MET O 11 TECHNICAL NOTE NO. 108

SIMULATION EXPERIMENTS TO DETERMINE THE EFFECT OF VARYING THE WEIGHTS OF OBSERVATIONS IN DATA ASSIMILATION

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July 1978

1. Introduction

The simulation experiments described in this technical note are a continuation of the work presented by Lyne (1978) and to avoid unnecessary duplication only the briefest description of the simulation model is given below. Further details are obtainable from the above reference and the references cited therein.

In the simulation experiments, data generated from the Met O 20 11 level model for a simulated FGGE observing system is assimilated into the 5 level model over a ten day period. This is achieved by updating the model over periods of three hours with corrections of observed minus forecast values. These corrections can be calculated with or without optimum interpolation, and are scaled by a factor λ.

A recent paper by Davies and Turner (1977) and the work of Anthes (1974) and Hoke and Anthes (1976 and 1977) suggest that in data assimilation it is not necessary nor perhaps desirable to replace model variables with the observed values, but rather all that is required is to "nudge" the variables in the right direction during the assimilation. In fact their schemes are very similar to that used in the simulation experiments described above, the main difference being their assumption that a complete time history for each observation is available. For a practical assimilation scheme this is unlikely, but in any case it is argued in the next section that this is not necessary.

Essentially the main parameter in all the updating schemes mentioned here is the factor λ , and the purpose of this note is to determine the effect of varying λ within the ranges suggested by these various authors.

2. Comparison of the Updating Methods

The methods of Anthes (1974), Hoke and Anthes (1976 and 1977) and the Newtonian relaxation method of Davies and Turner (1977) are summarised by the equation

$$\frac{\partial h}{\partial t} + c\frac{\partial h}{\partial x} = K\left(h_{true} - h\right) \qquad , K > 0$$

where h_{true} is the true value of h and also satisfies

$$\frac{\partial h_{true}}{\partial t} + c\frac{\partial h_{true}}{\partial x} = 0$$

Thus h is dynamically relaxed towards the true value h_{true}, which for an assimilation scheme would normally be the observational value or an estimate formed by interpolation from several observations. For a fuller theoretical treatment the reader is referred to the above papers.

The time stepping method suggested by these authors is the following implicit shceme

$$\frac{h_{n+1} - h_{n-1}}{2\Delta t} + c\frac{\partial h_n}{\partial x} = K\left(h_{true} - h\right)_{n+1}$$

where the subscript n is the time level. If Δt is small enough to satisfy the standard CFL condition (with $K = 0$), then it is easily shown that the above scheme is unconditionally stable ($K > 0$) with damping factor

$$[1 + 2\Delta t K]^{-1/2}$$

The updating method for the simulation experiments is summarised by the equations

$$\frac{h^*_{n+1} - h_{n-1}}{2\Delta t} + c\frac{\partial h_n}{\partial x} = 0$$

$$h_{n+1} = h^*_{n+1} + \lambda\left(h_{true} - h^*_{n+1}\right)$$

Thus by a straightforward rearrangement of terms it can be seen that the two methods are equivalent with

$$\lambda = \frac{2K\Delta t}{1 + 2K\Delta t}$$

Thus the following values of λ and K are equivalent for the time step of the 5 level model (Δt = 600 s)

$K = \infty$, $\lambda = 1$ (direct replacement)
$K = 10^{-3}\ s^{-1}$, $\lambda = 1.2/2.2$
$K = 10^{-4}\ s^{-1}$, $\lambda = 0.12/1.12$
$K = 10^{-5}\ s^{-1}$, $\lambda = 0.012/1.012$

The experiments described in the next section use the above values of λ.

In developing the theory of this method, the authors cited above assumed a knowledge of the time history of each observation, but this is not essential. All that need be assumed is that the observations describe a state of the atmosphere which, at their time of validity, satisfy the equations used by the model. Then in the period preceding this time of validity, the model variables are relaxed towards this state. For this reason, in the experiments reported here, the variables are updated for the 3 hour period preceding the time of validity, and the value of λ is held constant. This is in contrast to previously reported simulation experiments where λ had a triangular distribution centred at the observation time.

3. The Experiments and Results

Six experiments were performed and are summarised in the following table

Exp No.	Optimum Interpolation ?	Period of Assimilation (hn)	$K(s^{-1})$
Y501	No	3	∞
Y502	No	3	10^{-3}
Y503	No	3	10^{-4}
Y504	No	3	10^{-5}
Y505	No	6	10^{-4}
Y506	Yes	3	10^{-4}

Note that Y505 has an assimilation period of 6 hours preceding each observation time (00, 06, 12 and 18) and that Y506 includes optimum interpolation).

The main features of the results can be obtained from figures 1(a) to (d), which depict graphs of r.m.s. error in surface pressure over the ten days of assimilation for the latitude bands 90N - 90S, 90N - 30N, 30N - 30S and 30S - 90S. Thus it can be seen that the value $K = 10^{-4}\ s^{-1}$ is markedly superior to the other values except perhaps in the comparatively data rich area of the Northern hemisphere. In addition the inclusion of optimum interpolation considerably improves the assimilation, whereas assimilating over a six hour period has no significant effect. On the other hand plots of wind and temperature errors in figures 2 and 3 (90N - 90S only) imply that, for the former, assimilating over a six hour period has a detrimental effect, whilst for the latter a similar deterioration is caused by optimum interpolation. Essentially the same properties are exhibited in the other latitude bands (not shown).

Some useful comparison may also be made between experiments Y501, Y503, Y506 and experiments X342, V40, V41 of Lyne (1978). The main details of the latter three experiments are summarised in the table below but note that their assimilation period is centred on the observation time.

Exp. No.	Optimum Interpolation ?	Period of Assimilation	
X342	Yes	3	Triangular distribution
V40	No	3	Triangular distribution
V41	No	3	$\lambda = 1$, $(K = \infty)$

The global r.m.s error in surface pressure is plotted in figure 4 for these six experiments. V41 and Y501 differ only in that the former's period of assimilation is centred on the observation time whilst the latter immediately precedes it. As can be seen the error level of Y501 is substantially reduced from that of V41. Furthermore, the error level of Y503 ($K = 10^{-4}$) is significantly less than for V40 (triangular distribution of λ, min value $\lambda = 0$ $(K = 0)$, maximum value $\lambda = 1$ $(K = \infty)$). However the error levels of Y506 are slightly inferior to those of X342. A reasonable conclusion to draw from this in the light of the above comparisons is that optimum interpolation allows a larger value of λ (or K) to be used with a consequent improvement in assimilation.

Because of the clear superiority of the value $K = 10^{-4}\ s^{-1}$, attention will now be focused on experiments Y503, Y505 and Y506. In figures 5(a) to (c) and 6(a) to (c) the cross-sections of zonally averaged rms errors of wind and temperature for the last five days of each assimilation are presented. These broadly confirm the conclusions drawn above, that the wind analyses are worse with a six hour assimilation period, and that optimum interpolation has a slightly deleterious effect on the temperature analyses, except in certain regions of the mid-troposphere. One feature common to all three temperature cross-sections is the high error level in the topmost level between 30N and 40N. Detailed inspection of the r.m.s. error fields reveal that this is almost entirely due to the region over the Himalayas, but the precise reason why the effect is so large in these experiments is not clear.

Figures 7(a) to (c) contain the PMSL charts valid for the end of the assimilation period (day 20) with the "truth" field depicted in figure 7(d). The corresponding rms surface pressure error charts for the last five days of the assimilations are shown in figures 8(a) to (c).

As in Lyne (1978) attention is focussed on the four depressions in the Southern hemisphere marked A,B,C and D in figure 7(d), and the following ranking in standard of assimilation can be produced.

Standard	B	C	D
Best	Y505	Y506	Y505
	Y503	Y505	Y503
Worst	Y506	Y503	Y506

The picture for depression A is confusing. The pressure field for day 20 (figure 7) seems best described by Y503, but the rms errors over the last five days as a whole (figure 8) are lowest for Y506. In any case the picture for these depressions as a whole is confused, in that no single experiment gives significantly better or worse results than any other.

4. Conclusions

It has been demonstrated that considerable changes can be made in the standard of assimilation by varying the weight given to the observation. It would appear that a value for K of $10^{-4}\ s^{-1}$ is near the optimum with a higher value possible if optimum interpolation is used. In addition an improvement seems possible if the data is assimilated for a 3 hour period ahead of each synoptic hour instead of surrounding it, but that no significant improvement ensues if the assimilation period is extended to 6 hours. In connection with the last observation it must be observed that in Lyne (1978) it was concluded that, at least for this model generated data, merely inserting the data for two timesteps was superior to continuously assimilating it over 3 hours. However, the asymptotic error level of that "double insertion" experiment (V43) is greater than the present experiment Y503. In the two series of experiments as a whole, the lowest rms error levels were obtained with X342, that is optimum interpolation with the triangular distribution of λ.

Finally it is interesting to speculate on what could be the effect of including the "diffusive relaxation" of Davies and Turner (1977). Their theoretical analysis implies that it should be more effective than the Newtonian relaxation described here, though their numerical experiments are inconclusive on this point. However, optimum interpolation can be regarded as a means by which the effect of observations is "diffused" to surrounding grid points and the question naturally arises as to whether the beneficial effects of interpolation can be achieved much more cheaply by the inclusion of this relaxation term. It is not proposed at this juncture to alter the suite of programs used here to include this term, but it is intended that the FGGE data assimilation scheme will have this option available.

References

Anthes, R A 1974 'Data assimilation and initialisation of hurricane prediction models'. J Atmos. Sci. 31, 702-719

Davies, H C and Turner, R E 1977 'Updating prediction models by dynamical relaxation: an examination of the technique'. Quart. J R Met. Soc. 103, 225-245

Hoke, J E and Anthes, R A 1976 'The initialisation of numerical models by a dynamic-initialisation technique' Mon. Weather Rev. 12, 1551-1556

Hoke, J E and Anthes, R A 1977 'Dynamic initialisation of a three-dimensional primitive equation model of hurricane Alma of 1962'. Mon. Weather Rev. 105, 1266-1280

Lyne, W H 1978 'Simulation experiments to compare the relative merits of optimum interpolation and repeated insertion'. Met O 20 Tech. Note No II/124.

Figure Legends

Figure 1 Graphs of rms error in analyses of surface pressure during assimilation period

(a) 90N - 90S

(b) 90N - 30N

(c) 30N - 30S

(d) 30S - 90S

Figure 2 Graphs of global rms error in anlyses of wind during assimilation period

Figure 3 Graphs of global rms error in analyses of temperature during assimilation period.

