher temperature, 280a, and below about 60mb. at 220a. The even hundreds are the natural values to use for p_0 , etc., but as many as ten points are not required. Also 1,000mb. is a point that cannot be used, because when the balloon falls and reaches the ground the jar and rough usage of the fall make a blur which will efface any pressure-mark that is near the surface pressure; the space that may be covered by the blur is from about 950 to 1,050mb. But it is necessary to have a reliable pressure-mark as near the bottom as possible, because the scale is not linear and is

changing rapidly as the bottom is approached; p_0 should therefore be about 950mb. The lowest value, p_n , should be at 75mb. for the low temperatures, but 200mb. will suffice for the high temperature mark, because at a height in the ascent corresponding with 200mb. the temperature is certain to be below 250a.

Whatever value be taken for the lowest pressure, p_n , marked by the electric striker, reduce the pressure exactly to 10mb. below p_n but no further. The reason is this. The pressure-marks (see fig. 5) consist of small blurs ranging

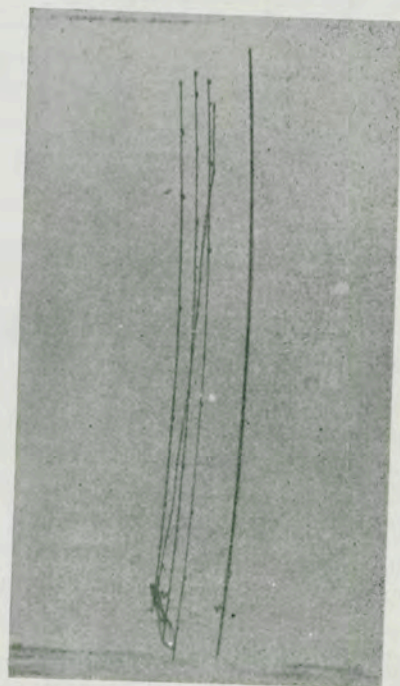


FIG. 5.—Enlarged copy of record from the balloon meteorograph with lines of calibration.

over 4 or 5mb. of pressure, hence the calibration-mark is not a definite scratch. But the actual end of the scratch at 10mb. below p_n is perfectly definite, and from it and the mark next to p_n a definite position for the p_n mark can be assigned. Comparing this assigned position with the centre of the blur it can be seen whether a little above the centre, or a little below, ought to be taken as the proper position for the corresponding pressure. Since all the blurs on the same

plate have a strong family likeness the proper part of each one that should be used is found.*

For the most accurate work the pressure should be decreased slowly so that the decrease in a given time should correspond with that which occurs in the actual ascent. Careful experiment has shown that with the form of aneroid box used a rapid decrease of pressure makes the aneroid box open about 0.3 per cent. less than it would do with a slow decrease. The instruments are calibrated quickly, but allowance is made for the 0.3 per cent. on the pressure scale.

In determining the pressures a mercury-gauge is used and it is convenient to get rid of the correction caused by the expansion of mercury with heat in the following way.

It is obvious that a pressure-gauge may be constructed to suit, when vertical, a liquid of any density, and that when the glass tube of the gauge is tilted out of the vertical it will suit a liquid of greater density. If, therefore, a mercury gauge is arranged to suit mercury at 300a, it will suit mercury at any lower temperature provided it is inclined to the vertical at the proper angle.

The proper angle is readily calculated, for if the vertical position is right for a temperature T_0 , and θ is the inclination to the vertical for T , then we have:—

$$\text{density at } T_0 : \text{density at } T = \cos \theta : 1.$$

The coefficient of expansion of mercury is .00018 on the Absolute Scale and therefore

$$\cos \theta = 1 - .00018 (T_0 - T).$$

From a table of cosines it is found that if $T_0 = 300$ then for temperatures $T = 300, 295, 290, 285, 280$, and 275 . θ must be given the values $0^\circ, 2.5^\circ, 3.5^\circ, 4.3^\circ, 4.9^\circ, 5.5^\circ$, respectively.†

The gauge is mounted on a board pivoted near the centre and provided with a clamp, and also with a pointer moving over the temperature scale. The capacity of the cistern is allowed for in the pressure scale and all connexions may be

* An improved procedure has been adopted in the Canadian Meteorological Service. The calibrating apparatus contains a pen lifter and each of the fiducial pressures is shown as the end of a scratch instead of as a blur. (Patterson Upper Air Investigation in Canada. Ottawa, 1915.)

† If θ be in circular measure $1 - \theta^2/2 = 1 - .00018 (T_0 - T)$ so that $\theta = \sqrt{.00036 (T_0 - T)} = .019 \sqrt{(T_0 - T)}$.

If θ be in degrees $\theta = 1.1^\circ \times \sqrt{(T_0 - T)}$.

This formula provides the explanation of the open spacing of the scale of temperature near 300a.

made with thick-walled rubber tube. The pressure scale slides by the side of the gauge, and can be set and clamped with the reading on the scale corresponding with the barometric pressure, at the time, level with the mercury in the cistern. The correction for the pressure of the petrol is made at the same time. Then on working the air-pump the pressure shown on the gauge is the true pressure prevailing inside the calibrating vessel and no correction is required for temperature, height above sea level, or barometric pressure at the time.

The chief difficulty of the calibration lies in the accurate determination of the temperature of the petrol at the instant when the marks are being made. There is no trouble at the high temperatures, but at the low temperatures the value is constantly changing. The air-bulb inside is very useful in this connexion because it can be used as a constant-volume gas-thermometer; but if this is considered too troublesome it can be connected with a small U-tube pressure gauge, which will show the nature and magnitude of any change of temperature that is occurring.

The apparatus might perhaps be improved by having some automatic device for constantly stirring the liquid inside the vessel and an electrical method of reading off the temperature of the petrol, but under present conditions it seems fairly certain that the temperatures, as finally published, have an instrumental probable error of less than 1a.

Figure 6 is from a photograph of the calibrating apparatus used at the Observatory, Benson, Oxon.

In the centre is the vacuum-chamber which is kept exhausted by the air-pump, and on the left the small taps which make communication between the calibrating vessel and the vacuum-chamber and the open air. Below is a shelf that is convenient for holding small tools, etc., and on the left hand, hanging below the shelf but with its mouth above, is the calibrating vessel. The CO_2 cylinder is not shown, but the lagged metal pipe through which the Carbonic Acid is introduced appears on the left hand at the bottom.

The pressure gauge is on the left, with its pivot and clamp combined rather more than half way up. The U is not seen as its plane is perpendicular to the photograph. The sliding scale is behind the glass tube. The mercury cistern is also behind and invisible. The U-tubes and pipes, just on the right of the pressure gauge, belong to the constant-volume air-thermometer.

An electric bell is shown at the top, a condenser, to reduce sparking, below the shelf, and a switch and contact maker

just over the left end of the vacuum-chamber. The bell is used to give audible evidence of proper electric contact being made.

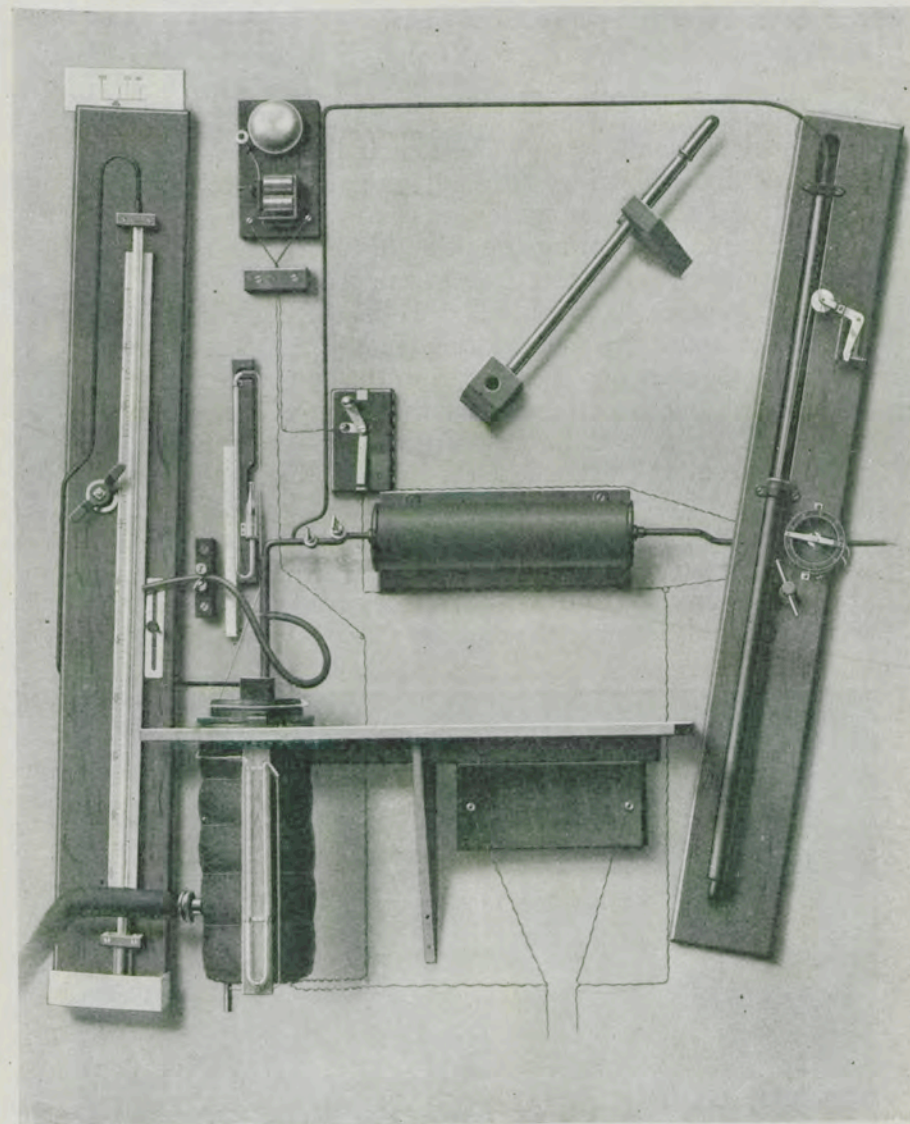


FIG. 6.—General view of the apparatus for calibrating meteorographs.

On the right an automatic arrangement for making electrical contact at suitable pressures is shown. It is not an essential part of the apparatus but saves time and trouble.

About 2 hours suffice with this apparatus for calibrating a batch of 8 to 10 instruments.

Preparing the Instrument for the Ascent.

Place the instrument in its own numbered cylindrical case with the writing end against the cross wires of the case, and wire it in by putting a piece of aluminium wire through the holes provided at the other end and then turning over the ends. The finder seldom meddles with the wire but will untie string.

Prepare a small wooden wedge and tie to it some six inches of fine red string with half a match tied to the other end. Insert the wedge by means of a small pair of pliers in its place so as to bring the pens down on the plate. Take great care not to shift the plate in the process.

Next prepare the "spider." Take three strips of split bamboo of about $\frac{1}{8}$ th inch section and 3 feet 6 inches long. Tie them together about 1 foot from the ends and place them so that each one is perpendicular to the cross formed by the other two. Fix them in this position by ties of fine cotton running from the extreme ends of each to the ends of the cross formed by the other two. Then tie a small red silk flag on each of the six ends. The result is that this "spider" when lying on the ground must have three red flags well above the ground and is, therefore, readily visible. Its weight should not exceed an ounce (fig. 7).

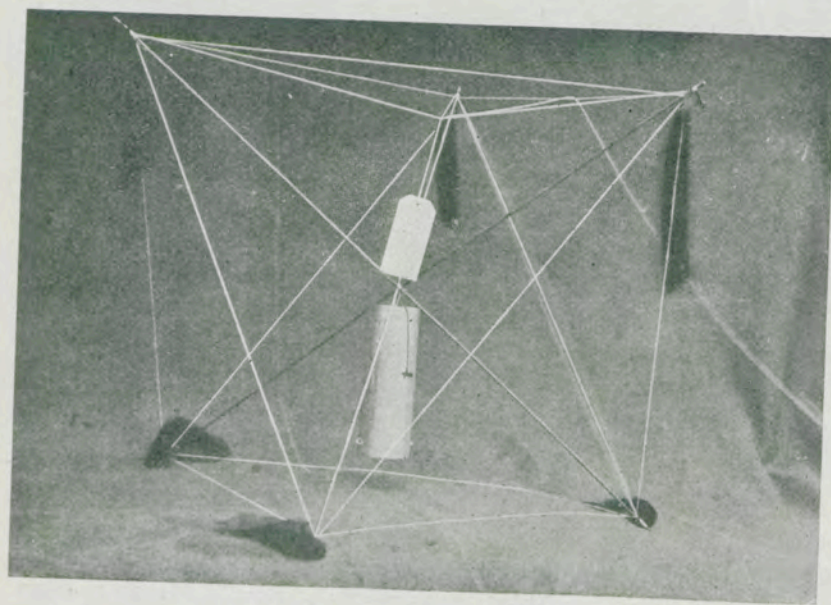


FIG. 7.—Bamboo frame or "spider" inside which the meteorograph is suspended during an ascent. The cotton was unduly coarse so that it might show in the photograph.

Tie the instrument to the "spider" by string from the cross wires of the case to the centre of the "spider." Tie securely so that the case is jammed tightly against the three bamboo strips. Neglect of this precaution will make a blurred trace. Do not forget to attach the label offering the reward: that used in England reads as follows:—

M.O. 074.

O. H. M. S.

INTERNATIONAL INVESTIGATION OF THE
UPPER AIR.

5 SHILLINGS REWARD.

DELICATE METEOROLOGICAL APPARATUS.

This instrument is the property of the Meteorological Office, London. The above reward will be paid for the instrument if it is not tampered with. The finder is requested to pull out the piece of red string (with the match-end attached), to put the instrument away in a safe place and to write to the Director, Meteorological Office, London, S.W., when instructions, and if desired, information, will be sent.

The balloon need not be returned.

Copy of label attached to the meteorograph before the ascent.

The label is soaked in melted paraffin-wax to protect it against wet. Secure the apparatus to the balloon by strong thread, so arranged that the axis of the case of the instrument must be vertical during the ascent. In England the distance between the case of the instrument and the centre of the balloon at starting used to be made equal to 13 metres but is now increased to 40 metres. The angle which this distance subtends is measured on a micrometer scale at the focus of the theodolite telescope, and by this means a fairly accurate calculation of the distance of the balloon, and therefore of the rate of ascent and of the velocity of the wind, can be made. [*Vide* Section I. of this Handbook.]

It is necessary when a balloon is sent up in strong sunshine—and since there are no clouds above 8 or 9k. this means when it is sent up between sunrise and sunset—that the instrument should not hang near to the balloon, for if it does so it will lie in the wake of air that has previously been in contact with the balloon. The balloon itself is very strongly heated by the sun, and an experiment made at

Manchester by sending up an instrument inside a balloon showed that this heating might reach 50a above the air temperature outside. There is, therefore, good reason for thinking that a wake of heated air follows the balloon and the instrument ought to be well clear of this. On the other hand it is impossible to start a balloon with an instrument hanging by a long string in windy weather. To meet this difficulty the device shown in fig. 8 is adopted.

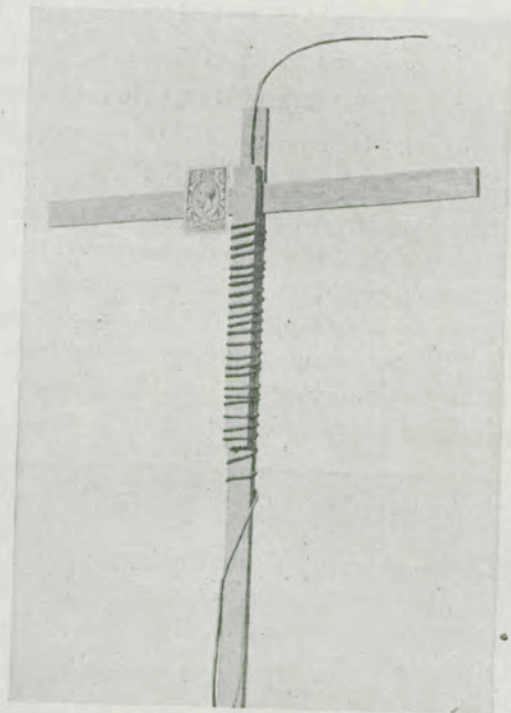


FIG. 8.—Arrangement of loose strips of wood held together at the beginning of the ascent by the thread by which the meteorograph is attached to the balloon.

The piece of thread, some 40 metres or so long, is coiled round a long thin strip of wood, and cannot uncoil or slip off over the end without imparting a spin to the strip. A rapid spin is prevented by the air resistance of a cross piece, and so soon as the instrument is hanging freely the thread begins to uncoil at the bottom and the instrument drops gradually down to its full distance below.

It is not desirable to give the balloon any extra weight to carry and hence it is arranged that when the thread has completely uncoiled the wood shall fall away.

The plan is shown in the figure, the stamp showing the

scale. The long vertical strip, some 75cm. long, the bottom of which is not shown, and the cross piece have notches cut across them so that they may lie loosely over one another at right angles in the same plane. There is a short piece with a longitudinal groove cut in the upper part in which the thread lies until it is below the cross piece, it is then coiled round and round and holds the short piece in position. When the uncoiling is complete there is nothing left to hold the three pieces together, and they fall to the ground free from the thread and from each other.

It is essential that the thread should leave the arrangement symmetrically at the top; hence the longitudinal groove is cut. The coil should not be unduly tight, the wood should be smooth, and care should be taken that there is no place where the thread can hitch itself, but no special skill is required in making it, and the coil need not be a single one. Only the upper part is shown in the figure.

The plan has been in use about two years and so far as the writer knows has never failed. The pieces may usually be seen to drop away a minute or two after the start, the time being dependent on the length of the cross piece. About 20cm. is a suitable length, and the notch must be in the middle so that the centre of gravity of the whole arrangement may be in the vertical strip.

Reading the Record.

Before describing the method of working up the traces the special microscope-stage which it is convenient to use must be described.

The extension of the aneroid box without change of temperature produces a scratch which is an arc of a circle of $4\frac{1}{2}$ inches radius, and accurate measurements of lengths along this arc have to be made. The stage itself is, therefore, pivoted (A in fig. 9) at $4\frac{1}{2}$ inches from the optical axis of the microscope, and is turned about the pivot by the endless screw (B) shown plainly at the bottom. Whole millimetres are shown by either scale engraved on the frame, and hundredths by the divided head (K) of the screw. This screw governs the motion of the plate-holder from right to left. Transverse motion, which is necessary for setting, is obtained by a sliding piece (S) which can be clamped in any position by the screw (C) and further by a screw and spring (D) for fine adjustment, which comes into action after the slide is clamped. This screw in the figure has a divided head (E) but it is not used for measurement. The plate holder is also pivoted at its centre to the sliding piece so that

it has the possibility of slight angular motion. Measurements in this direction are not made by moving the stage

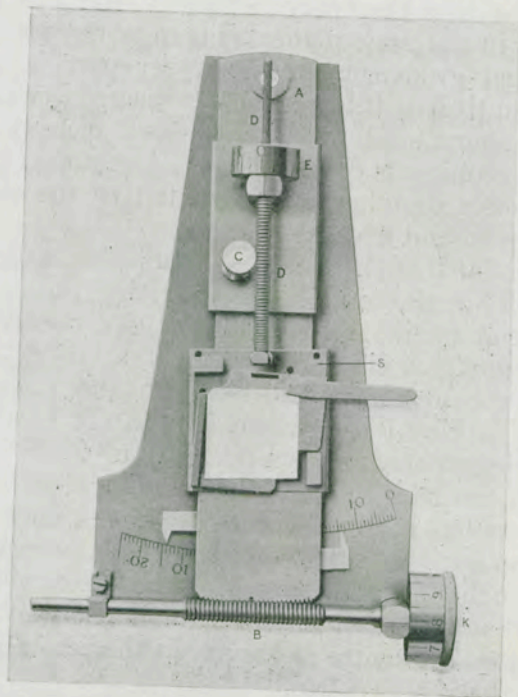


FIG. 9.—Stage for holding and moving the record under a microscope.

but by a micrometer placed at the focus of the eye piece, thus the extension of the aneroid box, and hence indirectly the air pressure is read by the measured movement of the stage and temperature-differences by the micrometer-scale at the focus.

Working up the Traces.

The procedure consists in transferring the calibration marks and the marks made on the ascent and descent accurately to squared paper. Fig. 5 is from a photograph of an actual trace magnified between four and five times. The left-hand parallel lines with the small blurs at unequal distances on them are the three calibration marks; the left-hand line showing the ordinary temperature, the right-hand line a temperature of about 220a. The coarse line on the right hand is made by the fixed pen; it is not used in the working up in the general way, but the fact that it is not duplicated proves conclusively that the plate had not shifted between the calibration and the ascent. The sloping line beginning on the left hand at the bottom and running

up to nearly the top on the right is the actual trace. Seen through the microscope, this line is a duplicated one, plainly crossing here and there, but the two parts never differ by more than 2° C. Both the ascent and the descent provide a trace; hence the duplication. This trace was obtained after sunset with the meteorograph 13 metres from the balloon; experience shows that if the ascent had been made in full sunshine, the up and down traces would have differed more, since the instrument would have ascended in air heated by contact with the hot balloon. Assume that the calibration marks were made at 280a, 250a and 220a. All three marks are arcs of a circle. Mount the plate on the carrier and turn the carrier about its pivot until it is found that on moving the stage by turning the micrometer screw the temperature lines retain exactly their relative position in the field of view. Adjust the micrometer at the focus by turning the eye-piece round so that the micrometer division marks are parallel to the calibration marks. Now shift the stage by the transverse motion E till the 280a calibration mark agrees with the micrometer division numbered 80. The 220a mark will now be somewhere in the neighbourhood of the 20 division of the micrometer scale. The magnification of the microscope can be altered to make the two fit by pulling out or pushing in the tube that carries the eye-piece. Make them so fit, and then the temperature indicated by any part of the trace can be read off at once in absolute measure by means of the micrometer scale.

It may happen that the scale is not quite linear, and that when the temperatures of 280a and 220a fit on the micrometer, the temperature of 250a may not quite fall on the 50 line as it should do. In this case make the 280a and 250a lines fit for temperatures above 250a and the 250a and 220a lines for temperatures below 250a.

The trace is now transferred to squared paper (Fig. 11). Take vertical lines on the paper to denote absolute temperatures. Plot on the temperature lines of 280a, 250a and 220a the position of the small blurs on those lines which indicate the pressures of p_0, p_1, p_2 , etc., using the micrometer screw and scale of the microscope stage for the purpose. The divided screw-head gives the movement of the writing point to .01 mm., and the usual range is about 1.5 mm. per 100 mb. pressure. It will be plainly seen from Fig. 5 that the scale is not linear. This is because the movement of the aneroid box is naturally greater when the faces of the box are flat than when they are extended or compressed. Now join the dots showing the same pressures and we have

a series of sloping lines on the chart marking the pressures of p_0, p_1 , etc. The slope of the lines is more apparent on the actual plate (see Fig. 5) than on the chart reproduced in Fig. 10, because in the chart the horizontal distances are

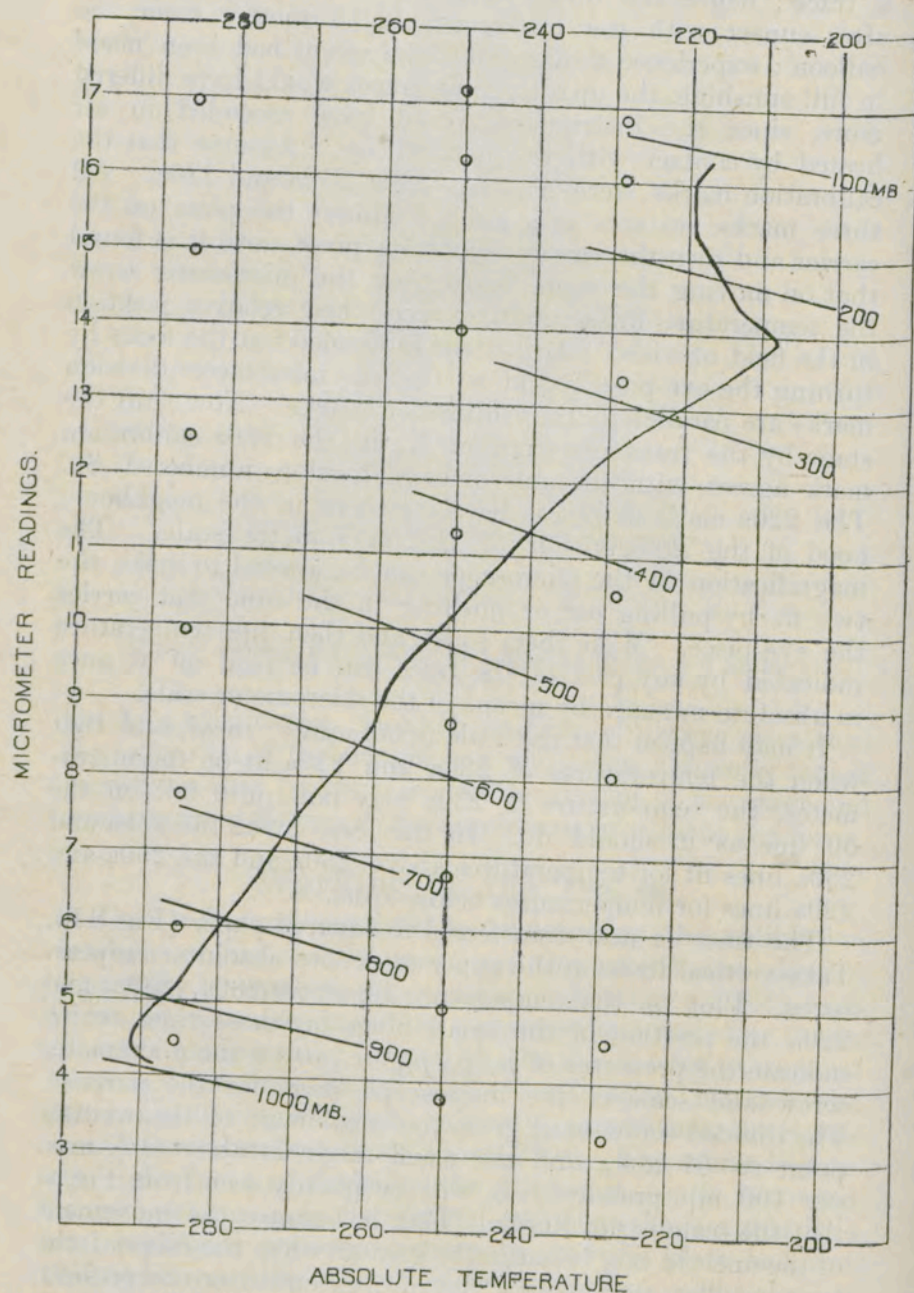


FIG. 10.—Interpretation of Balloon Soundings.
Stage 1. Pressures found from $\frac{1}{4}$ Micrometer Readings.

multiplied more than the vertical. This large slope is due to the temperature correction required when air is left in the aneroid box. Exhausted boxes have been used, but those containing air give the most accurate result, because that result is less dependent on the fatigue of the metal and also the scale is larger. It is not possible to avoid the temperature correction entirely, and that being so, its magnitude within wide limits is immaterial, and hence it is better to use a box containing air. It also makes the trace more nearly perpendicular to the lines of equal pressure, which is an advantage.

Next plot the actual trace on the chart, using the micrometer-screw and scale to read the aneroid-box-extension and the micrometer divisions at the focus to read the temperatures.

The remainder of the process is tedious, but presents no special difficulty. Interpolation between the lines of equal pressure gives the pressure corresponding to each temperature. When the pressure-scale departs widely from linearity—and this is especially the case in the trace shown—some trouble is required for correct interpolation. The following plan is adopted:—The actual extensions of a large number of aneroid-boxes over the pressure-scale have been plotted, and a mean curve drawn from them. From this curve a scale has been prepared marked to equal steps of pressure. On this scale each division is of rather different length from that of its immediate neighbours, the divisions being widest at pressures of about 400mb. and decreasing on both sides. This scale is drawn on a uniform strip of thin sheet rubber, about 8 inches long, held in a frame so that it may be stretched. Since it is stretched uniformly the proportionality of the divisions is maintained, the scale only being altered. By suitably stretching and selecting the right part of the scale, it is always possible to make a fairly good fit over any consecutive 200mb. of pressure and thus perform the interpolation to an accuracy of half a millibar at least. Where the slope of the trace is uniform it suffices to find the pressure for each even 5a of temperature, but all inversions and irregularities should be specially noted. In general the two traces, viz., that made during the ascent, and that made during the descent, do not differ by more than 2a; this is almost invariably the case in an ascent made at night. They may be treated as a single trace and the mean given. Sometimes in the daytime the difference may reach 4a or even more, and in such cases the two values should be given.

Computation of the Heights from the Pressures.

If p_0 and p be the pressure at two points in the atmosphere and h the vertical distance in kilometres between them, Laplace's formula gives $h = .06740 T (\log_{10} p_0 - \log_{10} p)$, where T is the absolute temperature of the intervening air. The humidity may be neglected without appreciable error.*

Obviously, since $\log p_0 - \log p = \log \frac{p_0}{p}$, the units in which p is expressed are immaterial.

The formula holds for any values of p provided T is constant, and it will hold with sufficient accuracy for two or three kilometres, but not further, if the mean value of T with regard to height be taken.

Suppose T_0, T_1, T_2, \dots to be a series of temperatures at equal height-intervals and T_m their arithmetical mean then

$$T'_m = (T_0 + T_1 + T_2 + \dots + T_{m-1})/m$$

but the correct value to use in the formula is given by the harmonic mean T ,

$$\frac{m}{T} = \frac{1}{T_0} + \frac{1}{T_1} + \frac{1}{T_2} + \dots + \frac{1}{T_{m-1}}$$

If the T_0 , etc., cover a large range of temperature the values of T and T'_m may differ by 2 or 3 degrees.

There are various ways in which the heights may be calculated, but on the whole a graphical method is the shortest and most convenient; this depends on the use of semi-logarithmic paper, and if the scale is large enough accuracy to the nearest millibar is easily obtained.

The paper, which is printed at the Meteorological Office, has equally-spaced horizontal lines, but the vertical lines correspond with the logs of 1.0, 1.1, 1.2, etc., up to 10. On this paper the graph of $h = k T \log p_0/p$ is a straight line, having a definite slope for each value of T . It is only necessary, therefore, to find the slope and devise some means of drawing a line at the correct inclination.

The vertical ruling on the paper stretches from 100 to 1000, and $\log 1000 - \log 100 = 1$, hence for a temperature of 273 the slope is such that it runs from 0k on the bottom left-hand corner (see Fig. 11) to 18.4k on the right-hand edge. This fixes the slope for the freezing point, and obviously the tangent of the slope is proportional to the absolute temperature so that the slope for any other temperature is readily obtained.

To draw the graph two plans are available. The simplest requires a piece of transparent celluloid with one straight edge. If $\theta_0, \theta_1, \theta_2$, etc., are the angles of slope corresponding

* The corrections for the variation in gravity with latitude and height are also neglected, cf. pp. 34, 35.

to T_0, T_1 , etc., and lines are engraved on the celluloid making angles of $\theta_0, \theta_1, \theta_2$ with the straight edge, these lines

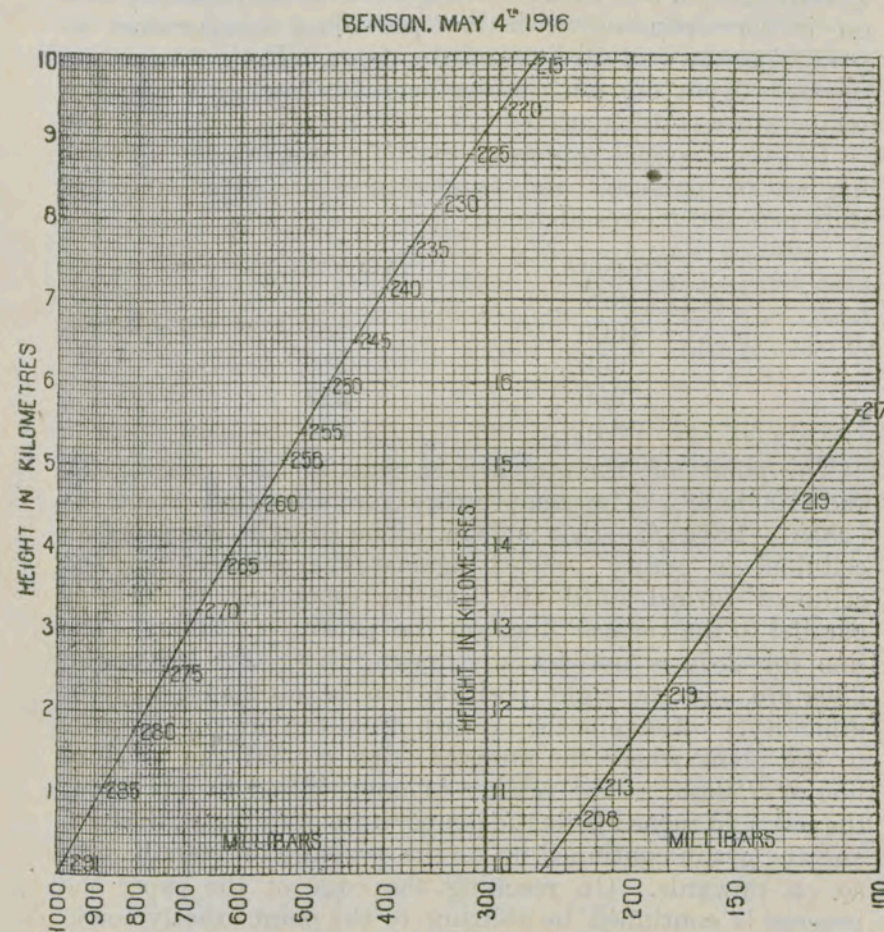


FIG. 11.—Interpretation of Balloon Soundings.
Stage 2. Heights found from Temperatures and Pressures.

placed parallel to the vertical lines on the paper enable one to draw the graph. The simplest way to fix the lines on the celluloid is to draw by direct calculation lines on the paper corresponding to the temperatures of 300a, 290a, 280a, and so on,* then to place the straight edge in coincidence with these lines and engrave them on the celluloid by tracing over a vertical line, marking each one with its proper temperature.

Having obtained a celluloid scale thus marked proceed as follows:—Place the semi-logarithmic paper on a drawing board and insert a pin or small pricker at the point corres-

* cf. M.O. Form 4110a referred to on p. 25.

ponding to the height above sea-level of the station and the barometric pressure. Place the straight edge of the celluloid against the pin and turn the celluloid until the engraved line on it corresponding with the prevailing temperature is parallel to the vertical lines on the paper. Then draw a line upwards from the pin until a height is reached where the temperature is different.

To take a special case as an example, suppose the following set of pressures and temperatures are given and the station is 200 metres above sea level :—

Pressure.	Temperature.
mb.	a.
1000	288
950	286
900	289
846	285

The point of the pricker is put at the point .2k and 1000mb. The temperature for the first stage is $(288a + 286a)/2$ —that is 287a. The straight edge of the celluloid is gently pressed against the pricker and the celluloid turned until the line corresponding to 287a is parallel to the vertical lines on the paper. (Although the line for 287a is not actually drawn, those for 290a and 280a are, and the right position can be estimated without difficulty.) A line is then drawn from the pricker upwards to the point where the straight edge crosses the pressure line of 950mb. The pricker is now shifted to this point, the celluloid turned to fit the temperature of $(286a + 289a)/2$, and the graph continued up to the pressure of 900mb., and so on upwards. On reaching the edge of the paper the process is continued by shifting to the point exactly corresponding on the opposite edge.

This plan is the simplest and will give with care and skill quite good results, but the following method is better.

Fix the paper on an ordinary drawing-board, with drawing-pins or otherwise, so that the vertical pressure-lines are exactly at right angles to the edge of the board. Then instead of celluloid use an adjustable T-square, and prepare a scale by which the adjustable square can be set for any temperature. Then use the square in the same way as the celluloid excepting that no pricker is required, the graph for any temperature-step being simply continued from the end of the preceding step.

Whatever plan may be adopted for making the temperature-scale for the celluloid or for the adjustable square the following considerations are of importance. The tangent

of the slope is proportional to the absolute temperature. If therefore a line be drawn on the semi-logarithmic paper starting from sea-level and at the right slope for a temperature of 280a say and if this line cuts any vertical line at a height h , then the heights at which sloping lines from the same bottom point cut the vertical line will be $270h/280$, $260h/280$ for temperatures of 270a, 260a, and so on.

The accuracy that can be obtained depends entirely on the scale of the paper; with the logarithmic scale 10 inches long, the size of the ordinary slide rule, an accuracy to rather under 1mb. is obtainable, but only with great care. Recently a logarithmic scale 1 metre long has been used, and this gives an accuracy within half a millibar over the whole scale.

The great convenience of the method is that the graph being once drawn, the height corresponding to any pressure or the pressure corresponding to any height is read off without the slightest trouble, and the temperature is given at once by interpolation.

The following figures are of interest, and are useful for making the celluloid scale or graduating the T-square:—

Step of Height.	Mean temperature of the step.	Ratio of Pressures at top and bottom.
k.	a	
10	311.0	3 to 1
5	246.5	2 to 1
4	267.5	10 to 6
3	252.8	3 to 2
2	237.5	4 to 3
1	255.9	8 to 7

Semi-logarithmic paper ruled for a square of 25 cm. (10 inches) is issued by the Meteorological Office as Form 4110 (526) and can be supplied on application.

For the special purpose of the graphical computation of pressure-differences from height-differences and *vice versa* for mean temperatures of the air-column which come within the range of meteorological practice, i.e. from 200a to 300a, a special issue is made of the semi-logarithmic paper in which the graduation is extended beyond the "1000 line" to 1050 to provide for surface pressures above 1000mb. and the slopes for 200a and 300a are set out in such a way that the slope for any temperature can be transferred directly to any part of the sheet by means of a pair of set squares or a parallel ruler. The special issue is numbered as Form 4110a.

Example of the Method.

As an example of the method of working up the data, the following figures from an ascent made on May 4th, 1916, at Benson, are given.

The readings of the micrometer on the microscope-stage corresponding to the blurs on the calibration-lines are given in the first Table :—

Pressure.	Temperature.		
	285·5a	250·0a	228·5a
mb.	Micrometer Readings.		
954	447?	379	328
820	598	497	452
687	776	672	610
553	992	875	808
420	1250	1129	1052
287	1497	1398	1337
153	1694	1626	1604
87	—	1718	1683

The readings from the trace follow :—

Micrometer Reading.	Temperature.	Micrometer Reading.	Temperature.
	a		a
433	291	1074	245
455	291	1138	240
495	288	1188	235
514	285	1237	230
599	280	1272	225
665	275	1305	220
746	270	1336	215
809	265	1371	210
877	260	1393	208
929	258	1436	213
967	255	1532	219
1020	250	1608	219
		1630	217

These two sets of readings are plotted on squared paper (see Fig. 10). The small circles show the position of the blurs. Interpolating between the circles on the vertical lines a new series of points are obtained giving the microscope stage readings for the temperatures 288·5, 250, and 228·5a at 100mb., 200mb., etc. By drawing straight lines through these points the pressure-lines for 100mb., etc., are obtained. Thence by interpolating again the following table, giving the temperatures corresponding with the pressures, is found.

mb	a	mb	a	mb	a	mb	a
980	291	571	260	400	240	233	208
880	285	533	258	372	235	220	213
804	280	509	255	344	230	184	219
740	275	473	250	315	225	127	219
676	270	437	245	289	220	107	217
620	265			265	215		

The next process is to obtain the heights corresponding to the pressures, taking into account the special distribution of temperature prevailing at the time.

This is done on the semi-logarithmic paper (Fig. 11) in the manner already described. On this occasion the barometric pressure reduced to M.S.L. was 998mb. and the surface temperature was 288a. The mean temperature from 998 to 880mb may be put at 289a. The line drawn from the point $h = 0, p = 998$ at the temperature 289a reaches the pressure 880mb. at a height of 1·06k. The temperature for the next step is 282·5a and the pressure 804mb. is reached at 1·80k, and so on. The temperatures 285a, 280a, etc., are then written in against the heights 1·06, 1·80, etc., and the temperatures at other points are estimated by interpolation.

Values are only given to the nearest whole degree, and to avoid a systematic error in mean values the case of ·5 is met by writing the nearest odd whole number.

Proceeding thus the final values as set out below are obtained.

Temperature.	Pressure.	Height.
a	mb	k
217	106	15.7
218	117	15.0
219	137	14.0
219	160	13.0
219	188	12.0
213	220	11.0
208	233	10.6
216	258	10.0
223	303	9.0
231	352	8.0
241	407	7.0
250	468	6.0
258	536	5.0
264	611	4.0
272	693	3.0
279	785	2.0
286	886	1.0
291	980	.16
288	992	.06

In this particular instance the positions of the blurs are consistent with each other excepting that the micrometer reading corresponding to 153mb at 228.5a seems to be too high. Extra cold was encountered and the calibration did not occur at as low a temperature as usual so that considerable extrapolation is required, but the instrument had been used before and was known to have a linear scale.

The relation between temperature and height obtained from these figures is shown in Figure 12.

The inversion of 11a between 10.6 to 12k is unusually large.

This table may be compared with a table on p. II 41 which has been compiled from the same original data in illustration of a different method of computation. The main difference between the two versions of the interpretation of the same results is that in the graphic method of computation no allowance is made for the variation of gravity with height, but that variation is taken into account in the arithmetical process described later on.

BENSON. MAY 4th 1916.

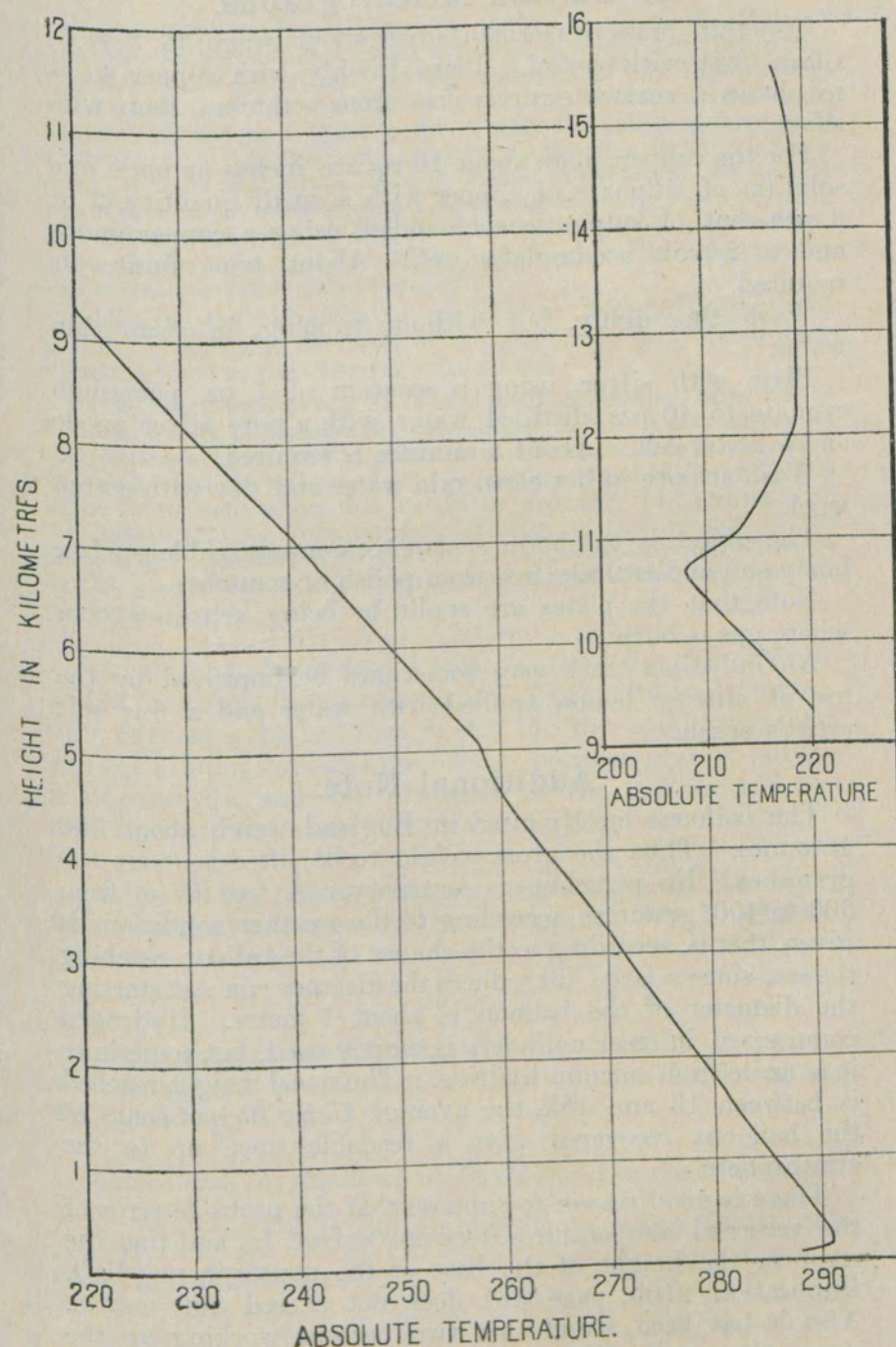


FIG. 12.—Interpretation of Balloon Soundings.
Stage 3. Temperature as a function of Height.

Instructions for making the Plated Metal for Balloon Meteorographs.

Use thin brass or German silver about gauge 35, S.W.G. Clean first with petrol. Plate thickly with copper so as to obtain a surface entirely free from scratches, then with silver.

For the copper, plate about 16 square inches at once in a solution of sulphate of copper with a small quantity, 2 or 3 per cent. of sulphuric acid added, using a copper anode and a 2 volt accumulator cell. About ten minutes is required.

Wash thoroughly, but without rubbing, in clean rain water.

Plate with silver, using a solution of 1 oz. potassium cyanide to 40 ozs. distilled water, with a pure silver anode and a 2 volt cell. About 5 minutes is required.

Wash at once in hot clean rain water and dry with cotton wool.

The object is to obtain a smooth, non-corrodible surface fairly soft and entirely free from polish or scratches.

Note that the plates are spoilt by being kept in a room where gas is burnt.

An indistinct trace may sometimes be improved by the use of salts of lemon applied with water and a fine soft artist's-brush.

Additional Note.

The balloons mostly used in England weigh about 300 grammes. Thus the gross weight to be lifted is about 400 grammes. No parachute is necessary. A free lift of from 300 to 400 grammes according to the weather conditions is given, that is, according to the chance of the balloon reaching the sea, since a large lift reduces the distance run; at starting the diameter of the balloon is about 1 metre. Hydrogen compressed in steel cylinders is mostly used, but sometimes it is made from calcium hydride. The usual height reached is between 13 and 18k, the average 15·5: 95 per cent. of the balloons recovered give a readable trace up to the stratosphere.

There is good reason to suppose that the probable error of the recorded temperatures does not exceed 1a, and that the error in the height of the base of the stratosphere (M.O. Publication 210b, page 36) does not exceed 200 metres. Also it has been found that two persons working up the same trace quite independently will obtain results that seldom differ by more than 1a.

THE BAROMETRIC FORMULA.

NOTE.—Formulae expressing for various purposes the lapse of pressure with height have already appeared in a considerable number of places in this Handbook. In the summary of the International tables which is given in the Introduction, the formula quoted from Laplace's *Mécanique Céleste* is given with the values of the special constants employed. In Section I, § 2 p. 19 the same formula is again quoted and the application given by means of tables for the special case of determination of depths below a datum level corresponding with given difference of height at certain specified temperatures, tables which can be used for making reductions for barometer readings at small elevations above sea-level.

Tables for the reduction to sea-level at selected temperatures at three pressures at the level of the station up to 500 metres are already printed in the Observer's Handbook.

In the same section p. I. 25 is explained the method of using the formula which is found most convenient in the great variety of circumstances represented by the values used in the *Réseau Mondial*, in which auxiliary tables and a slide-rule are found to be preferable to the voluminous tables that would be necessary for dealing with the figures in the ordinary way. Finally in this Section II the relation of differences of pressure to the corresponding differences of height is dealt with in the more general aspect of the study of the upper air.

The formula, known as Laplace's, used for the determination of height from observations of pressure and temperature, expresses the fact that in still air the equilibrium of a vertical column indicates the balance between gravity pulling it downwards, and the pressures at the bottom and top pushing it upwards and downwards respectively.

Let p be the pressure at any height h , Δ the density of the air, g the local value of the acceleration due to gravity.

In this notation

$$dp = -g \Delta dh \dots\dots (1).$$

In this preliminary investigation the variation of gravity with height and the effect of humidity on the density of the air will be neglected.

Accordingly by the laws of Boyle and Charles

$$\Delta = \frac{1}{R} \frac{p}{T} \text{ where } T \text{ is the absolute temperature (centigrade scale) and } R \text{ a constant.}$$

$$\text{Hence } \frac{dp}{p} = -\frac{g}{RT} dh \dots (2).$$

$$\text{Accordingly } d(\log_e p) = -\frac{g}{RT} dh \dots (3).$$

In terms of logarithms with base 10 this formula becomes

$$d(\log p) = -\frac{g}{R} \log_e \frac{dh}{T} \dots (4).$$

If we use d, t, s (decametre, tonne, second), units and suppose that the latitude is 45° we have

$$g = .980617, R^{-1} = .348394, * \log_e = .43429$$

so that when the decametre is the unit of height

$$d(\log p) = -.14837 dh/T \dots (5)$$

when the metre is the unit

$$d(\log p) = -.014837 \cdot dh/T \dots (6)$$

and when the kilometre is the unit

$$d(\log p) = -14.837 dh/T \dots (7)$$

In the simple case of an isothermal atmosphere T is constant and this equation can be integrated. The integral is

$$\log p_0 - \log p_1 = 14.837 h/T \dots (8)$$

where h is the interval in kilometres between two layers where the pressures are p_0 and p_1 .

In general T is dependent on the height and the integral $\int \frac{dh}{T}$ has to be evaluated.

This process is equivalent to taking the harmonic mean of the temperature and the word "mean" which is used in the International Tables and quoted in this Handbook (Introduction, p. 14) should be interpreted in that sense. For small variations of temperature up to say 10 per cent., and therefore for ranges in height of two or three kilometres, the difference between the harmonic and arithmetic means may be neglected but for greater ranges the difference is important.

* For R^{-1} the value .34831 would be derived from the Table on p. 7 of the introduction. This figure applies to air without carbon dioxide. For ordinary air $R^{-1} = .348394$ (cf. International Tables, p. B35).

For example with a uniform lapse-rate of 6 degrees absolute per kilometre from the temperature 300a at the ground to 240a at 10k., the harmonic mean temperature can be shown to be $60/\log_e(300/240)$ or 268.9a whereas the arithmetic mean is 270a.

The form of the equation (8), which may be written

$$\log(p_0/p_1) = 14.837 h/T$$

indicates that the ratio of the pressures at two points one over the other is independent of the pressures themselves. The ratio p_1/p_0 is given in Table A (p. 44) for $h = 1$ and for integral values of T from 190 to 310. The logarithm, $\log(p_0/p_1)$ is given in Table B. When natural logarithms are used (8) takes the forms:

$$\log_e(p_0/p) = 34.16h/T = h/(.0293T) \dots (8a)$$

An Approximate formula without Logarithms.

$$\text{By writing } p_0 = \frac{p_0 + p_1}{2} + \frac{p_0 - p_1}{2}$$

and using the logarithmic series, we can derive from (8) the approximate formula

$$h = 29.3 \frac{p_0 - p_1}{p_0 + p_1} (T_0 + T_1) \dots (9)$$

which gives the height in metres.

A similar formula due to Babinet (Comptes Rendus, 1850, p. 570) and frequently quoted is (in the same notation)

$$h = 32 \frac{p_0 - p_1}{p_0 + p_1} (T_0 + T_1 - 46) \dots (10)$$

The factor $T_0 + T_1 - 46$ is quoted by Babinet from Laplace (Mécanique Céleste, Livre X., p. 293). Laplace mentions that the proportion of water-vapour generally increases with the temperature and as damp air is lighter than dry air he makes a rough allowance for the water-vapour by taking .004 as the coefficient of expansion of air at 0° C. instead of .00366 the coefficient for dry air. Comparison with formula (11) below shows that when $p = 1000$ mb. this is equivalent to taking the relative humidity as 0 per cent. at 273a, 51 per cent. at 278a, 73 per cent. at 283a, 77 per cent. at 293a, and 63 per cent. at 303a.

The Allowance for Humidity.

Let e be the partial pressure of the water vapour, p the barometric pressure, then $p - e$ is the pressure exerted by the permanent gases.

The density of water vapour is 0.622 of the density of dry air under the same pressure, and at the same temperature. Writing Δ_1 for the density of the permanent gases and Δ_2 for the density of the water-vapour, we have

$$\Delta_1 = \frac{1}{R} \frac{p - e}{T}, \Delta_2 = \frac{0.622}{R} \frac{e}{T}$$

Hence the density of the moist air is

$$\Delta = \Delta_1 + \Delta_2 = \frac{1}{RT} (p - 0.378 e)$$

A good approximation is

$$\Delta = \frac{p - \frac{3}{8}e}{RT}$$

If this expression be substituted in place of $p/(RT)$ in the previous discussion, it is found that, if the height be expressed in metres,

$$d(\log p) = -0.014837 \left(1 - \frac{3}{8} \frac{e}{p}\right) \frac{dh}{T} \dots (11)$$

It will be noticed that the pressure lapse-rate is smaller for damp air than for dry.

The factor $1 - \frac{3}{8} \frac{e}{p}$ is very near to unity when the air is cold but it may be as low as .99 in warm damp air. [Cf. the Tables, p. I 33, ante.]

It is difficult to estimate the mean value of e as the humidities of neighbouring strata may differ considerably.

In Bjerknes' systematic treatment of upper air observations the name "virtual temperature" is given to the quotient

$T / \left(1 - \frac{3}{8} \frac{e}{p}\right)$. Evidently the lapse-rate for $\log p$ can be found at once from the virtual temperature.

An example of the application of the correction for humidity in the problem of the determination of the depth of a mine will be found (see I p. 20) above. The Dines balloon-apparatus does not generally include a hygrograph and no correction for humidity has been applied in the examples based on observations made with this instrument.

The Allowance for the variation of Gravity.

The value of g , which has been regarded as constant, really depends on the latitude and the height. It is given with sufficient accuracy by the formula

$$g = g_{45} (1 - k \cos 2\lambda) E^2 / (E + h)^2$$

where λ is the latitude, E the mean radius of the earth and k a constant which is equal to .00259. If E and h are expressed in metres

$$\pi E = 2 \times 10^7 \text{ and } E^2 / (E + h)^2 \doteq 1 - 2h/E = 1 - \pi h 10^{-7}$$

It follows that the formula for decrease of pressure is

$$d(\log p) = -0.014837 (1 - 0.00259 \cos 2\lambda) \left(1 - \frac{\pi h}{10^7}\right) \frac{dh}{T} \dots (12)$$

where h is measured in metres, T in degrees (centigrade) from the Absolute Zero.

The equation may be written

$$d(\log p) = -f(\lambda, h) dh/T$$

$f(\lambda, h)$ is given for various values of λ and h in Tables C_1 , C_2 and C_3 .

In dealing with the two practical problems which present themselves the departure of $f(\lambda, h)$ from the constant value .014837 is seldom of much importance.

The difference $f(\lambda, h) - f(\lambda, 0)$ amounts to .000047 for $h = 10k$. The proportion this difference bears to $f(\lambda, 0)$ is about 1/300, so that neglecting the difference may lead to an error of 70 metres in the estimation of a height of 20k. This is a systematic error comparable with an error of 1a in the estimation of all the temperatures.

The neglect of the latitude correction leads to errors of the same order of magnitude.

It should always be stated whether the corrections for the variations in gravity with latitude and height have been introduced in working up the results of balloon observations. The whole difficulty of dealing with these corrections is turned by Bjerknes, who works in terms of geopotential instead of height.

If U be written for geopotential

$$dU = g dh.$$

The unit of geopotential, the leo-metre is the increase in potential of a particle raised through one metre in a place where the intensity of gravity is 1 leo, *i.e.*, 1 decametre per second per second. The barometric equation

$$\text{can be written.} \quad d(\log p) = -\frac{g}{R} \log e \frac{dh}{T}$$

$$d(\log p) = -\frac{\log e}{R} \frac{dU}{T} \quad \dots \quad 13.$$

As g does not appear in this formula there is no occasion to allow for its variation in the course of the integration.

When pressure and temperature observations are given and the geopotential is required as well as the height, it would appear to be desirable to follow Bjerknes' procedure and find the geopotential first and then the height.

If the procedure were reversed the variation in gravity would have to be allowed for twice over. If the variation were ignored throughout, then the geopotentials would be more accurate than the heights.

It is necessary to insert a special caution against neglecting the variation in gravity in determining the heights and then using Bjerknes' Table for finding the corresponding geopotentials.

For convenience of reference the complete pressure—height—temperature formula may be written down here. It is—

$$\log p_1 - \log p_2 = \cdot 014837 \left[1 - \frac{3}{8} \left(\frac{e}{p} \right)_m \right]$$

$$(1 - \cdot 00259 \cos 2\lambda) \left(1 - \frac{\pi}{2} \frac{h_1 + h_2}{10^7} \right) \frac{h_2 - h_1}{T_m}$$

where p_1 and p_2 are pressures at heights h_1 and h_2 metres $(e/p)_m$ is the mean value of e/p , the ratio of vapour pressure to barometric pressure, λ is the latitude and T_m is the harmonic mean of the temperatures taken at equal intervals from h_1 to h_2 .

NOTE.—In the International Tables it is assumed that the pressures are given in terms of the height of a mercurial barometer and a term, ϵ , which is introduced to allow for the variation of gravity with height appears in the formula on page 34 and affects Table IX A, page 228.

It is assumed in this Handbook that the pressures are given at once in millibars. If Table IX A were used for such pressures the heights would be in excess by 1·5 parts in 1000.

It may be noticed in passing that the analysis adopted in the International Tables is not consistent. The intensity of gravity is assumed to fall off according to the inverse square law when it acts on air, but according to Poisson's Law when it acts on mercury (pp. B 32, 33).

Calculation of Pressures from Temperatures at given Heights.

The formula proved above for the relation between increments of height and pressure is

$$d(\log p) = -\frac{14\cdot 837}{T} (1 - \cdot 00259 \cos 2\lambda) \left(1 - \frac{\pi z}{10^7} \right) \frac{dz}{1000}.$$

Where p is pressure in any units,

λ is latitude,

z is height in metres,

T is absolute temperature.

If we suppose that the height is given in kilometres the formula becomes

$$d(\log p) = -\frac{14\cdot 837}{T} (1 - \cdot 00259 \cos 2\lambda) \left(1 - \frac{\pi z}{10^4} \right) dz.$$

The expression $\frac{14\cdot 837}{T}$ is tabulated in Table B₁ for values of T differing by $\frac{1}{2}$ ° from 190 to 310. $\cdot 00259 \cos 2\lambda$ is given in Table B₂, and $\frac{\pi z}{10^4}$ in Table B₃.

In the following example the increments dz are exact kilometres and the mean temperatures for successive kilometres are known. $14\cdot 837/T$ is taken from Table B₁, and the allowances for the other factors in the formula are made as corrections.

Thus the correct value of $d(\log p)$ for each kilometre is found. By successive subtractions $\log p$ is found for each height and hence p is determined.

Example of determination of pressures from temperatures at given heights.

PARIS—YEARLY MEANS.

The temperatures from 1k. upwards are those given by Gold (G.M. Vol. I., p. 85), those above 8k. being obtained from mean gradients. The surface temperature and pressure are normals. The latitude of Paris is 49° so that the Gravity Correction given by Table B₂ is $+0.4$: the corrections given in Table B₂ are all negative and are combined with this $+0.4$. Thus at 15k. the proportional correction is $-4.6 + 0.4$ or -4.2 per thousand.

Height k.	T		$\frac{14.837}{T}$	Gravity Correction		$d(\log p)$	$\log p.$	$p.$
	a.	a.		per 1000.	net.			mb.
15	219.2	219.1	.06772	-4.2	-28	.06744	2.08934	122.8
14	219.0	219.0	.06775	3.8	26	.06749	2.15678	143.5
13	219.0	219.1	.06772	3.5	24	.06748	2.22427	167.6
12	219.2	219.6	.06756	3.2	22	.06734	2.29175	195.8
11	220.0	222.1	.06680	2.9	19	.06661	2.35909	228.6
10	224.2	227.0	.06536	2.6	17	.06519	2.42570	266.5
9	229.9	233.4	.06357	2.3	15	.06342	2.49089	309.7
8	236.9	240.6	.06166	2.0	12	.06154	2.55431	358.4
7	244.3	247.8	.05988	1.6	10	.05978	2.61585	412.9
6	251.4	254.7	.05825	1.3	8	.05817	2.67563	473.8
5	258.1	261.2	.05681	1.0	6	.05675	2.73380	541.8
4	264.3	267.0	.05557	0.7	4	.05553	2.79055	617.4
3	269.8	272.1	.05453	0.4	2	.05451	2.84608	701.6
2	274.5	276.5	.05366	-0.1	-1	.05365	2.90059	795.4
1	278.5	280.7	.05286	+0.2	+1	.05287	2.95424	900.0
Ground	283.0						3.00711	1016.5
				Sum		.91776		

The example shows the small effect of the allowance for variation in gravity. If the variation had been ignored the pressure at 15k. would have been found to be 122.3mb. instead of 122.9mb.

In most cases it would not be profitable to take the temperatures as accurate to .1a.: if only 3 significant figures are used in the temperatures then 4-figure logarithms will suffice.

In the selected example the latitude correction is almost negligible, but this is not the case in very high or low latitudes where the two corrections for variation in gravity are of equal importance.

It may be noticed that if the pressures at intervening heights are not required the work may be simplified. The sum of the numbers in the fourth column is .91970. The gravity correction for 7.5k. is 2.0 per thousand and .91970

$$\left(1 - \frac{2}{1000}\right) = .91786.$$

Hence $\log p_{15} = \log p_o - .91786 = 2.08925$ and $p_{15} = 122.8$ so that the shortened procedure leads to no appreciable error.

Formulae for Checking Pressures at certain Heights.

The pressure p_h is given by the formula

$$\log p_o - \log p_h = 14.837 \frac{h}{T}$$

where p_o is the pressure at sea-level, h is the height in kilometres and T is the mean temperature from sea-level to the height h . It has been pointed out already that the formula is only approximate as the variation of gravity with height and latitude, the correction for humidity and the distinction between the arithmetic and harmonic mean temperatures are ignored. When one ascent is compared with another the differences in p_o , p_h , and T are generally small. They are connected by the equation

$$\frac{dp_o}{p_o} - \frac{dp_h}{p_h} = - \frac{14.837}{\log_{10} e} \frac{dT}{T^2}$$

which may be rewritten in the form

$$dp_h = \frac{p_h}{p_o} dp_o + 34.16 \frac{h}{T^2} dp_h$$

For convenience of reference we write a and β for the coefficients in this equation which becomes

$$dp_h = a dp_o + \beta dT.$$

As the standard value of p_0 we take 1000mb and for T we take the normal for S.E. England. If the corresponding standard values of p_h are computed from $p_0 = 1000\text{mb}$ and the normal temperatures at all heights the following table is obtained.

h	p_h	T	α	β
k		a		
15	119	243	.12	1.04
12	189	248	.19	1.26
9	300	257	.30	1.40

As an example we take the ascent at Manchester, February 8th, 1913. (Geophysical Journal, 1913, p. 13.)

