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Turbulence and Diffusion Note No 15 (May 1971)

ANALYSIS OF HIGH-FREQUENCY TURBULENCE DATA OBTAINED DURING JASIN, JUNE 1970

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1. Introduction

When plans for a Royal Society-sponsored air-sea interaction experiment were first discussed it was considered that because of the effects of ship motion on measured turbulence it would be difficult, if not impossible, to make satisfactory direct measurements of turbulent fluxes by instruments attached to a ship or a tethered balloon cable. It was suggested then that measurements of only high-frequency turbulent fluctuations (ie at frequencies higher than the characteristic ship motions) might be used to estimate turbulent fluxes provided certain terms in the relevant energy equations could be ignored or estimated conveniently. Some of the data obtained during the Joint Air-Sea Interaction Experiment (JASIN) in June 1970 provided the means of investigating the applicability of the high-frequency flux method over the sea in this context and are discussed in this note.

2. Theory

In the inertial subrange the following relations may be used to calculate rates of dissipation of turbulent fluctuations

$$S_{uu}(k) = C_1 \epsilon^{2/3} k^{-5/3} \quad (1)$$

$$S_{vv,ww}(k) = C_2 \epsilon^{2/3} k^{-5/3} \quad (2)$$

$$S_{\theta\theta}(k) = C_3 \chi_\theta \epsilon^{-1/3} k^{-5/3} \quad (3)$$

$$S_{qq}(k) = C_4 \chi_q \epsilon^{-1/3} k^{-5/3} \quad (4)$$

Here $C_1 = 3/4\ C_2 \simeq 0.47$ (Panofsky and Pasquill 1963) and $C_3 \simeq 0.7$ (Panofsky 1969) if k is in radians per metre. C_4 would be expected to have a value similar to that for C_3. The conventional Taylor transformation ($k = 2\pi n/\bar{u}$) will be used: we may then write

$$k S_{uu}(k) = n S_{uu}(n) = 0.47 \epsilon^{2/3} (2\pi n/\bar{u})^{-2/3}$$

or

$$\epsilon_u = \left(n\, S_{uu}(n)/0.47\right)^{3/2} \; 2\pi n/\bar{U} \qquad (5)$$

and alternatively

$$\epsilon_w = \left(n\, S_{ww}(n)/0.63\right)^{3/2} \; 2\pi n/\bar{U} \qquad (6)$$

Thus two estimates for ϵ can be obtained, which are unlikely to agree even if the measured data are of very high quality because of the lack of isotropy usually observed in the -5/3 region for the atmosphere, and also because the horizontal velocity rather than the alongwind component of this velocity is that normally used in eq (5), leading presumably to overestimates for ϵ_u. Dissipation rates for the temperature and humidity fluctuations may then be calculated from the relation

$$\chi_{\theta,q} = \left(n\, S_{\theta\theta,qq}(n)/0.7\right)\, \epsilon^{-1/3} \left(2\pi n/\bar{U}\right)^{2/3} \qquad (7)$$

The turbulent energy equations are, for horizontally homogeneous and stationary turbulence,

$$-\overline{u'w'}\frac{\partial \bar{u}}{\partial z} - \overline{v'w'}\frac{\partial \bar{v}}{\partial z} + g\overline{\rho' w'}/\bar{\rho} - \epsilon - \frac{\partial}{\partial z}\overline{E'w'} - \frac{\partial}{\partial z}\overline{p'w'}/\bar{\rho} = 0 \qquad (8)$$

$$-\overline{\theta' w'}\frac{\partial \bar{\theta}}{\partial z} - \chi_\theta - \frac{\partial}{\partial z}\overline{\theta'^2 w'}/2 = 0 \qquad (9)$$

$$-\overline{q'w'}\frac{\partial \bar{q}}{\partial z} - \chi_q - \frac{\partial}{\partial z}\overline{q'^2 w'}/2 = 0 \qquad (10)$$

