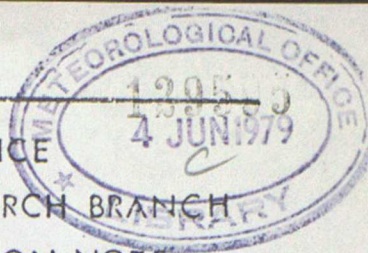


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LINEAR INVISCID FLOWS OVER TOPOGRAPHY IN A PERIODIC
DOMAIN

by

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1. Introduction

The problem of flow over hills has been extensively discussed in the literature, see for example Miles (1969) for a review of earlier work. However most of this effort has been devoted to the two-dimensional problem and the results are well known.

The linear inviscid case is considered here. It is important to appreciate the nature of the fluid flow in this simple case so that the expressly non-linear and viscous effects can be identified in more comprehensive models.

The fundamental solution for a three-dimensional obstacle is due to Wurtele (1956) who deduced the vertical velocity distribution produced by a ground profile given by

$$h(x, y) = h_0 \quad x > 0, |y| < b \\ = 0 \quad \text{otherwise}$$

and his solution is illustrated in Figure 1.

Crapper (1959) succeeded in finding a solution for a hemispherical hill, with a number of interesting properties. In particular suggested that the disturbance to the lee of his isolated obstacle was confined to a "corridor" of width equal to the hill diameter.

Little other work on three dimensions appears in the literature. The results discussed below add a family of periodic solutions, calculated numerically by Fourier Series methods.

For sufficiently long periodicities compared to the hill diameter the results are seen to be very similar to the isolated hump solutions, but for shorter periodicities the flow can change remarkably as the number of free wave modes is restricted.

The variation in the drag on the fluid above is also briefly examined and shows the marked similarity between stratification and rotation effects.

The Model

Consider the dynamical equations for an inviscid incompressible Boussinesq fluid

$$\frac{d\mathbf{u}^*}{dt} + f \hat{\mathbf{e}}_3 \times \mathbf{u}^* + \nabla p^* + \sigma \hat{\mathbf{e}}_3 = 0$$

$$\nabla \cdot \mathbf{u}^* = 0$$

$$d\rho^*/dt = 0$$

where $\sigma^* = g(\rho^* - \rho_0^*)/\rho_0^*$, p^* is the reduced pressure and $\hat{\mathbf{e}}_i, i=1,2,3$ are unit vectors in the coordinate directions, with $\hat{\mathbf{e}}_3$ vertical. All quantities are dimensional.

Then let the basic (zero-order) flow be given by

$$\mathbf{u}^* = u_0 \hat{\mathbf{e}}_1$$

with a flat topography and some stable (or neutral) stratification $\rho_0^*(z)$

We will consider solutions periodic on the boundaries of the square box $[0, L]^2$, with the kinematic boundary condition at the ground surface where $\mathbf{u}^* \cdot \mathbf{n} = 0$, \mathbf{n} is the normal vector.

We transform to non-dimensional variables, putting $\mathbf{u}^* = u_0 \mathbf{u}$, $p^* = u_0^2 p$ etc., using u_0 and L as characteristic scales.

Then considering the topography

$$h = h_0 H(x, y) \quad (x, y) \in [0, L]^2$$

where all quantities are now dimensionless and h_0 is assumed small, we expand the fields as, for example,

$$u = u_0 + h_0 u_1 + h_0^2 u_2 + \dots$$

The first order equations, ie. for u_1, p_1, σ_1 become

$$u_t + u_x - Rv + p_x = 0$$

$$v_t + v_x + Ru + p_y = 0$$

$$w_t + w_x + \sigma + p_z = 0$$

$$\sigma_t + \sigma_x - F^2 w = 0$$

$$u_x + v_y + w_z = 0$$

(1)

subject to the boundary condition (applied consistently at $z=0$),

$$W(x, y, 0) = H_x$$

where $R = Lf/u_0$ (inverse box Rossby number), and $F = LN/u_0$

(inverse box Froude number): all quantities are dimensionless first order perturbations, from which the subscript 1 has been dropped.

We now represent all variables as Fourier Series, eg.

$$W = \sum_{k, l} \bar{W}_{kl} \exp 2\pi i (kx + ly) \quad (2a)$$

where

$$\bar{W}_{kl}(z, t) = \int_0^L \int_0^{L_y} W(x, y, z, t) e^{-2\pi i (kx + ly)} dx dy \quad (2b)$$

Applying the Fourier transform (2b) to the equations (1) and writing

$$\bar{W}_{kl}(z, t) = \tilde{W}_{kl} e^{2\pi i (\omega z - \omega t)}$$

gives a set of linear algebraic equations to solve for \tilde{u}, \tilde{v} etc.

A solution will exist provided the dispersion relation

$$|\hat{K}|^2 (\hat{K} - \hat{\omega})^2 = F^2 (\hat{K}^2 + \hat{l}^2) + R^2 \hat{\omega}^2$$

is satisfied, where $\hat{K} = 2\pi k$, etc.

For steady state solutions, $\omega = 0$, and the solutions become

$$\begin{aligned} \tilde{w} &= i k \tilde{q} & \tilde{u} &= m \frac{(i R l - k^2)}{k} \cdot \frac{\tilde{w}}{k^2 + l^2} \\ \tilde{\sigma} &= -\frac{i F^2 \tilde{w}}{k} & \tilde{v} &= -m (l + i R) \cdot \frac{\tilde{w}}{k^2 + l^2} \\ \tilde{p} &= \frac{k^2 - R^2}{k} \cdot \frac{m \tilde{w}}{k^2 + l^2} \end{aligned}$$

Subject to

$$m^2 (R^2 - k^2) = - (k^2 + l^2) (F^2 - k^2) \quad (3)$$

where

$$H(x, y) = \sum_{k, l} \tilde{h}_{kl} e^{i(kx + ly)}$$

and \wedge have been dropped.

Now for given k, l there are two solutions corresponding to the roots of (3) and both satisfy the lower boundary condition. Thus the problem is indeterminate.

We will use a "radiation condition" that the disturbance either tends to zero as $z \rightarrow \infty$ or only propagates energy upwards, in particular

$$\frac{\partial \omega}{\partial m} = \frac{m}{k} \cdot \frac{k^2 - R^2}{|k|^2} > 0$$

the dimensionless vertical component of group velocity.

The solutions presented here are not critically dependent on the shape of topography and the form of h used is given by

$$\begin{aligned} h(x, y) &= h_0 \cos^2 \frac{\pi r}{(L_i/L)} & 0 < r < \frac{L_i}{2} \\ &= 0 & \text{elsewhere} \end{aligned}$$

and $r^2 = (x - \frac{1}{2})^2 + (y - \frac{1}{2})^2$, where L_i is the dimensional hill diameter, and we consider a square domain

In this case, notice that the first order flow can only depend on $(R, F, L/L_i)$. Variations of h_0 only change the amplitude scaling of the flow.

Section 2 will present stratified flow types ($R=0$) and Section 3 similar rotational solutions ($F=0$). Mixed modes ($R \neq 0 \neq F$) will be discussed in Section 4 but consideration of the drag force will be deferred will be deferred until Section 5.

2. Solutions in the absence of Rotation

In the absence of rotation, F determines the free wave modes capable of being supported by the system. We can identify three distinct types of motion: the case of F large when many waves are permitted and a classical lee-wave structure develops; that in which only one or two streamwise wavenumbers correspond to waves and the flow field takes an interesting form described below; and for small F with all waves evanescent, the disturbance decays rapidly with height.