Figure 4 Graphs of global rms error in analyses of surface pressure during assimilation period (experiments Y501,Y503,Y506,V40,V41 and X342)

Figure 5 Cross-sections of zonal rms wind errors of analyses for last five days of assimilation period

(a) Y503

(b) Y505

(c) Y506

Figure 6 Cross-sections of zonal rms temperature errors of analyses for last five days of assimilation period

(a) Y503

(b) Y505

(c) Y506

Figure 7 Charts of PMSL for 00 GMT of day 20 at end of assimilation period

(a) Y503

(b) Y505

(c) Y506

(d) TRUTH

Figure 8 Charts of rms surface pressure error of analyses for last five days of assimilation period

(a) Y503

(b) Y505

(c) Y506

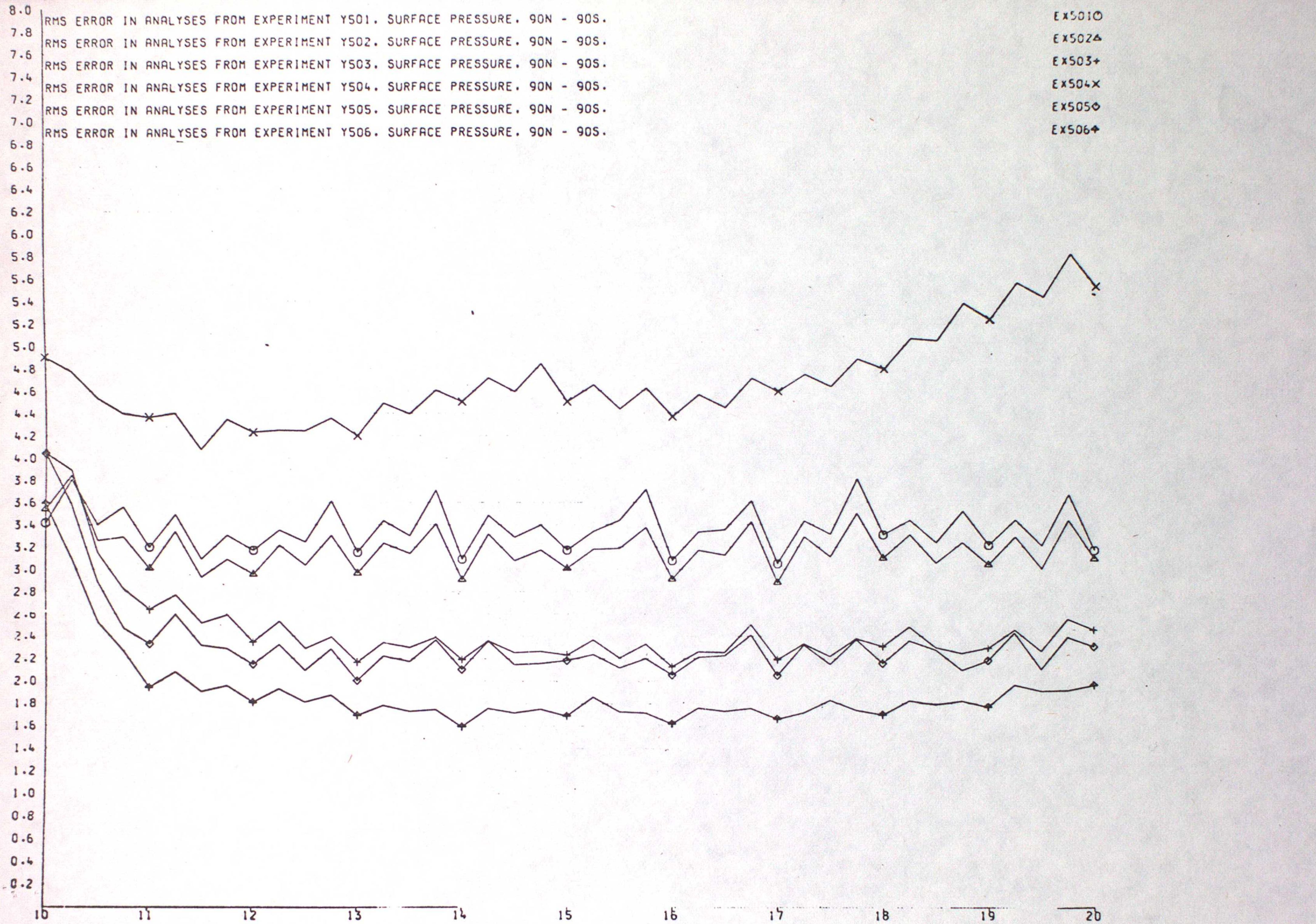
RMS ERROR IN ANALYSES FROM EXPERIMENT YS01. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS02. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS03. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS04. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS05. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS06. SURFACE PRESSURE. 90N - 90S.
EXS01○
EXS02△
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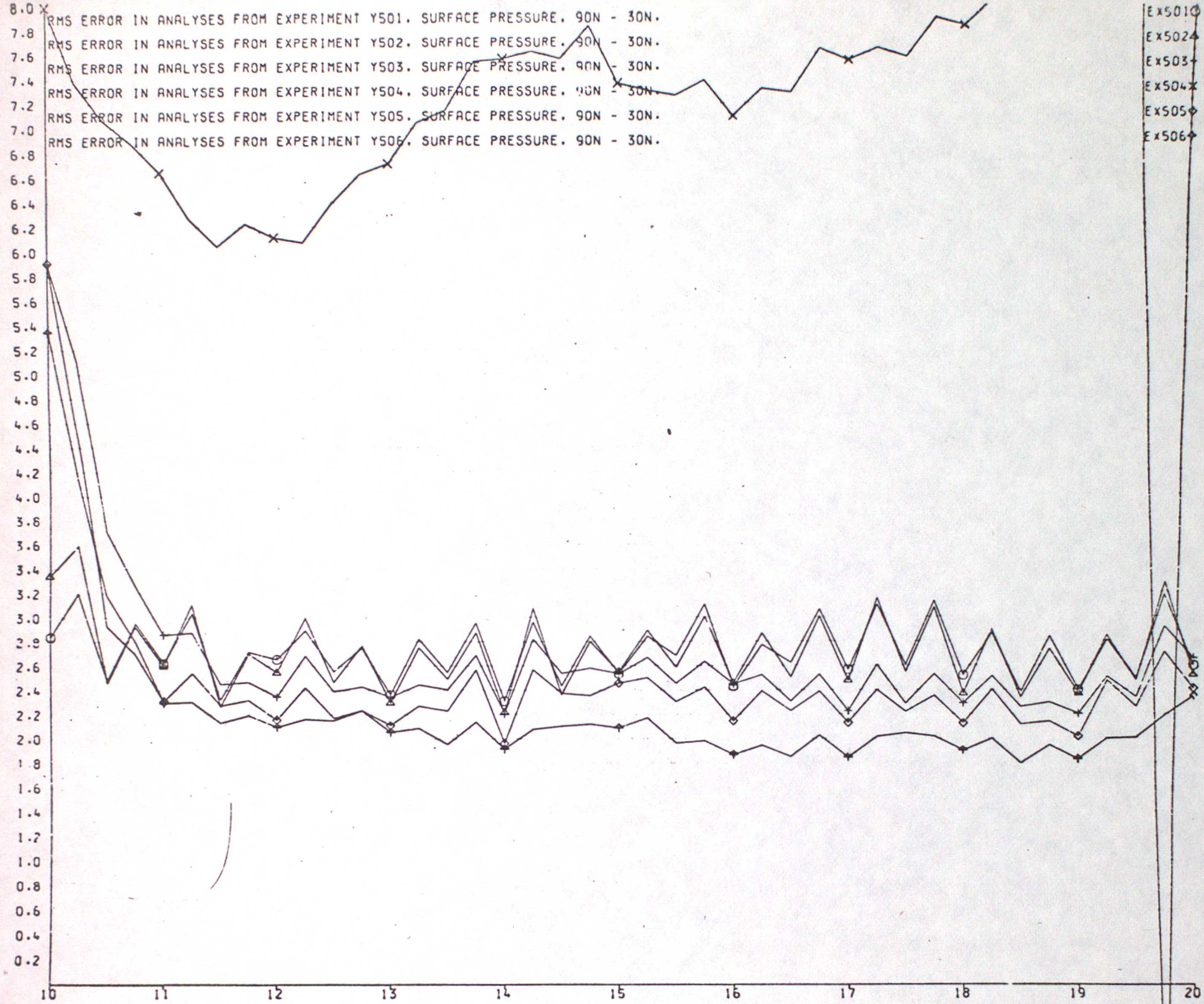
FIGURE 1a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RMS ERROR IN ANALYSES FROM EXPERIMENT Y503. SURFACE PRESSURE. 90N - 30N.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y504. SURFACE PRESSURE. 90N - 30N.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y505. SURFACE PRESSURE. 90N - 30N.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y506. SURFACE PRESSURE. 90N - 30N.
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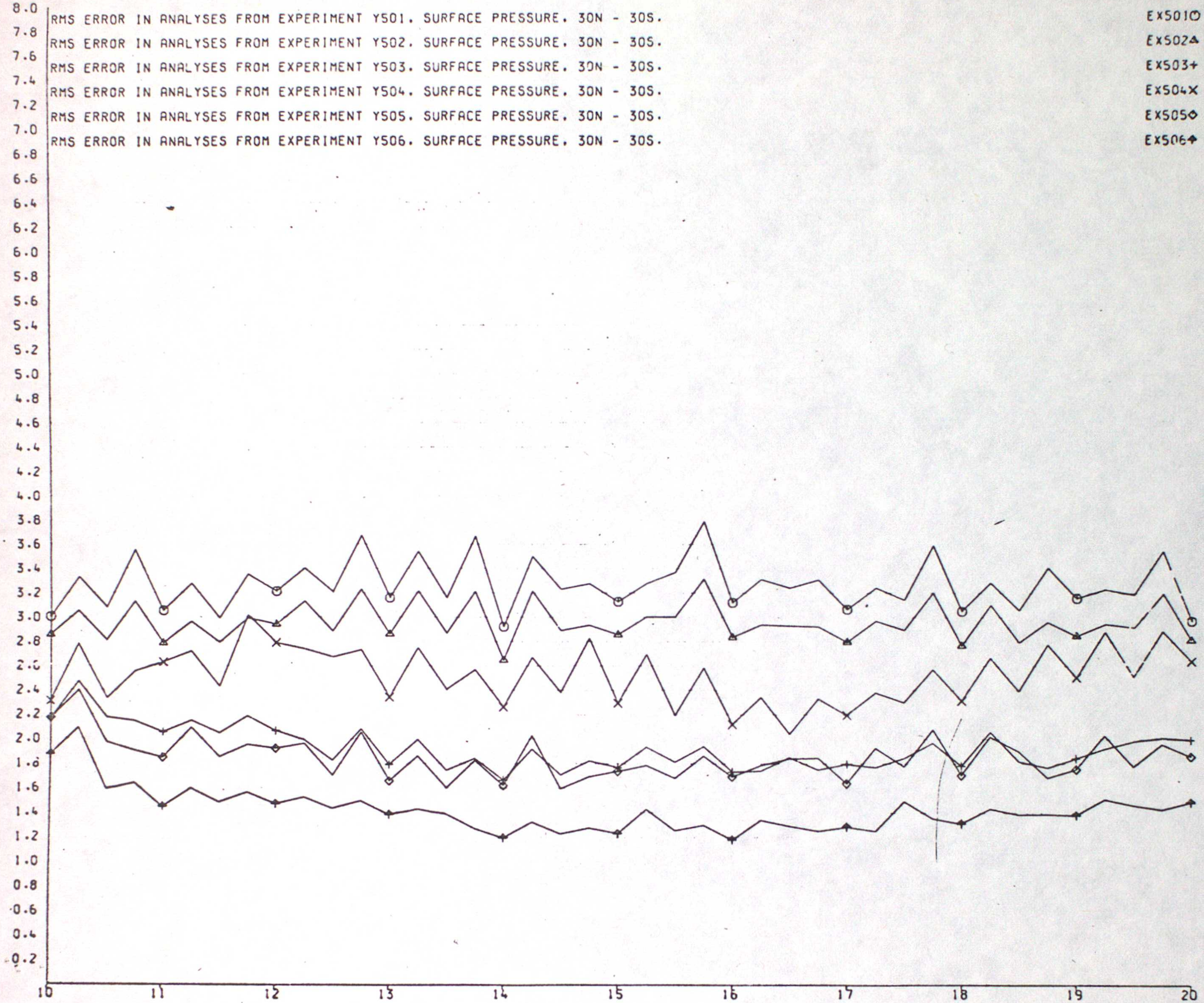
FIGURE 1b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RMS ERROR IN ANALYSES FROM EXPERIMENT Y503. SURFACE PRESSURE. 30N - 30S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y504. SURFACE PRESSURE. 30N - 30S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y505. SURFACE PRESSURE. 30N - 30S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y506. SURFACE PRESSURE. 30N - 30S.