Here, $E' = \frac{1}{2}(u'^2 + v'^2 + w'^2)$. It is conventional, though not necessarily justifiable, to neglect the pressure transport term in (8), and as a first approximation the first term can be replaced by $-\overline{U'w'}\frac{\partial \bar{U}}{\partial z}$ where U is the horizontal velocity. Near the surface over the sea the second term can probably be neglected by comparison with the first especially in unstable conditions without strong baroclinicity. Taking into account the contribution of turbulent fluxes of both heat and moisture to the buoyant production of turbulent kinetic energy eq (8) can then be written

$$-\overline{U'w'}\frac{\partial \bar{U}}{\partial z} + g\left(\overline{\theta' w'} + 0.61\,\bar{T}\,\overline{q'w'}\right)/\bar{T} - \epsilon - \frac{\partial}{\partial z}\overline{E'w'} = 0 \qquad (11)$$

Further, as a first approximation $u'^2 + v'^2 \simeq 2U'^2$ and so $E' \simeq U'^2 + w'^2/2$

The assumption is sometimes made that the dissipation and mechanical production terms are equal (eg Lumley and Panofsky 1964) and eq (11) may then be written

$$-\overline{U'w'}\frac{\partial \bar{U}}{\partial z} = \epsilon \qquad (12)$$

or in the constant-flux layer

$$u_*^3 \phi_m / kz = \epsilon \qquad (13)$$

Alternatively the dissipation may be equated to the sum of mechanical and buoyant production terms, and then in the constant flux region

$$u_*^3 \phi_m / kz + g\left(\overline{\theta' w'} + 0.61 \overline{T}\, \overline{q' w'}\right)/\overline{T} = \epsilon \qquad (14)$$

Such a relation is supported by an analysis given by Busch and Panofsky (1968).

3. Data

During the initial digitising of data obtained during JASIN, three periods were selected where data were obtained simultaneously at two levels in unstable conditions. In each case ten sections equally spaced throughout the full length of record, each about $3\frac{1}{2}$ minutes long, were sampled by the higher-speed Cardington data logger at 5 scans/sec after filtering by 4-stage RC filters with cut-off (-12 db) at 2.5Hz. The parameters sampled were the magnitude of the wind vector, wind inclination and a high-pass filtered temperature signal (filter cut-off at 0.1Hz). The data were converted to horizontal and vertical wind speed, and temperature, and averaged spectra were obtained from the ten sets of data for each occasion, using the FFT subroutine described by Rayment in TDN9: the output spectral estimates included corrections for the low-pass filtering. Spectra were also calculated for the turbulent fluctuations of humidity, in this case using the full lengths of data sampled at 1Hz: again corrections were applied for the attenuation introduced by the low-pass filtering (cut-off at 0.5Hz) during the digitising stage.

4. Results

Summaries of data obtained during the three periods are given in table 1. Wind speeds were not measured at 10m but were calculated (very approximately in the third case) using the relation

$$U_{10} = U_z - \frac{u_*}{k}\left(\log z/10 + \psi_m(10/L) - \psi_m(z/L)\right) \qquad (15)$$

which applies in the constant flux layer: values for ψ_M were obtained from Dyer and Hicks (1970). For this purpose L was estimated with sufficient accuracy from the relation (TDN14)

$$L = -2.4\, U_{10}'^{2} / \left(\theta_s - \theta_{10} + 175(q_s - q_{10})\right) \qquad (16)$$

The subscript S refers to parameters derived from sea-temperature measurements and U'_{10} is a guess at the wind speed at 10m. The vertical fluxes of momentum, heat and moisture at the surface were obtained using the conventional formulae

$$\tau_0 = \rho C_D (U_{10} - U_0)^2 \qquad (17)$$

$$H_0 = \rho c_p C_{H,E} (\theta_S - \theta_{10})(U_{10} - U_0) \qquad (18)$$

$$E_0 = \rho C_{H,E} (q_S - q_{10})(U_{10} - U_0) \qquad (19)$$

C_D and $C_{H,E}$ were corrected for stability as described in TDN14 - it was assumed that $C_D(N) = C_{H,E}(N) = 1.3 \times 10^{-3}$. The listed values for L_0 were then calculated from

$$L_0 = - u_*^3 \overline{T} \rho c_p / kg (H_0 + 175 E_0) \qquad (20)$$

The measured fluxes were obtained by planimetry of the appropriate cospectra after smoothing of the obviously identifiable peaks due to ship motion.