The arithmetic mean temperature for the first 9k. is found to be 252a. The pressure at sea-level is given as 1014mb, the pressure at 9k. as 296mb. Comparison with the normal values 257a, 1000mb, 300mb, gives $dT = -5$, $dp_0 = 14$ $dp_h = -4$.

Substituting in the formula

$$dp_h = \alpha dp_0 + \beta dT$$

we get

$$-4 = .3 \times 14 - 1.4 \times 5.$$

The agreement is by no means exact. By recomputing the pressure by the method of p. 37 it is found that a pressure at 9k. of 298mb, instead of 296mb, would be in better agreement with the other published data.

Calculation of Heights from Pressures and Temperatures.

In the following pages arithmetical methods are applied to the example worked out graphically above (pp. 18-29).

It will be noticed that the table giving pressures and corresponding temperatures, derived by Mr. Dines's graphic process, is assumed. As the order of accuracy of the final results cannot be higher than that of the observations, the margin of error is so large that the wisdom of applying the correction for the variation in gravity with height may be questioned. As, however, the neglect of this correction would lead to a systematic error, the height of the balloon being overestimated on the average, and as, moreover, the correction is easy to introduce, it has been adopted in the example.

REGISTERING BALLOON ASCENT, BENSON, MAY 4TH, 1916.

Calculation of Heights from Pressures and Corresponding Temperatures.

T	p	$\log p$	$d(\log p)$	T	$f(\lambda, h)$	$\frac{Td(\log p)}{f(\lambda, h)}$	h
a	mb			a			k.
217	107	0294		218	14.78	1.098	15.655
			.0744	219		2.388	14.557
219	127	1038		219			12.169
			.1610	216	14.79	1.134	11.035
219	184	2648				.356	10.679
			.0776	210.5		.799	9.880
213	220	3424				.553	9.327
			250	222.5	14.80	.563	8.764
208	233	3674				.588	8.176
			559	227.5		.533	7.643
215	265	4233		232.5		.506	7.137
			376	237.5		.629	6.508
220	289	4609			14.81	.574	5.934
			374	242.5		.542	5.392
225	315	4983				.346	5.046
			383	247.5		.522	4.524
230	344	5366			14.82	.634	3.890
			339	252.5		.678	3.212
235	372	5705				.721	2.491
			316	256.5		.675	1.816
240	400	6021			14.83	.747	1.069
			384	259		.906	.163
245	437	6405				.103	.060
			344	262.5			
250	473	6749					
			318	267.5			
255	509	7067					
			200	272.5			
258	533	7267					
			299	277.5			
260	571	7566					
			358	282.5			
265	620	7924					
			376	288			
270	676	8300					
			392	289.5	14.84		
275	740	8692					
			361	992			
280	804	9053					
			392				
285	880	9445					
			467				
291	980	9912					
			.0053				
288	992	9965					

NOTES.—The temperatures corresponding to particular pressures are known from the readings of the meteorograph (*cf.* p. 26).

The third column contains the logarithms of the pressures and the differences between successive logarithms are written down in the fourth column. The fifth column gives the mean temperatures for each pressure interval. The value of $f(\lambda, h)$ is taken from Table C₃. The quotient in the next column is obtained by a slide-rule. The height of the starting point .060k is written at the foot of the last column, and the numbers in the preceding column are added in succession.

When the calculation is complete the graphs showing temperature and pressure as functions of the height can be drawn. These graphs differ slightly from those shown in Figs. 11, 12, obtained without regard to the variation of gravity with height. The differences would be too small to show clearly on the reduced scale of the illustrations.

REGISTERING BALLOON ASCENT, BENSON, MAY 4TH, 1916.

Temperatures and Pressures found from the graphs (Figs. 11, 12) drawn from the data provided by the foregoing calculation.

Height.	Temperature.	Pressure.
k.	a	mb
15.66	217	107
15	218	118
14	219	138
13	219	161
12	218	189
11	212	221
10.68	208	233
10	214	260
9	223	304
8	231	353
7	241	408
6	249	469
5	258	536
4	264	611
3	271	694
2	279	786
1	285	887
.16	291	980
.06	288	992

REGISTERING BALLOON ASCENT, BENSON, MAY 4TH, 1916.

Check of pressures from heights and temperatures.

Height.	T	T'	$\frac{14.837}{T}$	Gravity Corr.		$d(\log p)$	$\log p$	p
Km.	a	a		per 1000	net.			mb.
15.66	217	217.5	.06821	-4.2	29	.06792 × .66 = .04483	.02793	107
15	218	218.5	6790	4.0	27	.06763	.07276	118
14	219	219	6775	3.6	24	.06751	.14039	138
13	219	218.5	6790	3.3	22	.06768	.20790	161
12	218	215	6901	3.0	21	.06880	.27558	189
11	212	210	7065	2.8	20	.07045 × .32 = .02254	.34438	221
10.68	208	211	7032	2.6	18	.07014 × .68 = .04770	.36692	233
10	214	218.5	6790	2.4	16	.06774	.41462	260
9	223	227	6536	2.1	14	.06522	.48236	304
8	231	236	6287	1.8	11	.06276	.54758	353
7	241	245	6056	1.4	8	.06048	.61034	408
6	249	253.5	5853	1.1	6	.05847	.67082	469
5	258	261	5685	0.8	5	.05680	.72929	536
4	264	267.5	5547	0.5	3	.05544	.78609	611
3	271	275	5395	-0.2	-1	.05394	.84153	694
2	279	282	5261	+0.1	+1	.05262	.89547	786
1	285	288	5152	0.3	+2	.05154 × .84 = .04329	.94809	887
.16	291	289.5	.05125	0.6	+3	.05128 × .1 = .00513	.99138	980
.06	288					Sum = .96858	.99651	992

NOTES.—The temperatures at given heights are taken from the graph (Fig. 12), *vide* p. 42. The mean temperature for each height interval is written down at once. The values of $14.837/T$ for each of these mean temperatures is taken from Table B₁. The gravity correction for each height is given as so many parts per 1000 in Table C₂. The actual correction is found by multiplication and written down, the unit being 10^{-5} . After the correction has been applied we have the value of $d(\log p)$ for a kilometre. If the height interval is not exactly 1k. we multiply by the appropriate factor. The pressure 992mb at .06k is known. The corresponding value of $\log p$ is written down. The value of $\log p$ at .16k is found by subtracting .00513 from .99651. As a check on the subtractions the sum of increments $d(\log p)$, viz., .96858 is found and compared with the difference between .99651 and .02793.

TABLES FOR THE
CALCULATION OF PRESSURES FROM TEMPERATURES AT GIVEN
HEIGHTS.

TABLE A, FOR CALCULATION BY MULTIPLICATION.

Table of values of p_1/p_0 , the ratio of the pressure at any height to the pressure at a point one k lower for values of T , the mean temperature of the column of air, from 190 to 310a. (Variation of gravity with latitude and height neglected.)

T	p_1/p_0	T	p_1/p_0	T	p_1/p_0	T	p_1/p_0
190	.8354	220	.8562	250	.8723	280	.8851
191	62	221	68	251	28	281	55
192	70	222	74	252	32	282	59
193	78	223	80	253	37	283	63
194	85	224	85	254	42	284	67
195	93	225	91	255	46	285	70
196	.8400	226	97	256	51	286	74
197	08	227	.8603	257	55	287	78
198	15	228	09	258	60	288	81
199	22	229	14	259	64	289	85
200	30	230	20	260	69	290	89
201	37	231	25	261	73	291	92
202	44	232	31	262	77	292	96
203	51	233	36	263	82	293	99
204	58	234	42	264	86	294	.8903
205	65	235	47	265	90	295	07
206	72	236	52	266	95	296	10
207	79	237	57	267	99	297	13
208	85	238	63	268	.8803	298	17
209	92	239	68	269	07	299	20
210	99	240	73	270	12	300	24
211	.8505	241	78	271	16	301	27
212	11	242	84	272	20	302	30
213	18	243	88	273	24	303	34
214	25	244	93	274	28	304	37
215	31	245	98	275	32	305	40
216	37	246	.8703	276	36	306	44
217	43	247	08	277	40	307	47
218	49	248	13	278	44	308	50
219	56	249	18	279	47	309	53
220	.8562	250	.8723	280	.8851	310	.8957

Note.—For the calculation of the pressure-differences corresponding with small differences in height the Tables in the Observer's Handbook may be used.

TABLE B₁. Vide pp. 37, 43.

T	$\frac{14.837}{T}$	T	$\frac{14.837}{T}$	T	$\frac{14.837}{T}$	T	$\frac{14.837}{T}$	T	$\frac{14.837}{T}$
190	.07800	210	.07065	230	.06451	250	.05935	270	.05495
191	.5	211	.5	231	.5	251	.5	271	.5
192	.5	212	.5	232	.5	252	.5	272	.5
193	.5	213	.5	233	.5	253	.5	273	.5
194	.5	214	.5	234	.5	254	.5	274	.5
195	.5	215	.5	235	.5	255	.5	275	.5
196	.5	216	.5	236	.5	256	.5	276	.5
197	.5	217	.5	237	.5	257	.5	277	.5
198	.5	218	.5	238	.5	258	.5	278	.5
199	.5	219	.5	239	.5	259	.5	279	.5
200	.5	220	.5	240	.5	260	.5	280	.5
201	.5	221	.5	241	.5	261	.5	281	.5
202	.5	222	.5	242	.5	262	.5	282	.5
203	.5	223	.5	243	.5	263	.5	283	.5
204	.5	224	.5	244	.5	264	.5	284	.5
205	.5	225	.5	245	.5	265	.5	285	.5
206	.5	226	.5	246	.5	266	.5	286	.5
207	.5	227	.5	247	.5	267	.5	287	.5
208	.5	228	.5	248	.5	268	.5	288	.5
209	.5	229	.5	249	.5	269	.5	289	.5
210	.5	230	.5	250	.5	270	.5	290	.5
211	.5	231	.5	251	.5	271	.5	291	.5
212	.5	232	.5	252	.5	272	.5	292	.5
213	.5	233	.5	253	.5	273	.5	293	.5
214	.5	234	.5	254	.5	274	.5	294	.5
215	.5	235	.5	255	.5	275	.5	295	.5
216	.5	236	.5	256	.5	276	.5	296	.5
217	.5	237	.5	257	.5	277	.5	297	.5
218	.5	238	.5	258	.5	278	.5	298	.5
219	.5	239	.5	259	.5	279	.5	299	.5
220	.5	240	.5	260	.5	280	.5	300	.5
221	.5	241	.5	261	.5	281	.5	301	.5
222	.5	242	.5	262	.5	282	.5	302	.5
223	.5	243	.5	263	.5	283	.5	303	.5
224	.5	244	.5	264	.5	284	.5	304	.5
225	.5	245	.5	265	.5	285	.5	305	.5
226	.5	246	.5	266	.5	286	.5	306	.5
227	.5	247	.5	267	.5	287	.5	307	.5
228	.5	248	.5	268	.5	288	.5	308	.5
229	.5	249	.5	269	.5	289	.5	309	.5
230	.5	250	.5	270	.5	290	.5	310	.5

Table B₂. Latitude correction to be added in latitudes above 45°, subtracted in latitudes below 45°.

Latitude—	0	10	20	30	40	50	60	70	80	90
Correction per 1000—	-2.6	-2.4	-1.9	-1.2	-0.4	+0.4	+1.2	+1.9	+2.4	+2.6

Table B₃. Height Correction to be subtracted.

Height k ...	19.5	18.5	17.5	16.5	15.5	14.5	13.5	12.5	11.5	10.5
Correction per 1000.	6.1	5.8	5.5	5.2	4.9	4.6	4.2	3.9	3.6	3.3
Height k ...	9.5	8.5	7.5	6.5	5.5	4.5	3.5	2.5	1.5	0.5
Correction per 1000.	3.0	2.7	2.4	2.0	1.7	1.4	1.1	0.8	0.5	0.2

TABLES FOR THE DETERMINATION OF HEIGHTS FROM PRESSURES AND TEMPERATURES. *Vide pp. 35, 41.*

$$dh = \frac{T}{f(\lambda, h)} d(\log p)$$

where h is the height in metres, θ is absolute temperature, logarithms are to base 10, p is in any units, and $f(\lambda, h)$ is a function of latitude and height.

TABLE C₁.

λ	$f(\lambda, 0)$	λ	$f(\lambda, 0)$	λ	$f(\lambda, 0)$
0	·014799	35	·014824	65	·014862
5	99	40	30	70	66
10	·014800	45	37	75	70
15	04	50	44	80	73
20	08	55	50	85	75
25	12	60	·014856	90	·014875
30	·014818				

TABLE C₂.

Correction to be subtracted from $f(\lambda, 0)$ to obtain $f(\lambda, h)$ to allow for the variation of gravity with height.

k.	k.	k.	k.	k.
1 ·00005	5 ·00023	9 ·00042	13 ·00061	17 ·00079
2 9	6 28	10 47	14 65	18 84
3 14	7 33	11 51	15 70	19 89
4 ·00019	8 ·00037	12 ·00056	16 ·00075	20 ·00093

TABLE C₃.

Table of $f(\lambda, h)$ for $\lambda = 52\frac{1}{2}^\circ$.

h	$f(\lambda, h)$	h	$f(\lambda, h)$	h	$f(\lambda, h)$	h	$f(\lambda, h)$
k.		k.		k.		k.	
0 ·014847		5 ·014824		10 ·014800		15 ·014777	
1 42		6 19		11 14796		16 72	
2 38		7 14		12 91		17 68	
3 33		8 10		13 86		18 63	
4 ·014828		9 ·014805		14 ·014782		19 ·014758	

COMPUTER'S HANDBOOK.

SECTION II. § 3.

DYNAMICAL METEOROLOGY. CALCULUS OF THE UPPER AIR.

FORMULAE AND TABLES OF CONSTANTS FOR USE IN THE DISCUSSION OF THE RESULTS OF SOUNDINGS WITH REGISTERING BALLOONS AND PILOT BALLOONS.

Prepared in connexion with a paper entitled Principia Atmospherica published in the Proceedings of the Royal Society of Edinburgh 1914, of which Notes are given in Appendix I.

1. Components of horizontal acceleration due to the rotation of the earth.

The formulae for the acceleration of a particle along and perpendicular to a meridian of a rotating earth are as follows :

$$A_{SN} = H_{SN} - 2\omega \sin \lambda V_{WE},$$

$$A_{WE} = H_{WE} + 2\omega \sin \lambda V_{SN},$$

where A_{SN} and A_{WE} are the components of the acceleration towards the North and towards the East respectively. H_{SN} and H_{WE} are the corresponding impressed forces per unit mass; V_{SN} and V_{WE} are the components of the velocity relative to the earth, ω is the earth's angular velocity and λ is the latitude.

If there is no acceleration and no impressed force except the pressure the components of the **slope of pressure** γ_{SN} to Northward and γ_{WE} to Eastward are

$$\gamma_{SN} = 2\omega \rho \sin \lambda V_{WE},$$

$$\gamma_{WE} = -2\omega \rho \sin \lambda V_{SN}.$$

Correction for Curvature.

If the path is curved to the left, the geodesic radius of curvature being r , and if the acceleration in the direction of motion may be neglected, then

$$\gamma_{SN} = \rho \left[2\omega \sin \lambda + \frac{V}{r} \right] V_{WE},$$

$$\gamma_{WE} = -\rho \left[2\omega \sin \lambda + \frac{V}{r} \right] V_{SN},$$

where V is written for the velocity so that $V^2 = V_{SN}^2 + V_{WE}^2$. If the path curves to the right the sign of r must be changed.

See Appendix II, p. 66.

2. Calculation of the amount of outflow from a South to North current and of inflow towards a North to South current which is required in order to maintain the system unchanged.

If H is the vertical depth of the current, λ the latitude, then

$$\frac{1}{H} \frac{dH}{d\lambda} = \frac{\pi}{180} \cot \lambda = 0.0175 \cot \lambda.$$

Whence it follows that for the maintenance of the thickness of a current of breadth l degrees of longitude, flowing to the North, the amount of air to be removed is equivalent to a crossflow of

$$0.0175 \frac{\cos^2 \lambda}{\sin \lambda} l V,$$

where V is the velocity of the current.

For a current flowing southward with the same velocity a similar amount of air must be supplied in order to maintain the thickness of the current as it flows on.

3. Characteristic equation.

$$p = R \rho T$$

for pressure in millibars and temperature in degrees absolute and density in grammes per cubic metre:—

For dry air* $R = 2.870 \times 10^{-3}$.

For air saturated at 273a with a pressure of 6.10 mb. of water-vapour

$$R = 2876 \times 10^{-6}.$$

For air saturated at 283a with a pressure of 12.24 mb. of water-vapour

$$R = 2883 \times 10^{-6}.$$

For air saturated at 300a with a pressure of 35.34 mb. of water-vapour

$$R = 2905 \times 10^{-6}.$$

These values of R are computed from the formula

$$\frac{R_w}{R_o} = \frac{p_o}{p_o - \frac{3}{8} T_o p_w},$$

where R_o is the value of R for dry air; R_w is the value of R for a mixture of air and water-vapour which is saturated at

* Computer's Handbook, Introduction.

the temperature T and which has a partial pressure of dry air of 1000 mb. at the freezing point.

Other values of R computed on the same plan are given in Table I., p. 53.

4. Change of pressure-difference between two verticals at different levels.* Rate of increase of pressure-difference (in millibars per metre increase of height)

$$= 0.0342 \frac{p}{T} \left(\frac{\Delta T}{T} - \frac{\Delta p}{p} \right).$$

5. Wind Velocity at different levels. Calculation of the gradient wind at any level at which the pressure is p millibars and the absolute temperature T , in terms of the pressure-gradient in millibars per 100 kilometres.

$$V = \frac{R}{2\omega \sin \lambda} \cdot \frac{T}{p} \cdot \frac{\Delta p}{L}$$

$$= K \cdot \frac{T}{p} \cdot \frac{\Delta p}{L}.$$

V = velocity in metres per second,

R = constant of characteristic equation,

λ = latitude,

ω = angular velocity of earth's rotation, in radians per second,

Δp = pressure-difference between two points on same level,

L = their distance apart in hundred-kilometres.

[The curvature of the path is neglected.]

For the values of $R/(2\omega \sin \lambda)$ see Table II.

For average values of p/T see Table III.

6. Calculation of the pressure-differences per 100 kilometres from the wind components, W . to E . and S . to N . at different levels.

If the pressure gradient northward be $\Delta_N p$ per 100 kilometres, then

$$\Delta_N p = \frac{1}{K} \cdot \frac{p}{T} (W. \text{ to } E.).$$

If the pressure gradient westward be $\Delta_W p$ per 100 kilometres, then

$$\Delta_W p = \frac{1}{K} \cdot \frac{p}{T} (S. \text{ to } N.).$$

For values of K see Table II.

For average values of p/T see Table III.

For the factors of wind components see Traverse Tables, Introduction, p. 23.

* Deduced from 8a, p. 33.

7. Calculation of the pressure-difference and temperature-difference per 100 kilometres from the components of wind velocity.

First compute the slope of pressure per 100 kilometres to North from the West-East component by the formula

$$\Delta_N p = \frac{1}{K} \cdot \frac{p}{T} \cdot (W. \text{ to } E.),$$

and slope of pressure per 100 kilometres to East from the North-South component by the formula

$$\Delta_W p = \frac{1}{K} \cdot \frac{p}{T} \cdot (S. \text{ to } N.).$$

Thence compute by subtraction of successive lines the change of pressure-difference per kilometre of elevation.

Then for the slope of temperature to the North

$$\Delta_N T = \frac{T}{p} \left(\frac{\text{increase per kilometre of pressure slope to North}}{34.2} \times T + \Delta_N p \right),$$

and for the slope of temperature to the West

$$\Delta_W T = \frac{T}{p} \left(\frac{\text{increase per kilometre of pressure slope to West}}{34.2} \times T + \Delta_W p \right),$$

If the values of T and T/p are not known for the special occasion an estimate must be used which may be taken from the tables of *average values* of T from Table IV and of T/p from Table III.

In these equations Δp and ΔT signify a **slope** of pressure and temperature, that is the value of Δ is positive to the North when the quantity is smaller Northward. This is unfortunately in contradiction with the usual interpretation of Δ , and such a contradiction ought to be avoided. Another symbol that would avoid the ambiguity has not yet been found.

8. Calculation of the change of wind-velocity with height in terms of the vertical and horizontal temperature gradients.

The formulae for absolute units are:—

For geostrophic winds

$$\frac{dv}{dh} = \frac{v}{T} \frac{dT}{dh} + \frac{g}{2\omega \sin \lambda} \frac{q}{T}$$

where $q = \frac{dT}{dl}$ and represents the horizontal temperature gradient.

For cyclostrophic winds

$$\frac{dv}{dh} = \frac{v}{T} \frac{dT}{dh} + \frac{gq}{T} \frac{E}{v \cot r}.$$

These formulae hold when v and q are regarded as vectors in a horizontal plane.

Note.—Geostrophic winds are those in which the pressure gradient is balanced by the rotation of the earth according to the formula $s = \frac{\Delta p}{\Delta l} = 2\omega v \sin \lambda$ in temperate and polar latitudes. Cyclostrophic winds are those in which the pressure gradient is balanced by the motion in a small circle, radius according to the formula $s = \frac{\Delta p}{\Delta l} = \frac{v^2}{E} \cot r$ where E is the earth's radius. Such are approximately the winds of the equatorial region and those of tropical hurricanes and tornadoes.

The values of vapour pressure for Table I are taken from Tables 57 and 58 of Landolt and Börnstein's *Physikalisch-chemische Tabellen*.

The modification of the characteristic equation on account of water vapour, keeping R constant, is dealt with by V. Bjerknes by means of an alteration from the actual to the "virtual temperature." See *Dynamic Meteorology and Hydrography*, vol. I. p. 26, Carnegie Institution, Washington, 1910.

An example of applying the foregoing formulae is appended. The data are published in C. J. P. Cave's *Structure of the Atmosphere*, for a pilot balloon ascent at Ditcham Park on April 29th, 1908, and are represented in his model of a N.W. current aloft crossing a S.W. current at the surface.

Height in kilo- metres	Velocity (metres per sec.)	Direc- tion	W to E comp.	$\frac{W \text{ to } E}{K}$ $K=25.4$	$\frac{p}{T}$	$\frac{W \text{ to } E}{K} \times \frac{p}{T}$ $=\Delta_{vp}$	Change per k. (increase)	Incr. per k. $\frac{\text{Incr. per k.}}{34.2}$	T	$\frac{\text{Incr. per k.}}{34.2} \times T$	$\frac{\text{Incr. per k.}}{34.2} \times \frac{T}{T+\Delta_{vp}}$	$\left(\frac{\text{Incr. per k.}}{34.2} \times \frac{T}{T+\Delta_{vp}} \right) \div \frac{p}{T}$	$100 \div \Delta_{vp} T$
6	20.5	300	+17.7	+70	1.90	+1.33	+25	+0.0073	249	+1.82	+3.03	+1.66	66
5	15.0	300	+13.0	+51	2.12	+1.08	+30	+0.0088	255	+2.24	+3.17	+1.55	+70
4	8.5	280	+8.4	+33	2.36	+78	+12	+0.0035	262	+92	+1.64	+74	+152
3	6.5	265	+6.4	+25	2.63	+66	+22	-0.0064	267	-1.71	-0.94	-41	-294
2	8.0	250	+7.5	+30	2.92	+88	+33	+0.0097	273	+2.65	+3.37	+1.08	+92
1	5.0	240	+4.3	+17	3.23	+55	+09	+0.0026	279	+73	+1.24	+37	+270
0	5.0	220	+3.2	+13	3.55	+46							
Height in kilo- metres	Velocity (metres per sec.)	Direc- tion	S to N comp.	$\frac{S \text{ to } N}{K}$ $K=25.4$	$\frac{p}{T}$	$\frac{S \text{ to } N}{K} \times \frac{p}{T}$ $=\Delta_{wp}$	Change per k. (increase)	Incr. per k. $\frac{\text{Incr. per k.}}{34.2}$	T	$\frac{\text{Incr. per k.}}{34.2} \times T$	$\frac{\text{Incr. per k.}}{34.2} \times \frac{T}{T+\Delta_{wp}}$	$\left(\frac{\text{Incr. per k.}}{34.2} \times \frac{T}{T+\Delta_{wp}} \right) \div \frac{p}{T}$	$100 \div \Delta_{wp} T$
6	20.5	300	-10.3	-41	1.90	.78	.14	-0.0041	249	-1.02	-1.73	-.86	-116
5	15.0	300	-7.5	-30	2.12	.64	.50	-0.0146	255	-3.72	-4.11	-.184	-54
4	8.5	280	-1.5	-06	2.36	.14	.19	-0.0056	262	-1.47	-1.51	-.61	-164
3	6.5	265	+0.6	+02	2.63	.05	.27	-0.0079	267	-2.11	-1.92	-.69	-145
2	8.0	250	+2.7	+11	2.92	.32	0	0	273	0	+.32	+.10	+1000
1	5.0	240	+2.5	+10	3.23	.32	+.21	-0.0061	279	-1.70	-1.27	-.37	-270
0	5.0	220	+3.8	+15	3.55	.53							

TABLES FOR FACILITATING COMPUTATION.

TABLE I. SATURATION-PRESSURES IN MILLIBARS OF AQUEOUS VAPOUR OVER ICE AND WATER AT DIFFERENT TEMPERATURES AND THE CORRESPONDING VALUES OF R IN THE CHARACTERISTIC EQUATION FOR AIR.

(i) Temperatures below the freezing point.

Temperature in degrees Absolute	ICE-VAPOUR		WATER-VAPOUR	
	Saturation pressure in millibars	R	Saturation pressure in millibars	R
223	0.05	2870		
225	0.06	2870		
230	0.10	2870		
235	0.17	2870		
240	0.29	2870		
245	0.48	2870		
250	0.79	2871		
255	1.27	2871	1.51	2872
260	2.01	2872	2.27	2872
265	3.13	2873	3.36	2874
270	4.78	2875	4.90	2875
273	6.10	2876	6.10	2876

(ii) Temperatures above the freezing point.

Temperature in degrees Absolute	Saturation pressure in millibars	R	Temperature in degrees Absolute	Saturation pressure in millibars	R
273	6.10	2876	308	55.80	2924
275	7.05	2877	309	58.97	2927
280	9.99	2880	310	62.30	2930
285	13.97	2884	311	65.79	2933
290	19.26	2890	312	69.45	2937
295	26.21	2895	313	73.29	2940
300	35.34	2905	314	77.30	2944
301	37.47	2907	315	81.50	2948
302	39.71	2909	316	85.90	2952
303	42.07	2911	317	90.51	2956
304	44.55	2913	318	95.32	2961
305	47.16	2915	319	100.36	2965
306	49.90	2918	320	105.63	2970
307	52.78	2921			

TABLE II. K . VALUES OF $R/2\omega \sin \lambda$ FOR DIFFERENT LATITUDES.

DRY AIR.—For the correction for humidity see Table I.

Latitude	$R/2\omega \sin \lambda$	Latitude	$R/2\omega \sin \lambda$	Latitude	$R/2\omega \sin \lambda$
1	1127.1	31	38.3	61	22.6
2	565.2	32	37.2	62	22.3
3	376.9	33	36.2	63	22.1
4	282.8	34	35.3	64	22.0
5	226.3	35	34.4	65	21.8
6	188.7	36	33.6	66	21.6
7	161.9	37	32.8	67	21.4
8	141.7	38	32.0	68	21.3
9	126.1	39	31.3	69	21.1
10	113.8	40	30.7	70	21.0
11	103.3	41	30.1	71	20.9
12	94.9	42	29.5	72	20.7
13	87.7	43	28.9	73	20.6
14	81.5	44	28.4	74	20.5
15	76.2	45	27.9	75	20.4
16	71.6	46	27.4	76	20.3
17	67.5	47	27.0	77	20.2
18	63.8	48	26.5	78	20.2
19	60.6	49	26.1	79	20.1
20	57.7	50	25.8	80	20.0
21	55.0	51	25.4	81	20.0
22	52.7	52	25.0	82	19.9
23	50.5	53	24.7	83	19.9
24	48.5	54	24.4	84	19.8
25	46.7	55	24.1	85	19.8
26	45.0	56	23.8	86	19.8
27	43.4	57	23.5	87	19.7
28	42.0	58	23.3	88	19.7
29	40.7	59	23.0	89	19.7
30	39.4	60	22.8	90	19.7

TABLE III. MEAN VALUES OF p/T AT DIFFERENT LEVELS—
(Compiled from the *Geophysical Journal*, 1912.)

Height	p/T	Height	p/T
K.		K.	
20	.26	10	1.19
19	.28	9	1.35
18	.32	8	1.52
17	.38	7	1.70
16	.45	6	1.90
15	.54	5	2.12
14	.65	4	2.36
13	.77	3	2.63
12	.90	2	2.92
11	1.04	1	3.23
		Ground	3.55

A table of mean values of p/T for the several months is given in the *Meteorological Glossary* under "Buoyancy," p. 53.

The "Traverse Tables" of pp. 24-31 can be used for obtaining components of wind.

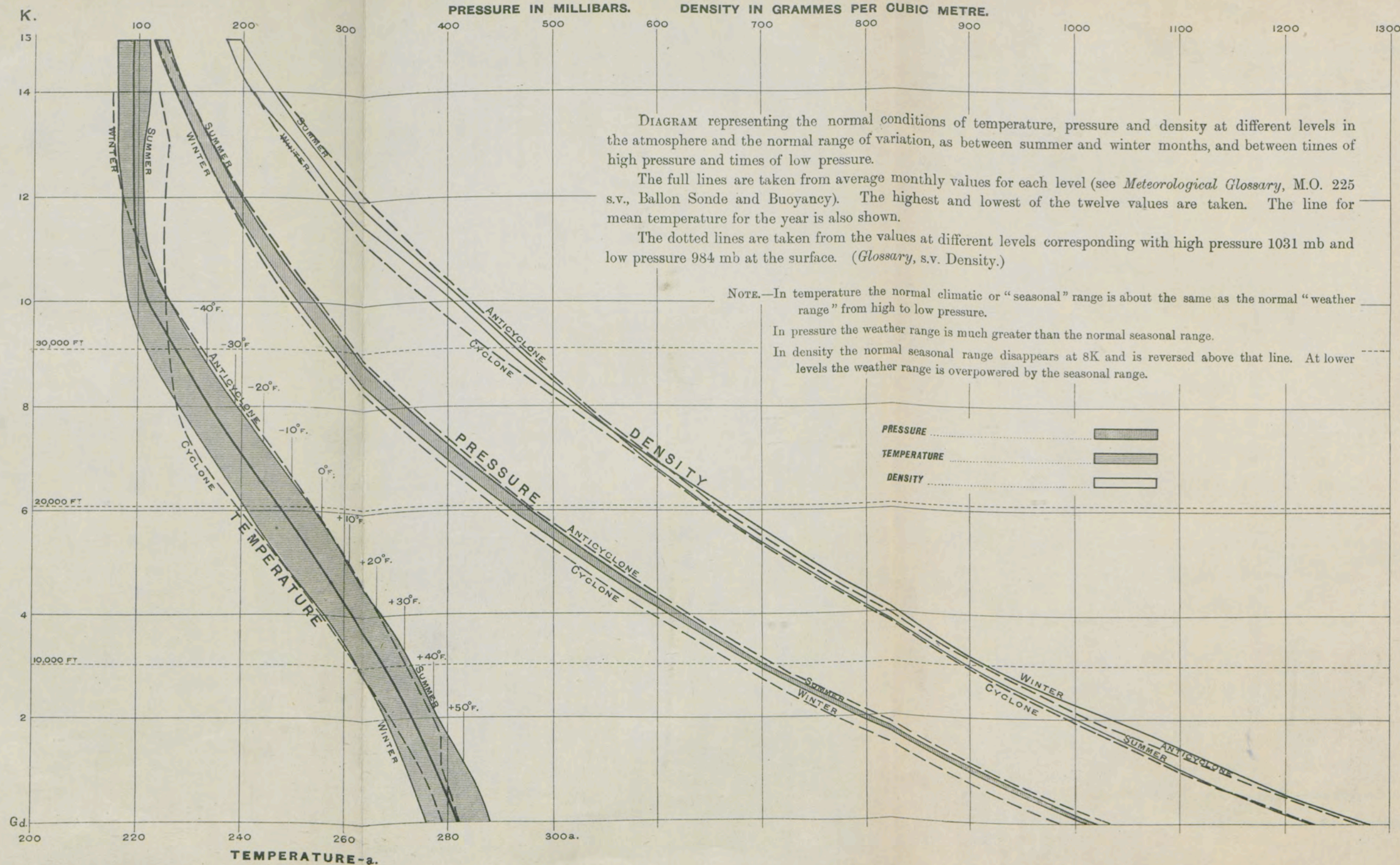


DIAGRAM representing the normal conditions of temperature, pressure and density at different levels in the atmosphere and the normal range of variation, as between summer and winter months, and between times of high pressure and times of low pressure.

The full lines are taken from average monthly values for each level (see *Meteorological Glossary*, M.O. 225 s.v., Ballon Sonde and Buoyancy). The highest and lowest of the twelve values are taken. The line for mean temperature for the year is also shown.

The dotted lines are taken from the values at different levels corresponding with high pressure 1031 mb and low pressure 984 mb at the surface. (*Glossary*, s.v. Density.)

NOTE.—In temperature the normal climatic or “seasonal” range is about the same as the normal “weather range” from high to low pressure.

In pressure the weather range is much greater than the normal seasonal range.

In density the normal seasonal range disappears at 8K and is reversed above that line. At lower levels the weather range is overpowered by the seasonal range.

TABLE IV. MEAN TEMPERATURE OF THE UPPER AIR IN *ENGLAND, SCOTLAND AND IRELAND.

Computed by W. H. Dines, F.R.S.

FOR SOUTH-EAST OF ENGLAND (PYRTON HILL, BENSON AND DITCHAM).

1. Unadjusted Mean Temperatures.

Height.	Number of Observations.												Year.
	Jan. (13)	Feb. (15)	Mar. (8)	April (11)	May (28)	June (13)	July (15)	Aug. (13)	Sept. (17)	Oct. (16)	Nov. (8)	Dec. (10)	
K.	°	°	°	°	°	°	°	°	°	°	°	°	
15	216	213	219	221	221	222	221	222	220	215	219	218	
14	16	13	19	21	20	21	21	21	19	16	20	19	
13	16	12	19	21	20	21	21	21	19	18	21	19	
12	15	12	18	21	19	19	21	21	22	21	23	20	
11	18	12	17	21	18	20	25	25	26	25	25	22	
10	21	16	18	20	21	24	25	25	33	32	27	28	
9	25	22	23	23	26	31	31	32	40	39	30	34	
8	31	30	29	28	33	38	38	40	47	46	36	41	
7	38	38	36	34	41	46	46	47	54	54	44	48	
6	45	45	42	41	49	54	53	54	61	60	52	55	
5	51	53	49	48	56	61	59	61	67	67	60	62	
4	58	60	56	54	63	67	66	68	73	73	66	68	
3	64	67	62	60	69	73	73	72	78	78	71	73	
2	69	72	68	66	75	78	78	77	82	84	75	78	
1	74	76	72	71	80	83	84	86	87	85	81	82	
Ground	76	77	78	78	84	87	88	88	87	85	81	82	

2. Mean temperatures adjusted by smoothing, by means of a sine curve, the values given in Table 1.

Height.	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
K.	°	°	°	°	°	°	°	°	°	°	°	°	
15	216	216	217	218	220	221	222	221	220	219	218	217	
14	16	16	17	18	20	21	22	21	20	19	18	17	
13	16	17	17	18	20	21	21	21	20	19	18	17	
12	17	17	17	18	19	20	21	22	22	21	20	18	
11	17	17	17	18	19	21	22	25	26	25	23	21	
10	19	18	18	20	22	24	32	32	32	30	27	25	
9	23	22	23	25	28	30	32	39	38	35	32	30	
8	28	28	29	32	35	37	39	46	45	42	39	36	
7	35	35	36	39	42	45	47	54	53	51	47	43	
6	42	41	42	44	48	51	54	61	60	57	53	50	
5	49	48	50	52	56	59	68	68	66	64	60	58	
4	56	56	57	59	63	66	73	73	72	75	72	69	
3	63	63	64	66	68	71	73	78	77	75	77	74	
2	68	68	69	71	74	77	84	84	82	80	77	74	
1	72	72	73	76	79	82	88	88	86	84	80	78	
Ground	76	76	77	80	84	86	88	88	86	84	80	78	

* For other information as to temperature in the British Isles and elsewhere, see “Geophysical Memoirs,” No. 5, Vol. I., International Kite and Balloon Ascents. By Ernest Gold, M.A.

FOR MANCHESTER.

3. Unadjusted Mean Temperature.

Height.	Number of Observations.												Year.
	Jan. 7	Feb. 4	Mar. 6	April 5	May 9	June 7	July 5	Aug. 6	Sept. 9	Oct. 5	Nov. 6	Dec. 4	
K.	°	°	°	°	°	°	°	°	°	°	°	°	
15	219	217	222	222	221	221	226	222	220	221	219	216	
14	19	17	22	22	21	21	27	22	20	21	20	16	
13	19	17	21	22	20	22	27	21	20	21	21	17	
12	19	17	18	20	20	20	25	21	21	21	21	17	
11	20	17	18	21	21	19	26	23	22	22	24	18	
10	22	21	19	20	22	21	27	29	26	24	27	20	
9	28	27	23	24	25	27	33	35	33	30	30	22	
8	34	31	29	30	32	35	40	42	41	37	31	24	
7	39	38	36	35	39	42	47	50	48	44	35	28	
6	45	45	43	42	46	50	54	56	55	51	42	35	
5	52	52	50	49	53	57	60	62	62	58	49	42	
4	58	59	56	55	60	64	66	68	67	65	56	51	
3	65	65	63	62	65	70	72	75	73	69	63	58	
2	69	70	69	68	71	74	77	78	79	73	69	63	
1	74	75	75	74	77	81	82	83	83	77	74	69	
Ground	79	77	80	82	83	88	89	89	88	85	80	76	

4. Mean temperatures adjusted by smoothing, by means of a sine curve, the values given in Table 3.

Height.	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year.
K.	°	°	°	°	°	°	°	°	°	°	°	°	
15	218	219	220	222	223	223	223	222	220	219	218	218	
14	18	19	20	22	23	23	23	22	20	19	18	18	
13	18	19	21	22	23	23	23	22	20	19	18	18	
12	17	17	18	20	21	22	23	23	22	20	19	18	
11	18	18	18	20	22	23	24	24	23	22	20	19	
10	20	20	20	21	23	25	26	27	27	26	24	22	
9	25	24	24	25	27	30	32	33	32	31	29	26	
8	28	28	30	32	35	38	39	39	38	35	32	29	
7	34	34	36	39	42	45	47	46	44	41	37	35	
6	40	41	42	45	49	52	54	53	51	48	44	42	
5	48	48	50	53	56	58	60	60	58	55	51	49	
4	55	54	56	59	62	64	65	66	65	62	58	56	
3	62	62	63	66	68	71	72	71	70	67	64	62	
2	67	67	68	71	74	76	77	76	75	72	70	68	
1	72	72	74	77	80	82	82	82	80	77	74	72	
Ground	78	78	80	83	86	88	88	88	86	83	80	78	

5. Mean Temperature at each kilometre height.

Height.	Mean temperatures for the whole year.				Amplitude of Seasonal Variation.		Date of lowest temperature in days from Jan. 1st.	
K.	Scot-land.	Ireland.	Man-chester.	S.E. England.	Man-chester.	S.E. England.	Man-chester.	S.E. England.
0	80.2	80.3	83.0	81.8	5.2	6.2	15	38
1	75.3	75.3	77.0	78.0	5.2	6.1	14	35
2	70.3	71.7	71.7	73.1	5.0	5.4	25	37
3	64.0	66.3	66.6	67.7	5.2	5.2	26	37
4	58.4	59.4	60.4	61.7	6.2	6.2	32	35
5	52.0	52.2	53.8	54.8	6.2	6.6	26	36
6	45.0	45.4	47.0	47.8	6.8	6.6	26	31
7	38.0	37.7	40.2	40.7	6.7	6.3	27	30
8	30.2	30.9	33.8	33.6	5.8	6.0	30	35
9	24.8	25.5	28.2	27.5	4.5	5.2	55	44
10	21.2	21.7	23.2	22.2	3.7	4.1	56	56
11	20.5	19.9	20.9	19.6	3.2	2.9	35	54
12	21.6	19.3	20.0	18.8	2.8	2.2	26	29
13	21.8	19.2	20.6	18.7	2.5	2.5	—20	14
14	22.0	19.0	20.5	18.9	2.6	2.8	—12	27
15-20	22.0	19.4	20.5	18.8	2.7	3.0	—10	23
H _p	9.86	10.13	10.35	10.68	5.6	5.9	19	47
No. of obs.	29	27	73	167	73	167	73	167

TABLE V. AVERAGE VALUES OF PRESSURE AND TEMPERATURE AT DIFFERENT LEVELS OVER HIGH PRESSURE (1033 MB.) AND LOW PRESSURE (982 MB.) AT THE SURFACE; WITH PRESSURE-DIFFERENCES AND TEMPERATURE-DIFFERENCES AT EACH LEVEL. COMPILED FROM THE DIAGRAM AND TABLES OF W. H. DINES, F.R.S., IN M. O. 210 B.

Height.	Pressure.		Diff.	Diff.	Temperature.	
	Low.	High.	Δp .	ΔT .	Low 984 mb	High 1031 mb.
k.	mb.	mb.	mb.	°a.	°a.	°a.
15	116	123	7	—	—	—
14	135	146	11	— 9	224	215
13	157	171	14	— 11	226	215
12	183	201	18	— 8	225	217
11	212	235	22	— 4	225	221
10	247	273	26	+ 1	225	226
9	288	317	29	+ 7	226	233
8	335	366	31	+ 13	227	240
7	388	422	34	+ 15	232	247
6	449	483	34	+ 14	240	254
5	516	552	36	+ 13	248	261
4	591	628	37	+ 12	255	267
3	675	713	38	+ 9	263	272
2	767	807	40	+ 8	269	277
1	870	913	43	+ 4	275	279
0	984	1031	47	+ 3	279	282

Standard deviation of P_0 13.8 mb.Standard deviation of P_s 14.1.Correlation coefficient between the variations of P_0 and P_s from the general European observations = .65.

TABLES OF MEAN TEMPERATURE OF THE UPPER AIR
IN ENGLAND, SCOTLAND AND IRELAND.

Prepared by W. H. DINES, F.R.S.

Explanatory Notes.—Figures showing the mean temperatures for the British Isles as a whole have been published by more than one author in recent years, but the number of observations that are now available permit of rather greater detail, and my assistant, Mr. Newnham, has averaged the values separately for Ireland, Scotland, Manchester, and the South East of England. The results are given in the accompanying tables.

The same pattern of meteorograph has been used in all the observations, the system of calibrating and working up the traces has been the same for the Scottish, Irish, and English stations, excepting Manchester, and I think I am responsible for the accuracy of all these results. At Manchester the traces have been worked up by Miss Margaret White on a slightly different but equally accurate system. Various methods of investigation agree in showing that the probable error of a single observation of temperature does not exceed 1°C . and for the value of H_c , the height of the troposphere does not exceed 200 metres.

The majority of the ascents used have reached a height of at least 14 k. and all have penetrated well into the stratosphere. The very considerable number of ascents which have reached 18 to 20 k. prove conclusively that on the yearly mean there is no definite change of temperature with change of height in the stratosphere and it is indeed impossible to say whether the mean temperature at 20 k. is greater or less than that at 13 k., hence in forming the means of the higher strata the last reliable temperature recorded has been accepted as the value from that point up to 20 k. Ascents exceeding 20 k. are too few to justify any conclusions as to the temperature above that height.

The ascents in the South-East of England are those made at Ditcham Park, Pyrton Hill and Benson. Strictly, Pyrton Hill and Benson are in the "Midland Counties", District IV of the Meteorological Office, but the three stations represent quite fairly a district some 30 miles west of London. The Scottish ascents were mostly at Crinan, in latitude $56^{\circ}6'\text{N}$. and $5^{\circ}35'\text{W}$., but about a third were at Eskdalemuir, close to the English border.

The Irish ascents were from various localities, but chiefly from Mungret College, near Limerick. In the group, England, S.E., the 167 observations used are fairly well distributed over the year; about half were made at 7 a.m., and about half just before sunset. A similar remark applies to Manchester, since the two sets of consecutive hourly ascents that afforded such valuable information about the daily period were utilised only to the extent of two observations each, but at Manchester the observations were chiefly at sunset. In both districts the number and distribution justify an attempt to find the seasonal change. Both for Scotland and for Ireland the ascents are much less numerous; they were made chiefly in the summer in the international weeks, and they do not suffice to show the seasonal change. This fact causes a difficulty in estimating the mean values, and the assumption has been made that the seasonal variation is the same as it is in England, and a correction for the preponderance of summer observations has been applied accordingly. For all the stations the period covered has been from January, 1908, to the present date, July, 1915.

Table IV 1 gives the monthly mean values as deduced directly from the observations for the S.E. of England. In Table IV 3 the values for Manchester are seen to be more discordant than those for the South-East of England, especially in November and December. This is what might be expected since they depend upon much fewer observations, and the ascents in November and December occurred under unusual barometric conditions, the pressure on the days of observation in November being low and in December very low. Better results might have been obtained by correcting for pressure, but this has not been done.

The mean annual values at each height for each district are given in Table 5, including also the height of the beginning of the stratosphere. For the two districts in which the number of observations permits, the amplitude of the seasonal change and the date of the minimum are given.

Table 5.—It is desirable to ascertain the probable error of the values. Apart from the seasonal variation, which is allowed for, the casual error of an observation of temperature is between 4° and 5°C ., say $4^{\circ}5\text{C}$., and the casual error of a mean of n such independent observations in $4.5/\sqrt{n}$. This gives standard errors of .35 for England S.E., .53 for Manchester and .85 for Scotland and Ireland. This is on the assumption that the observations are strictly independent of each other, but that is not the case.

There are 17 simultaneous ascents at Pyrton Hill and Ditcham Park and the results for each pair are almost identical. There are also many pairs of observations at twelve-hour intervals. The errors will therefore be larger than those given above, but it is safe to say they will not exceed them by 50 per cent., and if we take the values given as the probable instead of the standard errors we shall be on the safe side.

Apart from the seasonal variation the standard deviation of H_c is 1.40 k., the probable error of the means for Scotland or Ireland is about .25 k., for Manchester .15, and for England S.E., .10 k.

The probable errors of the amplitudes are 40 per cent. ($\sqrt{2}:1$) above those of the means, or about $0^{\circ}5\text{C}$. for England S.E., and $0^{\circ}75\text{C}$. for Manchester.

The values given in Table 5 seem quite consistent and show the variations that the difference in latitude between the South of England and Scotland would lead us to expect. In the South the mean temperature from 1 to 9 k. is 54° ; in Scotland it is 51° , but in the higher strata from 12 k. to 20 k. Scotland is 3 degrees the warmer. This is in agreement with the usual rule. Also the difference of $-.82\text{ k}$., 9.86 k . against 10.68 k ., in the value of H_c is almost beyond the range of a casual error, but shows the usual decrease of H_c with higher latitudes; a similar difference is shown between Petrograd and Central Europe. The values for Manchester and Ireland both for H_c and for the temperature are intermediate like the latitudes.

There is a discrepancy between the surface temperatures in England S.E. and Manchester, Manchester being the warmer, but this is because fewer ascents in the early morning occurred at Manchester when the temperature is lower than at sunset. The discrepancy has disappeared at 1 k., thus showing the small height to which the daily temperature range extends.

The other discrepancies, such as the difference in the values of H_c at Manchester and in Ireland and the irregularities of the phase angles are probably fortuitous.

APPENDIX I.

PRINCIPIA ATMOSPHERICA.

NOTES OF AN ADDRESS GIVEN AT THE MEETING OF
THE MATHEMATICAL ASSOCIATION ON WEDNES-
DAY, JANUARY 7, 1914, BY SIR NAPIER SHAW, Sc.D.,
F.R.S., DIRECTOR OF THE METEOROLOGICAL OFFICE.

To invite the attention of the Mathematical Association to Meteorology must be regarded as a venturesome proceeding. I shall endeavour to show that it is justified.

In order that the methods of mathematical calculation may be applied to the atmosphere it is necessary that the laws which it is proposed to use and the postulates which will be accepted should be clearly set forth. I shall begin by stating the laws or axioms which, in my opinion, may be applied in the calculation of atmospheric motion. I shall state two postulates based on observation and then ask your attention to some propositions, setting out the proofs in due form and order so far as time will allow.

I shall not attempt any exhaustive demonstration of the laws. That part of the subject is not, strictly speaking, mathematical; though I am aware that, from time to time, the axioms of various sciences have engaged the attention of mathematicians. But I shall shelter myself behind two great authorities. The first is Plato, who in the seventh book of the *Republic* makes Socrates speak as follows with reference to Astronomy: "And do you not think that the genuine astronomer will view with the same feelings the motion of the stars? That is to say, will he not regard the heaven itself, and the bodies which it contains, as framed by the heavenly Architect with the utmost beauty of which such works are susceptible? But as to the proportion which the day bears to the night, both to the month, the month to the year, and the other stars to the sun and moon, and to one another—will he not, think you, look down upon the man who believes such corporeal and visible objects to be changeless and exempt from all perturbations; and will he not hold it absurd to bestow extraordinary pains on the endeavour to apprehend their true condition? Hence we shall pursue astronomy with the help of problems, just as we pursue geometry; but we shall let the heavenly bodies

alone, if it be our design to become really acquainted with astronomy, and by that means to convert the natural intelligence of the soul from a useless into a useful possession." This is so truly descriptive of what I will call the mathematical method that you will forgive my quoting it again.

The second is Newton. Imagine an astronomer endeavouring to explain the motions of the heavenly bodies. Why is the moon moving? Why does the earth move? Why should the planets move? And think of the result of the enunciation of the Laws of Motion which just altered the questions to these: Why is the moon changing its motion? Why does the earth change its motion? and Why do the planets change their motion? The whole of mathematical astronomy is contained in that small change in the point of view. For the purposes of mathematical astronomy the laws of motion are incontrovertible. Yet who really believes them? Take the first law and say to any audience you please, "If you leave a body alone it will go on moving." Every man in the street would think you were simply romancing; and if your classes in mechanics really accept *ex animo* the first law of motion they have changed a good deal since I had the pleasure of examining them. No. You will not try to *prove* even that simple law. Sooner or later you will say "You must leave the mundane bodies alone (in the platonic sense) if you wish to become really acquainted with mundane mechanics."

Or take the second and third laws. Let me show you that either they are contradictory or the mass of the earth is equal to the mass of the sun. By Law III the force of the earth on the sun is equal to the force of the sun on the earth; by Law II the forces are the products of the mass and acceleration; and by the fundamental principles of geometry motion is simply relative and the accelerations are equal, being opposite aspects of the same motion. Hence the masses are equal. I leave you to get out of that dilemma, if you can, in words suitable for an introductory chapter on Mechanics.

Meanwhile let me state my laws or axioms of atmospheric motion and ask you, while you listen to them, to forget the treasured atmospheric experiences, which I know you all possess. For in spite of the fact that the study of the atmosphere receives little or no encouragement from the great seats of learning there is, I am sure, no one in this room—there is indeed no one on this planet—who is not somehow or other interested in the subject; and anyone, at least of my

acquaintance, will put in five minutes as many searching, penetrating questions about the weather as will take a lifetime, and more than that, to answer.

I have made out five laws. I do not pretend that I have discovered them or that they are new. Their categorical form is, however, a new departure about which, as I have said, I do not now propose to argue.

Note. The subject referred to in the title of this address is dealt with in greater detail in two papers: (1) "Principia Atmospherica"—a paper communicated to the Royal Society of Edinburgh on 1 December, 1913, and (2) "Upper Air Calculus," a paper published in the *Journal of the Scottish Meteorological Society*, vol. XVI. 1913.

AXIOMS OR LAWS OF ATMOSPHERIC MOTION.

1. The Law of the Relation of Motion to Pressure.

In the upper layers of the atmosphere the steady horizontal motion of the air at any level is along the horizontal section of the isobaric surfaces at that level, and the velocity is inversely proportional to the separation of the isobaric lines in the level of the section.

The line of argument in favour of this law, which cannot strictly speaking be either verified or contradicted by any available process of observation, is as follows: The condition specified in the law is the condition of kinematic equilibrium towards which all atmospheric motions tend, and have tended either since the earth began to rotate as it does now, or the atmosphere was first formed, whichever of those events is the later in time. Any deviation from the equilibrium state is by infinitesimal steps during which readjustment to the equilibrium condition has been taking place automatically. Hence any finite difference from the equilibrium state can only occur in quite exceptional conditions. Consequently if there is an ascertained difference from the equilibrium condition it requires explanation just as the divergences from the uniformity contemplated by the First Law of Motion require explanation.

The allowance for "curvature of path" (see p. 47) is one of the differences of which account may have to be taken. Its importance depends upon the latitude. For the half of the globe north of 30° N. and south of 30° S. it is generally negligible, but near the equator it becomes the paramount consideration in the question of the persistence of distribution. Thus rotary systems, small or large, are the only possible isobars for a synchronous chart of an equatorial region, if one were drawn. The long sweeps of "parallel isobars" with which we are concerned in this paper would be impossible there.

Near the surface there is always a component of motion along the gradient from high pressure to low pressure. In this region the friction due to obstacles and to the viscosity of the air prevents the steady state being reached and, in consequence, the centrifugal force due to the velocity of motion is not adequate to balance the pressure.

2. The Law of the Computation of Pressure and of the Application of the Gaseous Laws.

The pressure at any point in the atmosphere and at any instant is the weight of the column of air which stands upon one unit of horizontal area containing the point. The numerical values of pressure, temperature and density at any point of the atmosphere are therefore related by the usual formulæ for the gaseous laws.

3. The Law of Convection.

Convection in the atmosphere is the descent of colder air in contiguity with air relatively warmer.

4. The Law of the Limit of Convection.

Convection in the atmosphere is limited to that portion of it called the troposphere, in which there exists a sensible fall of temperature with height. The upper layer of the atmosphere in which there is no sensible fall of temperature with height, and therefore no convection, is called the stratosphere.

5. The Law of Saturation.

The amount of water vapour contained in a given volume of air cannot exceed a certain limit which depends upon the temperature and upon nothing else.

LEMMAS OR POSTULATES.

Lemma 1. In the stratosphere, from 11 kilometres upwards, it is colder in the high pressure than in the low pressure at the same level; and in the troposphere, from 9 kilometres downwards to 1 kilometre, it is warmer in the high pressure than in the low pressure at the same level.

[W. H. Dines, *Geophysical Memoirs*, No. 2.]

Lemma 2. The average horizontal circulation in the Northern hemisphere in January between 4 kilometres and 8 kilometres consists of a figure-of-eight orbit from west to east along isobars round the pole, with lobes over the continents and bights over the oceans.

The average circulation at the surface is the resultant of the circulation at 4 kilometres combined with a circulation in the opposite direction of similar shape due to the distribution of temperature near the surface.

[L. Teisserenc de Bort, *Ann. du Bureau Central Météorologique*, 1887, and W. N. Shaw, *Proc. Roy. Soc.* vol. LXXIV. p. 20. 1904.]

PROPOSITIONS.

Proposition 1. Conditions of Persistence of Existing Atmospheric Motions.

To show that in order that an atmospheric current may be maintained of definite thickness along South to North isobars it must be relieved of air equivalent to a cross velocity of

$$0.175 \frac{\cos \lambda}{\sin \lambda} lV,$$

where V is the velocity of the South to North current, l the width of the current in degrees of longitude and λ the latitude, and conversely:

For the maintenance of a current V from North to South a similar quantity of air must be supplied to it.

Discussed in the paper "Principia Atmospherica" communicated to the Royal Society of Edinburgh.

Proposition 2. Influence of the Stratosphere and Troposphere upon the Surface Distribution of Pressure.

To show that the rate of increase of pressure-difference per kilometre of height is $34.2 \frac{p}{T} \left(\frac{\Delta T}{T} - \frac{\Delta p}{p} \right)$; and hence that the distribution of pressure in the stratosphere is the dominant factor in the circulation of the air at the surface; that the intermediate layers between 2 kilometres and 8 kilometres exert little influence upon the distribution of pressure.

Proposition 3. Computation of Wind Velocity.

To show that the wind velocity across the slope of pressure at any level is proportional to $T \frac{\Delta p}{p}$; and thence to show how to utilise observations of the pressure and temperature to calculate the wind velocity at any level.

Proposition 4. Variation of Wind Velocity with Height.

To show that the wind velocity generally increases with height until the substratosphere is reached and falls off with increase of height in the stratosphere.

Propositions 2, 3 and 4 are discussed in the paper on the "Upper Air Calculus" in the *Journal of the Scottish Meteorological Society*, vol. XVI. 1913.

Proposition 5. Dependence of Wind Velocity upon the structure of the Atmosphere and vice versa.

To show how the distribution of pressure and temperature in the upper air can be calculated from the observations of structure represented by a sounding with a pilot balloon, and thence to account for the local distribution of rainfall when an upper current from the North-west crosses a lower current from the South-west.

See Formulae and Tables, pp. II. §3 47-57.

Proposition 6. The General Circulation in the Upper Air.

To account for the average general circulation over the Northern hemisphere in the 4 kilometre level as set out in Lemma 2.

This proposition consists in tracing the consequence of the persistent action of convection on the land slopes of the polar regions in accordance with Law III. (Royal Society of Edinburgh Proceedings, vol. XXXIV., p. 81.)

Proposition 7. The relation between Temperature Gradient in the Stratosphere and rate of change of Geostrophic Wind Velocity with Height.

From the first equation of No. 8 (p. II. §3 50), if the change of temperature in the vertical is regarded as negligible in the stratosphere, we get, substituting suitable units,

$$\left. \begin{array}{l} \text{change of velocity in metres per} \\ \text{second per kilometre of height} \end{array} \right\} = 673 \frac{Q}{T}$$

where Q is the temperature gradient per kilometre.

Proposition 8. In a Cyclotrophic Region in the Stratosphere the Wind Velocity diminishes to Zero within a Finite Distance.

This follows from the second equation of No. 8 (p. II. §3 51).

APPENDIX II.

THE FUNDAMENTAL EQUATIONS OF DYNAMICAL METEOROLOGY.

As a preliminary to the study of the Dynamics of the Air it is useful to consider the motion of a particle moving freely on the surface of a rotating globe.

The equations of motion of such a particle are found in treatises on Dynamics as a particular case of the general equations referred to moving axes. It is possible to find them, however, by more elementary methods in which familiar mechanical principles are used.

The first of the methods adopted here is the simpler. It is open to criticism, however, and should be regarded as providing an explanation rather than a proof of the validity of the fundamental equation.

Motion of a Particle on the Earth's Surface.

First Method.—The rotating globe with which we are concerned is supposed to be bounded by a smooth level surface. By a level surface we mean one which is such that a particle placed at relative rest on it has no tendency to motion. The forces acting on such a particle are the gravitational attraction of the globe and the upward pressure of the surface. These forces are not in opposite directions, they have a resultant which keeps the particle in its circular path round the axis of rotation and overcomes the centrifugal tendency. Another way of expressing the same thing is to say that the resultant of gravity and centrifugal force is along the normal to the surface. When we speak of "*g*," the acceleration of a particle falling freely, as the acceleration due to gravity we ignore the centrifugal force. More strictly "*g*" is the relative acceleration due to the resultant of gravity and centrifugal force.

When a particle is moving on the surface of the globe in the neighbourhood of a point *O* which is fixed on the surface, the motion of the particle may be regarded as compounded of the motion relative to *O* and the motion of *O* as it is carried round the axis of the globe. We know that if the particle were at rest relative to *O* the forces acting on it would not produce any relative motion, so we shall assume that only the motion relative to *O* need be considered.

Now let *OTN* be the tangent plane touching the surface of the globe at *O* and let *OT*, *ON* be axes in this plane, *OT* along the tangent to the path traced out by the particle and *ON* at right angles to the path. The plane *OTN* is rotating in space. The angular velocity ω may be resolved into two components; $\omega \cos \lambda$, the rate at which the plane is tipping and $\omega \sin \lambda$, the rate at which the axes *OT*, *ON* are spinning about the vertical. We shall assume that the tipping of the tangent plane does not affect the motion of the particle in the plane. It would have to be considered if we wanted to discuss the pressure of the particle on the earth's surface.

To find the acceleration in the tangent plane we may regard the surface of the earth as a flat disc rotating with angular velocity $\omega \sin \lambda$.

Fundamental Equations of Dynamical Meteorology. II §3.

If the particle is acted on by no forces parallel to this disc it moves onward in space with a constant velocity *v* whilst the disc rotates beneath it. At the end of a time *t* the particle has moved through a distance *vt* from the position *O* to the position *P*. In the same time the axis in the rotating "disc" has moved through an angle $\omega t \sin \lambda$ from *OP* to *OT* and therefore, the distance of *P* from *OT* is equal to $vt \sin (\omega \sin \lambda t)$ or since *t* is small to $vt^2 \omega \sin \lambda$.

The apparent acceleration is therefore $d^2/dt^2 (\omega v t^2 \sin \lambda)$ or $2 \omega v \sin \lambda$ in the direction *ON* at right angles to *OT*.

Thus it appears that when a particle moves freely on the surface of a smooth rotating globe its velocity is constant and its acceleration relative to the surface is $2 \omega v \sin \lambda$ at right angles to the velocity.

When the particle is acted on by a force with components H_v , H_{LR} in the direction of motion and at right angles to it from left to right the acceleration produced by the force must be added to the relative acceleration due to the rotation of the earth. Accordingly the total acceleration has components A_v , A_{LR} given by

$$mA_v = H_v \\ mA_{LR} = H_{LR} + m 2 \omega v \sin \lambda.$$

Technical objection may be raised on mathematical grounds to the foregoing demonstration, but it is of sufficient interest to warrant its inclusion here. It is based on a suggestion by Sir John Eliot. (See Meteorological Glossary, third issue, pp. 135, 138, where it is shown that when the curvature of the path is taken into account the equation takes the form

$$\frac{\gamma}{\rho} = 2 \omega v \sin \lambda \pm \frac{v^2}{E} \cot r.$$

where γ is the pressure gradient, *E* the radius of the earth and *r* the small circle representing the curvature of the path.)

Second Method.—A somewhat similar demonstration has been suggested by Gold (*R. S. Proc. Vol. 80, 1908*).

If a particle at rest relative to the earth is at a point *P* at a distance *r* from the fixed point *O*, then the velocity of *P* relative to *O* has for its component in the tangent plane at *O*, $\omega r \sin \lambda$. The corresponding acceleration is $(\omega r \sin \lambda)^2/r$ or $\omega^2 r \sin^2 \lambda$, and this acceleration is produced by the appropriate component of gravity at *P*.

Now suppose that the particle is moving on the earth's surface and that *O* is the centre of curvature. The velocity relative to *O* has $\omega r \sin \lambda + v$ for its component in the tangent plane at *O*. Accordingly the equation of motion is

$$m \omega^2 r \sin^2 \lambda - H_{LR} = m (\omega r \sin \lambda + v)^2/r \\ \text{or } H_{LR} + m 2 \omega v \sin \lambda = -m v^2/r = m A_{LR}.$$

The curvature has been taken as cyclonic; a slight alteration in the argument is required for anticyclonic curvature.

Third Method.—Let the velocity of the particle relative to the earth's surface be *V*. This velocity has components towards the east and towards the north, which may be denoted by v_{WE} and v_{SN} . In this section we write *u* for v_{WE} and *v* for v_{SN} .