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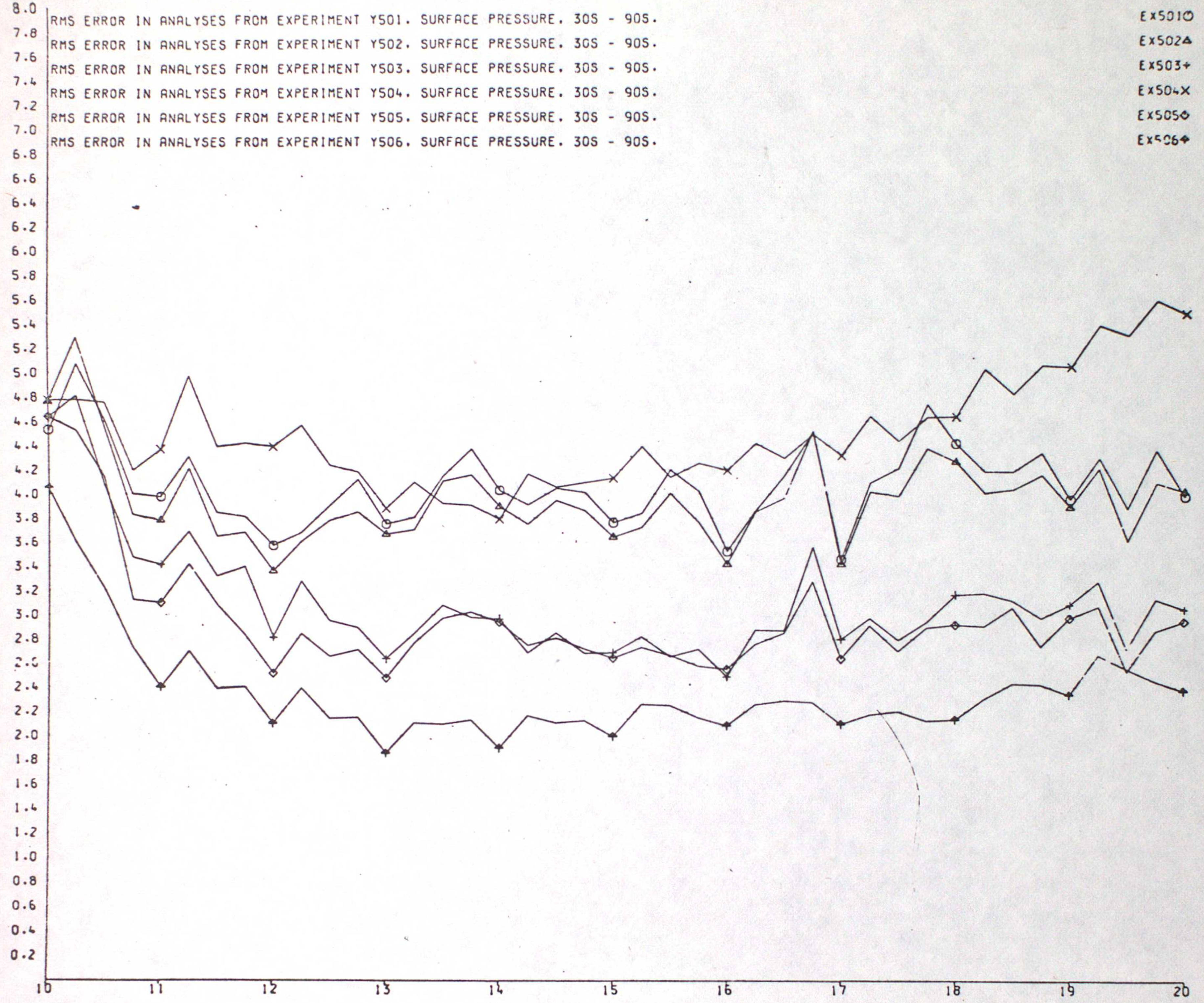
FIGURE 1c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RMS ERROR IN ANALYSES FROM EXPERIMENT Y502. SURFACE PRESSURE. 30S - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y503. SURFACE PRESSURE. 30S - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y504. SURFACE PRESSURE. 30S - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y505. SURFACE PRESSURE. 30S - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y506. SURFACE PRESSURE. 30S - 90S.
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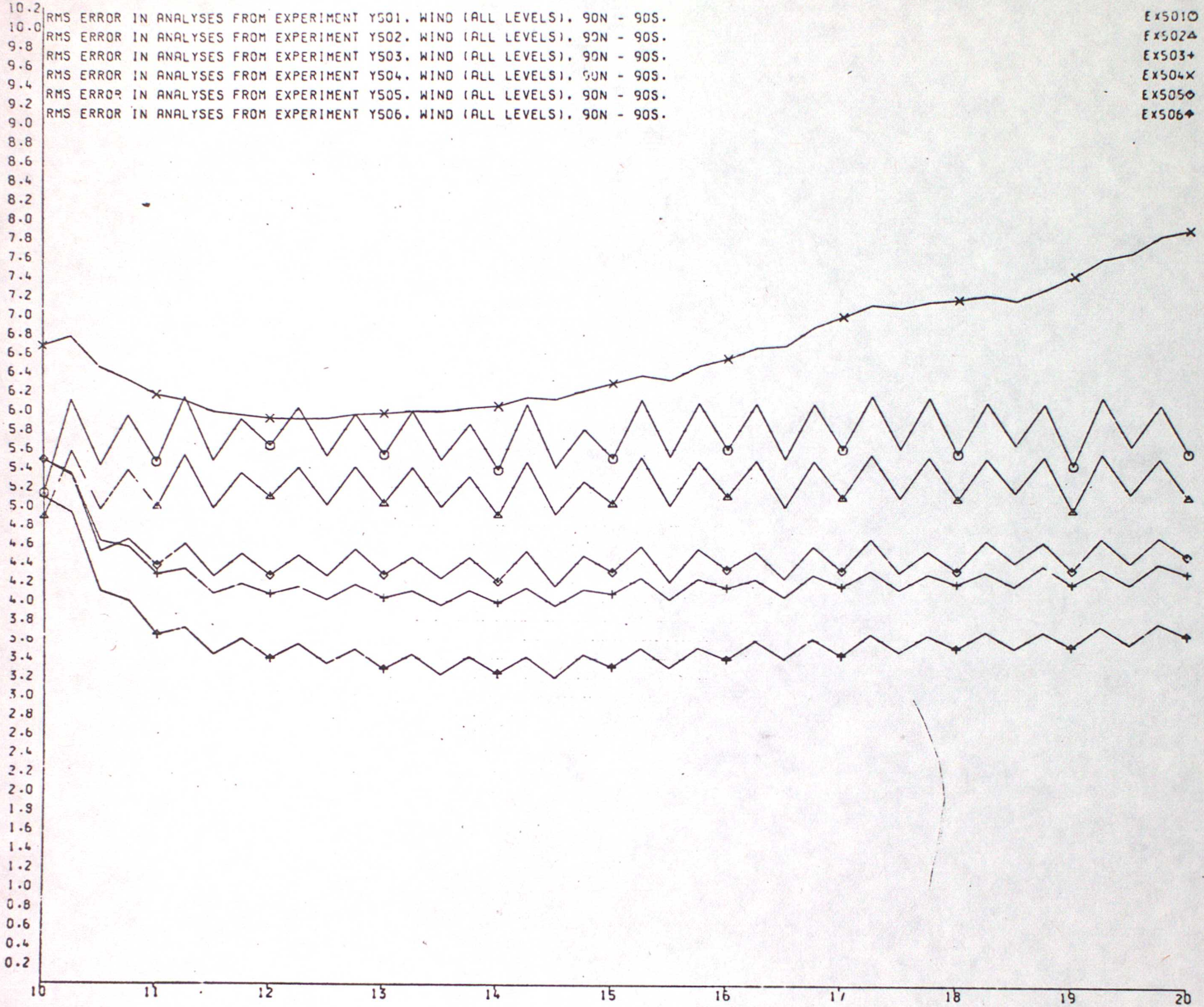
FIGURE 1d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RMS ERROR IN ANALYSES FROM EXPERIMENT YS02. WIND (ALL LEVELS). 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS03. WIND (ALL LEVELS). 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS04. WIND (ALL LEVELS). 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS05. WIND (ALL LEVELS). 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT YS06. WIND (ALL LEVELS). 90N - 90S.
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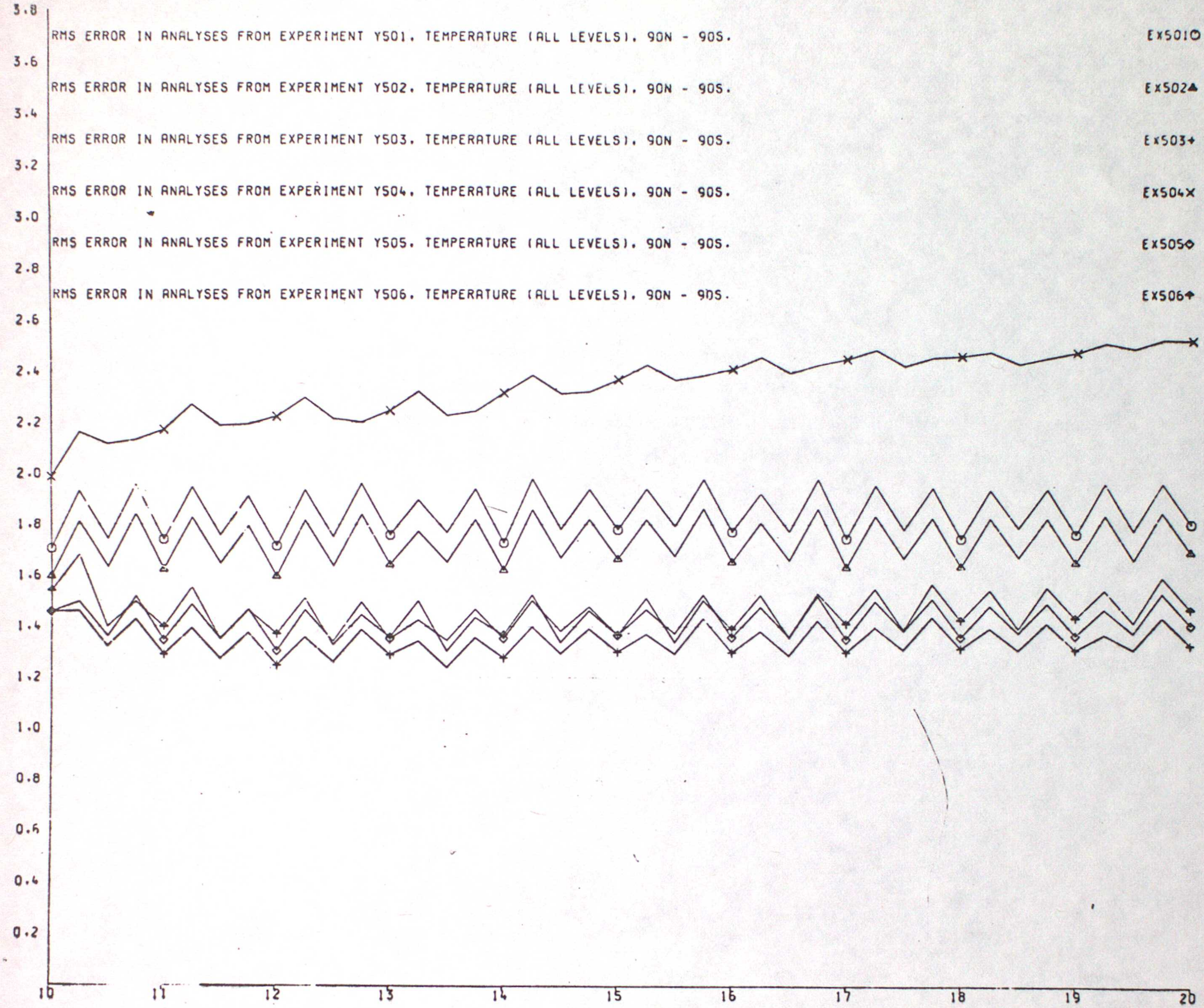
FIGURE 2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EX501
RMS ERROR IN ANALYSES FROM EXPERIMENT Y502. TEMPERATURE (ALL LEVELS). 90N - 90S.
EX502
RMS ERROR IN ANALYSES FROM EXPERIMENT Y503. TEMPERATURE (ALL LEVELS). 90N - 90S.
EX503
RMS ERROR IN ANALYSES FROM EXPERIMENT Y504. TEMPERATURE (ALL LEVELS). 90N - 90S.
EX504
RMS ERROR IN ANALYSES FROM EXPERIMENT Y505. TEMPERATURE (ALL LEVELS). 90N - 90S.
EX505
RMS ERROR IN ANALYSES FROM EXPERIMENT Y506. TEMPERATURE (ALL LEVELS). 90N - 90S.
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FIGURE 3

