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On Magnetic Fluctuations and Sunspot Frequency

A Discussion based primarily on the daily ranges of Declination as recorded at Kew Observatory, Richmond, during the 67 years 1858-1924

By J. M. STAGG, M.A., B.Sc.

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ABSTRACT

Absolute daily ranges of declination at Kew Observatory have been measured from 1916 to 1924, and fresh means for the years 1911 to 1915 derived from the published values in the Meteorological Office Year Books for these years. Using a factor determined from measurement for eight months in 1873 and 1875 of Greenwich magnetograms a mean value for each month of 1874 has been derived and thus a complete and homogeneous series of absolute daily ranges provided for the 67 years 1858–1924. A subsequent comparison with declination traces at Greenwich Observatory affords no evidence that any error was introduced in the recent values of Kew ranges by local artificial disturbances.

A short analysis by Fourier methods of the final annual variation of the range for the entire 67 year series and separately for the 21 years of greatest sunspot frequency and 21 years of least frequency in the period has been made.

With another long range measure of magnetic fluctuations in the form of interdiurnal variability of horizontal force available, comparison between large scale variations in solar activity (as judged by sunspot frequency) and variations in D-range and H-variability becomes possible over six sunspot cycles. Outstanding features in the comparison have been noted and the more conspicuous differences in behaviour throughout individual cycles examined.

Assuming the validity of such a relation as $R=a+bs$ connecting sunspot frequency on the one hand and range (or variability) statistics on the other, the 67 years' data have been treated numerically. By "least square" methods correlation coefficients and values of the parameters a and b in the equation have been obtained for each mean monthly and annual series. The results have been compared with those derived by the method of differences. Some features of interest in the annual variation of the two quantities together with that of the ratio $10^2b/a$ are discussed.

Values of the parameters have also been obtained for each of the separate cycles and investigations made to see how these quantities vary with the sunspot frequency for each cycle. This has been done from the interdiurnal variability of H statistics, for the international character figures from 1906, and absolute range data from Cheltenham Observatory from 1902 to 1922. A similarity of general behaviour results from the treatment of these diverse measures of magnetic variations but an apparent divergence from the conclusions drawn from the treatment of similar material by Dr. Bauer and Mr. Duvall seems necessary.

The possible existence of a secular change in means of declination range (and also H-variability) ther than that due to solar activity has been examined, using the values of the parameters a and b previously deduced to set each cycle-mean on a common sunspot basis. Though means derived from the substitution of force equivalents ($H\triangle D$) in γ for the ranges are used, no definite conclusions can be drawn regarding the existence of a progressive secular change in either of the magnetic measures used.

PART I.

COMPLETION AND DISCUSSION OF THE SIXTY-SEVEN YEARS' SEQUENCE OF MEASURES OF DECLINATION RANGE

§ 1. Introduction.—Long Range Statistics of Magnetic Variations.

It has been known that magnetic events on the earth's surface are more or less intimately related with changes which are observed to take place on the sun, particularly those outbursts of activity associated with the appearance of sunspots. Whether the magnetic variations owe their origin solely and immediately to the sunspots or, which is a more likely contingency, both phenomena are related to a common source still remains obscure. Various measures of the solar changes have been correlated with magnetic data but the results shed no more hopeful light than that derived from the use of sunspot "Relativzahlen" published by Wolf and his successor Wolfer at the Zürich Observatory. One difficulty which has hampered

(33.) Wt. 25548—15278/1698. 625. 7/27. T.S. 66.

research in this direction has been the lack of a homogeneous and sufficiently extensive series of magnetic measures; for, with the exception of the discussion of some Greenwich statistics by W. Ellis (1) and those published by Rajna (2) for Milan Observatory, investigations have in general been confined to the events of a few decades. Recently, however, some magnetic data have become available which extend over a longer period than any hitherto published and it is proposed to utilize these in an examination of the relationship between the solar and terrestrial variations.

Working on a suggestion of Dr. A. Schmidt at Potsdam Observatory to utilize day to day fluctuations in the mean value of the horizontal component of the earth's field as a measure of magnetic activity, Dr. J. Bartels (3) has had these daily estimates made for the five stations, Bombay, Batavia, Honolulu, Porto Rico, and Potsdam for each month of the years 1872-1920 and for Potsdam alone for the three years 1921 to 1923. Further, annual means based on Wolf's values of the daily range of declination at various continental observatories from 1836 to 1871 have been standardized to form a continuous sequence with Bartels's figures and thus furnish a reasonably consistent measure of fluctuations of magnetic activity from 1836 to 1923.

Although not professing to furnish a value of " activity " with the same claims to real physical significance as, say, the sum of the squares of the hourly ranges of the three orthogonal components or even the squares of the daily ranges as computed by Dr. A. C. Mitchell, it has been shown that means of absolute daily ranges of a single component (read simply as instantaneous maximum *minus* minimum values for a day) provide over a set of years a good indication of the variations of magnetic activity in those years. Hence since absolute ranges of declination at Kew Observatory had already been measured for the years 1858 to 1900 (4) and since the series was subsequently extended to 1910 (5) it was thought that the extension of the series up to the time of discontinuation of magnetic work at the Observatory in January 1925 would provide material for a good basis of comparison with the figures for the interdiurnal variability of H published by Bartels. If a similarity of sequence resulted, the rigour of the examination of the parallelism of sunspot measures and magnetic phenomenon from either or both " activity " measures would be enhanced. Daily ranges of declination were accordingly measured for the remaining years 1911 to 1924 and monthly means formed. The daily values for a few of the earlier of these years had been previously determined and the monthly means appeared in the *British Meteorological and Magnetic Year Book*, Part III, Section 2, 1911-15. Fresh means to two places of decimals were computed from these to bring the entire series into line. Thus with the exception of one year, 1874, monthly values of the absolute range of D at Kew Observatory existed for the 67 years 1858 to 1924.

§ 2. The Hiatus of 1874 in the Kew Data.

During the greater part of 1874 the Kew magnetograph system was out of action, and since at most about 50 days were available no authoritative means for individual months could be formed. Now it is known that the progress of magnetic events does not differ materially between the Kew and Greenwich Observatories; hence by measuring the range of each day of 1874 from the Greenwich records and allowing an overlap of four months of 1873 and four of 1875 for comparison a fairly safe factor could be obtained by which to interpolate monthly means. Through the kind permission of the Astronomer Royal this was made possible and the gap in the series satisfactorily filled. The mean value of the Kew/Greenwich factor deduced from the four months September, October, November and December of 1873 was 1·041; that for the first four months of 1875, 1·039, giving a mean value for the whole

(1) W. Ellis *Proc. R. Soc.*, London. Vol. LXIII, 1898, pp. 64-78.
(2) S. C. M. Rajna, Milano *Rend. Ist, lomb.* Serie II, Vol. XXXV. 1902.
(3) *Veröff. des Preuss Met. Inst.*, Nr. 332 *Abhand.* Bd. VIII Nr. 2. Berlin, 1925.
(4) *Geophysical Memoirs*, Vol. III, No. 22, 1923.
(5) *Geophysical Memoirs*, Vol. III, No. 29, 1926.

overlap of 1·040. The fact that this ratio does differ from unity is hardly surprising when it is considered :—

(i) that the magnets used at the two observatories differed so markedly in geometrical design and magnetic properties—that at Greenwich being the old Gaussian type whereas the Kew magnet was a smaller Adie pattern design, and

(ii) that the district in which the two stations are situated is well known for its anomalous magnetic nature.

§ 3. The Effects of local artificial disturbance on D Ranges at Kew Observatory.

After 1901 the charts at Kew Observatory were disturbed by artificial fields due to electrical traction in the neighbourhood. Though up till 1916 the effects were very small, subsequent to that date the phenomenon became decidedly more pronounced. In order to determine whether any serious error had been introduced into the measurements of absolute daily ranges by the locally produced oscillations a comparison with Greenwich Observatory was instituted covering some months before and after 1916, after which date no increase in disturbance was noticed. 1915 and 1924 were the years chosen for the comparison for two reasons :—

(i) 1924 was the last available year for Kew data and presumably would show the effects at least as well as any of its predecessors, since there were no reasons to believe that any contrivance had been introduced by the tram and railway authorities to diminish them.

(ii) In January 1915 a new declination variometer was brought into action at Greenwich and it therefore seemed that a fair test of the magnitude of the influence of disturbance before and after 1916 could best be arrived at by comparing sets of records each from the same magnetograph. Up to 1924 at least the Greenwich traces were only very slightly disturbed compared with those from the Kew instruments.

The results for the four months of 1915 gave a final ratio Kew/Greenwich of 1·036 and those from the five months of 1924 gave a value 1·038. From these figures it may be assumed that the effects on absolute ranges of declination of the disturbance due to local electrical traction were almost if not entirely negligible.

A more surprising aspect of the comparison is the correspondence between these last ratios and that which subsisted in 1874 with the previous Greenwich declinometer. The expectation of a smaller value from the more recent traces seemed to be a natural corrollary of the differences in the magnets used before and after 1914. The agreement to within one per cent is at present unaccountable.

Monthly means for the interpolated Kew ranges are given in the first line of Table I. In view of the apparently large decrease in magnitude of the absolute range between 1873 and 1875 in the existing figures, the gap had caused "some uneasiness."* The final mean for 1874 seems, however, to indicate that the decrease was a natural one and not in any way an artificial discontinuity introduced during the examination and readjustment of the Adie magnetograph in that year.

§ 4. The complete series of Means of D Range for Sixty-seven years, 1858-1924.

As already mentioned the individual monthly means of ranges from 1858 to 1900 (exclusive of 1874) and 1901 to 1910 appear in the *Geophysical Memoirs* of the Meteorological Office cited above. The ranges for the nine years 1916 to 1924 have, however been measured expressly for the purpose of this investigation and since the figures for the intervening years 1911-15 are only to be found in the appropriate separate volumes of the Meteorological Office publications, the figures for the 24 years 1901 to 1924 are collected here. Thus Table I together with the memoirs cited above furnish the mean monthly values of D ranges from 1858 to 1924 covering the entire life of the magnetograph at Kew Observatory.

(*) Loc. cit. *Geophysical Memoirs*, No. 22, p. 22.

TABLE I. MEAN MONTHLY AND ANNUAL VALUES OF *D* RANGE AT KEW OBSERVATORY.