The components of velocity relative to the centre of the earth are $u + \omega y$ towards the east and *v* towards the north, where ω is the angular velocity of the earth and *y* is the perpendicular from the moving particle on to the earth's axis.

The reaction of the smooth surface is along the vertical. A particle at relative rest on the surface is subject to the attraction of the earth,

which attraction has a horizontal component just sufficient to keep the particle in its circular track round the pole. The acceleration is $\omega^2 y$ towards the earth's axis. The surface component of this acceleration is equivalent to g^1 the horizontal component of gravity.

Therefore, $\omega^2 y \sin \lambda = g^1$ where λ is the latitude.

We are now in a position to consider the equations of motion of the moving particle.

Let the horizontal force acting on the particle be H the components from West to East and South to North being H_{WE} and H_{SN} and let the mass of the particle be m .

The change in the momentum of the particle about the earth's axis is caused by the couple $y H_{WE}$ so that

$$m \frac{d}{dt} [y(u + \omega y)] = y H_{WE}.$$

Now y changes with the time according to the law

$$dy = -v \sin \lambda dt.$$

Hence,

$$m \left[\frac{du}{dt} - \frac{uv}{y} \sin \lambda - 2\omega v \sin \lambda \right] = H_{WE}. \quad (1)$$

Again, we see the movement in a circle of latitude with velocity $u + \omega y$ involves an acceleration towards the earth's axis $(u + \omega y)^2/y$. In writing down the equation of motion along the meridian we must allow for this acceleration.

Thus,

$$m \left[\frac{dv}{dt} + \sin \lambda (u + \omega y)^2/y \right] = mg^1 + H_{SN}.$$

But we have proved that $\omega^2 y \sin \lambda = g^1$.

Therefore,

$$m \left[\frac{dv}{dt} + \sin \lambda \left(\frac{u}{y} + 2\omega \right) u \right] = H_{SN}. \quad (2)$$

In interpreting the equations 1, 2 we observe that the velocity components v_{WE} and v_{SN} are not in fixed directions because the meridians at different points are not parallel to one another. Accordingly du/dt and dv/dt are not true accelerations.

The following device may be used for finding the true relative accelerations A_{WE} and A_{SN} .

If the earth were at rest a force $m A_{WE}$ would produce the acceleration A_{WE} .

Accordingly by repeating the above analysis we should arrive at the equation

$$m \left[\frac{du}{dt} - \frac{uv}{y} \sin \lambda \right] = m A_{WE}$$

so that the component of acceleration to the east is not

$$du/dt \text{ but } du/dt - (uv/y) \sin \lambda.$$

On substitution in equation (1) we get

$$m (A_{WE} - 2\omega \sin \lambda v_{SN}) = H_{WE}$$

$$\text{or } A_{WE} = H_{WE}/m + 2\omega \sin \lambda v_{SN}. \quad (3)$$

In the same way it follows from (2) that

$$A_{SN} = H_{SN}/m - 2\omega \sin \lambda v_{WE}. \quad (4)$$

Hence A the acceleration of a particle relative to the earth's surface

is compounded of H/m and an acceleration $2\omega V \sin \lambda$ at right angles to V and to the right of V .

By taking components in the direction of motion and at right angles thereto we get

$$A_v = H_v/m \quad (5)$$

$$\text{and } A_{RL} = H_{RL}/m - 2\omega V \sin \lambda. \quad (6)$$

Curvature of Path.

Any path on the surface of the earth is curved. The curvature with which we are concerned when we notice whether a particle has an acceleration to the right or to the left is the curvature of the projection of the path on a tangent plane. This is called the geodesic curvature of the path.

Let a_L be the geodesic curvature to the left, then, just as in two-dimensional dynamics, the direction of motion is turning relative to axes fixed in the earth with angular velocity v/a_L about the vertical and the acceleration to the left is

$$A_{RL} = V^2/a_L.$$

Thus equations (5) and (6) may be written

$$\frac{dV}{dt} = H_v/m \quad (7)$$

and

$$V^2/a_L = H_{RL}/m - 2\omega V \sin \lambda. \quad (8)$$

The Equation of Motion for the Atmosphere.

The application of the foregoing argument to the meteorological problems requires no special comment.

If the influence of vertical currents and of viscosity may be ignored we can write down the equations

$$\left(\frac{d}{dt} + v \frac{d}{ds} \right) v = -\frac{1}{\rho} \frac{dp}{ds} \quad (9)$$

$$\frac{v^2}{a_L} = -\frac{1}{\rho} \frac{dp}{dn_L} - 2\omega v \sin \lambda \quad (10)$$

where p is the pressure, ds is an element of the path of the air, dn_L is at right angles thereto and towards the left, ρ is the density of the air and a_L is the radius of curvature to the left of the path of the air.

The **Gradient Wind** is determined by supposing the direction of v to coincide with the isobar so that both sides of equation (9) vanish.

The wind is **Geostrophic** if it is determined by supposing in addition that the curvature is negligible so that a_L is infinite and the left-hand side of the equation (10) becomes zero.

The wind is **Cyclostrophic** if it is determined by supposing that the influence of the earth's rotation is negligible so that the second term on the right-hand side of equation (10) becomes zero. It is legitimate to suppose that this may happen either when λ is very small, that is when the air is at or near the equator or when v is very large or a_L very small, because, in those cases, $\frac{v^2}{a_L}$ becomes large enough for the other term to be neglected in comparison herewith.

The fact that the earth is not spherical is recognised in the proof of the equations of motion, but in the application of the equations to practical problems the distinction can be ignored. When the earth is regarded as a sphere the case of steady motion in a small circle calls for consideration.

If r is the angular radius of the small circle, E the radius of the earth, the geodesic radius of curvature is the length of a generating line of the tangent cone which touches the globe along the small circle,

$$\text{i.e. } a_L = E \tan r.$$

If a_L is to be found from a map it can usually be measured directly on the scale of the map as the radius of curvature of the path which is represented there (see Gold M.O. 190. Barometric Gradient and Wind Force, p. 27).

The modification of the equation of motion when eddy motion is taken into account has been discussed by Taylor (Eddy Motion in the Atmosphere. Phil. Trans. A vol. 215 (1915)).

For an account of the possible paths of a particle moving freely on a smooth rotating globe reference may be made to a paper by Whipple. (Phil. Mag. Vol. 33, 1917 p. 457.)

COMPUTER'S HANDBOOK.

SECTION II. § 4.

Tables showing the distances apart in nautical miles of consecutive 10 millibar isobars corresponding with stated geostrophic wind-velocities in latitudes 52° , 40° , 30° .

The tables are computed to facilitate the formula $\gamma = V \times 2\omega D \sin \lambda$ where γ is the pressure-gradient in millibars per 60 nautical miles and V is the geostrophic velocity of the air in metres per second. ω is the angular velocity of rotation of the earth about its axis, D is the density of the moving air and λ is the latitude of the place at which γ , V and D are measured.

In order that the numerical equality may hold, the quantities represented must be measured in a consistent system of units, such as the C.G.S. system. In that system γ must be expressed in microbars per centimetre, V in centimetres per second and D in grammes per cubic centimetre. In that case the angular velocity ω can be taken at $2\pi/86164$ or $\cdot 00007292$ radians per second and $\sin \lambda$ may be taken from a table of sines. By taking γ in millibars per degree we multiply the left-hand side by the factor $10^5/9$, and by taking V in metres per second and D in grammes per cubic metre we multiply the right-hand side by 10^4 . Hence the numerical relation in the convenient units will be correct if we multiply the numerical factor 2ω by $9/10$, so that with the conventional measures $\gamma = \cdot 0001313 V D \sin \lambda$. We find it better to take the distance apart in nautical miles of consecutive isobars. If L is that distance for the interval of 10 millibars

$$\begin{aligned} 6000/L &= \cdot 001313 V D \sin \lambda \\ \text{or } 4570/L &= \cdot 001 V D \sin \lambda. \end{aligned}$$

From a numerical equation of this kind the following tables have been computed. It will be noticed that the temperature, pressure and humidity of the air have to be accounted for because they affect the density D . Neglecting the humidity as being insignificant, we notice that the same density may be found with any pair of pressures p_1 , p_2 and temperatures T_1 , T_2 if p_1/T_1 is equal to p_2/T_2 . Consequently the heading of the table gives the pressures and corresponding temperatures for the same values of p/T and therefore for the same values of the density.

TABLE SHOWING THE DISTANCES APART IN NAUTICAL MILES OF CONSECUTIVE 10 MILLIBAR ISOBARS CORRESPONDING WITH STATED GEOSTROPHIC WIND VELOCITIES IN LATITUDE 52°.

$\theta/p.$		·26	·27	·28	·29	·30	·31
Pressure $p.$		Temperature $T.$					
1050 millibars ...		273°	284°	294°	305°	315°	326°
1000 ...		260°	270°	280°	290°	300°	310°
950 ...		247°	256°	266°	275°	285°	294°
Gradient Wind Velocity.		Distances apart in Nautical Miles of Consecutive 10 Millibar Isobars.					
mi./hr.	m./s.						
2.2	1	3510	3645	3780	3915	4050	4185
4.5	2	1755	1823	1890	1958	2025	2093
6.7	3	1170	1215	1260	1305	1350	1395
9.0	4	878	911	945	979	1013	1046
11.2	5	702	729	756	783	810	837
13.4	6	585	608	630	653	675	698
15.7	7	501	521	540	559	579	598
17.9	8	439	456	473	489	506	523
20.1	9	390	405	420	435	450	465
22.4	10	351	365	378	392	405	419
24.6	11	319	331	344	356	368	380
26.8	12	293	304	315	326	338	349
29.1	13	270	280	291	301	312	322
31.3	14	251	260	270	280	289	299
33.6	15	234	243	252	261	270	279
35.8	16	219	228	236	245	253	262
38.0	17	206	214	222	230	238	246
40.3	18	195	203	210	218	225	233
42.5	19	185	192	199	206	213	220
44.7	20	176	182	189	196	203	209
47.0	21	167	174	180	186	193	199
49.2	22	160	166	172	178	184	190
51.5	23	153	158	164	170	176	182
53.7	24	146	152	158	163	169	174
55.9	25	140	146	151	157	162	167
58.2	26	135	140	145	151	156	161
60.4	27	130	135	140	145	150	155
62.6	28	125	130	135	140	145	149
64.9	29	121	126	130	135	140	144
67.1	30	117	122	126	131	135	140
69.3	31	113	118	122	126	131	135
71.6	32	110	114	118	122	127	131
73.8	33	106	110	115	119	123	127
76.1	34	103	107	111	115	119	123
78.3	35	100	104	108	112	116	120
80.5	36	98	101	105	109	113	116
82.8	37	95	99	102	106	109	113
85.0	38	92	96	99	103	107	110
87.2	39	90	93	97	100	104	107
89.5	40	88	91	95	98	101	105
91.7	41	86	89	92	95	99	102
94.0	42	84	87	90	93	96	100
96.2	43	82	85	88	91	94	97
98.4	44	80	83	86	89	92	95
100.7	45	78	81	84	87	90	93
102.9	46	76	79	82	85	88	91
105.1	47	75	78	80	83	86	89
107.4	48	73	76	79	82	84	87
109.6	49	72	74	77	80	83	86
111.8	50	70	73	76	78	81	84

Corrections:—For an increase of 1 mb. pressure subtract $\frac{1}{10}$ per cent. from the velocity; for 1° a. add $\frac{1}{2}$ per cent.; for 1° in latitude subtract $1\frac{1}{2}$ per cent.

TABLE SHOWING THE DISTANCES APART IN NAUTICAL MILES OF CONSECUTIVE 10 MILLIBAR ISOBARS IN LATITUDE 40°, CORRESPONDING WITH STATED GEOSTROPHIC WIND VELOCITIES.

$\theta/p.$		·26	·27	·28	·29	·30	·31
Pressure $p.$		Temperature $T.$					
1050 millibars ...		273°	284°	294°	305°	315°	326°
1000 ...		260°	270°	280°	290°	300°	310°
950 ...		247°	256°	266°	275°	285°	294°
Gradient Wind Velocity.		Distances apart in Nautical Miles of Consecutive 10 Millibar Isobars.					
mi./hr.	m./s.						
2.2	1	4361	4529	4697	4864	5032	5200
4.5	2	2180	2264	2348	2432	2516	2600
6.7	3	1454	1510	1566	1621	1677	1733
9.0	4	1090	1132	1174	1216	1258	1300
11.2	5	872	906	939	973	1006	1040
13.4	6	727	755	783	811	839	867
15.7	7	623	647	671	695	719	743
17.9	8	545	566	587	608	629	650
20.1	9	485	503	522	540	559	578
22.4	10	436	453	470	486	503	520
24.6	11	396	412	427	442	457	473
26.8	12	363	377	391	405	419	433
29.1	13	335	348	361	374	387	400
31.3	14	311	323	335	347	359	371
33.6	15	291	302	313	324	335	347
35.8	16	273	283	294	304	314	325
38.0	17	257	266	276	286	296	306
40.3	18	242	252	261	270	280	289
42.5	19	230	238	247	256	265	274
44.7	20	218	226	235	243	252	260
47.0	21	208	216	224	232	240	248
49.2	22	198	206	213	221	229	236
51.5	23	190	197	204	211	219	226
53.7	24	182	189	196	203	210	217
55.9	25	174	181	188	195	201	208
58.2	26	168	174	181	187	194	200
60.4	27	161	168	174	180	186	193
62.6	28	156	162	168	174	180	186
64.9	29	150	156	162	168	173	179
67.1	30	145	151	157	162	168	173
69.3	31	141	146	152	157	162	168
71.6	32	136	142	147	152	157	162
73.8	33	132	137	142	147	152	158
76.1	34	128	133	138	143	148	153
78.3	35	125	129	134	139	144	149
80.5	36	121	126	130	135	140	144
82.8	37	118	122	127	131	136	140
85.0	38	115	119	124	128	132	137
87.2	39	112	116	120	125	129	133
89.5	40	109	113	117	122	126	130
91.7	41	106	110	114	119	123	127
94.0	42	104	108	112	116	120	124
96.2	43	101	105	109	113	117	121
98.4	44	99	103	107	111	114	118
100.7	45	97	101	104	108	112	116
102.9	46	95	98	102	106	109	113
105.1	47	93	96	100	103	107	111
107.4	48	91	94	98	101	105	108
109.6	49	89	92	96	99	103	106
111.8	50	87	91	94	97	101	104

Corrections:—For an increase of 1 mb. pressure subtract $\frac{1}{10}$ per cent. from the velocity; for 1° a. add $\frac{1}{2}$ per cent.; for 1° in latitude subtract $1\frac{1}{2}$ per cent.

TABLE SHOWING THE DISTANCES APART IN NAUTICAL MILES OF CONSECUTIVE 10 MILLIBAR ISOBARS, IN LATITUDE 30°, CORRESPONDING WITH STATED GEOSTROPHIC WINDS.

$\theta/p.$		26	27	28	29	30	31
Pressure $p.$		Temperature $T.$					
1050 millibars ...		273°	284°	294°	305°	315°	326°
1000 ...		260°	270°	280°	290°	300°	310°
950 ...		247°	256°	266°	275°	285°	294°
Gradient Wind Velocity.		Distances apart in Nautical Miles of Consecutive 10 Millibar Isobars.					
mi./hr.	m./s.						
2.2	1	5607	5822	6039	6253	6471	6685
4.5	2	2803	2911	3019	3126	3235	3342
6.7	3	1869	1941	2013	2084	2157	2228
9.0	4	1402	1455	1510	1563	1618	1671
11.2	5	1121	1164	1208	1251	1294	1337
13.4	6	934	970	1006	1050	1078	1114
15.7	7	801	832	863	893	924	955
17.9	8	701	728	755	782	809	836
20.1	9	623	647	671	695	719	743
22.4	10	561	582	604	625	647	668
24.6	11	510	529	549	568	588	608
26.8	12	467	485	503	521	539	557
29.1	13	431	448	465	481	498	514
31.3	14	400	416	431	447	462	477
33.6	15	374	388	403	417	431	446
35.8	16	350	364	377	391	404	418
38.0	17	330	342	355	368	381	393
40.3	18	311	323	335	347	359	371
42.5	19	295	306	318	329	341	352
44.7	20	280	291	302	313	324	334
47.0	21	267	277	288	298	308	318
49.2	22	255	265	274	284	294	304
51.5	23	244	253	263	272	281	291
53.7	24	234	243	252	260	269	278
55.9	25	224	233	242	250	259	267
58.2	26	216	224	232	240	249	257
60.4	27	208	216	223	232	240	248
62.6	28	200	208	216	223	231	239
64.9	29	193	201	208	216	223	230
67.1	30	187	194	201	208	216	223
69.3	31	181	188	195	202	209	216
71.6	32	175	182	189	195	202	209
73.8	33	170	176	183	189	196	203
76.1	34	165	171	178	184	190	197
78.3	35	160	166	172	179	185	191
80.5	36	156	162	168	174	180	186
82.8	37	151	157	163	169	175	181
85.0	38	147	153	159	165	170	176
87.2	39	144	149	155	160	166	171
89.5	40	140	146	151	156	162	167
91.7	41	137	142	147	152	158	163
94.0	42	133	139	144	149	154	159
96.2	43	130	135	140	145	151	155
98.4	44	127	132	137	142	147	152
100.7	45	125	129	134	139	144	149
102.9	46	122	126	131	136	141	145
105.1	47	119	124	128	133	138	142
107.4	48	117	121	126	130	135	139
109.6	49	114	119	123	128	132	136
111.8	50	112	116	121	125	130	134

Corrections:—For an increase of 1 mb. pressure subtract $\frac{1}{10}$ per cent. from the velocity; for 1a. add $\frac{1}{2}$ per cent.; for 1° in latitude subtract $1\frac{1}{2}$ per cent.

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Section V.—Computations related to the Theory of Probabilities.

- 1.—Errors of Observations and Variations due to Accidental Causes with an Application to Errors of Means and Normals. By R. CORLESS, M.A.
- 2.—The Practical Application of Statistical Methods to Meteorology. By W. H. DINES, F.R.S.

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SECTION V.

COMPUTATIONS RELATING TO THE THEORY OF PROBABILITIES.

TABLE OF CONTENTS.

	PAGE.
SUB-SECTION 1.—ERRORS OF OBSERVATION AND VARIATIONS DUE TO ACCIDENTAL CAUSES, WITH AN APPLICATION TO ERRORS OF MEANS AND NORMALS. BY R. CORLESS, M.A.	
Systematic Errors	3
Accidental Errors	5
Mean Error of an Observation... ..	7
Error of Mean Square	7
Probable Error	8
Errors of Normal Values	10
Example of computation	10
SUB-SECTION 2.—THE PRACTICAL APPLICATION OF STATISTICAL METHODS TO METEOROLOGY. BY W. H. DINES, F.R.S.	
Prefatory Note	13
CHAPTER.	
I.—The mean value of a varying quantity... ..	14
II.—The standard deviation and the normal distribution... ..	15
III.—Practical method of finding a standard deviation	18
IV.—The standard error. Casual and systematic errors	21
V.—The standard error of a mean	22
VI.—The probable error. Sundry corrections	26
VII.—The normal distribution	28
VIII.—Correlation	29
IX.—Practical method of finding a correlation coefficient	32
X.—Correction for errors of observation	34
XI.—Correction for periodic variations	35
XII.—The meaning of the correlation coefficient	38
XIII.—Partial correlation	40
XIV.—Practical methods of finding partial correlation coefficients	45
XV.—The regression equations... ..	48

Sub-section 1.—Errors of Observation and Variations due to Accidental Causes, with an Application to Errors of Means and Normals. By R. Corless, M.A.

When making a meteorological observation, e.g. reading the barometer, thermometer, raingauge glass, etc. there are many causes which prevent the observer from obtaining the true reading which would accurately represent the pressure, temperature, rainfall, etc. which it is desired to record. Some of the most important of these causes will be discussed later, but it will be convenient at the outset to define what is meant by an error.

Let X be the *true value* of any measurable quantity. This value is unknown and generally cannot be found exactly; nevertheless we know that it exists, and we endeavour to arrive at a value differing from it as little as possible. Let A be the reading of an instrument which purports to measure the quantity under consideration, and suppose that

$$X = A + a \dots \dots \dots (1)$$

Then a is defined to be the *error of the reading* A .

If A is less than X then a is a positive number.

If A is greater than X then a is a negative number.

Since X is in general unknown, and A is known, it follows that, in general, the error of a reading is unknown.

We now proceed to consider some of the causes to which the origin of the error a may be traced.

Systematic Errors.

(a) **Constant error.**—This is the technical term given to a class of errors the characteristics of which can be illustrated by the "index error" of a barometer. This is really the amount by which the brass scale is vertically displaced from its true position, and consequently is constant throughout the whole range of barometer readings. In the certificates of the National Physical Laboratory issued with instruments which have been tested there the barometer correction includes index error, error due to capillarity (another constant error), calibration and capacity errors, as well as other minor errors so that the index error does not appear separately. The important features about this error are that it is constant for each instrument throughout the scale, that it can be determined with some accuracy, and can then be allowed for.

Other examples of common constant errors are (i) the reading of the barometer as taken from a barograph chart which differs from the true reading by an amount due to error of setting of the pen of the instrument; similar errors arise in the use of thermographs and hygrographs. (ii) Error made by the recording pen of a wind-direction recorder owing to inaccurate determination of true north. (iii) Error of a thermometer which was graduated correctly at first, but owing to shrinkage of the glass, develops an error which is constant throughout the scale.

(b) **Graduated or regular error.**—This may be illustrated by the following example. The graduation of a thermometer is usually carried out by marking upon it the freezing and boiling points which are obtained respectively by immersing the instrument in melting ice, and in steam in equilibrium with water boiling under a pressure equivalent to that exerted by 760 mm. of mercury in latitude 45° (1013.3 millibar), and then dividing the interval on the stem between the marks thus found into 100 equal parts (Centigrade) or 180 equal parts (Fahrenheit). Now the law of variation of mercury with temperature is that any fixed mass of mercury increases in volume by the same amount for an increase of 1° temperature in any part of the temperature scale (within limits). Consequently when the operation of dividing the stem into equal parts is carried out, the assumption is tacitly made that the *volume* of mercury occupying the space in the bore of the tube between two consecutive divisions is constant for all such consecutive divisions; but this will only be the case if the bore of the tube is quite uniform throughout its length. A well-made thermometer is carefully calibrated before it is filled, so as to determine the allowance to be made for the error.

It will now be clear that errors of this class vary from one part of the scale of an instrument to another, but are *fixed* and determinable at every part. Thus they can be allowed for, and their effect on the reading removed.

Examples of errors of this type are extremely numerous. Thus, (i) all errors due to incorrect graduation, (ii) errors of construction in instruments, *e.g.*, making the barograph or thermograph arm too long or too short or using a chart ruled for an instrument of another pattern, using a rain gauge funnel too large or too small or not truly circular, (iii) errors due to our ignorance of natural truths, *e.g.*, error arising from an incorrect anemometer factor.

(c) **Variable errors which can usually be avoided.**—A typical example of this class is the condensation of spirit in the further end of a minimum thermometer, which has the effect of causing the instrument to read too low. The characteristic feature of the class is that the error is incapable of measurement, and may be large or small. This kind of error is one that occurs rarely or never in the readings of a careful observer.

Examples of the class are unfortunately very numerous.

(i) Failure to attend to instruments properly is a fruitful source of these errors, as everyone will know who has made meteorological observations. (ii) Errors of parallax must be included in this class, although sometimes they may fairly be classed among graduation errors; as when a barometer is mounted too high so that the observer's reading is *consistently* too high, or when thermometers are mounted in such a position as to preclude the observer from standing squarely in front of them. (iii) Hurried and carelessly recorded observations also give rise to variable but avoidable errors.

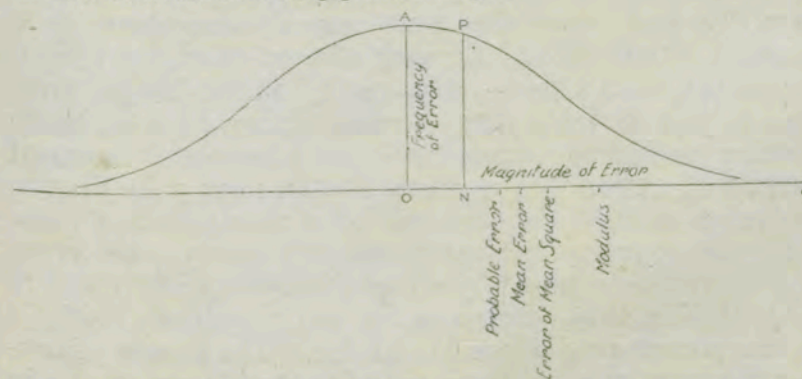
Thus far we have only considered errors of which the magnitude can be determined, or which ought seldom or never to occur. Those are all included in the general term *systematic error*. In what follows it is assumed that all such errors have been eliminated. It is found that errors still remain, which in any particular case can be traced to no definite cause, but are no doubt due to imperfections which always exist even in the best instruments, to the fallibility of the observer, and to a combination of conditions prevailing at the time of observation which had an effect on the observation, but which was not realised at all, or realised but imperfectly by the observer. These errors are known as—

Accidental errors.—They have been studied for a long time, and a method of dealing with them has been evolved of which a brief explanation will be given.

In our original equation (1)

$$X = A + a$$

we supposed that a was the total error of observation. Let us now suppose that all systematic errors of classes (a), (b) and (c) have been eliminated, so that a now represents an accidental error only. It is found that if a large number n of observations $A_1, A_2, A_3 \dots A_n$ are made of the quantity X , then the frequency of occurrence of the accidental errors of the several magnitudes $a_1, a_2, a_3 \dots a_n$ follows a certain law. In the first place it is evident that in practice accidental errors will usually be quite small, although large errors may occasionally occur. Consequently, provided n is large enough, several of the a 's will have equal values, and it might be foreseen that the number of a 's having the same small value $+a_1$ will be approximately equal to the number having the value $-a_1$, and that this number will be greater than the number of a 's having the same large value $+a_2$, which will approximate to the number with value $-a_2$. If we define the frequency f_p of an error equal to $\pm a_p$ to be the number of times that $+a_p$ or $-a_p$ occurs amongst the a 's, then we expect the curve connecting frequency with corresponding error, *i.e.*, the (f, a) curve to be of the following general shape.



Here PN represents f_p and ON represents a_p , also the curve is symmetrical about the f -axis (OA in the figure) so that f_p is also the frequency of the error $-a_p$.

As a matter of fact it is found that in the great majority of cases the curve of frequency is represented with some accuracy by an equation

$$f = Ke^{-a^2/c^2} \dots\dots\dots(2)$$

where K , c are constant for any particular set of observations, but vary greatly from one set to another, and e is the base of natural logarithms. Moreover, the same equation can be arrived at by theoretical considerations, and it is now recognised to represent the frequency curve so well that it has received the name of the "**normal curve of errors.**"

Putting $a=0$, we get $f=K$

$$f=0, \quad a=\pm\infty.$$

Thus $OA=K$, and the curve does not meet the Oa axis, although it rapidly approaches it as a increases numerically, and is distant from it by a very small amount when a is large. K is related to c by the equation $K = \frac{1}{c\sqrt{\pi}}$. c is called the modulus.

These remarks may appear to have little or no bearing on meteorological measurements, because it is not the custom to make a large number of measurements of the same element under a constant set of conditions. Indeed in many cases, only one really independent observation can be made, as in the case of rainfall; in others, as measurements of pressure and temperature, the quantity to be measured is very rarely sufficiently constant for a long series of observations to be carried out.

There is, however, a class of means to which the theory is found to be applicable and about which it gives important information. In the present state of our knowledge, we can assign no *a priori* reason to explain why, for example, the mean temperature at a place varies from year to year.* If we were not accustomed to expect it to vary we should naturally look upon the mean annual temperature as a constant number, fixed for each station. Consequently, it seems natural to treat variations in annual means from year to year as being due to causes similar to those which produce accidental errors, and to take the "**normal curve of error**" to represent the frequency distribution of variations from the "normal" of a long period of years. This natural procedure must be regarded merely as a working hypothesis, to be tested by actual examples, and if found to be applicable, to be adopted as the best hypothesis available in the present state of our knowledge. The process of testing the hypothesis has been applied to mean temperatures in

* See, however, Hildebrandsson "Quelques recherches sur les centres d'action de l'atmosphère."

a complete and critical way by M. Angot in his "Études sur le climat de France," with satisfactory results. He shows that the adoption of the curve of error and of the results arising therefrom, some of which are indicated below, leads to results in harmony with the observed figures. We may, therefore, premise that the method can be looked upon with considerable confidence to give trustworthy results. In the case of annual means, we have not a very long series of observations such as the method really requires, but we shall see how any false conclusion which might arise from this cause can be guarded against by calculating the "probable error" of the figure expressing the result.

We now proceed to define the positions of three important ordinates in Fig. 1, and to state the rules for arriving at their positions from a set of observations or means.

Mean error of an observation.—This is found by taking the mean of all the positive errors, and the mean of all the negative errors and then finding the numerical mean of the two. It is therefore the numerical mean of all the errors independent of sign. In our theoretical case, the mean of the positive errors will be the same as that of the negative errors, so that either of them will be the mean error. In practice, however, the two means are not identical, and the half of their sum is taken to be the mean error. Its value may be expressed by $\frac{\Sigma(a)}{n}$ which stands for $\frac{(a_1) + (a_2) + (a_3) + \dots + (a_n)}{n}$

where (a_p) means that the error a_p is to be taken with a + sign. In our alternative notation it is $\frac{f_1 a_1 + f_2 a_2 + \dots + f_p a_p}{f_1 + f_2 + \dots + f_p}$ where $f_1 + f_2 + \dots + f_p = n/2$. It can be shown that the mean error is theoretically $\frac{c}{\sqrt{\pi}}$ i.e., $c \times 0.564189$ ($\pi = 3.14159 \dots$). Its position on the Figure is found by measuring this length along the a axis on each side of O .

Error of Mean Square.—This expression introduces a measure of the error which avoids the inconvenience of dealing with positive and negative errors in different ways, as is necessary in calculating the mean error. It is found in the following way:—Add together the squares of all the errors, divide by the number of errors, and extract the square root of the quotient. It is thus the square root of the mean of the squares of the errors. It may be expressed

$$\frac{\Sigma a^2}{n} \text{ that is } \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}}$$

$$\text{or by } \sqrt{\frac{2f_1 a_1^2 + 2f_2 a_2^2 + \dots + 2f_p a_p^2}{2(f_1 + f_2 + \dots + f_p)}}$$

$$\text{which is } \sqrt{\frac{2(f_1 a_1^2 + f_2 a_2^2 + \dots + f_p a_p^2)}{n}}$$

$$\text{or } \sqrt{\frac{f_1 a_1^2 + f_2 a_2^2 + \dots + f_p a_p^2}{f_1 + f_2 + \dots + f_p}}$$

Theoretically for an indefinitely large series of observations the error of mean square is $\frac{c}{\sqrt{2}}$ i.e., $c \times 0.707107$.

Probable Error.—It has, however, become the general practice to employ another number, called the Probable Error, which is so chosen that among a very large number of observations there are as many errors larger than the probable error as there are smaller, thus, in long practice an error is as likely to exceed the probable error as it is to fall short of it. Another way of stating the same definition is that the odds for the occurrence of an error numerically exceeding the probable error are even. It will be observed that the word "probable" is used in a very special sense, and that the probable error is *not* the most likely value of an error, in fact the most likely value of an error is theoretically zero.

The definition means that the frequency of errors numerically less than the probable error is equal to the frequency of errors numerically greater, and hence that the probable error is that positive (or negative) error of which the corresponding ordinate divides the area of the half of figure 1 to the right (or left) of O into two equal portions. In this way it is found that

$$\text{Probable error} = c \times 0.476948.$$

Thus the magnitude of the different numbers used to express the amount of spread of the observations from the mean value increases from "Probable error" through "mean error" to "error of mean square." Also it is evident that having found one of the three numbers, then the other two can be immediately deduced from it.

$$\text{For } \frac{\text{Probable error}}{0.476948} = \frac{\text{Mean error}}{0.564189} = \frac{\text{Error of mean square}}{0.707107} \dots (3)$$

It will be seen from equation (3) that the probable error can be computed *either* from the mean error *or* from the error of mean square. In practice the two values will not be identical, and we cannot say which is to be preferred. It has been the almost universal custom, however, first to find the error of mean square, and then to use the relation

$$\begin{aligned} \text{Probable error} &= \frac{0.476948}{0.707107} \times \text{Error of mean square} \\ &= 0.674506 \times \text{Error of mean square.} \end{aligned}$$

The final formula for probable error is therefore

$$\begin{aligned} \text{P.E.} &= 0.67 \times \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}} = 0.67 \sqrt{\frac{\sum a^2}{n}} \\ \text{or } 0.67 \times \sqrt{\frac{f_1 a_1^2 + f_2 a_2^2 + \dots + f_p a_p^2}{n/2}} &= 0.67 \sqrt{\frac{\sum f a^2}{n/2}} \end{aligned}$$

according to convenience.

Throughout the above, we have treated X as if its value were accurately known, although it was stated at first that in general its value cannot be certainly found. In forming the mean error or error of mean square it is necessary to have a value of X to work with, in order to obtain the individual errors. What is done is to calculate from the observations the *most likely value* of X and to use this for the true value. In some cases the determination of the most-likely

value is a tedious and difficult matter, and it depends upon the computer's opinion to some extent. But by far the most important case occurring in meteorology is that in which there is no reason why one observation or mean should be regarded as more or less trustworthy than the others. This is so often the case that no other need be considered here. With equally trustworthy observations the most probable value is the ordinary mean, and the differences or *deviations* from the mean, each with its proper sign, are taken to be the errors of the observations. It will be readily understood, however, that in taking the mean of the observations to be the true value of X we are introducing a further error, namely, the error in the mean (r say) and hence that some of the deviations require a correction $+r$ and the remainder $-r$. This will obviously not affect the mean error, but it *does* affect the error of mean square, and it can be shewn that the

$$\text{Error of mean square of a measure} = \sqrt{\frac{\text{Sum of squares of deviations}}{n-1}}$$

$$\text{and that error of mean square of the mean of the measures} = \sqrt{\frac{\text{Sum of squares of deviations}}{n(n-1)}}$$

Hence probable error of a measure

$$\begin{aligned} &= 0.67 \times \sqrt{\frac{\text{Sum of squares of deviations}}{n-1}} \\ &= 0.67 \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n-1}} \dots \dots \dots (4) \end{aligned}$$

And probable error of the mean of the measures

$$\begin{aligned} &= 0.67 \times \sqrt{\frac{\text{Sum of squares of deviations}}{n(n-1)}} \\ &= 0.67 \times \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n(n-1)}} \dots \dots \dots (5) \end{aligned}$$

It will be noticed that the only effect of allowing for the fact that the mean of the observations is in general not the true value sought, is to change the error of mean square of a measure from

$$\sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}}$$

which was the value given before, to

$$\sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n-1}}$$

and the practical effect will be very small in most cases. The change in the probable error will be 0.67 of the change in the error of mean square.

It will further be observed that the probable error of the mean of a number of measures each of which is subject to the same probable error has been written down as $\frac{1}{\sqrt{n}}$ of that error. This is a very important theorem, and it may be well to state it in more general form:—

Probable error of a mean.—Suppose we have a number n of observations, each of which is as trustworthy as the rest as far as we

know, then each observation will have a definite probable error given by the formula (4). Let this error be E . Then the theorem states that the probable error of the *mean* of the observations = $\frac{E}{\sqrt{n}}$.

Errors of normal values.—The theorem has an important bearing on the question as to how long it is necessary to continue a series of observations in order to obtain a reliable normal value. For we may say that the probable error of a year's mean (say) must become approximately constant as the number of years increases *if we assume that no slow secular variations of climate are taking place.* We have at present little definite evidence of any such change in any place on the earth's surface, so that we shall assume that none occur. The outstanding variations from the mean from year to year are then subject to changes which we may describe as casual and arbitrary, so that there is no reason why the probable error of one year's mean should differ from any other. Expressed in symbols this is
$$\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n - 1}$$
 = constant for all values of n , or sum of squares of deviations varies as $(n - 1)$.

We can thus assume that in the above theorem E is constant, and we can now calculate what the probable error of the normal will be at the end of any period that we like to name, having given the probable error of the normal for n years say. For

$$\frac{\text{probable error of } n \text{ years' normal}}{\begin{matrix} & & m \\ \text{"} & \text{"} & \end{matrix}} = \frac{E}{\sqrt{n}} / \frac{E}{\sqrt{m}} = \sqrt{\frac{m}{n}} \dots(6)$$

or, in words, the probable error of the normal value of any element varies inversely as the square root of the number of years used in calculating the normal.

Example.

The following are the figures giving the mean height of the barometer read at 9 a.m. each day at St. Matthew's Vicarage, St. Helena. The second column gives the yearly means, and the mean of all the years 1893-1907 is inserted at the foot of the column. The third column gives the deviations from the mean in thousandths of an inch, *i.e.*, the several differences between the yearly means and the long period mean; and the fourth column gives the squares of these deviations. The sum of these squares is 1471, and the number of years is 15, so that the formula

$$\text{probable error of mean} = 0.67 \times \sqrt{\frac{\text{Sum of squares of deviations}}{n(n-1)}}$$

becomes

probable error in thousandths of an inch of the mean value
28.054 inches

$$= 0.67 \times \sqrt{\frac{1471}{15 \times 14}}$$

$$= 1.78$$

i.e., probable error of the mean value is .00178 inch or approximately 0.002 inch.

PRESSURE AT 9 A.M. AT ST. HELENA.

—	Pressures.	Deviation from mean in thousandths of an inch.	(Déviation). ²
	ins.		
1893... ..	28·038	—16	256
1894... ..	8·048	— 6	36
1895... ..	8·046	— 8	64
1896... ..	8·053	— 1	1
1897... ..	8·058	+ 4	16
1898... ..	8·041	—13	169
1899... ..	8·039	—15	225
1900... ..	8·059	+ 5	25
1901... ..	8·060	+ 6	36
1902... ..	8·059	+ 5	25
1903... ..	8·056	+ 2	4
1904... ..	8·066	+12	144
1905... ..	8·047	— 7	49
1906... ..	8·069	+15	225
1907... ..	8·068	+14	196
Mean 1893–1907 ...	28·054		1471

The probable error in thousandths of an inch of each year's mean will be

$$\begin{aligned} & 0.67 \times \sqrt{\frac{1471}{14}} \\ &= 0.67 \times \sqrt{105} \\ &= 6.91 \end{aligned}$$

or approximately

$$= 0.007 \text{ inch.}$$

This is the quantity that may be assumed to be constant when we are dealing with the question of accuracy of the observations.

Let n be the number of years required to give a normal of which the probable error does not exceed 0.001 inch. Then by equation (6), putting $m = 15$, n must not be less than the value given by

$$\frac{.001}{.002} = \sqrt{\frac{15}{n}}$$

i.e., $\frac{1}{4} = \frac{15}{n}$

$n = 60.$

and

Thus it will necessary to continue the observations for 45 years after 1907 in order to attain to the degree of exactness supposed.

It now becomes clear that the natural development of the theory compels us to take account of the number of years used in computing the normal, and hence that no difficulty need be felt on the score of the comparatively short series of

means to which the theory is applied, provided that the "normal curve of error" fits the observations reasonably well. In our actual example the series is so short, that it is impracticable to draw a regular frequency curve, and it would be unreasonable to expect this to be possible. The actual frequencies of the deviations are given in the following table:—

Deviation in 1000 in.	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Frequency...	1	1	0	1	0	0	0	0	1	1	1	0	0	0	0	1	0

Deviation in 1000 in.	+16	+15	+14	+13	+12	+11	+10	+9	+8	+7	+6	+5	+4	+3	+2	+1
Frequency...	0	1	1	0	1	0	0	0	0	0	1	2	1	0	1	0

The number of + deviations is 8 and of negative 7.

A fair test to apply to the deviations would be to ascertain whether the computed probable error agrees with the definition that errors numerically greater than the probable error are as likely to occur as errors less than the probable error. We found the probable error of a mean to be 0.007 i.e. 7 in the above table. The number of + and - deviations greater than 7 numerically is 7, and the number less than 7 is also 7, so that the agreement in this example, chosen quite at haphazard, is perfect.

Sub-section 2.—The Practical Application of Statistical Methods to Meteorology, by W. H. Dines, F.R.S.

PREFATORY NOTE.

The elementary formulæ of the science of statistics deserve to be much better known than is commonly the case, and it is with the hope of helping to bring about this desirable result that the following short notes and explanations have been written.

Perhaps no science has suffered so much from the want of what may be called statistical common sense as meteorology, and, since these notes are written for meteorologists, most of the examples have been taken from it, but the formulæ are perfectly general.

An attempt has been made to state the elementary formulæ relating to standard deviations, standard errors of a mean, and correlation coefficients, with as little mathematics as possible, but proofs have been given where it could be done simply. It is inevitable that some statements, the standard error of a correlation coefficient for example, should be made without proof. Also for the sake of brevity and to prevent confusion, certain difficulties, such for example, as that of the regression equation not being necessarily linear have been ignored.

It has seemed better to express a regression equation as the relationship between two quantities that we wish to find, and then to deduce the correlation coefficient as a means to that end, rather than to define the coefficient first and then deduce the equation.

Probably the most frequent use of statistics is to furnish arguments in favour of certain statements, and in very many cases, owing to the ordinary casual error, or to some unmentioned systematic error, the argument is fallacious. Many so-called educated people still believe in weather almanacks, simply on the ground that the special almanack they favour was right on such and such past occasions, the actual fact being that in our uncertain climate any statement in an almanack is bound to be right about as often as it is wrong from pure and simple chance alone. When all is said and done in statistical matters the element of chance still remains, but the use of proper methods will lead to statements such as the following: "The chances that these two things are in some way mutually connected are many millions to one,"

or it may be "are many hundreds to one," or, again: "This set of figures affords no proof whatever of any connection between the quantities, for they are grouped just as purely chance values might be." Where a strict analysis would produce this last statement, that which often appears is of the following nature: "Looking at these two curves we cannot fail to see the agreement between the quantities."

Where the chances are millions to one, for all practical purposes the case is one of certainty, the safety of a railway journey for example. But it is different if the chances are only hundreds to one, the case, say, of a deaf and short-sighted man going over a level railway crossing by himself.

Hence it is very desirable to know in statistical matters exactly what the chances are in favour of or against certain figures being significant, and in so far as casual errors are concerned this knowledge is attainable. It is much more difficult to locate and eliminate systematic errors.

It is very commonly supposed that the arithmetic required for obtaining standard deviations, correlation coefficients, etc., is very long and tedious, this is not the case in reality, for an accuracy that is quite ample for the purpose can be obtained with a moderate amount of work.

The method of forming a correlation table is not given, because in general in meteorological matters the number of observations is not great enough to make it worth while to treat them otherwise than separately. A competent computer will find no difficulty in collecting the observations in classes when the numbers justify such a procedure.

No references have been given, but I must express here my indebtedness to G. Udny Yule's book, "An Introduction to the Theory of Statistics."

CHAPTER I.—THE MEAN VALUE OF A VARYING QUANTITY.

It seems superfluous to explain the method by which the numerical mean of a set of observations can be obtained, since the explanation will be found in any book on arithmetic, but it is well to point out that the term "**numerical mean**" may in some cases be ambiguous. Thus the mean temperature of the atmosphere over a certain place should, perhaps, always denote the mean taken with regard to mass, but in reality it more often denotes the mean between certain limits of the values taken with regard to height, and the two values will in general differ widely. Hence it is always well when using the term "mean" to make sure that the context leaves no doubt as to what is meant.

In certain cases the actual numerical mean is not what is required. As an example, the mean annual temperature of a

place at which only irregular observations are obtainable may be taken. If in this case the observations are more or less concentrated into one season, the numerical mean represents that season rather than the year. If either summer or winter observations are absent or almost absent, then the figures are incapable of yielding a useful mean. No fixed rule can be laid down in such cases and no technical knowledge of statistical science will help; the application of a little common sense will in most cases show the best plan to adopt.

The term "**weighted mean**" is also used. Observations may be collected from various sources and it may be known that some sources are more reliable than others. If double weight be given to observations from one source, this means that each of such observations will be treated as though it were duplicated. In some instances, however, the so-called weighted mean is the true numerical mean. To obtain the average value of a bushel of corn, say, the market prices per bushel in various markets are taken. These prices are not equal and if a very high price ruled in a small market it would obviously be unfair to give this price equal weight with that of a low price prevailing in a large market where ten times as much corn was sold. A weight, therefore, proportional to the quantity sold in each market is given to the price for that market and the mean so formed. Obviously, the so-formed weighted mean is the true numerical mean value of the whole number of bushels of corn sold.

But the real difficulty of forming a true mean value in meteorological matters comes from the paucity of observations that are certainly free from systematic error. In the following six chapters it will be shown, not how the difficulty can be met, for that is impossible except by increasing the observations, but how much reliance may be placed upon a mean value deduced from a given set of observations.

CHAPTER II.—THE STANDARD DEVIATION AND THE NORMAL DISTRIBUTION.

In dealing with any varying quantity the three things that it is desirable to know are the mean value, the extent of the variation, and the manner of the variation.

Thus, if a person had to spend three months in a foreign country or a colony he would like to know, in order to provide himself with a suitable outfit, first, the mean temperature, secondly, the extent to which variations from that mean might go, and thirdly, whether extreme variations were frequent or rare.

The commonest standard for measuring the variation is known in statistical science as the **standard deviation**, and is found as follows:—

Suppose there are n values of a quantity, and that the mean value is X . Let the departures from the mean be $x_1, x_2, x_3 \dots x_n$, where $x_1, x_2 \dots$ &c., are numbers that may be either positive or negative. Then the standard deviation which we will denote by σ_x is

$$\sqrt{\frac{x_1^2 + x_2^2 + \dots + x_n^2}{n}}$$

As an example we will calculate the standard deviation of the following numbers 0, 1, 2, 3, 4, 5, 6, 7, and 8. The mean is obviously 4, and the departures from the mean, the $x_1, x_2 \dots$ &c., are

$$-4, -3, -2, -1, 0, 1, 2, 3, 4.$$

$$\therefore \text{the S.D.} = \sqrt{\frac{2(16+9+4+1)}{9}} = \sqrt{\frac{60}{9}} = 2.58.$$

It will be seen that if we had included one more number, 9, the mean value would have been 4.5, all the numbers to be squared would have had two figures, and the process would have been much longer and more troublesome. A method of avoiding extra work of this kind will be shown further on.

The reader will naturally wonder why the trouble of squaring the departures from the mean should be required when the numbers themselves show the variation quite plainly, and their arithmetical mean, apart from sign, is easily obtained. The reason is that the standard deviation as defined above has certain valuable properties, as will subsequently appear. Also, very little trouble is really required to obtain the value of the standard deviation to a degree of accuracy that is ample for all practical purposes.

The manner of the variation is also of some importance, but it will be assumed in the following pages that what is called the **normal distribution** prevails.

Suppose twenty coins thrown up and the number of heads counted, and further suppose the process repeated 10,000 times. The number of heads in each set must lie between 0 and 20 *inclusive*, and since heads and tails are equally likely, it is fair to conclude that in the long run the mean would be 10. Also the numbers 0 and 20 would very rarely occur, because it is extremely unlikely that a man should throw either heads or tails 20 times running, and so far as the chances are concerned he might equally well throw one coin 20 consecutive times to form a set as throw 20 coins at once. Now 0 or 20 show a departure of 10 from the mean, 1 or 19 departures of 9, and so on. It is fairly evident, therefore, that if we arranged the numbers of heads in each set as departures from 10, which is the mean, we should have 10,000 values all lying between +10 and -10, that amongst these numbers 10 would hardly appear at all, that nines would be very scarce, eights less so, but still scarce, whereas nothings, ones, twos and threes would be plentiful. This particular form of distribution is called the **normal distribution**. It is characteristic

of most cases of great groups of samples when a value above the mean is just as likely and no more so than a corresponding one below the mean.

The chance of any special number appearing can be ascertained theoretically, and it can also be shown that the standard deviation for this special case will be $\sqrt{\frac{1}{2} \times \frac{1}{2} \times 20} = \sqrt{5} = 2.23$. Apart from theory, many actual trials of this sort have been made, and theory and practice have been found to agree perfectly.

The result is that we get the following rough rules. If a quantity varies in the normal manner departures from the mean equal to 4 times the standard deviation will be very scarce, for practical purposes non-existent; against as much as 3 times or more the chances are 370 to 1, against twice the standard deviation they are 21 to 1, while it is an equal chance whether any special variation exceeds or is less than two-thirds of the standard deviation.

These rules must be accepted here, for proofs the reader must consult a text book on the theory of probabilities.

Before applying these rules it must of course be known that we are dealing with a normal distribution, but in the majority of cases the form of distribution approximates sufficiently to the normal to make the rules applicable.

The **standard deviation** is, so to speak, the unit to be employed in estimating the chances, hence its great importance in statistical matters.

The meaning of the Symbol Σx . The reader will understand that the process of obtaining a mean value involves the summation of a number of numerical values representing the successive readings or results of calculations for the same quantity. Mathematicians have a special notation to indicate the summation of a series of quantities of this kind which depend upon the symbol Σ (). The Greek letter Σ before the bracket shows that all the series of quantities represented in general by the symbol in the bracket are to be added. So the $\Sigma(x)$ means the summation of all the readings or results represented generally by x and specifically by the individual values $x_1, x_2, x_3, x_4 \dots$ and so on, for however many values there may be, if there are n such values x_n will finish the series. Thus $\Sigma(x) = x_1 + x_2 + \dots + x_n$. In the same way $\Sigma(x^2)$ means $x_1^2 + x_2^2 + x_3^2 + \dots + x_n^2$ and $\Sigma(xy)$ means $x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_n y_n$. It must be remembered that the summation which is here spoken of is *algebraical* summation, that is to say, the individual terms of which the series is composed may be positive or negative and must be treated accordingly in the summation. Thus the result of the whole summation may be positive if the positive terms are predominant or negative or zero. For example it follows from the definition of a mean value that the sum of the differences of a number of quantities from

their mean value is zero—we should express that in the notation here indicated as follows.— $(\bar{x}-x_1) + (\bar{x}-x_2) + (\bar{x}-x_3) + \dots + (\bar{x}-x_n) = 0$, which is written $\Sigma (\bar{x} - x) = 0$.

CHAPTER III.—PRACTICAL METHOD OF FINDING A STANDARD DEVIATION.

Take the quantities, which to be of much use should not be less than 30, and estimate or roughly calculate an approximate mean value, then write down in a column the departure of each value from this mean with its proper sign + or - before it.

It is of great importance that these differences should be expressed as whole numbers in suitable units, for if the unit be too large accuracy will be impaired, if too small, so that the numbers in the columns are large, the work is unnecessarily increased. It is of course useless to multiply the numbers to make them larger, because their accuracy cannot be thus increased, but it is often worth while to reduce them by dividing all by the same number. It is convenient to have the highest values in the column not much exceeding 20 or 25 since this will give sufficient accuracy, and it is easy to remember the squares of all whole numbers up to 20 or 25. If, therefore, these departures from the mean range up to 100 or so, divide them all by 3, if up to 300 divide them all by 10, and write down the whole number nearest to the third or tenth as the case may be, *i.e.*, $11 \div 3$ should be written as 4, not 3, and $198 \div 10$ as 20, not 19. Should the last figure be 5 omit it but put the nearest odd digit for the preceding figure, *i.e.*, write either 195, or 185 as 19, 115 as 11 and 125 as 13. The reason of course is to avoid the systematic error of either always adding or of always subtracting the fives.

The next step is to get the true mean. Add up all the positive numbers and separately add up all the negative numbers. If the two sums are identical the true mean has been chosen, but this is unlikely, and assuming the original values to be whole numbers the mean must be found to one decimal place. Let M be the mean already adopted, $M + c$ the true mean. Let the sum of the positive numbers be a , and of the negative numbers b . Then c will be equal to the difference between a and b divided by the total number of values dealt with. Obviously also if the negative values preponderate the estimated mean value M was too high and c must be subtracted. If the positive values preponderate c must be added. Also if we originally divided by 10 or 3 we must now multiply c by 10 or 3 before adding or subtracting.

The true mean is now found, and we have the figures necessary for getting out the **standard deviation**, but

before doing so it is a useful check on the arithmetic to see if it coincides with the mean found in the ordinary way.

Suppose we are dealing with n numbers, that n is large, say 100 or over, and c not greater than .5, the range as before not much exceeding 60. The standard deviation will not differ much from $\frac{5}{4}$ of $\frac{a+b}{n}$ where $\frac{a+b}{n}$ is the average departure from the mean without regard to sign. For many purposes this value will suffice, but if we require the value for purposes of correlation and it should chance that a correlation coefficient in which it is concerned should be nearly one, then a more accurate value is required.

To find it, proceed thus. If the numbers to be squared are mostly under 10 or 15, square them and add, but if a fair proportion of them exceed 15 or 20 the work may be abbreviated. Omit the last digit of the square, but if that digit equal or exceed 5 add one to the preceding digit. Thus the squares of 1 and 2 are neglected entirely, 3^2 counts as 1, 4^2 as 2, 5^2 as 3, ... 8^2 as 6, 11^2 as 12, 13^2 as 17, and so on. Now add the values so found and place a nought at the end, divide by n and extract the square root, both of which operations only take a few seconds if a slide rule be employed.

The principle on which this abbreviation is based is fairly obvious. If Σx^2 , the sum of the squares, is a large number running into thousands, and it must be so if many numbers of two digits have to be squared to form it, then its last two figures are of comparatively little consequence. It is hardly possible that the error caused by the abbreviation of the work can reach .5 %, whereas the casual error as we shall see subsequently can seldom be less than 5 %, hence to trouble about the .5 % is unnecessary.

The value thus obtained is the standard deviation only if M is the true mean. If not, then c^2 must be subtracted from $\frac{1}{n} \Sigma x^2$ before the square root is extracted.

For let x_1, x_2, \dots etc., be the numbers to be squared, y_1, y_2, \dots etc., the corresponding numbers when the true mean $M + c$ is used. Then $y_1 = x_1 - c$, $y_2 = x_2 - c$, and so on. $\Sigma (x)^2 = \Sigma (y + c)^2 = \Sigma (y^2) + 2c \Sigma (y) + \Sigma (c^2)$.

Now $\Sigma (y) = 0$ since the algebraical sum of the y 's must equal 0 the y 's being the departure from the mean; and $\Sigma (c^2) = n c^2$.

Hence $\Sigma (y^2) = \Sigma (x^2) - n c^2$, and we have the rule given above.

The reader may perhaps think that it would take him longer to adopt and use the above process than to go through the whole formal calculation, and if he is only going to work out two or three deviations that view is correct. The writer has worked out many hundred, perhaps thousands, of cases, and finds that by using the process and by increased practice he can now do in ten minutes or so what used to take two hours or more. A slide rule is of course a great saving of time, and since from the nature of the case a standard deviation

or a correlation coefficient cannot be reliable to more than two significant figures, the accuracy given by a slide rule is ample.

EXAMPLE OF METHOD.

Standard deviation of September rainfall. London. 25 years:—

Year.	Amount.	Departure from 1.50 in. divided by 5.	Squares.
	Ins.		
1888	1.14	- 7	5
1889	1.59	+ 2	—
189067	-17	29
189197	-11	12
1892	2.28	16	27
1893	1.09	- 8	6
1894	1.29	- 4	2
1895	1.11	- 8	6
1896	5.43	79	624
1897	2.93	29	84
189834	-23	53
1899	2.58	22	48
190078	-14	20
1901	1.46	- 1	—
1902	1.80	6	4
1903	2.62	22	48
1904	1.28	- 4	2
1905	1.78	6	4
1906	1.90	8	6
190757	-19	36
1908	1.39	- 2	—
1909	2.83	27	73
191059	-18	32
1911	1.34	- 3	1
1912	2.12	12	14
Total	41.88	-139 +229	11,360
Mean	1.68 in.	+ .0360	454

In the above figures the amounts were written down and a guess at the mean of 1.50 in. was made. In the second and third columns the figures which are entered are the differences, disregarding the decimal point, divided by 5, since this gives numbers of about the right size. Adding these columns it is found that 1.50 was too low a value by $5 \times .036$ or .18. The mean value is therefore 1.68.

The sum of the squares is approximately 11,360, and the mean square is 454.4. The root is 21.3 and multiplying by 5 the S.D. is 106. This must be corrected because it is based on a mean of 1.50 instead of 1.68. The true S.D. is $\sqrt{106^2 - 18^2} = 104$, i.e., 1.04 inches.

CHAPTER IV.—THE STANDARD ERROR. CASUAL AND SYSTEMATIC ERROR.

The quantity which has been described under the name of the standard deviation is also, in certain cases, known as the **standard error**.

Suppose a joiner were given a pattern and told to cut fifty pieces of wood of the same length. Those fifty pieces of wood would be of approximately the same length, but if measured by refined methods differences would appear and the *standard deviation* of their lengths could be calculated. This might equally well be called a *standard error*. Or the original pattern might be measured by fifty different people and if each person had attempted to go to the nearest thousandth of an inch fifty more or less different numbers would be obtained and their *standard deviation* would be the *standard error* of measurement or observation.

Errors of this sort are known as **casual errors**, that is to say they occur by chance, each one is entirely independent of every other one, they are just as likely to be positive as negative, and in magnitude they follow the rule of the normal distribution. They involve much trouble and uncertainty, but if the number of values dealt with is sufficiently large they always neutralize each other in taking the mean.

A **systematic error** is of a different character and must be carefully guarded against; suppose in the case mentioned above that the rule or measuring machine used by the fifty people in measuring the patterns were itself incorrect, no increase in the number of measurements would eliminate the error, which would be called "*systematic*."

Now, unfortunately, false deductions from statistics are quite common, so much so that it has been said that statistics will prove anything, but the fallacy of these deductions generally rests either upon the casual error of a mean value, a point that will be considered in the next chapter, or upon a systematic error.

The whole science is full of pitfalls for the unwary, and with the best intention possible it is easy to make a mistake, still statistics will not "prove anything," and while preventable ignorance will account for much, it is not easy always to acquit the users of statistics of something worse than ignorance. If all the parliamentary electors of the kingdom became experts in the science of statistics political speeches would necessarily take rather a different form.

One or two cases of systematic errors are given below as examples of the misleading effect that may be inadvertently produced by figures which are in themselves perfectly accurate, and which are quoted in perfect good faith.

The County Medical Officer for Surrey states in his official report that the death rate for Surrey for the year 1910 was 9.6 per thousand of the population, 11.0 for 1911 and 9.7 for 1912. It is a well recognised rule and easily proved that if, in a stable community, we divide 1,000 by the death rate we obtain the average length of life, and hence judging by the death rate in Surrey for the last three years, the average length of life in Surrey is about 100 years, a palpably absurd statement. Yet the figures are no doubt correct. In several of our colonies equally low or even lower death rates are found, yet neither in Canada, nor in New Zealand nor Australia do men, as a rule, live to be 100 years old.

The truth is that before a death rate can be used as a test of the health of one locality compared with another, as it constantly is, we must be certain that the conditions as to age and sex, but chiefly as to age, in the two places are the same. In Surrey a large proportion of the population consists of domestic servants between the ages of 15 and 50 between which ages deaths are rare, hence the death rate is low. That affords no reason to suppose that the climate of Surrey is especially favourable, or the sanitation better than in other counties; that may or may not be the case. Similarly with the colonies. Old people do not emigrate as a rule nor quite young children, the age distribution in a growing colony is very favourable for a low death rate, and introduces a systematic error. The inducement of a low death rate ought not therefore to be held out to intending emigrants as it often is; to do so is to make improper use of statistics.

Another instance may be taken from meteorology. The maximum or minimum temperature at a place A is compared with the mean of another place B, the statement in a newspaper often running in this form. The maximum temperature at — yesterday was — and was — above that of the preceding 50 years mean. If the mean referred to is that of another station, as it often is, the statement made may be very misleading for there is no security that the two stations even though near together have the same mean maximum temperature. There may be a systematic error of 2° or even more in the standard of comparison, and an entirely false view of the character of the season is conveyed to the reader if he reads the statements day after day.

CHAPTER V.—THE STANDARD ERROR OF A MEAN.

This might equally well be called the *error of sampling* and in some books it is so described. The problem to be solved is this: a 20 or a 30 years' mean is formed, to what extent may this reasonably be expected to differ from a 100 years' mean. Or, supposing 100 men from a certain district are measured in order to ascertain the average height of the men of that district, if another 100 were measured, to what extent might the two means differ?

By making certain assumptions an answer can be given to questions of this sort. If σ be the standard deviation

of the quantity considered, and n the number from which the mean value is formed, then $\frac{\sigma}{\sqrt{n}}$ is the **standard error of the mean value**.

The meaning of this statement is as follows. If we had a very long series of observations, let us say 1,000 and divided them into groups of 20 and then took the mean of each group we should have 50 values, having, when combined, the same mean value as the 1,000 but differing between themselves. Their standard deviation, or in this case we might better term it the standard error of these means would be

$\frac{\sigma}{\sqrt{20}}$, and the distribution of the variations, in most cases, would be normal, and therefore we might even get a value as far as $\frac{4\sigma}{\sqrt{20}}$ or $.89\sigma$ from the true mean by the rule that has been given; but that amount is hardly likely, though as much as $\frac{3\sigma}{\sqrt{20}}$ or $.66\sigma$ might well occur.

Putting the matter the other way, if we have a mean value from a sample of 20 years the true mean may differ from that of the sample by as much as $.66\sigma$ or even possibly by as much as $.9\sigma$. In this case, however, we do not know σ , but have to use its approximate value derived from the 20 figures.

To take a definite example, a 25 years' mean value of the rainfall for a London station for a month. We shall not make any serious error in taking the standard deviation of a month's rain in London as 1.00, it would certainly lie between .90 and 1.20 in. The standard error of a 25 years' mean is therefore $1.00 \div \sqrt{25} = .20$ in. Hence there need be no surprise at finding a 25 years' mean monthly rainfall as much as .60 in. out. Similarly the 100 years' mean might be .30 in. out.

The formula, $SD \text{ of mean} = \frac{\sigma}{\sqrt{n}}$, has been given without proof. It may be proved, and the assumption that must be made pointed out.

Let Z , X , and Y be three variable quantities such that $Z = X + Y$, σ_x , σ_y , and σ_z their standard deviations. Let x , y , and z be the corresponding departures of each from its mean value. Then, by definition, $\sigma_x = \sqrt{\frac{\sum x^2}{n}}$, $\sigma_y = \sqrt{\frac{\sum y^2}{n}}$, $\sigma_z = \sqrt{\frac{\sum z^2}{n}}$. The departures from the mean are so related that $z_1 = x_1 + y_1$, $z_2 = x_2 + y_2 \dots$ &c.

Squaring and adding $\sum (z^2) = \sum (x^2) + \sum (y^2) + 2 \sum (xy)$, or, dividing by n , $\sigma_z^2 = \sigma_x^2 + \sigma_y^2 + \frac{2}{n} \sum (xy)$. If X and Y are quantities that have no connection with each other, the term $\sum (xy)$ will be very small, for $\sum (xy) = x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots$ &c. In this set of terms some products will be positive and some will be negative, some small, some large, but since X and Y are independent of each other there will be no bias either way, and, in the long run, n being a large number, the positive values will neutralize the negative values,

and the algebraical sum $\Sigma(xy)$ will be quite small in comparison with the other terms of the equation. Hence we may write $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$. Now let X and Y be exactly similar quantities so that $\sigma_x = \sigma_y$, and we have $\sigma_z^2 = 2\sigma_x^2$.