Note that the Fourier Series have been truncated after the 64th wavenumbers in the x - and y - directions in the flows presented below.

a) Figs (2) and (3) show two vertical velocity fields for large values of F . The flow is imilar to the Wurtele solution of Fig.(1) and the maximum disturbance seems to lie about a line from the hill running downstream and making some angle with the horizontal. In the 2-D case of a ridge this angle increases steadily with increasing F (L/L_i held constant) and the results presented have shown the same trend over the range of parameters considered, (cf. Section 5). The flow close to the ground responds most directly to the forcing and is disturbed only locally by the hill. The fluid ascends and descends on the upstream and downstream faces.

b) For F not much greater than 2π only $k = 1$ waves can exist. This produces an interesting flow pattern which is dominated except at very low heights by these waves. Fig (5) shows the flow at an intermediate height where the evanescent modes are still important but the flow is not constrained too tightly by the bottom topography. At greater heights the evanescent modes become negligible relative to the $k = 1$ waves and the flow takes the form shown in Fig (4). The disturbance is then confined to the neighbourhood of two planes intersecting at the hill centre line and inclined at a certain angle to the horizontal. This angle appears to be given by and there is a suggestion that this is indeed the correct form from theoretical considerations given in the Appendix. Due to the periodic nature of the problem the planes originating from other hills in the cross-stream direction will enter the domain of integration at greater heights. However it is the periodicity in the x -direction which is crucial for the flow as the Appendix makes clear.

c) For $F < 2\pi$ there can be no wave modes and the flow is dominated by the bottom topography at low heights as in Fig. 6, the perturbation decaying rapidly with height.

3. Solutions in the absence of stratification

In this case R has a similar role to that of F in the previous section, ie. for small R very few wave modes will be permitted.

- a) For large R many waves are possible and the inertial wave structure shown in Fig. 7 will develop. The pattern is rather different from the gravity wave case of Fig. 3, say, and the disturbance propagates upwards at an angle which increases with R . At sufficiently large heights the disturbance shape splits (see Section 4). Near the hill itself the flow is dominated by the surface impulse and flows up and down the faces.
- b) For R not much greater than 2π the flow is dominated except very low down by the $k = 1$ modes. Again the disturbance lies about planes which slope outwards from the hill as in the stratified case.
- c) For small R all the modes are evanescent and the disturbance decays rapidly with height. Fig. 10 shows the vertical velocity high above the hill where the amplitude of the perturbation is much diminished from its surface value and the horizontal scale of the disturbance spreads to take up the whole box.

4. Solutions in the presence of rotation and stratification

In the general case the number of free wave modes are determined by $|F - R|$. Waves will be possible for (k, l) such that k lies strictly between F and R , where k is the angular wavenumber. Thus for $|F - R|$ small or F and R sufficiently large that there is negligible amplitude in the wave modes the flow is essentially the evanescent pattern shown in Figs. 6 and 10, and for an actual mixed case in Fig. 14.

Consider the sequence Figs. 15-17 where a fully developed gravity wave pattern is compared with the flow resulting from a case with the same stratification but a moderate rotation. Fig. 15(a)-(c) shows the disturbance propagating upwards and spreading in the absence of rotation. Fig. 16 shows how rotation can modify this flow - the disturbance propagates at a smaller angle and the characteristic "Bow wave" is straightened out. In this case the perturbation does not spread laterally either, until the height is sufficiently large when the disturbance apparently "splits" as the dominant waves subtly change. This will also occur in the purely rotating case, as in Fig. 17(c).

Fig. 17(a) clearly shows the effect that stratification in Fig. 16 has on the flow pattern. The disturbance tilts at a shallower angle and is "straightened out" from the purely rotating case.

Finally Figs. 18(a)-(d) show a clear example of the change in the shape of the perturbation with height in a mixed case.

5. Variation of drag force with R and F

The drag force is given by equation (4)

$$F_{\text{drag}} = \rho_0 u_0^2 (h_0 L)^2 (Im) \sum_{k,l} k \tilde{p}_{ke} \tilde{h}_{-k,-l}$$

in the x direction, taken positive when the fluid is being retarded by the obstacle.

Now substituting for \tilde{p}_{ke} and using $\tilde{h}_{-k,-l} = \tilde{h}_{kl}$ for a symmetric hill,

$$\frac{F_{\text{drag}}}{\rho_0 u_0^2 (h_0 L)^2} = \sum_{k,l}^{\text{wave modes}} \frac{|(R^2 - k^2)(F^2 - k^2)|^{\frac{1}{2}} \cdot |k| \cdot \tilde{h}_{kl}^2}{(k^2 + l^2)^{\frac{1}{2}}} \quad (5)$$

where the evanescent terms do not appear as they do not contribute to the drag.

Notice that the right hand side of (5) is symmetric in F and R and for the case of either F or R zero it is sufficient to consider R=0 only.

Fig 11(a) shows the drag on a hill ($L/L_i = 20$) as a function of F_i . The drag is zero below $F=2\pi$ when the first wave modes appear. Then the drag steadily increases tending to infinity as $F \rightarrow \infty$. Fig 11(b) shows F_{drag}/F_i as a function of F_i and suggests that as $F \rightarrow \infty$

$$F_{\text{drag}} \sim \phi(L/L_i) \cdot F_i$$

where ϕ is an unknown function.

Fig 13 shows the variation of drag for $L/L_i = 10$ as a function of both F & R. The drag is zero about the line $F=R$ depending on whether wave modes are permitted - waves can exist only for k lying strictly between F and R. Hence the drag increases as $|F-R|$ increases.

6. Group velocity considerations

In many cases some understanding of the behaviour of the flow can be deduced from consideration of the kinematics of the waves excited by the obstacle. We may assume that the hill is the natural source of any waves excited, and then the group velocity of some characteristic mode gives the direction of energy propagation.

A number of related results are collected below. The group velocity of a stationary wave with angular wavenumber $\underline{k} = (k, l, m) = \underline{k}_H + m \hat{k}$ where \hat{k} is the unit vertical vector, is given in dimensionless form by

$$\underline{v}_g = \hat{i} - \frac{m^2(F^2 - R^2) \cdot \underline{k}_H}{k |\underline{k}|^4} - \frac{|\underline{k}_H|^2 (R^2 - F^2) \cdot m \hat{k}}{k |\underline{k}|^4}$$

The angle θ this vector makes with the horizontal is given by

$$\tan^2 \theta = \frac{-(F^2 - k^2)(R^2 - k^2)^3}{k^2 |\underline{k}_H|^2 (R^2 - F^2)^2 + (F^2 - k^2)(R^2 - k^2)(k^4 + k^2(F^2 - 3R^2) + F^2 R^2)}$$

Examination of the distribution of Fourier coefficients of the vertical velocity field suggests that when a spectrum is excited by a hill of length L_1 it can be characterised by the mode with horizontal wavenumber $\underline{k}_H \sim (\frac{L_1}{L}, 0)$

Then we have

$$\tan^2 \theta \sim F_i^2 - 1 \quad \text{if } R = 0$$

$$\sim (R_i^2 - 1)^3 / (4R_i^4 - 4R_i^2 + 1) \quad \text{if } F = 0$$

Note that the propagation direction tends to the vertical as the stratification or rotation increases in each case. In the general case we have

$$\tan^2 \theta \sim \frac{(F_i^2 - 1)(1 - R_i^2)^3}{(1 - 4R_i^2 + 4R_i^4) + F_i^2 R_i^2 (2 - 4R_i^2 + F_i^2 R_i^2)}$$

which has the interesting property that

$$\theta \rightarrow \pi/2 \quad \text{as } R_i \rightarrow \infty, \forall F_i \neq 1$$

but $\theta \rightarrow 0$ as $F_i \rightarrow \infty, \forall R_i \neq 0$.