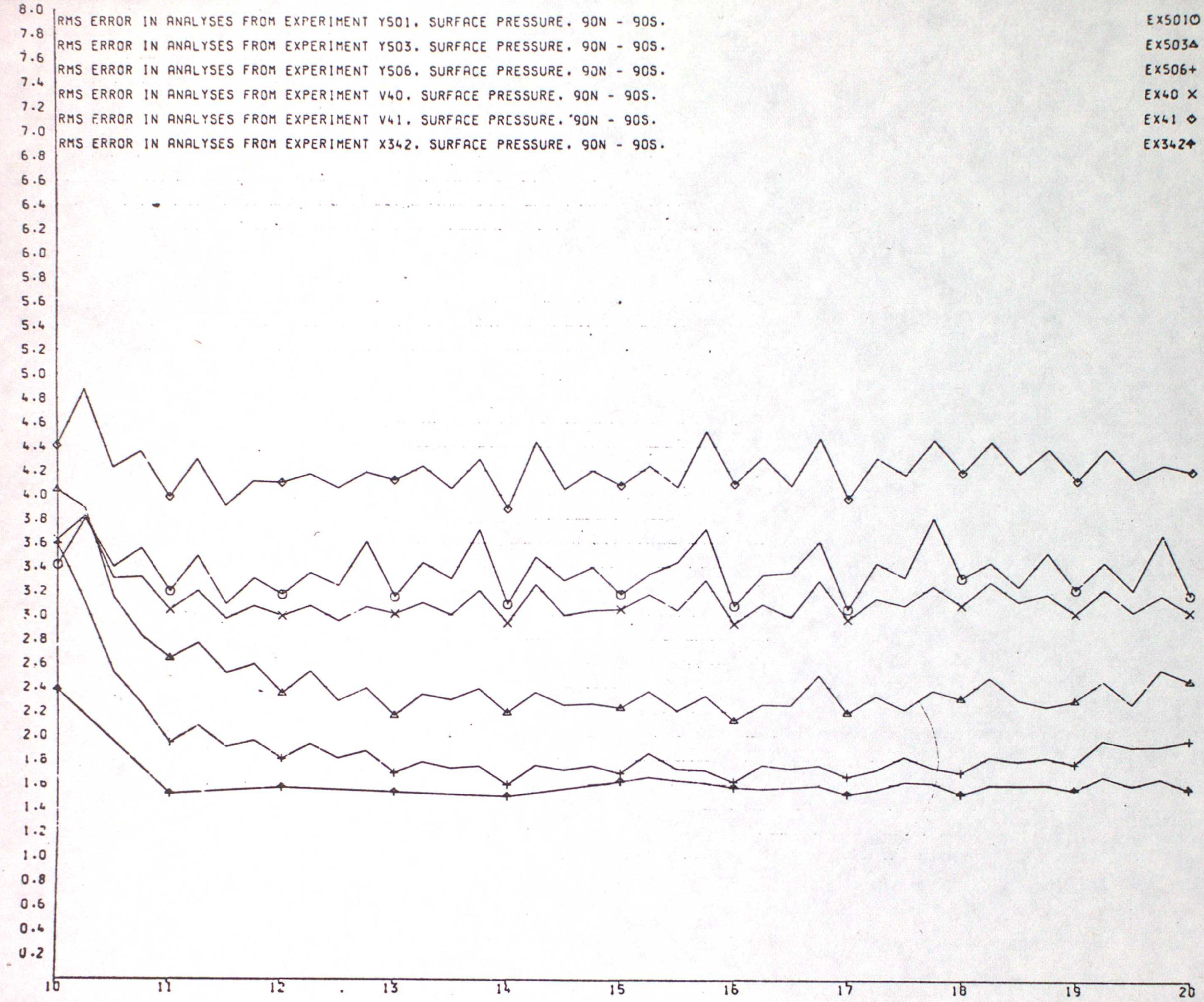
RMS ERROR IN ANALYSES FROM EXPERIMENT Y501. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y503. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT Y506. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT V40. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT V41. SURFACE PRESSURE. 90N - 90S.
RMS ERROR IN ANALYSES FROM EXPERIMENT X342. SURFACE PRESSURE. 90N - 90S.
EX501○
EX503△
EX506+
EX40 ×
EX41 ◇
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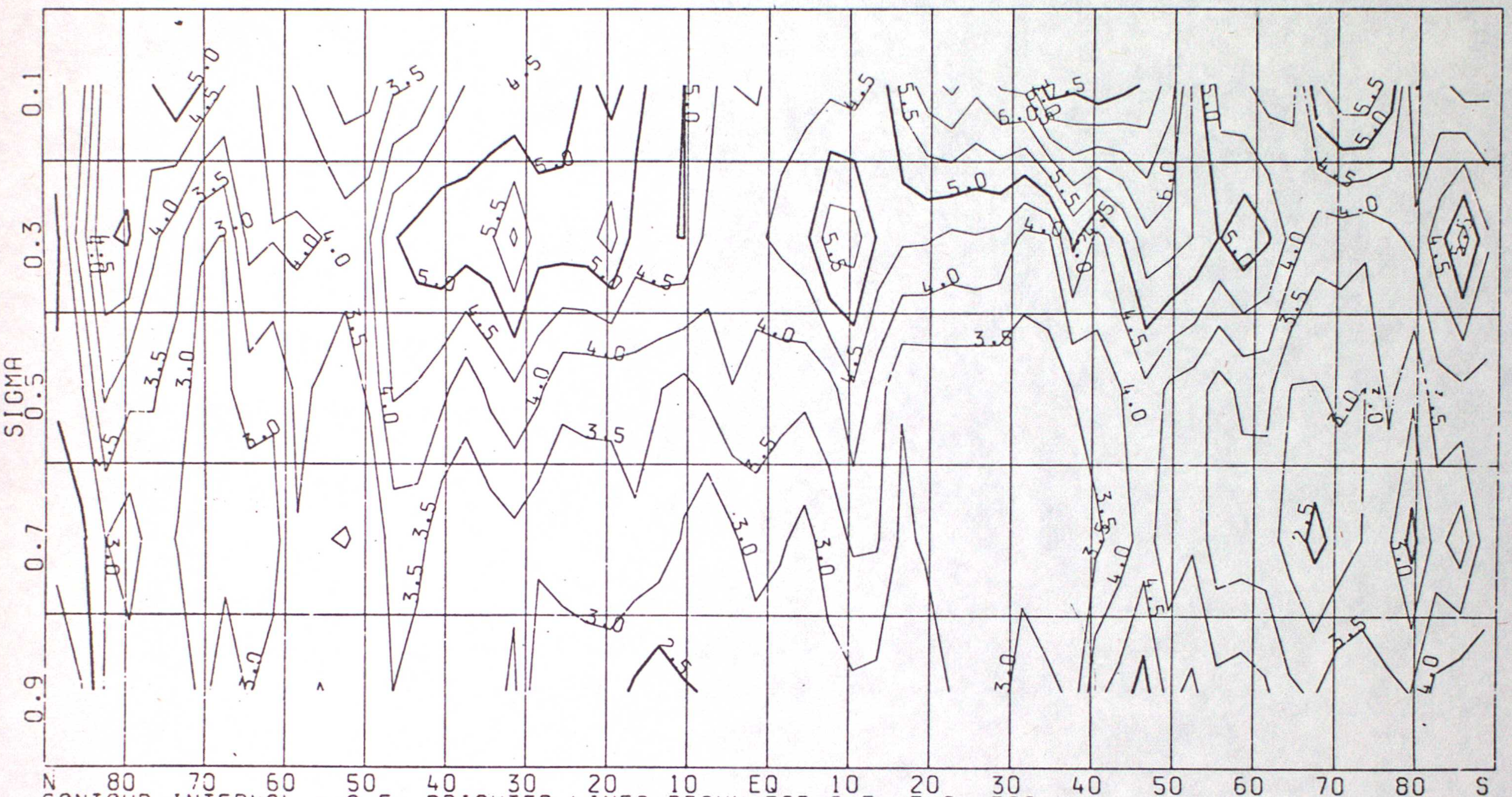
FIGURE 4