Introducing another similar quantity X by the same process $\sigma_z^2 = 3\sigma_x^2$ and so on. This may be written $\sigma_z = \sigma_x \sqrt{3}$ and in general $\sigma_z = \sigma_x \sqrt{n}$, where σ_z is the standard deviation of the sum of n similar quantities each of which has a standard deviation σ_x . Dividing by n we have the formula for the standard deviation of a mean used above, for Z being the sum of the n quantities, $\frac{Z}{n}$ is their mean; $\frac{\sigma_z}{n}$ is the standard deviation of the mean and equals $\frac{\sigma_x \sqrt{n}}{n} = \frac{\sigma_x}{\sqrt{n}}$.

The assumption made is most important. If x and y were not independent $\Sigma(xy)$ could not be taken as so small that we might neglect it, and our proof would fail.

In practice the difficulty often is to know whether or no the values we are dealing with are really independent each one of the other. If they are then the formula may be used with the greatest confidence.

An example may be taken from the variation of temperature. In the neighbourhood of London, confining our attention to one special month of the year, the standard deviation of the mean daily temperature is about 7°F . The mean of a month is taken from 30 or 31 days, and applying the formula the standard deviation for a monthly mean is $7/\sqrt{30}$ and equals 1.3°F . nearly. But in reality the *S.D.* of a monthly mean is considerably more than this, 3.0° perhaps, the formula will not apply. The reason is that daily temperatures on consecutive days are not independent of each other; if it has been warm for a week there is a decided bias in favour of the following days being warm, and conversely. But the formula would be applicable to a 30 years' mean of a given date, for one cannot suppose that the temperature on a given date in one year can be dependent in any way upon the temperature on the same date in a preceding or succeeding year.

The extent to which an average obtained from a small number of cases may depart from the true average on account of the number of cases being insufficient has been discussed above, but in most cases of statistics the true average is unknown, and the practical question comes to this. The means of two samples are more different than was expected, can their difference be due entirely to chance or is there some genuine cause for it? The usual tendency of mankind is to assign a cause for purely casual differences, or to use the differences to support some preconceived idea. Hence it is very useful to have some criterion by which to know how great the casual difference may be between two means that ought to be the same.

It was shown above that the standard deviation of the sum of two similar variables each with an *S.D.* σ is equal to $\sigma\sqrt{2}$. Exactly the same proof holds for the difference as well as the sum of the quantities. Hence if there are two values each derived from n cases the standard error of the difference will be $\frac{\sigma\sqrt{2}}{\sqrt{n}}$ and unless the difference in question exceeds some three times this there is no special reason to consider it as significant.

The number of cases on which the two mean values depend may be different, m and n , say, the standard error then becomes $\sigma \sqrt{\frac{1}{n} + \frac{1}{m}}$.

The difficulty in these cases is that we do not know the value of σ ; the $2n$, or the $m+n$, cases can only give us an approximate value; the σ of the two samples may be different. The choice lies between the weighted mean of the two samples, or using $\sqrt{\frac{\sigma_n^2}{n} + \frac{\sigma_m^2}{m}}$ instead of $\sigma \sqrt{\frac{1}{n} + \frac{1}{m}}$.

As an example, consider the case of our winter temperature. Taking the observations for the last 100 years, what difference between the mean of the first 50 and of the last 50 years is necessary to prove that the winters are becoming milder? The standard error of the difference between two 50 years' means is $\sigma \sqrt{\frac{2}{50}} = \frac{\sigma}{5}$. To make fairly certain that a secular change is occurring the difference must reach $\frac{3}{5}\sigma$, and to make practically

certain it must reach $\frac{4}{5}\sigma$. The difficulty is to determine σ . If a secular change is really occurring and σ be calculated from the whole 100 years in the usual way, the value so found will obviously be too high and it would be better to calculate the σ separately for each part and take the mean. An approximate value of σ may be taken as 2°F ., but records from properly verified instruments with identical exposure and environment for 100 years are not available. What evidence there is goes to show that a difference of as much as 2°F . between the earlier and later mean has occurred, whereas accepting the standard deviation as two degrees F ., $\frac{8}{5}$ is sufficient to prove the change, hence the statistical evidence agrees with the popular belief of the country districts in saying that the English winter is becoming milder.

CHAPTER VI.—THE PROBABLE ERROR. SUNDRY CORRECTIONS.

In a previous chapter the error of a mean value was discussed, but all statistical quantities are liable to error, the standard error itself included.

Firstly it is desirable to explain the term 'probable error,' a term frequently used in statistics.

If a man measures the same quantity fifty times and does so carefully and conscientiously, his measurements will not agree, some will be larger some smaller than others, the differences may be very little but they will exist, and it is obviously desirable to know what the magnitude of the error may be. The standard error, as has been already explained, is the standard deviation from the mean of the set of numbers that represent his various measurements, and the probable error is two-thirds, strictly $\cdot 674$, of this. The meaning of the term is this. An error of observation follows the rule called the *normal distribution* and in such a case in the long run, one half the errors will differ from the mean by less than $2/3$ of the standard deviation and the other half by more. Hence the term. If M be the mean, and E the probable error, then half the measurements will lie between $M + E$ and $M - E$ and half will lie without the range $M + E$ to $M - E$ altogether. The chances therefore are equal whether or no a measurement differ from the mean by more or less than E .

The term was first employed with reference to errors of observation but has now become extended to all cases. It is only applicable when the distribution is normal, whereas the standard error is applicable to all cases.

It may be stated here without proof that the standard error of a standard deviation from n observations, n not being too small, is $\frac{\sigma}{\sqrt{2n}}$. This is the purely casual error.

But there are other sources of error which may sometimes be allowed for.

Correction for the Observational Error.

Returning to a previous illustration let us consider the standard deviation of 50 pieces of wood cut by a man A and measured by a man B. The standard deviation is that given by the figures written down by B, but two extreme cases are possible. A may have cut the pieces into different lengths, and B measured them with great accuracy; in this case σ would be the standard deviation of the

length; or, secondly, A may have cut all the pieces carefully to one length, and B may have measured them badly, here the same quantity σ is the standard error of B's measurement.

In general these two cases are combined, and the standard deviation σ_m is dependent on the real deviation σ , and the standard error of measurement σ_e . It has already been proved in such a case that if the error of observation or measurement is independent of the real variation, then σ_m^2 , the apparent deviation $= \sigma^2$, the real deviation, $+ \sigma_e^2$ the deviation of the error, or $\sigma^2 = \sigma_m^2 - \sigma_e^2$. Thus, if we know σ_e we can correct σ_m and obtain the true value σ .

Another case is where a quantity is subject to a steady progressive change, or to a periodic variation. The death rate of England, or, indeed, of almost any European country, is an instance of the first kind, the annual change of temperature an instance of the second. The death rate of Great Britain has decreased steadily for the last forty years at a rate of about 1 per thousand for each ten years. This steady decline may be ascribed to increasing wealth and improving sanitation, the fluctuations may be ascribed chiefly to the weather, and it is desirable to be able to separate the standard deviation of a period of say 40 years into its two separate parts. Considering the steady decline of a 41 years' period alone, the departures from the mean will run thus 0, $\cdot 1$, $\cdot 2$, $\cdot 3$. . . &c., up to $2\cdot 0$, and also $-\cdot 1$, $-\cdot 2$, $-\cdot 3$. . . &c., down to $-2\cdot 0$. Squaring and adding, the sum, by a well-known formula for the sum of the squares of the first n natural numbers, becomes $\frac{2}{100} \left(\frac{20 \times 21 \times 41}{6} \right)$, the mean square value is $1\cdot 4$, and taking the square root the standard deviation is $1\cdot 18$. If we assume that the casual variations are entirely independent of the steady decline, the rule $\sigma_z^2 = \sigma_r^2 + \sigma_e^2$ will hold. Thus, taking σ_z as the value calculated from the figures in the usual way, the *SD* of the casual variation, apart from the steady decline, is given by the equation $\sigma^2 = \sigma_z^2 - (1\cdot 18)^2$. In this particular instance the correction would be a large one.

The other case is a periodic variation, the annual temperature change at a station in England for example. The value may be put in the form of a sine curve, $T = a \sin x$, where a is the amplitude. Making the reasonable assumption that casual variations of temperature from day to day are independent of the time of year, the *SD* of a year's observations is formed in two independent ways, one the regular variation with standard deviation σ_a say; the other the casual variation σ_e ; then, if σ_o be the observed deviation $\sigma_o^2 = \sigma_a^2 + \sigma_e^2$. If the observations are distributed with fair uniformity over the year σ_a can be calculated by summing a trigonometrical series, and we shall get $\sigma_a^2 = \frac{a^2}{2}$ where a is the amplitude. (See page 37.)

In this special case σ_o is about 6°C , a is also about 6°C , hence both σ_a and σ_e lie between 4° and 5°C .

In both cases it would be possible first to correct the quantity for the date, and then to obtain the standard deviation from the corrected values. For a small number of observations this would be the simplest plan, but it would not be so for a large number. In a later chapter this mode of correction has been applied to a column of figures where a steady decline occurs, before the figures are entered.

CHAPTER VII.—THE NORMAL DISTRIBUTION.

It has been stated that departures from a mean or expected value are generally distributed in a certain way called the **normal distribution**, or **law of facility**, but it is necessary to explain this somewhat more fully. The matter is put most simply by means of an example.

Any person who has added up many columns of figures of definite length, say thirty to the column, to represent daily values for a month, has perhaps noticed the following peculiarities. If the numbers to be added mostly reach at least three figures, the total of the digits column will generally be between about 100 and 170, but very rarely as low a number as 80 or as high as 190 may appear. He will notice too that the mean of the sums comes to about 135 and that numbers between 120 and 150 are most frequent. The reason is this: If a number reaches three figures most probably the last figure is purely casual, thus 0 is just as likely as 9, 1 as 8, and so on. The mean of these numbers will in the long run be 4·5, and the mean of 30 of them will be 135. The standard deviation of these digits works out as 2·87, and by the rule already given the standard deviation of the sum of 30 of them is $2·87 \times \sqrt{30} = 15·7$. If then the distribution is normal, in accordance with the rule already given, half the totals of the digits column will lie between $135 \pm \frac{2}{3}$ of 15·7, that is between 124 and 146, and few only will be outside the limits $135 \pm 3 \times 15·7$, that is below 88 or above 182.

Any person who will honestly give this a fair trial, will soon become convinced of the truth of the formula for a mean value and of the rule concerning the chances of certain values occurring, but he must make sure that his last figure is purely casual. In this particular case the distribution of the totals is normal or very nearly so, but the distribution of the digits is not normal, for it is equal for all values from 0 to 9, beyond which limits no value is possible.

The general rule is that where a value depends upon a fair number of independent causes none of which greatly preponderate, its distribution will be normal, even though the distribution of the individual causes is not normal.

A common case is when either the positive or the negative values are less numerous but of greater magnitude than those of opposite sign. The height of the barometer at sea level is an example. In England the range may be from about 2·00 inches below to about 1·00 inches above the mean. Still the departure from the normal, if we use the total range as the test, is not much out. The standard deviation is about ·35 to ·40 inches. A range of six times this, 2·20 inches, covers almost every case, 28·50 to 30·70, while eight times covers every recorded instance. (The recorded instances for Scotland rather exceed the eight times, but the standard deviation for Scotland is greater than that for the South-East of England.)

The daily rainfall is another example, but here the range is enormously long on the positive side. For London the mean is ·07,

the most common entry is zero, so that the extreme and also the most frequent value is --·07. Values of +2·00 are on record and given a sufficiently long period it is known that even 6·00 inches in the day might occur. But taking the monthly mean, 30 or 31 days, the values belong approximately to a normal distribution, for the standard deviation is about 1·00, and six times this covers the ordinary range. The positive departures are still greater and therefore less numerous than the negative, but the discrepancy is greatly reduced from that prevailing for the daily fall.

As a general rule, therefore, mean values will follow the rule of the normal distribution, and for other values a rough test may be applied by noting whether the range of six times the standard deviation covers nearly all the observations, and whether about half the whole set are included within the limits, mean $\pm \frac{2}{3}$ standard deviation.

Of course printer's errors or clerical errors do not come within the scope of these rules, they may be of any magnitude, and in the absence of any precise information it is best to cut out any very discordant value or pair of values.

It has been stated that the standard deviation may be taken as equal to 1·25 times the average value of the departures from the mean taken without regard to sign. This rule depends upon the distribution being normal and also upon the number of observations being sufficiently large to reduce the standard error to reasonable proportions. Conversely, if the given ratio, 4:5, is found between the quantities, or a fair approximation to it, it is a reasonable inference that the distribution is normal or nearly so.

CHAPTER VIII.—CORRELATION.

In previous chapters the variation of one single quantity has been considered, but the interest of statistics more often lies in the question whether there is any connection between two varying quantities, and, if so, how much. Innumerable questions of this sort arise, a correct answer to which is often of great importance, and if the necessary statistical tables are available and only two variables are involved, a decisive answer can be given, but often more than two variables are involved, or the necessary data are not available.

The simplest and best method of dealing with two variables is to plot them on squared paper. This affords more information than any other method, but the result cannot be tabulated and cannot practically be extended to more than two variables. The alternative is to form the so-called regression equations and correlation coefficient.

The problem to be solved is this: there are two variables, represented by X and Y , their departures from these mean values are denoted by x and y in general, but by $x_1 y_1, x_2 y_2, \dots$ &c. for special occasions. Can we find some definite relationship between X and Y so that if we know x_1 we can make a fairly good guess at y_1 , or if we know y_3 we can make a fairly good guess at x_3 ? A clearer idea will be given by taking a special instance. Can we discover from the tabulated results of the last 50 years any connection between the weather and the general health of the community, so that if we knew, for example, what the mean temperature of next summer was to be we could also foretell the death rate? In this case, X would denote the mean summer temperature, Y the death rate for the summer quarter as published by the Registrar General, x_n and y_n the departures of these quantities from their means for some special year denoted by n . Looking back at the records of past years it would be seen that there was a rough sort of proportionality between x_n and y_n , that in general they both had the same sign, and that when x_n was large y_n was also large. A fair representation would be given by the relationship $y = bx$, and the question would be what value of b best fits the general conditions. If $y_1 = bx_1$ exactly, then $y_1 - bx_1 = 0$, but in general $y - bx$ would have a certain value either positive or negative, mostly small if X and Y were so related that, knowing one, the other was also almost known, mostly large if X and Y were independent of each other. Now $y - bx$ may be positive or negative but $(y - bx)^2$ must be positive and b will have its most suitable value if it be chosen so that $\Sigma(y - bx)^2$ may be as small as possible.

Hence put $\Sigma(y - bx)^2 = a$ minimum.

Differentiating with regard to b and equating to zero

$$\Sigma x(y - bx) = 0$$

$$\Sigma(xy) = b \Sigma(x^2)$$

$$b = \frac{\Sigma(xy)}{\Sigma(x^2)}$$

This relation determines b , for $\Sigma(xy)$ is easily computed, but it is convenient, as will be seen later on, to put it in a rather different form. $\sqrt{\frac{\Sigma(x^2)}{n}}$ or σ_x is the standard deviation of X and σ_y that of Y .

Write the equation in the form $b = \frac{\Sigma(xy)}{n \sigma_x \sigma_y} \times \frac{\sigma_y}{\sigma_x}$, and let r stand for $\frac{\Sigma(xy)}{n \sigma_x \sigma_y}$, so that $b = r \frac{\sigma_y}{\sigma_x}$, then r is called the correlation coefficient between X and Y .

The reader who cannot follow the method of the differential calculus can still see that the correlation coefficient r is a very convenient measure of the connection between X and Y . The sum of the products, $\Sigma(xy)$ is small if the variables X and Y are independent, for in that case, as has already been pointed out, the positive and the negative values of the individual products neutralize each other. Also r is independent of the units adopted to measure the quantities, an important consideration. For if $r = \frac{\Sigma(xy)}{n \sigma_x \sigma_y}$ and x is measured in a new unit, inches, say, instead of feet, then $\Sigma(xy)$ becomes twelve times as great as before, but σ_x too is measured in the new unit and it also becomes twelve times as great, and r remains unaltered by the change. Similarly for y , and thus r is seen to be a

pure number or ratio. It cannot exceed numerically $+1$ or -1 , it will be nearly one if the connection between the quantities is very close, actually one if they are strictly proportional to each other. The interpretation of its meaning may in some cases be difficult but a correlation coefficient is an impartial measure of the relationship between the two quantities as shown by the set of observations considered.

The correlation coefficient was first suggested by Sir Francis Galton, and for some time was called Galton's function. It is a most important quantity in statistical matters.

The type of equation $y = bx$ used above was called by Galton a "regression equation."

The usual practice is to define a correlation coefficient as equal to $\frac{\Sigma(xy)}{n \sigma_x \sigma_y}$ and then deduce its properties, but it seems better to deduce the correlation from the fact that it makes the standard deviation of $y - bx$ a minimum. This expression is the error of the estimate made when Y is estimated from the known value of X ; to do this we use the regression equation $y = bx$, or as it may be written $Y = Y_m + bx$, where Y_m is the mean value. Obviously this standard error ought to be as small as possible, and the value of r as determined above makes it so.

The sum of the squares of the error of estimate may be put into another form.

$$\begin{aligned} \Sigma(y - bx)^2 &= \Sigma(y^2) - 2b \Sigma(xy) + b^2 \Sigma(x^2) \\ &= n \sigma_y^2 - 2 \frac{\{\Sigma(xy)\}^2}{n \sigma_x^2} + \frac{\{\Sigma(xy)\}^2}{n^2 \sigma_x^4} n \sigma_x^2 \\ &= n \sigma_y^2 - \frac{\{\Sigma(xy)\}^2}{n \sigma_x^2} = n \sigma_y^2 (1 - r^2). \end{aligned}$$

That is, the standard error of the estimate is $\sigma_y \sqrt{1 - r^2}$. Obviously, in the whole of the above investigation, the letters x and y might have been interchanged, so that if we were estimating X from the known value of Y the standard error would be $\sigma_x \sqrt{1 - r^2}$.

There are two ways of looking at the correlation coefficient, it may be considered in connection with the products $\Sigma(xy)$ from which it is formed, or it may be considered from the point of view of the regression equation.

If there is a close relationship between the quantities then r will be nearly 1, if the relationship is perfect then $r = \pm 1$. Thus, to return to our special case, if a hot summer is always associated with a high death rate and a cold summer with a low death rate, then the positive x 's will be associated with the positive y 's, and the negative x 's with the negative y 's.

Since the negative quantities when multiplied together give a positive product the terms in the sum $\Sigma(xy)$ will all be of the same sign and the total will be positive. Conversely, if a hot summer is healthy and a cold one unhealthy then the positive x 's are associated with the negative y 's, the numbers in the column forming $\Sigma(xy)$ will nearly all be negative numbers, and the total will give a large negative number. Then again, if there is no connection neither positive nor negative will preponderate in the products and $\Sigma(xy)$ will be small. Now r must be nearly 1 numerically, or small, according as $\frac{\Sigma(xy)}{n}$ is large

or small, and thus the decimal defining r is a good measure of the association between the quantities. But a caution must be added. The value of r is useless as a criterion unless it depends on a reasonably large number of independent observations, 25 perhaps as a minimum.

A precisely similar conclusion will be reached if we look at the matter from the point of view of the regression equations. These are usually written in the form $x = r \frac{\sigma_x}{\sigma_y} y$ and $y = r \frac{\sigma_y}{\sigma_x} x$, but this is apt to convey a false impression for it implies at first sight that x and y are subject to a fixed relationship.

It is better to write $x = r \frac{\sigma_x}{\sigma_y} y + \epsilon$, where r , σ_x and σ_y are constants and x , y , and ϵ variable. Then $\epsilon^2 = (x - r \frac{\sigma_x}{\sigma_y} y)^2$.

It has been proved above that $\Sigma(x - r \frac{\sigma_x}{\sigma_y} y)^2 = n \sigma^2 (1 - r^2)$, thus $\sigma_\epsilon = \sigma_x \sqrt{1 - r^2}$. If $r = 0$, $\sigma_\epsilon = \sigma_x$; if $r = 1$, $\sigma_\epsilon = 0$. Now the regression equation is useful if ϵ is mostly small compared with x , useless if ϵ is mostly as large as x ; in the first case $\sigma_\epsilon / \sigma_x$ is small, in the second case $\sigma_\epsilon / \sigma_x$ nearly equals 1. (It cannot exceed 1.) But the ratio $\sigma_\epsilon / \sigma_x = \sqrt{1 - r^2}$. Hence the same conclusion as before is reached, if r is small there is no relationship between X and Y , for knowing Y does not help us to estimate X , but if r is nearly 1 there is a relationship, because then if we know Y we also know X with but a small error.

CHAPTER IX.—PRACTICAL METHOD OF OBTAINING A CORRELATION COEFFICIENT.

The quantity to be calculated is $\frac{\Sigma(xy)}{n \sigma_x \sigma_y}$ and it is merely a matter of multiplying, adding, etc. It has been shown how to obtain the values of the standard deviations σ_x and σ_y , and the process for getting $\Sigma(xy)$ is very similar. Here, also, it is desirable to choose units so that numbers much exceeding 20 shall not have to be multiplied together. It is not necessary to find the exact mean values at first, any more than it is in the case of the standard deviations, but the necessary correction if this is not done may be of far more importance.

In forming the product sum $\Sigma(xy)$ it is well to use two columns, one for the positive values the other for the negative values. The sign of the coefficient + or - of course depends upon whether the positive or the negative total is the greater. If the estimated means for the two variables are both too high or both too low, that is to say, if they are both of the same sign, the positive products will evidently be increased, and $\Sigma(xy)$ will, in the algebraical sense, be too great, and conversely if the departures from the mean are of different signs. It is easy to obtain the necessary correction. Let the estimated mean of the X quantity be a too low and of the Y quantity

b too low. $\Sigma(xy)$ is what has been obtained, but $\Sigma(x - a)(y - b)$ is what ought to have been obtained.

$\Sigma(x - a)(y - b) = \Sigma xy - a \Sigma y - b \Sigma x + \Sigma(ab)$
 $= \Sigma(xy) - a \Sigma(y - b) - b \Sigma(x - a) - 2 \Sigma(ab) + \Sigma(ab)$,
 but $\Sigma(y - b)$ and $\Sigma(x - a)$ are zero because they are algebraical sums of the departures from the true mean and $\Sigma(ab) = nab$.

Hence $\Sigma(x - a)(y - b) = \Sigma(xy) - nab$.

Abbreviated methods of forming the products may be employed, unless it is obvious that the coefficient is going to be nearly one, but it must be remembered that no square root has to be extracted here as it must be in finding a standard deviation, and extracting a square root reduces a small percentage error to half its size.

The following is worked out as an example:—Column A gives the mean temperature for June–August at Oxford from 1881 to 1912, and column B the death rate of the summer quarter July to September corrected to the year 1885 for its steady decrease.

Year.	A.	B.	C.	D.	C ² .	D ² .	C × D
1880 ...	60.0	19.3	0	1.3	0	1.7	0
1 ...	59.7	15.8	−.3	−2.2	.1	4.8	.7
2 ...	58.4	17.0	−1.6	−1.0	2.7	1.0	1.6
3 ...	59.0	16.3	−1.0	−1.7	1.0	2.9	1.7
4 ...	61.4	19.5	1.4	1.5	1.9	2.3	2.1
5 ...	59.6	16.5	−.4	−1.5	.2	2.3	.1
6 ...	60.3	18.1	.3	.1	.1	0	0
7 ...	62.1	18.1	2.1	.1	4.4	0	0
8 ...	56.8	15.6	−3.2	−2.4	10.2	5.8	7.7
9 ...	59.4	17.7	−.6	−.3	.4	.1	.2
1890 ...	58.0	17.7	−2.0	−.3	4.0	.1	.1
1 ...	58.5	16.7	−1.5	−1.3	2.3	1.7	2.0
2 ...	58.5	16.6	−1.5	−1.4	2.3	2.0	2.1
3 ...	62.7	20.3	2.7	2.3	7.3	5.3	6.2
4 ...	58.6	15.5	−1.4	−2.5	1.9	6.3	3.5
5 ...	60.9	19.0	.9	1.0	.8	1.0	.9
6 ...	61.4	18.1	1.4	.1	1.9	0	.1
7 ...	61.8	19.7	1.8	1.7	3.2	2.9	3.1
8 ...	60.6	20.0	.6	2.0	.4	4.0	1.2
9 ...	64.3	21.5	4.3	3.5	18.5	12.3	15.0
1900 ...	61.3	19.0	1.3	1.0	1.7	1.0	1.3
1 ...	61.2	19.6	1.2	1.6	1.4	2.6	1.9
2 ...	58.8	16.7	−1.2	−1.3	1.4	1.7	1.6
3 ...	58.1	16.9	−1.9	−1.1	3.6	1.2	2.1
4 ...	60.2	19.0	.2	1.0	0	1.0	.2
5 ...	60.8	17.5	.8	−.5	.6	.3	−.4
6 ...	61.8	18.5	1.8	.5	3.2	.2	.9
7 ...	57.3	16.0	−2.7	−2.0	7.3	4.0	5.4
8 ...	60.2	16.9	.2	−1.1	0	1.2	−.2
9 ...	57.9	15.9	−2.1	−2.1	4.4	4.4	4.4
1910 ...	58.8	15.9	−1.2	−2.1	1.4	4.4	2.3
1 ...	64.7	20.1	4.7	2.1	22.2	4.4	9.9
2 ...	58.3	15.8	−1.7	−2.2	2.9	4.8	3.7
Total ...			+1.4	−8.2	113.7	87.7	82.0
Means ...	60.0	17.8			3.44	2.66	−.6
							2.47

The first two columns A and B being written down, the first step is to calculate or estimate their mean values. Obviously 60° F. for the temperature and 18 for the death rate will not be far out. Assume these as provisional values and put in the figures in columns C and D, which are the departures from the estimated means. If preferred both C and D can be written as double columns, the positive values on the left, the negative on the right, this facilitates addition. Then form the columns C² and D² leaving out the last figure in the manner already explained, and then the column of products C × D.

Now add up the columns, noting however that it is not necessary to add up A and B except as a check on the arithmetic. The total of C is found to be 50.0 taken apart from sign but +1.4 taken algebraically. D comes to 46.8 and -8.2. Since 1.4 ÷ 33 (there are 33 observations) is less than .05, the estimate of 60° is correct to the first decimal place. Since the sum of column D is -8.2 and 8.2 ÷ 33 = just under .25 the estimated mean of 18 was too high by .2 and the mean correct to the first place is 17.8.

The column C² comes to 113.7, dividing by 33 the mean is 3.44 and the root mean 1.85. Since the mean of 60° was found correct there is no correction and the standard deviation of the three months' temperature is 1.85 F. The total of column D² is 87.7 and the mean is 2.66, and the root mean is 1.63. This is not the standard deviation because there is a correction. The standard deviation is $\sqrt{1.63^2 - .2^2} = \sqrt{2.62} = 1.62$.

The sum of the products comes to 82.0 - .6 = 81.4. The mean product is 2.47 and there is no correction because one of the means was correctly estimated. Had it chanced that the mean temperature had been 59.7 or .3 below the estimated value it would have been necessary to correct the 2.47 by subtracting .2 × .3 or .06 from it.

The correlation coefficient = $\frac{\Sigma(xy)}{n} \div$ the standard deviations = $2.47 \div (1.85 \times 1.62) = .82$, a distinctly high value.

For this particular case therefore the value of b is $.82 \times \frac{1.62}{1.85} = .72$ and the relationship we have been searching for is such that if the summer temperature is 1° F. above its usual value the death rate is most likely .72 per thousand above its usual value. On 35 millions, the population of England, this is equivalent to 25,200 deaths per degree per annum or 6,300 per degree per quarter. During the three months of a summer notorious for its heat, such as 1911, some 50,000 more deaths may be expected than during a summer notorious for its cold, such as 1888.

The extent to which this and similar statements founded on regression equations may be relied on is discussed in a subsequent chapter (page 49).

CHAPTER X.—CORRECTION FOR ERRORS OF OBSERVATION.

A correlation coefficient is always lowered in the numerical sense by the **errors of observation** and a correction may sometimes be usefully employed.

Let $a_1 a_2 \dots b_1 b_2 \dots$ &c., be the errors of observation corresponding to $x_1 x_2 \dots y_1 y_2 \dots$ &c., then instead of the product

sum $\Sigma(xy)$ we ought to have used $\Sigma(x-a)(y-b)$. But $\Sigma(x-a)(y-b) = \Sigma(xy)$ because $\Sigma(ay)$, $\Sigma(bx)$ and $\Sigma(ab)$ being product sums between quantities that are mutually independent of each other will be too small to matter in comparison with $\Sigma(xy)$. But the cor-

relation coefficient is $\frac{\Sigma(xy)}{n \sigma_x \sigma_y}$ and σ_x and σ_y are altered; $\sigma_x^2 = \Sigma(x-a)^2$ instead of Σx^2 , and it has been already shown that $(\sigma_x^2 - \sigma_a^2)$ is the true sum of the squares instead of σ_x^2 , hence the correlation coefficient must be altered in the ratio of $\sigma_x \sigma_y : \sqrt{\sigma_x^2 - \sigma_a^2} \sqrt{\sigma_y^2 - \sigma_b^2}$. This correction appears as a percentage on the correlation coefficient r , and it will be shown subsequently that when r is large, small changes in it are most important (see page 39), hence it is desirable to correct, if possible, for the observational error when r has a high value.

A curious result appears from this correction. In general the difficulty is that σ_a and σ_b are unknown, but if the correlation coefficient is fairly high a limit is set to the magnitude of σ_a and σ_b . For a correlation coefficient cannot exceed one and therefore the correction must not raise the observed value beyond one. Let r be

the observed value, the corrected value is $r \frac{\sigma_x \sigma_y}{\sqrt{\sigma_x^2 - \sigma_a^2} \sqrt{\sigma_y^2 - \sigma_b^2}}$, and this must be less than one. This fixes a maximum value for the ratios σ_a / σ_x and σ_b / σ_y .

These ratios are proper fractions, for σ_a is of necessity less than σ_x , and, if we assume they are fairly small we may write

$$\frac{r \sigma_x \sigma_y}{\sqrt{\sigma_x^2 - \sigma_a^2} \sqrt{\sigma_y^2 - \sigma_b^2}} = r \left\{ 1 + \frac{1}{2} \left(\frac{\sigma_a}{\sigma_x} \right)^2 + \frac{1}{2} \left(\frac{\sigma_b}{\sigma_y} \right)^2 + \dots \right\}$$

neglecting higher terms

$$\text{hence } \sigma_a^2 / 2 \sigma_x^2 + \sigma_b^2 / 2 \sigma_y^2 \text{ must be less than } \frac{1-r}{r}.$$

As a special case, assume $\sigma_a / \sigma_x = \sigma_b / \sigma_y = m$, then reverting to the exact expression

$$m^2 \text{ is equal to or less than } 1 - r.$$

Put $r = .90, .80$, &c. in succession—

$$m^2 < 1 - .90, m < .32, \sigma_a < .32 \sigma_x.$$

$$m^2 < 1 - .80, m < .45, \sigma_a < .45 \sigma_x.$$

$$m^2 < 1 - .70, m < .55, \sigma_a < .55 \sigma_x.$$

$$m^2 < 1 - .60, m < .63, \sigma_a < .63 \sigma_x.$$

This merely sets a limit; the observational error may be indefinitely small, but it cannot exceed the limit given.

Apart from any algebraical investigation it is obvious that an observational error must reduce the correlation; for if a quantity cannot be measured correctly its variations must be partly hidden under the errors of its measurement, and its connection with another quantity thereby masked.

CHAPTER XI.—CORRECTION FOR A PERIODIC VARIATION.

A correlation coefficient is sometimes required between two quantities which are subject to steady or **periodic variation**, thus questions dealing with a death rate

have to reckon with the steady decline that has occurred, questions dealing with meteorological matters have to reckon with the seasonal variation. It is possible in such cases to correct each separate observation first as was done in the example given, and then form the correlation coefficient, but the process is tedious and often may be avoided. Each case must be treated on its merits, but the common case of the seasonal variation is worked out below as an example.

Let X and Y be two quantities, both subject to an annual variation. Then X is subject to two variations, one x due possibly to the influence of Y and one due simply to the seasonal change. Thus we may write

$$X = X_m + x + a \frac{\sin(t + \alpha)}{12} 2\pi$$

$$Y = Y_m + y + b \frac{\sin(t + \beta)}{12} 2\pi$$

where X_m and Y_m are the mean values, a and b the amplitudes of the seasonal variations, t the date measured in months from January 1st, α and β the phase angles.

The correlation coefficient has been obtained in a crude form direct from the observations; it is desired to correct it so that the effect of the periodic terms may be omitted, it being assumed that the observations are evenly distributed over the year or years.

The product sum that has been used is

$$\sum \left(x + a \sin \frac{t + \alpha}{6} \pi \right) \left(y + b \sin \frac{t + \beta}{6} \pi \right) = A_x B_y, \text{ say.}$$

Multiplying up we get two terms of the form $a \sum y \sin \frac{t + \beta}{6} \pi$, but these are insignificant, since y being independent of t , $y \sin \frac{t + \beta}{6} \pi$ is just as likely to be positive as negative.

Hence $A_x B_y = \sum xy + \sum a b \sin \frac{t + \alpha}{6} \pi \sin \frac{t + \beta}{6} \pi$, and by a known formula this

$$= \sum xy + \frac{a b}{2} \sum \left(\cos \frac{\alpha - \beta}{6} \pi - \cos \frac{2t + \alpha + \beta}{6} \pi \right).$$

The second of these terms vanishes since the positive and negative values of the cosine are equally distributed and neutralize each other, and hence

$$A_x B_y = \sum (xy) + n \frac{a b}{2} \cos(\alpha - \beta) \frac{\pi}{6}.$$

The standard deviations are also altered. The original crude S.D. was $\sqrt{\frac{1}{n} \sum \left(x + a \sin \left(\frac{t + \alpha}{6} \right) \pi \right)^2} = \sigma_{xa}$ say. Squaring we get

$$\begin{aligned} & \sqrt{\sum \left(\frac{x^2}{n} + \frac{2ax}{n} \sin \frac{t + \alpha}{6} \pi + \frac{a^2}{n} \sin^2 \frac{t + \alpha}{6} \pi \right)} \\ &= \sqrt{\sum \left(\frac{x^2}{n} + 0 + \frac{a^2}{2n} \left(1 - \cos \frac{t + \alpha}{3} \pi \right) \right)}, \end{aligned}$$

The sum of the periodic term vanishes and we have

$$\begin{aligned} \sigma_{xa}^2 &= \sum \frac{x^2}{n} + \frac{a^2}{2} \\ \therefore \sum \frac{x^2}{n} &= \sigma_{xa}^2 - \frac{a^2}{2}. \end{aligned}$$

Thus the crude correlation coefficient was $\frac{A_x B_y}{n \sigma_{xa} \sigma_{yb}}$.

The corrected coefficient is $\frac{A_x B_y - n \frac{a b}{2} \cos(\alpha - \beta) \frac{\pi}{6}}{n \sqrt{\sigma_{xa}^2 - \frac{a^2}{2}} \sqrt{\sigma_{yb}^2 - \frac{b^2}{2}}}$.

The following case taken from the investigation of the upper air is an example.

As we rise in the atmosphere the temperature falls, but at a varying height of about 11 kilometres, or $6\frac{1}{2}$ miles, the fall ceases abruptly. This height is denoted by H_c and its standard deviation may be taken as 1.5 km. It has in England an annual variation with an amplitude of about .6 km. and with a minimum value at the beginning of March. This quantity is closely correlated with the barometric pressure at a height of about 9 km. ($5\frac{1}{2}$ miles), the coefficient being about .80 to .85. This pressure has also an annual variation with an amplitude of about 6 mm. and a minimum in the middle of February, it is denoted by P_9 . Its standard deviation is about 8 mm. These S.D.'s include the annual variations. Taking the correlation coefficient as .85, what will it be with the annual variations excluded?

The new S.D.'s are $\sqrt{15^2 - \frac{36}{2}} = 14.4$ using 100 metres as the unit of height,

$$\text{and } \sqrt{8^2 - \frac{36}{2}} = 6.8 \text{ mm.}$$

The difference of phase is about three weeks, so

$$\cos(\alpha - \beta) \frac{\pi}{6} = \cos 22\frac{1}{2}^\circ = .92.$$

Hence the corrected coefficient is

$$\frac{.85 \times 15 \times 8 - 6 \times 3 \times .92}{14.4 \times 6.8} = .87.$$

Ought .85 or .87 to be considered the true coefficient? If the variation of H_c depends directly upon that of P_9 , the annual variation ought not to be excluded, but if the annual variation of H_c occurs quite independently, then, in discussing the connection between H_c and P_9 , it ought to be excluded. The similar value for either case tends to show that the first supposition is the more correct.

CHAPTER XII.—THE MEANING OF THE CORRELATION COEFFICIENT.

The inferences which may be drawn from the **value of a correlation coefficient** are subject to certain limitations, which must be carefully borne in mind.

Like all other statistical quantities a correlation coefficient is subject to error and its standard error is equal to $\frac{1-r^2}{\sqrt{n}}$, but this formula itself is not valid if n is too small. Suppose $r = .5$, a fairly high value, and $n = 25$, then the standard error is .15. To be quite certain that the result is not a chance one a value equal to some four times the standard error is necessary so that a coefficient of .50 on 25 observations need not be significant.

Also it is very necessary to make sure that the observations really are independent, thus observations of temperature or any other meteorological element taken at the same time at stations near together, or at short intervals of time at the same station, are not independent. The formulæ for the standard error are based on the assumption of perfect independence and fail if the condition fails.

As an example, the case of the international observations on the upper air may be cited. At fixed times on 23 dates in the year at some 15 stations in Europe observations are made by sending up small free balloons carrying self-recording instruments, six of the days are consecutive and there are three sets of three each that are consecutive, also some of the stations are comparatively close together. A considerable number of the figures have been worked up statistically and it is found that the standard errors of the various means, standard deviations, &c. are much larger than would be inferred from the number of observations used. So much is this the case that in order to find a standard deviation or a correlation coefficient, from six to even ten times as many observations are necessary to attain a definite degree of accuracy as would be required if the individual observations were strictly independent.

On obtaining a correlation coefficient it is further desirable to see that no secular or periodic changes in the variables have been concerned in its production. Thus the correlation between the amount of coal used in warming a greenhouse and the temperature of the greenhouse for a year would be of little interest, because the natural seasonal change of temperature would partly conceal the effect that was being looked for. Similarly, if r were the correlation coefficient between the simultaneous temperatures of two places, one in the northern and one in the southern hemisphere, a small value of r , especially if it were positive, would be of more significance than a large negative value, because the fact of summer in one place

coinciding with winter in the other would by itself give a large negative value.

No precise classification of correlation coefficients can be made. Assuming that they depend on as many as 50 independent cases one may say roughly that values under .30 are hardly significant, values between .30 and .70 prove a moderate connection, values between .70 and .90 a close connection, and values over .90 a very intimate connection indeed. It is apparent too that if the coefficient is very high fewer observations are necessary to establish it since $1 - r^2$ is then small.

Let σ_1 be the standard deviation of a certain quantity which is denoted by the suffix 1. If a person knowing only the mean value were asked to make a guess at the value of this quantity on a definite occasion, he would make the best guess possible by quoting a value close to the mean but he might be as much as $4\sigma_1$ out, and most probably he would be as much as $\frac{2}{3}\sigma_1$ out. If he knew the value of another quantity, suffix 2 on that occasion, and the correlation r_{12} between the two quantities, he could make a better guess. Its standard error, σ_{12} , would be reduced in the ratio of $\sqrt{1-r^2} : 1$, since it has been proved that $\sigma_{12} = \sigma_1 \sqrt{1-r^2}$. (p.31). If $r = .60$ this ratio becomes .8, for $r = .80$ the ratio is .6 while as high a value as $\sqrt{.75}$ or .87 is required to make the ratio .5, that is to halve the error of the estimate. Looked at from this point of view, small differences in the value of a correlation coefficient are of no consequence whatever while that coefficient is small, but become important only when it is large. Thus it matters little if $r = .30$ or if $r = .40$, but it makes a great difference if $r = .95$ instead of .90.

As a matter of fact values over .70 are decidedly rare.

It might seem from the above remarks as though correlation coefficients were useless unless they were very high. This is so for the individual case, but is not so for mean values. If a coefficient exceeds some four times its standard error there is very strong proof that either one quantity influences the other, or that both are conjointly influenced by something else. This latter case will be considered under the heading Partial Correlation. The correlation coefficient itself affords no clue as to which is cause and which is effect. This must be sought for elsewhere. Thus in the example that has been worked out it is impossible to suppose that the death rate could influence the temperature, but it is likely enough that the temperature should influence the death rate, and the sign and magnitude of the coefficient show the form and the extent of the influence. In this case it is hardly possible to suppose that both quantities are influenced by some third cause, so that it is fairly certain that if we could reduce the temperature of a summer quarter we could reduce the death rate. The information is of interest, but

of very little practical use, since there is no power of altering the temperature, but there are many cases in which the knowledge of the value of a correlation coefficient might be of the greatest use.

For example, suppose the correlation were known between the customs tariff on certain articles and the average rate of wages in a community measured by their purchasing power. The problem is extremely complicated because so many other variables are involved, but the large mass of statistics in existence might perhaps afford some kind of answer.

A **correlation coefficient** denotes the magnitude of the relationship between two quantities shown in a certain definite set of observations, observations let us say made in a certain year. Since similar causes produce similar effects it may be inferred that the observations of some other year will give much the same value of the coefficient. If it were not so, the correlation coefficient would be of no more value than the opinion of a man who changed his mind with every passing wind. Values of coefficients obtained from different samples of the same material, or from observations made in different years, will as a rule lie within the range of six times the standard error of the coefficient. Even with 100 observations, but a small coefficient, the error reaches .10, the range covers .60, and it is on this ground that the statement was made above that values of less than .30 were not as a rule significant. It is very fortunate that the error due to chance is reduced as the value rises, so that fairly high values even from as few as 25 observations may be considered significant.

CHAPTER XIII.—PARTIAL CORRELATION.

It has been shown that the correlation between two quantities does not by itself show what is cause and what is effect, it only shows a relationship which may be due to some other reason. **Partial correlation** will often throw considerable light upon a question of this sort.

It is important to get a clear idea of the meaning and scope of correlation both total and partial, and perhaps the following illustration may help.

Suppose standing upon a table a set of vessels A, B, C , etc., each containing water and each communicating with one or more of the others by pipes of various sizes hidden beneath the table. Is it possible to obtain more or less full details about the system of hidden communicating pipes?

If the water levels are subject to variation and can also be measured at definite periods, then broadly the magnitude of the correlation coefficient between two vessels A and B will be an index of the magnitude of the communication between A and B for if A and B are connected either directly, or through C or D say, by a large pipe, then plainly their water levels will rise and fall together, and the correlation r_{ab} will be large.

But the magnitude of r_{ab} will afford no clue as to whether communication between A and B is direct or indirect. This will be afforded by the partial correlation coefficients. For the partial coefficient $r_{ab.c}$ is the index to the communication which does not pass by means of the vessel C . Suppose for the moment that r_{ab} is an exact measure of the total communication between A and B and let there be n vessels in which the water levels can be measured. There will be $n(n-1)$ total correlation coefficients and there will be $n(n-1)$ direct communicating pipes, if each vessel is supposed to be connected with every other one. The sizes of these pipes are the unknown quantities, and theoretically these sizes can be found since the $n(n-1)$ values of r_{ab}, r_{ac}, \dots , etc., give $n(n-1)$ equations. The method of partial correlation is a practical way of obtaining a solution.

Of course the illustration is not exact in all respects; in it the correlations are necessarily positive, but it fairly represents the general statistical case. Whether the question be one of sociology, biology, meteorology or otherwise, the varying quantities may be represented by the water levels. The quantities vary partly on account of external and perhaps unknown causes, and partly through their mutual interdependence. The statistical problem is to discover the nature of this mutual dependence of one quantity upon another, and partial correlation coefficients are of very great use in its solution, for they enable us to ascertain in some cases whether the apparent action of one quantity upon another is real, or merely due to the fact that both are influenced by some third cause. They also lead to valuable regression equations, and if all the various factors which produce a certain variation can be introduced and the errors of observation eliminated, they afford a complete solution.

Before proceeding to state and prove the formulæ required, it is necessary to explain the notation. The following notation is due to Mr. Udney Yule (Proc. Roy. Soc., Series A, vol. LXXIX, p. 182, 1907), and is very convenient. Each variable quantity X_1, X_2, X_3, \dots &c., is denoted by a number, 1, 2, 3, &c., which appears as a suffix, x_1 represents the departure from the mean of the quantity X_1 , r_{12} the

total correlation coefficient between X_1 and X_2 , r_{12} that between X_2 and X_3 , and so on. For the regression coefficients b is used, and we have

$$x_1 = b_{12} x_2, \\ \text{but } x_2 = b_{21} x_1.$$

Taking two quantities into account r_{123} or r_{213} , for in the case of correlation coefficients the order of the suffixes before the dot is indifferent, denotes the partial correlation coefficient between X_1 and X_2 , and $b_{12.3}$ is explained by its position in the regression equation $x_1 = b_{12.3} x_2 + b_{13.2} x_3$.

Also $\sigma_{1.2}$ denotes the standard deviation of the expression $(x_1 - b_{12} x_2)$. (In a preceding page this expression was denoted by ϵ , but in future it will be denoted by $x_{1.2}$.) $\sigma_{1.23}$ is the standard deviation of $x_1 - b_{12.3} x_2 - b_{13.2} x_3$, and so on.

In this notation, dealing with two quantities only, the regression equations may be written thus—

$$\left. \begin{aligned} x_1 - b_{12} x_2 &= x_{1.2} \\ x_2 - b_{21} x_1 &= x_{2.1} \end{aligned} \right\} \text{I}$$

and the following relations have been proved :—(page 31.)

$$\left. \begin{aligned} \Sigma x_{1.2}^2 &= n \sigma_{1.2}^2 = n \sigma_1^2 (1 - r_{12}^2) \\ \Sigma x_{2.1}^2 &= n \sigma_{2.1}^2 = n \sigma_2^2 (1 - r_{12}^2) \end{aligned} \right\} \text{II}$$

where

$$r_{12} = \frac{\Sigma(x_1 x_2)}{n \sigma_1 \sigma_2} \dots \text{III}$$

$$b_{12} = r_{12} \frac{\sigma_1}{\sigma_2}, \quad b_{21} = r_{12} \frac{\sigma_2}{\sigma_1} \text{IV}$$

and the quantities b_{12} and b_{21} were determined, so that the errors of estimate $x_{1.2}$ and $x_{2.1}$ might be as small as possible.

The corresponding set of equations may now be obtained for three quantities.

Firstly, take the equation $x_1 = b_{12.3} x_2 + b_{13.2} x_3 + x_{1.23}$. In this equation x_1 is the variation of a quantity X_1 , which is dependent upon the variation of many other quantities $X_2, X_3, X_4, X_5 \dots$ &c. In the equation $b_{12.3} x_2$ is by its definition the part that depends upon changes in X_2 and upon changes in X_3 alone; $b_{13.2} x_3$ depends upon X_3 alone, and $x_{1.23}$ is the part left over which is dependent upon all the other quantities. There is a set of observations giving the simultaneous values of X_1, X_2 , and X_3 , and the problem is to determine $b_{12.3}$ and $b_{13.2}$ so that $\Sigma x_{1.23}^2$ may have the smallest possible value. Nothing is known about the mutual relations of X_1, X_2, X_3 , that is the point to be determined if possible, but the manner in which the variations on special occasions have occurred is known and has been tabulated.

Write the equations thus—

$$\left. \begin{aligned} x_1 - b_{12.3} x_2 - b_{13.2} x_3 &= x_{1.23} \dots \\ x_2 - b_{23.1} x_3 - b_{21.3} x_1 &= x_{2.31} \dots \\ x_3 - b_{31.2} x_1 - b_{32.1} x_2 &= x_{3.12} \dots \end{aligned} \right\} \text{V.}$$

The problem is to find the best values to give $b_{12.3} \dots$ &c.

$\Sigma x_{1.23}^2 = \Sigma(x_1 - b_{12.3} x_2 - b_{13.2} x_3)^2 = a$ minimum subject to the variation of $b_{12.3}$ and $b_{13.2}$.

Hence differentiating with regard to $b_{12.3}$, and equating to 0.

$$\Sigma x_2(x_1 - b_{12.3} x_2 - b_{13.2} x_3) = \Sigma(x_2 x_{1.23}) = 0.$$

Differentiating with regard to $b_{13.2}$,

$$\Sigma(x_3 x_{1.23}) = 0.$$

Hence

$$\begin{aligned} \Sigma(x_2 x_{1.23}) &= \Sigma(x_3 x_{1.23}) = \Sigma(x_3 x_{2.31}) \\ &= \Sigma(x_1 x_{2.31}) = \Sigma(x_1 x_{3.12}) = \Sigma(x_2 x_{3.12}) = 0 \dots \text{VI.} \end{aligned}$$

That is to say, if the suffix of the left hand x is found after the dot in the suffix of the right hand x the product sum is zero, and this is true for any number of variables from two upwards.

We have

$$\begin{aligned} \Sigma(x_{1.2} x_{1.23}) &= \Sigma x_{1.23}(x_1 - b_{12} x_2) \\ &= \Sigma(x_1 x_{1.23}) - b_{12} \Sigma(x_2 x_{1.23}) \\ &= \Sigma(x_1 x_{1.23}) - 0 = \Sigma(x_1 x_{1.23}) \dots \text{by VI.} \end{aligned}$$

Also

$$\begin{aligned} \Sigma x_{1.23}^2 &= \Sigma(x_{1.23} x_{1.23}) \\ &= \Sigma x_{1.23}(x_1 - b_{12.3} x_2 - b_{13.2} x_3) \\ &= \Sigma(x_1 x_{1.23}) - 0 - 0 \dots \dots \dots \text{by VI} \end{aligned}$$

$$\therefore \Sigma(x_1 x_{1.23}) = \Sigma(x_{1.23} x_{1.23}) = \Sigma(x_{1.23})^2 \dots \text{VII}$$

and similarly for $\Sigma(x_2 x_{2.31})$ and $\Sigma(x_3 x_{3.12})$.

That is to say, that any figure which occurs after the dot in the suffix of one x may be either inserted or left out after the dot of the suffix of the other x without altering the value of the product sum.

These two transformations, VI and VII, enable us to find the value of $b_{12.3}$ thus—

$$\Sigma(x_{23} x_{1.23}) = 0 \dots \dots \dots \text{by VI}$$

$$\therefore \Sigma x_{23}(x_1 - b_{12.3} x_2 - b_{13.2} x_3) = 0$$

$$\Sigma(x_1 x_{23}) - b_{12.3} \Sigma(x_2 x_{23}) - 0 = 0 \dots \text{by VI}$$

$$\therefore \Sigma(x_{1.3} x_{23}) - b_{12.3} \Sigma(x_{23})^2 = 0 \dots \dots \dots \text{by VII}$$

$$b_{12.3} = \frac{\Sigma(x_{1.3} x_{23})}{\Sigma(x_{23})^2} \dots \dots \dots \text{VIII.}$$

As in the case of simple correlation write this in the form

$$b_{12.3} = \frac{\Sigma(x_{1.3} x_{23})}{n \sigma_{23} \sigma_{1.3}} \frac{\sigma_{1.3}}{\sigma_{23}},$$

and denote the first fraction by r_{123} , so that

$$b_{12.3} = r_{123} \frac{\sigma_{1.3}}{\sigma_{23}} \dots \dots \dots \text{IX.}$$

Similarly

$$b_{21.3} = r_{123} \frac{\sigma_{23}}{\sigma_{1.3}} \dots \dots \dots \text{IX}$$

where

$$r_{123} = \frac{\Sigma(x_{1.3} x_{23})}{n \sigma_{1.3} \sigma_{23}} \dots \dots \dots \text{X.}$$

Thus the partial correlation coefficient r_{123} is a quantity that depends upon the sum of the products of the form $x_{1.3}$ and x_{23} , and we come across this quantity r_{123} when we seek the best value for $b_{12.3}$ and $b_{21.3}$. Now $x_{1.3}$ is the error of the estimate that is made in X_1 by causes other than that due to the variation of X_3 , and $X_{2.3}$ is the same quantity for X_2 . Hence r_{123} is the correlation between X_1 and X_2 after any influence that X_3 may have upon either is allowed for. Obviously a partial, like a total coefficient, cannot exceed 1.

We proceed to find the value of r_{123} , by first establishing a relation between r_{123} and σ_{123} .

$$\Sigma x_{123}^2 = \Sigma x_{13} x_{123} \dots \dots \dots \text{by VII}$$

$$= \Sigma x_{13} (x_1 - b_{123} x_2 - b_{132} x_3)$$

$$= \Sigma (x_1 x_{13}) - b_{123} \Sigma (x_2 x_{13}) \dots \dots \dots \text{by VI}$$

$$= \Sigma x_{13}^2 - b_{123} \Sigma (x_{23} x_{13}) \dots \dots \dots \text{by VII}$$

$$\therefore n \sigma_{123}^2 = n \sigma_{13}^2 - b_{123} \Sigma (x_{23} x_{13})$$

$$= n \sigma_{13}^2 - \frac{\Sigma (x_{13} x_{23})^2}{n \sigma_{23}^2} \dots \dots \dots \text{by VIII}$$

$$= n \sigma_{13}^2 - n r_{123}^2 \frac{\sigma_{13}^2 \sigma_{23}^2}{\sigma_{23}^2} \dots \dots \dots \text{by X}$$

$$\therefore \sigma_{123}^2 = \sigma_{13}^2 (1 - r_{123}^2) \dots \dots \dots \text{XI.}$$

Similarly it could be proved that

$$\sigma_{123}^2 = \sigma_{12}^2 (1 - r_{123}^2) \dots \dots \dots \text{XI}$$

$$\therefore \sigma_{123}^2 = \sigma_1^2 (1 - r_{12}^2) (1 - r_{13}^2) \dots \dots \dots \text{XII by II.}$$

$$\text{or } = \sigma_1^2 (1 - r_{13}^2) (1 - r_{12}^2) \dots \dots \dots \text{XII by II.}$$

Now r_{123} can be found.

$$\Sigma (x_{13} x_{23}) = \Sigma (x_1 - b_{123} x_2 - b_{132} x_3) (x_2 - b_{231} x_1 - b_{213} x_3)$$

$$= \Sigma x_1 x_2 - \Sigma b_{123} (x_2 x_3) - \Sigma b_{231} (x_1 x_3) + b_{123} b_{231} \Sigma x_3^2$$

$$= n \left\{ r_{12} \sigma_1 \sigma_2 - r_{13} \frac{\sigma_1}{\sigma_3} r_{23} \sigma_2 \sigma_3 - r_{23} \frac{\sigma_2}{\sigma_3} r_{13} \sigma_1 \sigma_3 + r_{12} \frac{\sigma_1}{\sigma_3} r_{23} \frac{\sigma_2}{\sigma_3} \sigma_3^2 \right\}$$

$$= n \sigma_1 \sigma_2 \{ r_{12} - r_{13} r_{23} - r_{23} r_{13} + r_{13} r_{23} \}$$

$$= \sigma_1 \sigma_2 (r_{12} - r_{13} r_{23})$$

$$\therefore r_{123} = \frac{\sigma_1 \sigma_2}{\sigma_{13} \sigma_{23}} (r_{12} - r_{13} r_{23}) \dots \text{by X}$$

$$= \frac{r_{12} - r_{13} r_{23}}{\sqrt{1 - r_{13}^2} \sqrt{1 - r_{23}^2}} \dots \text{XIII by II.}$$

If b 's instead of r 's had been substituted in the place of the product sums, we should have found

$$b_{123} = \frac{b_{12} - b_{13} b_{32}}{1 - b_{23} b_{32}} \dots \text{XIV.}$$

The formula for r_{231} and r_{312} obviously follow by writing the following figure of the suffix in cyclic order.

Also a similar proof holds for more than three variables so that in general

$$r_{123 \dots n} = \frac{r_{124 \dots n} - r_{134 \dots n} r_{234 \dots n}}{\sqrt{1 - r_{134 \dots n}^2} \sqrt{1 - r_{234 \dots n}^2}}.$$

These formulæ suffice to give numerical coefficients in the regression equations, V, but there are two modes of procedure. Knowing the primary regression coefficients b_{12} , b_{21} , b_{23} , b_{32} , b_{31} and b_{13} we can use equations XIV to obtain the numerical values of the six first order regression coefficients of the equation (V). This is the shortest but not the most convenient method. It is better to proceed by means of XIII, IX and XII, because by this means the necessity of finding b_{12} , b_{21} , &c. is avoided, and the useful quantities σ_{123} , σ_{231} and σ_{312} are found, but more particularly because the identity of equation XII affords an absolute check on the accuracy of the arithmetic.

To use this method it must be shown that

$$\frac{\sigma_{123}}{\sigma_{231}} = \frac{\sigma_{13}}{\sigma_{23}}.$$

We have

$$\frac{\sigma_{123}^2}{\sigma_{231}^2} = \frac{\sigma_1^2 (1 - r_{13}^2) (1 - r_{12}^2)}{\sigma_2^2 (1 - r_{23}^2) (1 - r_{21}^2)} \dots \text{by XII}$$

$$= \frac{\sigma_1^2 (1 - r_{13}^2)}{\sigma_2^2 (1 - r_{23}^2)} = \frac{\sigma_{13}^2}{\sigma_{23}^2} \dots \text{by II}$$

$$\therefore b_{123} = r_{123} \frac{\sigma_{13}}{\sigma_{23}} = r_{123} \frac{\sigma_{123}}{\sigma_{231}} \dots \text{XV.}$$

The method is best shown by an example, and it will be seen how little trouble is required. It is of course assumed that the three primary coefficients and the three standard deviations are already found.

Before, however, leaving this part of the subject it may be well to give the following formula:—

In the regression equation $x_1 = b_{12} x_2$ it has been shown that

$1 - r_{12}^2 = \frac{\sigma_{12}^2}{\sigma_1^2}$, and here r_{12} is the correlation between the two

sides of the equation. Similarly if R is the correlation between the two sides of the general equation, $1 - R^2 = \frac{\sigma_{123 \dots n}^2}{\sigma_1^2}$.

The error of estimate continually decreases, and the correlation R continually increases as more variables are taken into account, but the error is not greatly reduced unless the partial correlation coefficients are large, since $\sigma_{123 \dots n}^2 = \sigma_1^2 (1 - r_{12}^2) (1 - r_{13}^2) \dots$ and each factor is less than 1, but not greatly less unless the corresponding r is nearly one.

In fact, the term 'total correlation' applied to the case of two variables seems to be a misnomer, for the b_{12} in the 'total' regression equation is just as fully a 'partial' coefficient as the $b_{1234 \dots n}$ in the partial regression equation, while in the latter case the correlation between the two sides is more complete than in the former.

CHAPTER XIV.—PRACTICAL METHODS OF FINDING PARTIAL CORRELATION COEFFICIENTS.

The following instance of the method of working out a partial correlation coefficient is given as an example.

It has been found that as a balloon rises in the atmosphere the temperature falls, with interruptions sometimes, it is true, in the lower strata, but still on the whole with a steady fall up to a height of six or seven miles. At somewhere about this height the fall ceases abruptly and for the remainder of the ascent the temperature shows but little variation. This height is denoted by H_c and by the suffix 4. It is an interesting subject of enquiry to find the cause of

this phenomenon, and the method of total correlation shows that the value of H_c is closely correlated with the temperature of the atmosphere below it, and also with the pressure that prevails at about five or six miles. We will denote the mean temperature of the air from 1 up to 9 kilometers by T_m and by the suffix 2, and the atmospheric pressure at 9 kilometers by P_9 and the suffix 3. (Eight kilometers = five miles, and the metric system is used because all the observations have been published in that system.)

From some 400 observations the following primary coefficients and standard deviations have been deduced after correction for the observational error—

$$r_{23} = .95, r_{24} = .79, r_{34} = .84.$$

The values to be worked out from these are—

$$\left. \begin{aligned} r_{234} &= \frac{r_{23} - r_{24} r_{34}}{\sqrt{1 - r_{24}^2} \sqrt{1 - r_{34}^2}} \\ r_{342} &= \frac{r_{34} - r_{32} r_{42}}{\sqrt{1 - r_{32}^2} \sqrt{1 - r_{42}^2}} \\ r_{423} &= \frac{r_{42} - r_{43} r_{23}}{\sqrt{1 - r_{43}^2} \sqrt{1 - r_{23}^2}} \end{aligned} \right\} \text{XIII. p. 44.}$$

The observations also give $\sigma_2 = 6^\circ\text{C.}$, $\sigma_3 = 8\text{ mm.}$, $\sigma_4 = 15$ (hundred metres), but it will be better, to begin with, for reasons subsequently explained, to take the standard deviation of each quantity as the unit of measurement of that quantity so that σ_2 , σ_3 and σ_4 each equal 1.

Obviously the first requirement is to find the values of $\sqrt{1 - r^2}$ in the three cases. Since $\sqrt{1 - r^2} = \sqrt{1 - r} \sqrt{1 + r}$, this is readily done by a slide rule, but it is better to make or obtain a table giving the value to three decimal places of $\sqrt{1 - r^2}$ from $r = .01$ to $r = .99$.

From a slide rule the values are found to be .312, .542 and .613.

Arrange thus, leaving the third line blank for the present, the .312 or $\sqrt{1 - r_{23}^2}$ standing under the value of r_{23} and so on—

$$\begin{array}{ccc} r_{23} = .95 & r_{34} = .84 & r_{42} = .79 \\ .312 & .542 & .613 \\ .510 & .882 & .998 \\ r_{234} = \frac{.95 - .664}{.542 \cdot .613} = \frac{.286}{.332} = .86 \\ r_{342} = \frac{.84 - .75}{.312 \cdot .613} = \frac{.09}{.191} = .47 \\ r_{423} = \frac{.79 - .798}{.312 \cdot .542} = \frac{-.008}{.169} = -.04 \end{array}$$

Using a slide rule these are all the figures that need be written to find the three partial coefficients. Thus in the first equation the .664 is the product of .84 and .79, the subtraction is done mentally and the result .286 is successively divided by .542 and .612 and gives .86, similarly for the other two.

Now write in the third line by putting the value of $\sqrt{1 - r_{23}^2}$ under the .312 and so on. Then—

$$\left. \begin{aligned} \sigma_{234} &= .613 \times .510 \text{ or } .312 \times .998 = .31 \times \sigma_2 \\ \sigma_{342} &= .312 \times .882 \text{ or } .542 \times .510 = .27 \times \sigma_3 \\ \sigma_{423} &= .542 \times .998 \text{ or } .613 \times .882 = .54 \times \sigma_4 \end{aligned} \right\} \text{by XII.}$$

The alternative methods of calculation afford a check on the accuracy of the previous arithmetic, so that when the two products are found to be identical to the second decimal place the values of r_{234} . . &c., may be accepted as correct.

The following three lines, making a total of only twelve, complete the work

$$\begin{aligned} b_{234} &= .86 \times \frac{31}{27} = .98 & b_{324} &= .86 \times \frac{27}{31} = .74 \\ b_{342} &= .47 \times \frac{27}{54} = .24 & b_{432} &= .47 \times \frac{54}{27} = .94 \\ b_{423} &= -.04 \times \frac{54}{31} = -.07 & b_{243} &= -.04 \times \frac{31}{54} = -.02 \end{aligned}$$

It is convenient to place a δ before a quantity to denote the amount of the variation of that quantity. Thus δT_m denotes the departure, positive or negative, of the temperature of the air column from its mean value. With this notation the regression equations are

$$\begin{aligned} \delta T_m &= b_{234} \delta P_9 + b_{243} \delta H_c \\ \delta P_9 &= b_{342} \delta H_c + b_{324} \delta T_m \\ \delta H_c &= b_{423} \delta T_m + b_{432} \delta P_9 \end{aligned}$$

The suffix denoting the left hand number stands first in each term of the right hand side, and the second suffix of any b refers to the variable with which it is associated, thus with the middle term $b_{342} \delta H_c$, the 3 refers to the P_9 on the left and the 4 to the H_c .