Thus the $R=0$ behaviour appears to be singular.

This will not be explored further here.

Finally we note that in the rotational case all wave modes satisfy

$$\theta \rightarrow \pi/2 \quad \text{as } R_i \rightarrow \infty$$

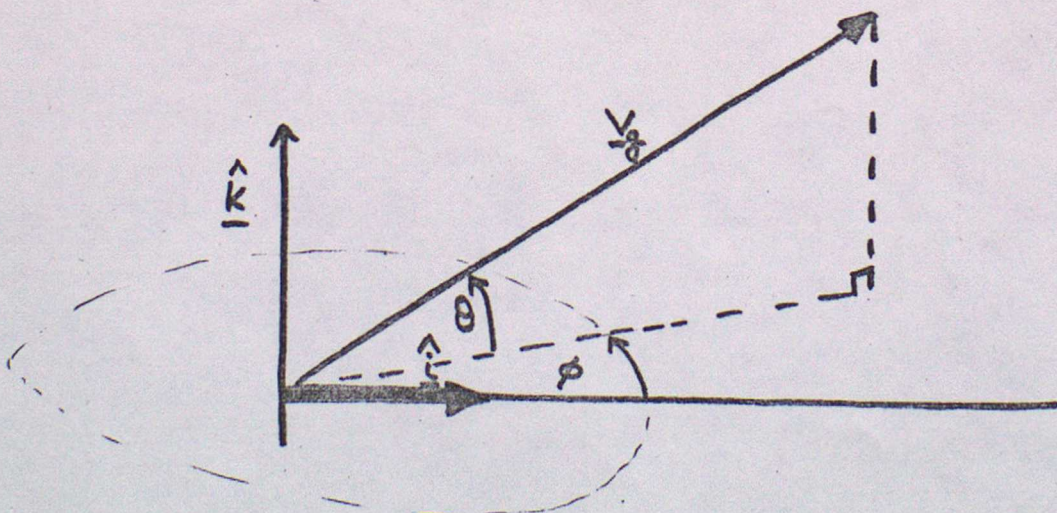
Numerical results suggest that as L increases the original dominant mode relaxes and two symmetric modes can be identified propagating upwards and at an angle to the streamwise direction - cf. Figs. 17-18.

Clearly to fully characterise the propagation direction we must have another angle. It is convenient to choose the angular displacement ϕ of the projection of \underline{v}_g on the horizontal from the \hat{e} direction.

Then

$$\tan \phi = -\frac{e}{k} \cdot \frac{1}{1 - \frac{|\underline{k}_H|^2 (R^2 - F^2)}{(F^2 - k^2)(R^2 - k^2)}}$$

Thus in general only $l=0$ waves propagate in the streamwise direction.



DEFINITION SKETCH FOR θ, ϕ .

Appendix 1. Strongly periodic flow.

Consider the model of Section 1 with periodic boundary conditions in x only and the solution bounded as $|y| \rightarrow \infty$

Let the topography be given by

$$h = h_0 H_x(x) H_y(y) \quad x \in [0, 1]$$

Then clearly the Fourier representation will involve an integral transform in y so that (2) becomes

$$W = \sum_k \int_{-\infty}^{\infty} \tilde{W}_{kl}(z) e^{2\pi i(kx + ly)} dk dl \quad (6a)$$

where

$$\tilde{W}_{kl} = \int_0^1 \int_{-\infty}^{\infty} w(x, y, z) e^{-2\pi i(kx + ly)} dx dy$$

there are no implicit factors of 2π , and we have non-dimensionalised with the length L .

If we choose $F \in (2\pi, 4\pi)$ then only one wave mode can exist. Provided we consider z such that the evanescent modes are negligible (this requires $|F^2 - 16\pi^2|^{1/2} z \gg 1$) then (6a) is given approximately by

$$W = -8\pi \operatorname{Re} \int_0^{\infty} \tilde{h}(1, l) \sin(2\pi x + mz) e^{2\pi i ly} dl$$

where

$$m = (F^2 - 4\pi^2)^{1/2} (1 + l^2)^{1/2} = \alpha (1 + l^2)^{1/2}$$

Factoring \tilde{h} and choosing $H_y(y) = K_0(|y|)$, where K_0 is the Kelvin function of order zero, we find

$$W = \begin{cases} +4\pi^2 \tilde{H}_x(1) [\sin 2\pi x \cdot J_0(Y) + \cos 2\pi x \cdot Y_0(Y)] & 2\pi y < \alpha z \\ -8\pi \tilde{H}_x(1) \cos 2\pi x \cdot K_0(Y) & 2\pi y > \alpha z \end{cases} \quad (7)$$

$$Y = |\alpha^2 z^2 - 4\pi^2 y^2|^{1/2}.$$

and

where J_0, Y_0 are Bessel functions of the first and second kind.

A section through this solution is shown in Fig 12. We note that this solution has intensity in sheets which slope at an angle θ given by $\tan \theta = 2\pi/\alpha$ from the horizontal.

Solutions for the \cos^2 topography for a range of F show that the amplitude peaks sharply in narrow slabs inclined at approximately this angle.

For more general topography, a stationary phase approximation to the integral gives

$$W = \frac{\sqrt{\pi}}{2\beta} \phi(l_0) \sin \left(f(l_0) + \frac{\pi}{4} + 2\pi x \right) \quad 2\pi y < \alpha z \quad (7)$$

$$= 0 \quad 2\pi y > \alpha z$$

where

$$\beta = (\alpha^2 z^2 - 4\pi^2 y^2)^{3/4} / \sqrt{2} \alpha z$$

$$\phi(l) = -8\pi \cdot \tilde{H}_x(l) \cdot \tilde{H}_y(l)$$

$$l_0 = 2\pi |y| (\alpha^2 z^2 - 4\pi^2 y^2)^{-1/2}$$

$$f(l_0) = \frac{4\pi^2 y^2 + \alpha^2 z^2}{(\alpha^2 z^2 - 4\pi^2 y^2)^{1/2}}$$

which also shows a tendency to singular behaviour approaching the plane with tilt $\tan^{-1} 2\pi/\alpha$ depending on the rate at which $\phi(l) \rightarrow 0$ as $l \rightarrow \infty$. This approximation cannot be uniformly valid as $y \rightarrow \alpha z/2\pi$ but clearly illustrates the peculiar importance of this plane in the strongly periodic problem.

Note that (7) suggests a sufficiently slowly decaying topography as $z \rightarrow \infty$ will not have a maximum at $\tan \theta = 2\pi/\alpha$.

Appendix 2

THE MODEL AS AN APPROXIMATE DESCRIPTION OF A REAL FLUID.

The assumptions on which the model rests and the conditions that must be satisfied for the first order solution to give a good approximation to the solution of the non-linear equation are generally well known. However a brief summary is included below.

- (i) Inviscid, incompressible, Boussinesq.
- (ii) Rotation about vertical; constant f .
- (iii) Constant N .
- (iv) Unbounded atmosphere.
- (v) Periodic horizontal boundary conditions.
- (vi) Radiation condition at upper boundary.
- (vii) Steady problem.

