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A conceptual model of a cold front based on vorticity "conservation"

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1 Introduction

Cold fronts in the vicinity of the British Isles are often observed to be, for a range of spatial scales, quasi two-dimensional (although the pressure field varies in the along-front direction) and, for a range of time scales, quasi steady-state (in a reference frame moving with the front). If one can assume exact two-dimensionality and time-invariance, or can parametrize the effects of the violations of those two assumptions, then it is possible to construct mathematical (as opposed to numerical) models of fronts which can be used to explain some of the observed characteristics on the relevant scales. Specifically, one can use the equations representing "conservation" of vorticity and momentum to infer constraints on the flow regimes that are possible, and this was the motivation behind the present work. Although it is recognised that three-dimensionality, time evolution and other factors which the model does not include may be significant, the roles of all these factors will only be understood if simple models are constructed in the context of which more complex behaviour can be examined.

2 Theory

2.1 The relevant component of vorticity

The general form of the vorticity equation is

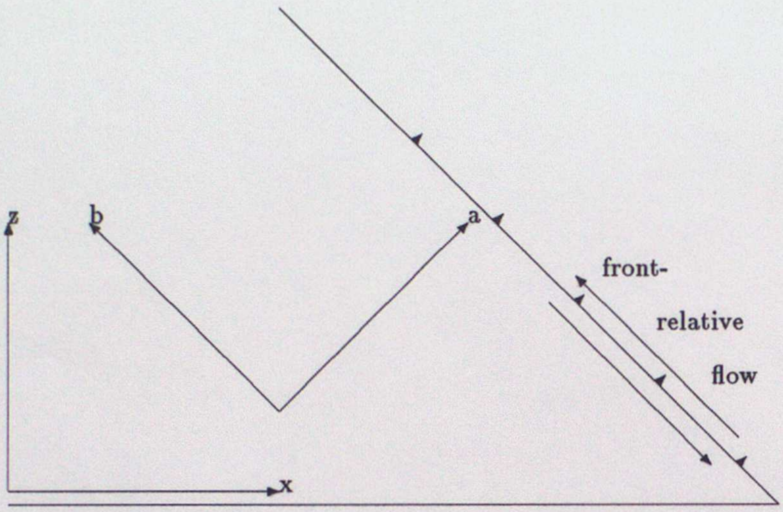
$$\partial\varsigma/\partial t + (\mathbf{V}.\nabla)(\varsigma + 2\mathbf{\Omega}) - ((\varsigma + 2\mathbf{\Omega}).\nabla)\mathbf{V} + (\varsigma + 2\mathbf{\Omega})(\nabla.\mathbf{V}) = -c_p(\nabla\theta \wedge \nabla\Pi) + (\nabla \wedge \mathbf{F}) \qquad (1)$$

Here ς is the vorticity *vector*, $\mathbf{\Omega}$ is the planetary rotation vector, Π is the Exner function, $(= (p/p_{ref})^{\kappa})$, $\mathbf{F}$ is momentum dissipation per unit mass and other symbols have their usual meaning.

In the following, we imagine a front to be a flat inclined plane parallel to the y-axis moving horizontally at a constant speed u_{front}. In the frame moving with the front, the flow immediately on either side of the front has zero component in the direction normal to the frontal plane: this is to be regarded as the definition of the front (rather than a boundary between air having distinct values of a quasi-conserved property).

It is useful to consider the "conservation" of a particular component of vorticity in the x-z plane for which a transformation of axes is desirable. Figure 1 illustrates the transformed set.

Figure 1. Coordinate axes in the x-z plane



It will be shown that air parcels approximately conserve the sign of ς_a, which is the component of *absolute* vorticity perpendicular to the frontal surface. For this component equation (1) reduces to:

$$D\varsigma_a/Dt - (\varsigma_a \partial u_a/\partial a + \varsigma_b \partial u_a/\partial b + \varsigma_y \partial u_a/\partial y) + \varsigma_a(\nabla.\mathbf{V})$$
$$= -c_p(\partial\theta/\partial y\ \partial\Pi/\partial b - \partial\theta/\partial b\ \partial\Pi/\partial y) + (\partial F_b/\partial y - \partial F_y/\partial b) \qquad (2)$$

If we assume that $\partial/\partial y = 0$ for all variables except Π, then equation (2) becomes

$$D\varsigma_a/Dt - \varsigma_a \partial u_a/\partial a - \varsigma_b \partial u_a/\partial b - \varsigma_a(\nabla.\mathbf{V}) = c_p \partial\theta/\partial b\ \partial\Pi/\partial y + \partial F_y/\partial b \qquad (3)$$

(i) (ii) (iii) (iv) (v) (vi)

Note that although we have rotated the coordinate axes we have not applied a Gallilean transformation.