Note also, more particularly during the process of working out, the great convenience of keeping the cyclic order unimpaired.

Substituting the numerical values, we get

$$\begin{aligned} \delta T_m &= .98 \delta P_9 - .02 \delta H_c & \sigma_{234} &= .31 \dots \text{I.} \\ \delta P_9 &= .24 \delta H_c + .74 \delta T_m & \sigma_{342} &= .27 \dots \text{II.} \\ \delta H_c &= -.07 \delta T_m + .94 \delta P_9 & \sigma_{423} &= .54 \dots \text{III.} \end{aligned}$$

In these equations the standard deviations have been taken as units. The numerical values of the b_{234} . . &c., depend upon the units and the numerical coefficients cannot be in any way compared with each other if the ratio depends upon such an arbitrary matter as the choice of a unit. Take for example equation I as it stands. For all practical purposes the .98 may be taken as equivalent to 1 and the .02 as equivalent to nothing. Hence the meaning is that a variation of P_9 is accompanied by a corresponding variation of T_m of just the same magnitude, using the average amount of departure in both cases as the standard, *i.e.*, if a variation of P_9 of exceptional magnitude occurs it will produce a variation of exceptional magnitude and of the same sign in T_m , if the variation of P_9 is moderate then that of T_m will be moderate, and so on. The low value of the coefficient of δH_c further shows that variations of H_c are entirely without effect upon T_m . Similarly from III it appears that P_9 has very nearly a full effect upon H_c and T_m a trifling effect, but if the more natural unit of 1 mm. instead of 8 mm. had been adopted the equation would have stood thus

$$\delta H_c = -.07 \delta T_m + .12 \delta P_9$$

and the large effect of P_9 compared with the small effect of T would have been hidden. Hence the advantage of using the standard deviations as units.

If, however, it is desired to express the equations in ordinary units the following rule for changing is applicable. Suppose the new unit used is less than the old, and let m be their ratio, so that m is a proper fraction. Multiply each term by the ratio m referring to it.

Thus changing (1) to degrees centigrade, millimetres of mercury, and 100 metres, the ratios are $\frac{1}{6}$, $\frac{1}{8}$ and $\frac{1}{15}$ and I becomes

$$\frac{1}{6} \delta T_m = .98 \times \frac{1}{8} \delta P_o - .02 \times \frac{1}{15} \delta H_c \times \frac{1}{8} \delta A_p - .02 \times \frac{1}{15} \delta A_c.$$

Further remarks on these equations will be found on page 51.

CHAPTER XV.—THE REGRESSION EQUATIONS.

In the preceding pages in dealing with correlation, whether total or partial, it has been assumed that the object of the enquiry has been to obtain certain equations called **regression equations**, and it is desirable to have a clear idea of the meaning and use of these equations.

The name is due to the following considerations.—The most probable value for a varying quantity is in general one close to its mean value, and if for any reason it has departed from the mean, the tendency is for it to fall back or regress towards it. But when it is correlated with some second quantity and the other quantity differs from its own mean, this fact makes it likely that the first quantity will not have regressed fully but only partly, and the 'regression' equation expresses the extent to which it has most likely gone back. For example, if one day is very hot the chances are that the next day will be hot, for there is a high correlation between the temperatures of two consecutive days. The most probable temperature for Sunday next is the mean for that special day, but if on Saturday the temperature is much above its mean that will make it likely that it will also be above on Sunday. The regression equation will show the most probable temperature for Sunday as being between that of Saturday and the mean for past Sundays at the same date, *i.e.*, it will show how much it is likely to have regressed from Saturday's value. The term was first used by Galton to denote the amount by which the height of the son of a tall father was likely to have fallen back towards the general average.

In the form in which they are commonly written regression equations are not equations at all in the ordinary sense of

the word, because the two sides are not as a rule, and are not meant to be, equal. It is only when the correlation coefficients concerned in the production of a regression equation are equal to ± 1 that the term "equation" is strictly accurate.

From the special example given on page 33 the regression equation

$$\delta(DR) = .72 \delta T$$

is deduced, where $\delta(DR)$ denotes the departure of the death rate per thousand per annum from its mean value for the summer quarter, and δT denotes the departure of the temperature in degrees F from its mean.

The indicated relationship is by no means exact, and that for two reasons. First, but not by any means of most importance, is the fact that the decimal .72 is only an approximate value, it is based on observations running from 1880 to 1912, and had it been based on some other period it might have been perhaps .60, perhaps .80, instead of .72. Where large numbers of observation are obtainable, this part of the inexactness can be reduced to small proportions, for the standard error of a regression coefficient b_{12} for a normal distribution is $\frac{\sigma_1}{\sigma_2} \frac{\sqrt{1-r^2_{12}}}{\sqrt{n}}$ where n is the number of observations, and if n is large the error is small. For this special case the probable error is .06.

But the chief amount of the inexactness is due to another cause altogether and cannot be reduced by any number of observations. The regression equation professedly excludes all causes of variation (observational errors included, unless they have been allowed for), save those dependent in some way upon the variable specified upon the right-hand side, in this special case on the temperature. Let us use the standard deviations as units and write the equation in the form

$$\delta(DR) = .82 \delta T \dots (A).$$

This shows that 82/100, say four-fifths of the variation, is in some way associated with temperature, but that there are other causes that are operative but are not shown in the equation, which account for the remainder, and it is these other causes mostly which make the estimate inexact.

The interpretation of this particular equation is fairly simple, though very often this is not the case.

The correlation coefficient is high, and after any reasonable allowance for a casual error due to paucity of observations must remain high, hence there is certainly an intimate connection. There are three alternatives with regard to this or to any other similar regression equation—

- (1) The death rate may be influenced by the temperature;
- (2) The temperature may be influenced by the death rate. An impossible conclusion;
- (3) Both quantities may be independent of each other, but influenced by some third quantity that is unknown or at least unconsidered. This, also, seems extremely unlikely, so that (1) must be accepted as the true explanation.

There are various uses to which a regression equation may be put. The following is an example:—

There is in England and indeed in all European countries a very great mortality among infants under one year of age, and of recent years steps have been taken to check this mortality. The question is, have these steps been efficacious? There is no doubt about the fact that during the last ten years a decided reduction is shown, but it is perfectly well known that the weather since 1903 has been on the whole more favourable than usual to child life. The mortality occurs chiefly through diarrhoea which is prevalent in the summer—it is very greatly increased if the summer is hot, and hardly occurs in a noticeable form if the summer is cold. From the table which is given on page 33, column A, it will be seen that only two hot summers have occurred since 1902, namely, 1906 and 1911. This has reduced the infant mortality, but is it responsible for the whole reduction or only for part?

Now, if a regression equation be formed between the temperature and the infant mortality, and there is ample material to do this, and then the mortality of recent years be corrected by means of this equation to the values that would presumably have occurred had the temperature been normal, these corrected values become comparable with the long years' average which prevailed before the remedial measures were introduced, and a better judgment as to the efficacy of the measures can be formed.

The remarks which have been made above and which, for the sake of clearness, have been based on the case of one special total or primary regression equation are also applicable to partial regression equations in general with three or more variables, but a partial regression equation will often give a large amount of information besides. If the reader doubts this let him make the following trial:—

Take three or more quantities x_1, x_2, x_3 , and write down one value for each in sets of three, subject only to the condition for each set that $ax_1 + bx_2 + cx_3 = 0$, where a, b , and c are numerical constants.

Now, let him form the six primary regression equations between the quantities. These equations may take almost any form, and are not likely to be noticeable, but let him proceed to form the three partial regression equations and he will find that he gets three identical equations, namely, $ax_1 + bx_2 + cx_3 = 0$, with each term divided either by a, b , or c . If the relation were not originally known it might well escape notice in the original tables, and in forming the primary correlation coefficients, but it would be revealed at once on finding the partial correlation coefficients, because each partial coefficient would be either $+1$ or -1 . A similar result would be obtained with a fixed linear relationship between four or

more quantities. Hence partial correlation may bring out a previously unsuspected relationship.

More often partial regression equations show the agency through which one quantity may act upon another, and this is well shown in the special example that has been worked out.

On reference to the formal proof of the formulæ employed in finding the coefficients of a partial regression equation it will be seen that r_{123} , and therefore also b_{123} and b_{213} depend on the product sum $\Sigma(x_{13}x_{23})$. These terms, x_{13} and x_{23} , are the variations of x_1 and x_2 which are left after any influence that the quantity denoted by the suffix 3 may exert is excluded. It follows, therefore, that the terms b_{123} and b_{213} are absolutely independent of (3); and, in general, that each term on the right hand side of a partial regression equation with three, four, or any number of terms is entirely independent of every other term.

Now, let us take the special example already worked out and given on page 47.

$$\begin{aligned}\partial T_m &= \cdot98 \partial P_9 - \cdot02 \partial H_c & \sigma_{231} &= \cdot31 \dots \text{I.} \\ \partial P_9 &= \cdot24 \partial H_c + \cdot74 \partial T_m & \sigma_{342} &= \cdot27 \dots \text{II.} \\ \partial H_c &= -\cdot07 \partial T_m + \cdot94 \partial P_9 & \sigma_{123} &= \cdot54 \dots \text{III.}\end{aligned}$$

Here the standard deviations are units, and the standard errors of estimate made by using the equations are shown on the right hand.

These particular equations were worked out with a definite purpose, and have fully repaid the trouble involved in the calculation. The primary correlation coefficients are $\cdot95$ between T_m and P_9 , $\cdot79$ between T_m and H_c , and $\cdot84$ between P_9 and H_c , and the question that arose was whether H_c was more dependent upon T_m or upon P_9 . The answer given by III. is clear and precise. H_c is dependent to a very large extent on P_9 , and is practically independent of T_m . Also the large primary correlation coefficient of $\cdot79$ is readily explained by I. and III., for I. shows that T_m is also very greatly influenced by P_9 , and thus the two quantities T_m and H_c which vary together appear to do so because, although really independent of each other, their variations are both due to that of another quantity, namely P_9 . The case is similar to the following:—

The water evaporated in a steam boiler is dependent chiefly upon the coal burnt in the furnace, so also is the amount and temperature of the air that leaves the top of the chimney. These two quantities would have a very high correlation coefficient, but yet they are independent of each other, or negatively correlated, so much so indeed that the lower the amount and temperature of the chimney gases, the more efficient is the boiler. Just so T_m and H_c appear to be independent of each other but both dependent on P_9 . This is the most prominent result that stands out from these regression equations, but more may be inferred.

The $\cdot98$ of equation I. and the $\cdot94$ of equation III. are so nearly equal to 1 that it may be said that the whole variations both of T_m and of H_c are accounted for by the variation of P_9 ; which, since T_m and H_c are independent of each other, is almost equivalent to saying that P_9 is the sole cause, or nearly so, of the variations of the others. Equation II. gives $\partial P_9 = \cdot24 \partial H_c + \dots$, and shows that the converse action of H_c on P_9 is comparatively small. Furthermore, II. gives $\partial P_9 = \dots \cdot74 \partial T_m$. Now it is known from the elementary laws of physics that P_9 must depend upon T_m and upon the barometric pressure at mean sea level, and

the correct value of this coefficient can be calculated. It is .77, a sufficiently good agreement with .74.

In conclusion, the following **standard errors** of various quantities are given. The proofs mostly lie beyond the range of an elementary book.

Standard error of a mean from n observations $= \frac{\sigma}{\sqrt{n}}$.

Of a standard deviation $\frac{\sigma}{\sqrt{2n}}$.

Of a correlation coefficient r , total or partial $= \frac{1-r^2}{\sqrt{n}}$.

Of a regression coefficient $b_{12} = \frac{\sigma_{12}}{\sigma_2 \sqrt{n}}$.

Of a regression coefficient $b_{123} = \frac{\sigma_{123}}{\sigma_{23} \sqrt{n}}$.

Of the amplitude in any term in a Fourier Series obtained from n well distributed observations $= \frac{\sigma \sqrt{2}}{\sqrt{n}}$.

This last value only holds when the genuine amplitude is several times as large as the error $\frac{\sigma \sqrt{2}}{\sqrt{n}}$. If there be no genuine sine curve at all, or if the real amplitude is very small, the root mean square of the fictitious amplitudes will be $\frac{2\sigma}{\sqrt{n}}$. (This is not the standard error because it is not taken about the mean value.)

The result is that no significance must be attached to the terms of a Fourier Series which have a small amplitude until it has been ascertained that the amplitudes are well in excess of $\frac{2\sigma}{\sqrt{n}}$, for a set of purely chance values are almost certain to show a considerable amplitude.

The normal distribution is assumed, and in such a case the probable error is two-thirds of the standard error.

METEOROLOGICAL OFFICE.

THE COMPUTER'S HANDBOOK.

Section V.—Computations related to the Theory of Probabilities (continuation).

- 3.—A Collection of Correlation Coefficients from Meteorological Papers and a Note on the Partial Correlation Coefficient. By CAPTAIN E. H. CHAPMAN, R.E., with an Introduction by SIR NAPIER SHAW, F.R.S.

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CONTENTS.

	PAGE
INTRODUCTION BY SIR NAPIER SHAW, F.R.S.	55
INDEX	57
A COLLECTION OF CORRELATION COEFFICIENTS FROM METEOROLOGICAL PAPERS BY CAPTAIN E. H. CHAPMAN, R.E.	60
NOTE ON THE PARTIAL CORRELATION COEFFICIENT BY CAPTAIN E. H. CHAPMAN, R.E.	144

LIST OF AUTHORS.

	PAGE
T. AKATU Climatic Correlations	77
E. G. BILHAM Rainfall	107
T. A. BLAIR Weather and Crops... ..	143
C. E. P. BROOKS Temperature	100, 101, 102
A. HAMPION BROWN Rainfall	75
E. H. CHAPMAN Pressure	93
... .. Rainfall	105, 107
... .. Upper Air	67
N. A. COMISSOPOLUS Rainfall	106
J. I. CRAIG Sunspots	108
... .. Temperature	95-97
W. H. DINES Temperature	95
... .. Upper Air	61, 64-66
F. M. EXNER Pressure	78-90
P. H. GALLÉ Temperature	98
R. H. HOOKER Rainfall	103
... .. Weather and Crops... ..	112-117
S. M. JACOB Rainfall and Crops (India)	125
Dr. ALICE LEE Pressure	77
H. L. MOORE Weather and Crops	118
R. C. MOSSMAN Climatic Correlations	74
E. V. NEWNHAM Upper Air	69, 70
T. OKADA Climatic Correlations	75
... .. Weather and Crops	127
Prof. KARL PEARSON Pressure	77
J. PECK Rainfall	103
Sir NAPIER SHAW Pressure	92
J. WARREN SMITH Weather and Crops	118-125
E. C. SNOW Rainfall	103
J. P. VAN DER STOK Pressure	90
J. F. VOORHEES Weather and Crops	127
G. T. WALKER Climatic Correlations	71-74
... .. Sunspots	108-111
AXEL WALLÉN Weather and Crops	128-143

INTRODUCTION.

CORRELATION.

A proposal for a meteorological conference in Edinburgh in September 1914, promised meteorologists an opportunity of discussing a number of questions of very general interest. Among them was the place of correlation in the meteorology of the globe. I had undertaken to introduce the subject in place of Mr. Mossman who had found himself unable to do so, and when the conference had been abandoned it formed the basis of the opening discussion of the session 1914-15 at the Meteorological Office. The following remarks are taken from the introduction to the discussion.

"The subject lends itself very profitably to discussion. Statistical methods have been very extensively developed in the last twenty years and have been applied to a number of meteorological problems, but for the most part the results have been unproductive, and we should like to know why. For the most part they have only confirmed by a numerical expression what we knew already.

"Correlation deals with figures: it is an arithmetical process and may be compared or contrasted with the diagrammatic method, the method of plotting and thinking about the result. The two methods are very well represented by the papers which form the nominal subject of discussion:—Dr. Walker's Studies of the Relationships of the Indian Monsoon Rainfall and Professor Hildebrandsson's fifth and final contribution to research on Centres of Action of the Atmosphere. For the purpose of discussion we may regard Dr. Walker as the exponent of the statistical method and Professor Hildebrandsson as the exponent of the method of charts and diagrams. Too much stress must not be laid upon this distinction, for both papers are fully illustrated by diagrams.

"Dr. Walker gives us diagrams for:—

- Monsoon rainfall and annual pressure of the previous year, 1865-1913.
- Departures from normal of monsoon rainfall and of annual pressure of the same year reversed, 1865-1912.
- Correlation coefficients of monsoon rainfall with monthly pressures of the two preceding, the actual and the following year.
- Departures of monsoon rainfall and of the pressure of ten months ending nine months before its arrival.
- Actual monsoon rainfall and that calculated from the pressure and rainfall of the previous year (the latter a remarkably colourless curve), and
- The comparison of the increments from one year to the next of the data illustrated in figure I. (a very striking diagram).

"The text deals with these diagrams from the point of view of statistical methods. The highest correlation coefficient is .6, and it is suggested that the diagrams show more relationship than the correlation figures express.

"Professor Hildebrandsson, on the other hand, gives a resumé of all the relationships to which his study of diagrams has led him, mostly vague and hazy relationships, which, somehow break the word of promise to our hope. He adds diagrams of comparison of Java rainfall October-March

with the barometer at the Cape of the preceding year.

- " at Mauritius, April-September preceding.
- " " at Sydney and Melbourne, October-March of the preceding year.
- " " at Cordoba and Santiago the following April-September.

"All these show, more or less, an inverse relationship and suggest that the characteristics of the seasons go round the world from West to East taking, say, a year and a half to get from the Cape westward to South America.

"He adds world-maps of temperature-departures from the normal for January and July for each year from 1894 to 1903, and mean departures for the groups of years 1884, 1889, 1895 and 1905, times of sunspot maximum; and 1885, 1889, 1902 and 1903 times of sunspot minimum which lead to the attractive but rather vague conclusion that in times of sunspot maximum the equatorial regions are cold and the polar regions warm and vice versa.

"The conclusions from the statistical method are numerically precise but disappointing, those from the pictorial method seductive and stimulating but vague.

"I want to make some remarks upon these two methods of tackling the problem of the meteorology of the globe that may be a guide to future action.

"First as to the dangers which lie in wait even for the wary in the statistical method.

"When one hands oneself over to arithmetic there is no escape from its grip. When once the arithmetical engine is set to work one cannot afterwards escape the results. There is no means of correcting errors and certainly the compiler of meteorological data takes a full share of humanity's liability to error. Let me give you an example. Take Lockyer's tables of monthly and annual pressure at 75 stations distributed over the globe. It includes the values for Valencia and Aberdeen, two of our own observatories. Now, it is the established practice of meteorological institutes to give the values of the pressure at station level, and the station level at Valencia was changed in 1892. So the values between 1867 and 1892 are not on all fours for statistical purposes with those from 1893 to 1905, and any correlation coefficient based on them is depreciated. What the amount of the effect would be I do not know, but it is not negligible.

"The same thing has occurred with Falmouth. There is a break there too. It has affected our own published mean values and I hardly know how to warn prospective investigators against such accidents. We are not more liable to such accidents than other Institutes and it is quite possible that there may be many such the world over. If you deal with the diagrams the eye can make allowances but the result of the arithmetic is inexorable. The only conclusion I can draw is that the arithmetical methods probably give us too low an idea of relationships on account of these accidents.

"It is extremely difficult to avoid them. I have given you an example of an error which may creep into a long series, though every year taken by itself is correct. That there are others that are less excusable no one can deny. Within the last week I have noticed that we have given the height of an anemometer at Kew as 35 feet, when it should be 70 feet, and we dated a report 14th October which gave results for the 18th, a prophetic power to which we lay no claim. Again, I doubt if we are exceptional. I only suggest these things as an incidental disadvantage of the arithmetical method. And yet the arithmetical method has in it the germ of finality. If you prove a relationship by statistical process, in spite of these liabilities to error, it is established. Any incidental errors depress the force of the reasoning and if it asserts itself the ground of confidence is much stronger and more convincing than the mere inspection of curves. Unfortunately, as I have said, most of the statistical results hitherto obtained have only confirmed one's confidence in what was already accepted with less rigid proof.

"Dr. Walker, by the method of partial correlation coefficients and regression equations, has shown how to calculate the coming monsoon rainfall from various elements, but the result may be disappointing, and for this reason: the regression equations express only the most probable numerical relation between the quantity sought and the variables upon which it depends. There may be, and apparently are, other variables which affect the result for any particular occasion, so that it is only on the average of a series of years, so long that the other causes cancel out, that the truth of the statistical relations asserts itself. But this is not good enough for the practical man who wants to know what the coming season will be like, and is not satisfied with the assurance that if this year is out in one way another year will be as much out in the opposite way. In fact, for practical purposes we can only deal with correlation coefficients of high magnitude, say .7 or above, and the problem we have to set before us is to find out variables which are related to the one we are interested in by high correlation coefficients.

"But we cannot guess these relationships beforehand, generally speaking we have not the data. Suppose, for example, we wish to attribute the seasonal weather in tropical countries to the extent of ice in the polar regions, we simply do not know the extent of ice in the polar regions in any year, and to remain idle until we do is much too fatalistic.

"The only way seems to me to be first to deal with the world as a whole so far as we know it and next to track down our quarry by a combination of the statistical with the geographical method.

"In pursuit of this general idea I have tried to find to what extent the mean pressure for the month at Kew Observatory was related to the pressure in the same or the preceding month in any other part of the globe. I was enabled to obtain the computations by the fortunate coincidence that a correspondent unknown to me, Mr. C. N. C. Evers, offered his services as a voluntary computer.

"The results are not exhaustive but so far as they go they tend to show that the pressure of the atmosphere in the British Isles is partially dependent on the pressure in the surrounding region and is not subject to controls which can be specified by the pressure of some distant locality."

Other persons, beginning with Mr. J. I. Craig (Ref. 22, p. V 95), had followed a similar line of inquiry and gradually our knowledge of the relationships is taking shape. In order to avoid recomputing the same data and to bring together the results of great numerical labour it seemed desirable to collect the correlation coefficients already available which were based upon a sufficient number of data. They have accordingly been collected by Captain Chapman as represented in the following list. He has adopted 25 as the normal minimum of the number of pairs of figures to be correlated, but in a few cases, for special reasons, coefficients based upon a smaller number have been given.

NAPIER SHAW.

INDEX OF CORRELATION COEFFICIENTS.

I. UPPER AIR.

	PAGE
Barometric change in past twelve hours with height of isothermal ...	66, 70
Gradient of pressure with height of isothermal ...	66, 70
wind velocity ...	65
Isothermal, height of, at same station, different intervals of time ...	69
simultaneous observations of, at different stations ...	69
with barometric change in past twelve hours ...	66, 70
gradient of pressure ...	66, 70
pressure ...	61, 64
temperature ...	61, 63, 64, 66
water vapour ...	66
wind velocity ...	63
temperature of, with height of ...	61, 64
pressure ...	61, 64
temperature (mean) ...	61, 64
Pressure, readings of, 1-10 days apart ...	69, 70
simultaneous readings of, at different stations ...	70
with isothermal, height of ...	61, 64
temperature of ...	61, 64
pressure at different heights ...	61
temperature ...	61, 64-66
water vapour ...	66
Temperature, mean, from 1-9k, with isothermal, height of ...	61, 64
temperature of ...	61, 64
pressure ...	61, 64
with pressure ...	65, 66
water vapour ...	66
wind velocity ...	63
Water vapour with height of isothermal ...	66
pressure ...	66
temperature ...	66
Wind velocity, at different heights ...	67, 70
components of ...	63, 68, 71
general upper ...	63
geostrophic ...	63, 67, 70
surface ...	63, 67, 70

II. CLIMATIC CORRELATIONS.

PAGE

Gradient of pressure, China	75
Monsoon rainfall, India	71-73
Nile flood	72, 74
Pressure, America, South	72
Argentina	72
Australia	72
Cairo	72
Chili	72
China	76
Indian, mean	71, 73
Japan	76
Mauritius	72
Ruikiu Islands	77
Sydney	76
Toronto	76
Rainfall, Australian, Summer	72
Baltimore	74
Beirut	74
Chili	74
England, S.W.	75
Havana	75
India	74
Ireland, South	75
Java	74
Malta	74
Persia	74
San Fernando	74
Seychelles	72
Smyrna	74
Trinidad	74
Zanzibar	72
Snow, India	71, 72, 74
Sunshine, duration of, Japan	75, 77
Sunspot, number	73
Temperature, Australia, Central	73
India	73
Irkutsk	76
Japan	75, 76
San Francisco	76

III. ATMOSPHERIC PRESSURE.

Azores and Iceland, mean monthly pressure at	90
Barometric change at Cronkbourne with different stations	93
Contemporaneous observations of pressure at different stations	77
Iceland and Azores, mean monthly pressure at	90
Kew, relation of pressure to simultaneous and previous pressures in different parts of the globe	93
Polar pressure with air movement at Bremen	78
pressure at different latitudes	87
at different stations same month	79-81, 87
different months	88, 89
temperature	79-81, 88
westerly component of wind at Potsdam	78
Pressure difference at different pairs of stations	91
with pressure at different stations	90, 91
same month	79, 83-88
different months	88-90
Temperature at different latitudes with polar pressure	87
with mean monthly pressure at different stations, same month	78, 79, 83, 84
diff. months	88, 89
with polar pressure	79-81, 88
temperature at different stations	86
Wind velocity with difference of pressure	91, 92

IV. TEMPERATURE.

PAGE

Africa, diurnal variation of temperature.	100
Cairo and England, S.W., annual mean temperature at	95
Continentality and temperature	101, 102
Death rate with temperature and sunshine	95
Egypt, lower, average temperature in, and different stations	96
and England, S.W., diurnal deviations of temperature	86
England, S.W., and Cairo, annual mean temperatures at	95
and Egypt diurnal deviations of temperature	96
Sunshine with temperature and death rate	95
Trade winds, fluctuations in strength of, with winter temperature	98

V. RAINFALL, Etc.

Altitude of station with rainfall	106
Barometric change with rainfall for Woolacombe	105, 106
change of underground water level	107
England, S.E., rainfall returns for	103, 104
Pressure, monthly, with rainfall for same month	107
Underground water level, change of, with barometric change	107
Well, height of water in, at Kew, with height of Thames at Richmond Lock	107
with lunar semi-diurnal oscillation	107
rainfall	103
solar semi-diurnal oscillation	107

VI. SUNSPOTS.

Nile flood with pressure at St. Helena	108
rainfall in England	108
spotted area of sun	108
sunspots	108
Thames, flow of	108
Pressure with sunspots, different stations	108-111
Rainfall	108-111
Temperature with sunspots, different stations	108-111
Thames, flow of, with Nile flood	108
sunspots	108

VII. WEATHER AND CROPS.

America, South, pressure in, with Japan rice crop	127
British Isles, rainfall with crops, different periods	112-117
temperature with crops, different periods	112-117
India, monthly rainfall with crops which fail to come to maturity	126
irrigated crops	125, 126
unirrigated crops	125-127
Japan, rice crop, with pressure	127
in South America	127
Sweden, rainfall and temperature with crops, different stations	129-142
sea temperature, Norwegian Atlantic coast, with air temperature at Orebro	143
United States, heat, total effective, with crops	123, 124
rainfall, total effective, with crops	118
with crops, different periods	118-125
temperature, with crops	118, 120-125, 127
thermal constants, with crops	120

COMPUTER'S HANDBOOK.

SECTION V.

COMPUTATIONS RELATED TO THE THEORY OF PROBABILITIES (*continuation*).

A COLLECTION OF CORRELATION COEFFICIENTS FROM METEOROLOGICAL PAPERS.

By

CAPTAIN E. H. CHAPMAN, R.E.

The object of the present publication is to bring together as many as possible of the correlation coefficients which have been obtained in work on meteorological data.

The correlation coefficient has been used extensively in Meteorology, but with varying success. There is no finer example of its successful use than that which is found in the work of Mr. W. H. Dines on the upper air. In the study of the effect of weather on crops the correlation coefficient has also been used with success. But in other meteorological problems its use has not led to definite results.

There is no doubt that failure to obtain results by the use of the correlation coefficient is more often than not failure to recognise the fact that the coefficient is based on linear regression.

W. G. Reed¹, of the United States Weather Bureau, in an excellent exposition of the correlation coefficient says :—

"The early statements of the use of the coefficient of correlation indicate clearly that the attempt to obtain such a coefficient from miscellaneous material is an abuse of this method of measuring relationship. The material in hand should be investigated carefully before any attempt is made to determine the relationship by the use of the coefficient of correlation. This investigation may take the form of a correlation table, or a dot chart after Galton's graphic method of correlation. The table or chart should be carefully examined to see whether the points may be generalised to a straight line. The coefficient should never be attempted without

¹ W. G. Reed, The Coefficient of Correlation. Quarterly Publications of the Amer. Stat. Assoc., June, 1917.

first investigating the relationship far enough to see if it follows a straight line."

The necessity for a preliminary investigation of the data as suggested by Reed cannot be too strongly insisted upon. If this investigation shows that the relationship between the variables does not approximate to a simple straight line, the coefficient of correlation is **not** a measure of the relationship.

Correlation coefficients taken from the papers consulted have been arranged according to subject in the following groups :—

- I. Upper Air.
- II. Seasonal Correlations.
- III. Atmospheric Pressure.
- IV. Temperature.
- V. Rainfall.
- VI. Sunspots.
- VII. Weather and Crops.

A subject index is given.

I. UPPER AIR.

Reference I.

W. H. DINES, F.R.S. ... Total and Partial Correlation Coefficients between Sundry Variables of the Upper Air, Meteorological Office, London. Geophysical Memoirs, No. 2, 1912.

Notation.

Variables correlated.	Suffix by which denoted throughout the paper.
Pressure in mm. at sea-level.	1
Mean temperature of the air column from 1 to 9 kilometres, formed from the temperatures at 2.5, 5.0 and 7.5 kilometres, degrees C.	2
Pressure at 9 kilometres.	3
Height of the isothermal.	4
Temperature of the isothermal.	5

Observations used.

Observations.	Denoted by
106 observations, the total available on the Continent for the year 1907.	106 C.
100 observations on the Continent taken backwards from June, 1909.	100 C.
66 observations made in British Isles, July 1909-June 1911.	66 E.
The same set of 66 observations, but taken as departures from the mean for the month in which each occurred instead of as departures from the mean of the whole set.	E. A.
29 observations England, winter months, Dec. 1-March 7 (2 from November).	29 E.

Total Correlation Coefficients.

	106 C.	100 C.	66 E.	29 E.	E. A.	Mean
r_{12}	.04	.52	.47	.74	.55	.46
r_{13}	.29	.67	.68	.88	.80	.66
r_{14}	.34	.64	.73	.92	.81	.69
r_{15}	-.55	-.38	-.71	-.65	-.65	-.59
r_{23}	.88	.96	.90	.90	.94	.92
r_{24}	.78	.80	.74	.87	.68	.78
r_{25}	-.02	-.31	-.28	-.65	-.60	-.39
r_{34}	.82	.79	.86	.92	.77	.83
r_{35}	-.22	-.27	-.48	-.76	-.71	-.49
r_{45}	-.46	-.60	-.60	-.86	-.73	-.65

Partial Correlation Coefficients. First Order.

	106 C.	100 C.	66 E.	29 E.	E. A.	Mean.	Mean.
$3r_{12}$	-.47	-.59	-.45	-.25	-.96	-.54	} -.08
$4r_{12}$	-.38	-.39	-.16	-.31	-.02	-.25	
$5r_{12}$.03	.64	.40	.55	.21	+.37	
$2r_{13}$.49	.71	.67	.74	.98	+.72	} +.51
$4r_{13}$.02	.35	.15	.22	.47	+.24	
$5r_{13}$.21	.64	.55	.78	.63	+.56	
$2r_{14}$.49	.49	.64	.84	.72	+.64	} +.53
$3r_{14}$.19	.25	.39	.59	.51	+.39	
$5r_{14}$.12	.56	.54	.94	.65	+.56	
$2r_{15}$	-.55	-.25	-.68	-.33	-.48	-.46	} -.31
$3r_{15}$	-.52	-.28	-.60	.06	-.20	-.37	
$4r_{15}$	-.47	.01	-.50	.71	-.15	-.09	
$1r_{23}$.90	.96	.90	.78	.99	+.91	} +.59
$4r_{23}$.67	.89	.76	.52	.88	+.74	
$5r_{23}$.90	.96	.91	.82	.91	+.90	
$1r_{24}$.82	.82	.66	.72	.46	+.70	} +.63
$3r_{24}$.22	.26	-.18	.24	.24	+.16	
$5r_{24}$.87	.83	.75	.80	.43	+.74	
$1r_{25}$.00	-.08	.09	-.33	-.38	-.14	} +.07
$3r_{25}$.34	-.18	.40	.12	.27	+.19	
$4r_{25}$.61	.44	.30	.39	-.22	+.30	
$1r_{34}$.82	.64	.73	.59	.62	+.64	} -.14
$2r_{34}$.45	.16	.67	.64	.52	+.49	
$5r_{34}$.83	.81	.81	.80	.55	+.76	
$1r_{35}$	-.08	-.02	.01	.52	-.33	-.19	} -.52
$2r_{35}$	-.39	.11	-.54	-.53	.53	-.36	
$4r_{35}$.31	.42	.08	.16	-.34	+.12	
$1r_{45}$	-.35	-.50	-.17	-.88	-.46	-.47	} -.52
$2r_{45}$	-.71	-.67	-.61	-.79	-.55	-.67	
$3r_{45}$	-.50	-.67	-.41	-.64	-.41	-.53	

Partial Correlation Coefficients. Second Order.

	106 C.	66 E.	29 E.	E. A.	Means.
$45r_{12}$	-.14	0	-.95	-.07	-.29
$45r_{13}$.20	.32	.12	.44	.27
$25r_{14}$.17	.39	.99	.63	.53
$35r_{14}$	-.10	.20	.82	.48	.35
$24r_{15}$	-.33	-.48	.95	-.15	—
$34r_{15}$	-.50	-.52	.70	.01	—
$45r_{23}$.63	.78	.50	.89	.70
$15r_{24}$.88	.67	.97	.36	.70
$35r_{24}$.47	.02	.43	-.13	.20
$14r_{25}$.53	.27	.93	-.22	.37
$34r_{25}$.54	.37	.37	.18	.38
$14r_{35}$.83	.74	.33	.22	.53
$25r_{34}$.25	.51	.42	.37	.39
$15r_{34}$.36	.19	.01	.31	.19
$24r_{35}$	-.12	-.23	-.02	-.33	-.18
$12r_{45}$	-.61	-.31	-.97	-.34	-.56
$13r_{45}$.36	.19	.01	-.31	.06
$23r_{45}$	-.64	-.38	-.69	-.38	-.52

Correlation Coefficients between the following variables :—

N Northerly component of (i) the general drift of the air up to 12 to 20 kilometres, (ii) the geostrophic wind for sea level, or (iii) the surface wind. Unit for (i) kilometres, (ii) metres per second, (iii) Beaufort numbers.

E Easterly component.

T_0 Temperature in °C. at surface.

T_4 Temperature in °C. at 4 kilometres.

T_8 Temperature in °C. at 8 kilometres.

H_0 Height of isothermal in kilometres.

Winter = November to April. Summer = May to October.

Winter = November to April. Summer = May to October.

	General drift of upper Air.						Mean.	Geostrophic Wind.		Surface Wind
	English, 1908-11.			Continental.				Winter.	Summer.	Winter.
	Winter.	Summer.	Combined.	1909.	Winter, 1907-8.	Summer, 1907-8.				
N and T_0 ...	·16	·01	·18	·26	·01	·09	·06	·22	·16	·53
N and T_4 ...	·13	·16	·10	·18	·08	·00	·02	·01	·12	—
N and T_8 ...	·17	0	·06	·21	·18	·08	·03	·12	·14	—
N and H_c ...	·26	·11	·06	·04	·05	·18	·10	·03	·01	—
E and T_0 ...	·18	·07	·13	·12	·36	·24	·06	·57	·17	·42
E and T_4 ...	·17	·12	·07	·28	·09	·31	·08	·31	·05	—
E and T_8 ...	·08	·17	·16	·26	·06	·13	·03	·02	·18	—
E and H_c ...	·31	·08	·09	·08	·06	·23	·09	·11	·11	—
H_c and T_0 ...	·22	·42	·43	·49	·09	·15	·30	—	—	—
H_c and T_4 ...	·60	·57	·67	·75	·68	·55	·64	—	—	—
H_c and T_8 ...	·75	·72	·71	·76	·82	·66	·74	—	—	—
Number of Observations	46	93	139	80	83	105	407	38	55	64

In the above table the signs of the coefficients under geostrophic wind have been corrected according to Reference 4.

Reference 2.

W. H. DINES, F.R.S. ... Correlation and Regression Tables for the Upper Air. Beiträge zur Physik der freien Atmosphäre. V Band, 1913, p. 213.

Suffix notation as in Reference 1.

Observations used.	
106C, 100C, 66E, E.A., 29E.	
85 observations in England 1911-1912.	
Observations during International Week December, 1909.	As in Reference 1.
" " " " August, 1910.	Additional to those in Reference 1.

Total Correlation Coefficients.

	r_{12}	r_{13}	r_{14}	r_{15}	r_{23}	r_{24}	r_{25}	r_{34}	r_{35}	r_{45}
Extremes {	.45	.65	.65	-.50	.90	.75	-.35	.80	-.45	-.65
	.04	.29	.34	-.12*	.78*	.68	-.02	.69*	-.08*	-.46
	.74	.88	.92	-.71	.96	.87	-.65	.92	-.76	-.86

The values in the top row (given to nearest .05) are those which the author judges to be the most likely. They are very near the actual means, but equal weight has not been given to all the observations. The four coefficients marked with an asterisk are outside the range of the corresponding total correlation coefficients in Reference 1.

Partial Correlation Coefficients. First Order.

	Actual.	Corrected Value.		Actual.	Corrected Value.		Actual.	Corrected Value.
$3r_{12}$	-.48	-.75	$3r_{15}$	-.31	-.30	$4r_{25}$.28	.39
$4r_{12}$	-.07	-.14	$4r_{15}$	-.11	-.11	$1r_{34}$.65	.71
$5r_{12}$.37	.35	$1r_{23}$.90	.97	$2r_{34}$.44	.48
$2r_{13}$.63	.83	$4r_{23}$.76	.87	$3r_{34}$.75	.81
$4r_{13}$.29	.28	$5r_{23}$.90	.94	$1r_{35}$	-.18	-.18
$5r_{13}$.55	.58	$1r_{24}$.68	.76	$2r_{35}$	-.35	-.42
$2r_{14}$.53	.58	$3r_{24}$.11	-.05	$4r_{35}$.15	.24
$3r_{14}$.29	.28	$5r_{24}$.74	.80	$1r_{45}$	-.50	-.51
$5r_{14}$.51	.51	$1r_{25}$	-.16	-.16	$2r_{45}$	-.66	-.69
$2r_{15}$	-.41	-.41	$3r_{25}$.14	.27	$3r_{45}$	-.55	-.59

The actual values of the partial correlation coefficients are calculated from the total correlation coefficients in the usual way.

In order to correct for the systematic error caused by errors of observation the author increases the value of each total correlation coefficient by 5 per cent. The values of the partial correlation coefficients calculated from these increased total correlation coefficients are given in the above table under "Corrected Value."

Values of R .

In the regression equation $x_1 = ax_2 + bx_3$; ${}_{23}R_1$ is the correlation between the two sides of the equation.

${}_{23}R_1$.88	${}_{13}R_2$.98	${}_{12}R_3$.98	${}_{12}R_4$.87	${}_{12}R_5$.54
${}_{24}R_1$.69	${}_{14}R_2$.80	${}_{14}R_3$.86	${}_{13}R_4$.85	${}_{13}R_5$.55
${}_{25}R_1$.61	${}_{15}R_2$.51	${}_{15}R_3$.69	${}_{15}R_4$.78	${}_{14}R_5$.68
${}_{34}R_1$.71	${}_{34}R_2$.95	${}_{24}R_3$.96	${}_{23}R_4$.84	${}_{23}R_5$.53
${}_{35}R_1$.72	${}_{35}R_2$.95	${}_{25}R_3$.95	${}_{25}R_4$.90	${}_{24}R_5$.74
${}_{45}R_1$.69	${}_{45}R_2$.82	${}_{45}R_3$.85	${}_{35}R_4$.90	${}_{34}R_5$.70

Partial Correlation Coefficients. Second Order.

${}_{45}r_{12}$	-.07	${}_{45}r_{23}$.85	${}_{34}r_{25}$.34
${}_{45}r_{13}$.28	${}_{15}r_{24}$.79	${}_{15}r_{34}$.30
${}_{25}r_{14}$.42	${}_{35}r_{24}$.17	${}_{24}r_{35}$	-.20
${}_{35}r_{14}$.10	${}_{14}r_{35}$.74	${}_{12}r_{45}$	-.61
${}_{24}r_{15}$	-.03	${}_{25}r_{34}$.28	${}_{13}r_{45}$	-.61
${}_{34}r_{15}$	-.19	${}_{14}r_{25}$.39	${}_{23}r_{45}$	-.56

Partial Correlation Coefficient. Third Order.

$${}_{123}r_{45} = -.64.$$

Barometric Gradient and Wind Velocity.

The correlation coefficient between the barometric gradient and the velocity of the wind is about .70. Four calculations, each based on 100 observations, two at a coast station and two inland, gave values which all lie between .65 and .75.

Reference 3.

W. H. DINES, F.R.S. ... Daily Temperature Change at Great Heights. Quar. Jour. Roy. Met. Soc., 1914, p. 11.

Correlation between pressure at 9 kilometres and temperature at various heights.

Data.—92 observations at Pyrton Hill and Ditcham Park, mostly during the years 1910-1912 inclusive.

Height in kms.	0	0.5	1.0	1.5	2.0	3.0	5.0	9.0
Correlation Coefficient	.28	.60	.68	.73	.74	.82	.82	.40

Correlation between pressure and temperature at same height.

Data.—36 observations at Pyrton Hill, selected to be independent and taken in the winter half-year, November-April.

Height in kms.	0	2	4	6	8	10	12	14
Correlation Coefficient	.16	.50	.69	.78	.82	-.08	-.60	-.44

Correlation between pressure and temperature at same height.

Data.—50 observations at Pyrton Hill, selected to be independent and taken in the summer half-year, May to October.

Height in kms.	0	2	4	6	8	10	12	14
Correlation Coefficient05	.51	.62	.62	.63	.48	-.11	-.05

Reference 4.

W. H. DINES, F.R.S. ... The Characteristics of the Free Atmosphere. Meteorological Office, London. Geophysical Memoirs, No. 13, 1919.

Correlation Coefficients between Pressure and Temperature at Heights up to 13 kilometres.

Data.—All available observations from Pyrton Hill, Ditcham Park and Benson.

Height in kms.	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Jan.-March	-.02	.54	.82	.79	.86	.85	.84	.87	.91	.81	.35	-.32	-.38	-.37
April-June	.14	.28	.49	.79	.89	.92	.87	.81	.45	.20	-.12	-.24	-.01	
July-Sept...	-.02	.31	.56	.72	.75	.81	.83	.87	.87	.88	.43	-.08	-.41	-.19
Oct.-Dec.33	.56	.76	.77	.83	.87	.85	.85	.86	.78	.29	-.24	-.34	-.50

P_s the barometer pressure at M.S.L.

H_c the thickness of the troposphere, or the height of the tropopause (called the height of the isothermal in Reference 1).

$T_{0.4}$ the mean temperature up to 4k.

V the total water vapour contents of the atmosphere.

γ the steepness of the barometric gradient.

B the barometric rise in the past 12 hours.

Correlation Coefficients.

		H_c	P_s	$T_{0.4}$
V39	.08	.73
$T_{0.4}$66	.34	—
γ22	—	—
B17	—	—

Reference 5.

E. H. CHAPMAN ... Work on Upper Air Data, N.E. France. Meteorological Office, London, M.S. 1918.

Correlations between wind velocities at various heights—

(i) above diagonal. 31 pilot balloon ascents at Avesnes le Comte, November, 1916, to January, 1917, at 7h., surface velocity 5 feet above ground ;

(ii) below diagonal, 30 pilot balloon ascents near St. Omer, December, 1915, to March, 1916, at 7h., surface velocity 30 feet above ground.

		Height in feet.					
		Surface.	250	500	1,000	2,000	3,000
Height in feet.	Surface.		.80	.82	.81	.74	.74
"	250	.88		.93	.82	.74	.77
"	500	.70	.87		.95	.87	.90
"	1,000	.68	.74	.88		.95	.95
"	2,000	.74	.68	.65	.83		.86
"	3,000	.70	.77	.67	.74	.86	
"	4,000	.79	.70	.69	.72	.88	.90

Correlations between surface wind velocity, geostrophic wind velocity at sea level and wind velocities at various heights.

Data.—67 pilot balloon ascents made near St. Omer, December, 1916, to March, 1917, 7h. Surface velocities from Dines' pressure tube anemometer 30 feet high.

Total Correlation Coefficients.

Velocity of wind at	Velocity of	
	Surface Wind.	Geostrophic wind. Sea level.
3,000 ft.	.76	.71
2,000 "	.83	.71
1,000 "	.80	.68
500 "	.77	.60
200 "	.80	.57
Surface	—	.54

Partial Correlation Coefficients.

Velocity of wind at	Eliminating effect of geostrophic wind velocity at sea level, the correlations between velocity of surface wind and wind velocity at various heights are	Eliminating effect of surface wind velocity, the correlations between velocity of geostrophic wind at sea level and wind velocity at various heights are
3,000 ft.	.81	.53
2,000 "	.73	.54
1,000 "	.69	.50
500 "	.66	.33
200 "	.70	.27

Correlation coefficients between wind components V_{WE} , V_{SN} at various heights.

Pilot balloon data for period November, 1916, to January, 1917.

Station and Time of Day.	Number of Observations.	Correlations between Wind Components.	
		V_{WE} 1,500 ft. and V_{WE} 3,000 ft.	V_{SN} 1,500 ft. and V_{SN} 3,000 ft.
PONT NOYELLES, near AMIENS.			
(1) Morning	36	.94	.81
(2) Afternoon	33	.95	.84
AVESNES LE COMTE.			
Morning and afternoon ...	76	.92	.85
PONT NOYELLES.			
Morning and afternoon ...	32	.86	.80
AVESNES LE COMTE.			
Mostly morning	30	.92	.80
Near HAZEBROUCK.			
(1) Morning	20	.92	.76
(2) Afternoon	24	.91	.85

Correlation coefficients between wind components at surface, 500, 1,000 and 1,500 feet.

Components at surface denoted by suffix 1.

Mean of components at 500 and 1,000 ft. denoted by suffix 2.

Components at 3,000 ft. denoted by suffix 3.

Data.—Pilot balloon ascents November, 1916–January, 1917, and pilot balloon ascents May–June, 1916.

Station, Time and Period.	Number of Observations.	V_{WE}			V_{SN}		
		r_{12}	r_{13}	r_{23}	r_{12}	r_{13}	r_{23}
Near HAZEBROUCK.							
Morning, Nov. 16–Jan. 17	43	.82	.87	.94	.82	.86	.95
" May–June, 1916...	34	.77	.76	.88	.83	.85	.94
Afternoon, Nov. 16–Jan. 17	48	.91	.86	.89	.93	.97	.89
" May–June, 1916	39	.93	.92	.94	.96	.94	.97
Near AMIENS.							
Morning, Nov. 16–Jan. 17	33	.83	.87	.85	.71	.88	.77
Afternoon, Nov. 16–Jan. 17	30	.93	.93	.97	.78	.90	.91

Partial Correlation Coefficients.

Station, Time and Period.	Number of Observations.	V_{WE}		V_{SN}	
		r_{12}^{23}	r_{13}^{23}	r_{23}^{13}	r_{23}^{12}
Near HAZEBROUCK.					
Morning—					
Nov. 1916–June, 1917	43	.53	.82	.44	.83
May–June, 1916 ...	34	.26	.74	.37	.79
Afternoon—					
May–June, 1916 ...	48	.27	.50	.86	.15
May–June, 1916 ...	39	.37	.58	.7	.70
Near AMIENS.					
Morning—					
Nov. 1916–June, 1917	33	.55	.48	.75	.43
Afternoon—					
Nov. 1916–June, 1917	30	.26	.78	.74	.75

Reference 6.

E. V. NEWNHAM ... Further Contributions to the Statistical Analysis of Upper Air Observations. Meteorological Office, London. M.S. 1915.

Notation H_c height of tropopause.

P_s barometric pressure at M.S.L.

P_9 pressure at 9k.

Simultaneous Observations H_c .

Stations.	Distance apart miles.	Number of Observations.	Correlation Coefficient.
Pyrtton and Ditcham ...	45	17	.90
Paris and Brussels...	170	20	.70
Strasburg and Munich ...	180	30	.85
Pavia and Lindenberg ...	540	31	.35
Pavia and Pavlovsk ...	1400	28	.00

Correlation Coefficients between Values of H_c (and P_s) at the same Station one day apart.

	England.	Hamburg and Lindenberg.	Paris, Munich and Strassburg.
H_c70	.80	.75
P_s85	.70	.65
H_c corrected for P_s65	.85	.60
Number of observations ...	34	34½	34

Correlation Coefficients between Values of H_c (and P_s) at the same Station 2, 3 and 4 days apart.

	2 days apart.	3 days apart.	4 days apart.
H_c65	.15	.36
P_s40	.05	—
Number of observations (British and Continental).	52	45	36

British and Continental Observations 1908–1914.

Correlation Coefficients.

	S.-N. component of geostrophic wind and		
	H_c	P_s	H_c eliminating effect of P_s .
Winter 344 observations05	-.02	.08
Summer 418 observations10	-.13	.25

Correlation Coefficients.
Observations as in last Table.

					Barometric gradient and		
					H_c	P_s	H_c eliminating effect of P_s
Winter	-.15	-.16	-.06.
Summer	-.28	-.12	-.26

Correlation Coefficient.	Number of Observations.	Variables correlated.
.01	635	Steepness of barometric gradient and H_c (the values of H_c are corrected for P_s and seasonal variation).
.17	264	H_c and barometric change previous 12 hours for Lindenberg, Paris, Pyrtion and Manchester.
.76	1826 (1900-1904)	Evening barometer readings 24 hours apart, St. James' Park, London.
.81	364 (1907)	Evening barometer readings 24 hours apart, St. James' Park, London.
.74	358 (1907)	Simultaneous barometer readings 7h, London and Berlin.
.73	359 (1907)	Barometer reading London 7h with Berlin 24 hours later.
.76	1826	Kew 7h barometer readings 1900-1904.
.53	1823	1 day apart.
.10	608	2 days apart.
-.02	848	3 days apart.
		10 days apart.

Reference 7.

E. V. NEWNHAM ... The Relation between Winds at 1,500, 5,000, and 10,000 ft. Meteorological Office, London. M.S. 1917.

Data.—Pilot balloon ascents made at 7h, 13h, or 18h at one of the following stations in the British Isles:—Yarmouth, Llangefni, Sleaford, Longside, Farnborough, Helston, Pembroke, Luce Bay, Barrow, Athelstaneford.

Two sets of observations (i) December, 1916, 33 ascents.
(ii) January, 1917, 38 ascents.

Correlation Coefficients.

Observations.		Geostrophic Wind Velocity, Sea Level, and Wind Velocity at				
		1 000 ft.	2,000 ft.	3,000 ft.	4,000 ft.	5,000 ft.
December, 191675	.77	.76	.78	.82
January, 191751	.57	.67	.64	.57

Correlation Coefficients between Wind Components at 1,500 and 10,000 feet.

Observations.			Component.	
Places.	Season.	Number.	W.-E.	S.-N.
Pyrtion Hill, South Farnborough and Benson.	Winter, Nov.-Feb., 1914-15-16.	50	.87	.80
	Summer, May-Oct., 1914-15.	100	.59	.73

Correlation Coefficients between Wind Components at 5,000 and 10,000 feet.

Observations.			Component.	
Places.	Season.	Number.	W.-E.	S.-N.
Pyrtion Hill, South Farnborough and Benson.	Winter, Nov.-Feb., 1914-15-16.	50	.90	.86
	Winter, Jan.-Dec., 1914-15-16-17.	46	.87	.93
" "	Summer, May-Oct., 1914-15.	100	.82	.85
" "	Summer, June-Aug., 1914-15.	51	.84	.79

II. CLIMATIC CORRELATIONS.

Reference 8.

G. T. WALKER ... Correlation in Seasonal Variations of Weather. Mem. Indian Meteor. Depart., Vol. XXI., Part II., 1910.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
-.22 ± .15	*1876-1908 [1876-1890, half weight; 1891-1908 weight unity].	Indian Monsoon Rainfall and (i) Snowfall in May to N. and W. of India.
-.38 ± .14		(ii) Accumulation of snow at the end of May to N. and W. of India.
-.36 ± .11	33 (1876-1908)	(iii) Accumulation of snow at the end of May to N. and W. of India.
+ .17 ± .10	44 (1865-1908)	(iv) Mean Indian pressure of previous year.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
- '36 ± '10	34 (1875-1908)	<i>Indian Monsoon Rainfall and</i> (v) Mean pressure for May of same year at Mauritius.
- '30 ± '13	35 (1874-1908)	(vi) May rainfall at Zanzibar.
- '25 ± '13	"	(vii) April "
- '31 ± '13	"	(viii) April and May rainfall at Zanzibar.
		<i>Accumulation of snow at the end of May to N. and W. of India and</i>
+ '53 ± '11	As above*	(i) Snowfall in May to N. and W. of India.
+ '09 ± '13	"	(ii) Mauritius May pressure.
+ '31 ± '13	"	(iii) Zanzibar rainfall, April and May.
+ '39 ± '13	"	(iv) Seychelles rainfall, May.
		<i>Mean departure of the Argentine Republic and Chili pressure for March, April and May weighted $\frac{1}{2}$, 1, and 1 respectively with</i>
+ '42 ± '08	48 (1861-1908)	(i) Indian monsoon rainfall of same year.
- '37 ± '11	As above*	(ii) May snow accumulation to N. and W. of India.
- '12 ± '11	34 (1875-1908)	(iii) Mauritius pressure for May.
		<i>Total rainfall, April and May at Zanzibar with</i>
+ '15 ± '14	34 (1875-1908)	(i) Mauritius May pressure.
- '32 ± '13	"	(ii) South American "
		<i>Nile floods with</i>
'00 ± '12	34 (1875-1908)	(i) Mauritius pressure in May.
- '06 ± '11	"	(ii) " " July.
- '19 ± '10	"	(iii) " " September.
- '49 ± '08	42 (1867-1908)	(iv) South American pressure in March, April, May.
- '44 ± '12	35 (1874-1908)	(v) Zanzibar rainfall, April and May.
- '35 ± '11	As above*	(vi) Indian snowfall accumulation.
- '00 ± '12	34 (1875-1908)	(vii) Cairo pressure in April.
- '21 ± '11	"	(viii) " " May.
		<i>Australian summer rainfall (December to April) with</i>
- '41 ± '10	32 to 18 during years 1871-1905.	(i) Australian pressure previous November.
- '36 ± '11		(ii) " " " " October.
- '12 ± '12		(iii) " " " " August.
- '03 ± '13		(iv) " " " " June.
- '45 ± '10		(v) " " " " mean previous Oct.-Nov.
- '31 ± '11	11 (1892-1902)	(vi) Mauritius pressure mean previous Oct.-Nov.
- '37 ± '11		(vii) Rainfall of previous Indian monsoon.

Reference 9.

G. T. WALKER

A Further Study of Relationships with Indian Monsoon Rainfall. Mem. Indian Meteor. Depart., Vol. XXI., Part VIII., 1914.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
+ '50	32 (1875-1906)	Mean pressure for year in India with monsoon rainfall in following year.
+ '25	42 (1865-1906)	Mean pressure for year in India with monsoon rainfall in following year.
+ '20	48 (1865-1912)	Mean pressure for year in India with monsoon rainfall in following year.
+ '35	38 (1875-1912)	Mean pressure for year in India with monsoon rainfall in following year.
- '55	38 (1875-1912)	Mean pressure for year in India with monsoon rainfall in same year.
+ '25 ± '10	49 (1864-1912)	Indian monsoon rainfall with :— (i) Mean Indian pressure for the period October two years before to September one year before.
+ '25 ± '10	49 (1864-1912)	(ii) Mean Indian pressure November-August year before.
+ '60 ± '08	46 (1867-1912)	Successive differences in the annual values of Indian pressure and monsoon rainfall.
+ '05 ± '12	31 (1875-1905)	Average Indian temperature in May with the rainfall of the following monsoon.
- '02 ± '12	"	Average temperature of N.W. India and desert regions beyond, in May, with following Indian monsoon rainfall.
- '17	"	Average temperature as last for June and following Indian monsoon rainfall.
- '25	"	Indian mean temperature in January with annual sunspot number.
- '10	"	Average annual temperature of desert regions to N.W. of India and annual sunspot number.
+ '20	"	Mean temperature Central Australia for June, July, and August with annual sunspot number

Correlation Coefficients of the Monsoon Rainfall of any year with the monthly pressures of India in the second year before, year before, actual and following years.

Data for 1864-1912.

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Second year before	- '030	- '212	- '189	- '143	- '152	- '071	- '209	+ '325	- '050	+ '232	+ '137	+ '254
Year before	+ '243	+ '245	+ '170	+ '297	- '002	+ '223	+ '249	+ '163	+ '008	- '086	+ '034	+ '264
Actual year	- '167	- '228	- '240	- '349	- '120	- '304	- '123	+ '061	- '618	- '488	- '192	- '103
Following year	- '139	- '144	- '251	- '357	- '470	- '239	+ '066	- '062	- '057	- '071	+ '261	+ '288

Reference 10.

G. T. WALKER

... The Cold Weather Storms of Northern India (Part I.). Mem. Indian Meteor. Depart., Vol. XXI., Part VII., 1913.

Correlation Coefficient.	Number of Observations.	Variables correlated.
		Rainfall in North-west India, December to March, with rainfall November to March at
.30	20	(i) Bushire,
.21	18	(ii) Tehran,
.10	20	(iii) Ispahan,
.22	19	(iv) Baghdad,
.27	34	(v) Beirut,
.43	15	(vi) Smyrna.
.27	27	(vii) Malta (rainfall October to March).
.72	38	Winter snowfall in the mountains of Northern India, with winter rainfall of North-west India.
		Proportionate departures from means of rainfall in North-west India and snowfall in North India ('6 former + '4 latter), with rainfall November to March at
.28	20	(i) Bushire,
.32	18	(ii) Tehran,
.18	20	(iii) Ispahan,
.12	19	(iv) Baghdad,
.44	34	(v) Beirut,
.47	15	(vi) Smyrna,
.28	27	(vii) Malta.

Reference 11.

R. C. MOSSMAN

... Southern Hemisphere Seasonal Correlations. Symon's Meteorological Magazine, Vol. 48, 1913. (Papers.)

Correlation Coefficient.	Number of Observations.	Variables correlated.
-.62 ± .07	38 (1869-1906)	Nile Flood as measured by the volume of water passing Aswan between July 1st and October 31st, and winter rainfall May to August of the same year, at Santiago de Chile.
-.47 ± .11	30 (1880-1909)	Rainfall at Java October to March, with rainfall at Trinidad the following six months.
-.57 ± .05	30 (1851-1880)	Rainfall January to March at Baltimore, U.S.A., and rainfall at San Fernando, near Cadiz, for the same months.
-.21	54 (1851-1904)	As last.

Other correlation coefficients, based on 22 or less observations are given in the papers which are covered by this reference.

Reference 12.

A. HAMPTON BROWN

... A Cuban Rain Record and its Application. Quar. Jour. Roy. Met. Soc., Vol. XL., 1914.

Correlation Coefficient.	Number of Observations.	Variables correlated.
-.47 ± .09	36 1877-1912	Rainfall at Havana May to October with rainfall in South Ireland the following January to March.

Reference 13.

A. HAMPTON BROWN

... The Rainfall in Cuba and England, South West. Symon's Meteor. Mag., Feb., 1915.

Correlation Coefficient.	Number of Observations.	Variables correlated.
-.54 ± .08	36 (1877-1912)	Rainfall at Havana May to October with rainfall in S.W. England the following January to March.

Reference 14.

T. OKADA

... Some Researches in the Far Eastern Seasonal Correlations. Jour. Meteor. Soc. Japan. First Note, Dec., 1915; Second Note, May, 1917; Third Note, June, 1917; Fourth Note, July, 1917.

Data for 20 Years 1891-1910.

Station.	March barometric Gradient at Zikawei (China) and			
	Air Temperature.		Duration of bright sunshine.	
	July-August	July	July	August
Nemuro76 ± .07	.61 ± .10	.37 ± .13	.41 ± .13
Miyako78 ± .06	.75 ± .07		
Isinomaki68 ± .08	.61 ± .09		
Tokyo58 ± .10	.42 ± .12
Niigata47 ± .12	.47 ± .12
Station.	Jan.-March barometric gradient at Zikawei and August temperatures.		Pressure difference Zikawei-Nafa for Jan. to March and air temperature in August.	
Nemuro...	.61 ± .10		.58 ± .01	
Miyako50 ± .11		.49 ± .11	
Isinomaki59 ± .10		.60 ± .10	

Correlation Coefficient.	Number of Observations.	Variables correlated.
$-.34 \pm .12$	25 (1890-1914)	July air temperature at San Francisco and :— (i) July air temperature at Erimo, N. Japan.
$-.53 \pm .11$	21 (1887-1907)	(ii) April air temperature at Irkutsk.
$-.41 \pm .11$	25 (1886-1910)	Mean barometric pressures at Zikawei in January and February and at Nemuro in July and August.
$.32 \pm .11$	28 (1887-1914)	April pressure at Toronto and August temperature at Erimo in N. Japan.

Correlation between Series of First Differences.

Thus if $x_1, x_2, x_3, \dots, y_1, y_2, y_3, \dots$ are the two series the correlations obtained are those between the two series :—

$$x_2 - x_1, x_3 - x_2, \dots$$

$$\text{and } y_2 - y_1, y_3 - y_2, \dots$$

Correlation Coefficient.	Number of Observations.	Variables Correlated.
$-.51 \pm .11$	25 (1889-1914)	First differences of July temperatures at San Francisco and July temperatures at Erimo in N. Japan.

Other correlation coefficients :—

Variables.	Suffix.
First difference of series of mean August temperature at Nemuro and Miyako ...	q
" " " " " April pressure differences between Ponta Delgada and Stykkisholm ...	x
" " " " " March pressure difference between Zikawei and Miyazaki	y
" " " " " January Sydney pressure ...	z

Period.	Number of terms in series of first difference.	r_{qx}	r_{qy}	r_{xy}	r_{qz}	r_{xz}	r_{yz}
1883-1912	29	$.61 \pm .08$	$.40 \pm .11$	$.27 \pm .11$			
1883-1907	24	$.67 \pm .08$	$.41 \pm .12$	$.25 \pm .13$	$.49 \pm .11$	$.39 \pm .12$	$.20 \pm .13$

Reference 15.

T AKATU ... Correlation between the Barometric Pressure Gradient at Nafa in the Riukiu Islands for January to April, and the Rainfall, Air Temperature, and Sunshine Duration at Koti, on the South Coast of Japan. Jour. Meteor. Soc. Japan, April, 1917.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$.54 \pm .10$	24 (1891-1914)	Barometric pressure gradient at Nafa, Riukiu Islands January to April and (i) rainfall, (ii) air temperature, and (iii) sunshine duration for July at Koti on the South coast of Japan,
$-.45 \pm .11$	"	(i) rainfall,
$-.53 \pm .10$	"	(ii) air temperature, (iii) sunshine duration.