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	Sun-spot No.
	′	′	′	′	′	′	′	′	′	′	′	′	′	′
1874 ..	15·31	14·83	15·46	17·69	14·05	14·01	14·89	13·97	15·55	15·42	13·51	8·79	14·46	44·7
1911 ..	9·51	12·78	11·99	12·94	11·25	10·72	11·93	10·93	11·02	9·91	6·16	6·30	10·45	5·7
1912 ..	4·77	7·43	9·46	11·41	10·64	10·78	11·00	11·58	11·86	10·07	7·85	7·42	9·52	3·6
1913 ..	7·35	7·66	10·68	11·68	10·25	11·21	10·47	10·81	10·81	11·35	6·62	5·59	9·54	1·4
1914 ..	5·63	7·00	10·93	11·89	10·50	11·93	11·96	12·68	12·18	10·42	9·45	7·40	10·16	9·6
1915 ..	7·60	10·31	13·71	14·33	12·64	15·21	14·06	14·80	14·97	17·26	15·10	9·52	13·29	47·4
1916 ..	10·71	11·18	19·96	17·59	15·94	15·59	14·98	16·03	14·95	15·36	16·53	11·58	15·03	57·1
1917 ..	14·72	13·81	14·82	15·27	14·68	16·58	16·22	21·12	16·65	17·34	11·52	12·07	15·40	103·9
1918 ..	13·34	14·09	16·45	18·73	15·86	14·49	15·39	16·49	17·83	17·56	15·83	15·95	16·00	80·6
1919 ..	15·10	15·44	20·17	16·73	17·47	15·89	15·83	20·09	18·26	17·61	11·13	11·65	16·28	63·6
1920 ..	11·36	13·82	23·02	17·22	13·18	14·37	14·47	14·90	18·03	14·84	10·84	9·82	14·66	37·6
1921 ..	9·91	9·42	13·67	15·53	21·32	12·75	13·84	14·36	13·49	13·62	12·33	10·25	13·37	26·1
1922 ..	11·42	12·36	16·00	16·28	13·93	13·47	14·31	13·88	14·64	13·86	8·88	7·52	13·05	14·2
1923 ..	6·43	8·47	10·50	12·35	10·09	11·46	8·85	9·94	11·84	11·76	7·87	7·55	9·76	5·8
1924 ..	9·58	8·70	12·42	11·10	11·30	13·30	12·39	11·85	12·82	11·15	8·98	6·16	10·81	16·7
Mean 67 yrs.	11·06	13·15	15·35	15·79	14·19	14·01	14·02	15·10	15·32	14·75	12·08	9·94	13·73	42·1
21 Sun max.yrs	12·52	16·62	18·18	19·04	16·23	16·16	16·62	17·90	17·74	16·89	14·71	12·03	16·22	79·2
21 Sun min. yrs	8·64	10·21	12·09	12·33	11·55	11·77	11·42	12·00	12·24	11·49	8·88	7·18	10·82	8·4

Included in the same table are the general monthly means for the 67 years 1858-1924 together with means from the two groups each of 21 years representative of high and low sunspot development. The group of high frequency has a mean sunspot number of 79·2 and is comprised of the years 1859-61, 69-72, 82-84, 92-95, 1905-07 and 1916-19; while the corresponding series selected for their low mean sunspot numbers includes the groups of years 1866-67, 76-79, 87-90, 99-1902, 1911-14, 22-24 with a mean number of 8·4.

§ 5. THE ANNUAL VARIATION OF *D* RANGE—HARMONIC ANALYSIS.

Before discussing the main body of data the annual variation of magnetic activity as expressed by these final sets of monthly means of daily declination range is worth passing notice. Although the general (67 years) series presents the usual semi-annual period with maxima in the equinoxes and minima at the solstices, the uniformity of the three mid-summer monthly values is not a little surprising. In comparison with this series, the other two representative of high and low solar activity are relatively lacking in smoothness. Nevertheless rigorous analysis by Fourier methods seems warranted.

Assuming the three sequences expressible in either of the forms $\Sigma\ a_n \cos nt + b_n \sin nt$ or $\Sigma\ c_n \sin (nt + \alpha)$ where $t = \frac{2\pi r}{12}$ and $r = 0$ to 11, the values of the coefficients of the first and second harmonic components can be readily calculated by the usual methods. Table II* gives the results of the analysis. In this table the phase angles are stated for the middle of January. The dates of the maxima of the components are also given. Two points of interest arise from an examination of the figures;

(i). Variations of the range of the annual and semi-annual oscillations $2c_1$, and $2c_2$, respectively in the three groups of years are conspicuous. Expressed in terms of percentages, of the yearly mean for each of the three groups the figures are :—

All years—	annual range ($2c_1$) 25% ;	semi annual range ($2c_2$) 26%
Sunspot max. years—	,, ,, ,, 22%	,, ,, ,, ,, 28%
Sunspot min. years—	,, ,, ,, 33%	,, ,, ,, ,, 27%

(*) The values of a_n, b_n and c_n directly derived were increased in the ratio $\frac{n\pi}{12} / \sin \frac{n\pi}{12}$ to allow for the use of mean monthly instead of instantaneous values.

TABLE II. RESULTS OF HARMONIC ANALYSIS OF D RANGES.
(α stated for middle of January.)

	a_1	b_1	a_2	b_2	α_1	c_1	α_2	c_2	Epochs of Maxima	
									1st Harmonic	2nd Harmonic
	′	′	′	′	°	′	°	′		
All years(67)	−1·553	0·667	−1·366	1·315	293	1·69	314	1·81	June 22	Mar. 24, Sep. 24.
S max. years (21) ..	−1·572	0·880	−1·547	1·709	299	1·80	318	2·31	June 16	Mar. 22, Sep. 22.
S min. years (21) ..	−1·639	0·746	−0·971	1·082	294	1·80	318	1·45	June 21	Mar. 22, Sep. 21.

Thus the two components for the all-year mean have each a range equal to $\frac{1}{4}$ of the mean value for the year. The range of the first harmonic increases to $\frac{1}{3}$ of the annual oscillation in years of low solar activity, and diminishes to little more than $\frac{1}{5}$ for sunspot maximum years. Relatively (to the all-year figures) the second harmonic is less well developed in comparison with the first in the case of low mean frequency and an equal amount more highly developed in the case of large sunspot number. Recollecting the influence of equinoctial disturbance in years of frequent sunspots the latter results have a ready explanation and provide a basis of estimate of the relative contributions of the ordinary solar diurnal variation and the more purely disturbance effects to the resultant mean monthly figures.

(ii). The fixity of the epochs of maxima for both harmonics in the three groups of years is noteworthy. For years of apparent solar inactivity the maxima of annual and semi-annual oscillations are almost exactly coincident with the summer solstice and two equinoxes respectively. A slight retardation to the extent of two or three days in the case of all years for both components is scarcely large enough to be of physical significance; while the advanced maximum of the 12 monthly wave to June 16th is the only noticeable feature for sunspot maxima years.

§ 6. D RANGES AND H VARIABILITIES ON A PERCENTAGE BASIS.

With measures of activity so diversely obtained and producing figures of such a different order of magnitude and range as the "interdiurnal variability" of H and absolute range of D, it was necessary to put both series on a more purely numerical basis. Hence utilizing the 67-year means in Table I, each monthly value was reduced to a percentage of this mean. This was done for each of the 12 months of the 52 years for which "activity" (measured by the day-to-day fluctuations of H) figures are available and for the 67 years of absolute range of D. The series of annual means were treated in the same way.

The two series of monthly values as percentages are presented in Tables III and IV. With the absolute values of the means already given in Table I, reconversion of any part of the entire series to minutes of arc (or 10γ units) is facilitated. Detailed examination of *each* monthly array of percentages could serve no useful purpose but the comparative values for the variability of H, range of D and sunspot frequency measures for January, February and March over the 52 years common to both magnetic measures are given as sample runs in Plate I. The sunspot figures (as also the other elements) are expressed as percentages of the mean for the whole series of years.

Plate II allows the three series of 52 annual means to be simultaneously compared. A general parallelism cannot escape even passing attention and the conformity in broad outline is not detracted from on closer inspection, for the two series of magnetic measures show a degree of correspondence which is not a little surprising in view of the essential difference of the measures on which they are based.

TABLE III. MONTHLY MEANS OF MAGNETIC ACTIVITY* 1872–1923.

(As percentages of the 52-years' means, together with those means).

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1872	142	310	165	156	132	159	264	278	171	248	153	129	193
1873	198	104	172	148	100	134	115	99	87	87	110	116	121
1874	143	147	138	159	79	102	110	80	94	162	90	66	114
1875	83	121	94	88	80	78	89	56	89	57	72	49	80
1876	68	99	77	59	58	75	56	51	73	65	72	69	69
1877	86	80	76	61	112	84	77	62	61	74	94	67	77
1878	91	41	39	52	47	105	52	60	70	65	52	96	63
1879	49	62	66	67	72	99	52	49	78	64	40	82	64
1880	65	43	69	88	69	70	93	182	116	88	123	113	94
1881	115	133	90	93	59	94	109	72	143	100	134	100	104
1882	86	104	80	249	97	115	135	111	82	205	327	110	143
1883	71	104	126	132	82	117	126	88	157	82	161	104	113
1884	123	99	93	118	95	131	154	68	122	135	153	111	117
1885	124	108	121	118	148	137	119	102	94	84	92	90	111
1886	154	83	108	108	120	84	105	80	70	98	95	63	98
1887	76	61	66	96	85	95	99	92	99	74	79	106	86
1888	147	52	70	78	108	76	69	93	63	74	69	84	81
1889	68	61	77	66	79	103	65	60	81	65	91	78	74
1890	68	59	55	59	61	66	52	58	72	96	85	63	67
1891	85	90	87	99	104	75	120	104	124	81	93	121	99
1892	134	200	179	130	201	183	211	195	94	93	130	169	158
1893	123	139	108	104	86	145	105	186	116	123	119	117	123
1894	113	203	156	163	108	153	218	215	133	95	154	102	151
1895	120	168	104	105	73	112	114	91	108	127	111	100	111
1896	111	99	116	88	146	101	101	113	102	87	89	111	105
1897	150	93	81	120	91	91	98	71	72	78	70	149	95
1898	74	81	127	82	81	88	75	80	168	84	93	96	95
1899	86	114	76	69	101	76	69	74	61	52	49	66	74
1900	95	74	130	60	124	55	56	62	52	63	44	55	73
1901	63	53	69	47	52	74	71	58	48	49	51	71	58
1902	57	61	49	70	56	61	67	65	46	63	66	53	59
1903	60	63	48	91	62	66	77	89	66	156	148	144	89
1904	121	56	51	115	106	113	73	79	66	81	88	83	86
1905	98	103	104	78	61	103	94	108	102	88	158	113	101
1906	72	127	76	78	103	92	102	80	76	56	83	141	90
1907	108	203	98	72	108	104	105	73	113	114	120	89	109
1908	95	74	112	95	107	80	106	116	230	108	126	119	115
1909	150	93	121	72	154	88	93	79	211	103	86	124	117
1910	93	75	119	125	93	83	76	136	74	115	69	91	97
1911	71	65	73	96	104	82	88	70	68	82	58	112	79
1912	63	54	61	74	63	61	76	75	71	58	82	62	67
1913	75	56	55	66	53	62	51	53	59	65	43	54	57
1914	64	59	51	83	66	61	88	59	84	74	71	86	70
1915	79	81	88	89	67	162	97	75	89	151	127	113	101
1916	104	73	157	142	100	109	97	160	104	107	80	70	110
1917	157	103	86	115	100	142	167	283	111	132	82	144	133
1918	106	148	130	177	126	119	92	132	114	163	134	197	137
1919	136	132	132	105	171	132	126	155	177	184	155	150	146
1920	113	124	296	125	143	108	115	114	143	114	106	128	138
1921	93	83	97	127	357	115	72	87	91	116	102	124	123
1922	102	113	105	98	75	84	82	57	112	109	58	94	90
1923	68	69	79	50	82	94	82	66	97	77	65	53	74
Mean in 10γ units	·733	·852	·926	·867	·852	·759	·765	·862	·919	·888	·903	·758	·840

* Deduced from figures by J. Bartels. *Met. Zs.* Oct. 1925, pp. 400–2.