In equation (3) terms (ii) and (iv) clearly cannot change the sign of ς_a. Friction with the ground, included in term (vi), will in general have the effect of changing the magnitude of ς_a rather than its sign. Similarly internal dissipation is unlikely to change the sign of ς_a.

Under the "flat-plate" description of the frontal surface, u_a is zero everywhere along the front so $\partial u_a/\partial b$ is zero at the front. More generally, it can be asserted that a twisting-tilting term, (iii) ,acts to reorientate the vorticity vector so that the angle it makes with the flow is unchanged and thus it cannot change the sign of the component of vorticity which is perpendicular to the flow. For a curved frontal surface it is reasonable to associate ς_a with this component although equation (3) only holds rigorously if the frontal surface is flat. The evolution of vorticity is discussed by Haynes and McIntyre (1987).

The effect of term (v) on the sign of ς_a is more predictable. Of course, if the front is in a steady state *and* there is no diabatic heating, then $\partial\theta/\partial b = 0$ by definition. In practise, if θ_e is approximately constant along the frontal surface then $\partial\theta/\partial b > 0$ and as $\partial\Pi/\partial y$ is in general -ve term (v) is negative i.e. latent heat release can change +ve ς_a to -ve ς_a. However the rate at which this occurs is in general small.

The significance of the sign of ς_a is that, because for a two-dimensional front $\varsigma_a = -\partial M/\partial b$, it controls the sign of $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ (assuming the front-relative flow $\mathbf{V}_{\mathbf{f.r.}}$ is either as shown in figure 1 or consistently the reverse of that). Here M is the along-front component of absolute momentum ($= v + fx$) (strictly $M = v + fx - 2\Omega_x z$ but given the ratio of horizontal to vertical length scales for mid-latitude fronts the latter term can safely be ignored). To understand the relevance of the sign of $\mathbf{V_{f.r.}}.\nabla M$, we have to consider the absolute momentum budget of the frontal system.

That the sign of ς_a is conserved can be derived from the potential vorticity theorem with the additional assumption that $\partial\theta/\partial p < 0$, provided that $\varsigma_a \approx \varsigma_\theta$, where ς_θ is the vorticity evaluated on a θ surface.

2.2 The along-front absolute momentum budget

The equation governing the along-front absolute momentum can be written as follows:

$$\partial M/\partial t + \mathbf{V}.\nabla M + c_p\theta\partial\Pi/\partial y = F_y \tag{4}$$

However if we are interested in the rate of change of M in a frame moving with the front, we partition $\mathbf{V}$ into $\mathbf{V_{f.r.}} + u_{front}$ and thus the $\mathbf{V}.\nabla M$ term becomes

$$\mathbf{V_{f.r.}}.\nabla M + (f + \partial v/\partial x)u_{front}$$

However we can define

$$\partial_{f.r.}M/\partial t \equiv \partial M/\partial t + u_{front}\partial v/\partial x$$

Here $\partial_{f.r.}/\partial t$ means rate of change in a frame moving with the front. Note that we are not formally applying a Gallilean transformation, merely partitioning the various terms in $\mathbf{V}.\nabla M$. Thus equation (4) becomes

$$\partial_{f.r.}M/\partial t + \mathbf{V_{f.r.}}.\nabla M + (f.u_{front} + c_p\theta\partial\Pi/\partial y) = F_y \tag{5}$$

The significant things about the $(f.u_{front} + c_p\theta\partial\Pi/\partial y)$ term are that

- it essentially is the same on both sides of the frontal surface: the effect of the difference in θ is negligible.
- there is usually approximate balance between the two terms : indeed one might choose to define u_{front} to be such that there was exact balance. This was in fact done in the calculation whose results are shown later.