Table 2 shows the magnitudes of various terms in the turbulent energy equations. The "mechanical" production terms in column 3 were calculated assuming the lower level of measurement was within the constant flux layer, a very rough approximation especially for the third period. The values for ϕ_M were those given by Dyer and Hicks (1970): u_{*0} was deduced using eq (17). The buoyant production of turbulent kinetic energy (column 4) was calculated using measured fluxes at the appropriate heights. The next three columns list respectively the rates of dissipation deduced from the alongwind and vertical velocity spectra using eq (5) and (6), and their average values. The alongwind spectra had been corrected for the finite time constant of the anemometers (about 0.1 sec). The estimates are very unreliable because it was found that the spectra showed no well-defined -5/3 region, almost certainly because of noise introduced by ship motion. Figure 1 shows a typical spectrum. The crosses are spectral estimates obtained as described in § 3: the dots are estimates using the complete length of data sampled at 1 second intervals and then segmented as described in TDN11. The agreement in the overlap region is excellent, showing that the ten short samples used in the high-frequency analysis were a good representation of the full record length, and that the low-pass filtering reduced aliasing to very small amounts.

The distinctive features are major peaks around 0.16 and 0.08Hz, the latter being somewhat obscured by the smoothing applied before plotting the data. However there are also harmonically related peaks at .02, .04, .3, .6, and 1.2Hz which with the two major peaks produce a series spaced exactly one octave apart. The peak at 0.16Hz corresponds to the characteristic period of the ship motion. Inspection of the analogue records shows that the spurious inclination angle and speed fluctuations due to this motion are markedly non-sinusoidal so a series of harmonically related peaks in the power spectra might be expected. The subsidiary peaks are certainly not the result of "leakage" which results from a non-ideal spectralwindow. This has the shape $((\sin \pi T n)/\pi T n)^2$, (Jones 1964) where T is the length of record. Thus a particular frequency in the original record appears not as a single spectral line but as a series spaced $^1/T$ apart, with amplitude falling off by a factor of 4 for each octave. The peak at about 1.2Hz may also be reinforced by contributions due to oscillations of the pendulum mount of the yawmeter at its natural frequency which lies close to this. The dissipation values in columns 5 - 7 were obtained by subjectively fitting a line of slope $-^5/_3$ to data between 1 and 2Hz after smoothing out any peaks in this frequency range. It will be argued later that these values for ϵ are almost certainly substantial overestimates.

The vertical fluxes of turbulent kinetic energy were obtained from 8-sec averaged data but showed marked changes with increasing averaging time, suggesting that ship-induced motion was important even for this relatively low-frequency cut-off (0.05Hz): this is not altogether surprising because triple velocity correlations will be more sensitive to the effects of ship motion than say the momentum flux.

Column 9 lists values for the production term in the temperature variance equation (eq (9)), using the measured heat flux and assuming in order to determine the potential temperature gradient that the measurement level is within the constant flux layer. Column 10 shows dissipation rates for the temperature fluctuations obtained from eq (7). Vestigial $-^5/_3$ regions were usually discernible in the temperature spectra, but at frequencies above about 1Hz the spectra were contaminated by instrumental noise (due chiefly to tape flutter and the telemetry system) and so even apart from any errors in ϵ , the χ_θ values were not very accurate. Column 11 gives the vertical flux of temperature variance, again from 8-sec averaged data.

The final three columns list terms in the balance equation for moisture fluctuations. The production was estimated assuming a constant flux layer up to the lower level of measurement, and using the measured moisture flux. In calculating χ_q, $S_{qq}(n)$ was corrected for (calculated) instrumental response before trying to identify a $-5/3$ region, but because of small harmonically-related peaks in the spectrum due presumably to ship motion, and noise at higher frequencies, the resulting estimates for χ_q could well have been significantly in error even apart from errors in ϵ (some of the noise must result from the differing time constants of the wet bulb and "dry bulb" thermometers and the use of an equation of the form

$$q = f_1(T_w) - f_2(T - T_w)$$

to deduce specific humidity from the observations).