§ 7. LARGE SCALE COMPARISON.

Subsequent to the initial years of the Kew series the most prominent features are :—

(i) The double maximum of magnetic activity at 1870 and 1872 corresponding with the sunspot maximum at 1870. According to Dr. Bartels's figures

PLATE I

To face p. 10.

Monthly Means of Sunspot Frequency (S), D-Range (R), and H-Variability (V), 1872–1923.
(as percentages of their respective monthly means)

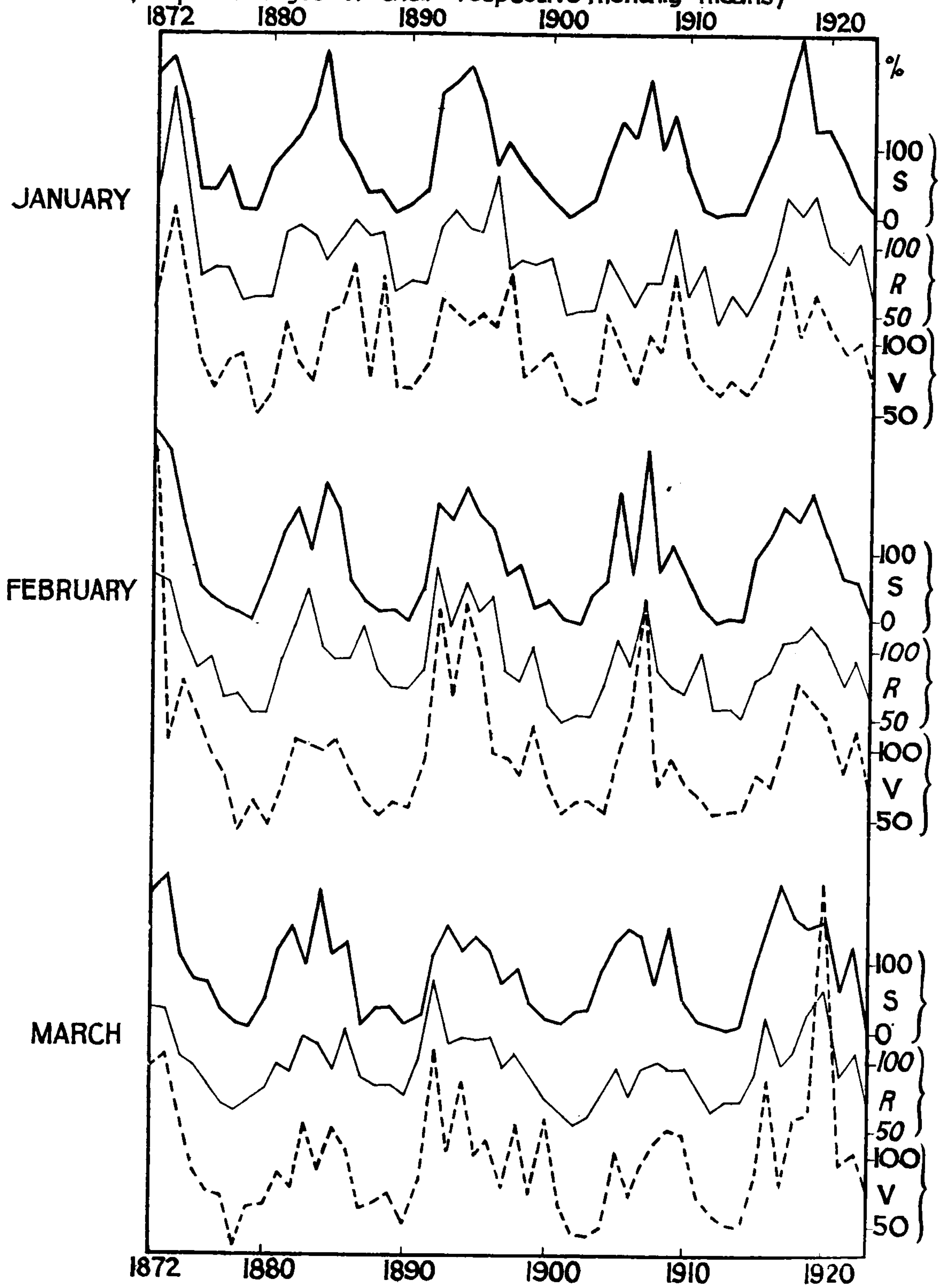

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PLATE II

To face p. 11.

Long Range Magnetic and Solar Variations.
(Magnetic Measures as percentages of Whole Series)

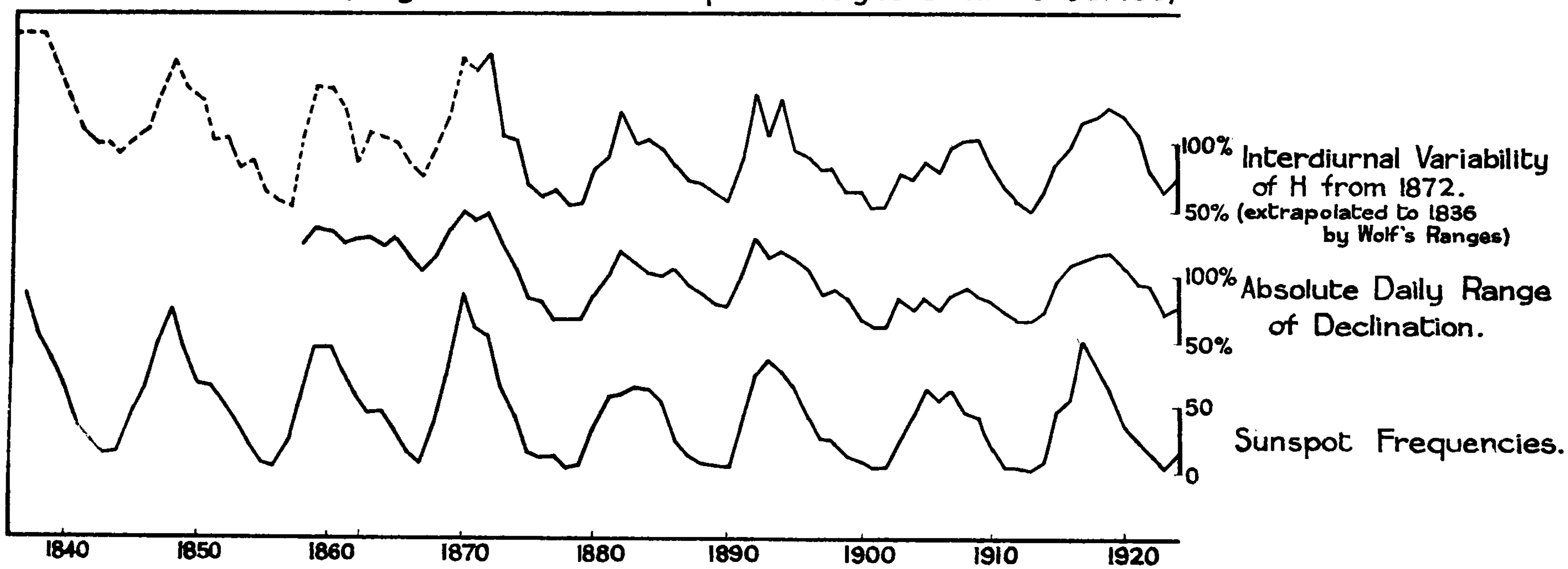

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TABLE IV.—MONTHLY MEANS OF *D* RANGES 1858-1924
(As percentages of the 67-years' means, together with those means).