In general it is appropriate to regard this term as being controlled by large (synoptic) scale processes: however if there was a significant $\partial_{f.r.}M/\partial t$ of the same sign on both sides of the front it is reasonable to suppose that the resulting flow parallel to the front would give rise to a change to $\partial\Pi/\partial y$ which in turn would reduce in magnitude the net $\partial_{f.r.}M/\partial t$. The effect of the F_y term is largely to redistribute momentum across the frontal surface: its contribution to the net $\partial_{f.r.}M/\partial t$ will be small. We therefore argue that for a quasi steady-state front there must be approximate balance between the $\mathbf{V_{f.r.}}.\nabla M$ terms on opposite sides of the front, or at least they must be of opposite signs. Because the sign of ς_a is conserved, the $\mathbf{V_{f.r.}}.\nabla M$ term can be regarded as being characteristic of the air masses on opposite sides of the frontal surface. If the absolute momentum were increasing through advection in the cold air and decreasing in the warm, the vertical component of vorticity would be decreasing and the flow would rapidly become inertially unstable. Thus it is argued that a cold front should be perceived as a boundary between a (warm) air mass having positive front-relative absolute momentum advection and a (cold) air mass having negative front-relative absolute momentum advection.

In the above conceptual model, it will be noted that the marker of the different air masses is only defined with respect to a pre-existing cold front. To some readers, this may seem a relatively unhelpful definition: a marker defined in absolute terms would be more useful for objectively locating a front. This clearly is unrealistic though because an air mass which is "cold" with respect to one front may be "warm" with respect to another. Furthermore a marker of an absolute nature may be useful for giving an indication as to where the air has come from but will not necessarily convey much information as to the subsequent evolution. Admittedly potential vorticity, if observed everywhere, can be used for future prediction, but if just the flow were observed everywhere that would be just as good. The value of the momentum advection parameter is that it clearly implies that the air mass is trying to accelerate in a direction parallel to the front.

2.3 The attainment of thermal wind balance

The acceleration in the air masses on either side of the frontal boundary should be regarded essentially as the process of establishing thermal wind balance. If the atmosphere was in thermal wind balance on the frontal scale then this acceleration would be zero: thus if the acceleration is positive the implication is that the vertical wind shear must be less in magnitude than that which thermally balances the horizontal temperature gradient. The question should be asked as to why this is the case.

2.3.1 Quasi-geostrophic processes

It has been remarked by Hoskins et al (1978) that *in relation to a fluid particle moving with the geostrophic flow, geostrophic motion destroys itself by changing the two parts of the thermal wind balance equally but in opposite directions*. In the context of a front forced by deformation it is straightforward to see this process in action. An imposed negative $\partial u_g/\partial x$ field, although it will intensify a pre-existing $\partial\theta/\partial x$ field, will also weaken a pre-existing $\partial v/\partial z$ field through the action of the stretching term in the ς_x budget.

2.3.2 Action on the required timescales

The ς_x field does not respond directly by the $\partial\theta/\partial x$ field: the direct effect of a positive $\partial\theta/\partial x$ field is to cause a local increase in the ς_y field, (although other terms in the ς_y budget may compensate for the increase caused by $\partial\theta/\partial x$). The ς_y field is usually dominated by the $\partial u/\partial z$ term and as we can write

$$\partial\varsigma_x\partial t = f.\partial u/\partial z + \ldots$$

it is clear that thermal wind balance can only become established on a timescale of f^{-1}. A further factor is as follows. If one imagines a steady state two-dimensional front with no diabatic processes, then the streamlines for flow in the x-z plane must be parallel to the isentropes. In such a scenario, for flow along a particular streamline, the maximum updraft will colocated with the maximum horizontal temperature gradient, the timescales necessary thermal wind balance to become established are unlikely to be achieved.

2.3.3 Effects of diffusion

Diffusion will act on both temperature and wind fields so it is not clear that its net effect will be to reduce $\partial v/\partial z$ from a conceptual "diffusion-free" value more than it will for the $\partial\theta/\partial x$ field. However, because vertical scales in the atmosphere are much smaller than horizontal, and because viscous diffusion effectively acts on ∇^2 of the field in question, the effect on the $\partial v/\partial z$ is the greater: in the final stage of frontal collapse Kelvin-Helmholtz instability will limit vertical windshear (see Roach 1970).

2.3.4 Effects of diabatic processes

Clearly these can affect the temperature field on timescales shorter than f^{-1} and thus the wind field will lag behind, assuming that latent heat release acts to intensify the existing temperature gradient.