The constraints under which the first order solution (as presented) would be expected to give a good approx. to the full solution:

- (i) $|h_0 u_1| \ll 1$ $|h_0 w_1| \ll u_1, u_2$ linearisation requirements.
- (ii) e'/e_0 ; Linearisation of $d\rho_0/dt = 0$.
- (iii) $h_0(u_1 \cdot \nabla H) \ll 1$ $h_0 \partial u_1 / \partial z \ll u_1$; boundary condition,

to which must be added the more general constraints.

- (iv) $L_z \ll H_s$, where H_s is a scale height for the ρ_0 density distribution; due to incompressibility and Boussinesq approximation.

- (v) $Re = \frac{uL}{\nu} \gg 1$ well known inviscid approximation. The effects of any boundary layers on the interior flow are ignored.

If we assume that α is a characteristic (dimensionless) horizontal hill scale, $\left(\frac{L_i}{L}\right)$ and that $K \sim \alpha^{-1} \sim l$, then

$$W_1 \sim \alpha^{-1}$$

$$u_1 \sim \left| \frac{(F^2 \alpha^2 - 1)(R \alpha - 1)}{R \alpha + 1} \right|^{\frac{1}{2}} \alpha^{-1} \sim v_1$$

$$G_1 \sim F^2$$

$$\phi_1 \sim |(R^2 \alpha^2 - 1)(F^2 \alpha^2 - 1)|^{\frac{1}{2}} \alpha^{-1}$$

provided $F \alpha \neq O(1) \neq R \alpha$, when the dominant coefficients are rapidly changing, and more care is required; (N.B. $F \alpha = F_i$, $R \alpha = R_i$).

Using m to indicate a vertical scale L_z we can deduce table 1 below.

Notice that the vertical scale decreases with increasing stratification F_i and that in the "homogeneous" regime $L_z \sim \alpha$, approaching the potential flow.

However the limits $R_i \rightarrow \infty$, and $F_i \rightarrow \infty$ may not be represented by this model for finite h_0 due to the linearisation constraints.

The consequences of these restrictions are summarised in Table 2. In particular, notice the stratification constraint $h_0 \ll \alpha \bar{F}_i$ which prevents consideration of the limit $F_i \rightarrow \infty$ for finite h_0 .

Table 1 Variation of the (dimensionless) vertical scale $L_z^* = L L_z$ with magnitude of $\alpha R (= R_i)$ and $\alpha F (= F_i)$.

L_z^* F_i	R_i	$> O(1)$	$< O(1)$
$> O(1)$	$\frac{L_i F}{N}$	$\frac{L_i F}{N}$	$\frac{L_i}{N}$
$< O(1)$	$R_i L_i$	L_i	L_i

Table 2. (i) Linearisation constraints on the magnitude of h_0 : h_0/α is the characteristic slope of the topography.

$F_i \backslash R_i$	$> O(1)$	$< O(1)$
$> O(1)$	$\frac{h_0}{\alpha} \ll F_i^{-1}$	$\frac{h_0}{\alpha} \ll F_i^{-1}$
$< O(1)$	$\frac{h_0}{\alpha} \ll 1$	$\frac{h_0}{\alpha} \ll 1$

(ii) $h_0^* \ll H_s$

(iv) $L_z^* \ll H_s$

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Fig 1.

Vertical velocity in the plane $z = 2$ from Wurtele, where $L = u_0/N$ and $W^* = 8\pi^2 h_0 b u_0 I$, and I is plotted.

Figs 2-10.

Vertical velocities in plane $z = z_0$, max modulus \hat{w} normalised to surface velocity quoted for each case, with parameter values chosen.

Fig 11.(a) and Fig. 11(b)

Variation of drag with F ; $R=0$

Fig 12.

Section in plane $x = \frac{1}{8}$, $y > 0$, through the solution given by equation (7)

Fig 13

Contours of drag as function of F and R with $L/L_1 = 10$

Figs 14-18

Vertical velocities in planes $z = z_0$, max modulus \hat{w} normalised to surface velocity quoted in each case, with parameter values chosen.

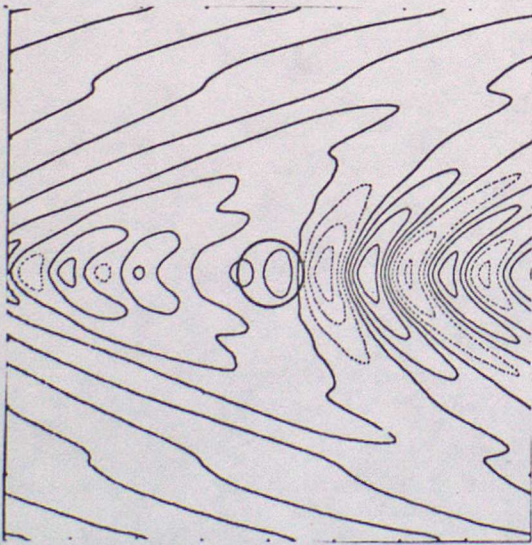
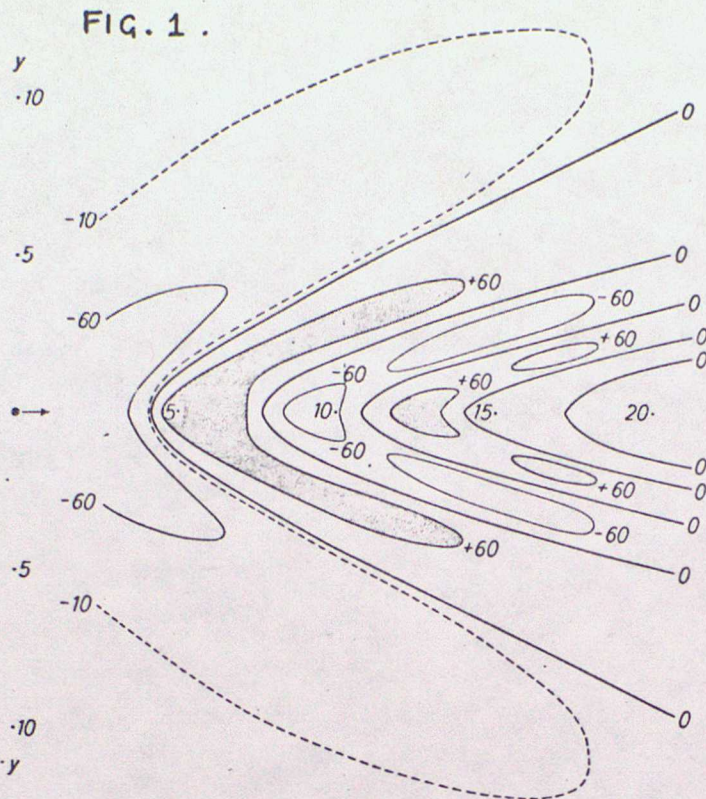