III. ATMOSPHERIC PRESSURE.

Reference 16.

Prof. KARL PEARSON On the Distribution of Frequency and Dr. ALICE LEE. (Variation and Correlation) of the Barometric Height at Divers Stations. Phil. Trans. A., Vol. 190, p. 423.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$.98$	8 years 1878-1885	Contemporaneous observation of barometric height at 9 a.m. at (i) Babbacombe and Churchstoke.
$.76$	8 years 1880-1887	(ii) Southampton and Laudale.
$.96$	8 years 1878-1885	(iii) Hillington and Churchstoke.

Reference 17.

F. M. EXNER Über monatliche Witterungsanomalien auf nördlichen Erdhalbkugel im Winter. Akad. Wiss. Wien. Ber 122 2a, pp. 1165-1240, June, 1913.

ata.—Mean monthly pressures and temperatures for the winter months December, January and February for periods of 10 years.

(i) 1897-1906 referred to as period I.

(ii) 1887-1896 " " " II.

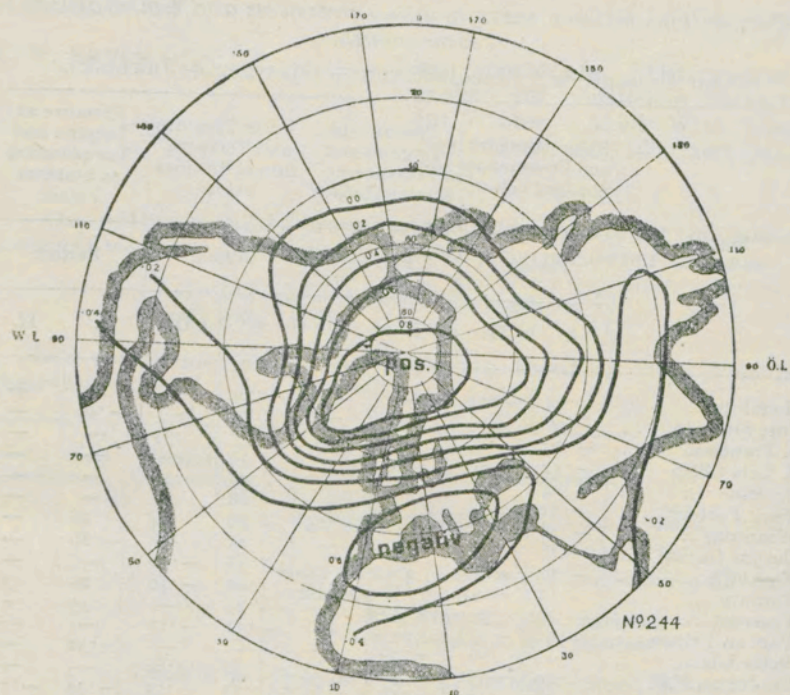
The correlation coefficients are thus based on $10 \times 3 = 30$ observations.

Correlation Coefficient.	Number of Observations.	Variables correlated.
		Mean monthly pressures and temperatures for same month— K = Kirensk. M = Markowa. S = Surgut. G = Gjesvaer.
-.74	Period I. 30	Pressure $\frac{1}{2}$ (K + S) and temperature $\frac{1}{2}$ (K + S).
-.35	Not stated.	As last, but for summer.
.43	Period I. 30	Pressure $\frac{1}{2}$ (M + G) and temperature $\frac{1}{2}$ (M + G).
.84	"	Pressure $\frac{1}{2}$ (M + G) and pressure Obdorsk.
-.37	"	Temperature $\frac{1}{2}$ (M + G) and temperature $\frac{1}{2}$ (K + S).
-.69	"	Pressure $\frac{1}{2}$ (M + G) and temperature $\frac{1}{2}$ (K + S).
-.76	"	Pressure $\frac{1}{4}$ (K + S + M + G) and temperature $[\frac{1}{2} (K + S) - \frac{1}{2} (M + G)]$.
-.39	"	Pressure $\frac{1}{2}$ (Winnipeg + Toronto + M + G) and temperature $[\frac{1}{2} (Winnipeg + Toronto) - \frac{1}{2} (M + G)]$.
.44	"	Pressure and temperature thus $[\frac{1}{2} (Valencia + Spokane + Vladivostok) - \frac{1}{2} (M + G)]$.
.57	"	Pressure and temperature thus $[\frac{1}{2} (Aden + Honolulu) - \frac{1}{2} (M + G)]$.
-.74	"	Pressure $\frac{1}{4}$ (Honolulu + Aden + Madras + Manila + St. Louis + Corpus Christi + Fort de France) and pressure $\frac{1}{2}$ (M + G).
-.39	24 (1899-1906)	Polar pressure and air movement at Bremen.
-.57	Period I. 30	" " " westerly component of wind at Potsdam.

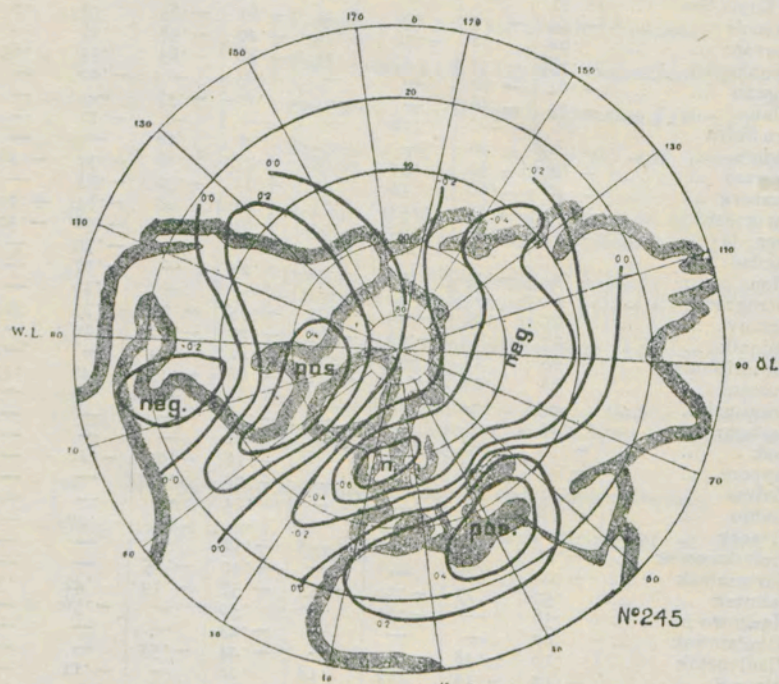
Correlations between mean monthly pressures and temperatures same month.

Polar Pressure is $\frac{1}{3}$ (Jacobshaven + Gjesvaer + Jakutsk).

Station.	Polar Pressure and Pressure at Stations below.		Pressure at Lugano and Pressure at Stations below.		Polar Pressure and Temperature at Stations below.		Pressure at Lugano and Temperature at Stations below.	
	Period.		Period.		Period.		Period.	
	I.	II.	I.	II.	I.	II.	I.	II.
Honolulu...	-.23	-.11	—	—	.29	—	—	—
Port Simpson ...	-.36	—	.26	—	.29	—	-.06	—
S. Francisco ...	-.10	-.04	—	—	.37	.33	—	—
S. Luis Obispo ...	-.12	—	.14	—	.51	—	—	—
Spokane ...	-.24	.21	—	—	.39	.12	—	—
York Factory ...	-.01	—	—	—	-.08	—	—	—
Winnipeg ...	-.16	-.10	-.21	-.05	-.00	.04	.16	—
Corpus Christi ...	-.16	—	—	—	.05	—	-.20	—
Nashville ...	-.20	-.28	—	—	-.18	-.10	—	—
Toronto ...	-.12	-.32	-.17	.16	.01	-.40	.29	.38
Prospect ...	-.27	—	.23	—	-.25	—	.35	—
Port au Prince ...	-.61	-.30	—	—	-.31	-.17	—	—
Belle Isle ...	—	—	-.49	—	—	—	-.32	—
St. Johns, N.F.30	-.21	-.38	-.23	.42	.66	—	—
Jacobshaven78	.84	-.50	-.63	.17	.72	-.34	-.79
Ivigut77	.79	-.63	-.77	.21	.74	—	—
Stykkisholm55	.80	-.56	-.65	.07	—	.14	—
P. Delgada ...	-.16	-.65	-.03	—	-.22	-.27	.34	.39
St. Louis Sen. ...	-.53	—	—	—	-.21	—	—	—
Valencia ...	-.44	-.13	.50	.25	-.61	-.65	.78	.70
Gjesvaer92	.82	-.78	-.59	-.20	-.58	.16	—
Copenhagen ...	-.28	-.33	.47	.71	-.46	-.65	.53	.63
Lugano ...	-.64	-.70	—	—	.05	—	-.02	—
Milan ...	-.75	-.56	—	—	.10	-.37	—	—
Cirn Sefra ...	—	—	.70	—	—	—	-.11	—
Algiers ...	-.73	-.66	—	—	.27	.28	—	—
Palermo ...	-.68	-.60	.94	.86	.45	.26	-.37	-.46
Lemberg ...	-.46	-.60	.71	.92	-.12	-.30	-.03	—
Novorossisk ...	-.33	-.73	.53	.75	.27	.40	-.54	-.70
Suez ...	-.60	-.09	.65	.03	.61	.55	-.58	-.50
Bagdad ...	—	—	—	—	—	—	-.59	—
Aden ...	-.54	-.11	—	—	.22	—	-.07	—
Petrograd ...	—	—	-.51	—	—	—	.46	—
Saratov65	.06	-.49	-.02	-.55	-.23	.37	.04
Obdorsk83	.60	-.68	-.61	-.22	-.50	.10	—
Ekaterinburg76	.42	—	—	-.42	-.49	—	—
Barnaul68	.24	-.45	-.42	-.68	-.47	.47	.37
Kozalinsk... ..	—	—	-.50	—	—	—	.45	—
Kashgar13	—	—	—	-.31	—	—	—
Jask ...	-.33	—	—	—	-.01	—	—	—
Jeypore ...	-.42	-.12	.21	.19	-.16	-.03	-.10	—
Madras ...	-.25	.10	—	—	-.01	.22	—	—
Lashio ...	-.38	—	—	—	.14	—	—	—
Kirensk ...	—	—	.02	—	—	—	.40	—
Troitskosawsk ...	-.11	—	—	—	-.32	—	—	—
Nertschinsk ...	-.01	.20	—	—	-.40	-.25	—	—
Jakutsk50	.77	-.30	-.53	-.57	-.19	.42	.31
Markowo36	—	-.31	—	.50	—	-.56	—
Nikolaiewsk07	—	—	—	-.23	—	—	—
Vladivostok10	-.38	—	—	-.44	-.53	—	—
Zikawei ...	-.15	-.18	.22	.18	.08	—	-.11	—
Manila ...	-.34	.07	—	—	.33	—	—	—



*FIG. 1.—WINTER.



*FIG. 2.—WINTER.

Correlations are found between mean polar pressure in December, January and February, and mean pressure for the same month at various stations in the Northern Hemisphere. This is done for two periods of 10 years, there being 30 months in each set. The isocorrelates are drawn for the average correlation coefficients from the two periods.

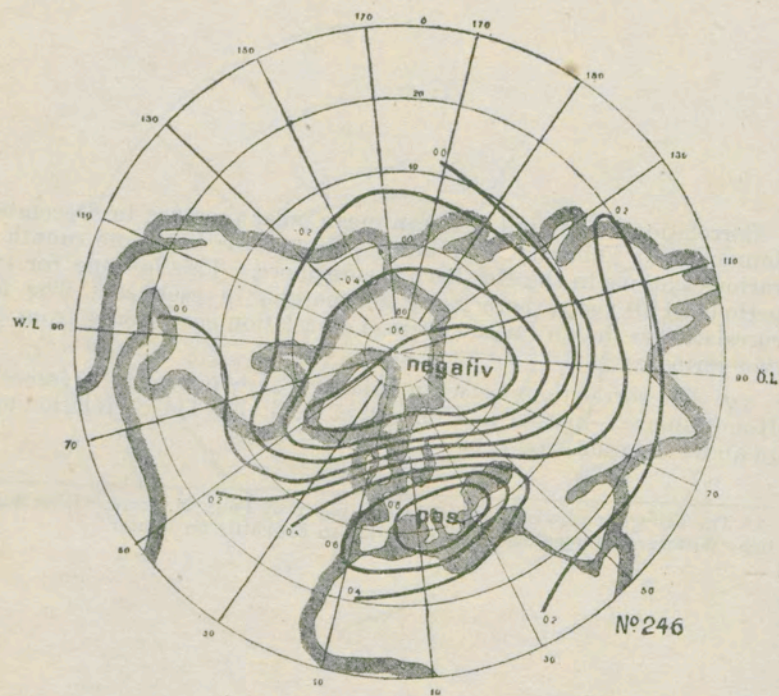
e.g. The correlations between polar pressure and mean pressure at Honolulu are :—period I.—.23, period II.—.11. The correlation used in above diagram would be $\frac{1}{2}(-.23 - .11)$.
= -.17.

* These diagrams are reproduced from a paper by Felix M. Exner "Über monatliche Witterungs anomalien auf der nördlichen Erdhälfte im Winter."

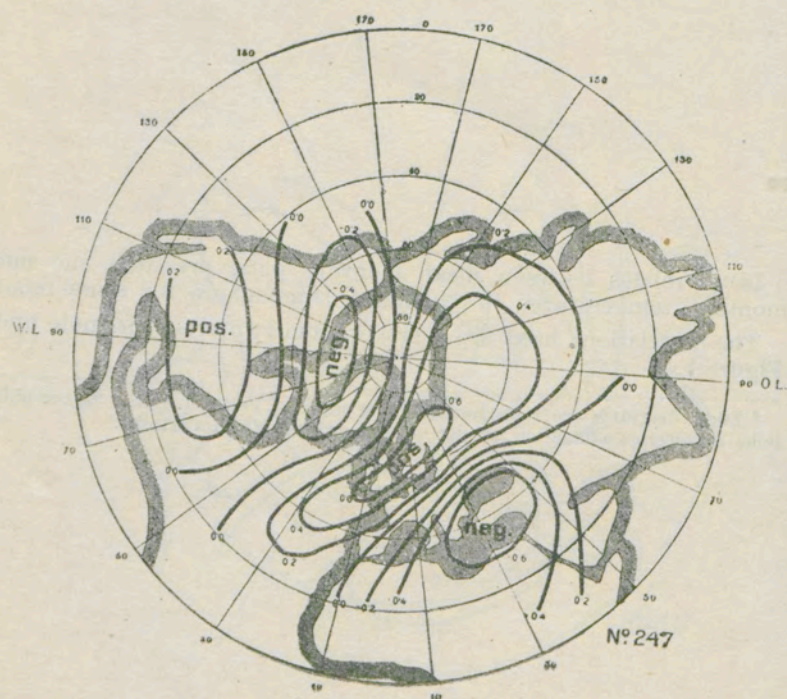
Iso-correlates between mean monthly polar pressures and mean monthly temperatures in Northern Hemisphere for same month.

The correlations used are averages from two sets. See note under Figure 1.

* These diagrams are reproduced from a paper by Felix M. Exner "Über monatliche Witterungs anomalien auf der nördlichen Erdhälfte im Winter."



*FIG. 3.—WINTER.



*FIG. 4.—WINTER.

Iso-correlates between mean monthly pressures at Lugano, and mean monthly pressures in Northern Hemisphere for same month.

The correlations used are averages from two sets. *See note under Figure 1.*

* Reproduced from a paper by Felix M. Exner (loc. cit.).

Iso-correlates between mean monthly pressures at Lugano, and mean monthly temperatures in Northern Hemisphere for same month.

The correlations used are averages from two sets. *See note under Figure 1.*

* Reproduced from a paper by Felix M. Exner (loc. cit.).

Correlations between Monthly Pressure and Temperatures for same Month.

Station.	Pressure at P. Delgada and			
	Pressure		Temperature	
	Period		Period	
	I.	II.	I.	II.
Port Simpson23	—	—	—
S. Francisco26	.10	—	—
S. Luis ...	-.36	—	—	—
Spokane12	—	.22	.03
Winnipeg42	-.20	—	—
Nashville... ..	.51	+.12	.29	—
Toronto66	.24	-.05	.24
Port au Prince ...	—	—	—	—
St. John's, N.F.55	.46	-.04	-.18
Jakobshaven ...	-.20	—	-.25	-.55
Ivigtut ...	—	—	-.48	-.55
Stykkisholm ...	-.39	-.57	—	—
P. Delgada ...	—	—	—	—
Valencia37	.10	.26	.62
Gjesvaer ...	-.16	—	.19	—
Bronno ...	—	—	—	—
Copenhagen ...	—	—	—	—
Lugano ...	—	—	—	—
Milan ...	—	—	—	—
Algiers21	.51	-.62	-.11
Palermo ...	—	—	-.30	.14
Lemberg ...	-.03	—	-.07	—
Novorossisk09	—	—	—
Suez11	—	-.33	—
Archangel ...	—	—	-.37	-.46
Saratov ...	-.03	—	-.18	—
Obdorsk14	—	.11	—
Barnaul08	—	-.20	.31
Kazalinsk14	—	—	—
Jeypore ...	-.15	—	—	—
Nertschinsk ...	—	—	—	—
Jakutsk01	—	.15	—
Markowo03	—	.17	—
Nikolaievsk ...	—	—	—	—
Vladivostok ...	-.35	.49	-.25	.21
Zikawei ...	-.18	—	—	—
Manila ...	—	—	—	—

Correlations between monthly pressures and temperatures for same month.

Station.	Pressure at St. John's, and				Pressure at Stykkisholm, and			
	Pressure		Temperature		Pressure		Temperature	
	Period		Period		Period		Period	
	I.	II.	I.	II.	I.	II.	I.	II.
Spokane ...	-.10	—	.34	-.05	—	—	—	—
Winnipeg ...	—	—	—	—	.18	—	—	—
Nashville42	-.08	.12	—	—	—	—	—
Toronto58	.16	.23	.44	—	—	—	—
Port au Prince ...	-.08	—	-.54	-.48	—	—	-.39	-.13
St. John's... ..	—	—	—	—	-.15	-.12	—	—
Jakobshaven55	.08	-.48	-.08	.51	.87	.67	.78
Ivigtut ...	—	—	—	—	—	—	.65	—
Stykkisholm ...	-.15	-.12	-.44	-.43	—	—	—	—
P. Delgada55	.46	—	—	-.39	-.57	—	—
Valencia04	—	.06	—	-.10	-.05	—	—
Gjesvaer27	-.26	.08	—	.47	.77	-.35	-.44
Bronno ...	—	—	—	—	—	—	-.58	—
Kopenhagen ...	—	—	—	—	—	—	-.67	-.66
Lugano ...	-.38	-.23	.33	—	—	—	—	—
Milan ...	—	—	.22	.25	—	—	—	—
Algiers ...	-.14	—	-.21	-.10	-.62	-.66	—	—
Lemberg ...	-.34	-.13	.20	—	-.33	-.50	—	—
Suez ...	-.16	—	.11	—	-.47	-.12	.55	.63
Saratov19	—	-.40	.19	.38	.12	—	—
Obdorsk ...	—	—	—	—	.27	.30	—	—
Barnaul10	—	-.47	-.02	—	—	—	—
Nertschinsk ...	-.39	-.34	—	—	-.19	—	—	—
Jakutsk01	—	-.06	—	—	—	—	—
Nikolaievsk ...	—	—	—	—	-.13	-.23	—	—
Vladivostok ...	-.20	.08	-.10	—	-.25	-.59	—	—

Correlations between mean monthly pressures and temperatures for same month.

Station.	Pressure at Winnipeg and pressure at other stations.		Station.	Pressure at Obdorsk and pressure at other stations	
	Period			Period	
	I.	II.		I.	II.
S. Francisco... ..	— '49	— '01	Port au Prince ...	— '55	— '35
Stykkisholm ...	'18	—	St. John's ...	'53	'02
Valencia ...	'35	'15	Ivigtut ...	'51	'36
Palermo ...	— '32	— '06	Algiers ...	— '58	— '37
Barnaul ...	— '27	— '23	Suez ...	— '48	'10
Vladivostok ...	— '53	— '38	Jeypore ...	— '51	— '24
Zikawei ...	— '49	— '15	Manila ...	— '43	— '22

Station.	Pressure at Saratov and pressure at other stations.		Station.	Pressure at Obdorsk and temperature at other stations.	
	Period			Period	
	I.	II.		I.	II.
Gjesvaer	'68	'27	Archangel	— '47	— '56
Milan	— '49	'00	Saratov	— '49	— '12
Algiers	— '64	— '24	Kazalinsk	— '38	—
Novorossisk	— '19	'40	Kirensk	— '79	—
Barnaul	— '80	'22			

Station.	Pressure at Vladivostok and pressure or temperature at other stations.		Station.	Pressure at Lemberg and pressure or temperature at other stations.	
	Period			Period	
	I.	II.		I.	II.
Jakobshaven, pres. ...	— '15	— '52	Invigtut, pressure ...	— '35	— '63
Valencia " ...	— '25	— '59	Markowo, temp. ...	— '66	—
Jakobshaven, temp....	— '24	— '53	Petrograd " ...	'52	—
Stykkisholm " ...	'51	'22	Bronno " ...	'78	—
Gjesvaer " ...	'31	'37			

Correlation Coefficients. Period		The variables correlated are monthly means for same month of
I.	II.	
— '67	— '60	Pressure at Saratov and temperature at Barnaul.
'07	'31	Pressure at Irkutsk and pressure at Ivigtut.
— '31	— '48	Pressure at Irkutsk and pressure at Lemberg.
— '13	— '32	Pressure at S. Francisco and pressure at Ekaterinberg.
— '12	—	Pressure at Port Simpson and pressure at Belle Isle.
— '33	—	Pressure at Port Simpson and pressure at Markowo.
— '47	— '33	Pressure at Palermo and pressure at Barnaul.
'44	'73	Pressure at Copenhagen and pressure at Novorossisk.
— '01	— '07	Pressure at Gjesvaer and pressure at Valencia.
— '80	— '63	Pressure at Gjesvaer and pressure at Algiers.
'56	—	Pressure at Gjesvaer and temperature at Markowo.
— '49	— '57	Temperature at St. John's and pressure at Palermo.
— '59	'23	Temperature at St. John's and pressure at Barnaul.
— '29	—	Temperature at Kirensk and temperature at Markowo.
'40	—	Temperature at Kirensk and temperature at Kazalinsk.
'04	'06	Temperature at Jakutsk and temperature at Gjesvaer.
— '28	— '08	Temperature at Jakutsk and temperature at Novorossisk.
— '54	— '43	Temperature at Gjesvaer and temperature at Novorossisk.

Latitude N°.	Correlation of Polar Pressure $\frac{1}{2}$ (Jakobshaven + Gjesvaer + Jakutsk) and			
	Air Pressure.		Temperature.	
	Number of Stations.	Correlation Coefficient.	Number of Stations.	Correlation Coefficient.
10—20	4	— '25	2	— '07
20—30	2	— '22	1	'09
30—40	7	— '37	6	'20
40—50	8	— '27	8	'02
50—60	6	'12	6	— '49
60—70	4	'70	3	— '09
Above 70	2	'84	2	'03

Partial correlation coefficients between mean monthly pressures for same month when the effect of the polar pressure is eliminated.

The total correlation coefficients used in the calculation of the partial correlation coefficients are the means of those for periods I. and II.

Lugano and St. Johns	— '37	Copenhagen and No-	'53	Obdorsk and Jeypore	— '27
" " Ivigtut	— '39	vorossisk.		" " Manila	— '32
" " Stykkisholm.	— '28	Stykkisholm and St.	— '22	Ivigtut and Jakutsk	— '62
" " Valencia	'26	Johns.		Saratov and Stykkisholm.	'10
" " Gjesvaer	— '30	Stykkisholm and Val-	— '17	" " Gjesvaer	'36
" " Copenhagen.	'55	encia.		" " Algiers	— '29
" " Palermo	'83	Stykkisholm and	'10	" " Novorossisk.	'61
" " Novorossisk.	'45	Gjesvaer.	— '35	" " Barnaul	'41
" " Barnaul	— '19	gier.	— '44	Algiers and Gjesvaer	— '31
" " Jeypore	'03	Stykkisholm and		Lemberg and Ivigtut	— '15
" " Zikawei	'12	Vladivostock.		" " Stykkisholm.	— '10
" " Jakutsk	'02	Winnipeg and Val-	'22	" " Lugano	'74
Ponta Delgada and	'25	encia.	— '05	" " Jakutsk	— '09
Nashville.		" " Palermo	— '35	Jakobshaven and St.	'47
Ponta Delgada and	'40	" " Barnaul	— '22	Johns.	
Toronto.		" " Vladivostock.	— '48	" " Stykkisholm.	'34
Ponta Delgada and	'57	" " Zikawei	— '35	" " Lu-	— '05
St. John's.		Obdorsk and Port au	— '20	gano.	
Ponta Delgada and	— '31	Prince.		" " Vladivostock.	— '36
Stykkisholm.		" " Ivigtut	— '21	San Francisco and	— '23
Ponta Delgada and	'12	" " Stykkisholm.	— '38	Ekaterinberg.	
Algier.		" " Lugano	— '32	Palermo and Barnaul	— '16
St. Johns and Nertschinsk.	— '34	" " Algiers	'05		

Correlation Coefficients. Period.		Variables correlated.
I.	II.	
-.58	-.34	Pressure $\frac{1}{2}$ (St. Johns + Valencia + Stykkisholm) and pressure $\frac{1}{2}$ (Zikawei + Honolulu + San Francisco) same month.
		Polar pressure $\frac{1}{2}$ (Jakobshaven + Gjesvaer + Jakutsk) and pressure a month earlier at
.43	.32	(i) Jakobshaven,
.49	.30	(ii) Stykkisholm,
-.29	-.14	(iii) Suez.
		Polar pressure and temperature a month earlier at
.25	.54	(i) Jakobshaven,
-.19	-.34	(ii) Nertschinsk,
-.43	-.63	(iii) Vladivostock.
.43	.51	Polar pressure one month with Polar pressure a month earlier.
		Pressure at Obdorsk and pressure a month later at
.27	.09	(i) Ivigtut,
-.24	-.14	(ii) Milan,
-.36	-.18	(iii) Algiers.
		Pressure at Obdorsk and temperature a month later at
-.53	-.29	(i) Jakutsk,
-.53	-.40	(ii) Vladivostock,
-.20	-.19	(iii) Zikawei.
		Pressure at Stykkisholm and pressure a month earlier at
.27	.13	(i) St. Johns,
.46	.44	(ii) Ivigtut,
.32	.44	(iii) Stykkisholm,
-.43	-.37	(iv) Copenhagen,
-.58	-.48	(v) Lemberg,
-.28	-.31	(vi) Algiers,
-.53	-.28	(vii) Novorossisk,
.33	.31	(viii) Jakutsk.
		Pressure at Stykkisholm and temperature a month earlier at
.49	.43	(i) Novorossisk,
.54	.40	(ii) Novorossisk-Ekaterinburg.
.45	.44	Pressure at Ponta Delgada and pressure a month earlier at Lemberg.
		Pressure at Ponta Delgada and temperature a month earlier at
.39	.35	(i) Jakutsk,
.38	.37	(ii) Gjesvaer,
-.48	-.54	(iii) Novorossisk.
		Pressure at Lemberg and pressure a month earlier at
-.19	-.09	(i) Archangel,
-.23	-.24	(ii) Ekaterinberg,
-.25	-.17	(iii) Saratov,
-.35	-.31	(iv) Barnaul,
.15	.22	(v) Milan,
.29	.26	(vi) Palermo.
.23	.21	(vii) Algiers.

Correlation Coefficients. Period.		Variables correlated
I.	II.	
		Pressure at Lemberg and temperature a month earlier at
-.31	-.21	(i) Winnipeg,
-.46	-.29	(ii) St. Johns,
.26	.21	(iii) Zikawei.
		Polar pressure and pressure one month earlier at
.49	.53	(i) Jakobshaven,
-.21	-.38	(ii) Milan.
.43	.51	Polar pressure with polar pressure one month earlier
.28	.15	" " " " two months "

Partial Correlation Coefficients.

Variables.	Suffix.
Mean monthly pressure at Stykkisholm ...	1
" " " Lemberg a month earlier ...	2
" " " Ivigtut " " ...	3
" " " Jakutsk " " ...	4

The total correlation coefficients for this and the next two sets of four variables are the mean coefficients obtained from those for periods I and II. Thus $r_{12} = \frac{1}{2} (-.58 - .48)$

$$= -.53$$

$3r_{12} = -.46$	$2r_{13} = .23$	$34r_{12} = -.41$
$3r_{14} = .27$	$2r_{14} = .13$	$24r_{13} = .24$
$3r_{24} = -.35$	$2r_{34} = -.00$	$23r_{14} = .13$

Variables.	Suffix.
Mean monthly pressure at Ponta Delgada...	1
" " temperature at Novorossisk a month earlier	2
" " " Gjesvaer " "	3
" " " Jakutsk " "	4

See note above.

$3r_{12} = -.40$	$2r_{13} = .16$	$34r_{12} = -.36$
$3r_{14} = .38$	$2r_{14} = .33$	$24r_{13} = .19$
$3r_{24} = -.18$	$2r_{34} = -.05$	$23r_{14} = .34$

Variables.	Suffix.
Mean monthly pressure at Lemberg ...	1
" " temperature at St. John's a month earlier	2
" " pressure at Palermo a month earlier	3
" " " Barnaul " "	4

See note above.

$3r_{12} = -.28$	$2r_{13} = .17$	$34r_{12} = -.23$
$3r_{14} = -.25$	$2r_{14} = -.20$	$24r_{13} = .12$
$3r_{24} = -.26$	$2r_{34} = -.23$	$23r_{14} = -.17$

Correlation Coefficient.	Number of Observations.	Variables correlated.
$.124 \pm .06$	33 (1874-1906)	January mean pressure at Jakobshaven, with following February mean pressure at (i) Jakobshaven,
$.31 \pm .06$	"	(ii) Stykkisholm.
$-.29 \pm .06$	"	January mean pressure at Vienna and following February mean pressure at (i) Stykkisholm,
$.24 \pm .06$	"	(ii) Ponta Delgada,
$.15 \pm .07$	"	(iii) Vienna.
$-.14 \pm .07$	"	January mean pressure at Barnaul and following February mean pressure at Vienna.

Reference 18.

J. P. VAN DER STOK

... On the Relation between Meteorological Conditions in the Netherlands and some Circumjacent Places.

(i) Atmospheric Pressure.

(ii) Atmospheric Pressure and Wind.

Amsterdam Verk K. Akad Wet
18, 1915. (i) p. 310, (ii) p. 321.

Correlation coefficients between monthly mean pressures at Azores and Iceland. Data for 36 years, 1875-1911.

Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	May to Oct.
$-.527$	$-.595$	$-.620$	$-.484$	$-.365$	$-.396$	$-.345$	$-.376$	$-.485$	$-.469$	$-.421$	$-.541$	$-.37$

Correlation coefficients between 10-day means of barometric height for the months of December, January and February.

Data from December, 1900, to February, 1914.

	Helder.	Val.	Cler.	Milan.	Neu.	Chris.
Helder	—	.770	.727	.511	.633	.609
Valencia		—	.704	.380	.247	.310
Clermont (S. France) ...			—	.645	.246	.058
Milan (N. Italy) ...				—	.370	.095
Neufahrwasser (Baltic Coast, Prussia)... ..					—	.746
Christiansund						—

Partial correlation coefficients between each pair of stations, the influence of the other four stations being eliminated.

	Helder.	Valencia.	Clermont.	Milan.	Neu.	Chris.
Helder ...	—	.470	.594	.020	.303	.521
Valencia ...		—	.213	-.085	.355	.084
Clermont ...			—	.379	.037	.526
Milan ...				—	.297	.150
Neufahrwasser					—	.476
Christiansund						—

Correlation coefficients between daily observations of height of barometer at 7h.

Data for January, February, December, of the years 1912-1914. 240 observations.

	Ile d'Aix.	Dresden.	Lerwick.	Valencia.	Mulhausen.	Sylt.
De Bilt709	.868	.579	.633	.818	.864
Ile d'Aix532	.148	.670	.888	.359
Dresden402	.360	.781	.848
Lerwick543	.134	.744
Valencia480	.433
Mulhausen528
Sylt ...						

Correlation coefficients between Difference of Atmospheric Pressure and Wind at Flushing.

Data for each day of the months January, February, December, of the years 1912, 1913, and January, February, 1914. 7h. observations.

Difference of Atmospheric Pressure and Wind at Flushing.				N. component.	E. component.
Flushing—Valencia	-.456	+.313
Flushing—Biarritz	+.256	+.765
Flushing—Munich	+.737	+.522
Flushing—Neufahrwasser	+.300	-.375
Flushing—Lerwick	-.561	-.463

Correlation coefficients between differences of atmospheric pressure.

	Flush.-Biar.	Flush.-Mun.	Flush.-Neu.	Flush.-Ler.
Flushing—Valencia383	-.185	-.354	.297
Flushing—Biarritz576	-.166	-.201
Flushing—Munich290	-.491
Flushing—Neufahrwasser				.197

	De Bilt-Dres.	De Bilt-Mul.	De Bilt-Ile d'Aix.	De Bilt-Val.	De Bilt-Ler.
De Bilt-Sylt366	-.466	-.520	-.253	.546
De Bilt-Dresden...		.354	-.061	-.411	-.271
De Bilt-Mulhausen			.782	.014	-.508
De Bilt-Ile d'Aix				.420	-.289
De Bilt-Valencia					.300

Correlation coefficients between difference of atmospheric pressure and average wind for the Netherlands.

Difference of Atmospheric Pressure and Wind.					
			N. component.	E. component.	
De Bilt-Dresden	+.663	-.246	
De Bilt-Ile d'Aix	+.243	+.763	
De Bilt-Sylt	-.168	-.820	
De Bilt-Mulhausen	+.624	+.687	
De Bilt-Valencia	-.398	+.323	
De Bilt-Lerwick	-.635	-.535	

Reference 19.

Sir NAPIER SHAW ... Geographical and Chronogeographical
(Computations by ... Coefficients of Correlation.
C. N. C. EVERS.) Atmospheric Pressure. Meteorological Office, London, M.S. 1914.

Data.—Mean monthly barometric pressures for the months of January, February, July, August, from the barometric pressure at 78 Stations.

Place.	Period.	Number of Observations.
Kew ...	1871-1910	40
Brussels ...	1871-1904	34
Valencia ...	1871-1910	40
Aberdeen ...	1871-1906	36
Madrid ...	1871-1901	31
Lisbon ...	1871-1903	33
Cairo ...	1871-1905	35
Zanzibar ...	1889-1906 or 1907	16 or 17 (1891 missing)
Bombay ...	1871-1907 or 1908	37 or 38
Ponta Delgada ...	1874-1906	33
Petrograd ...	1871-1903	33
Batavia ...	1871-1904	34
Cordoba ...	1873-1907	35
Sydney ...	1871-1906 or 1907	36 or 37
Tokio ...	1873-1905 or 1906	33 or 34
Toronto ...	1871-1906	36
Barnaul ...	1871-1903	33
Stykkisholm ...	1874-1904 or 1905	31 or 32
Jacobshaven ...	1871-1903 or 1904	33 or 34

Correlation coefficients between mean monthly pressures.

Station.	Kew, February, with various Stations.		Kew, August, with various Stations.		
	Same month, February.	Preceding January.	Same month, August.	Preceding July.	Preceding February.
Kew ...	—	.20±.10	—	.15±.10	-.16±.10
Brussels97±.01	.14±.11	.95±.01	.13±.11	-.18±.11
Valencia88±.02	.20±.10	.88±.02	.09±.11	-.16±.10
Aberdeen85±.03	.03±.11	.79±.04	.23±.11	-.17±.11
Madrid53±.09	.23±.12	.35±.11	.01±.12	-.21±.12
Lisbon43±.10	.24±.11	-.08±.12	.11±.12	-.26±.11
Cairo35±.10	.02±.11	.22±.11	.09±.11	-.25±.11
Zanzibar28±.16	.20±.16	.36±.14	.15±.16	-.38±.14
Bombay25±.10	.05±.11	.26±.10	.14±.11	-.29±.10
Ponta Delgada	.13±.12	.15±.12	.02±.12	.02±.12	-.17±.11
Petrograd11±.12	-.12±.12	.40±.10	.43±.10	+.41±.10
Batavia13±.11	.00±.16	-.17±.11	-.03±.12	-.37±.10
Cordoba13±.11	.07±.11	-.37±.10	-.14±.11	+.09±.11
Sydney12±.11	.07±.11	.00±.11	.05±.11	-.07±.11
Tokio ...	-.06±.12	-.25±.11	.17±.11	-.23±.11	-.15±.12
Toronto ...	-.17±.11	.16±.11	.37±.10	.06±.11	.03±.11
Barnaul ...	-.22±.11	-.25±.11	.30±.11	.05±.12	.18±.11
Stykkisholm ...	-.25±.11	-.08±.12	.03±.12	.11±.12	.01±.12
Jacobshaven ...	-.38±.10	.07±.12	-.16±.11	.09±.12	.04±.12

Reference 20.

E. H. CHAPMAN ... The Correlation between Changes in Barometric Height at Stations in the British Isles. Quart. Jour. Roy. Meteor. Soc., Vol. XLI., July, 1915.

Change in barometer at Cronkbourne (Isle of Man), 9h.—21h., with change in barometer at various stations the night before 21h.—9h. during the year 1900. (365 observations).

Station.	Co-efficient.	Station.	Co-efficient.	Station.	Co-efficient.
Deerness29	Cronkbourne36	Southampton16
Dunrobin Castle	.34	Stonyhurst22	Plymouth27
Braemar32	Hillington12	Falmouth38
Aberdeen28	Churchstoke26	Armagh52
Laudale40	Cambridge09	Markree Castle*	.53
Dundee29	St. David's41	Dublin49
Glasgow36	Kew11	Birr Castle59
Cockle Park20	Woolacombe35	Valencia66
Scarborough11				

* 310 observations.

Change in barometer at Cronkbourne from 9h.—21h. and change in barometer at the same and other stations for various 12-hour intervals. Year 1900 (365 observations).

Change in barometer at Cronkbourne, 9h.—21h., with

Change in barometer, 9h.—21h., day before.	Change in barometer, 21h.—9h., night after.	Change in barometer, 9h.—21h., day after.
Deerness - '01	Aberdeen '40	Deerness - '08
Cockle Park - '15	Cockle Park '41	Cronkbourne - '11
Cronkbourne - '12	Scarborough '56	Hillington - '01
Hillington - '20	Cronkbourne '17	Plymouth - '11
Plymouth - '10	Hillington '54	Valencia - '06
Valencia - '07	Plymouth '26	
	Valencia - '07	

Correlation coefficients between barometric change at Cronkbourne 9h.—21h., and barometric change at 12 stations 21h.—9h. night before (365 or 366 observations).

Years 1895–1904.

Station.	1895.	1896.	1897.	1898.	1899.	1900.	1901.	1902.	1903.	1904.	The 10 years 1895–1904 3652 observations.
Deerness ..	'35	'11	'26	'17	'18	'29	'30	'23	— '02	'13	'19
Glasgow ..	'43	'12	'33	'22	'22	'36	'36	'31	'14	'23	'27
Scarborough ..	'22 ¹	— '01	— '01	'02	'07 ²	'11	'15	'14	— '03	— '05	'06 ⁵
Cronkbourne ..	'40	'14	'26	'20	'25	'36	'31	'30	'24	'24	'27
Stonyhurst ..	'32	'07	'17	'11	'15	'22	'23	'23	'12	'05	'16
Hillington..	'21	— '02	— '02	'00	'04	'12	'13	'11	— '00	— '05	'05
St. David's ..	'39	'17	'38	'19	'28	'41	'29	'33	{ No obser- vations. }		'31 ⁰
Plymouth ..	'34	'12	'25	'15	'23	'27	'18	'33	'32	'16	'24
Armagh ..	'49	'33	'52	'36	'38	'52	'49	'50	'44	'45	'49
Markree Castle ..	'54	'46	'60	'46	'50	'53 ³	'55 ⁴	'58	'55	'60	'54 ⁷
Birr Castle ..	'52	'42	'59	'41	'48	'59	'47	'55	'52	'51	'51
Valencia ..	'57	'57	'63	'45	'58	'66	'54	'60	'63	'65	'60

¹ 346 observations.
² 304 " "
³ 310 " "

⁴ 334 observations.
⁵ 357² " "

⁶ 2921 observations.
⁷ 3566 " "

Results for the Twelve Calendar Months.

Change in barometer at Cronkbourne, 9h.—21h.	Observations.											
	Jan. 310	Feb. 282	Mar. 310	Apr. 300	May 310	June 300	July 310	Aug. 310	Sept. 300	Oct. 310	Nov. 300	Dec. 310
(i) change at Valencia, 21h.—9h. night before.	'51	'54	'51	'58	'61	'70	'77	'77	'66	'67	'55	'57
(ii) change at Hillington, 21h.—9h. night before.	'07	— '03	— '06	'04	'18	'05	'22	'04	— '13	'12	'14	— '00

IV.—TEMPERATURE.

Reference 21.

W. H. DINES, F.R.S. ... Correlation between Death Rate, Temperature and Sunshine. Meteorological Office, London. M.S. 1915.

Variables.

Denoted by

Temperature... Mean for seasonal quarters at Oxford ... *t*
Sunshine ... Ditto ... *s*
Death Rate ... Civil quarters. Rates per 1000 for England ... *d*

Data for the 33 years 1880–1912.

Correlation Coefficients.

	Total.			Partial.		
	<i>r_{ts}</i>	<i>r_{ds}</i>	<i>r_{dt}</i>	<i>d_{r_{ts}}</i>	<i>t_{r_{ds}}</i>	<i>s_{r_{dt}}</i>
Winter	— '06	'05	— '41	— '04	'03	— '41
Spring	'50	'14	— '10	'52	'22	— '20
Summer	'76	'64	'82	'54	'05	'67
Autumn	'50	'24	'09	'50	'20	— '03

Reference 22.

J. I. CRAIG ... A See-Saw of Temperature. Quart. Jour. Roy. Met. Soc., Vol. XLI., April, 1915, p. 89.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
— '0427 ± 0'0096	34 1877–1910	Annual mean temperature at Cairo (Abbassia), and annual mean temperature England, S.W., and South Wales. Quarterly mean temperature for the same station and district.
— '072 ± 0'006	"	First Quarter of Year.
— '028 ± 0'011	"	Second " " "
— '017 ± 0'011	"	Third " " "
— '054 ± 0'008	"	Fourth " " "

Monthly Correlations for the months of January, April, July, and October.

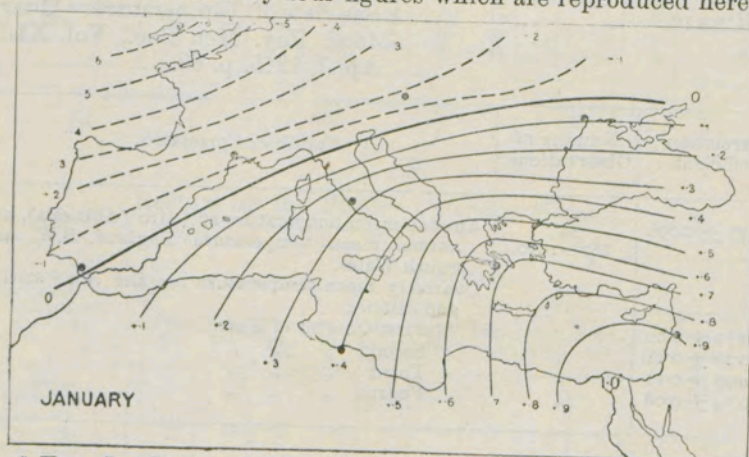
Temperature in Lower Egypt (average for Abbassia and Alexandria) and temperature at—	Number of Observations.	Jan.	April.	July.	Oct.
Beirut	28	.89	.52	.61	.78
Tripoli	15-17	.40	.38	.32	.23
Athens	34	.67	.70	.38	.29
Odessa	42	.12	.39	.28	.13
Rome	20	-.32	-.02	.01	-.30
San Fernando	36	-.04	-.50	-.31	-.22
South France	36	-.14	-.19	-.34	-.30
Vienna	35	-.19	-.14	-.05	-.21
S.W. England	40	-.56	-.24	-.19	-.40

South France = (Lyon + Toulouse + Marseilles + Montpellier + Perpignan)/5.
S.W. England = (Falmouth + Plymouth + Pembroke)/3.

Correlations between diurnal deviations of temperature for S.W. England (Pembroke, Newquay, and Portland), and for Egypt (Alexandria and Abbassia).

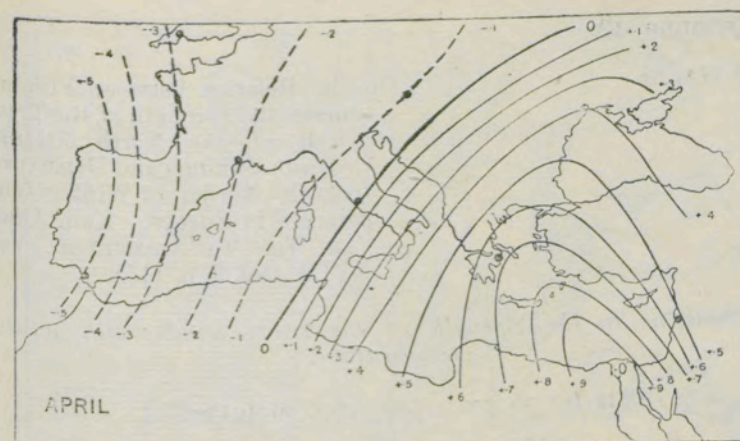
Lag.	January 1912 and 1913.	January 1910 and 1911.
- 4 days	.16	—
- 2 "	-.13	—
- 1 "	-.39	—
0 "	-.78	-.09
1 "	-.86	—
2 "	-.58	—
4 "	-.40	-.12
8 "	—	-.22
11 "	—	-.12

The table of monthly Correlations for January, April, July and October, is illustrated by four figures which are reproduced here.

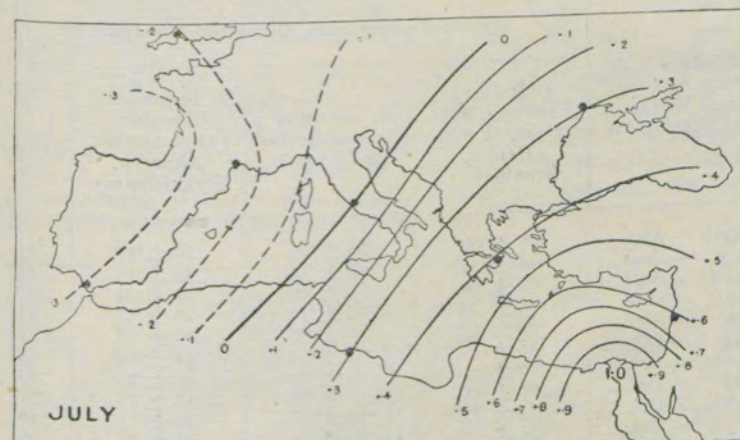


* FIG. 5.—Lines of Thermal Equi-Correlation with Egypt.

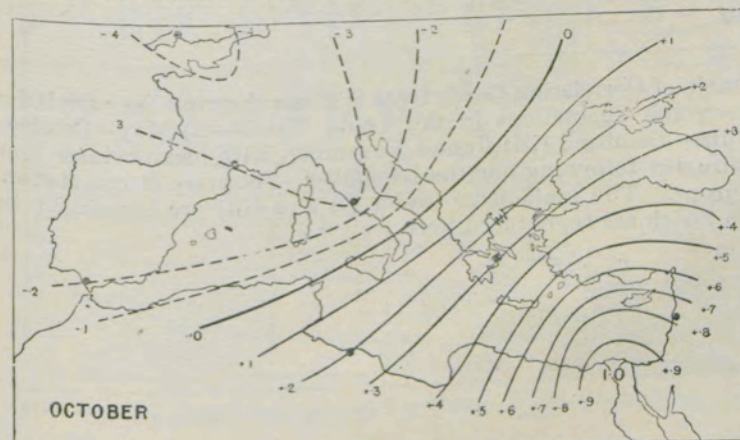
* This diagram is reproduced by permission from a paper by J. I. CRAIG, in the Quarterly Journal of the Royal Meteorological Society, Vol. XLI., April, 1915.



* FIG. 6.—Lines of Thermal Equi-Correlation with Egypt.



* FIG. 7.—Lines of Thermal Equi-Correlation with Egypt.



* FIG. 8.—Lines of Thermal Equi-Correlation with Egypt.

* These diagrams are reproduced by permission from a paper by J. I. CRAIG, in the Quarterly Journal of the Royal Meteorological Society, Vol. XLI., April, 1915.

Reference 23.

P. H. GALLÉ ... On the Relation between Fluctuations in the Strength of the Trade Winds of the North Atlantic Ocean in Summer and Departures from the Normal of Winter Temperature in Europe. Kon. Akad. Van Wet. Te Amsterdam, Vol. XVIII., No. 9, p. 1435.

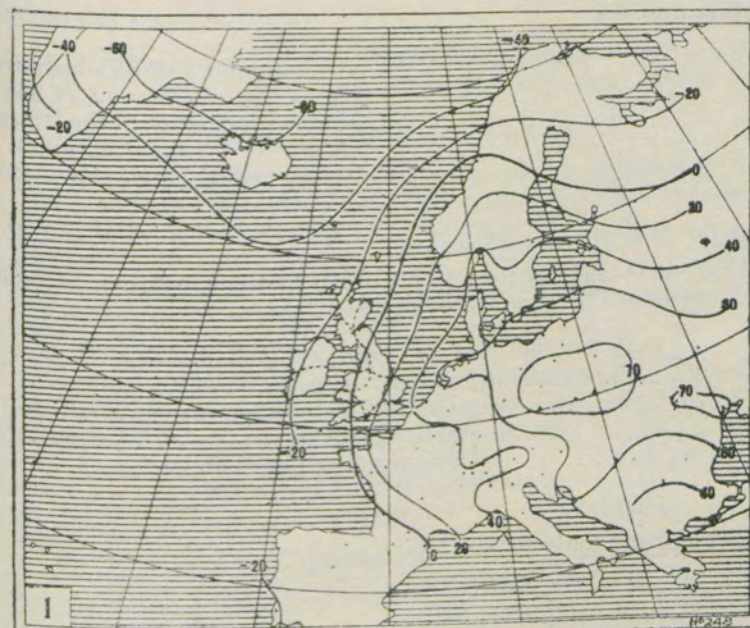
Fluctuations in the strength of the trade winds with winter temperature.

Data for 16 years 1899-1900 to 1913-1914.

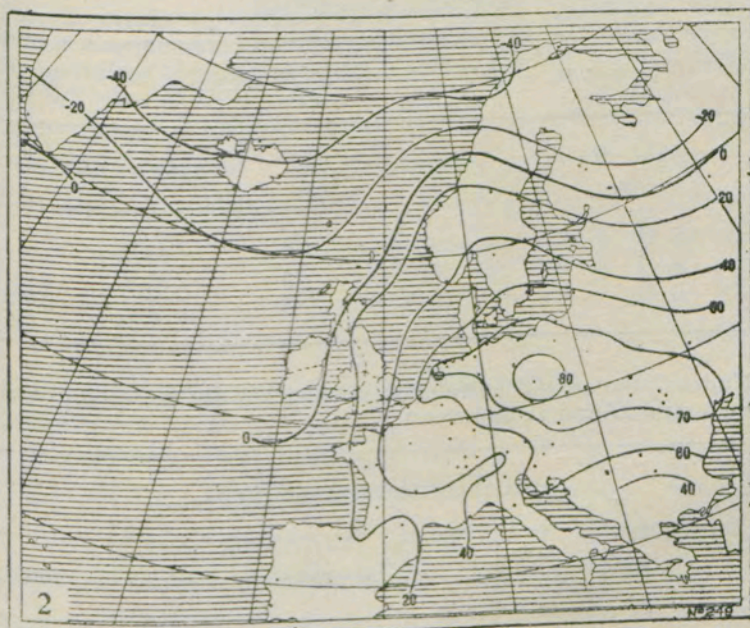
Correlation Coefficients.

Fluctuations in the Strength of the Trade Winds (15°-25°N., 25°-45°W.)	Winter Temperature, December—February in			
	a. Holland (Flushing, Maestricht, de Bilt, Helder, Groningen).	b. Germany (Gorlitz, Ratibor and Posen).	c. Iceland and Greenland (Angmagsalik, Akureyn, Stykkisholm).	$\frac{1}{2}(b+c)$.
1 March — Aug....	.76 ± .07	.68 ± .09	.16 ± .17	.42
2 July — Sept....	.28 ± .16	.47 ± .14	.38 ± .15	.43
3 April — Sept....	.70 ± .09	.70 ± .09	.47 ± .14	.58
4 Jan. — Sept....	.69 ± .09	.65 ± .10	.16 ± .17	.41
5 Oct. — Sept....	.60 ± .11	.56 ± .12	.06 ± .17	.31
6 Aug. — Oct.38 ± .15	.61 ± .11	.44 ± .14	.53
7 June — Oct.60 ± .11	.79 ± .07	.48 ± .13	.63
8 May — Oct.61 ± .11	.77 ± .07	.56 ± .12	.67
9 Feb. — Oct.61 ± .11	.72 ± .08	.40 ± .15	.56
10 Nov. — Oct.53 ± .12	.63 ± .10	.28 ± .16	.46
11 Dec. — Nov....	.62 ± .11	.63 ± .10	.19 ± .17	.41
12 June — Nov....	.69 ± .09	.81 ± .06	.43 ± .14	.62
13 $\frac{1}{2}(3+8+12)$70 ± .09	.80 ± .06	.51 ± .13	.66
14 July — Dec.40 ± .15	.49 ± .13	.08 ± .17	.29

A table of Correlation Coefficients is given showing the correlation between the fluctuations in the Trade Winds—(i) May—October; (ii) June—October; (iii) June—November, with mean winter temperature, the following months December—February at 135 stations in Europe. The coefficients under (i) and (iii) are illustrated by charts which are reproduced here.



*FIG. 9.—Iso-correlates showing correlation between fluctuations in the strength of the Trade Winds, May to October, and following winter temperature, December to February.



*FIG. 10.—As above, but period for Trade Wind fluctuations, June—November.

* Reproduced from a paper by P. H. Gallé. Proc. K. Akad. Wetenschap Amsterdam, Vol. XVIII. No. 9.

Reference 24.

C. E. P. BROOKS ... The Reduction of Temperature Observations to Mean of 24 hours, and the Elucidation of the Diurnal Variation on the Continent of Africa. Jour. Roy. Met. Soc., Oct., 1917.

Data for 32 stations in Africa:—

Analysis is made of the 24 hourly observations into their harmonic components. The form adopted is—

$T_H = a_0 + a_1 \sin(H + A_1) + a_2 \sin(2H + A_2) + \dots$
 where T_H is the temperature at any hour H .
 a_0 „ „ mean of the hourly observations of temperature,
 a_1, a_2, \dots are the amplitudes of the 1st, 2nd, ... harmonics,
 and A_1, A_2, \dots „ „ corresponding phase angles.

Other variables—

M mean daily maximum,

m „ „ minimum,

C cloudiness percentage,

L a measure of continentality,

$\phi = 2A_1 - A_2$ from above.

Correlation coefficient.

—		Total.	Partial.			
		$M-m$.	C	H	L	$\cos(\text{lat.})$.
a_196	-.33	.28	.43	
a_279	.07	.14	.37	
ϕ71	.35	.17	-.75

Under C the partial correlation coefficients are between a_1 and C for constant H and L , and so on.

Partial correlation coefficients.

ϕ with		C [$\cos(\text{lat.})$ const.]	$\cos(\text{lat.})$ [C const.]
January38	-.18
April46	-.18
July45	-.50
October56	-.45

Reference 25.

C. E. P. BROOKS ... Continentiality and Temperature. Jour. Roy. Met. Soc., April, 1917.

Continentiality and temperature.

Area considered is that part of Eurasia between latitudes 40° and 60° North and extending from the Atlantic coast and islands eastward to longitude 90° E. 56 stations were selected in this area.

Variables. Suffix.

C_5 percentage of land in a 5° circle—the station as centre	1
C_{10} „ „ „ „ 10° „ „ „ „ „	2
C_{20} „ „ „ „ 20° „ „ „ „ „	3
Height of station	4
Amount of radiation received	5
Temperature	6

Partial Correlation Coefficients.

—		2345 $^{\circ}$ 16	1345 $^{\circ}$ 26	1245 $^{\circ}$ 36	1235 $^{\circ}$ 46	1234 $^{\circ}$ 56
January395	-.680	-.521	-.723	.944
July217	-.245	.577	-.595	.840

Partial correlation coefficients, corrected for height and latitude, between continentality and temperature for 100 stations between the same meridians but ranging in latitude from Spitzbergen to latitude $22^\circ 30' N$.

Temperature and	January.	July.
C_5	$-.67 \pm .05$	$+.52 \pm .07$
C_{10}	$-.72 \pm .05$	$+.56 \pm .07$
C_{20}	$-.79 \pm .04$	$+.57 \pm .07$

First Order Correlation Coefficients.

—	January.					July.	
	C_{10}	C_{20}	H	R	T	R	T
C_591	.76	.32	.16	-.72	.26
C_{10}92	.30	.20	-.83	.30
C_{20}34	.31	-.77	.29
H44	-.23	.33
R29	.73

Reference 26.

C. E. P. BROOKS Continentality and Temperature.
Second Paper. The Effect of
Latitude on the Influence of
Continentality on Temperature.
Jour. Roy. Met. Soc., Oct., 1918.

Variables.	Suffix.
Mean temperature at sea-level in January	1
" " " " July	2
Area of ice in a ten degree circle round the point at which the temperature is taken, January	3
Area of ice in a ten degree circle round the point at which the temperature is taken, July	4
Amount of land in the western half of the ten degree circle	5
Amount of land in the eastern half of the ten degree circle	6

Correlation Coefficients. Zero Order.

Latitude.	r_{13}	r_{15}	r_{16}	r_{24}	r_{25}	r_{26}	r_{35}	r_{36}	r_{45}	r_{46}	r_{56}
70° N.	+·21	-.78	-.48	-.71	+·46	+·54	-.62	-.83	-.42	-.56	+·68
60°	+·21	-.88	-.51	-.53	+·53	+·91	-.35	-.69	-.45	-.61	+·63
50°	-.09	-.88	-.59		+·79	+·84	-.04	-.17			+·81
40°		-.83	-.60		+·83	+·85					+·84
30°		-.75	-.49		+·84	+·68					+·82
20°		-.24	-.25		+·90	+·81					+·83
10°		+·52	+·66		+·68	+·54					+·84
0°		+·21	+·19		+·17	+·02					+·75
10° S.		+·53	+·37		+·22	+·05					+·74
20°		+·72	+·56		+·11	+·02					+·75
30°		+·79	+·65		-.23	-.28					+·61
40°		+·53	+·06		-.09	-.27					+·41

36 observations.

Partial Correlation Coefficients.—First or Second Order.

Latitude.	First Order.				Second Order.					
	r_{15}	r_{16}	r_{25}	r_{26}	r_{35}	r_{36}	r_{45}	r_{46}	r_{56}	
70° N.	—	—	—	—	-.74	-.84	-.51	-.59	+·15	+·11
60°	—	—	-.10	+·87	-.18	-.83	-.04			
50°	—	—	+·35	+·55	-.17	-.84	+·41			
40°	-.74	+·29	+·39	+·52						
30°	-.69	+·31	+·68	-.04						
20°	-.07	-.09	+·71	+·23						
10°	-.08	+·48	+·35	-.06						
0°	+·11	+·05	+·22	-.15						
10° S.	+·41	-.05	+·38	-.33						
20°	+·56	+·02	+·19	-.16						
30°	+·66	+·34	-.08	-.18						
40°	+·56	-.20	+·02	-.25						

36 observations.

V.—RAINFALL, ETC.

Reference 27.

R. H. HOOKER An Elementary Explanation of Correlation Illustrated by Rainfall and Depth of Water in a Well. Jour. Roy. Met. Soc., Vol. XXXIV., 1908, p. 277.

Correlations between depth of water in a well at the end of August and rainfall during periods of eight weeks at different times of the year.

Data for the 20 years 1887-1906.

Period, Weeks of Year }	1-8	5-12	9-16	13-20	17-24	21-28	25-32
Correlation Coefficient }	-.28±·14	·02±·15	·35±·13	·63±·09	·50±·11	·28±·14	·42±·12

Reference 28.

J. PECK and E. C. SNOW ... The Correlation of Rainfall. Jour. Roy. Met. Soc., Vol. XXXIX., 1913.

Data used.—Rainfall returns for the 4 years 1908-1911 for 30 stations situated to the South East of a line drawn from the Wash to the Isle of Wight.

The correlation coefficients are obtained as follows:—

Let p_1, p_2, \dots, p_{30} represent the rainfalls at the 30 stations in the p^{th} month of the year.

and q_1, q_2, \dots, q_{30} represent the rainfalls at the 30 stations in the q^{th} month of the year.

$$\text{Let } p_1 + p_2 + \dots + p_{30} = 30P$$

$$\text{and } q_1 + q_2 + \dots + q_{30} = 30Q$$

$$\text{then } (P - p_1)^2 + (P - p_2)^2 + \dots + (P - p_{30})^2 = 30 \sigma_p^2$$

$$\text{and } (Q - q_1)^2 + (Q - q_2)^2 + \dots + (Q - q_{30})^2 = 30 \sigma_q^2$$

$$\text{and } (P - p_1)(Q - q_1) + (P - p_2)(Q - q_2) + \dots + (P - p_{30})(Q - q_{30}) = 30 \sigma_p \sigma_q r_{pq}$$

r_{pq} is the correlation between the distribution of rainfall in the p^{th} month and that in the q^{th} month

Tables of Monthly Correlations of Rainfall.

1908.

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
------	------	------	-------	-----	------	------	------	-------	------	------	------

Jan.		-.16	.79	.40	.62	.46	-.15	.73	.50	.46	-.10	.70	Jan.
Feb.	-.24		-.01	-.03	.08	-.05	.39	-.14	.08	.09	.15	-.24	Feb.
Mar.	.83	.14		.53	.71	-.03	-.42	.67	.57	.51	.31	.55	Mar.
April	.38	.18	.14		.48	.00	-.18	.22	.37	.11	.07	.03	April
May	.54	-.40	.26	.24		.09	-.17	.64	.52	.40	.15	.44	May
June	-.04	-.10	-.20	.03	.35		.26	.10	.01	-.14	-.23	.22	June
July	-.07	.23	-.17	.08	.05	.20		.07	-.26	-.18	-.02	-.30	July
Aug.	.28	.13	.33	.06	.03	-.20	-.26		.31	.44	.29	.69	Aug.
Sept.	.71	-.16	.39	.51	.45	-.07	-.23	.30		.48	.38	.52	Sept.
Oct.	.70	-.14	.67	.20	.42	-.33	-.29	.31	.71		.35	.43	Oct.
Nov.	-.09	.48	.06	-.08	-.29	-.42	-.02	.26	-.08	.14		.25	Nov.
Dec.	.51	.21	.58	-.10	-.04	-.56	-.14	.42	.19	.59	.50		Dec.

Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
------	------	------	-------	-----	------	------	------	-------	------	------	------

1909.

1909.

1909.

1910.