3 Observations

3.1 The basic dropsonde data

As a major component of the Mesoscale Frontal Dynamics Project (Clough and Testud, 1988), a large number of dropsondes were deployed in selected cold fronts as they approached the English Channel from the Atlantic. Figure 2 shows low-level winds from the sondes that were dropped during Intensive Observing Period (IOP) 7, on 9/1/1988. The winds are shown principally to indicate the domain and spacing of the measurements. The spacing of the sondes varied between 25km and 100km. All dropsonde parameters (temperature, humidity and wind) were analysed with continuous functions (Pedder, 1989) which were then evaluated on a 12.5 * 12.5 km grid with 31 levels in the vertical evenly distributed with respect to $\ln(p)$ between 400mb and 1000mb. Although spatial gradients can in principle be calculated anywhere within the grid, it was decided to calculate the advection of absolute momentum field for a cross-section along a line midway between the two intensively observed legs, as shown in figure 2.

3.2 Derivation of the absolute momentum advection field

In order to calculate $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ it is necessary to calculate vertical motion field w, as this is not observed directly. Although the front appeared from satellite imagery to be two-dimensional with the dropsonde runs perpendicular to the front, if w was calculated simply by integrating $\partial u/\partial x$ vertically the resulting field was clearly inconsistent with the relative humidity. Examination of the $\partial v/\partial y$ field showed that including it improved the w field. Nevertheless care has to be taken using $\partial/\partial y$ fields because an assumption has to be made about the speed of the front in order that the data from different dropsonde runs can be compared. The speed used in the calculation was that which minimised

$$\int\int(\partial v/\partial y)^2 dx\, dz.$$

Nevertheless, the $\partial v/\partial y$ field is the same order of magnitude as the $\partial u/\partial x$ (as one might expect if the front was forced by geostrophic deformation and the ageostrophic flow is small compared to the geostrophic).

Having calculated w as described above, the calculation of $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ is straightforward, although note that a $\partial v/\partial y$ term, calculated as above, is included. The results are shown in figure 3, and for comparison the θ_e field is also shown. θ_e is highly conservative (traditionally regarded as a good marker of fronts) and is observed more or less directly by the dropsondes. It will be noted that for a considerable portion of the frontal surface the isopleths of θ_e are approximately parallel to those of $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$, confirming the theoretical model described in section 1. The contour interval used for the $-\mathbf{V}_{\mathbf{f.r.}}.\nabla M$field is such that $f.u_{front}$ is 1 unit so that the relative sizes of the terms in equation (5) are apparent. There is an area of -ve $-\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ in the upper left portion of figure 3 which has a θ_e more characteristic of the warm air: satellite imagery and the w field (not shown) suggest that this is an area of ascent associated with the eastward propogation of the upper trough and it is likely that the -ve $-\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ is associated with that. Note that there are areas of +ve advection at low levels which correspond encouragingly to high θ_e air.

Examination of the fields contributing to $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$, (not shown), indicates that the x-component is the smallest, while there is considerable cancellation beween the y- and z- components. This implies that the combination is relatively insensitive to the choice of u_{front} (which experiments bear out) and also that the resultant may suffer unduly from small errors in the y- and z- components. Some of the small scale features of the $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ field are thought to be due to the latter.

A cautionary word should be expressed about the two-dimensionality and the time-invariance of the IOP 7 front . When the maximum horizontal gradient of $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ is compared with the corresponding horizontal gradient of M, the resulting timescale on which the existing M field could have been created by front-relative advection is 1 hour. Although dissipation will increase the timescale, it is clearly inappropriate to assume that $\partial_{f.r.}M/\partial t$ is small. In fact there is some evidence of a wave approaching the intensively observed part of the front at the time of the observations, which is consistent with $\partial_{f.r.}M/\partial t \neq 0$. However the fact that the contours of $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ are reasonably parallel to those of θ_e suggests that the conceptual model is more robust than the requirement for two-dimensionality and time invariance implies.