FIG. 2

$$F_i = 6 \quad \alpha = \frac{L}{L_i} = 8 \quad F = 48$$

$$\hat{W} = 0.22 \quad z_0 = .125$$

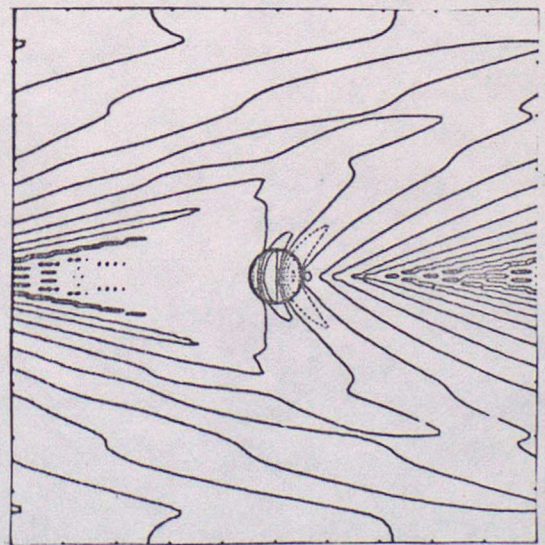


FIG. 3

$$F_i = 100 \quad \alpha = 10 \quad F = 1000$$

$$\hat{W} = 0.63 \quad z_0 = .005$$

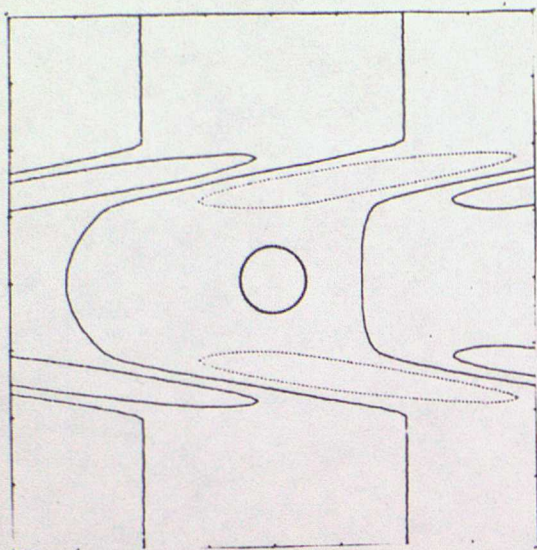


FIG. 4

$$F_i = 1 \quad \alpha = 8 \quad F = 8$$

$$\hat{W} = 0.015 \quad z_0 = .250$$

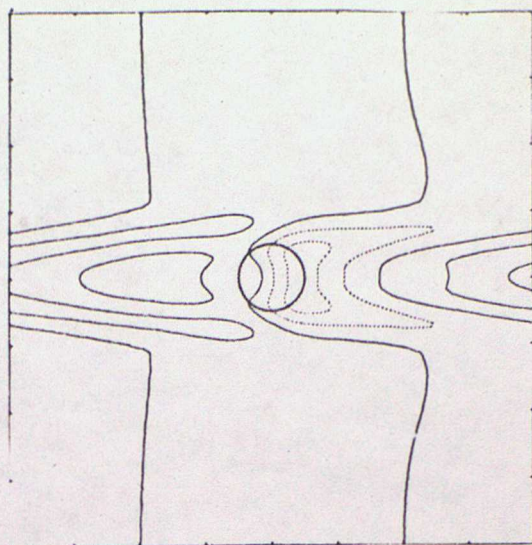


FIG. 5

$$F_i = 1 \quad \alpha = 8 \quad F = 8$$

$$\hat{W} = 0.015 \quad z_0 = .100$$

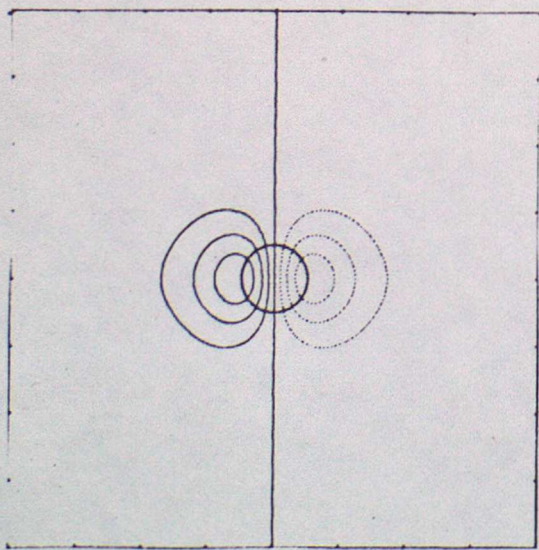


FIG. 6

$$F_i = 1/3 \quad \alpha = 8$$

$$F = 8/3$$

$$\hat{W} = 0.0093$$

$$z_0 = .125$$

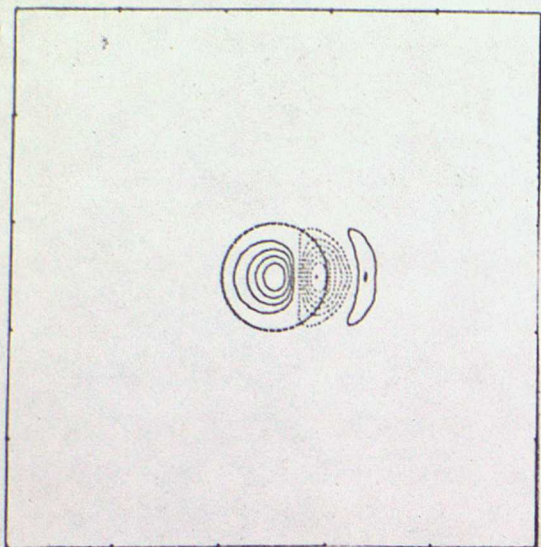