For the lower level of measurement in each experimental run it is seen that the calculated dissipation is substantially larger than the estimated production of turbulent kinetic energy. Because both τ and $\frac{\partial \bar{U}}{\partial z}$ decrease with height it appears that dissipation is larger than production at the higher level also but the observed rate of change with height of the vertical flux of turbulent kinetic energy is positive and hence of the wrong sign to balance the equation. This suggests that the estimates for dissipation are significantly in error (too large). χ_θ and χ_q are likely to be more reliable since the temperature and humidity spectra show less noise due to ship motion than the velocity spectra and because ϵ only enters as the one third power in their case, and in this context it is interesting to note the approximate balance between production and dissipation for the humidity fluctuations, but the striking imbalance for corresponding terms in the temperature variance budget.

It is clear therefore that using a pendulum-mounted yawmeter in order to obtain a satisfactory levelling of the yawmeter in the mean prevents the use of the resulting velocity data to estimate dissipation from spectral estimates at frequencies above that of the characteristic ship motion. Satisfactory "high-frequency" measurements could presumably be made with the present system only by clamping the yawmeter mount, and this would preclude any possibility of measurement of the vertical turbulent fluxes. On the other hand it may be possible to use spectral estimates at frequencies substantially lower than ship motion, where the signal-noise ratio is much more favourable, provided the $-^5/3$ region extends sufficiently far. Measurements of Miyake et al (1970) suggest the limit for w is around $nz/\bar{U} \sim 1$ (probably somewhat lower for U) or in the present circumstances around 0.16Hz at 45m and 0.07Hz at 90m. This allowed revised estimates for ϵ to be obtained (Table 3) except at 45m where the limit corresponded to the main noise peak in the U and w spectra. In these cases the observed spectra at low frequencies were extrapolated to a frequency of 0.16Hz and a $-^5/3$ relation assumed at higher frequencies: the resulting estimates cannot be very reliable but it is difficult to estimate the magnitude of the errors. Revised values for χ_θ and χ_q also appear in table 3, and estimates for the vertical fluxes of turbulent energy calculated from 32 sec averaged data (shown in parentheses).

It is seen now that at 45m the total production of turbulent kinetic energy exceeds dissipation, though there may be an approximate equality of dissipation, and mechanical production. However, because the mechanical production falls off approximately as the reciprocal of the height, the dissipation will substantially exceed total and mechanical production at 140m. The vertical flux of turbulent kinetic energy increases with height between 45 and 140m, with a net flux divergence of around +10 $cm^2\ sec^{-3}$, of correct sign but much too large to balance the equation at 45m, and too large and the wrong sign at 140m! On the third occasion the mechanical production was rather small (winds at 90 and 185m were nearly equal) and on average dissipation exceeded production but again the flux divergence was large and of the wrong sign to effect a balance. In general it appears that close to the surface (45m) the total production probably exceeds dissipation but the reverse is true a hundred or so metres higher: the divergence as calculated seems completely unreliable.

In the case of the temperature fluctuations, at the lower height of measurement in each case, the production is small compared to dissipation. At 140m on the first two occasions the potential temperature gradient was almost certainly positive, probably a few tenths of a degree per kilometre, and so the production, while positive also, was again smaller than dissipation. On the third occasion (185m) because of the small heat flux the production was again probably less than dissipation. However, on only two out of the three occasions was the divergence of the correct sign to balance the discrepancy.

The moisture fluctuation data suggest an approximate equality between production and dissipation at 45m. In the third case at 90m the calculated production may be significantly in error because it is based on the constant flux layer assumption. The average divergence between the two levels for all three occasions is around -2×10^{-11} (8-sec averaged data) which, assuming production is inversely proportional to height, is of the right sign to balance the equation. On the other hand the comparatively large values of $\frac{1}{2}\overline{w'q'^2}$ at the lower level suggests that production exceeds dissipation nearer the surface.