4 Conclusions

Given the comments in the last section, it seems reasonable to conclude that the conceptual model developed, i.e. that a cold front should be perceived as a boundary between an air mass with +ve $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ and one with -ve $\mathbf{V}_{\mathbf{f.r.}}.\nabla M$ is one that can certainly be used qualitatively, although to interpret the argument used to construct the conceptual model quantitatively would be dangerous. Given that the aquisition of along-front absolute momentum depends on the front-relative motion, it is possible to infer that, for an ana-front with ζ_a having the same sign as the planetary component, as was the case for IOP 7, both air masses will be deflected to the right in relation to their own front-relative motion. However for a kata-front with ς_a having this sign, both air masses will be deflected to the left, but because of the relative movement of the two air masses this will increase the cyclonic circulation at the front.

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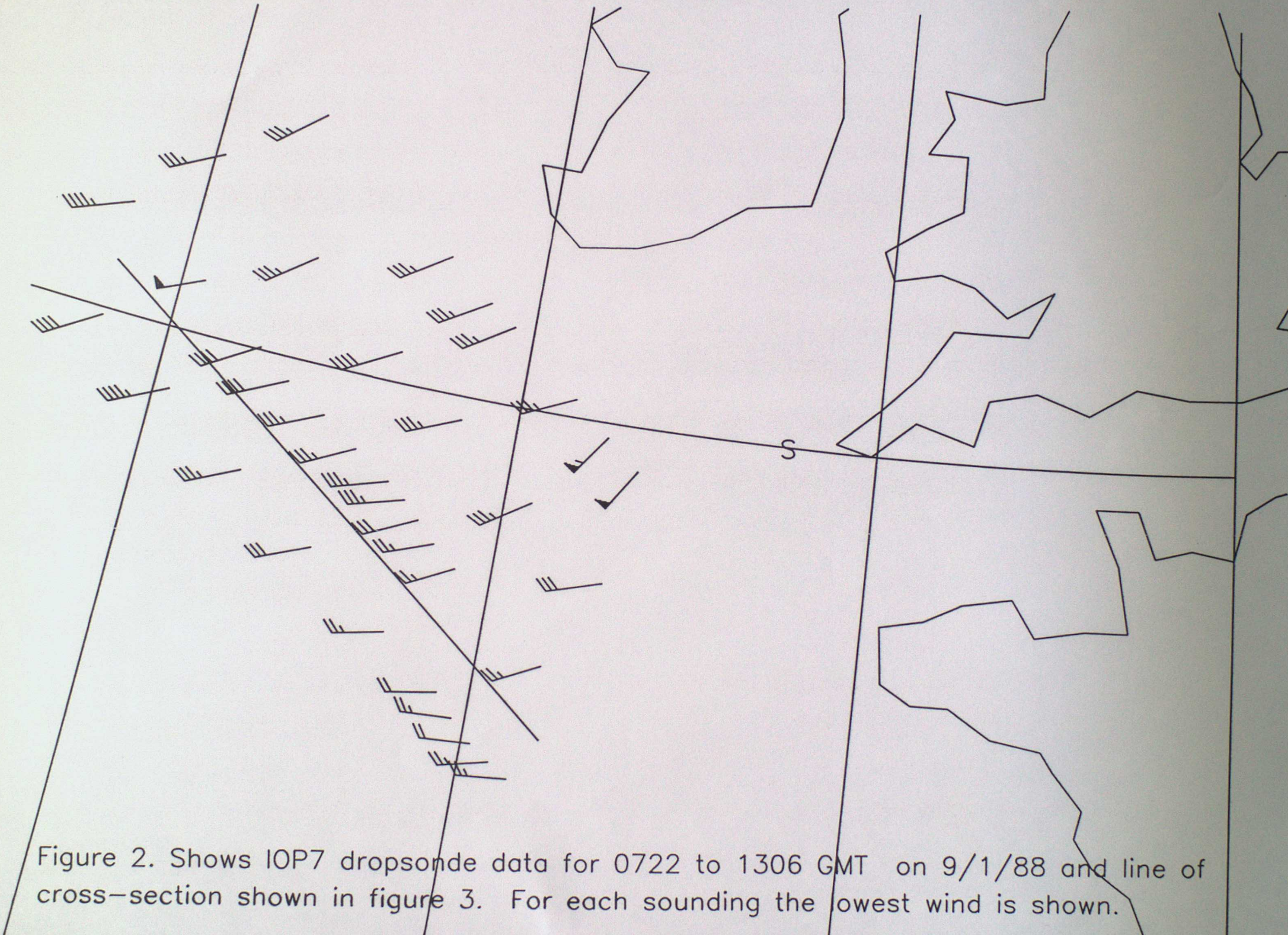

Figure 2. Shows IOP7 dropsonde data for 0722 to 1306 GMT on 9/1/88 and line of cross-section shown in figure 3. For each sounding the lowest wind is shown.