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year
1858	114	130	130	127	106	118	118	99	128	143	112	150	123
1859	101	131	130	151	113	127	123	151	170	154	122	164	137
1860	121	120	150	135	123	141	151	178	132	111	103	118	133
1861	170	128	128	125	112	118	112	128	116	114	110	165	126
1862	131	109	93	112	109	109	136	141	121	168	159	181	129
1863	189	147	120	120	119	109	117	119	130	126	138	156	130
1864	112	100	119	118	116	123	117	123	119	132	159	168	124
1865	172	167	112	106	111	117	104	143	123	154	158	102	130
1866	122	163	123	109	100	87	95	100	110	139	142	110	116
1867	97	105	104	91	99	103	94	99	134	122	101	80	103
1868	88	103	111	139	105	104	115	128	129	141	106	75	114
1869	96	151	130	166	139	127	126	124	153	120	119	119	132
1870.	154	162	142	147	147	141	135	147	178	150	146	147	150
1871	127	157	142	178	132	137	134	151	116	133	163	133	142
1872	146	151	137	141	123	131	154	153	134	173	160	173	147
1873	213	148	135	129	118	131	123	122	116	110	110	69	127
1874	138	113	101	112	99	100	106	93	102	105	112	88	105
1875	78	86	95	91	93	92	87	81	82	77	74	70	84
1876	83	92	84	70	70	84	85	82	76	78	77	78	80
1877	81	62	70	71	89	78	74	62	51	63	74	57	69
1878	60	67	66	73	73	93	72	71	71	55	54	73	69
1879	61	54	71	68	74	78	77	80	73	64	59	74	70
1880	61	55	79	80	87	86	90	120	91	97	100	102	88
1881	109	92	96	85	88	107	100	91	115	91	118	132	101
1882	111	115	91	136	121	106	94	105	97	120	230	129	120
1883	105	144	114	113	93	102	124	91	111	108	110	100	110
1884	89	101	110	111	100	107	106	93	99	96	109	110	103
1885	102	91	92	90	123	112	103	103	108	91	88	85	100
1886	118	91	122	106	111	103	109	96	90	106	118	132	108
1887	106	116	87	97	95	86	92	95	103	79	88	107	95
1888	109	86	82	87	95	91	89	86	85	86	88	88	89
1889	66	72	81	74	79	81	83	81	77	82	103	84	80
1890	75	70	75	70	74	77	78	76	85	89	89	71	78
1891	72	83	101	104	120	90	95	92	111	112	112	103	100
1892	115	157	156	114	143	124	148	119	107	119	107	138	129
1893	124	116	110	109	113	117	118	116	112	106	125	101	114
1894	111	148	115	111	117	109	131	117	124	113	137	109	120
1895	109	126	113	110	117	121	114	90	97	121	140	110	113
1896	150	129	114	101	110	91	97	100	99	89	89	110	106
1897	82	85	94	102	93	86	87	84	76	78	80	120	88
1898	90	77	103	82	92	88	87	88	100	93	80	94	90
1899	89	91	88	81	101	92	81	80	76	69	63	78	83
1900	92	61	73	63	74	75	74	74	60	59	43	47	67
1901	50	49	62	65	75	75	71	67	61	55	43	44	61
1902	53	52	54	66	63	71	69	77	59	62	59	41	61
1903	56	53	60	75	81	91	82	84	80	116	106	105	82
1904	93	72	73	92	94	98	90	67	51	55	60	72	76
1905	76	108	91	87	87	94	94	87	80	63	87	56	85
1906	57	89	72	85	81	83	89	69	78	57	50	83	75
1907	76	137	92	82	87	89	90	78	86	86	82	69	88
1908	74	84	98	94	103	93	89	101	124	84	77	62	92
1909	115	74	94	84	102	79	74	89	104	85	61	83	87
1910	66	69	94	83	78	86	72	86	86	97	68	76	81
1911	86	97	78	82	79	77	85	72	72	67	51	63	76
1912	43	57	62	72	75	77	78	77	77	68	65	74	69
1913	66	58	70	74	72	80	75	72	71	77	55	56	69
1914	51	53	71	75	74	85	85	84	80	71	78	74	74
1915	69	78	89	91	89	109	100	98	98	117	125	96	97
1916	97	85	130	111	112	111	107	106	98	104	137	116	109
1917	133	105	97	97	103	118	116	140	109	118	95	121	112
1918	121	107	107	119	112	103	110	109	116	119	130	160	117
1919	136	117	131	106	123	113	113	133	119	119	92	117	119
1920	103	105	150	109	93	103	103	99	118	101	90	98	107
1921	90	72	89	98	150	91	98	95	88	92	102	103	97
1922	103	93	104	103	98	96	102	92	96	94	74	75	95
1923	58	64	68	78	71	82	63	66	77	80	65	76	71
1924	87	66	81	70	80	95	88	78	84	76	74	62	79
Means for Entire Period	′ 11·06	′ 13·15	′ 15·35	′ 15·79	′ 14·19	′ 14·01	′ 14·02	′ 15·10	′ 15·32	′ 14·75	′ 12·08	′ 9·94	13·73

which up to 1871 are really Wolf's " täglichen Deklinationsvariationen " the second maximum, two years subsequent to the sunspot turning point, is the main one ; while the Kew figures have the higher value coincident with maximum sunspottedness in 1870. For both solar and magnetic data the smoothed maximum would be at 1871.

(ii) A similar and even more pronounced instance of a double outbreak of magnetic activity with an intervening maximum of sunspots occurs at 1892 and 1894. Here the variability and range figures agree in putting the principal maximum at the earlier year. Smoothed data for the three sequences show a common maximum in 1893.

(iii) Though the intervening cycle of sunspot activity has its turning value of 64.7 mean *Relatizahlen* in 1883, the following year 1884 is little less active, the mean number being 63.5. The magnetic data on the other hand both reached their highest value in 1882. Thus on the bases of these two dissimilar measures, the maximum of magnetic activity preceded that of solar activity by one year.

(iv) In this respect the statistics for the cycle 1856-67 are somewhat comparable. For there the sunspots reach their highest annual maximum in 1860, whereas both range and variability means indicate a maximum in the preceding year. The difference in sunspottedness between 1859 and 1860 is however, trifling.

Thus for these four cycles covering the period 1857 to 1901 the order of relation of magnetic to solar activity is :—

(*a*) 1857 - 1867, both single maximum, magnetic precedes solar.
(*b*) 1868 - 1878, magnetic double maximum (smoothed maxima sychronous).
(*c*) 1879 - 1890, magnetic precedes solar maximum.
(*d*) 1890 - 1901, double magnetic activity maximum (smoothed means have simultaneous maximum).

(v) For cycles prior to 1857 few magnetic data are available. In addition to the composite measures of declination range furnished by Wolf and incorporated in Bartels's variability statistics, the only other long series figures are those published by Ranja for the diurnal range of D at Milan and those from the Greenwich magnetograph used by Ellis. Though the uniformity and homogeneousness of all of these in the early stages are questionable, the fact that Wolf derived his measures from four or five stations in general, gives these latter a better claim to acceptance for the purpose on hand than those from individual stations. Since Wolf's ranges are inequality rather than absolute ranges, his figures are to be taken as a retrospective extension of day-to-day variability measures rather than measures of absolute declination range means prior to 1858, and in this light Bartels has standardized them with his horizontal force figures. It is evident that less weight can be attached to deductions from them than from the sequence subsequent to 1872.

The cycle 1844-56 just preceding the earliest of those discussed above shows a coincidence of maxima of solar and magnetic variations ; while the first for which any data are available, though incomplete, also indicates a simultaneity of smoothed maxima. 1837 and 1838 are almost equally active from a magnetic point of view, but the earlier year has the preponderance, the weighted values for 1836, 37, 38 and 39 being 17·6, 17·7, 17·7 and 15·3 10γ units, respectively. 1837 is the year of greatest sunspot development.

Hence from the investigation of these first six sunspot cycles with the possible exception of the earliest incomplete cycle, there is a suggestion of an alternation of coincidence and precedence of magnetic with solar activity.

(vi) An inspection of the results from the two most modern cycles will not permit an extension of this deduction. Rather, in that magnetic activity on both bases is decidedly retarded behind the maximum of sunspottedness, they show a marked divergence from the previous runs.

Wolfer's *Relatizahlen* for the earlier of the two cycles have a double crest at 1905 and 1907, 1905 being the higher of the two. This results in a smoothed maximum about the middle of 1906. But in contrast, the interdiurnal variability of H means reach their maximum at 1909 and the range of declination figures at 1908, that is, 4 and 3 years respectively subsequent to the year of highest sunspot number for the cycle and 3 and 2 years after the epoch of smoothed maximum. The retardation is no less prominent in the next cycle. For 1917 is a clean cut peak of sunspot means and 1919 shows an equally definite maximum of magnetic activity on both measures.

§ 8. The (possible) Existence of a Secular Change in Declination Range.

In addition to the anomalies of behaviour of the elements treated above within isolated cycles, the general downward trend of the curves representing the magnetic figures calls for further attention. The possibility of a secular change in the daily range of declination has been discussed in some detail elsewhere (1) but the extension of the data used in examining the supposed change together with the appearance of a long series of variability figures for another magnetic component recorded at different sites on the Earth's surface makes a further investigation desirable. For the question whether the change in declination range was to be attributed to a genuine progressive decline in magnetic activity or to be associated with the recognized secular changes in the controlling field remained undecided. Table V summarizes the available evidence both in respect of the variations of declination ranges and the day-to-day fluctuations in the mean value of the horizontal force.

Table V.—Variation of Magnetic Activity with Sunspot Cycle.

Group of years	Mean Sunspot Number	Mean Declination Range	Equivalent Force	Mean Variability of H
		′	γ	10γ
1836–1843(‡)	73·0			1·24
1844–1856	53·2			1·01
1856–1867	49·9			0·99
1858–67‡	52·6	17·18	88·0	1·04
1868–78	56·0	15·24	79·5	1·02
1879–1889	34·8	13·25	69·5	0·83
1890–1901	39·2	13·13	72·0(*)	0·85
1902–1912	33·7	10·90	57·5(*)	0·77
1913–1923	40·7	13·32	71·4	0·90

(‡) Incomplete cycles.
(*) These forces are for the two cycles 1890–1900, and 1901–1912 respectively.

Confining attention at present to the first, third and fifth columns of the table and to the years subsequent to 1857 it seems clear that relatively to the four later cycles, the two groups of years 1858-67 and 1868-78 have remarkably high mean ranges and that, further, the group mean ranges diminish systematically from the first to the fifth of this set of cycles. In these respects the data derived from the variability of the horizontal force closely agree, the only difference in behaviour between the two magnetic measures being that the period of almost stationary range is one of very slight increase in Dr. Bartels's activity figures. In both sequences the last cycle is the first which gives evidence of a genuine tendency to an increase of activity over its immediate five predecessors.

Now the two most obvious influences regulating a possible secular change in the range of declination are :—

(1) *Geo. Mem.*, Vol. III, No. 29, pp. 257–259.

(i) a secular change in mean sunspottedness
and (ii) a secular change in the magnetic field controlling the movement of the declination needle.

Reference to the second column of Table V shows that the mean sunspot numbers of the two cycles 1858-67 and 1868-78 were considerably in excess of those for any of the subsequent cycles. Indeed the mean number for the period 1868-78 is some 66 per cent greater than that for the period of least range 1902-12. This, with the generally recognized fact that an increase of 100 in the scale of sunspot frequency numbers results in an increase of approximately 50 per cent in mean range goes at least some way to account for the apparently excessive decrease in range during the first decade of the present century. The estimation of the precise extent for which the differences in mean sunspot activity are accountable must, however, be postponed till a later part of the investigation. (*See* § 20).

§ 9. Evidence of Secular Change in the Force Equivalent of the *D* Range.