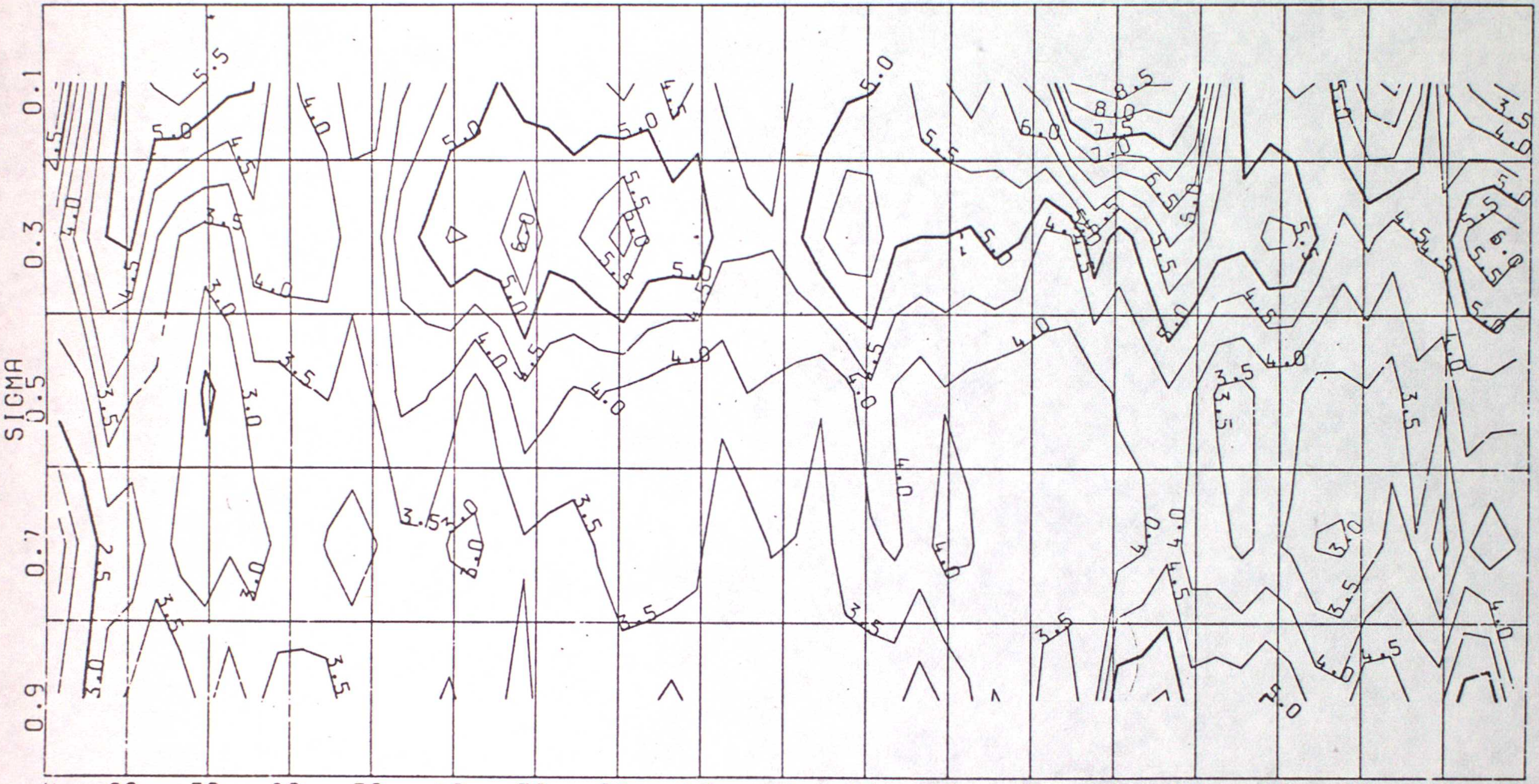
ZONAL RMS WIND ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y503



CONTOUR INTERVAL = 0.5 BRIGHTER LINES DRAWN FOR 2.5, 5.0, ETC.

FIGURE 5a

ZONAL RMS WIND ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y505



CONTOUR INTERVAL = 0.5 BRIGHTER LINES DRAWN FOR 2.5. 5.0. ETC.

FIGURE 5b

ZONAL RMS WIND ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y506

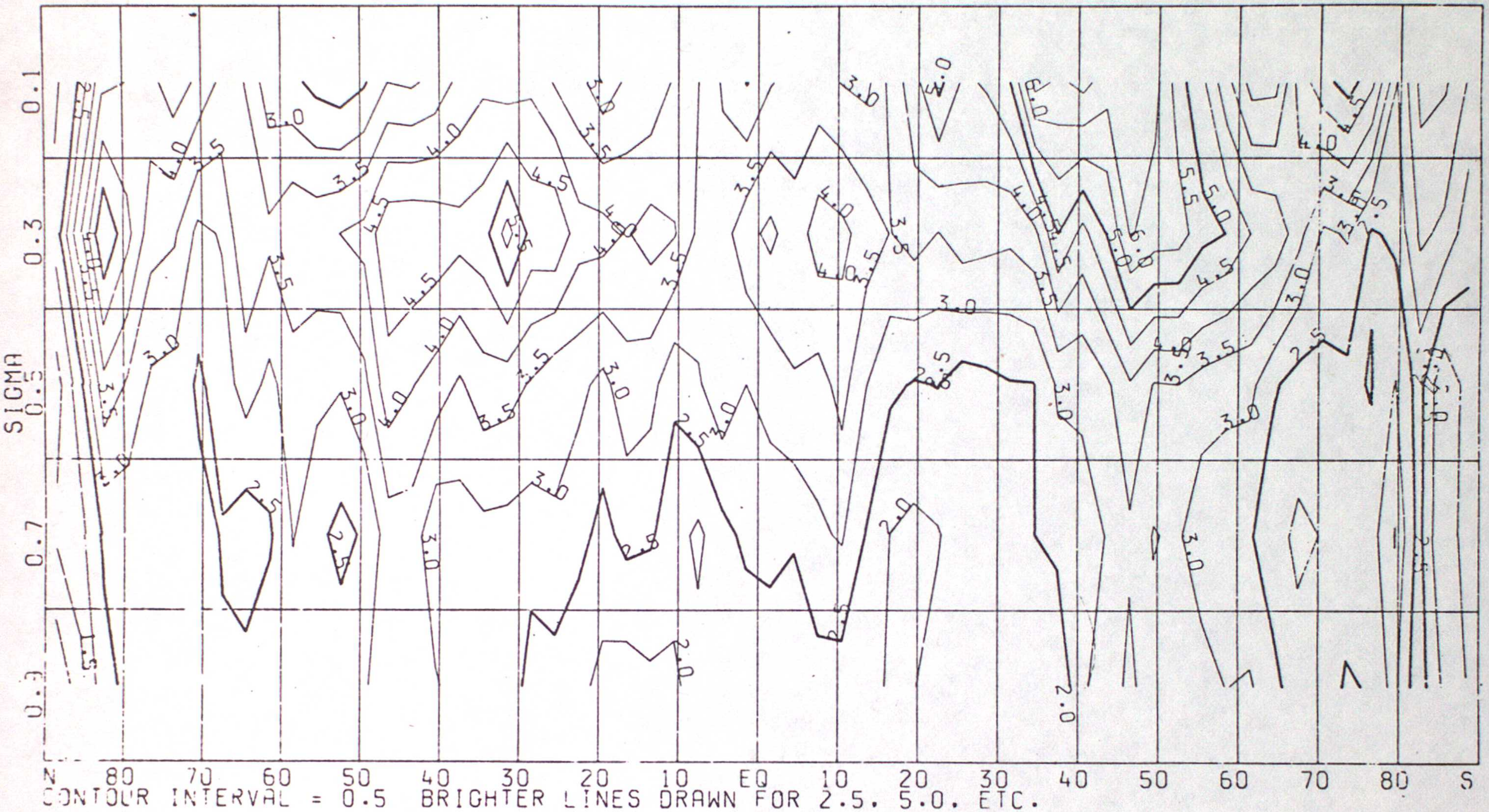
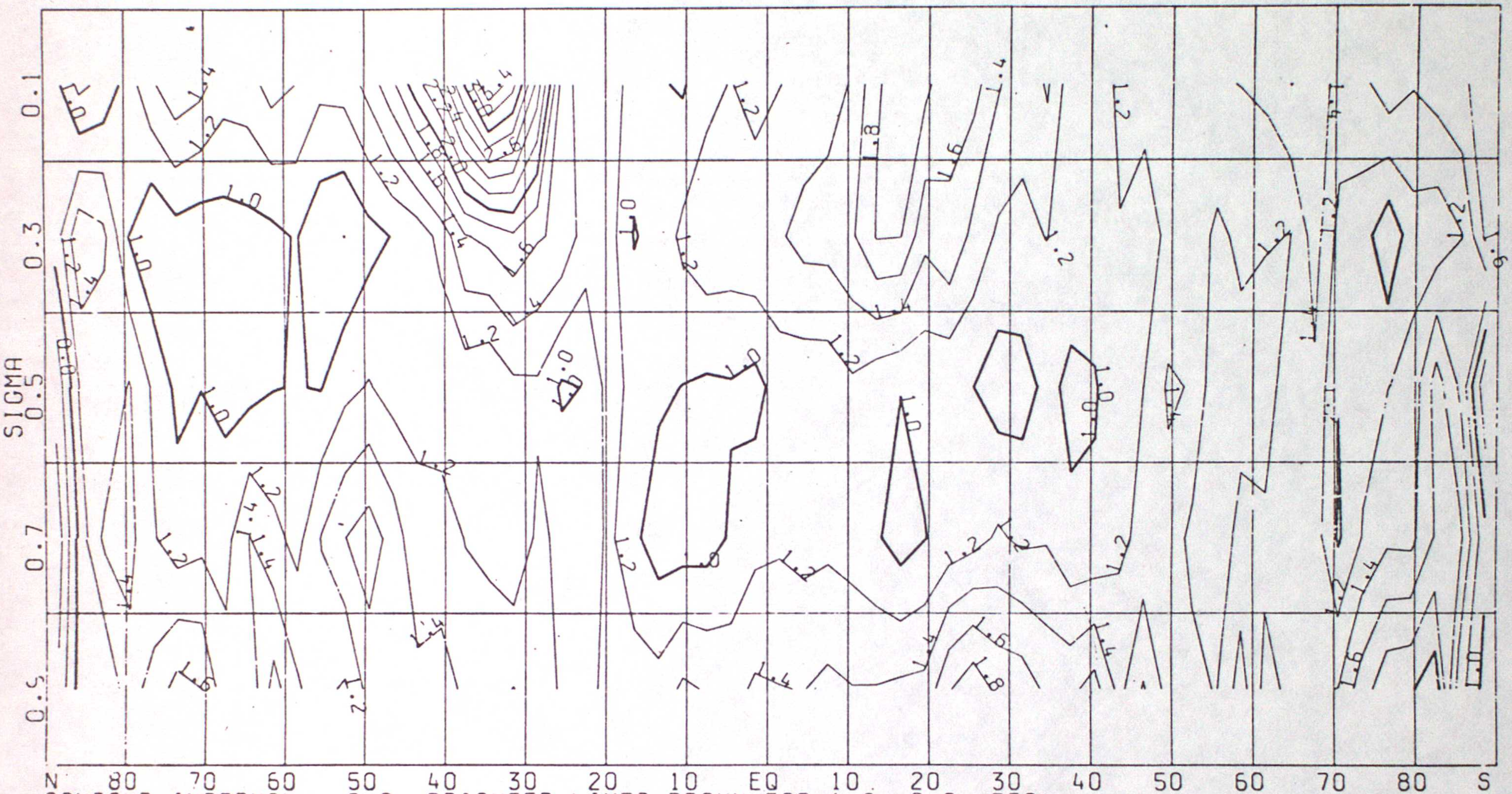