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	
Jan.		.77	.50	.15	-.29	.38	.15	.24	-.41	.65	.84	.43	Jan.
Feb.	.34		.70	.27	-.39	.46	-.46	.34	-.56	.72	.55	.64	Feb.
Mar.	.52	.33		-.05	-.03	.29	-.01	.10	.40	.43	.74	.06	Mar.
April	.26	.54	.31		.12	.05	.30	.28	.03	.25	.12	.21	April
May	.05	.62	.16	.27		-.35	.36	.09	.78	-.61	-.05	-.30	May
June	.53	-.02	.24	-.05	.01		-.22	.09	-.41	.56	.20	.53	June
July	.29	.19	.00	.09	.32	.43		.19	.50	-.19	.03	.18	July
Aug.	-.06	-.12	.00	-.10	-.22	.04	-.33		.09	.09	.02	.58	Aug.
Sept.	.53	-.04	.00	-.14	-.26	.31	.17	.02		-.76	-.33	-.28	Sept.
Oct.	.37	.31	.50	.73	-.01	-.05	-.04	-.09	-.13		.49	.50	Oct.
Nov.	.42	.49	.36	.72	.02	-.01	-.01	.03	.11	.79		.11	Nov.
Dec.	.18	.73	.45	.73	.44	-.15	-.09	-.10	-.35	.57	.57		Dec.
	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	

1911.

1911.

1910.

1911.

Correlations between monthly rainfalls in successive years.

1909	1908.				1911.	1910.			
	May	June	July	Aug.		May	June	July	Aug.
May	.76	.17	-.20	.49	May	-.22	.23	-.16	.30
June	.18	-.05	.00	-.24	June	.51	-.18	.50	.24
July	.07	.13	.55	-.13	July	.28	-.15	.06	-.07
August	.21	-.20	-.13	.26	August	.12	-.22	-.16	-.10

Correlations between rainfall in yearly periods.

	1908	1909	1910	1911
1908	—	.69	.70	.57
1909	.69	—	.75	.64
1910	.70	.75	—	.71
1911	.57	.64	.71	—

Reference 29.

E. H. CHAPMAN ... Barometric Change and Rainfall.
 Jour. Roy. Met. Soc., Vol. XL.,
 Oct., 1914.

Correlations between Barometric Change and Rainfall.

Woolacombe, N. Devon, April-Sept., 1900. 183 observations.

Rainfall, gh.-gh., with barometric change.

12 hours before, 21h.-9h.	Same 24 hours, 9h.-9h.	12 hours after, 9h.-21h.
-.27 ± .05	-.30 ± .05	.36 ± .04

Height of barometer at 21h.	Rainfall, gh.-gh., with barometric change 12 hours before, 21h.-9h., according to height of barometer at 21h.							
	Woolacombe.				Laudale.			
	Summer.		Winter.		Summer.		Winter.	
	r	n	r	n	r	n	r	n
30.00 ins. & above	-.12	290	-.28	273	-.33	282	-.34	193
29.75-29.99 ins. ...	-.29	250	-.26	170	-.40	235	-.31	140
Below 29.75 ins.	-.29	192	-.25	286	-.29	215	-.10	305

Summer = April to September, years 1900-1903.
 Winter = October to March, years 1900-1 to 1903-4.
 r = Correlation coefficient.
 n = Number of observations.

Woolacombe, 1904-05.

		Rainfall, 9h.-9h., with barometric change 12 hours before, 21h.-9h.; observations grouped according to wind direction at 21h. and 9h. Wind group.			
		S.W.	N.W.	N.E.	S.E.
Correlation Coefficients	-.21	.27	-.16	-.30
No. of observations	140	84	108	88

Woolacombe, 1900-06.

		Rainfall, 9h.-9h., with barometric change 12 hours before, 21h.-9h.; observations grouped according to wind direction at 21h. and 9h. Wind group.							
		S.W.		N.W.		N.E.		S.E.	
		r	n	r	n	r	n	r	n
All observations...		-.10	713	.03	338	-.07	534	-.34	340
Barometer falling 21h.-9h.		.10	330	-.02	95	.10	219	-.33	213

r = correlation coefficient.
 n = number of observations.

In the paper from which the above correlation coefficients are extracted, use is made of the coefficient of contingency and of the correlation ratio.

Reference 30.

N. A. COMISSOPULUS

... On the Seasonal Variability of Rainfall over the British Isles. Jour. Roy. Met. Soc., Vol. XLII., Jan., 1916.

Correlation Coefficient.	Number of Observations.	Variables correlated.
-.09 ± .05	Data for 35 years, 1875-1909, for a large number of stations in British Isles.	The variability of rainfall (British Isles) and (i) Altitude of station.
40 ± .04		(ii) Annual total amount of rain.

Reference 31.

E. H. CHAPMAN ... The Relation between Atmospheric Pressure and Rainfall at a Single Station. Jour. Roy. Met. Soc., Vol. XLII., Oct., 1916.

Correlation Coefficients between mean monthly pressure and rainfall totals for the same month.

Observations for 47 years 1869-1915.

	Kew.	Valencia.		Kew.	Valencia.
January ...	-.68	-.65	July ...	-.80	-.59
February ...	-.71	-.79	August ...	-.81	-.68
March ...	-.81	-.70	September ...	-.66	-.74
April ...	-.72	-.79	October ...	-.67	-.63
May ...	-.66	-.75	November ...	-.56	-.67
June ...	-.39	-.76	December ...	-.65	-.72
Whole year ...	-.53	-.59			

Reference 32.

E. G. BILHAM ... The Lunar and Solar Diurnal Variations of Water-Level in a Well at Kew Observatory, Richmond. Proc. Roy. Soc. A. 94.

Correlation Coefficients.	Variables Correlated.
	Height of water in a Well at Kew and range of:
.89	(i) Lunar semi-diurnal oscillation in the well.
.90	(ii) Solar " " "
-.88	Change in barometric pressure 3 hours, 4 a.m. to 7 a.m., and change in underground water-level during same period.

Reference 33.

E. G. BILHAM ... On the Variation of Underground Water-Level near a Tidal River. Quart. Jour. Roy. Meteor. Soc. Vol. XLIV. 1918.

Correlation Coefficient.	Number of Observations.	
.93	24	Height of Thames at Richmond Lock and level of water in a Well at Kew Observatory.

VI.—SUNSPOTS.

Reference 34.

J. I. CRAIG England—Abyssinia—The South Atlantic; A Meteorological Triangle. Quart. Jour. Roy. Met. Soc., Vol. XXXVI., 1910, p. 341.

Correlation Coefficient.	Number of Observations.	Variables correlated.
		Nile flood as measured by the area of the Nile Gauge Diagram and
$\cdot 36 \pm \cdot 11$	30 (1849-1878)	(i) Sunspots measured by the mean daily area of the spots.
$\cdot 15 \pm \cdot 12$	25 (1885-1907)	(ii) Sunspots, Wolf's number.
$\cdot 29 \pm \cdot 11$	34 (1869-1903)	Spotted area of sun and volume of flood passing Aswan.
$\cdot 09 \pm \cdot 15$	20	Sunspots and flow of River Thames in Summer.
$\cdot 61 \pm \cdot 11$	16	Pressure June to August at St. Helena and Nile flood.
		Nile flood and relative intensity of rainfall in
$\cdot 26 \pm \cdot 12$	31	(i) S.W. England.
$\cdot 23 \pm \cdot 12$	(1878-1908)	(ii) Western districts of Britain.
Insignificant	39 (1869-1907)	Nile flood and rainfall in Exe Valley.
$\cdot 10 \pm \cdot 13$	25 (1883-1907)	Nile flood and Summer flow of River Thames.

Reference 35.

G. T. WALKER Correlation of Sunspots with Rainfall, Temperature, and Pressure. Mems. Indian Meteor. Dept., Vol. XXI., Parts 10, 11, 12. 1915.

Sunspots with Rainfall, Temperature, and Pressure.

Station.	Country.	Sunspots and					
		Rainfall.		Temperature.		Pressure.	
		Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.
Abbassia ...	Egypt ...	23	- '05	42	- '16	42	- '20
Aburi ...	Gold Coast ...	27	+ '14	—	—	—	—
Accra ...	Gold Coast ...	23	- '19	—	—	—	—
Adelaide ...	South Australia	74	+ '13	38	- '29	55	- '31
Aden ...	Arabia ...	33	+ '01	30	- '11	32	- '18
Agra ...	India ...	53	+ '19	39	- '43	39	0
Akmolinsk ...	Siberia ...	28	+ '07	—	—	—	—
Albany, N.Y. ...	U.S.A. ...	38	- '33	39	- '03	39	+ '24
Albany ...	West Australia	—	—	—	—	—	—
Algiers ...	Algeria ...	22	- '23	24	+ '13	24	- '19
Alice Springs ...	South Australia	25	- '30	35	- '04	31	- '30
Apia ...	Samoa ...	19	+ '05	—	—	—	—
Archangelsk ...	Russia ...	38	- '02	39	- '08	34	- '08
Astrachan ...	Russia ...	—	—	54	- '19	30	- '03
Auckland ...	New Zealand ...	47	+ '17	51	+ '27	27	+ '14
Azo ...	Argentina ...	55	- '19	—	—	—	—

Station.	Country.	Sunspots and					
		Rainfall.		Temperature.		Pressure.	
		Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.
Baghdad ...	Asiatic Turkey	25	+ '09	21	- '30	17	- '20
Bahia ...	Brazil ...	20	+ '31	20	- '29	20	- '08
Banda Neira ...	East Indies ...	33	- '07	—	—	—	—
Barbadoes ...	West Indies ...	—	—	43	- '14	29	+ '15
Barnaul ...	Siberia ...	33	+ '03	39	- '17	57	+ '05
Basel ...	Switzerland ...	—	—	38	+ '19	62	+ '06
Batavia ...	Java ...	48	+ '24	45	- '40	45	- '27
Bathurst ...	Gambia ...	29	+ '51	—	—	—	—
Berlin ...	Germany ...	55	+ '19	—	—	—	—
Bermuda ...	Bermudas ...	39	- '15	35	+ '17	29	- '03
Blumenau ...	Brazil ...	21	+ '14	20	- '07	20	- '43
Bombay ...	India ...	66	+ '11	36	- '30	64	- '37
Bordeaux ...	France ...	—	—	18	- '24	18	+ '16
Brisbane ...	Queensland ...	59	+ '08	27	- '42	27	- '39
Buenos Ayres ...	Argentina ...	47	- '20	38	- '12	42	+ '24
Bushire ...	Persia ...	34	+ '17	34	+ '01	34	- '12
Calcutta ...	India ...	83	- '03	36	- '44	59	- '27
Cape Town ...	Cape Colony ...	60	+ '11	31	- '10	55	- '47
Carnarvon ...	West Australia	26	+ '17	19	+ '07	22	- '21
Cayenne ...	Guayana ...	—	—	15	- '21	—	—
Charlotte Waters	South Australia	37	- '17	—	—	—	—
Colombo ...	Ceylon ...	43	- '24	39	- '18	44	- '38
Colon ...	Colombia ...	32	+ '11	—	—	—	—
Constantinople	Turkey ...	48	- '06	—	—	—	—
Copenhagen ...	Denmark ...	55	- '03	—	—	—	—
Cordoba ...	Argentina ...	26	- '07	41	- '33	41	+ '12
Christiansund ...	Norway ...	50	+ '10	37	+ '20	45	- '01
Curytiba ...	Brazil ...	19	- '21	—	—	—	—
Denver, Colo. ...	U.S.A. ...	40	+ '26	40	- '03	40	- '15
Derby ...	West Australia	23	- '14	19	- '27	20	- '28
Dunedin ...	New Zealand ...	42	- '12	—	—	—	—
Durban ...	Natal ...	37	+ '37	34	- '29	34	- '19
Edinburgh ...	Scotland ...	55	- '01	—	—	—	—
Edmonton ...	Canada ...	21	- '50	—	—	—	—
Ekaterinburg ...	Russia ...	39	- '38	39	- '23	57	+ '26
Eniseisk ...	Siberia ...	36	+ '09	34	- '16	34	+ '08
Eyre ...	West Australia	24	- '33	—	—	—	—
Fortaleza ...	Brazil ...	40	+ '08	—	—	—	—
Galveston, Ten. ...	U.S.A. ...	40	- '10	40	+ '10	40	+ '30
Genoa ...	Italy ...	55	- '06	—	—	—	—
Gouriev ...	Siberia ...	22	+ '35	—	—	—	—
Greenwich ...	British Isles	63	- '04	62	+ '09	58	- '09
Gütersloh ...	Germany ...	55	- '11	—	—	—	—
Hamburg ...	Germany ...	—	—	36	- '05	36	+ '13
Helena, Mont. ...	U.S.A. ...	31	+ '22	33	- '09	31	- '07
Hermanstadt ...	Austria ...	55	+ '21	—	—	—	—
Hobart ...	Tasmania ...	68	+ '11	—	—	—	—
Hongkong ...	China ...	42	+ '02	28	- '45	28	+ '28
Honolulu ...	Hawaii Is. ...	30	- '17	38	- '20	30	+ '25
Irgis ...	Siberia ...	—	—	37	- '26	—	—
Irkutsk ...	Siberia ...	29	- '10	35	- '09	35	- '14
Jacobabad ...	India ...	52	+ '14	—	—	—	—

Station.	Country.	Sunspots and					
		Rainfall.		Temperature.		Pressure.	
		Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.
Jacobshaven ...	Greenland ...	36	-.12	39	-.16	47	+.22
Jakutsk... ..	Siberia	19	-.02	31	-.35	19	+.23
Jerusalem ...	Palestine	59	-.22	—	—	—	—
Kasalinsk ...	Siberia	27	-.35	—	—	—	—
Key West, Fla. ...	U.S.A.	54	-.09	40	+.11	40	+.20
Kiev	Russia	53	+.03	53	-.11	—	—
Kimberley ...	Cape Colony ...	26	-.02	—	—	—	—
Koepang	Timor	32	+.19	—	—	—	—
Koeta Radja ...	Sumatra	32	+.13	—	—	—	—
Königsberg ...	Germany	55	-.28	—	—	—	—
Lagos	Nigeria	27	+.09	—	—	—	—
Leh	India	37	+.32	36	+.01	34	-.27
Lisbon	Portugal	55	+.02	28	+.12	56	-.18
Lugansk	Russia	69	-.20	56	-.09	34	-.04
Madras	India	100	+.19	36	-.20	64	-.28
Madrid	Spain	52	-.08	—	—	—	—
Mahe	Seychelles	22	-.25	—	—	—	—
Makassar	Celebes	33	-.18	—	—	—	—
Malden I.	Polynesia	18	-.23	—	—	—	—
Malta	Malta	42	-.07	38	+.20	—	—
Manila	Philippine Is. ...	43	-.13	34	0	34	+.02
Menado	Celebes	32	+.23	—	—	—	—
Meshed	Persia	17	-.12	—	—	—	—
Mexico	Mexico	32	-.19	34	-.13	34	+.04
Mogodor	Morocco	19	+.27	—	—	—	—
Moscow	Russia	51	+.10	38	0	38	+.04
Moulmein	India	63	+.03	—	—	—	—
Mumias	Brit. East Africa	14	+.05	—	—	—	—
Nagasaki	Japan	—	—	33	-.04	26	+.14
Nagpur	Italy	58	+.03	—	—	—	—
Naples	Italy	55	+.01	—	—	—	—
Nashville, Tenn. ...	U.S.A.	—	—	—	—	—	—
Nerchinsk	Siberia	65	+.05	—	—	40	0
Newcastle	Jamaica	34	-.35	23	-.38	23	-.13
Nikolaevsk	Siberia	34	-.14	44	-.28	24	+.23
Noumea	New Caledonia	13	+.34	—	—	—	—
Oaxaca	Mexico	21	-.23	—	—	—	—
Obdorsk	Siberia	25	-.14	25	+.16	—	—
Palermo	Italy	—	—	—	—	—	—
Para	Brazil	15	+.00	15	-.25	15	-.09
Paramaribo	Dutch Guinea...	25	+.04	—	—	—	+.29
Paris	France	55	-.07	—	—	—	—
Pekin	China	32	+.11	27	-.23	—	—
Pelotas	Brazil	15	-.36	15	-.58	15	+.36
Perth	West Australia	38	-.01	36	-.01	29	+.13
Petrograd	Russia	39	+.25	39	-.10	58	+.18
Ponta Delgada... ..	Azores	38	-.06	25	-.18	19	+.21
Port Darwin	South Australia	35	+.03	36	+.03	32	-.32
Port Elizabeth... ..	Cape Colony ...	22	-.07	—	—	—	—
Port Louis	Mauritius	42	+.07	38	-.07	39	-.09
Punta Arenas	Chili	20	-.43	20	-.02	19	-.05
Rangoon	India	—	—	38	-.15	38	-.16
Rawalpindi	India	52	+.03	—	—	—	—

Station.	Country.	Sunspots and					
		Rainfall.		Temperature.		Pressure.	
		Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.	Number of Years.	Correlation Co-efficient.
Recife	Brazil	20	+.26	22	-.45	—	—
Rio de Janeiro...	Brazil	59	-.01	19	-.25	59	-.27
Rothersey	Scotland	55	+.18	—	—	—	—
Rome	Italy	—	—	61	0	—	—
Sacramento, Cal. ...	U.S.A.	60	-.05	—	—	—	—
Saint Christopher	The Lesser Antilles.	30	+.03	—	—	—	—
Saint Helena	Saint Helena ...	16	+.14	16	-.17	16	-.09
Saint Louis, Mo. ...	U.S.A.	58	-.05	40	-.20	—	—
Saint Louis	Senegal	19	+.38	—	—	—	—
San Diego, Cal. ...	U.S.A.	60	-.18	40	-.17	40	+.02
Santiago	Chili	59	-.20	53	-.38	53	+.35
Sao Paulo de Loanda.	Angola	25	-.05	—	—	—	—
Scutari	Asia Minor	—	—	39	-.05	38	+.13
Seychelles	Seychelles	—	—	20	+.10	19	0
Sierra Leone	West Africa ...	38	-.14	23	-.16	20	+.03
Singapore	Strait Settlements.	36	+.01	—	—	—	—
Sintang	Borneo	32	-.01	—	—	—	—
Sitka	Alaska	35	-.25	—	—	—	—
Stykkisholm	Iceland	36	-.25	63	-.03	63	-.04
Suakin	Red Sea Coast ...	21	-.31	—	—	—	—
Sucre	Bolivia	15	-.15	—	—	—	—
Sydney	New S. Wales...	69	-.05	55	-.49	55	-.07
Sydney	Nova Scotia ...	37	+.17	36	-.35	36	+.29
Tashkend	Siberia	27	-.12	30	-.34	33	-.05
Tananarive	Madagascar ...	24	+.27	—	—	—	—
Tezpur	India	49	+.02	—	—	—	—
Thargomindah... ..	Queensland	29	+.12	—	—	—	—
Thorsavn	Faroe Island ...	35	+.03	39	+.03	46	-.07
Tiflis	Russia	39	-.27	39	-.10	58	-.02
Tokio	Japan	36	-.30	36	-.22	39	+.12
Toronto	Canada	37	-.06	57	-.10	63	-.10
Trinidad	The Lesser Antilles.	39	-.03	—	—	—	—
Turukhansk	Siberia	23	-.24	22	+.24	—	—
Upsala	Sweden	55	+.11	—	—	—	—
Valencia	Ireland	38	+.16	27	-.02	45	+.03
Vardöe	Norway	18	-.27	39	-.02	47	+.04
Viatica	Russia	32	-.09	—	—	—	—
Victoria, B.C.	Canada	34	+.31	22	-.35	21	-.09
Vienna	Austria	67	-.18	61	-.01	60	+.08
Vladivostock	Siberia	28	+.03	—	—	—	—
Warsaw	Poland	—	—	59	0	59	+.13
Washington, D.C. ...	U.S.A.	27	-.24	40	-.17	40	+.19
Wellington	New Zealand ...	48	+.32	—	—	—	—
Winnipeg, Man.	Canada	20	-.10	20	-.55	20	-.17
Zanzibar	Brit. East Africa	32	-.15	22	-.22	22	-.46
Zi-ka-Wei	China	36	-.03	37	-.12	36	+.13

VII WEATHER AND CROPS.

Reference 36.

R. H. HOOKER ... The Correlation of the Weather and Crops. Jour. Roy. Stat. Soc., Vol. LXX., Part I, March, 1907.

The district dealt with comprised the following counties:—
Lincoln, Huntingdon, Cambridge, Norfolk, Suffolk, Essex, Bedford, and Hertford.

Meteorological returns from the following stations were used:—
Hillington, Yarmouth, Geldeston, Cambridge, Felixstowe, Clacton-on-Sea, Rothamsted, Shoeburyness.

The Correlation Coefficients are based on 21 observations 1885-1905. Values below 0.3 are ignored. Coefficients between 0.3 and 0.5 are regarded as suggestive of dependence. Real connection is assumed between the variables correlated whenever the Correlation Coefficient exceeds 0.5.

SEASONAL CORRELATIONS.

Rainfall and Crops.

Rainfall during	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to crop:										
Previous—spring...	-.25	-.09	+.05	+.07	+.42	-.04	-.08	+.04	+.18	+.12
" summer...	-.34	-.33	-.26	-.39	-.18	+.27	-.14	+.48	-.04	-.26
" autumn...	-.61	-.20	-.23	-.45	-.04	+.23	+.23	-.08	-.29	-.21
" winter...	-.38	-.30	-.28	-.36	-.34	-.12	-.10	-.06	-.24	-.23
Year of crop:										
Spring ...	+.52	+.53	+.74	+.46	+.17	-.15	+.18	+.60	+.80	+.79
Summer ...	-.28	+.06	+.27	+.37	+.22	-.46	+.50	+.14	+.31	+.33
Cereal Year ...	-.65	-.04	+.10	-.05	+.04	-.29	+.57	+.25	+.20	+.27

Accumulated temperature above 42° F. and Crops.

Accumulated Temperature above 42° F.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to crop:										
Previous—spring...	+.14	+.11	-.02	+.12	-.21	+.04	+.03	-.21	-.06	-.05
" summer...	+.37	+.30	+.13	+.23	-.11	-.25	+.07	+.28	-.13	+.07
" autumn...	+.36	+.04	-.04	-.08	-.01	+.08	-.44	+.01	-.01	-.17
" winter...	+.53	+.28	+.31	+.27	+.03	-.00	-.41	-.09	+.09	+.09
Year of crop:										
Spring ...	-.51	-.49	-.52	-.31	-.16	+.21	-.04	-.59	-.40	-.49
Summer ...	-.06	-.57	-.70	-.16	-.13	+.13	-.52	-.36	-.42	-.49
Cereal year ...	+.10	-.38	-.49	-.17	-.12	+.19	-.62	-.54	-.34	-.49

Accumulated temperature below 42° F. and Crops.

Accumulated temperature below 42° F.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to crop:										
Previous spring ...	-.27	-.29	-.26	-.27	+.13	+.44	+.01	-.19	-.36	-.27
" autumn	-.01	+.34	+.32	+.18	+.03	-.21	+.53	+.20	+.08	+.32
Year of crop:										
Winter ...	-.39	-.10	-.16	-.22	+.03	+.25	+.39	+.09	-.04	-.13
Spring ...	+.02	+.34	+.29	-.30	+.00	+.27	+.07	+.23	+.04	+.07
Cereal year ...	-.30	+.10	+.04	-.22	+.03	+.23	+.44	+.18	+.00	-.01

The Correlation between Rainfall and certain Crops.

Correlation Coefficients based on 21 observations, years 1885-1905.

Rainfall during periods of eight weeks as follows:—	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to Crop:—										
Weeks 9-16 ...	+.02	-.17	-.13	+.17	+.31	-.17	+.24	+.21	+.44	+.36
" 13-20 ...	-.21	-.12	-.00	-.02	+.20	-.18	-.15	-.03	+.07	+.06
" 17-24 ...	-.27	-.04	+.03	-.04	+.16	+.14	-.30	-.16	+.11	+.05
" 21-28 ...	-.49	-.37	-.18	-.17	+.07	+.24	-.10	-.34	+.05	-.18
" 25-32 ...	-.23	-.29	-.33	-.15	+.24	-.06	-.06	-.38	-.11	-.33
" 29-36 ...	-.15	-.11	-.20	-.52	+.40	+.28	-.18	-.43	-.06	-.21
" 33-40 ...	-.55	-.23	-.39	-.13	+.03	+.26	-.22	-.23	-.23	-.16
" 37-44 ...	-.66	-.30	-.32	+.30	+.14	+.05	+.25	+.12	-.39	-.25
" 41-48 ...	-.47	-.12	-.29	-.47	+.02	+.38	+.09	+.09	-.32	-.29
" 45-52 ...	+.21	+.22	+.08	-.21	-.12	+.24	-.12	+.12	+.12	+.01
Year of Crop:—										
Weeks 49-4 ...	+.01	-.00	-.06	-.17	-.18	+.06	-.26	+.06	+.00	-.08
" 1-8 ...	-.58	-.41	-.40	-.36	-.29	-.02	+.03	-.09	-.31	-.28
" 5-12 ...	-.30	-.16	-.20	-.13	-.20	-.34	+.16	-.13	-.32	-.14
" 9-16 ...	+.32	+.27	+.13	+.39	+.24	-.22	+.08	+.26	+.04	+.15
" 13-20 ...	+.36	+.30	+.64	+.43	+.21	-.18	-.06	+.50	+.78	+.74
" 17-24 ...	+.26	+.27	+.62	+.38	-.05	-.41	+.25	+.37	+.73	+.68
" 21-28 ...	+.01	+.32	+.53	+.32	+.06	-.43	+.60	+.36	+.37	+.42
" 25-32 ...	-.37	+.09	+.20	+.24	-.13	+.39	+.39	+.01	+.09	+.13
" 29-36 ...	-.31	-.02	+.13	+.25	+.14	-.32	+.34	+.05	+.29	+.25
" 33-40 ...	+.01	+.29	+.17	-.21	-.18	+.00	+.03	-.01	—	—
" 37-44 ...	+.19	+.40	+.22	-.40	-.39	+.36	-.13	-.09	—	—

Accumulated Temperature and Crops.

Accumulated temps. above 42° F. (day degrees) during eight week periods as follows :		Crop.									
		Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Previous Year :—											
Weeks	9-16 ...	+.00	+.07	-.03	+.06	-.28	+.06	-.06	-.12	+.05	+.04
"	13-20 ...	+.12	+.09	-.11	+.11	-.24	+.10	+.07	-.10	-.06	-.11
"	17-24 ...	+.23	+.25	+.15	+.33	+.22	-.10	+.36	+.09	-.09	+.04
"	21-28 ...	+.12	+.28	+.16	+.25	+.11	-.31	+.25	+.07	-.23	+.04
"	25-32 ...	+.31	+.32	+.25	+.26	-.12	-.35	+.19	+.44	+.08	+.21
"	29-36 ...	+.52	+.22	+.07	+.25	-.07	-.25	-.12	+.28	-.11	+.08
"	33-40 ...	+.42	+.19	-.05	-.17	-.13	+.07	-.38	-.05	-.32	-.30
"	37-44 ...	+.36	+.09	+.04	-.16	-.10	+.12	-.45	+.04	+.00	-.17
"	41-48 ...	+.20	-.18	-.12	+.06	-.04	+.03	-.46	-.03	+.16	-.07
"	45-52 ...	+.24	-.08	-.11	-.02	-.03	+.29	-.41	-.11	+.08	-.15
Year of Crop :—											
Weeks	49- 4 ...	+.46	+.27	+.33	+.38	+.22	+.04	-.29	+.09	+.16	+.13
"	1- 8 ...	+.52	+.31	+.36	+.44	+.18	-.26	-.31	-.10	+.13	+.20
"	5-12 ...	+.17	-.04	-.06	+.09	-.22	-.07	-.26	-.45	-.14	-.14
"	9-16 ...	-.12	-.24	-.32	-.10	-.10	+.14	-.11	-.47	-.38	-.38
"	13-20 ...	-.44	-.40	-.52	-.31	-.16	+.40	-.10	-.59	-.48	-.50
"	17-24 ...	-.49	-.57	-.68	-.54	-.35	+.33	-.13	-.56	-.41	-.54
"	21-28 ...	-.16	-.55	-.65	-.35	-.24	+.09	-.32	-.34	-.31	-.49
"	25-32 ...	-.01	-.52	-.59	-.01	-.11	-.07	-.49	-.23	-.35	-.38
"	29-36 ...	-.07	-.49	-.53	-.02	+.07	+.19	-.47	-.18	-.27	-.31
"	33-40 ...	+.02	-.14	-.16	+.14	+.31	+.27	-.09	+.05	—	—
"	37-44 ...	-.05	-.07	+.07	+.35	+.49	-.10	+.20	+.21	—	—

Accumulated Temperatures below 42° F. and Crops.

Accumulated temps. below 42° F. during periods of eight weeks as follows :		Crop.									
		Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to Crop :—											
Weeks	9-16 ...	-.18	-.33	-.35	-.15	+.15	+.10	-.05	-.27	-.41	-.33
"	13-20 ...	-.22	-.12	+.07	-.25	+.08	+.12	+.07	-.12	-.09	-.01
"	17-24 ...	-.13	-.10	+.05	-.32	+.26	+.21	-.11	-.18	+.01	+.02
"	21-28 ...	-.07	+.00	-.04	+.09	+.10	+.04	+.46	+.01	-.01	+.15
"	25-32 ...	+.01	+.37	+.28	+.16	+.02	-.20	+.48	+.17	-.03	+.24
"	29-36 ...	-.07	+.19	+.14	+.11	+.16	-.21	+.42	+.13	-.10	+.04
Year of Crop :—											
Weeks	49- 4 ...	-.32	-.06	-.15	-.26	-.07	+.18	+.30	-.08	-.20	-.12
"	1- 8 ...	-.39	-.13	-.19	-.37	-.13	+.38	+.28	+.08	-.00	-.17
"	5-12 ...	-.24	+.01	+.02	-.13	+.16	+.32	+.21	+.27	+.19	+.09
"	9-16 ...	-.15	+.20	+.23	-.28	+.09	+.30	+.04	+.16	+.15	+.16
"	13-20 ...	+.10	+.30	+.25	-.23	-.11	-.05	+.18	+.14	-.03	-.03
"	17-24 ...	+.26	+.58	+.46	+.06	+.06	-.04	+.35	+.44	+.04	+.15
"	21-28 ...	+.21	+.16	-.24	-.18	-.34	+.31	-.32	-.34	—	—

Partial correlations between rainfall and crops when the effect of
accumulated temperature is eliminated.

Values of the Partial Correlation Coefficient

$$r_{cr} = \frac{r_{cr} - r_{ca} r_{ra}}{\sqrt{(1 - r_{ca}^2)(1 - r_{ra}^2)}}$$

where c = crop (wheat, barley, etc.), r = rainfall, a = accumulated temperature above 42° F.

Seasons.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Previous—spring ...	-.21	-.03	+.04	+.17	+.38	-.02	-.07	-.10	+.18	+.11
" summer ...	-.15	-.19	-.24	-.33	-.32	+.15	-.12	-.41	-.16	-.26
" autumn ...	-.53	-.21	-.33	-.53	-.05	+.29	+.06	-.08	-.32	-.31
Winter ...	-.44	-.31	-.29	-.37	-.34	-.12	-.11	-.06	-.24	-.22
Spring ...	+.32	+.35	+.70	+.37	+.10	-.04	+.19	+.39	+.76	+.71
Summer ...	-.40	-.43	-.24	+.34	+.19	-.48	+.28	-.10	+.03	-.03
Cereal year ...	-.69	-.29	-.19	-.15	-.02	-.23	+.39	-.02	+.03	+.03

Correlations between accumulated temperatures and crops when the
effect of rainfall is eliminated.Values of r_{ca}

Seasons.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Previous spring ...	-.00	+.08	+.00	+.19	+.03	+.02	-.02	-.23	+.05	+.02
" summer ...	+.21	+.12	-.05	-.03	-.29	-.10	-.02	-.03	-.20	-.12
" autumn ...	+.14	+.05	-.18	-.33	-.03	+.20	-.39	-.02	-.15	-.29
" winter ...	+.57	+.29	+.32	+.29	+.03	-.00	-.41	-.09	-.07	+.06
Spring ...	-.30	-.26	-.17	-.06	-.07	+.15	+.07	-.38	+.10	-.09
Summer ...	-.31	-.67	-.69	+.07	+.01	-.18	-.32	-.34	-.30	-.38
Cereal year ...	-.33	-.46	-.52	-.22	-.12	+.05	-.47	-.49	-.28	-.43

The joint influence of the rainfall and accumulated temperature upon the crop.

Values of R where

$$R = \sqrt{\frac{r_{cr}^2 + r_{ca}^2 - 2r_{cr}r_{ca}r_{ra}}{1 - r_{ra}^2}}$$

Seasons.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Previous spring ...	+25	+12	+05	+20	+42	+06	+08	+24	+18	+12
" summer ...	+39	+35	+27	+49	+34	+29	+14	+48	+21	+27
" autumn ...	+61	+21	+33	+54	+05	+30	+45	+08	+32	+35
Winter ...	+67	+41	+41	+45	+34	+12	+42	+10	+25	+23
Spring ...	+59	+57	+79	+46	+19	+22	+19	+67	+42	+79
Summer ...	+40	+67	+72	+38	+22	+49	+57	+37	+42	+48
Cereal year ...	+69	+52	+52	+22	+13	+29	+69	+54	+35	+50

Partial Correlations. Values of r_{cr} .

Period.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to Crop:—										
Weeks 9-16 ...	+02	-16	+13	+22	+21	-16	+24	+18	+51	+42
" 13-20 ...	-17	-09	-06	+04	+09	-15	-14	-03	+05	-00
" 17-24 ...	-22	+06	+10	+09	+27	+11	-19	-14	+08	+08
" 21-28 ...	-49	-29	-13	-08	+12	+13	-01	-33	-05	-17
" 25-32 ...	-07	-15	-19	-23	-27	+05	+06	-17	-09	-27
" 29-36 ...	+08	-02	-19	-48	-47	+20	-25	-37	-12	-19
" 33-40 ...	-44	-16	-29	-56	-22	+08	+08	-25	-50	-40
" 37-44 ...	-62	-30	-36	-47	+10	+14	+00	-12	-45	-40
" 41-48 ...	-44	-18	-34	-48	-03	+41	-04	-11	-29	-32
" 45-52 ...	+19	+23	+09	-20	-12	+22	-09	+13	+11	+02
Year of Crop:—										
Weeks 49-4 ...	-01	-02	-08	-20	-19	+06	-26	+06	-05	-09
" 1-8 ...	-55	-35	-35	-30	-26	-09	-05	-12	-29	-24
" 5-12 ...	-31	-15	-20	-14	-20	-34	+17	-13	-32	-11
" 9-16 ...	+32	+20	+01	+39	+22	-18	+04	+09	-14	-01
" 13-20 ...	+19	+12	+52	+32	+15	+04	-13	+28	+70	+64
" 17-24 ...	+09	+06	+52	+22	-21	-32	+21	+20	+68	+61
" 21-28 ...	-04	+19	+45	+23	-03	-42	+55	+28	+30	+31
" 25-32 ...	-46	-55	-41	+24	+22	-21	+14	-19	-22	-21
" 29-36 ...	-39	-32	-15	+27	+19	-26	+16	-04	+17	+10
" 33-40 ...	+03	+26	+10	-17	-02	+18	-02	+02	—	—
" 37-44 ...	+19	+41	+30	-27	-20	+36	-04	-01	—	—

Partial Correlations. Values of r_{ca} .

Period.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to Crop:—										
Weeks 9-16 ...	+01	-01	+04	+15	-17	-02	+06	-02	+30	+22
" 13-20 ...	+01	+03	-12	+11	-16	-00	-02	-10	-03	-09
" 17-24 ...	+13	+26	+18	+34	+31	-05	+25	+03	-05	+07
" 21-28 ...	-11	+16	+10	+20	+15	-24	+23	-07	-23	-03
" 25-32 ...	+22	+19	+10	+09	-26	-27	+19	+30	+02	+04
" 29-36 ...	+51	+20	+09	+05	-28	-16	-22	+12	-15	-01
" 33-40 ...	+21	+09	-20	-46	-22	-10	-30	-19	-54	-45
" 37-44 ...	-00	-10	-17	-39	-03	+17	-39	-03	-26	-36
" 41-48 ...	+09	-23	-22	-10	-05	+16	-45	-06	+08	-16
" 45-52 ...	+22	-11	-12	+01	-02	+27	-40	-12	+06	-15
Year of Crop:—										
Weeks 49-4 ...	+46	+27	+33	+39	+23	+03	-29	+09	+16	+14
" 1-8 ...	+49	+24	+29	+40	+12	-27	-31	-13	+05	+14
" 5-12 ...	+18	-03	-06	+12	-22	-07	-37	-45	-15	-11
" 9-16 ...	+01	-16	-29	+06	-01	+05	-09	-41	-40	-35
" 13-20 ...	-28	-30	-27	-11	-06	+36	-16	-45	-13	-19
" 17-24 ...	-44	-51	-60	-45	-40	+19	-04	-49	-19	-41
" 21-28 ...	-16	-50	-60	-28	-23	-05	-17	-26	-22	-42
" 25-32 ...	-30	-70	-68	+13	+04	-18	-36	-32	-40	-41
" 29-36 ...	-25	-56	-53	+12	+15	+05	-37	-18	-16	-22
" 33-40 ...	+03	+02	-08	+04	+25	+32	-09	+05	—	—
" 37-44 ...	+06	+15	+24	+20	+38	+09	+16	+17	—	—

The joint influence of the rainfall and accumulated temperature on the crop.

$$\text{Values of } R \text{ where } R = \sqrt{\frac{r_{cr}^2 + r_{ca}^2 - 2r_{cr}r_{ca}r_{ra}}{1 - r_{ra}^2}}$$

Period.	Wheat.	Barley.	Oats.	Beans.	Peas.	Potatoes.	Turnips and Swedes.	Mangolds.	Hay from Clover and Rotation Grass.	Hay from Permanent Grass.
Year previous to Crop:—										
Weeks 9-16 ...	+02	+17	13	23	34	16	25	21	51	42
" 13-20 ...	+21	13	12	11	26	18	15	10	08	11
" 17-24 ...	+31	26	18	34	35	14	40	16	12	09
" 21-28 ...	+50	39	20	26	16	33	26	34	23	18
" 25-32 ...	+31	35	29	34	30	35	20	47	12	34
" 29-36 ...	+53	23	20	52	47	31	28	45	16	21
" 33-40 ...	+56	25	30	57	26	10	39	26	57	49
" 37-44 ...	+31	36	49	14	18	45	13	45	43	33
" 41-48 ...	+47	25	36	48	05	41	46	11	32	15
" 45-52 ...	+31	24	14	20	14	36	42	17	14	15
Year of Crop:—										
Weeks 49-4 ...	+46	27	33	42	29	08	38	11	16	17
" 1-8 ...	+68	45	49	52	31	27	31	15	32	31
" 5-12 ...	+34	16	21	17	30	35	29	47	35	20
" 9-16 ...	+32	25	32	40	24	22	12	47	40	38
" 13-20 ...	+45	42	68	44	40	45	25	59	75	74
" 17-24 ...	+50	57	78	56	40	45	61	52	42	56
" 21-28 ...	+16	57	73	41	24	43	50	32	41	43
" 25-32 ...	+47	70	68	24	24	22	49	19	32	33
" 29-36 ...	+39	56	54	27	21	32	10	07	—	—
" 33-40 ...	+03	29	18	22	31	32	20	21	—	—
" 37-44 ...	+20	41	30	43	52	37	20	21	—	—

Reference 37.

H. L. MOORE

... Economic Cycles, Their Law and Cause. Macmillan, New York, 1914.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$-.227 \pm .075$	72 (1839-1910)	Amount of rainfall in Ohio Valley and time.
$.382 \pm .090$	41 (1870-1910)	Yield of corn per acre in Illinois and time.
$.013 \pm .105$	"	" " hay " " " " " "
$.043 \pm .105$	"	" " oats " " " " " "
$.122 \pm .104$	"	" " potatoes per acre in Illinois and time.
		The above 5 coefficients were given to show any tendency to secular change in rainfall and the four crops considered.
.069	"	Yield of corn per acre in Illinois in June and rainfall for June.
.496	"	Yield of corn per acre in Illinois in July and rainfall for July.
.293	"	Yield of corn per acre in Illinois in August and rainfall for August.
.087	"	Yield of corn per acre in Illinois in September and rainfall for September.
.589	"	Yield of corn per acre in Illinois in July and August and rainfall for July and August.
.290	"	Yield of oats per acre in Illinois in May, June, July, and rainfall for May, June, July.
.620	"	Yield of hay per acre in Illinois from March to June, and rainfall from March to June.
.666	"	Yield of potatoes per acre in Illinois in July and August and rainfall for July and August.
.600	"	Annual rainfall in Illinois with annual rainfall in Ohio.
-.181	"	Yield of oats per acre in Illinois and rainfall in March.
-.147	"	" " " " " " April.
.120	"	" " " " " " May.
.297	"	" " " " " " June.
.140	"	" " " " " " July.
.584	"	An index of the fluctuation of the four crops: corn, oats, hay, potatoes and an index of the mean effective rainfall of the critical periods of the crops.
		The fluctuation of any crop is $\frac{\Delta}{\sigma}$ where Δ = deviation of the yield of any year from the mean yield of the whole period, and σ is the standard deviation. The index of the fluctuation of the four crops is the average of the four terms $\frac{\Delta}{\sigma}$ one for each crop. The index of the mean effective rainfall of the critical periods of the crops is the total rainfall for the proper months divided by the number of such months.

Reference 38.

J. WARREN SMITH ...

... Correlation. Monthly Weather Rev., Wash., May, 1911.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$.50 \pm .10$	27 (1883-1909)	Potato yield in Ohio and— (i) Total rainfall June and July.
$-.69 \pm .10$	"	(ii) Mean temperature June and July.

Reference 39.

J. WARREN SMITH ...

... The Effect of Weather upon the Yield of Corn. Month. Weather Rev., Wash., Feb., 1914.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$+0.59 \pm .06$	60 (1854-1913)	July rainfall and yield of corn in Ohio.
		Corn yield in Wayne County and rainfall in Ohio—
$+ .31$	20 (1891-1910)	June 21-30.
$+ .12$		July 1-10.
$+ .71$		" 11-20.
$+ .16$		" 21-31.
$+ .56$		August 1-10.
$+ .46$		" 11-20.
$+ .14$		" 21-31.
$+ .36$		September 1-10.
		Corn yield and rainfall in Central Ohio for 10-day periods—
$-.09$	20 (1891-1910)	June 1-10.
$+ .12$		" 11-20.
$-.04$		" 21-30.
$+ .16$		July 1-10.
$+ .36$		" 11-20.
$+ .36$		" 21-31.
$+ .52$		August 1-10.
$+ .29$		" 11-20.
$-.06$		" 21-31.
		Corn yield and rainfall in Central Ohio for 20-day periods—
$+ .03$	20 (1891-1910)	June 1-20.
$-.10$		" 11-30.
$+ .07$		" 21-July 10.
$+ .36$		July 1-20.
$+ .41$		" 11-30.
$+ .50$		" 21-August 10.
$+ .45$		August 1-20.
$+ .20$		" 11-31.
		Corn yield and rainfall in Central Ohio for 30-day periods—
$-.02$	20 (1891-1910)	June 1-30.
$+ .11$		" 11-July 10.
$+ .26$		" 21- " 20.
$+ .43$		July 1-31.
$+ .49$		" 11-August 10.
$+ .48$		" 21- " 20.
$+ .37$		August 1-31.
		Corn yield and rainfall in Central Ohio for 40-day periods—
$+ .07$	20 (1891-1910)	June 1-July 10.
$+ .24$		" 11- " 20.
$+ .36$		" 21- " 31.
$+ .53$		July 1-August 10.
$+ .60$		" 11- " 20.
$+ .52$		" 21- " 31.
		Corn yield and rainfall in Central Ohio for 50-day periods—
$+ .17$	20 (1891-1910)	June 1-July 20.
$+ .36$		" 11- " 31.
$+ .49$		" 21-August 10.
$+ .59$		July 1- " 20.
$+ .55$		" 11- " 31.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
+ '42	30 (1883-1912)	Thermal constants and rainfall and also other factors Wauseon, Ohio— Thermal constants from date above ground to date of blossoming of corn and total rainfall for same period.
+ '28		Thermal constants and total rainfall from date of blossoming of corn to date ripe.
- '11		Thermal constants and total rainfall from date of planting of corn to date that it is ripe.
+ '37		Thermal constants from date of planting of corn to date it is ripe and the total number of days in same period.
- '001		Relation of total number of days from date of planting of corn to date it is ripe and the yield of corn.
- '01	30 (1883-1912)	Corn yield and mean minimum temperatures at Wauseon, Ohio— Mean minimum temperatures for 10 days before blossoming and yield.
- '01		Mean minimum temperatures for 10 days after blossoming and yield.
- '07		Mean minimum temperatures from date corn appears above ground to date of blossoming and yield.
+ '33		Mean minimum temperatures from date of blossoming to date corn is ripe and yield.
'48	20	Yield of corn in Columbia and Franklin Counties and (1) Rainfall for July.
'50	20	(2) Rainfall, June 21st-Aug. 10th.
		Yield of corn as above and factor determined for the amounts of rainfall given below according to the formula $\frac{ab}{c}$ where a = total rainfall for period. b = the number of days of the specified rainfall. c = total number of days in the period—
'61	20	(i) days with '01 inch or more.
'61		(ii) " '10 "
'61		(iii) " '20 "
'64		(iv) " '25 "
'59		(v) " '30 "
'61		(vi) " '40 "
'70		(vii) " '50 "
'55		(viii) " '60 "
'56		(ix) " '70 "
'57		(x) " '75 "
'38		(xi) " '80 "
'59		(xii) " '90 "
'41		(xiii) " 1'00 "
'50	20 (1891-1910)	Corn yield in Central Ohio and rainfall, July 21-Aug. 10.
		Corn yield in Central Ohio and factor determined for the amounts of rainfall below according to the formula $\frac{ab}{c}$ given above—
'44	20 (1891-1910)	(i) days with '01 inch or more.
'51		(ii) " '10 "
'43		(iii) " '20 "
'49		(iv) " '25 "
'50		(v) " '30 "
'47		(vi) " '40 "
'64		(vii) " '50 "

Correlation Coefficient.	Number of Observations.	Variables correlated.
	24 (1888-1911)	Corn yield for Indiana, Illinois, Iowa and Missouri, and rainfall for— (i) June. (ii) July. (iii) August. (iv) June and July. (v) July and August. (vi) June, July and August. (vii) Mean temperature for July over the four states.
'34 '73 '48 '68 '69 '69 '61 '78 ± '05	25 (1888-1912)	Yield of corn in Ohio, Indiana, Illinois, Iowa, Nebraska, Kansas, Missouri, Kentucky and July rainfall over the eight states.

Reference 40.

J. WARREN SMITH ... The Effect of Weather upon the Yield of Potatoes. Month. Weather Rev., Wash., May, 1915.

Correlation Coefficient.	Number of Observations.	Variables correlated.
- '51 ± '07	55 (1860-1914)	July mean temperature and yield of potatoes in Ohio.
- 0'10 ± 0'09	55 (1860-1914)	Potato yield in Ohio and mean temperature for— May.
- 0'22 ± 0'09		June.
- 0'51 ± 0'07		July.
- 0'31 ± 0'08		August.
- 0'21 ± 0'09		September.
- 0'11 ± 0'09		October.
- 0'50 ± 0'07		June and July combined.
- 0'50 ± 0'07		July and August combined.
- 0'49 ± 0'07		June, July and August combined.
- '21 ± '09	55 (1860-1914)	Yield of Potatoes in Ohio and rainfall in— April.
'06 ± '10		May.
'10 ± '09		June.
'33 ± '08		July.
'22 ± '09		August.
- '13 ± '09		September.
'07 ± '10		October.
'37 ± '08		July and August combined.
'29 ± '12	20 (1891-1910)	Yield of Potatoes in Central Ohio and Rainfall for 10-day periods.
'32 ± '12		June 1-10.
'16 ± '13		" 11-20.
'48 ± '10		" 21-30.
- '29 ± '12		July 1-10.
- '12 ± '13		" 11-20.
		" 21-31.
'48 ± 0'10	20 (1891-1910)	Correlation of rainfall for 20-day periods with potato yield in Central Ohio.
'30 ± 0'12		June 1-20.
'44 ± 0'11		" 11-30.
'03 ± 0'13		" 21-July 10.
- '23 ± 0'12		July 1-20.
- '08 ± 0'13		" 11-31.
'29 ± 0'12		" 21-August 10
'22 ± 0'12		August 1-20.
		" 11-31.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$.42 \pm 0.11$ $.58 \pm 0.09$ $.26 \pm 0.12$ $.002 \pm 0.13$ $-.20 \pm 0.13$ $.19 \pm 0.13$ $.11 \pm 0.13$	20 (1891-1910)	Yield of potatoes in Central Ohio and rainfall for 30-day periods. June 1-30. " 11-July 10. " 21- " 20. July 1-31. " 11-August 10. " 21- " 20. August 1-31.
$.59 \pm 0.09$ $.35 \pm 0.11$ $.09 \pm 0.13$ $.02 \pm 0.13$ $.02 \pm 0.13$ $.06 \pm 0.13$	20 (1891-1910)	Yield of potatoes in Central Ohio and rainfall for 40-day periods. June 1-July 10. " 11- " 20. " 21- " 31. July 1-August 10. " 11- " 20. " 21- " 31.
0.44 ± 0.11 0.20 ± 0.13 0.09 ± 0.13 0.17 ± 0.13 -0.05 ± 0.13	20 (1891-1910)	Yield of potatoes in Central Ohio and rainfall for 50-day periods. June 1-July 20. " 11- " 31. " 21-August 10. July 1- " 20. " 11- " 31.
$-.12 \pm 0.13$ $-.17 \pm 0.13$ $-.28 \pm 0.12$ $-.44 \pm 0.11$ $-.33 \pm 0.12$ $-.33 \pm 0.12$ $-.23 \pm 0.13$ $-.36 \pm 0.11$ $-.38 \pm 0.11$	20 (1891-1910)	Yield of potatoes in Central Ohio and mean temperature at Columbus, Ohio, for 10-day periods. June 1-10. " 11-20. " 21-30. July 1-10. " 11-20. " 21-31. August 1-10. " 11-20. " 21-31.
$-.19 \pm 0.11$ $-.27 \pm 0.12$ $-.61 \pm 0.09$ $-.54 \pm 0.10$ $-.41 \pm 0.11$ $-.42 \pm 0.11$ $-.36 \pm 0.11$ $-.43 \pm 0.11$	20 (1891-1910)	Yield of potatoes in Central Ohio and temperature for 20-day periods. June 1-20. " 11-30. " 21-July 10. July 1- " 20. " 11- " 31. " 21-August 10. August 1-20. " 11-31.
$-.33 \pm 0.12$ $-.53 \pm 0.11$ $-.61 \pm 0.10$ $-.57 \pm 0.09$ $-.51 \pm 0.10$ $-.49 \pm 0.10$ $-.35 \pm 0.11$	20 (1891-1910)	Yield of potatoes in Central Ohio and temperature for 30-day periods. June 1-30. " 11-July 10. " 21- " 20. July 1- " 31. " 11-August 10. " 21- " 20. August 1-31.
$-.58 \pm 0.09$ $-.58 \pm 0.09$ $-.62 \pm 0.09$ $-.63 \pm 0.09$ $-.54 \pm 0.10$ $-.51 \pm 0.10$	20 (1891-1910)	Yield of potatoes in Central Ohio and temperature for 40-day periods. June 1-10. " 11-20. " 21-31. July 1-10. " 11-20. " 21-31.

Correlation Coefficient.	Number of Observations.	Variables correlated.
$-.58 \pm 0.09$ $-.54 \pm 0.10$ $-.65 \pm 0.08$ $-.67 \pm 0.08$ $-.52 \pm 0.10$	20 (1891-1910)	Yield of potatoes in Central Ohio and temperature for 50-day periods. June 1-July 20. " 11- " 31. " 21-August 10. July 1- " 20. " 11- " 31.
0.03 0.24 0.16 0.25 0.17 -0.30	30 (1883-1912)	Yield of potatoes at Wauseon, Ohio, and total effective temperature. From date of planting to date above ground. From date above ground to date of bloom. From date of bloom to date ripe. From date planted to date ripe. For 10 days before blooming. For 10 days after blooming.
0.02 -0.06 0.33 0.18 0.09 -0.07	30 (1883-1912)	Yield of potatoes at Wauseon, Ohio, and rainfall. For 10 days before planting. From date planted to date above ground. From date above ground to date in bloom. From date in bloom to date ripe. For 10 days before blooming. For 10 days after blooming.
0.13 0.06 0.19 -0.14 0.14 -0.22 -0.02 -0.04 -0.12 0.20 0.22	30 (1883-1912)	Yield of potatoes in Portage County, Ohio, and average temperature for— April. May. June. July. August. September. October. June and July combined. July and August combined. June and July with July reversed.* July and August with July reversed.*
-0.23 0.06 0.38 -0.03	54 (1860-1913)	Yield of potatoes in Portage County, Ohio, and rainfall— June. July. August. September.

* Sign of departure from mean reversed.

Correlation Coefficient.		Number of Observations.	Variables correlated.
Temperature.	Rainfall.		
-0.02 -0.48 -0.35 -0.27 -0.02 -0.04 -0.59 -0.87 -0.27 -0.55 -0.38	0.02 0.38 0.21 -0.34 -0.17 -0.14 0.41 $*0.18$ -0.12 0.35 $*-0.24$	30 (1884-1913)	Yield of Potatoes in Licking County, Ohio and rainfall and temperature— May. June. July. August. September. October. June and July combined. July and August combined. August and September combined. June, July and August. July, August and September.

* Departure sign for July reversed.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
-0.01 0.06 -0.29 -0.36 -0.28 -0.23 -0.15 -0.44 -0.44 -0.15	54 (1860-1913)	Yield of Potatoes in Licking County, Ohio, and temperature— April. May. June. July. August. September. October. June and July combined. July and August combined. August and September combined.

Reference 41.

J. WARREN SMITH ... Agricultural Meteorology. Proc. Sec. Pan Amer. Sci. Cong., Wash., 1917, p. 75.

Correlation Coefficient.	Number of Observations.	Variables correlated.
.12 ± .09 .59 ± .06 .37 ± .08 .48 ± .07 .67 ± .05 .57 ± .06	60 (1854-1913)	Yield of corn in Ohio with rainfall in— (i) June. (ii) July. (iii) August. (iv) June and July combined. (v) July and August combined. (vi) June, July, and August combined.
.52 ± .10 .50 ± .10 .49 ± .10 .60 ± .09 .59 ± .09	20 (1891-1910)	Corn crop in Central Ohio with rainfall— (i) August 1 to 10, 10 days. (ii) July 21 to August 10, 20 days. (iii) July 11 to August 10, 30 days. (iv) July 11 to August 20, 40 days. (v) July 1 to August 20, 50 days.
-.03 -.03 .18 .08 -.003 -.28	30 (1883-1912)	Corn yield at Wauseon, Ohio, with total effective heat (sum of daily temperatures above 43° F.)— (i) 10 days before planting. (ii) Planting to above ground. (iii) Above ground to blossom. (iv) Blossom to ripening. (v) 10 days before blossom. (vi) 10 days after blossom.
.01 -.06 -.03 .29 .45 .20 .74 .57 .46	30 (1883-1912)	Yield of corn at Wauseon, Ohio, with rainfall— (i) 10 days before planting. (ii) Planting to above ground. (iii) Above ground to blossom. (iv) Blossom to ripening. (v) 5 days before to 5 days after blossoming. (vi) 10 " " blossom. (vii) 10 " after " " (viii) 20 " " " " (ix) 30 " " " "

Correlation coefficients.	Number of observations.	Variables correlated.
(a) — -.10 -.22 -.51 -.31 -.21 -.11 -.50 -.50 -.49	(b) -.21 .06 .10 .33 .22 -.13 .07 — — -.37 —	55 (1860-1914) Yield of potatoes in Ohio and (a) temperature, (b) rainfall for the periods— (i) April. (ii) May. (iii) June. (iv) July. (v) August. (vi) September. (vii) October. (viii) June and July combined. (ix) July and August combined. (x) June, July, and August combined.
-.12 -.17 -.28 -.44 -.33 -.33 -.23 -.36 -.38	.29 .32 .16 .48 -.29 -.12 .06 .37 -.26	20 (1891-1910) Yield of potatoes in Central Ohio, and (a) temperature, (b) rainfall for the periods— (i) June 1 to 10. (ii) " 11 to 20. (iii) " 21 to 30. (iv) July 1 to 10. (v) " 11 to 20. (vi) " 21 to 31. (vii) August 1 to 10. (viii) " 11 to 20. (ix) " 21 to 31.
.16 .09 .14 .05 .21 .26 .46 -.10 -.11 -.03 .17 .19	.04 .16 -.02 -.17 .09 .01 .06 .02 .02 .17 -.17 .15	From 30 to 60 Yield of winter wheat in Ohio with (a) temperature (b) rainfall for the periods— (i) September. (ii) October. (iii) November. (iv) December. (v) January. (vi) February. (vii) March. (viii) April. (ix) May. (x) Autumn (September to November). (xi) Winter (December to February). (xii) Spring (March to May).

Reference 42.

S. M. JACOB ... Correlation of Rainfall and the Succeeding Crops with Special Reference to the Punjab. Mem. Indian Meteor. Depart., Vol. XXI., Part XIV., 1916.

Total correlation coefficients between rainfall for months July-October and area of crop sown in autumn in three different districts.

Data for 30 years 1886-1915.

Crop.	District.	July.	Aug.	Sept.	Oct.
Well irrigated wheat	Dona Lehnda	-.38	-.25	-.52	—
	" Charhda	—	-.28	-.53	-.34
Unirrigated mixed wheat and grain.	Sirwal	—	-.01	-.50	-.39
	" Charhda	.18	.21	.54	—
All unirrigated crops	Dona Lehnda	—	.13	.54	.19
	" Charhda	—	.16	.27	—
	Sirwal	.21	.04	.37	—
	" Charhda	—	.17	.48	.33
	Sirwal	—	-.06	.48	—

Correlations between monthly rainfalls—

August with September	— '37
" " October	— '10
September with October	— '13

Partial correlation coefficients between area of well-irrigated wheat sown in autumn in the Dona Charhda district (30 years data) and rainfall—

- (i) for August when effect of September and October rainfall is eliminated — '79
- (ii) for September when effect of August and October rainfall is eliminated — '86
- (iii) for October when effect of August and September rainfall is eliminated — '74

Partial correlation coefficients between area of all unirrigated spring crops sown in autumn in the Dona Charhda district (30 years data) and rainfall—

- (i) for August when effect of September and October rainfall is eliminated + '57
- (ii) for September when effect of August and October rainfall is eliminated + '72
- (iii) for October when effect of August and September rainfall is eliminated + '59

Correlative between monthly rainfalls and area of crops which fails to come to maturity.

The estimate of crop failure is made by personal estimate of various observers.

Crop.	District.	Sept.	Dec.	Jan.	Feb.	March.
Well-irrigated wheat ...	Dona Lehnda ...		— '36	— '19	— '27	— '27
	Dona Charhda...	— '41	— '46	+ '01	— '21	— '36
All well-irrigated crops	Dona Charhda...	— '46	— '39	— '18	— '25	— '43
	Sirwal ...	— '08	— '05	+ '17	— '02	— '15
Unirrigated wheat ...	Dona Lehnda ...		— '31	— '16	— '19	— '07
	Dona Charhda...	— '37	— '40	— '16	— '26	— '26
	Sirwal ...	— '12	— '07	+ '23	— '02	— '14
Unirrigated wheat gram	Dona Lehnda ...		— '13	— '14	— '19	— '23
	Dona Charhda...	— '37	— '34	— '19	— '31	— '35
	Sirwal ...	— '54	— '33	— '11	— '37	— '41
Unirrigated gram ...	Dona Charhda...	— '43	— '51	— '19	— '31	— '35
All unirrigated crops...	Dona Lehnda ...		— '23	— '20	— '14	— '22
	Dona Charhda...	— '40	— '40	— '17	— '32	— '34
	Sirwal ...	— '42	— '32	— '07	— '26	— '28

In the above table the Dona Lehnda district results are for the 30 years 1886-1915. For the other two districts the results are for the 15 years 1901-1915.

Correlation coefficients between monthly rainfalls and unirrigated wheat crop in the Dona Charhda District. 30 or 15 observations.

	Sept.	Dec.	Jan.	Feb.	March.
Whole period 1886-1915 ...	— '30	— '30	— '27	— '19	— '24
First " 1886-1900 ...	— '25	— '25	— '46	— '02	— '19
Second " 1901-1915 ...	— '37	— '40	— '16	— '26	— '26

Reference 43.

T. OKADA On the Possibility of Forecasting the Approximate Yield of Rice-Crop for Northern Japan. Jour. Meteor. Soc. Japan, Nov., 1917.

Correlation Coefficient.	Number of Observations.	Variables correlated.
'69 ± '07	24 (1884-1908)	First difference series of yield of rice crop for Hokkaido, North Japan, and (i) South American pressure (mean of monthly pressures at Santiago and Buenos Ayres for March, April, May, weighted 1, 1, 1, respectively). (ii) Pressure difference between Zikawei and Miyazaki for March.
'64 ± '08	19 (1893-1912)	First difference series of yield of rice crop for North-East Japan and (i) South American pressure, as above.
'56 ± '09	24 (1884-1908)	(ii) Pressure difference, Zikawei and Miyazaki, for March, as above.
'51 ± '10	24 (1884-1908)	(iii) As for (ii).
'52 ± '09	29 (1884-1913)	

Reference 44.

J. F. VOORHEES Climatic Control of Cropping Systems and Farm Operations. Proc. Sec. Pan Amer. Sci. Cong., Wash., 1917, p. 127.

Correlation Coefficient.	Number of Observations.	Variables Correlated.
— '76 ± '05	50	Mean temperature and length of time from planting to blooming of yellow soy bean.
— '79 ± '05	50	Mean temperature and length of time from emergence to blooming of Indian corn at Wauseon, Ohio.
— '93 ± '013	50	Mean temperature and length of period of incubation of the Texas fever cattle tick, <i>margaropus annulatus</i> .

Reference 45.

AXEL WALLÉN... Sur la Corrélation entre les Récoltes et les Variations de la Température et de l'Eau Tombée en Suède. Kung. Sven. Vet. Hand., Band 57, No. 8, Stockholm, 1917.

Correlation coefficients.