The mean value of the horizontal force at any place is dependent on the epoch of the measurement. Like the other magnetic elements it suffers a secular change and this immediately affects the extent of the extreme excursions of the declination needle throughout any interval. Although the best values of the magnitude of the effect are obtained in association with mean ranges reduced to a common sunspot basis, the general run can be gauged from column 4 of Table V. There are given the force equivalents of the corresponding ranges in column 3 obtained by multiplying the range in radians by the average value of the horizontal force over the period concerned. The figures for the first four periods under discussion are taken directly from Table VI of *Geophysical Memoirs, No.* 22, where Dr. Chree drew attention to the phenomenon. It is to be remarked that the equivalent forces for the two cycles 1890-1901 and 1902-1912 are really calculated for the two sets of years in which 1901 is included in the earlier cycle, this being done with a view to the subsequent discussion of this question in which the sunspot cycles are taken as ending with the year immediately preceding the year of lowest sunspot number in a group of years. The change in the position of the single year 1901 in the present approximate examination has little effect on the final sequence of figures in column 4. This sequence plainly indicates that the relative magnetic positions of the six most modern cycles are altered but slightly, in fact the chief result arising from the use of force equivalents in place of the means themselves is the closer approximation of the final sequence of group means to that for the interdiurnal variability of *H*—an approximation which is especially conspicuous in the last five cycles. The slight rise from the third to the fourth of the groups beginning 1858 in the mean variability of *H* is exactly mirrored in the rise in the equivalent in γ of the mean range, and the decided sag in the fifth cycle is equally noticeable in both measures.

Hence taking account of the secular change in *H*, the question whether a secular change in declination range, independent of change in solar activity, still remains outstanding, and whether the difference between the second and fourth and the much smaller difference between the fourth and sixth cycles owe their origin to the differences between the corresponding entries in the second column of Table V cannot be definitely decided till the sunspot effect on mean range can be precisely computed.—(*See* § 20).

§ 10. Long Period Changes in Sunspot Frequency.

In connection with these and other results to be discussed, some comments on the apparent fluctuations of solar activity as judged by the run of mean annual sunspot number for various groups of years seem necessary at this point. The three cycles whose maxima were reached in 1883, 1893 and 1905 were decidedly less well developed from a sunspot point of view than the previous three, weighted means of the relative sunspot numbers for the two sets of three cycles being 35·8 and 53·0 respectively. Indeed, though solar activity in the most modern cycle (1913-1923) showed a tendency to better development than its three precursors, the weighted mean sunspot frequency for these four latest cycles amounts to only 37·0 as compared with 55·7 for the previous four. Thus there is evidence of a decrease in general sunspottedness—the break taking place in the latter part of the cycle whose maximum was attained in 1870. This feature is also noticeable both in the magnetic range and variability means as Plate II shows.

TABLE VI.—CHANGES IN SOLAR ACTIVITY, 1756–1923.

Group of Years used	Number of of Years used	Mean Sunspot Number	Weighted Group mean of Sunspot Numbers
1756–1766	11	42·5	
1767–75	9	59·4	
1776–84	9	68·6	62·3 (1767–98)
1785–98	14	60·0	
1799–1810	12	23·5	
1811–23	13	18·2	26·1 (1799–1833)
1824–33	10	39·5	
1834–43	10	65·4	
1844–56	13	53·2	
1857–67	11	49·9	53·0 (1844–78); 55·7 (1834–78)
1868–78	11	56·0	
1879–89	11	34·8	
1890–1901	12	39·2	35·8 (1879–1913); 37·0 (1879–1923)
1902–13	11	33·7	
1914–23	10	40·7	

Mean sunspot figures for the complete cycles beginning 1767 and ending 1833 were computed from the numbers published by Wolfer (See Table VI) to see whether the previous history of the solar periods showed any tendency for cycles to occur in groups. The results seemed to point to as distinct an increase in the general frequency of spots immediately following the 1830 maximum as was the decrease subsequent to 1873 or 4. On further retrospect an apparently equally marked increase had set in after the maximum of 1761. The weighted mean for the period 1767-1798 was 62·3 while the succeeding set of cycles covering the years 1799-1833 fell to the conspicuously low mean value of 26·1 *Relativzahlen*.

While it is to be kept in mind that the same reliance cannot be placed on the early solar statistics compiled by Wolf as on those for more recent variations, these results may throw some light on meteorological and geophysical phenomena other than terrestrial magnetism.

§ 11. INTRA-CYCLE COMPARISON OF SOLAR AND MAGNETIC VARIATIONS.

Since the above analysis shows that a mean range or mean variability derived from the entire series cannot be equally representative of the state of affairs for each solar cycle, means for each of the cycles from 1858 onwards have been computed for both magnetic and solar activity measures. The yearly means alone were treated. These were converted into percentages of the cycle-means and tabulated as shown below (Table VII). The graphical representations appear in Plate III. Features which are difficult to detect in the general treatment of the data are made evident in these separated figures.

If, for greater convenience, numbers be affixed to the cycles in their chronological order—No. 1 to the first (incomplete) cycle 1858-67, No. 2 to 1868-78 and No. 6 to the last complete cycle—it now becomes more obvious that there exists a greater closeness of parallelism between the three sequences in cycles 2, 3 and 4 than in the others considered and that, further, of these three cycles numbers 2 and 4 are more intimately related than the third. The depression of magnetic activity in 1893

TABLE VII.—SUNSPOT FREQUENCIES AND MAGNETIC DATA AS PERCENTAGES OF THEIR RESPECTIVE GROUP MEANS.

Year	Sunspot Number	Absolute Declination Range	Inter-diurnal Variability of H	Year	Sunspot Number	Absolute Declination Range	Inter-diurnal Variability of H
	As percentages of their respective Group Means				As percentages of their respective Group Means		
1857	— 46*	—	52*	1890.. ..	18	81	66
1858	104 110	96	97 102	1891.. ..	91	105	98
1859	178 188	109	131 137	1892.. ..	186	135	156
1860	182 192	106	129 135	1893.. ..	217	119	121
1861	147 155	100	118 124	1894.. ..	199	126	149
1862	112 118	103	82 86	1895.. ..	163	118	109
1863	84 88	104	101 106	1896.. ..	107	111	104
1864	89 94	99	97 102	1897.. ..	67	93	94
1865	58 61	103	93 98	•1898.. ..	68	94	94
1866	31 33	93	82 86	1899.. ..	31	86	73
1867	14 15	83	72 76	1900.. ..	24	70	72
				1901.. ..	7	68	58
1868	66	103	91	1902.. ..	15	77	65
1869	132	119	113	1903.. ..	72	103	97
1870	249	135	156	1904.. ..	125	96	94
1871	199	128	146	1905.. ..	188	107	110
1872	181	133	159	1906.. ..	160	94	99
1873	118	114	100	1907.. ..	184	111	119
1874	80	95	95	1908.. ..	144	116	126
1875	31	76	66	1909.. ..	130	110	127
1876	20	72	57	1910.. ..	55	102	105
1877	22	62	64	1911.. ..	17	96	86
1878	6	62	52	1912.. ..	11	87	73
1879	17	72	65	1913.. ..	3	71	54
1880	93	91	95	1914.. ..	24	76	66
1881	156	104	105	1915.. ..	116	100	94
1882	171	124	145	1916.. ..	140	113	102
1883	182	114	114	1917.. ..	255	116	124
1884	180	106	118	1918.. ..	198	120	128
1885	149	103	112	1919.. ..	156	122	137
1886	102	112	99	1920.. ..	92	110	129
1887	38	99	87	1921.. ..	64	100	114
1888	20	92	82	1922.. ..	35	98	84
1889	18	83	75	1923.. ..	14	73	69

* Means for the complete cycle. 1857–1867.

coincident with the maximum of sunspottedness for the cycle stands out as a marked departure from the usual course. For though cycles 2 and 5 both show a well defined recrudescence of magnetic activity, one of the maxima in cycle 2 occurred simultaneously with that of sunspots and in the other, the resuscitation was accompanied by a similar feature in the solar data.

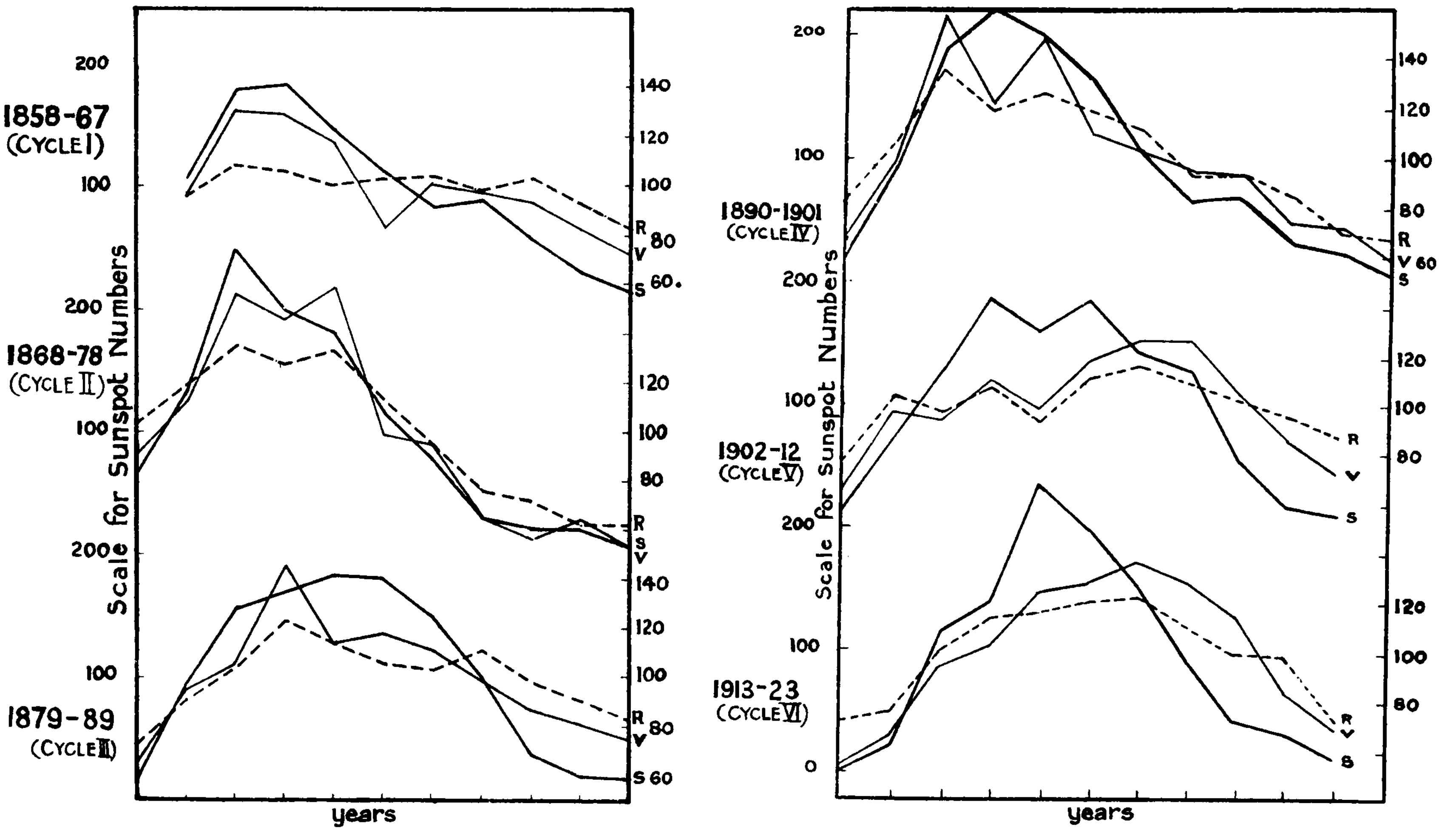
Although following the general trend of the sun's superficial variations measured by Wolfer's numbers, the range data for the first cycle show an unwarranted uniformity, this may be partly accounted for by the incompleteness of the magnetic ranges over the entire cycle. For the lack of the first two years resulted in a mean range which was decidedly higher than it otherwise would have been and since further the set of years 1858-66 was intrinsically one of comparatively high solar activity this produced a series of annual values closely arranged about the correspondingly high mean. While it is difficult to conceive how the ordinary ailments of magnetograph systems could produce traces characterized at the same time by an *enhancement* and a *uniformity* of daily amplitude, the features of Plate II certainly hint that the data derived from the Kew instrument suffered slightly from the state of knowledge of magnetograph and photographic manipulation at that early date.