FIG. 7

$$R_i = 100 \quad \alpha = 5 \quad R = 500$$

$$\hat{W} = 1.1$$

$$z_0 = 0.5$$

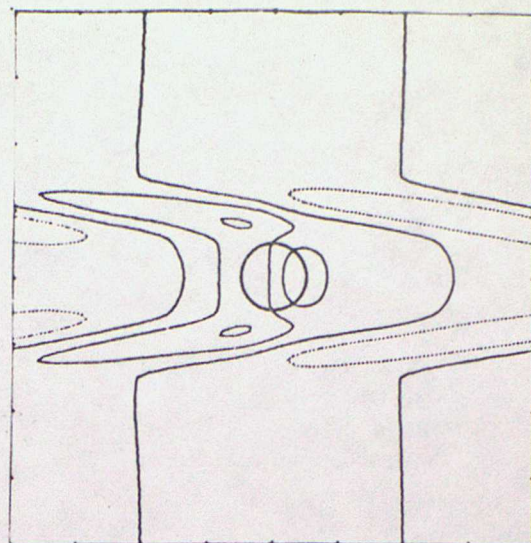


FIG. 8

$$R_i = 1 \quad \alpha = 8 \quad R = 8$$

$$\hat{W} = 0.018$$

$$z_0 = 0.1$$

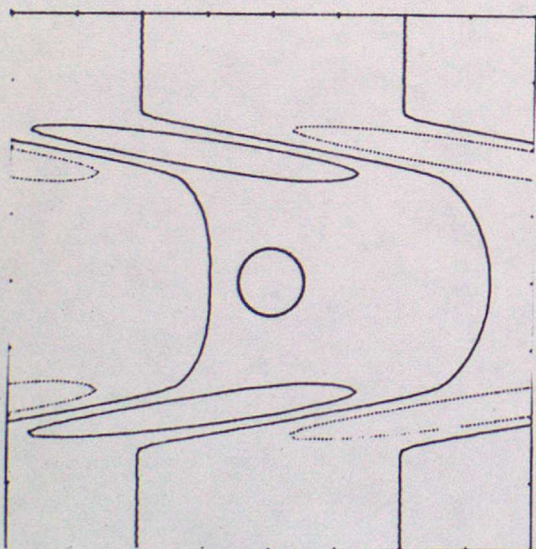


FIG. 9

$$R_i = 1 \quad \alpha = 8 \quad R = 8$$

$$\hat{W} = 0.015$$

$$z_0 = 0.2$$

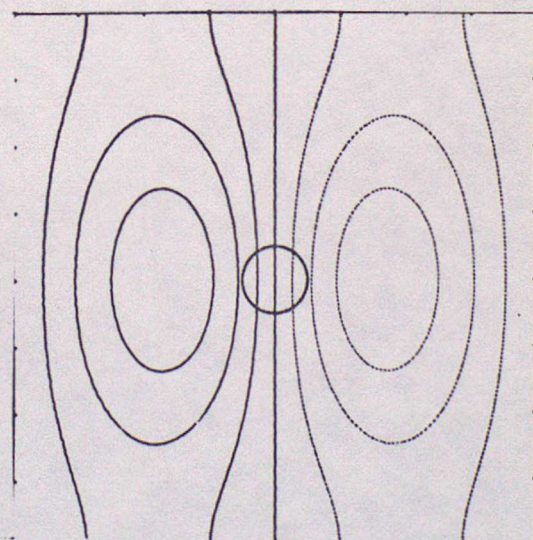


FIG. 10

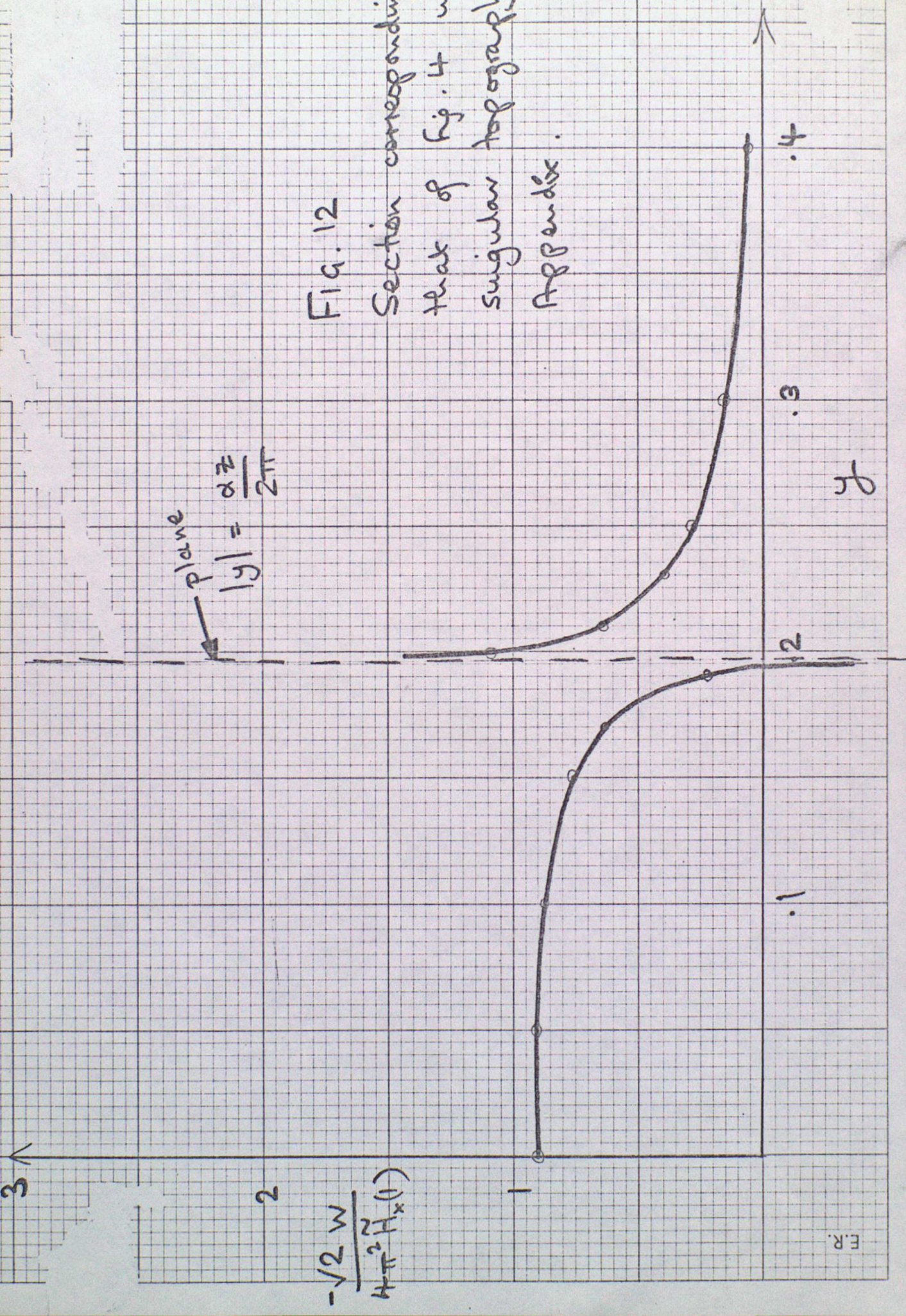
$$R_i = 1/3 \quad \alpha = 8 \quad R = 8/3$$

$$\hat{W} = 8.7 \times 10^{-5}$$

$$z_0 = 0.5$$

Fig. 12

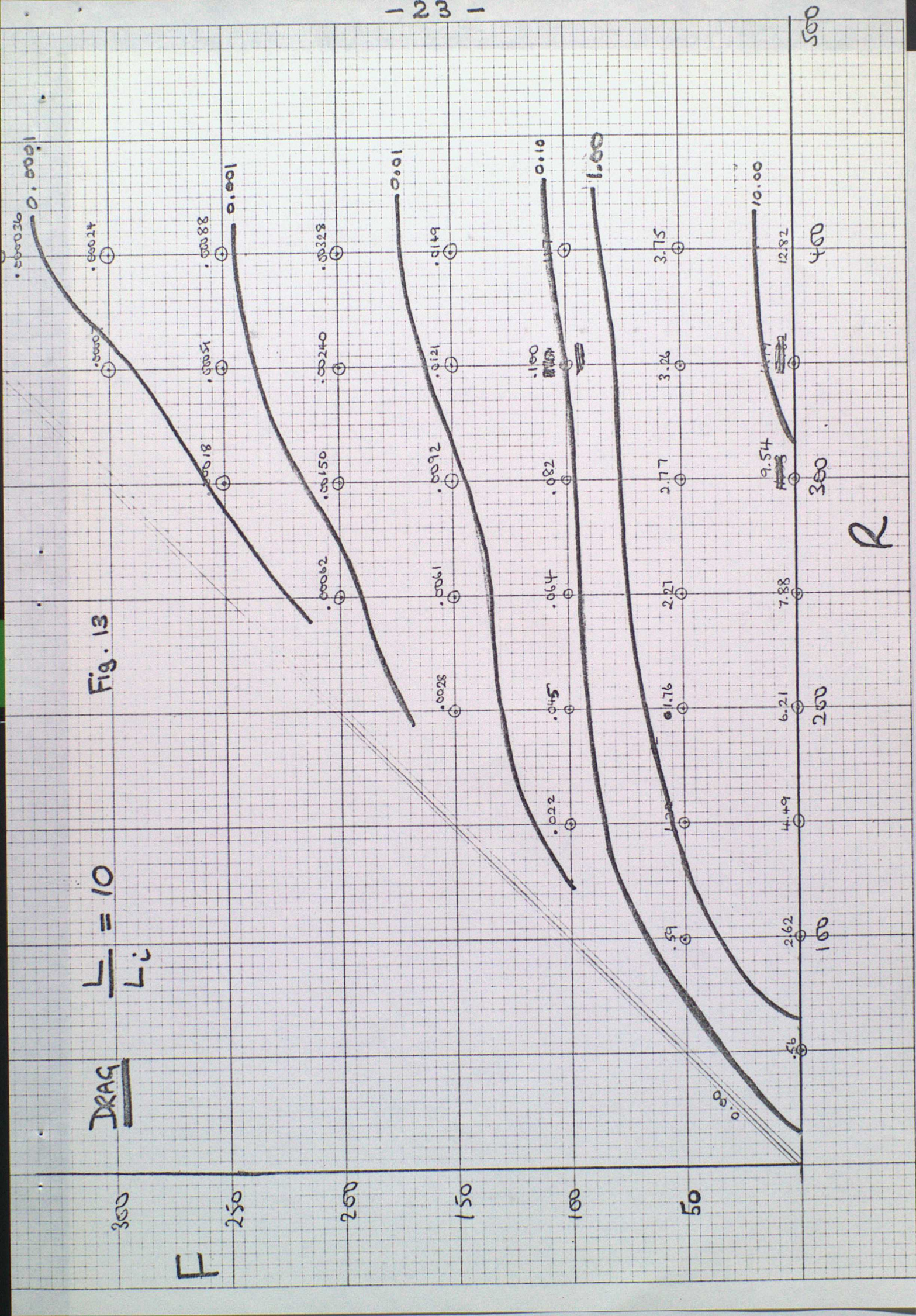
Section corresponding to
that of Fig. 4 with
singular topography of
Appendix.