FIGURE 5c

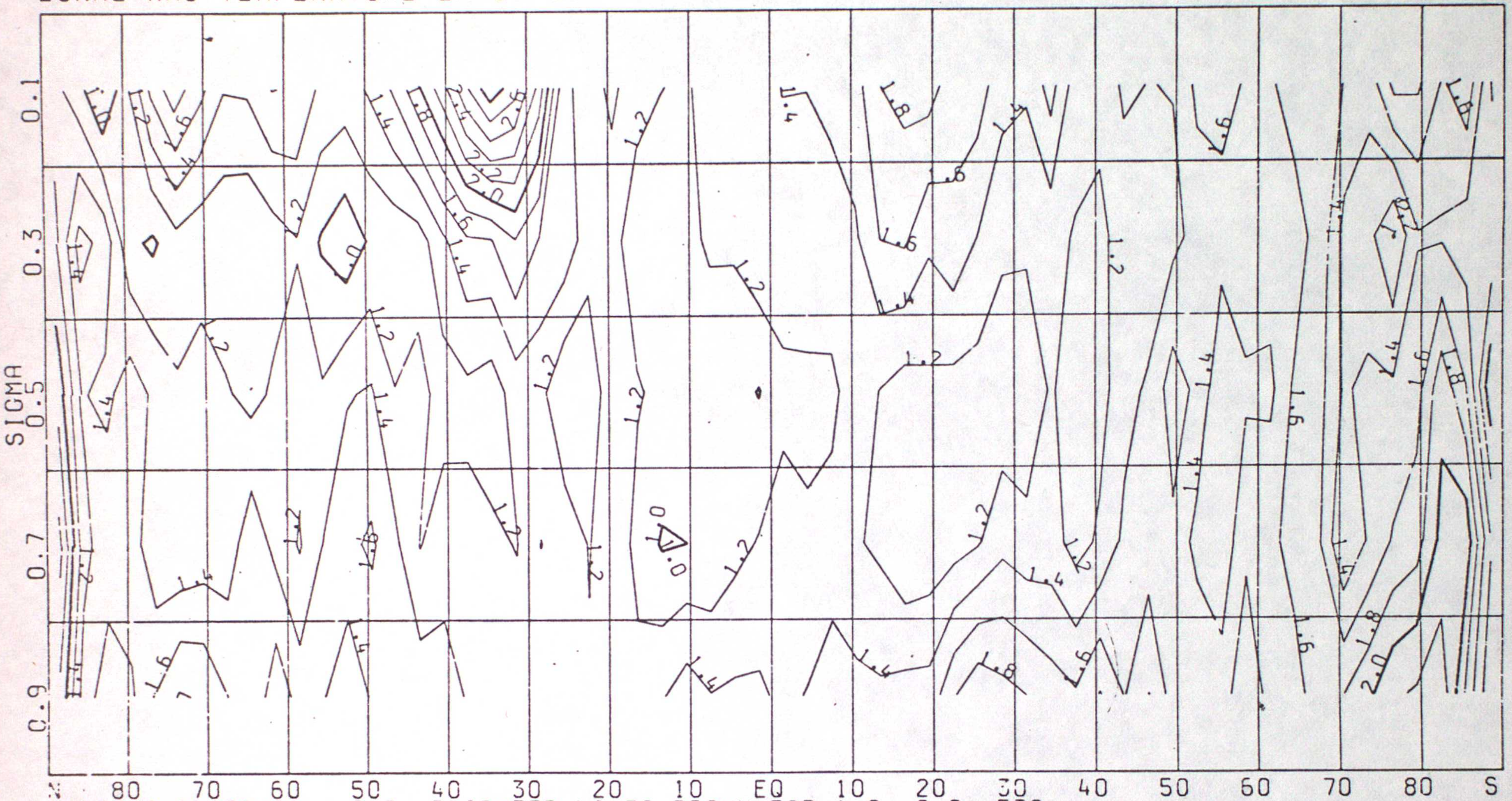
ZONAL RMS TEMPERATURE ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y503



CONTOUR INTERVAL = 0.2 BRIGHTER LINES DRAWN FOR 1.0. 2.0. ETC.

FIGURE 6a

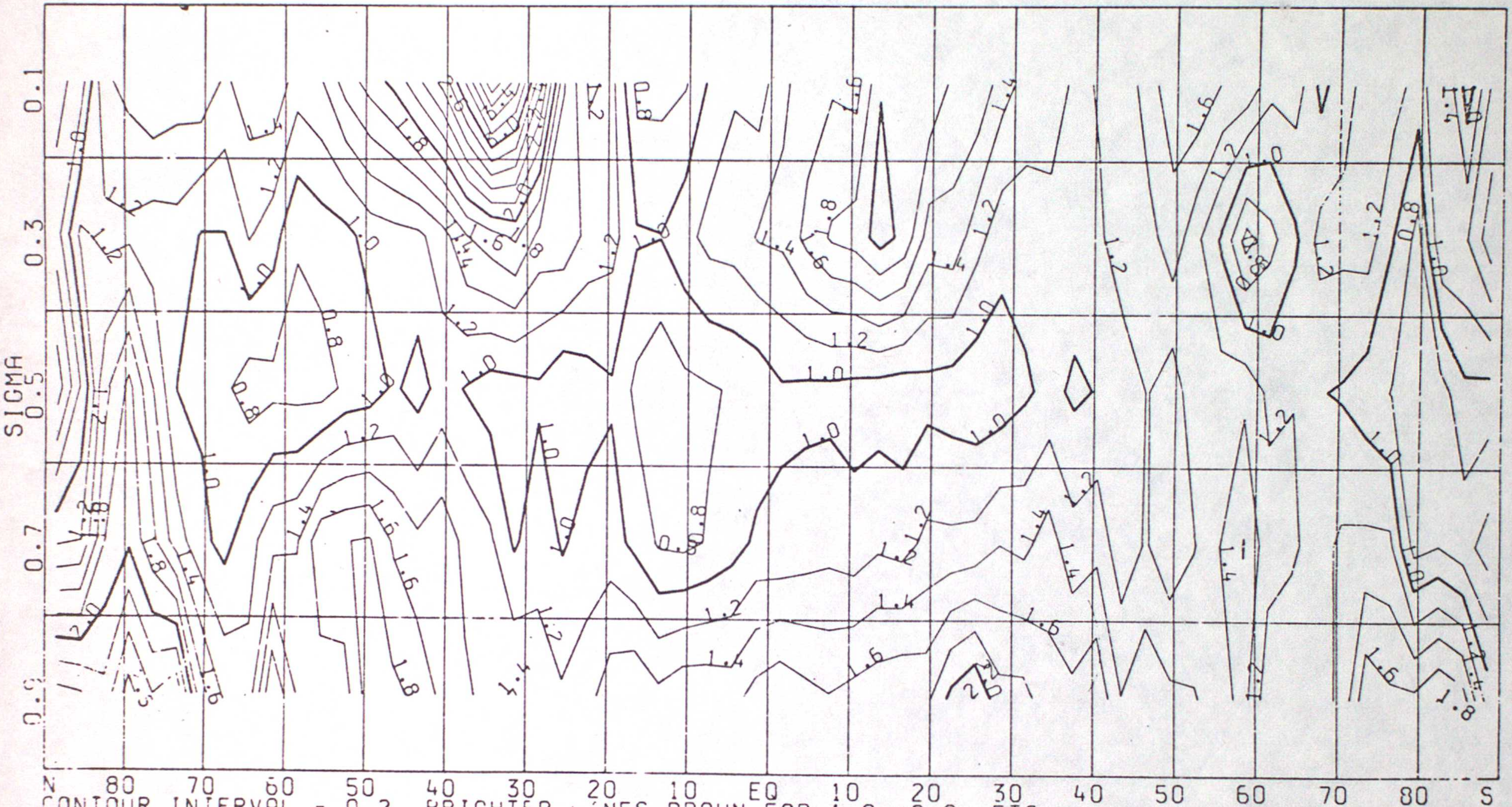
ZONAL RMS TEMPERATURE ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y505



CONTOUR INTERVAL = 0.2 BRIGHTER LINES DRAWN FOR 1.0, 2.0, ETC.

FIGURE 6b

ZONAL RMS TEMPERATURE ERROR IN ANALYSES FOR LAST FIVE DAYS OF Y506



CONTOUR INTERVAL = 0.2 BRIGHTER LINES DRAWN FOR 1.0, 2.0, ETC.