PLATE III

To face p. 16.

Simultaneous Variations of D-Range, H-Variability & Sunspot Frequency over 6 Solar Cycles.

All measures shown as percentages of their Cycle Means



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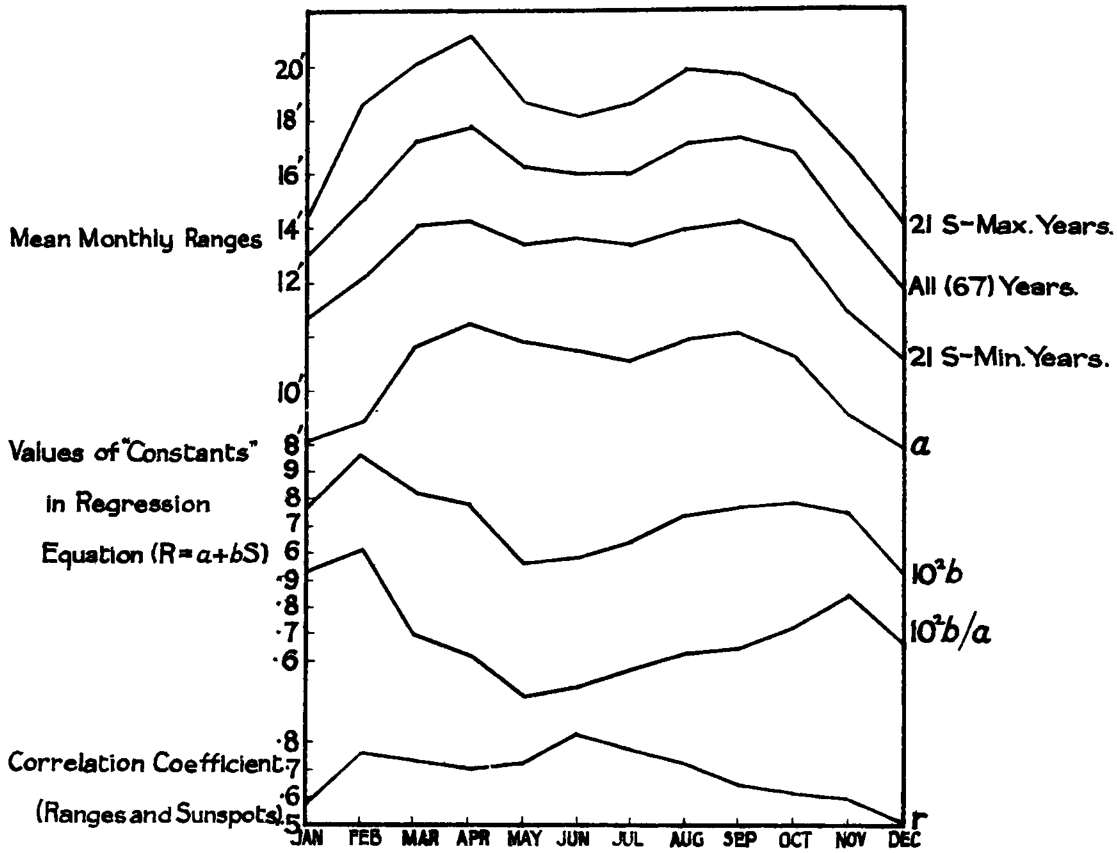
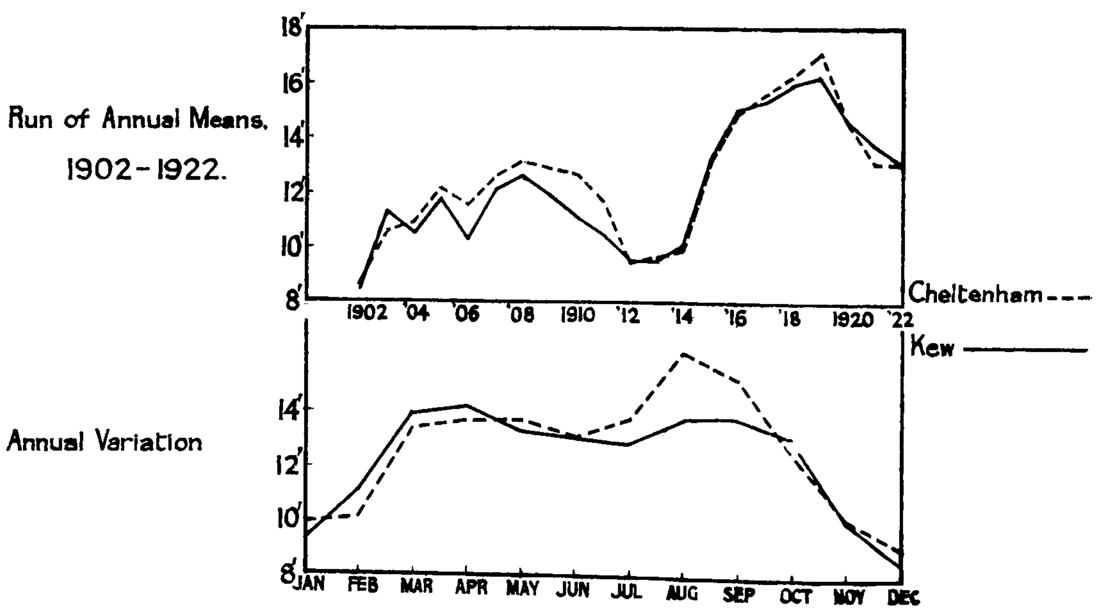
Annual Variation of Quantities relating to Absolute Daily Range of Declination.



Kew and Cheltenham Declination Ranges.



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PART II.

THE RELATIONS BETWEEN SUNSPOT FREQUENCY AND MEASURES OF MAGNETIC FLUCTUATIONS TREATED NUMERICALLY

§ 12. THE REGRESSION EQUATION $R = a + bs$.

The work of Dr. R. Wolf at Zürich and that of Dr. C. Chree at Kew Observatory showed that such a formula as $R = a + bs$ adequately described the kind of relationship existing between sunspot frequency figures and daily ranges of declination—the significance of the latter being confined to regular diurnal rather than the absolute ranges used in this investigation. At first sight it looked as if the approximate validity of such an equation for regular ranges would preclude its extension to measures which differed so widely in their nature, but it was later found that, to a first approximation at least, the same formula held for absolute ranges. Hence if the relation be treated not as having a rigorously precise physical interpretation but simply regarded as the best simple working relation connecting the two variables, some interesting deductions may be made from the values assumed by the parameters a and b under different conditions of time and place.

§ 13. VALUES OF THE PARAMETERS a AND b BY THE "METHOD OF DIFFERENCES."

Approaching the subject from this point of view, a set of values has been computed by the method of differences by which two groups of years, one of high and one of low sunspot values were formed and sequences of mean monthly values for each group deduced. Thus, if R_{Jx} represents the mean of the several Januarys in the years of high sunspot frequency and R_{Jn} the mean for the Januarys of relatively low frequency with S_{Jx} and S_{Jn} the corresponding mean sunspot values for the two groups, a measure of the second parameter could be derived from the relation $b_J = \frac{R_{Jx} - R_{Jn}}{S_{Jx} - S_{Jn}}$. Twelve values of the quantity b could thus be computed, one for each month, and another derived from the annual means. It was then an easy matter to calculate the corresponding a's. This has been done for the 67 years of declination range material now available, using the two groups each of 21 years detailed in § 4 as representative of the two classes of years. The mean sunspot frequencies 79·2 and 8·4 computed for the two sets are sufficiently separated to warrant their acceptance for the present use. The values obtained for a and b are entered in Table VIII (a.).

§ 14. DETERMINATION OF a AND b BY THE METHOD OF "LEAST SQUARES."

In spite of the large number of years used in each group it can be contended that the best use is not being made of the available data. For the linear relation so derived has been adapted to suit extreme conditions and the resulting values of the parameters may therefore be unsuited to a moderate state of affairs. Since no data for additional years can become available for continuation of the Kew series, it was thought that the determination of a final series of "best" values for the regression co-efficients would be of some value. Denoting by δx the algebraic departure of any monthly value of sunspottedness from the mean for the entire series of months of the same name, and by δy the corresponding departure for declination ranges, the co-efficient b was computed from the usual relation $b = \frac{\Sigma\, \delta x\, \delta y}{\Sigma \delta x^2}$ and a was then deduced as in the previous method. As a measure of the relative increase of R per one hundred per cent increase of S the values of the quotient $10^2 b/a$ are also illuminating. They have been computed from each pair of monthly values of a and b.