$$\frac{L}{L_i} = 10$$

DRAG

Fig. 13



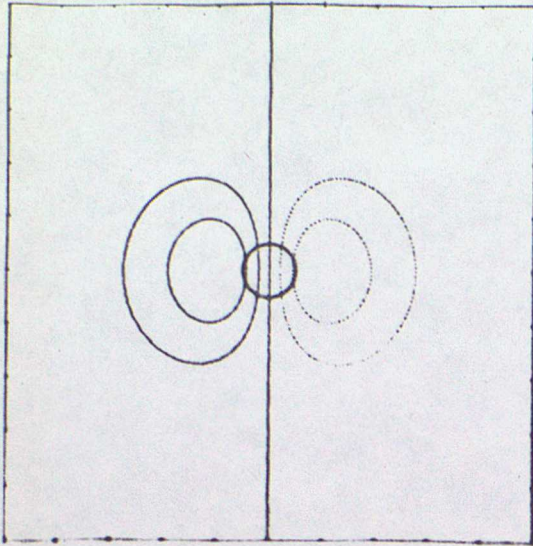


FIG. 14

$$F_i = 200 \quad R_i = 400$$

$$\alpha = 10 \quad z_0 = .4$$

$$\hat{\omega} = 1.22 \times 10^{-3}$$

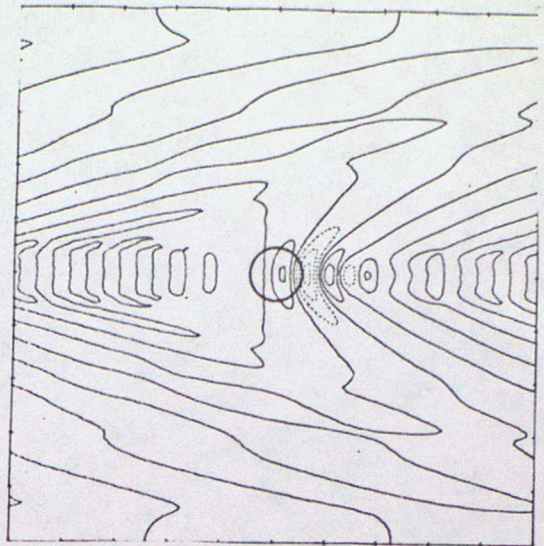


FIG. 15(a)

$$F_i = 10 \quad \alpha = 10 \quad R_i = 0$$

$$z_0 = .05$$

$$\hat{\omega} = .59$$

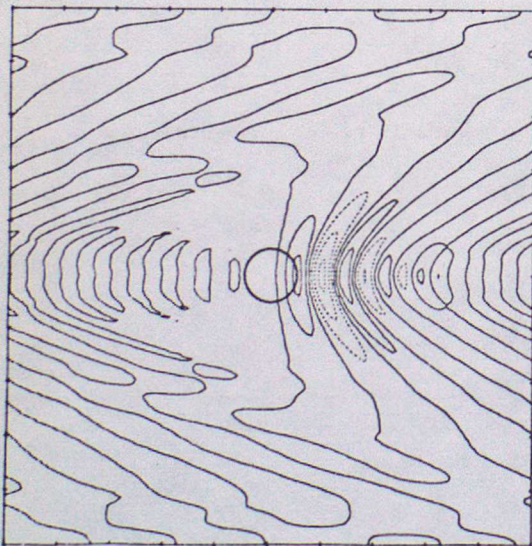


FIG. 15(b)

$$z_0 = .1$$

$$\hat{\omega} = .38$$

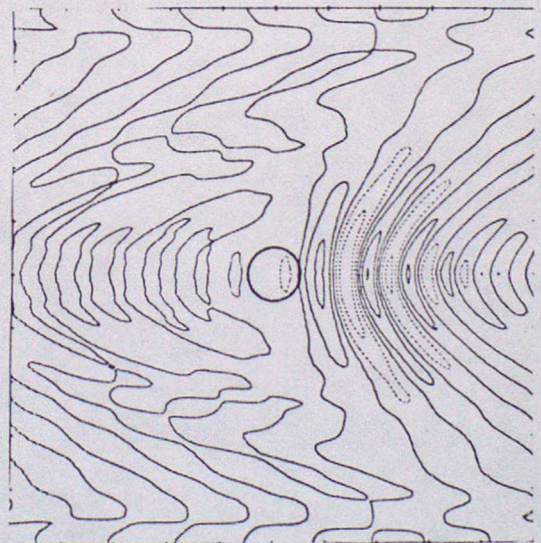


FIG. 15(c)

$$z_0 = .15$$

$$\hat{\omega} = .27$$

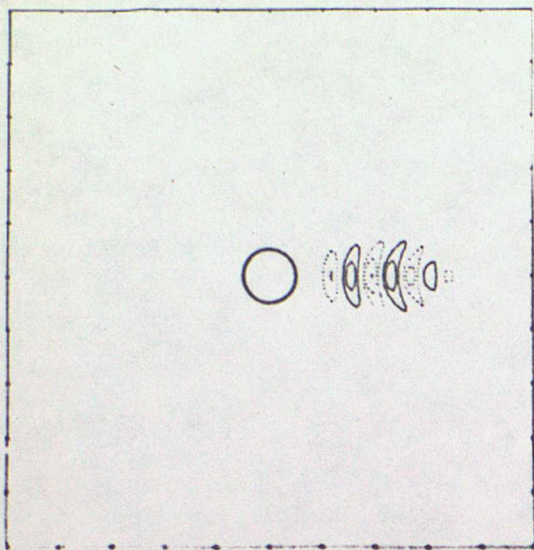


Fig. 16

$$F_i = 10 \quad R_i = 6 \quad \alpha = 10$$

$$z_0 = .05 \quad \hat{w} = .308$$

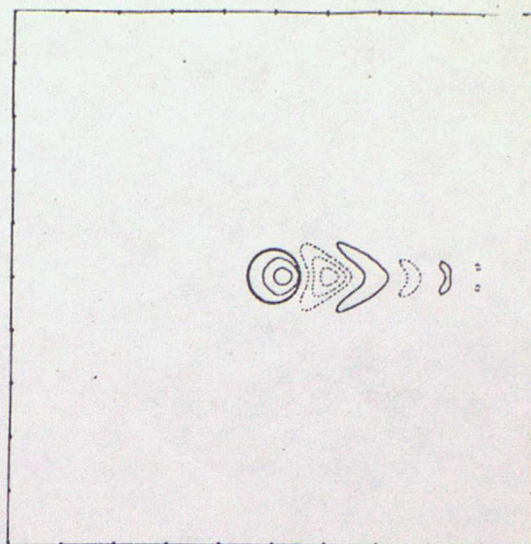


Fig. 17(a)