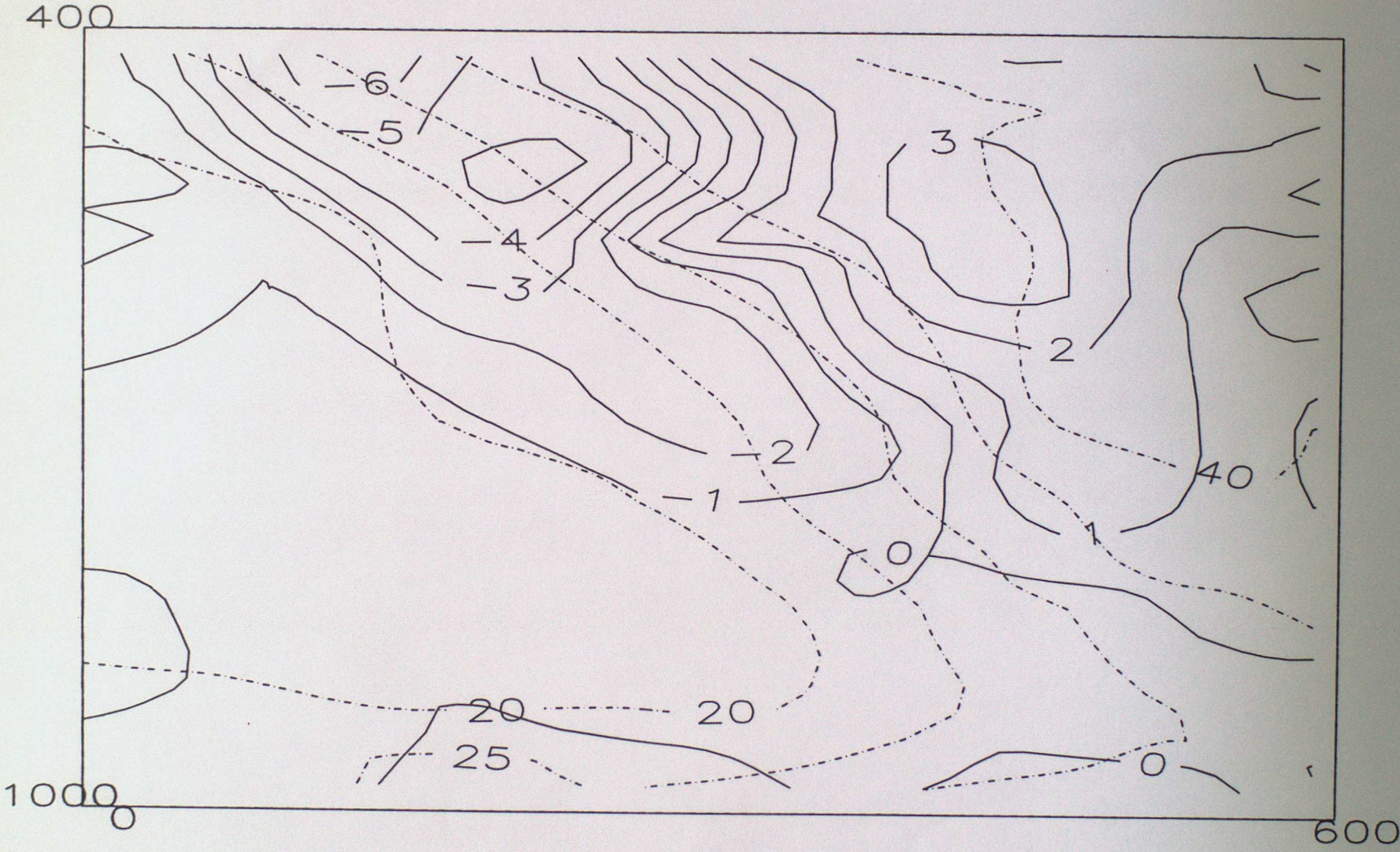

Figure 3. Shows contours of front−relative advection of absolute momentum (continuous)
(One contour interval = f*Ufront) and
ϑe (dot−dashed)(contour interval 5 degrees)