TABLE VIII.—ANNUAL VARIATION OF REGRESSION AND CORRELATION CO-EFFICIENTS.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	Arithmetic Means from		
														Winter	Equinox	Summer
(a) By "Differences"—																
a ..	8′·15	9′·47	11′·10	11′·62	11′·06	11′·30	10′·86	11′·47	11′·52	10′·75	8′·18	6′·59	10′·18			
10^2b ..	6·28	8·97	9·05	9·07	6·09	5·86	6·87	7·67	8·01	8·18	8·39	7·24	7·63			
(b) By "Least Squares"—																
a ..	8′·13	8′·99	11′·69	12′·53	11′·80	11′·46	11′·15	11′·85	12′·05	11′·36	9′·02	7′·88	10′·39	8′·51	11′·91	11′·57
10^2b ..	7·63	9·61	8·29	7·83	5·62	5·83	6·43	7·38	7·74	8·20	7·57	5·28	7·95	7·52	8·01	6·31
$10^2b/a$	0·939	1·068	0·709	0·625	0·476	0·509	0·577	0·623	0·642	0·722	0·840	0·670	0·765	0·879	0·675	0·546
r ..	0·58	0·76	0·74	0·71	0·73	0·83	0·78	0·73	0·65	0·62	0·60	0·51	0·80	0·61	0·68	0·77

To discover whether the different months showed any tendency to vary in the closeness of parallelism between the sunspot and magnetic data the correlation co-efficient $r = \frac{\Sigma\ \delta x,\ \Sigma\ \delta y}{\{\Sigma\ (\delta x)^2,\ \Sigma\ (\delta y)^2\}^{\frac{1}{2}}}$ was computed. The squares for the standard deviation of sunspots were taken to two places and those for ranges to three places of decimals—probably an unwarranted excess of refinement so far as the correlation co-efficients were concerned but a mentionable fact for a later part of the discussion. The final sets of values of a, b, $10^2b/a$ and r thus derived by rigorous methods are given in Table VIII (b) and graphed in Plate IV.

§ 15. THE ANNUAL VARIATION IN THE "CONSTANTS" OF THE REGRESSION EQUATION.

The "constant" a of the relation $R = a + bs$ ostensibly indicates the mean range to be expected when no sunspots are visible. The table shows it to vary throughout the year, having a semi-annual period with maxima in the neighbourhood of the equinoxes and with the principal of these (12′.53) in April and a slightly less pronounced maximum of 12′.05 in September. The real minimum occurs in December with a value of 7′.88; that occurring in the summer in July is a poor secondary of 11·15 minutes of arc. Mean seasonal values derived, as in the case of other variants, from the mean monthly values show an increase of 36 per cent from the winter to the summer months.

Increase of range for unit increase of sunspot numbers is indicated by the values of the second parameter b. This also has a semi-annual variation of the same general type as that of a but differing conspicuously in detail. For example the spring maximum is considerably advanced to February (the chief maximum), while the autumn peak is retarded to October. May, with a value ·0562, is a better approximation to the real minimum (·0528) of December than was the corresponding summer minimum for a. Indeed if judged on the basis of the mean seasonal values this mid-winter minimum appears to be in the nature of an "accident" although derived from 67 years data. For the general winter mean deduced from the means of the four months November, December, January and February exceeds the summer mean derived from May, June, July and August by 19 per cent, 27 per cent being the figure for the excess of the maximum equinoctial values over those for summer. Hence although the relative positions of winter and summer have been interchanged with respect to the equinoctial values, the variations of the parameters a and b are not inversely related; the principal minima have simply changed places from winter for a to summer for mean b values.

§ 16. The Annual Variation in $\frac{10^2 b}{a}$ and its Significance.

For the efficiency ratio $\frac{10^2 b}{a}$ the monthly values show a further progress in the change of type of variation seen in passing from a to b. The two equinoctial maxima have given place to maxima which though both situated in the "winter" months February and November are separate and distinct. May is again the month of lowest value for the ratio with December a poor secondary minimum. Seasonal means make the change in type clear; for now the percentage excesses of the winter and equinoctial means over the summer mean are 61 per cent and 24 per cent, respectively.

Though the decrease in length of the earth-sun line is relatively small, slightly increased efficiency of sunspots (or their accompanying effects) in enhancing the magnetic range when the earth is in perihelion as compared with aphelion is from *a priori* reasoning to be expected. Reasons for the different behaviours of November and February in comparison with December and January are not, however, so apparent except in that they are partly to be found in the values of the correlation co-efficients for the months in question in the last line of Table VIII. These give a mean of 0·68 for the former pair of months and 0·55 for January and December and so provide evidence of a more intimate relationship existing in the months near the equinoxes. This would in turn indicate that the effect is a combination of two influences; the first, as described above, arises from the winter solstice being the epoch of greatest proximity of the earth to the sun and therefore if any effect of sunspots in producing increased magnetic movements on the earth's surface is to be looked for at all, it should be most appreciable about this time of year. The second influence depends on the orientation of the earth's axis with respect to the earth-sun line. This has its optimum effect at the equinoxes when the axis is at right angles to the radius vector from fairly low solar latitudes and presumably accounts for the generally enhanced magnetic disturbances near the equinoctial months. Hence February and November being between the times when the earth is astronomically in an intermediate position between the perihelion and its disposition at the equinoxes derives the best combined effect from both sources.

As indicated above the magnetic and solar variations experience the lowest mutual relationship in the winter months, the four-monthly mean of the correlation co-efficients being 0·61. In summer the connexion is closest as shown by the mean 0·77, while the equinoctial months have the intermediate value of 0·68. It is noticeable, though to be expected, that the co-efficients derived directly from the series of annual values is higher than any individual monthly value with the exception of June.

As a matter of statistics, the differences in the two series of b's computed by the two essentially different processes are worthy of examination. The residues remaining after subtracting the value computed by the "difference" method from that calculated by "least squares" are tabulated below (Table IX), together with the monthly values of the co-efficients of correlation already given in Table VIII. Inspection of the two sequences discloses a tendency of the residues, 10 out of 12 of which are negative, to decrease numerically, that is, towards zero, as the correlation co-efficient increases. The seasonal means bear this out clearly.

Table IX.—Variation in the Difference ($R \times 10^2$) of the values of b derived by two methods with the corresponding Correlation Co-efficients.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Year	Seasonal Means		
														Winter	Equinox	Summer
	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
$R \times 10^2$	−1·35	+10·64	−0·76	−1·24	−0·47	−0·03	−0·44	−0·29	−0·27	+0·02	−0·82	−1·96	+0·32	−0·87	−0·56	−0·31
r ..	0·58	0·76	0·74	0·71	0·73	0·83	0·78	0·73	0·65	0·62	0·60	0·51	0·80	0·61	0·68	0·77

§ 17. The Equation $R = a + bs$ applied to separate Groups of Years.

Rigorous methods of deriving measures of intimacy of relation between mean annual sunspot numbers and yearly values of magnetic activity on any basis entail the computation of further correlations between the variables concerned. For although the graphs of Plate III are indispensable in diagnosing the actual points of divergence in the sequences under comparison, they cannot furnish a numerical value of the extent of the discrepancies from one cycle to another.

Defining a sunspot cycle in the same way as Wolf in his extensive compendium of sunspot figures (1), the annual values of magnetic data, both absolute declination ranges and interdiurnal variability of horizontal force, were grouped to correspond with the solar activity numbers. Since the earliest available annual mean of declination range is that for 1858 the Kew material covered six cycles, all except the first being complete and homogeneous. This first cycle lacked only two years 1856 and 1857. Values for these years might have been supplied like 1874 from the Greenwich data but, as already mentioned and as will be seen from the results of the partial investigation, the value of the rest of the data for the cycle did not seem to warrant this procedure. The measures of activity published by Dr. Bartels are slightly more heterogeneous than the range data. For the years prior to 1872 recourse had been had to Wolf's statistics of declination ranges at several European observatories and subsequent to 1920, Potsdam horizontal force variations alone provide the data. Hence, although Bartels's figures have been dealt with back to 1856, the beginning of the first cycle treated for the Kew data, it will be understood that the first two cycles are not strictly homogeneous with the other cycles considered.

As in the previous work on correlation, §§ 14-17, standard deviations of x (sunspot numbers) were computed to two places of decimals and of y (ranges) to the third place of decimals. Use was made of the deviations to evaluate the two parameters a and b of the regression equation $R = a + bs$ for each separate cycle. The results are given in Table X below.

Table X.—Values of Regression and Correlation Co-efficients for Groups of Years.

Group of Years used	Mean Sunspot No. for Group (S_n)	Absolute Declination Range (Kew)			Interdiurnal Variability of H (various stations)			International Character Figure		Cheltenham Absolute D Ranges †		
		$b \times 10^2$	a	r	$b \times 10^2$	a	r	$b \times 10^2$	S_n	$b \times 10^2$	a	r
		′	′		10γ					′	′	
1856–66	49·6	..	..	..	0·82	0·58	0·89					
1858–66*	57·6	2·19	16·25	0·70	..	..	..					
1867–77	56·6	7·79	11·25	0·92	0·81	0·58	0·97					
1878–88	34·6	6·86	10·74	0·78	0·72	0·57	0·90					
1889–1900	38·8	8·50	10·06	0·94	0·79	0·55	0·81					
1901–12	31·1	4·35	9·34	0·74	0·50	0·59	0·70	0·17	33·7 (1906-12)	4·32	10·03	0·67
1913–23	40·7	6·39	10·72	0·85	0·62	0·65	0·79	0·22	40·7	6·27	10·98	0·83

* Incomplete cycle. † Cycles 1902–12 and 1913–22.

Confining attention for the present to the columns containing the Kew declination range data, several points of importance emerge.

(i) The first (incomplete) cycle gives a value of b less than one half the least of those computed for any of the remaining cycles. This in conjunction with the fact that the value of a for this cycle exceeds the highest of the other five values by 44 per cent. seems to point to the same conclusion as hinted at in § 11, viz., that the data for this early cycle has suffered somewhat from the state of magnetic science at that time.

(1) Shown by the lines of demarcation separating successive groups of years. *Terr. Mag.*, Vol. XXX, No. 2, 1925, pp. 83-86.

(ii) The variations of the co-efficient *b* are definite and, when examined in connexion with the mean sunspot numbers for each of the cycles—due allowance being made for the results from the (possibly defective) years 1858-66—seem to indicate a direct proportionality with solar activity if the fluctuations of the latter be gauged by the numbers.

§ 18. The Results Obtained by Other Investigators—Cheltenham (U.S.A.) D Ranges.