$$R_i = 6 \quad \alpha = 10 \quad F_i = 0$$

$$z_0 = .05 \quad \hat{w} = .369$$

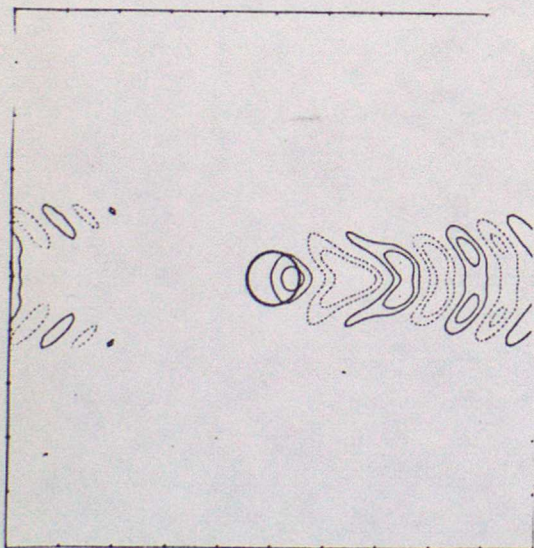


Fig. 17(b)

$$z_0 = .1$$

$$\hat{w} = .185$$

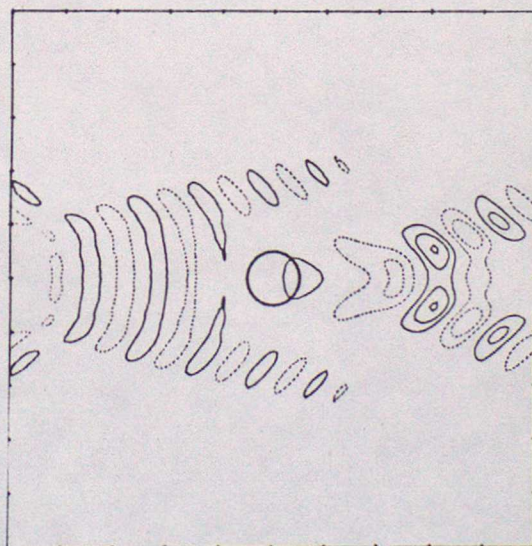


Fig. 17(c)

$$z_0 = .2$$

$$\hat{w} = .158$$

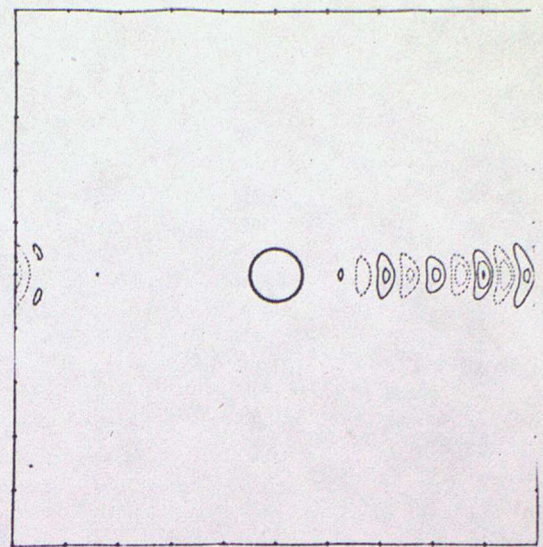
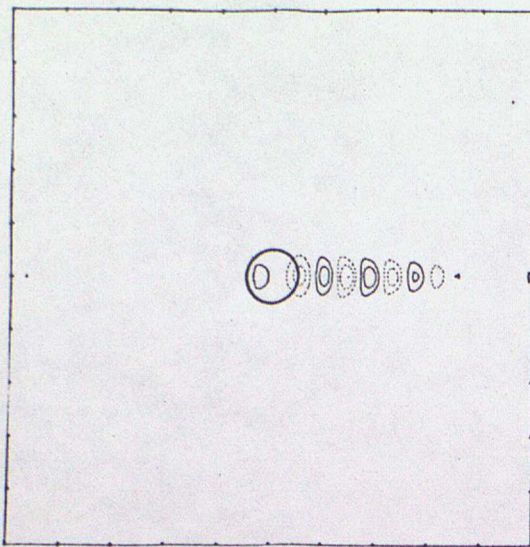


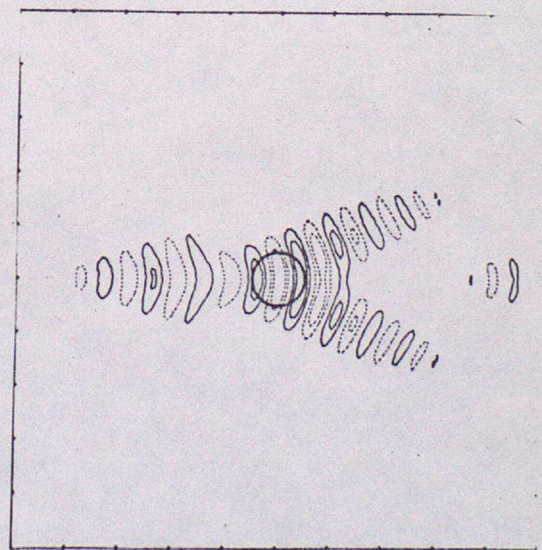
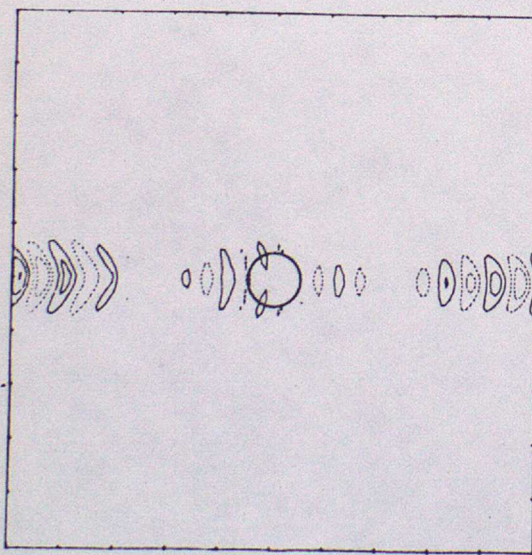
Fig. 18 (a)

$$F_i = 6 \quad \alpha = 10 \quad R_i = 10$$

$$z_0 = .05 \quad \hat{w} = .376$$

(b)

$$z_0 = .1 \quad \hat{w} = .305$$



(c)

$$z_0 = .15$$

$$\hat{w} = .232$$

(d)

$$z_0 = .3$$

$$\hat{w} = .170$$