FIGURE 6C

SMOOTHED PMSL OF D020H00 FOR EXP. Y503

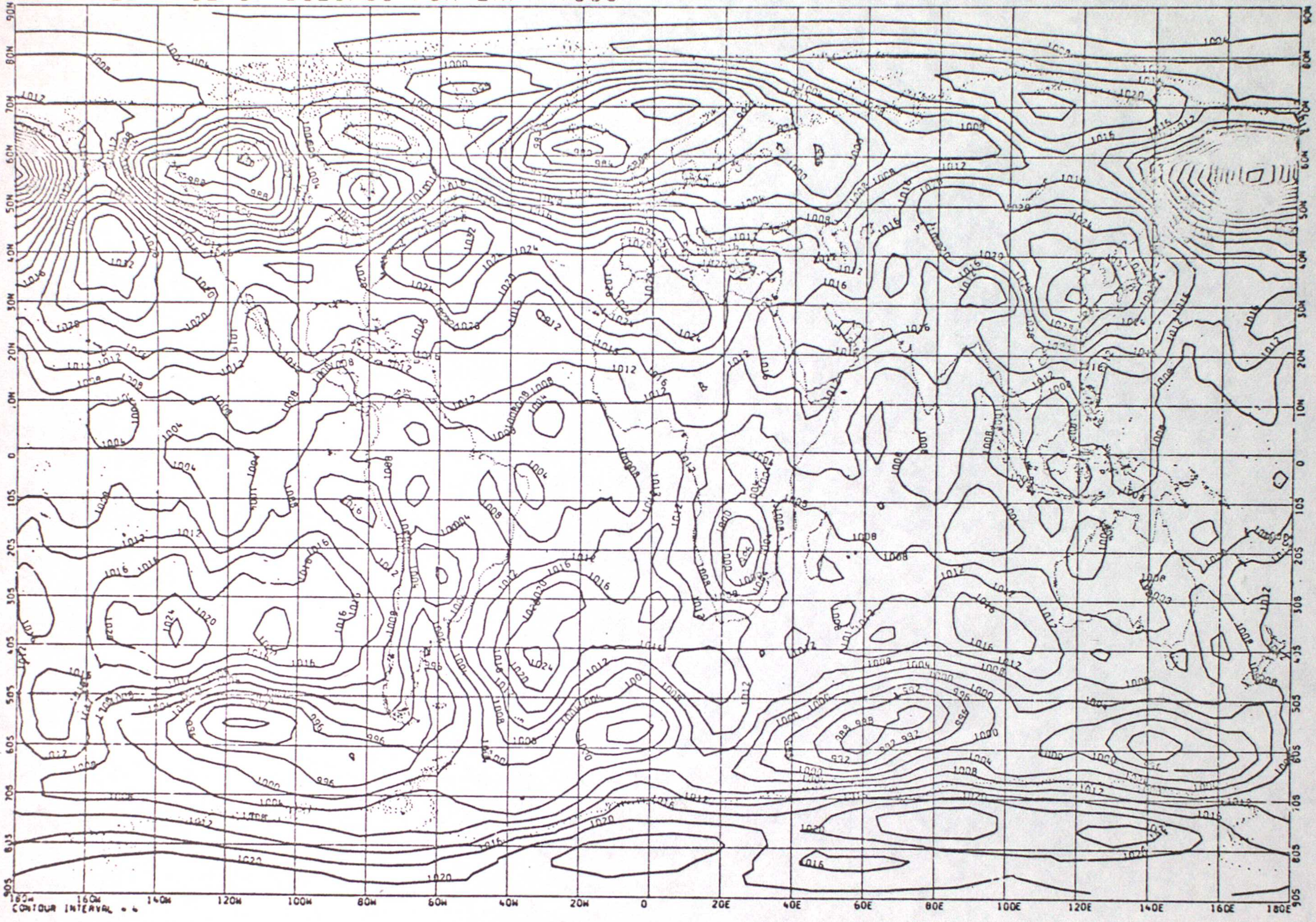


FIGURE 7a

SMOOTHED PMSL OF D020H00 FOR EXP. Y505

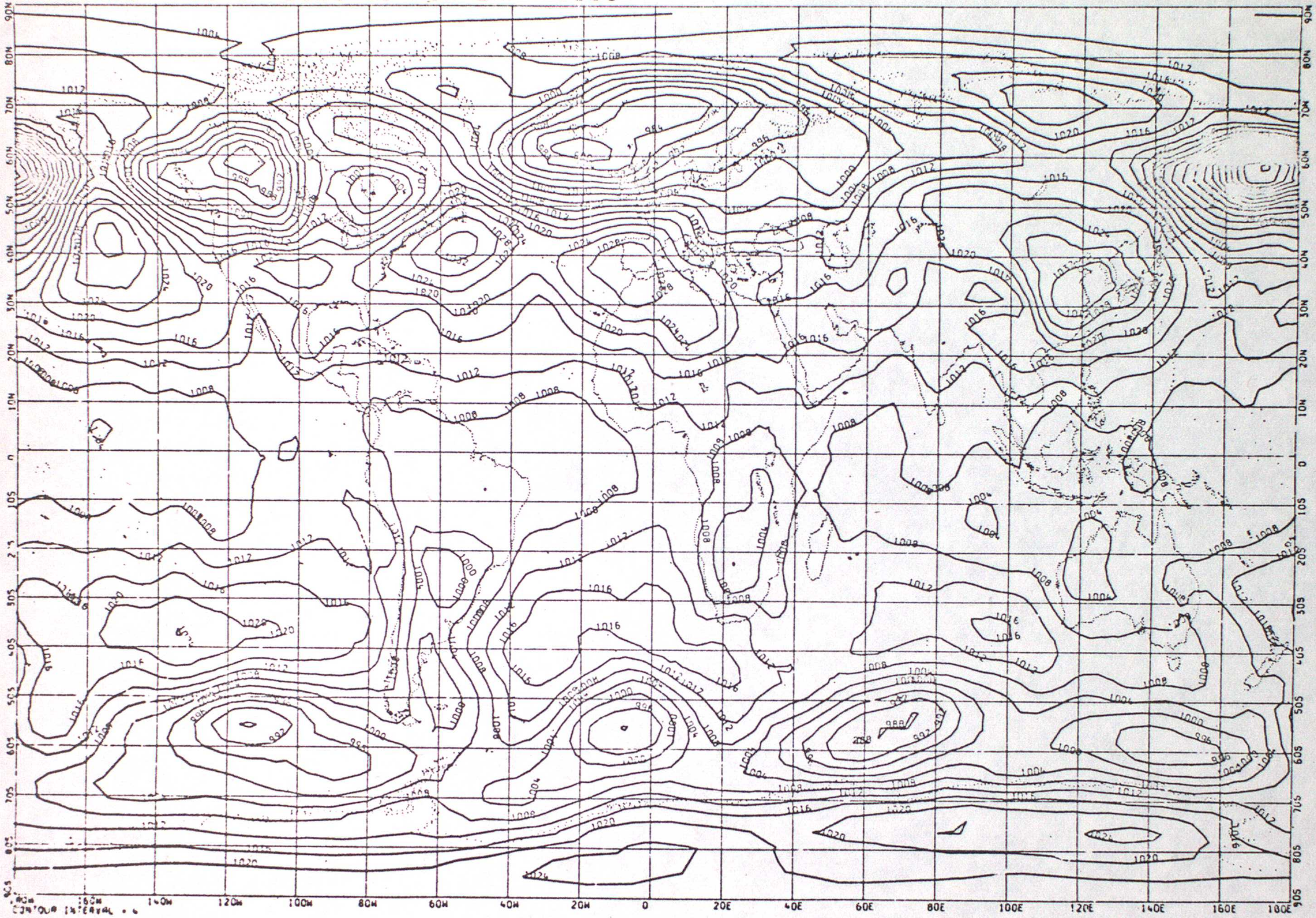


FIGURE 7b

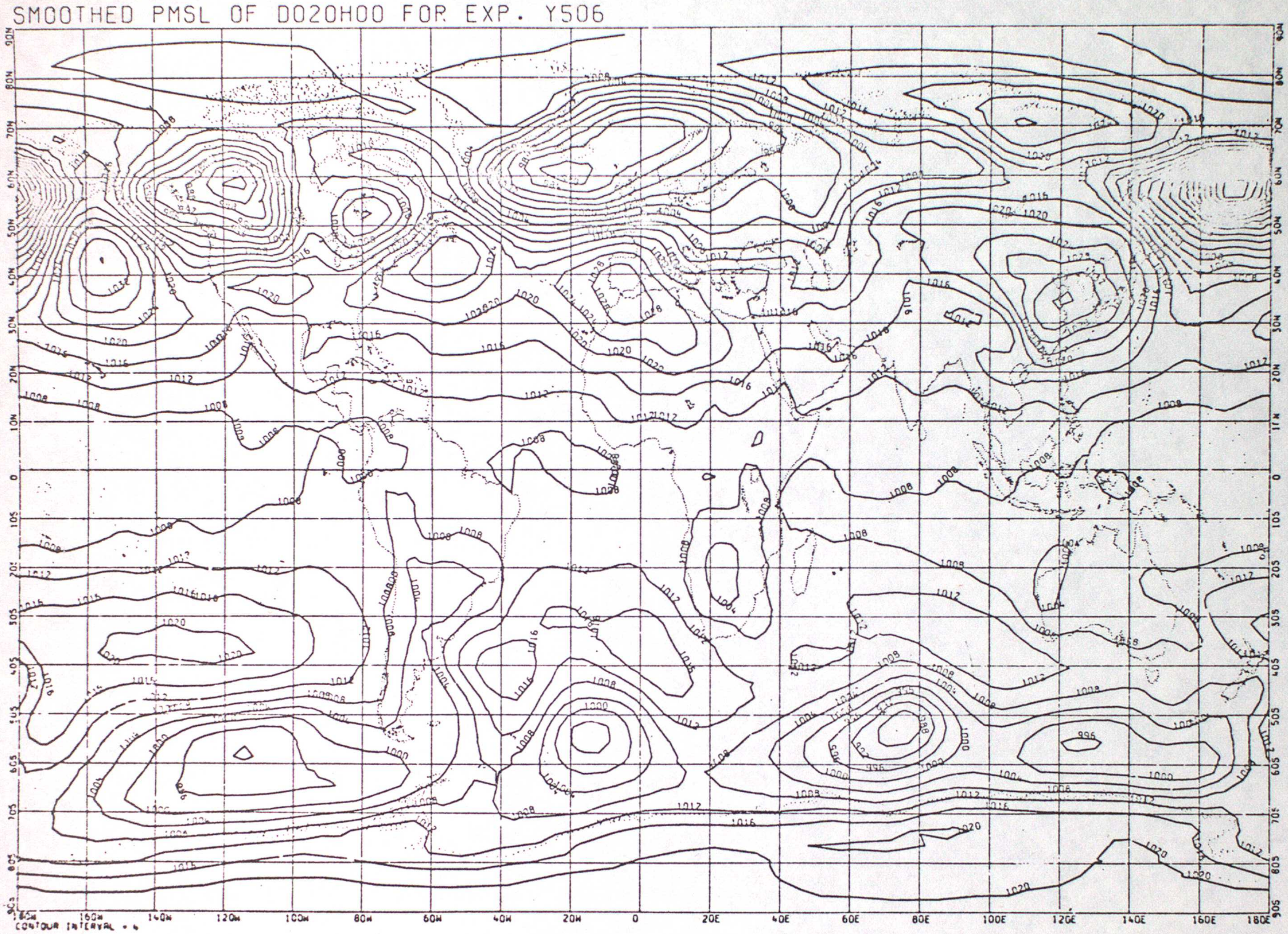


FIGURE 7c

SMOOTHED PMSL OF D020H00 OF 'TRUTH'.