This result seems to be in partial conflict with the results obtained by Dr. L. A. Bauer and C. R. Duvall (1) from a discussion of data which, except for the introduction of a factor depending on the square of the value of the horizontal component of the magnetic field, are almost identical with those used here for the period covering the first four cycles. The outcome of their analysis is that " it would appear as though a_y (the mean value of a) may vary directly and b_y inversely with sunspot activity " (2). A similar conclusion is reached in a more recent publication (3) by the same authors which has just come to hand. In this Kew ranges up to 1915 have been used and by the application of a reduction factor, the entire series of 57 years' means—1874 being filled up by reference to " Greenwich results for close by years "—was put on a basis supplied by data from the Magnetic Observatory at Cheltenham, U.S.A., and continued till 1925. Direct comparison of the annual means of declination ranges measured from the Kew curves for the years 1916-1924 and the Cheltenham data on to which they have been grafted is frustrated by the introduction by the authors of the factor involving H^2 as mentioned above. Further, it is not stated whether the finally accepted values of W_D published in Table I (p. 39, *Terr. Mag.*, March, 1926) have incorporated the Kew ranges up to 1915 or whether the Cheltenham statistics have been introduced from 1902, but division of the annual values of W_D by the actually measured means of Kew range suggests that the latter has been the course adopted. For Table XI which shows that the values of the ratio W_D/Kew range from 1880 till 1924 progresses steadily up to 1900 as would be expected from a fairly uniform secular change in H, but subsequently to 1901 the variations in the ratio are such as can only be explained by the introduction of some extraneous material whose continuity with the Kew statistics is somewhat doubtful.

TABLE XI.—VARIATION OF THE RATIO W_D/KEW RANGE FOR 45 YEARS.

Unit of the Decade	0	1	2	3	4	5	6	7	8	9
1880 ..	1·124	1·128	1·132	1·135	1·136	1·140	1·142	1·145	1·147	1·156
1890 ..	1·153	1·156	1·157	1·164	1·164	1·167	1·171	1·178	1·178	1·182
1900 ..	1·189	1·166	1·199	1·143	1·214	1·206	1·256	1·197	1·206	1·219
1910 ..	1·248	1·234	1·156	1·164	1·141	1·137	1·078	1·098	1·094	1·124
1920 ..	1·064	1·032	1·058	1·137	1·045					

With the exception of the years 1901-1904 the mean monthly values of absolute D range at Cheltenham are readily obtainable from the results published for that Observatory. Those for the first three years and nine months (the Observatory started recording in April, 1901) can be computed from the values of the ordinates of instantaneous maxima and minima given for each day in the results for these years. Table XII gives the simultaneous values of the mean annual ranges at Cheltenham and Kew from 1902 to 1922 together with the ratios and Plate IV presents the annual sequence graphically. These afford a direct comparison between the march of events at the two stations. The third line of Table XII gives no evidence of any systematic variation in the value of the factor Cheltenham/Kew apart from an indication that there has been a tendency for it to decrease since the installation of the Cheltenham magnetograph. A fluctuation of some 20 per cent about the mean value does not

(1) *Terr. Mag.*, Vol. XXX, No. 4, Dec., 1925, pp. 191-213.
(2) *Loc. cit.*, p. 195.
(3) *Terr. Mag.*, Vol. XXXI, No. 1, March, 1926, pp. 37-47.

TABLE XII.—CHELTENHAM AND KEW MEAN ANNUAL *D* RANGE 1902-22 AND THE FACTOR. $^C/_K$.

Unit of the Decade		0	1	2	3	4	5	6	7	8	9
1900	Cheltenham	..	..	8·59′	10·56′	10·95′	12·15′	11·57′	12·53′	13·19′	12·90′
	Kew.. ..	..	..	8·42	11·28	10·46	11·69	10·27	12·11	12·60	11·97
	C/K. ..	..	..	1·020	0·936	1·047	1·040	1·126	1·034	1·046	1·077
1910	Cheltenham	12·69′	11·75′	9·43′	9·65′	9·99′	13·17′	14·87′	15·61′	16·25′	17·10′
	Kew.. ..	11·13	10·45	9·52	9·54	10·16	13·29	15·03	15·40	16·00	16·28
	C/K. ..	1·140	1·124	0·991	1·012	0·983	0·992	0·990	1·014	1·015	1·050
1920	Cheltenham	14·67′	13·04′	13·11′							
	Kew ..	14·66	13·37	13·05							
	C/K. ..	1·001	0·978	1·005							

appear to have any obvious connexion with disturbance in the two localities though it is to be noted that the inclination at Cheltenham is approximately 4° (1) in excess of that at Kew and the intensity of the vertical component of the earth's field some 25 per cent greater. Since an immediate corrollary of the Stewart-Schuster hypothesis concerning the diurnal variation of the Earth's magnetic field is that the vertical, rather than the horizontal component of force should be the controlling factor influencing the diurnal changes, the differences in the values of the vertical intensity between the two stations may go some way to account for the variations in the ratio from year to year.

§ 19. VALUES OF REGRESSION CO-EFFICIENTS FROM OTHER DATA FOR TWO OR MORE SUNSPOT CYCLES.

(a) *Interdiurnal variability of H.*—It was obviously of interest to see whether the other data used by Dr. Bauer and Mr. Duvall allowed similar conclusions to be drawn if treated in the same way as the declination ranges above. The series of annual values of Dr. Bartels's means of interdiurnal variability of *H* derived from five stations and extrapolated by material recruited from Wolf's range figures were treated in cycles in a manner precisely similar to that employed for the Kew ranges. Columns 6 to 8 of Table X containing the results of this work tend to indicate that an increase of mean sunspot frequency is associated with an increase rather than a decrease in the value of the quantity *b*. Consideration of Table VI p. 45, of the more recent publication of Messrs. Bauer and Duvall (*Terr. Mag.*, March, 1926) although dealing with variability data restricted to Batavia and Potsdam and standardized to the latter observatory, would alone afford evidence of a direct proportionality between the parameter *b* and sunspot frequency and an inverse relation connecting this latter with the quantity a_y except in the final group of years 1913-1922. This aspect of the table is not commented on by the authors.

(b) *International character figures* 1906-23.—When international character figures which are available from 1906 onwards are divided into two groups, those from the years 1906-12 and those for the group 1913-23 and treated by the "least square" methods to give values of the quantity *b* for each of the two groups, they provide results altogether in agreement with those for the declination range and variability of *H* data shown in Table X. For an increase of 21 per cent in the mean sunspot frequency from the first to the second group of years is associated with a 28 per cent increase in the value of *b*, as is seen from Columns 9 and 10 of Table X in which the figures are entered.

(c) *D Ranges at Cheltenham Observatory,* 1902-22.—The Cheltenham data have been submitted to a similar analysis. The outcome as shown in Table X does not suggest

(1) *Terr. Mag.*, Vol. XXXI, No. 1, March, 1926, pp. 27-31.

that any change should be made in the deductions already drawn from the other material used. Only the two periods 1902-1912 and 1913-22 are available and each cycle is therefore incomplete to the extent of one year but the difference in mean sunspottedness for the two groups, being 33·7 on the Wolf-Wolfer scale for the earlier and 44·1 for the latter, is sufficiently marked to warrant a definite conclusion. An activity would from 0·043 to 0·063 for the 31 per cent rise in the indicated solar these two scarcely incline one to postulate an inverse proportionality between increase of b variables.

§ 20. Further note on the apparent Secular Change in the Magnetic Measures used in this Investigation.

In § 8 further examination of the apparent diminution in the mean range of declination had to be abandoned for the lack of a means of referring each cycle to a single standard of sunspot activity. With the measures of the calculated increase of range per unit increase of "frequency" now available it is possible to continue the investigation. The results are presented in Table XIII. To form this table values of the mean range for each cycle reduced to a common sunspot number 42·5 which was the mean for the entire period 1858-1923, have been determined using the values of a previously calculated for separate cycles. Corresponding results for the inter-diurnal variability of H whose large scale changes showed a marked parallelism with those for the Kew D-ranges are included in the table. Since the use of the extra-polated values of variability allows the first cycle to be completed the standard mean sunspot frequency in the case of this material covers the period 1856-1923. This mean is 41·7. The final column in the first section of the table gives the force equivalents of each of the mean ranges after reduction to the common basis of solar activity.

Table XIII.—The Changes in the Means of D Range and H Variability through Six Cycles.

Cycle Number.	Absolute Daily range of Declination (Kew)					Interdiurnal Variability of H (Potsdam, etc.)			
	$S_n - 42·5$	bs	a	Mean Range on Common basis	Equivalent Force	$S_n - 41·7$	bs	a	Mean Activity of H on Common basis
		′	′	′	γ		10γ	10γ	10γ
1	+15·1	−0·33	16·25	15·92	81·6	+ 7·9	−0·06	0·58	0·52
2	+14·1	−1·10	11·25	10·15	52·8	+14·9	−0·12	0·58	0·46
3	− 7·9	+0·54	10·74	11·28	59·3	− 7·1	+0·05	0·57	0·62
4	− 3·7	+0·31	10·06	10·37	67·8	− 2·9	+0·02	0·55	0·57
5	−11·4	+0·50	9·34	9·84	53·0	−10·6	+0·05	0·59	0·64
6	− 1·8	+0·12	10·73	10·85	58·2	− 1·0	+0·01	0·65	0·66

Some conspicuous features emerge from a scrutiny of each of the final sequences of group means of range and variability—features which are not, however, similar either in the relative magnitude or direction of the variations in the two magnetic measures. They are

(i) A fall and subsequent rise in both range and variability means within the first three groups of years. For the declination data the fall of nearly 30γ in the equivalent force greatly exceeds any subsequent inter-cycle fluctuations. Though the possibility of a slight inferiority of the ranges for that cycle has been already indicated, it is to be noted that the first two values of mean activity of H might equally well be in error of the true means for reasons previously mentioned.

(ii) While the increase in the mean force equivalent of the range from the second to the third cycle is markedly continued into the fourth, the variability figures show an opposite tendency.

(iii) A rise from the fifth to the sixth cycle is common to both measures.

Hence on the evidence of the material here available it would seem that no definite conclusion may be drawn as to the existence of a secular change either in range of D or variability of H other than that due to changes in solar activity. It is to be remembered, however, that entirely different deductions may be drawn from results obtained by processes differing in slight detail from those used in the present investigation. Corroboration of this may be seen in Table VI, p. 30, of *Geophysical Memoirs*, No. 29, in the formation of which a uniform b has been accepted for the entire set of years in place of a series of b's derived each from its own cycle. In view of the intrinsic nature of the quantity b and the relations in which it is involved it is difficult to say which procedure gives the best representations of the facts.

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ERRATA

§ 14, *page* 19

Line 6.—*For* $c_1 \sin (15° t + a_1)$ *read* $c_1 \sin (15° t + \alpha_1)$
For $c_2 \sin (30° t + a_2)$ *read* $c_2 \sin (30° t + \alpha_2)$
Line 8.—*For* the c, a constants *read* the c, α constants
Line 10.—*For* $a_n = \tan^{-1} (a_n/b_n)$ *read* $\alpha_n = \tan^{-1} (a_n/b_n)$
Line 14.—*For* the values of a_1 *read* the values of α_1