Data for the 30 years 1881-1910, the data for which district are

In the tables of correlation coefficients r is used when one when one of the variables is crop amount

District.	Year previous to Crop.									
	August.		September.		October.		November.		December.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
Gottland ...	+ '26	+ '26	+ '23	+ '30	+ '26	+ '21	+ '04	+ '05	+ '09	+ '12
Malmöhus ...	- '29	- '33	- '27	+ '00	+ '38	+ '07	- '11	- '27	- '24	- '42
Kristianstad ...	- '09	- '19	- '06	+ '10	+ '41	+ '07	+ '15	- '31	+ '10	- '12
Halland ...	- '20	- '45	- '36	- '06	+ '38	- '01	+ '04	- '04	+ '08	- '08
Blekinge ...	- '06	+ '01	- '13	+ '18	+ '66	+ '58	- '47	- '57	- '16	- '24
S. Kalmar ...	+ '02	+ '06	- '32	- '02	+ '35	+ '16	- '19	- '24	- '02	+ '04
N. Kalmar ...	- '22	- '19	- '20	- '09	+ '45	+ '24	- '09	- '07	+ '20	+ '17
Jönköping ...	+ '00	+ '07	+ '03	+ '16	+ '32	+ '02	- '12	- '20	+ '12	+ '08
Kronoberg ...	- '16	- '17	+ '10	+ '10	- '15	- '18	- '02	- '02	- '10	- '12
S. Alvsborg ...	- '21	-	+ '22	-	+ '20	-	- '09	-	- '07	-
Göteborg ...	+ '18	+ '19	- '10	- '15	+ '12	+ '16	+ '25	+ '26	+ '08	+ '09

District.	Year of Crop.							
	January.		February.		March.		April.	
	r	ρ	r	ρ	r	ρ	r	ρ
Gottland ...	+ '80	+ '74	+ '42	+ '40	+ '13	+ '07	- '12	- '08
Malmöhus ...	+ '29	+ '23	+ '20	+ '29	+ '27	+ '03	+ '02	+ '01
Kristianstad ...	+ '25	+ '10	+ '22	+ '18	+ '23	- '11	+ '18	- '00
Halland ...	+ '18	+ '11	+ '08	+ '09	+ '31	+ '08	+ '24	+ '24
Blekinge ...	+ '23	+ '16	+ '26	+ '33	+ '23	+ '10	+ '07	+ '02
S. Kalmar ...	+ '55	+ '56	+ '33	+ '43	+ '39	+ '31	+ '19	+ '10
N. Kalmar ...	+ '34	+ '34	+ '22	+ '27	+ '41	+ '37	+ '39	+ '40
Jönköping ...	+ '27	+ '06	+ '23	+ '26	+ '34	+ '17	+ '19	+ '24
Kronoberg ...	- '25	- '32	+ '05	+ '04	+ '22	+ '14	+ '24	+ '21
S. Alvsborg ...	+ '41	-	+ '51	-	+ '32	-	+ '16	-
Göteborg ...	+ '03	+ '05	- '00	- '00	+ '17	+ '19	+ '13	+ '15
N. Alvsborg ...	+ '08	- '06	+ '18	+ '18	+ '27	+ '18	+ '21	+ '13
Skaraborg ...	+ '15	+ '04	+ '07	+ '09	+ '17	- '03	+ '10	- '02
Östergötland ...	+ '22	+ '14	+ '16	+ '15	+ '30	+ '16	+ '34	+ '32
Södermanland ...	-	+ '22	-	+ '41	-	+ '20	+ '29	+ '24
Stockholm ...	-	+ '18	-	+ '31	-	+ '28	+ '48	+ '48
Upsala ...	-	+ '26	-	+ '52	-	+ '34	+ '38	+ '00
Västmanland ...	-	+ '21	-	+ '44	-	+ '33	+ '35	+ '33
Orebro ...	-	+ '02	-	+ '16	-	+ '32	+ '56	+ '40
Värmland ...	+ '16	+ '07	+ '12	+ '14	+ '32	+ '22	+ '29	+ '25
Kopparberg ...	-	+ '15	-	+ '23	-	+ '30	-	+ '22
Gävleborg ...	+ '09	- '01	+ '17	+ '08	+ '27	+ '20	+ '15	-

Wheat and Temperature.

except for Gottland, for the 26 years 1885-1910.

of the variables is crop amount actual figures, ρ is used corrected for secular variation.

District.	Year previous to Crop.									
	August.		September.		October.		November.		December.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
N. Alvsborg ...	- '17	- '12	- '05	+ '12	+ '47	+ '24	+ '03	+ '04	- '08	- '17
Skaraborg ...	- '17	- '04	- '16	+ '27	+ '50	+ '28	- '15	- '20	- '06	- '18
Östergötland ...	- '26	- '31	- '12	+ '02	+ '52	+ '25	- '03	- '00	+ '09	+ '01
Södermanland ...	- '43	- '48	- '36	- '33	+ '32	+ '13	+ '20	+ '21	-	+ '21
Stockholm ...	- '49	- '49	- '38	- '27	+ '17	- '01	+ '21	+ '26	-	+ '30
Upsala ...	- '59	- '59	- '28	- '19	+ '25	+ '09	+ '24	+ '21	-	+ '27
Västmanland ...	- '54	- '52	- '39	- '34	+ '26	+ '16	+ '30	+ '32	-	+ '14
Orebro ...	- '55	- '53	- '34	- '24	+ '19	+ '18	+ '17	+ '19	-	+ '18
Värmland ...	- '14	- '13	- '29	- '19	+ '54	+ '28	+ '23	+ '32	+ '22	+ '32
Kopparberg ...	-	- '45	-	- '40	-	- '02	-	+ '19	-	+ '11
Gävleborg ...	- '26	- '25	- '39	- '27	+ '42	+ '10	+ '19	+ '32	+ '03	-

District.	Year of Crop.							
	May.		June.		July.		August.	
	r	ρ	r	ρ	r	ρ	r	ρ
Gottland.	- '41	- '40	- '53	- '55	- '22	- '06	- '34	- '35
Malmöhus.	- '20	- '31	+ '09	+ '07	- '24	- '33	- '08	- '27
Kristianstad.	+ '04	- '01	+ '25	+ '22	+ '02	- '11	+ '05	- '19
Halland.	+ '08	- '08	+ '18	- '00	+ '03	- '12	- '03	- '36
Blekinge.	- '30	- '33	- '07	- '12	- '17	- '02	+ '03	- '04
S. Kalmar.	- '08	- '08	+ '07	+ '03	- '14	- '02	+ '08	+ '11
N. Kalmar.	- '28	- '18	+ '15	+ '19	- '04	+ '05	+ '25	+ '23
Jönköping.	- '08	- '04	+ '39	+ '41	+ '18	+ '19	+ '12	+ '09
Kronoberg.	+ '08	+ '07	+ '28	+ '26	+ '16	+ '14	+ '12	-
S. Alvsborg.	- '04	-	- '05	-	+ '10	-	+ '32	-
Göteborg.	- '14	- '14	+ '11	+ '13	+ '09	+ '09	+ '01	+ '02
N. Alvsborg.	+ '02	- '01	+ '21	+ '15	- '02	- '06	+ '10	+ '06
Skaraborg.	- '19	- '24	+ '21	+ '13	- '14	- '23	- '05	- '03
Östergötland.	- '11	- '08	+ '21	+ '22	+ '04	+ '09	+ '15	+ '21
Södermanland.	- '01	- '02	+ '09	+ '00	+ '14	+ '12	+ '15	+ '09
Stockholm.	+ '15	+ '22	+ '17	+ '13	+ '31	+ '36	+ '17	+ '18
Upsala.	+ '11	+ '16	+ '09	+ '04	+ '27	+ '26	+ '13	+ '17
Västmanland.	+ '04	+ '05	+ '03	- '00	+ '23	+ '22	+ '11	+ '13
Orebro.	+ '07	+ '10	+ '18	+ '16	+ '07	+ '08	+ '17	+ '17
Värmland.	+ '05	+ '05	+ '19	+ '15	+ '10	+ '10	+ '20	+ '21
Kopparberg.	-	+ '29	-	+ '17	-	+ '15	-	+ '30
Gävleborg.	+ '01	+ '17	+ '13	+ '00	+ '37	+ '47	+ '25	+ '42

Correlation Coefficients.

Districts.	Year previous to Crop.							
	August.		September.		October.		November.	
	<i>r</i>	ρ	<i>r</i>	ρ	<i>r</i>	ρ	<i>r</i>	ρ
Gottland ...	-.23	-.25	+.14	+.18	-.36	-.29	+.16	+.10
Malmöhus ...	+.14	-.01	-.31	-.11	-.46	-.21	-.32	-.30
Kristianstad ...	+.20	-.15	-.20	-.15	-.39	-.11	-.11	-.06
Halland ...	+.17	-.03	-.31	-.42	-.09	+.10	-.24	-.22
Blekinge ...	+.21	+.10	-.11	-.03	-.39	-.21	-.03	+.01
S. Kalmar ...	+.11	+.10	-.17	-.10	-.40	-.21	-.10	-.08
N. Kalmar ...	+.07	-.04	+.11	+.36	-.43	-.33	-.26	-.24
Jönköping ...	+.20	-.02	+.00	+.20	-.22	-.05	-.32	-.13
Kronoberg ...	+.19	+.16	-.08	-.06	+.03	+.12	-.10	-.09
S. Älvsborg ...	+.27	—	+.13	—	+.12	—	-.03	—
Bohus ...	—	-.09	—	+.03	—	-.16	—	+.06
N. Älvsborg ...	+.03	-.04	-.12	-.05	+.01	-.06	-.16	-.06
Skaraborg ...	-.04	-.39	-.12	+.13	-.12	-.18	-.27	-.14
Ostergötland ...	+.29	-.06	-.30	+.21	-.24	-.21	-.10	-.15
Södermanland ...	+.26	+.19	+.17	+.23	-.07	-.01	-.12	+.11
Stockholm ...	+.39	+.35	+.21	+.36	+.01	+.09	-.18	-.14
Upsala ...	+.38	+.30	+.14	+.32	+.10	+.07	-.03	+.02
Västmanland ...	+.26	+.31	+.38	+.42	+.09	+.08	+.08	+.11
Orebrö ...	+.12	+.00	+.09	+.21	+.13	+.09	-.16	-.11
Varmland ...	+.14	-.08	+.09	+.37	+.23	+.07	-.17	+.04
Kopparberg ...	—	+.21	—	+.34	—	+.02	—	-.24
Gävleborg ...	+.11	-.01	-.05	+.14	-.12	-.18	-.16	+.00

—Wheat and Rainfall.

Districts.	Year of Crop.									
	April.		May.		June.		July.		August.	
	<i>r</i>	ρ	<i>r</i>	ρ	<i>r</i>	ρ	<i>r</i>	ρ	<i>r</i>	ρ
Gottland.	+.07	+.03	+.53	+.56	+.31	+.30	-.00	-.05	-.21	-.21
Malmöhus.	+.28	-.21	+.07	-.06	-.07	-.38	-.50	-.13	+.05	-.05
Kristianstad.	+.31	-.18	+.23	+.10	+.01	-.24	-.04	+.25	+.20	-.09
Halland.	+.21	-.20	-.03	-.06	+.09	-.19	-.19	-.25	+.12	-.08
Blekinge.	+.34	+.07	+.51	+.52	+.02	+.01	-.05	+.08	-.08	-.22
S. Kalmar.	+.37	+.16	+.63	+.59	+.06	+.04	-.12	+.01	-.09	-.18
N. Kalmar.	+.04	-.09	+.31	+.32	+.01	-.15	-.19	+.02	+.19	+.11
Jönköping.	+.16	-.02	-.18	-.18	+.07	-.19	-.42	-.20	+.08	+.08
Kronoberg.	-.12	-.25	-.20	-.23	+.04	-.05	-.08	+.02	+.10	+.09
S. Älvsborg.	+.07	—	+.05	—	+.05	—	-.01	—	+.12	—
Bohus.	-.22	-.21	-.11	-.10	+.15	+.16	+.07	+.04	-.19	-.17
N. Älvsborg.	-.01	-.12	+.02	-.02	-.00	-.16	-.14	+.05	+.12	-.03
Skaraborg.	-.06	-.24	+.01	+.18	-.04	-.31	-.17	+.10	+.03	-.13
Ostergötland.	+.32	+.09	+.02	+.15	+.01	-.26	-.24	+.02	+.14	-.13
Södermanland.	+.47	+.44	+.13	+.22	-.17	-.14	-.15	-.08	+.06	-.12
Stockholm.	+.43	+.39	+.16	+.18	-.36	-.20	-.29	-.25	+.12	+.11
Upsala.	+.40	+.25	+.32	+.29	+.11	+.11	-.19	-.11	-.01	-.04
Västmanland.	+.34	+.30	+.33	+.34	+.24	+.25	-.05	-.03	-.09	-.13
Orebrö.	+.30	+.23	+.15	+.27	-.16	-.19	-.25	-.16	+.03	-.12
Varmland.	-.00	-.10	+.20	+.14	+.15	+.13	-.24	+.11	+.01	-.17
Kopparberg.	—	+.41	—	+.26	—	-.27	—	-.08	—	-.21
Gävleborg.	+.21	+.20	-.16	-.23	+.00	+.05	-.20	+.06	-.18	-.32

Correlation Coefficients.

District.	Year previous to Crop.									
	August.		September.		October.		November.		December.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
Gottland ...	+ '25	+ '24	+ '13	+ '18	+ '15	+ '10	+ '26	+ '27	+ '14	+ '16
Malmöhus ...	+ '32	+ '33	+ '11	+ '17	+ '31	+ '04	+ '11	+ '20	+ '26	+ '41
Kristianstad ...	+ '05	+ '07	+ '25	+ '37	+ '41	+ '18	+ '32	+ '49	+ '03	+ '10
Halland ...	+ '51	+ '41	+ '37	+ '29	+ '46	+ '38	+ '11	+ '18	+ '14	+ '22
Blekinge ...	+ '12	+ '09	+ '07	+ '13	+ '50	+ '39	+ '29	+ '29	+ '12	+ '15
S. Kalmar ...	—	+ '07	—	+ '26	—	+ '01	—	+ '18	—	+ '16
N. " ...	+ '18	+ '14	+ '18	+ '07	+ '31	+ '02	+ '02	+ '10	+ '04	+ '03
Jönköping ...	+ '25	+ '24	+ '06	+ '11	+ '30	+ '31	+ '01	+ '04	+ '15	+ '18
Kronoberg ...	+ '11	+ '12	+ '29	+ '33	+ '12	+ '00	+ '33	+ '35	+ '27	+ '30
S. Alvsborg ...	+ '19	+ '20	+ '11	+ '21	+ '21	+ '06	+ '05	+ '07	+ '04	+ '07
Bohus ...	+ '19	+ '17	+ '13	+ '07	+ '08	+ '24	+ '05	+ '08	+ '24	+ '22
N. Alvsborg ...	+ '02	—	+ '19	—	+ '15	—	+ '11	—	+ '21	—
Skaraborg ...	+ '13	+ '09	+ '23	+ '30	+ '36	+ '27	+ '14	+ '14	+ '21	+ '23

District.	Year of Crop.							
	January.		February.		March.		April.	
	r	ρ	r	ρ	r	ρ	r	ρ
Gottland...	+ '56	+ '50	+ '12	+ '08	+ '12	+ '17	+ '21	+ '24
Malmöhus ...	+ '08	+ '01	+ '15	+ '15	+ '21	+ '01	+ '08	+ '16
Kristianstad ...	+ '07	+ '11	+ '26	+ '21	+ '17	+ '03	+ '14	+ '01
Halland ...	+ '29	+ '40	+ '14	+ '17	+ '29	+ '22	+ '09	+ '05
Blekinge ...	+ '42	+ '38	+ '53	+ '58	+ '48	+ '40	+ '15	+ '12
S. Kalmar ...	—	+ '19	—	+ '20	—	+ '11	—	+ '08
N. " ...	+ '02	+ '14	+ '05	+ '01	+ '25	+ '16	+ '06	+ '20
Jönköping ...	+ '18	+ '15	+ '20	+ '23	+ '33	+ '29	+ '04	+ '24
Kronoberg ...	+ '00	+ '10	+ '21	+ '20	+ '21	+ '13	+ '04	+ '01
S. Alvsborg ...	+ '20	+ '14	+ '23	+ '22	+ '20	+ '08	+ '04	+ '00
Bohus ...	+ '10	+ '05	+ '12	+ '14	+ '16	+ '26	+ '01	+ '07
N. Alvsborg ...	+ '09	—	+ '06	—	+ '06	—	+ '05	—
Skaraborg ...	+ '11	+ '18	+ '04	+ '04	+ '13	+ '05	+ '06	+ '03
Östergötland ...	+ '09	+ '07	+ '01	+ '10	+ '23	+ '06	+ '32	+ '13
Södermanland ...	—	+ '18	—	+ '06	—	+ '02	+ '03	+ '06
Stockholm ...	—	+ '12	—	+ '23	—	+ '24	+ '35	+ '35
Upsala ...	—	+ '12	—	+ '24	—	+ '20	+ '20	+ '26
Västmanland ...	—	+ '06	—	+ '09	—	+ '30	+ '19	+ '16
Orebrö ...	—	+ '17	—	+ '09	—	+ '01	+ '16	+ '13
Värmland ...	+ '04	+ '12	+ '03	+ '07	+ '13	+ '07	+ '07	+ '00
Kopparberg ...	+ '11	+ '17	+ '06	+ '00	+ '34	+ '28	+ '23	+ '23
Gävleborg ...	+ '01	+ '11	+ '17	+ '10	+ '36	+ '34	+ '37	+ '44
Jämtland ...	+ '13	+ '16	+ '20	+ '29	+ '20	+ '30	+ '55	+ '62
Västernorrland ...	—	+ '14	—	+ '19	—	+ '13	—	+ '16
Västerbotten ...	—	+ '23	—	+ '20	—	+ '21	—	+ '64
Norrbottn ...	—	+ '31	—	+ '46	—	+ '37	—	+ '23

Rye and Temperature.

District.	Year previous to Crop.									
	August.		September.		October.		November.		December.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
Östergötland ...	+ '20	+ '19	+ '06	+ '09	+ '41	+ '12	+ '02	+ '02	+ '05	+ '04
Södermanland ...	+ '45	+ '46	+ '27	+ '23	+ '15	+ '00	+ '17	+ '16	—	+ '06
Stockholm ...	+ '42	+ '43	+ '42	+ '29	+ '36	+ '15	+ '11	+ '17	—	+ '34
Upsala ...	+ '63	+ '69	+ '38	+ '26	+ '25	+ '01	+ '11	+ '19	—	+ '25
Västmanland ...	+ '59	+ '57	+ '41	+ '33	+ '20	+ '05	+ '21	+ '25	—	+ '04
Orebrö ...	+ '41	+ '39	+ '11	+ '04	+ '29	+ '11	+ '06	+ '05	—	+ '10
Värmland ...	+ '32	+ '33	+ '13	+ '05	+ '15	+ '06	+ '17	+ '18	+ '07	+ '14
Kopparberg ...	+ '37	+ '46	+ '42	+ '23	+ '29	+ '09	+ '20	+ '28	+ '10	+ '06
Gävleborg ...	+ '45	+ '49	+ '41	+ '37	+ '45	+ '21	+ '15	+ '23	+ '12	+ '04
Jämtland ...	+ '15	+ '27	+ '14	+ '27	+ '19	+ '10	+ '26	+ '27	+ '25	+ '31
Västernorrland ...	—	+ '28	—	+ '32	—	+ '04	—	+ '20	—	+ '34
Västerbotten ...	—	+ '02	—	+ '31	—	+ '04	—	+ '09	—	+ '41
Norrbottn ...	—	+ '17	—	+ '03	—	+ '01	—	+ '39	—	+ '25

District.	Year of Crop.							
	May.		June.		July.		August.	
	r	ρ	r	ρ	r	ρ	r	ρ
Gottland.	+ '26	+ '23	+ '26	+ '30	+ '25	+ '24	+ '24	+ '24
Malmöhus.	+ '04	+ '14	+ '04	+ '12	+ '08	+ '04	+ '07	+ '19
Kristianstad.	+ '12	+ '18	+ '14	+ '05	+ '04	+ '03	+ '19	+ '31
Halland.	+ '01	+ '06	+ '03	+ '12	+ '21	+ '32	+ '16	+ '32
Blekinge.	+ '41	+ '42	+ '25	+ '30	+ '01	+ '03	+ '01	+ '05
S. Kalmar.	—	+ '42	—	+ '02	—	+ '01	—	+ '08
N. " .	+ '16	+ '16	+ '28	+ '39	+ '01	+ '10	+ '19	+ '21
Jönköping.	+ '02	+ '17	+ '15	+ '16	+ '05	+ '03	+ '10	+ '05
Kronoberg.	+ '19	+ '25	+ '18	+ '15	+ '14	+ '12	+ '01	+ '06
S. Alvsborg.	+ '02	+ '00	+ '35	+ '25	+ '30	+ '28	+ '25	+ '16
Bohus.	+ '22	+ '24	+ '05	+ '03	+ '07	+ '05	+ '15	+ '13
N. Alvsborg.	+ '06	—	+ '15	—	+ '11	—	+ '05	—
Skaraborg.	+ '06	+ '05	+ '30	+ '27	+ '03	+ '04	+ '05	+ '15
Östergötland.	+ '13	+ '00	+ '08	+ '31	+ '02	+ '09	+ '18	+ '13
Södermanland.	+ '14	+ '14	+ '31	+ '26	+ '06	+ '04	+ '10	+ '13
Stockholm.	+ '17	+ '27	+ '23	+ '17	+ '01	+ '04	+ '17	+ '16
Upsala.	+ '09	+ '17	+ '24	+ '18	+ '22	+ '24	+ '15	+ '25
Västmanland.	+ '12	+ '15	+ '29	+ '22	+ '13	+ '12	+ '10	+ '15
Orebrö.	+ '06	+ '19	+ '28	+ '31	+ '01	+ '01	+ '02	+ '07
Värmland.	+ '02	+ '00	+ '14	+ '08	+ '22	+ '12	+ '04	+ '01
Kopparberg.	+ '16	+ '22	+ '25	+ '25	+ '31	+ '36	+ '24	+ '35
Gävleborg.	+ '18	+ '36	+ '32	+ '26	+ '13	+ '10	+ '36	+ '35
Jämtland.	+ '52	+ '50	+ '33	+ '40	+ '46	+ '54	+ '52	+ '57
Västernorrland.	—	+ '43	—	+ '20	—	+ '46	—	+ '49
Västerbotten.	—	+ '82	—	+ '66	—	+ '47	—	+ '48
Norrbottn.	—	+ '01	—	+ '08	—	+ '21	—	+ '40

Correlation Coefficients.

Districts.	Year previous to Crop.							
	August.		September.		October.		November.	
	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>
Gottland ...	-.24	-.29	+.01	+.03	-.69	-.62	+.02	-.03
Malmöhus ...	+.26	+.21	-.40	-.29	-.29	+.01	-.31	-.27
Kristianstad ...	+.12	-.09	-.25	-.16	-.25	-.01	+.02	-.07
Halland ...	+.18	+.13	-.21	-.33	-.19	-.16	-.27	-.27
Blekinge ...	+.02	+.12	-.19	+.03	-.34	-.20	-.12	-.10
S. Kalmar ...	—	-.23	—	-.19	—	-.26	—	-.14
N. Kalmar ...	+.07	-.06	+.02	+.28	-.49	-.36	-.19	-.16
Jönköping ...	+.25	+.15	-.07	+.04	-.26	+.08	-.28	-.19
Kronoberg ...	+.13	+.11	-.03	-.01	-.03	+.05	-.09	-.06
S. Älvsborg ...	+.07	-.05	-.00	+.03	+.14	+.10	-.13	-.17
Bohus ...	—	-.10	—	-.09	—	-.18	—	+.04
N. Älvsborg ...	-.35	—	-.17	—	-.17	—	-.14	—
Skaraborg ...	-.18	-.26	+.03	+.10	-.05	-.05	-.24	-.20
Östergötland ...	+.13	-.10	-.40	+.12	-.16	-.08	-.14	-.33
Södermanland ...	+.09	+.02	+.07	+.18	-.12	-.05	-.05	+.04
Stockholm ...	+.34	+.30	+.13	+.29	+.18	+.03	-.21	-.13
Upsala ...	+.35	+.25	+.13	+.32	+.18	+.17	-.21	-.16
Västmanland ...	+.12	+.03	+.06	+.14	+.18	+.17	-.18	-.12
Orebrö ...	+.03	-.11	-.05	+.07	+.26	+.23	-.28	-.23
Värmland ...	+.03	-.12	+.18	-.10	+.02	-.09	-.15	-.04
Kopparberg ...	+.15	+.04	+.10	+.20	+.17	+.05	-.40	-.21
Gävleborg ...	+.25	+.21	-.03	+.02	+.03	+.05	-.27	-.15
Jämtland ...	+.10	+.25	+.18	+.21	+.09	+.21	-.17	-.09
Västernorrland...	—	+.53	—	+.07	—	+.29	—	+.26
Västerbotten ...	—	+.32	—	-.09	—	+.46	—	+.21
Norrbotten ...	—	+.17	—	-.09	—	+.11	—	+.11

—Rye and Rainfall.

Year of Crop.										District
April.		May.		June.		July.		August.		
<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	<i>r</i>	<i>ρ</i>	
+·09	+·06	+·35	+·37	+·30	+·29	-·07	-·11	-·21	-·22	Gottland.
+·12	-·29	+·04	-·07	+·11	-·01	-·46	-·34	+·07	+·00	Malmöhus.
+·18	-·13	+·23	+·14	+·09	-·01	-·01	+·13	+·19	+·03	Kristianstad.
+·23	+·13	-·02	-·03	+·02	-·08	-·04	-·00	+·16	+·01	Halland.
+·28	+·09	+·47	+·43	+·28	+·30	-·26	-·20	+·01	-·05	Blekinge.
—	+·05	—	+·25	—	-·06	—	-·11	—	-·06	S. Kalmar.
+·06	-·09	+·16	+·13	-·12	-·31	-·20	+·03	+·19	-·12	N. Kalmar.
+·04	+·11	-·02	+·01	-·13	-·26	-·22	-·10	-·10	-·05	Jönköping.
-·11	-·23	-·08	-·11	+·09	+·01	-·19	-·09	+·01	-·01	Kronoberg.
+·32	+·16	-·15	-·16	-·29	-·45	-·54	-·49	-·18	-·32	S. Älvsborg.
-·41	-·35	-·15	-·13	-·03	-·01	+·20	+·09	-·20	-·13	Bohus.
-·14	—	-·05	—	-·28	—	+·04	—	-·07	—	N. Älvsborg.
-·14	-·20	+·04	+·10	-·29	-·34	+·03	-·16	-·05	-·09	Skaraborg.
+·32	+·14	-·13	-·09	-·08	-·31	-·39	-·26	+·18	-·08	Ostergötland.
+·26	+·21	-·05	-·01	-·33	-·33	-·29	-·25	+·10	+·07	Södermanland.
+·46	+·48	+·32	+·23	+·02	-·13	-·28	-·10	+·15	+·19	Stockholm.
+·46	+·39	+·32	+·33	+·02	+·02	-·28	-·23	+·15	+·08	Upsala.
+·22	+·16	+·11	+·14	-·09	-·08	-·30	-·24	+·10	+·08	Västmanland.
+·00	-·09	+·17	+·18	-·29	-·34	-·43	-·34	+·02	-·09	Orebro.
-·10	-·14	+·03	-·03	-·34	-·42	-·52	-·43	-·09	-·17	Värmland.
+·16	+·15	+·03	-·01	-·26	-·24	-·61	-·23	-·14	-·23	Kopparberg.
+·16	+·12	-·09	-·11	-·09	-·10	-·27	-·10	-·10	-·16	Gävleborg.
-·13	-·04	+·07	+·13	+·07	+·09	-·07	-·13	-·50	-·50	Jämtland.
—	+·10	—	+·04	—	+·30	—	-·05	—	-·56	Västernorrland.
—	-·27	—	-·03	—	+·22	—	-·02	—	-·44	Västerbotten.
—	+·33	—	-·08	—	+·32	—	-·03	—	-·40	Norrbotten.

Barley and Temperature.

Correlation

Districts.	March.		April.		May.		June.		July.		August.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
Gottland ...	-.07	-.12	-.04	-.02	-.25	-.24	-.73	-.76	-.39	-.48	-.16	-.18
Malmöhus ...	—	+.03	-.03	-.15	-.22	-.43	-.16	-.46	-.24	-.43	-.14	-.41
Kristianstad	+.15	-.10	+.12	-.04	-.08	-.15	-.12	-.30	-.02	-.06	-.01	-.18
Halland ...	—	+.30	+.12	-.02	+.06	+.06	-.08	-.42	-.11	-.34	-.06	-.11
Blekinge ...	+.10	+.30	+.02	+.05	-.04	-.10	-.32	-.38	-.31	-.36	-.19	-.17
S. Kalmar ...	+.21	+.16	+.07	+.03	+.05	+.06	-.36	-.39	-.48	-.50	-.08	-.11
N. Kalmar ...	+.15	—	+.18	—	-.05	—	-.14	—	-.34	—	+.17	—
Jönköping ...	+.30	+.13	+.15	+.05	+.28	+.40	+.14	+.05	-.00	-.02	+.22	+.16
Kronoberg ...	-.02	—	-.02	—	+.05	—	+.07	—	-.52	—	-.21	—
S. Älvsborg	—	+.04	+.11	+.07	+.09	+.07	+.16	+.03	+.34	+.27	+.50	+.42
Bohus ...	-.02	—	-.03	—	-.02	—	-.04	—	-.17	—	-.13	—
N. Älvsborg	+.10	+.05	+.03	+.04	-.11	-.11	-.17	-.21	+.00	-.01	+.15	+.13
Skaraborg ...	—	-.00	+.00	-.11	-.12	-.13	-.10	-.25	-.36	-.46	-.07	-.06
Östergötland	+.19	-.01	+.09	+.01	-.16	-.21	-.07	-.23	-.20	-.31	+.06	+.04
Söderman- land.	—	+.01	-.02	-.05	-.15	-.18	-.26	-.33	-.56	-.65	-.16	-.23
Stockholm ...	—	+.08	+.13	+.11	+.17	+.19	-.01	-.01	-.54	-.54	+.02	+.02
Upsala ...	+.23	—	+.19	—	+.11	—	-.08	—	-.29	—	+.15	—
Västmanland	—	+.30	+.24	+.22	-.02	-.00	+.03	-.02	-.20	-.22	+.01	+.02
Orebrö ...	—	+.16	+.21	+.19	+.04	+.07	+.18	+.17	-.17	-.14	+.12	+.13
Värmland ...	+.29	+.18	+.16	+.07	-.10	+.04	+.13	+.03	+.07	+.08	+.26	+.25
Kopparberg	+.34	+.29	+.32	+.32	+.24	+.28	+.21	+.04	+.32	+.34	+.32	+.38
Gävleborg ...	—	+.30	—	+.42	—	+.42	—	+.02	—	+.05	—	+.06
Jämtland ...	—	+.35	—	+.52	—	+.50	—	+.43	—	+.47	—	+.58
Västernorr- land.	—	+.01	—	-.00	—	+.39	—	+.19	—	+.30	—	+.54
Västerbotten	+.27	—	+.49	—	+.75	—	+.59	—	+.33	—	+.55	—
Norrbottn	+.04	—	+.32	—	+.30	—	+.32	—	+.39	—	+.21	—

Barley and Rainfall.

Coefficients.

April.		May.		June.		July.		August.		Districts.
r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	
+.07	+.06	+.48	+.49	+.50	+.49	+.12	+.08	-.41	-.31	Gottland.
+.38	-.08	+.08	-.09	+.19	+.49	-.48	-.46	+.01	-.15	Malmöhus.
+.23	-.14	+.40	+.33	+.14	+.04	-.00	+.19	+.06	-.20	Kristianstad.
+.47	+.27	-.05	-.10	+.18	-.00	-.08	-.00	+.28	+.23	Halland.
-.27	-.01	-.08	+.05	+.16	+.21	+.02	-.14	+.10	+.22	Blekinge.
+.47	+.40	+.61	+.56	+.16	+.15	-.05	+.00	-.17	-.16	S. Kalmar.
+.14	—	+.37	—	+.02	—	+.17	—	-.13	—	N. Kalmar.
+.16	-.02	-.14	-.11	+.21	-.02	-.11	+.20	+.22	+.11	Jönköping.
-.01	—	+.27	—	+.30	—	+.40	—	+.21	—	Kronoberg.
+.53	+.36	-.11	-.13	+.12	-.04	-.37	-.29	-.14	-.31	S. Älvsborg.
-.05	—	-.09	—	+.35	—	+.28	—	-.09	—	Bohus.
+.25	+.22	+.04	+.03	+.38	+.34	+.01	+.08	-.05	-.01	N. Älvsborg.
+.12	+.03	+.17	+.33	+.36	+.30	+.13	+.41	+.13	-.01	Skaraborg.
+.45	+.35	+.25	+.48	+.11	-.03	-.09	+.21	+.08	-.17	Östergötland.
+.11	+.12	+.21	+.24	+.09	+.12	+.44	+.53	-.04	-.05	Söderman- land.
+.14	+.11	+.17	+.18	+.38	+.42	+.38	+.39	-.09	-.10	Stockholm.
+.11	—	+.20	—	-.44	—	+.19	—	-.08	—	Upsala.
+.11	+.07	+.11	+.13	+.35	+.37	+.30	+.35	-.12	-.15	Västmanland.
+.11	+.03	+.16	+.18	-.06	-.02	+.06	+.18	-.02	-.11	Orebrö.
+.18	+.18	+.22	+.17	+.27	+.27	-.20	+.05	-.20	-.36	Värmland.
+.20	+.18	-.05	-.07	+.21	+.25	-.03	+.09	-.19	-.23	Kopparberg.
—	-.03	—	+.18	—	+.30	—	+.02	—	-.11	Gävleborg.
—	-.07	—	+.11	—	+.15	—	-.14	—	-.43	Jämtland.
—	-.01	—	-.15	—	+.14	—	+.07	—	-.24	Västernorr- land.
-.20	—	-.14	—	+.03	—	-.06	—	-.27	—	Västerbotten.
+.08	—	-.12	—	+.43	—	-.02	—	-.43	—	Norrbottn.

Oats and Temperature.

Correlation

Districts.	March.		April.		May.		June.		July.		August.	
	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	r	ρ
Gottland ...	-.11	-.18	-.04	-.02	-.13	-.12	-.68	-.65	-.42	-.40	-.28	-.30
Malmöhus ...	+.35	+.07	-.01	-.10	-.30	-.46	-.28	-.59	-.29	-.46	-.12	-.31
Kristianstad ...	-.01	-.17	+.03	-.07	-.20	-.24	-.29	-.40	-.10	-.16	—	-.11
Halland ...	—	+.21	+.10	-.01	-.18	-.38	-.23	-.70	-.18	-.38	-.06	-.29
Blekinge ...	+.01	+.12	-.15	-.12	-.19	-.24	-.35	-.37	-.28	-.29	-.20	-.18
S. Kalmar ...	+.25	+.19	+.17	+.13	-.05	-.05	-.26	-.30	-.28	-.28	+.04	+.01
N. Kalmar ...	+.15	+.07	+.24	+.02	-.15	-.06	-.26	-.29	-.25	-.26	+.25	+.29
Jönköping ...	+.21	+.08	+.00	-.12	+.19	+.20	+.00	-.11	-.12	-.13	+.34	+.32
Kronoberg ...	+.29	+.20	+.21	+.19	-.11	-.13	-.10	-.15	-.07	-.11	+.14	+.10
S. Älvsborg ...	—	+.19	+.06	+.00	-.01	-.06	-.08	-.33	+.00	-.16	+.30	+.17
Bohus ...	-.04	—	-.11	—	-.20	—	-.20	—	-.26	—	-.15	—
N. Älvsborg ...	-.03	-.07	-.19	-.24	-.30	-.29	-.52	-.44	-.48	-.51	-.22	-.25
Skaraborg ...	—	-.07	-.07	-.04	-.15	-.17	-.25	-.37	-.34	-.48	-.09	-.08
Östergötland ...	+.13	-.01	+.13	+.08	-.24	-.27	-.09	-.18	-.07	-.08	+.22	+.23
Södermanland ...	—	+.13	+.15	+.09	-.13	-.15	-.25	-.34	-.40	-.45	+.03	-.02
Stockholm ...	—	+.28	+.24	+.23	+.10	+.12	-.04	-.06	-.35	-.35	+.09	+.08
Upsala ...	—	+.39	+.39	+.61	+.22	+.20	-.15	-.09	-.31	-.29	+.18	+.19
Västmanland ...	—	+.34	+.28	+.25	-.03	±.00	-.04	-.16	-.20	-.24	+.00	+.03
Orebrö ...	—	+.06	+.23	+.21	-.07	-.05	-.03	-.07	-.20	-.22	+.10	+.10
Värmland ...	+.18	+.13	+.01	-.02	+.10	-.10	-.02	-.06	-.39	-.42	-.05	-.03
Kopparberg ...	-.43	+.38	+.37	+.39	+.19	+.32	+.07	-.05	+.10	+.11	+.05	+.24
Gävleborg ...	—	+.32	—	+.40	—	+.42	—	+.24	—	-.08	—	+.21
Jämtland ...	—	+.40	+.62	+.67	+.56	+.52	+.39	+.45	+.51	+.56	+.66	+.64
Västernorrland ...	+.07	—	-.05	—	+.33	—	+.20	—	+.26	—	+.51	—
Västerbotten ...	—	+.09	—	+.38	—	+.71	—	+.52	—	+.27	—	+.29

Oats and Rainfall.

Coefficients.

April.		May.		June.		July.		August.		Districts.
r	ρ	r	ρ	r	ρ	r	ρ	r	ρ	
+.08	-.02	+.48	+.51	+.50	+.49	+.27	+.22	-.38	-.38	Gottland.
-.40	-.10	+.18	+.15	+.26	+.24	-.42	-.29	-.04	-.21	Malmöhus.
+.03	-.19	+.39	+.33	+.19	+.13	+.01	+.11	-.06	-.21	Kristianstad.
+.39	+.16	+.07	+.10	+.23	+.09	+.03	-.01	+.18	+.04	Halland.
-.29	-.11	+.11	+.21	+.19	+.24	+.08	-.01	-.06	-.00	Blekinge.
+.26	+.00	+.48	+.42	+.23	+.23	-.12	-.06	-.35	-.39	S. Kalmar
+.19	+.11	+.42	+.43	+.16	+.10	+.04	+.19	-.28	-.44	N. Kalmar.
+.15	+.04	+.01	+.00	+.27	+.14	+.08	+.28	+.18	+.13	Jönköping.
-.07	-.19	+.20	+.19	+.31	+.23	+.00	+.10	+.01	-.01	Kronoberg.
+.38	+.13	+.31	+.42	+.38	+.24	+.08	+.18	+.36	+.25	S. Älvsborg.
-.05	—	+.13	—	+.37	—	+.38	—	-.03	—	Bohus.
+.15	+.13	+.21	+.20	+.56	+.54	+.40	+.48	+.14	+.10	N. Älvsborg.
+.06	-.02	+.22	+.33	+.55	+.52	+.24	+.45	+.14	+.04	Skaraborg.
+.17	+.00	+.30	+.42	+.17	+.08	+.04	+.25	-.03	-.21	Östergötland.
+.19	+.15	+.36	+.44	+.17	+.23	+.30	+.38	-.28	-.34	Södermanland.
+.24	+.16	+.24	+.25	+.33	+.37	+.31	+.34	-.21	-.22	Stockholm.
+.07	+.19	+.23	+.41	+.35	+.41	+.33	+.31	-.09	-.04	Upsala.
+.27	+.19	+.29	+.35	+.43	+.50	+.19	+.31	-.06	-.12	Västmanland.
+.05	-.06	+.18	+.20	+.24	+.25	+.18	+.36	-.00	-.12	Orebrö.
-.03	-.05	+.01	-.00	+.46	+.46	+.49	+.60	+.01	+.22	Värmland.
+.06	+.01	-.06	-.09	+.49	+.58	+.16	+.37	+.06	-.12	Kopparberg.
—	+.09	—	+.03	—	+.33	—	+.17	—	-.14	Gävleborg.
-.16	-.24	+.14	+.08	+.08	+.09	-.23	-.16	-.46	-.49	Jämtland.
-.19	—	-.21	—	+.01	—	+.07	—	-.22	—	Västernorrland.
—	-.27	—	-.24	—	-.03	—	-.00	—	-.03	Västerbotten.

Total and partial correlation coefficients.

District.	Variables.			Total.			Partial.	
	Crop. Suffix 1.	Temperature. Suffix 2.	Suffix 3.	r_{12}	r_{13}	r_{23}	r_{12}^*	r_{13}^*
Stockholm ..	Barley ..	July ..	Rainfall, June and July	-.54	.63	-.56	-.29	.47
S. Kalmar ..	Wheat ..	Jan., Feb. and March.	" May ..	.56	.59	.53	.36	.42
" ..	Barley ..	June and July	" April and May	-.56	.60	-.27	-.51	.56
S. Alvsborg ..	Rye ..	Feb. ..	" June and July	.22	-.65	-.08	.22	-.65
N. Alvsborg ..	Oats ..	June and July	" May, June and July.	-.59	.69	-.57	-.33	.54
Skaraborg ..	Barley ..	July ..	" " "	-.46	.57	-.47	-.26	.45
" ..	Oats ..	" ..	" " "	-.48	.69	-.47	-.23	.60
Södermanland ..	Barley ..	June and July	" July ..	-.65	.53	-.34	-.59	.43
Upsala ..	Oats ..	April ..	" May and June	.61	.46	.17	.61	.45
Västmanland ..	" ..	Mar. and April	" " "	.55	.60	.19	.55	.60
Värmland ..	" ..	July ..	" June and July	-.42	.65	-.35	-.27	.60
Kopparberg ..	" ..	March, April and May.	" June ..	.50	.58	.40	.36	.48
Upsala ..	Wheat ..	August ..	Temp., February ..	-.59	.52	-.18	-.60	.52
Jämtland ..	Oats ..	April ..	" July and August	.67	.71	.48	.54	.59

Suffix.	Variables.	Total.	First Order.	Second Order.
District—Upsala.				
1	Rye Crop ...	$r_{12} = -.69$	$3r_{12} = -.68$	$34r_{12} = -.63$
2	Temp. August ...	$r_{13} = +.32$	$2r_{13} = +.27$	$24r_{13} = .27$
3	Rainfall September ...	$r_{14} = +.44$	$4r_{12} = -.64$	$23r_{14} = .30$
4	" April and May ...	$r_{23} = -.18$	$2r_{14} = .31$	
		$r_{24} = -.33$	$4r_{13} = .30$	
		$r_{34} = +.11$	$3r_{14} = .43$	
			$4r_{23} = -.15$	
			$3r_{24} = -.32$	
			$2r_{34} = .05$	
District—Blekinge.				
1	Wheat Crop...	$r_{12} = .58$	$3r_{12} = .60$	$34r_{12} = .38$
2	Temp. October ...	$r_{13} = -.57$	$4r_{12} = .39$	$24r_{13} = -.63$
3	" November ...	$r_{14} = .52$	$2r_{13} = -.59$	$14r_{23} = .37$
4	Rainfall May ...	$r_{23} = -.17$	$4r_{13} = -.64$	
		$r_{24} = .60$	$2r_{14} = .27$	
		$r_{34} = .06$	$3r_{14} = .60$	
District—Halland.				
1	Oats Crop ...	$r_{12} = .21$	$3r_{12} = .41$	$34r_{12} = .40$
2	Temp. March ...	$r_{13} = -.70$	$4r_{12} = .19$	$24r_{13} = -.75$
3	" June ...	$r_{14} = .22$	$2r_{13} = -.75$	$23r_{14} = .20$
4	Rainfall April and May ...	$r_{23} = .11$	$4r_{13} = -.71$	
		$r_{24} = .11$	$2r_{14} = .20$	
		$r_{34} = -.08$	$3r_{14} = .23$	
District—S. Kalmar.				
1	Rye Crop ...	$r_{12} = .41$	$3r_{12} = .35$	$34r_{12} = .22$
2	Temp. October ...	$r_{13} = .59$	$4r_{12} = .23$	$24r_{13} = .52$
3	" Jan., Feb. and March	$r_{14} = .49$	$2r_{13} = .56$	$23r_{14} = .28$
4	Rainfall May and June ...	$r_{23} = .22$	$4r_{13} = .53$	
		$r_{24} = .49$	$2r_{14} = .36$	
		$r_{34} = .31$	$3r_{14} = .40$	
District—Södermanland.				
1	Wheat Crop...	$r_{12} = -.48$	$3r_{12} = -.47$	$34r_{12} = -.42$
2	Temp. August ...	$r_{13} = .41$	$4r_{12} = -.39$	$24r_{13} = .30$
3	" February ...	$r_{14} = .44$	$2r_{13} = .41$	$23r_{14} = .20$
4	Rainfall April ...	$r_{23} = -.13$	$4r_{13} = .27$	
		$r_{24} = -.32$	$2r_{14} = .34$	
		$r_{34} = .45$	$3r_{14} = .31$	

Suffix.	Variables.	Total.	First Order.	Second Order.
District—Stockholm.				
1	Wheat Crop...	$r_{12} = -.49$	$3r_{12} = -.36$	$34r_{12} = -.22$
2	Temp. August ...	$r_{13} = .48$	$4r_{12} = -.34$	$24r_{13} = .34$
3	" April ...	$r_{14} = .47$	$2r_{13} = .34$	$23r_{14} = .30$
4	Rainfall Aug. and Sept. ...	$r_{23} = -.44$	$4r_{13} = .42$	
		$r_{24} = -.49$	$2r_{14} = .31$	
		$r_{34} = .26$	$3r_{14} = .41$	
District—Västmanland.				
1	Wheat Crop...	$r_{12} = -.52$	$3r_{12} = -.48$	$34r_{12} = -.49$
2	Temp. August ...	$r_{13} = .45$	$4r_{12} = -.52$	$24r_{13} = .22$
3	" Feb., March and April	$r_{14} = .48$	$2r_{13} = -.39$	$23r_{14} = .35$
4	Rainfall April, May and June ...	$r_{23} = -.24$	$4r_{13} = .29$	
		$r_{24} = -.15$	$2r_{14} = .47$	
		$r_{34} = .48$	$3r_{14} = .33$	
District—Västerbotten.				
1	Rye Crop ...	$r_{12} = .41$	$3r_{12} = .26$	$34r_{12} = .33$
2	Temp. December ...	$r_{13} = .86$	$4r_{12} = .46$	$24r_{13} = .78$
3	" April, May and June	$r_{14} = .59$	$2r_{13} = .85$	$23r_{14} = .44$
4	" July and August ...	$r_{23} = .33$	$4r_{13} = .81$	
		$r_{24} = .07$	$2r_{14} = .62$	
		$r_{34} = .50$	$3r_{14} = .36$	

Suffix.	Variables.	Total.	First Order.	Second Order.	Third Order.
District—N. Kalmar.					
1	Wheat Crop ...	$r_{12} = .57$	$3r_{12} = .56$	$34r_{12} = .57$	$345r_{12} = .58$
2	Temp. Jan. to April ...	$r_{13} = .36$	$4r_{12} = .57$	$35r_{12} = .53$	$245r_{13} = .48$
3	Rainfall September ...	$r_{14} = -.37$	$5r_{12} = .49$	$45r_{12} = .52$	$235r_{14} = -.49$
4	" Oct. and Nov.	$r_{15} = .32$	$2r_{13} = .35$	$24r_{13} = .44$	$234r_{15} = -.23$
5	" May ...	$r_{23} = .13$	$4r_{13} = .44$	$25r_{13} = .36$	
		$r_{24} = -.10$	$5r_{13} = .30$	$45r_{13} = .39$	
		$r_{25} = .56$	$2r_{14} = -.38$	$23r_{14} = -.47$	
		$r_{34} = .13$	$3r_{14} = -.45$	$25r_{14} = -.38$	
		$r_{35} = .27$	$5r_{14} = -.33$	$35r_{14} = -.42$	
		$r_{45} = -.20$	$2r_{15} = .00$	$23r_{15} = .-$	
			$3r_{15} = .24$	$24r_{15} = -.07$	
			$4r_{15} = .27$	$34r_{15} = .15$	
District—Malmöhus.					
1	Wheat Crop ...	$r_{12} = -.33$	$3r_{12} = -.31$	$45r_{12} = -.25$	$345r_{12} = -.20$
2	Temp. August ...	$r_{13} = -.49$	$2r_{13} = -.48$	$25r_{14} = .19$	$245r_{13} = -.52$
3	" Nov. and Dec....	$r_{14} = .30$	$4r_{12} = -.29$	$35r_{12} = -.22$	$235r_{14} = .26$
4	" Jan. and Feb. ...	$r_{15} = -.33$	$2r_{14} = .25$	$25r_{13} = -.50$	$234r_{15} = -.24$
5	Rainfall Oct. and Nov.	$r_{23} = .14$	$5r_{12} = -.26$	$45r_{13} = -.54$	
		$r_{24} = -.19$	$2r_{15} = -.26$	$35r_{14} = .28$	
		$r_{25} = .30$	$4r_{13} = -.53$	$34r_{12} = -.26$	
		$r_{34} = .06$	$3r_{14} = .37$	$24r_{13} = -.51$	
		$r_{35} = .00$	$5r_{13} = -.51$	$24r_{15} = -.20$	
		$r_{45} = -.35$	$3r_{15} = -.38$	$34r_{15} = -.29$	
			$5r_{14} = -.20$	$23r_{14} = .33$	
			$4r_{15} = -.26$	$23r_{15} = -.31$	
			$4r_{23} = .15$		
			$3r_{24} = -.20$		
			$2r_{25} = .09$		
			$5r_{23} = .14$		
			$3r_{25} = .30$		
			$2r_{35} = -.04$		
			$5r_{24} = -.10$		
			$4r_{25} = .25$		
			$2r_{45} = -.31$		
			$5r_{34} = .06$		
			$4r_{35} = .02$		
			$3r_{45} = -.35$		

Correlation Coefficient.	District.	Variables correlated.
-.49	Stockholm ...	Temp. Aug. and Rainfall Aug. + Sept.
-.30	Malmöhus ...	" " " Oct. + Nov.
-.32	Södermanland ...	" " " April.
-.33	Upsala ...	" " " April + May.
-.44	Stockholm ...	" " " Temp. April.
-.35	S. Kalmar ...	Temp. Sept. and Rainfall April + May.
-.27	N. " ...	Rainfall Sept. and Rainfall May.
-.35	Malmöhus ...	" Sept. + Oct. + Nov. and Temp. Jan., Feb., March.
-.60	Blekinge ...	Temp. Oct. and Rainfall May.
-.22	S. Kalmar ...	" " Temp. Jan., Feb., March.
-.47	" ...	" " Rainfall April + May.
-.51	" ...	" " " April + May + June.
-.49	" ...	" " " May + June.
-.39	Gottland ...	Rainfall Oct. and Temp. Jan.
-.45	S. Kalmar ...	" " Rainfall April + May.
-.35	Malmöhus ...	" Oct. + Nov. and Temp. Jan., Feb.
-.33	Västerviken ...	Temp. Dec. and Temp. April, May, June.
-.55	Gottland ...	" Jan. and Rainfall April + May + June.
-.33	S. Kalmar ...	" Jan., Feb., March and Rainfall April + May + June.
-.53	" ...	Temp. Jan., Feb., March and Rainfall May.
-.31	" ...	" " " May + June.
-.56	N. " ...	Temp. Jan., Feb., March, April and Rainfall May.
-.45	Södermanland ...	" Feb. and Rainfall April.
-.39	Malmöhus ...	Minimum winter temp. and Rainfall April.
-.62	S. Kalmar ...	" " " " April + May.
-.29	Malmöhus ...	" " " " July.
-.48	Västmanland ...	Temp. Feb., March, April and Rainfall April, May, June.
-.40	Kopparberg ...	Temp. March, April, May and Rainfall June.
-.48	Jämtland ...	" April and Temp. July, Aug.
-.50	Västerviken ...	" April, May, June and Temp. July, Aug.
-.52	Gottland ...	Rainfall April + May and Temp. June.
-.27	S. Kalmar ...	" " " " June, July.
-.57	N. Älvsborg ...	" May + June " " July."
-.47	Skaraborg ...	" " " " July."
-.56	Stockholm ...	" June + July " "

Correlation Coefficients	Number of Observations.	Variables Correlated.
.21 ± .13	25	Mean temp. July-Sept. at the stations Gjesvaer and Vardo on the Arctic Coast and mean temp. March-May following at Grimsey and Berufjord.
.74	40 (1874-1913)	Temp. of the sea Norwegian Atlantic coast in February and the date of commencement of agricultural work in spring in the district of Upsala.

Correlation between sea temperature Norwegian Atlantic Coast and air temperature at Orebro.

40 years data 1874-1913.

Temperature of the sea in	Air temperature at Orebro.											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January70	.57	.45	.23	-.06							
February ...	—	.82	.52	.38	.00	-.19						
March ...	—	—	.77	.59	.13	-.20	.16					
April ...	—	—	—	.65	.00	-.28	.10	.06				
May ...	—	—	—	—	.42	-.16	.07	.12				
June ...	—	—	—	—	—	.52	.05	.29	-.00	.04		
July ...	—	—	—	—	—	—	.47	.25	-.10	-.09		
August ...	—	—	—	—	—	—	—	.50	-.25	-.14	-.30	
September ...	—	—	—	—	—	—	—	—	.62	-.14	-.28	-.29
October ...	—	—	—	—	—	—	—	—	—	.50	-.25	-.45
November ...	—	—	—	—	—	—	—	—	—	—	.37	-.09
December ...	—	—	—	—	—	—	—	—	—	—	—	.68
Air temp. at Orebro ...												
Jan. ...	1.00	.52	.36	.28								
Feb. ...	—	1.00	.65	.49	.17							

Reference 46.

T. A. BLAIR Partial Correlation Applied to Dakota Data on Weather and Wheat Yield. Month. Weather Rev., Wash., Feb., 1918.

Variables.	Denoted by
Rainfall for May and June	Suffix <i>p</i>
Mean temperature for June	" <i>t</i>
Yield of Corn	" <i>y</i>

Data for 26 Years 1892-1917.

Total Correlation Coefficients.				Partial Correlation Coefficients		
	r_{yy}	r_{ty}	r_{pt}	r'_{yy}	$y'r_{ty}$	$y'r_{pt}$
North Dakota61	-.45	-.38	North Dakota53	-.30
South "49	-.62	-.56	South "22	-.48

Now consider the case of four variables x, y, u, v . It is required to find the correlation between x and y when the effects of both u and v have been eliminated as far as possible. We can construct an equation of the form $x = pu + qv$ which expresses the best relationship between x, u and v . Take the series $x_k - pu_k - qv_k$ from $k=1$ to $k=n$. This is a series in which the effect of both u and v on x is eliminated as far as possible by the correlations between x and u and v . Substituting the known values of p and q we get—

$$x_k - \frac{r_{ux} - r_{uv} r_{vx}}{1 - r_{uv}^2} \frac{\sigma_x}{\sigma_u} u_k - \frac{r_{vx} - r_{uv} r_{ux}}{1 - r_{uv}^2} \frac{\sigma_x}{\sigma_v} v_k$$

We can write this as

$$\sigma_x \left[\frac{x_k}{\sigma_x} - \frac{r_{ux} - r_{uv} r_{vx}}{1 - r_{uv}^2} \frac{u_k}{\sigma_u} - \frac{r_{vx} - r_{uv} r_{ux}}{1 - r_{uv}^2} \frac{v_k}{\sigma_v} \right] \quad \text{(iv)}$$

Similarly the series in which the effect of both u and v on y is eliminated as far as possible by the correlations between y and both u and v can be written :—

$$\sigma_y \left[\frac{y_k}{\sigma_y} - \frac{r_{uy} - r_{uv} r_{vy}}{1 - r_{uv}^2} \frac{u_k}{\sigma_u} - \frac{r_{vy} - r_{uv} r_{uy}}{1 - r_{uv}^2} \frac{v_k}{\sigma_v} \right] \quad \text{(v)}$$

If we correlate the two series represented by (iv) and (v) we shall be obtaining the correlation between x and y with the effect of u and v on each of them eliminated as far as possible. That is we shall be obtaining an expression for ${}_{uv}r_{xy}$.

To get the standard deviation ${}_{uv}\sigma_x$ from (iv) we square and add all terms from $k=1$ to $k=n$.

We obtain

$$\begin{aligned} {}_{uv}\sigma_x^2 &= \sigma_x^2 \left[1 + \frac{(r_{ux} - r_{uv} r_{vx})^2}{(1 - r_{uv}^2)^2} - \frac{2(r_{ux} - r_{uv} r_{vx})r_{ux}}{1 - r_{uv}^2} \right. \\ &\quad + \frac{(r_{vx} - r_{uv} r_{ux})^2}{(1 - r_{uv}^2)^2} - \frac{2(r_{vx} - r_{uv} r_{ux})r_{vx}}{1 - r_{uv}^2} \\ &\quad \left. + \frac{2(r_{ux} - r_{uv} r_{vx})(r_{vx} - r_{uv} r_{ux})r_{uv}}{(1 - r_{uv}^2)^2} \right] \\ &= \sigma_x^2 \left[1 + \frac{(r_{ux} - r_{uv} r_{vx})(r_{ux} - r_{uv} r_{vx} + r_{uv} r_{vx} - r_{ux} r_{uv}^2)}{(1 - r_{uv}^2)^2} \right. \\ &\quad + \frac{(r_{vx} - r_{uv} r_{ux})(r_{vx} - r_{uv} r_{ux} + r_{uv} r_{ux} - r_{vx} r_{uv}^2)}{(1 - r_{uv}^2)^2} \\ &\quad \left. - \frac{2(r_{ux}^2 - 2r_{uv} r_{ux} r_{vx} + r_{vx}^2)}{1 - r_{uv}^2} \right] \end{aligned}$$

$$\begin{aligned} &= \sigma_x^2 \left[1 + \frac{(r_{ux} - r_{uv} r_{vx})r_{ux} + (r_{vx} - r_{uv} r_{ux})r_{vx}}{1 - r_{uv}^2} \right. \\ &\quad \left. - \frac{2(r_{ux}^2 - 2r_{uv} r_{ux} r_{vx} + r_{vx}^2)}{1 - r_{uv}^2} \right] \\ &= \sigma_x^2 \left[1 + \frac{r_{ux}^2 - 2r_{uv} r_{ux} r_{vx} + r_{vx}^2 - 2r_{ux}^2 + 4r_{uv} r_{ux} r_{vx} - 2r_{vx}^2}{1 - r_{uv}^2} \right] \\ &= \sigma_x^2 \left[1 - \frac{r_{ux}^2 + r_{vx}^2 - 2r_{uv} r_{ux} r_{vx}}{1 - r_{uv}^2} \right] \\ &= \sigma_x^2 (1 - r_{vx}^2) \left[\frac{1 - r_{uv}^2 - r_{vx}^2 + r_{uv}^2 r_{vx}^2 - r_{vx}^2 r_{vx}^2 + 2r_{uv} r_{ux} r_{vx} - r_{ux}^2}{(1 - r_{uv}^2)(1 - r_{vx}^2)} \right] \\ &= \sigma_x^2 (1 - r_{vx}^2) \left(1 - \frac{r_{ux}^2 - 2r_{uv} r_{ux} r_{vx} + r_{uv}^2 r_{vx}^2}{(1 - r_{uv}^2)(1 - r_{vx}^2)} \right) \\ &= \sigma_x^2 (1 - r_{vx}^2) (1 - r_{ux}^2) \quad \text{(vi)} \end{aligned}$$

Similarly we have

$${}_{uv}\sigma_y^2 = \sigma_y^2 (1 - r_{vy}^2) (1 - r_{uy}^2) \quad \text{(vii)}$$

We shall require the product of all such pairs of terms as (iv) and (v) from $k=1$ to $k=n$.

This will be

$$\begin{aligned} n\sigma_x\sigma_y &\left[r_{xy} - \frac{r_{ux} - r_{uv} r_{vx}}{1 - r_{uv}^2} r_{uy} + \frac{(r_{ux} - r_{uv} r_{vx})(r_{uy} - r_{uv} r_{vy})}{(1 - r_{uv}^2)^2} \right. \\ &\quad - \frac{r_{vx} - r_{uv} r_{ux}}{1 - r_{uv}^2} r_{vy} + \frac{(r_{uy} - r_{uv} r_{vy})(r_{vx} - r_{uv} r_{ux})r_{uv}}{(1 - r_{uv}^2)^2} \\ &\quad - \frac{r_{uy} - r_{uv} r_{vy}}{1 - r_{uv}^2} r_{ux} + \frac{(r_{ux} - r_{uv} r_{vx})(r_{vy} - r_{uv} r_{uy})r_{uv}}{(1 - r_{uv}^2)^2} \\ &\quad \left. - \frac{r_{vy} - r_{uv} r_{uy}}{1 - r_{uv}^2} r_{vx} + \frac{(r_{vx} - r_{uv} r_{ux})(r_{vy} - r_{uv} r_{uy})}{(1 - r_{uv}^2)^2} \right] \\ &= n\sigma_x\sigma_y \left[r_{xy} - \frac{2(r_{ux} r_{uy} + r_{vx} r_{vy} - r_{uv} r_{uy} r_{vx} - r_{uv} r_{ux} r_{vy})}{1 - r_{uv}^2} \right. \\ &\quad + \frac{(r_{ux} - r_{uv} r_{vx})(r_{uy} - r_{uv} r_{vy} + r_{uv} r_{vy} - r_{uv}^2 r_{uy})}{(1 - r_{uv}^2)^2} \\ &\quad \left. + \frac{(r_{vx} - r_{uv} r_{ux})(r_{vy} - r_{uv} r_{uy} + r_{uv} r_{uy} - r_{uv}^2 r_{vy})}{(1 - r_{uv}^2)^2} \right] \end{aligned}$$

$$\begin{aligned}
&= n\sigma_x\sigma_y \left[r_{xy} - \frac{2(r_{ux}r_{uy} + r_{vx}r_{vy} - r_{uv}r_{uy}r_{vx} - r_{uv}r_{ux}r_{vy})}{1 - r_{uv}^2} \right. \\
&\quad \left. + \frac{r_{ux}r_{uy} - r_{uv}r_{vy}r_{uy} + r_{vx}r_{vy} - r_{uv}r_{ux}r_{vy}}{1 - r_{uv}^2} \right] \\
&= n\sigma_x\sigma_y \left[r_{xy} - \frac{r_{vx}r_{vy}(1 - r_{uv}^2) + r_{ux}r_{uy}}{1 - r_{uv}^2} \right. \\
&\quad \left. - \frac{r_{uv}r_{uy}r_{vx} - r_{uv}r_{ux}r_{vy} + r_{vx}r_{vy}r_{uv}^2}{1 - r_{uv}^2} \right] \\
&= n\sigma_x\sigma_y \left[r_{xy} - r_{vx}r_{vy} - \frac{(r_{ux} - r_{vx}r_{uv})(r_{uy} - r_{vy}r_{uv})}{1 - r_{uv}^2} \right] \\
&= n\sigma_x\sigma_y \sqrt{(1 - r_{vx}^2)(1 - r_{vy}^2)} \left[\sqrt{(1 - r_{vx}^2)(1 - r_{vy}^2)} \right. \\
&\quad \left. - \frac{r_{ux} - r_{vx}r_{uv}}{\sqrt{(1 - r_{vx}^2)(1 - r_{uv}^2)}} \cdot \frac{r_{uy} - r_{vy}r_{uv}}{\sqrt{(1 - r_{vy}^2)(1 - r_{uv}^2)}} \right] \\
&= n\sigma_x\sigma_y \sqrt{(1 - r_{vx}^2)(1 - r_{vy}^2)} (r_{xy} - r_{vx}r_{vy}r_{uv})
\end{aligned}$$

We can now write down the correlation between the two series represented by (iv) and (v).

We have

$$\begin{aligned}
{}_{uv}r_{xy} &= \frac{n\sigma_x\sigma_y \sqrt{(1 - r_{vx}^2)(1 - r_{vy}^2)} (r_{xy} - r_{vx}r_{vy}r_{uv})}{n\sigma_x\sigma_y \sqrt{(1 - r_{vx}^2)(1 - r_{vy}^2)} \sqrt{(1 - r_{ux}^2)(1 - r_{uy}^2)}} \\
&\quad \text{using the standard deviations in (vi) and (vii).}
\end{aligned}$$

$$= \frac{r_{xy} - r_{vx}r_{vy}r_{uv}}{\sqrt{(1 - r_{ux}^2)(1 - r_{uy}^2)}}$$

which is the usual formula for ${}_{uv}r_{xy}$.