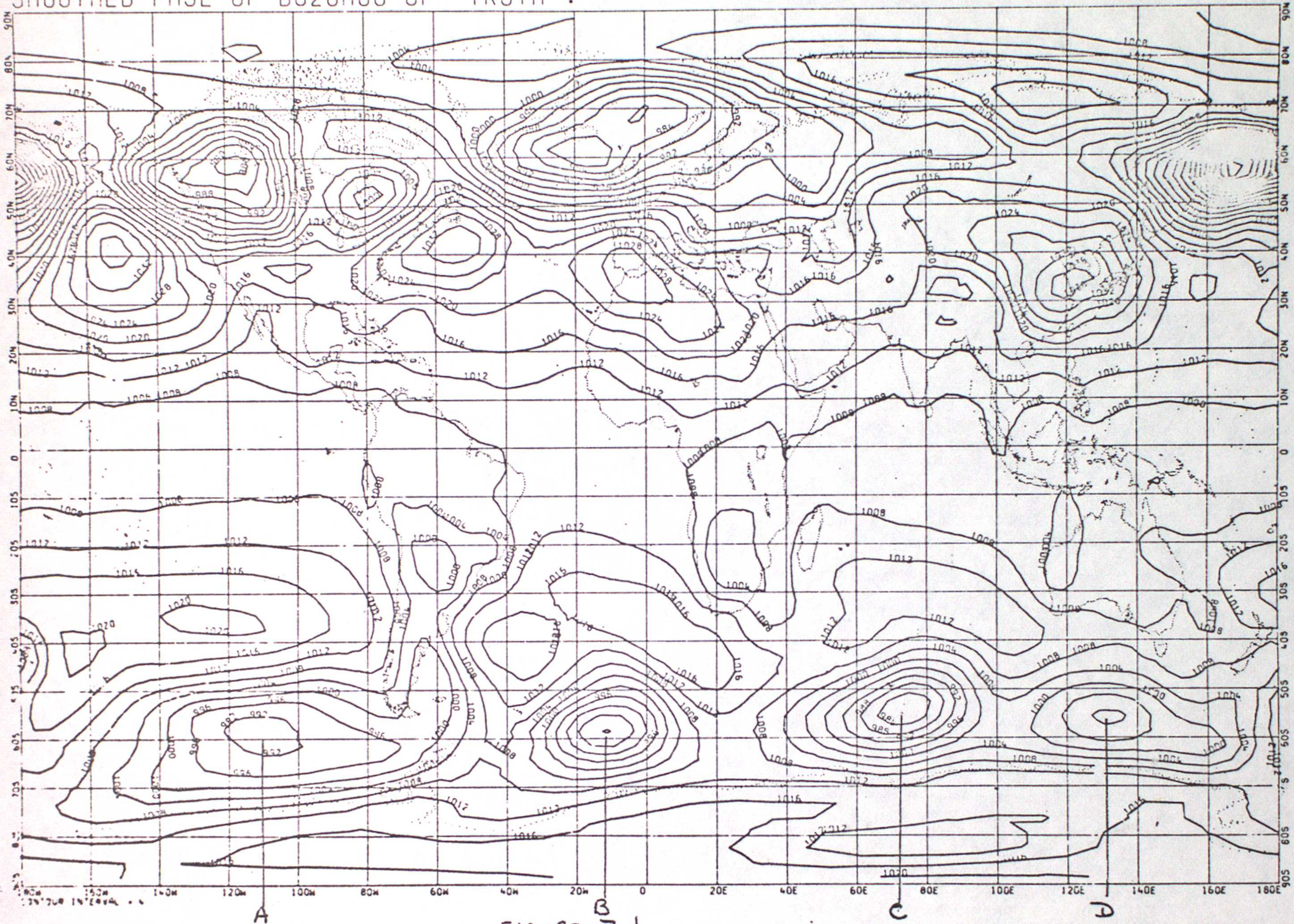


FIGURE 7d

RMS SURFACE PRESSURE ERROR OF ANALYSES FROM EXPERIMENT Y503

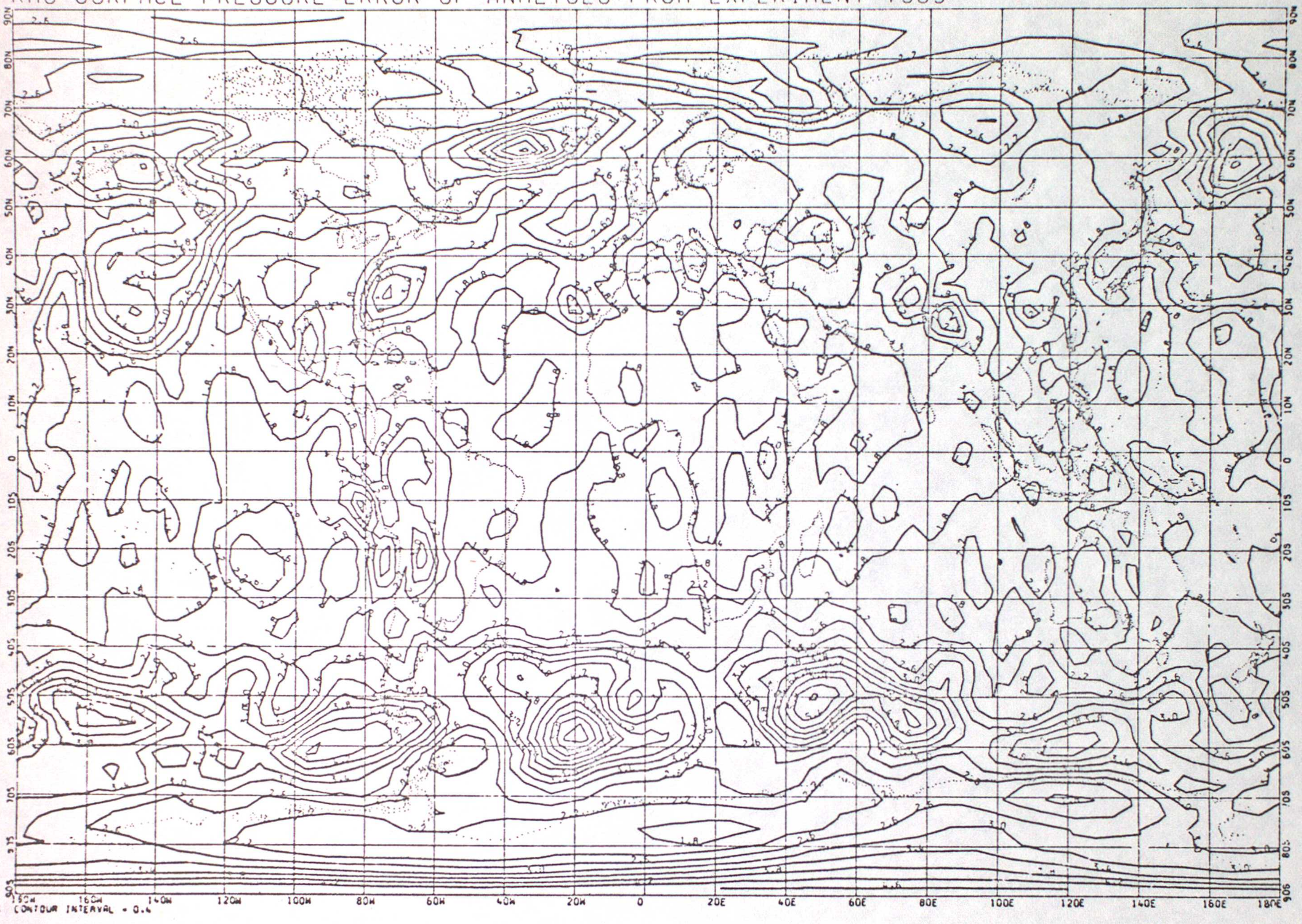


FIGURE 8a

RMS SURFACE PRESSURE ERROR OF ANALYSES FROM EXPERIMENT Y505

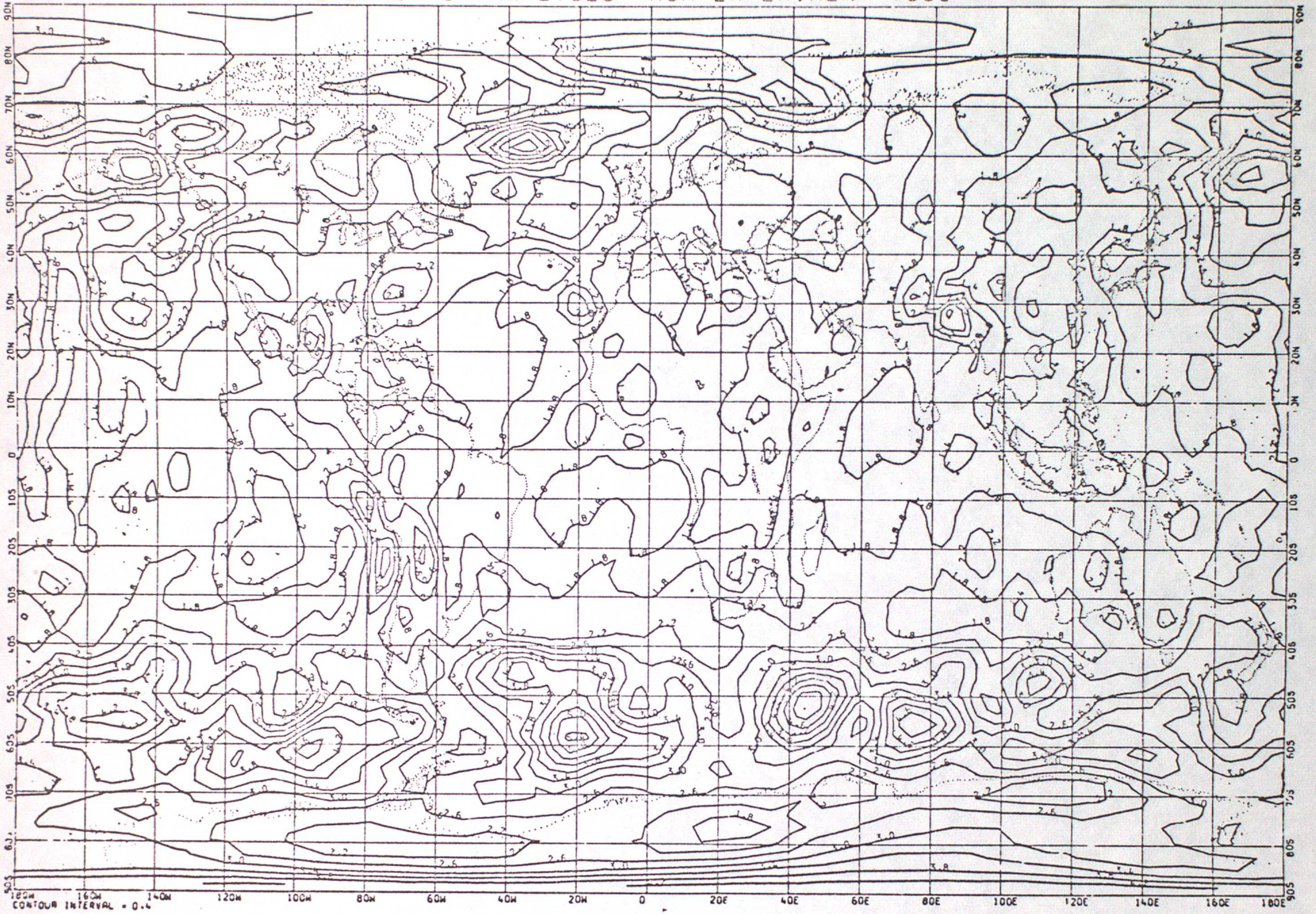


FIGURE 8b

RMS SURFACE PRESSURE ERROR OF ANALYSES FROM EXPERIMENT Y506

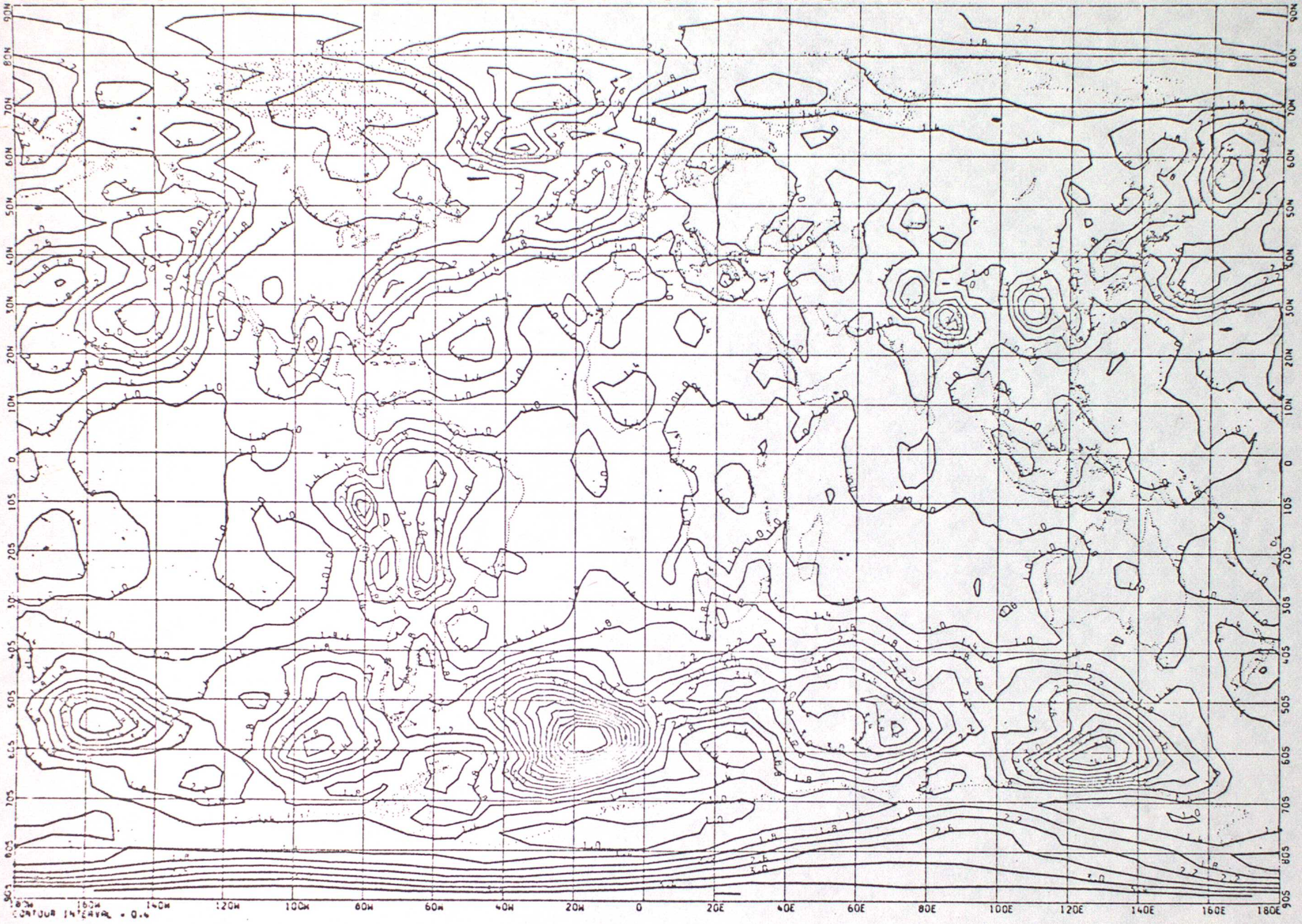


FIGURE 8c