5. Conclusions

At 45m the data are not inconsistent with approximate equality of mechanical production and dissipation of turbulent kinetic energy and production and dissipation of humidity fluctuations, but there is marked inequality in the corresponding quantities for temperature. At greater heights it is not possible to draw very firm conclusions since in the absence of accurately measured vertical gradients it is not possible to estimate accurately the production terms. However it appears that at the higher level on these three occasions dissipation exceeded production in all cases (turbulent kinetic energy, temperature and moisture variance).

It appears then that even if reliable values for ϵ , χ_θ and χ_q could be obtained, the divergence terms in the appropriate energy balance equations are so important that it would not be reasonable to neglect these, especially well above the surface, and equate production and dissipation to obtain fluxes if the appropriate vertical gradients were measured as well.

Unexpectedly, the effects of ship motion have been found to extend to harmonically related frequencies much higher than the characteristic ship motion and this makes the calculation of dissipation rates rather uncertain. In general a worthwhile improvement could only be obtained by using a servo-stabilising system to control the amount of tethering cable paid out, to reduce the effects of ship motion. However this would also much increase the reliability of estimates of vertical fluxes obtained by the conventional eddy correlation method (it will be shown in a later TDN that fluxes obtained in this way with the present system may already be plausible estimates) and hence diminish the need for a high-frequency method. Steps have already been taken to secure a suitable servo-system.

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Table 1. Measured fluxes and calculated surface fluxes

Date/time	U_{10} m sec^{-1}	H_0 mw cm^{-2}	E_0 g cm^{-2} sec^{-1}	τ_0 dynes cm^{-2}	L_0 m	Height m	Measured wind speed m sec^{-1} (a)	Measured fluxes H	$E \times 10^6$	τ
17/6/70 1610-1738	7.7	0.48	1.9 $\times 10^{-6}$	-0.90	-220	45 140	8.38 8.67	0.12 -0.76	2.1 0.6	-0.80 -0.59
17/6/70 1749-1922	6.9	0.43	2.1 $\times 10^{-6}$	-0.71	-145	45 140	7.48 7.86	0.75 -0.89	3.4 3.8	-0.39 -0.56
17/6/70 2143-0005	5.7	0.97	2.2 $\times 10^{-6}$	-0.52	- 55	90 185	6.05 6.10	0.68 0.19	2.0 1.9	-0.39 -0.23

Note (a) Speed not corrected for ships' drift velocity

Table 2. Turbulent energy balance

(1) Date/time	(2) Height m	(3) $u_*^3 \phi_M / kz$ $cm^2\ sec^{-3}$	(4) $\frac{g}{\rho c_p T}(H + 0.61 \bar{T} c_p E)$ $cm^2\ sec^{-3}$	(5) ϵ_u $cm^2\ sec^{-3}$	(6) ϵ_w $cm^2\ sec^{-3}$	(7) $\bar{\epsilon}$ $cm^2\ sec^{-3}$	(8) $\frac{1}{2}\overline{w'(2U'^2 + w'^2)}$ $cm^3\ sec^{-3}$	(9) $\frac{\overline{w'\theta'}^2}{k u_* z} \phi_H$ $^oC^2\ sec^{-1}$	(10) $\chi_\theta \times 10^5$ $^oC^2\ sec^{-1}$	(11) $\frac{1}{2}\overline{w'\theta'^2}$ cm $^oC^2\ sec^{-1}$	(12) $\frac{\overline{w'q'}^2}{k u_* z} \phi_E$ sec^{-1}	(13) $\chi_q \times 10^{11}$ sec^{-1}	(14) $\frac{1}{2}\overline{w'q'^2}$ cm sec^{-1}
17/6/70 1610-1738	45 140	7.4 -	+ 1.3 - 1.8	42 10.4	28 19.2	35 14.8	$+ 1.0 \times 10^5$ $+ 2.2 \times 10^5$	1×10^{-7} -	4.8 1.7	$- 0.8 \times 10^{-2}$ $+ 4.7 \times 10^{-2}$	2.5×10^{-11} -	4.1 3.3	$+ 2.0 \times 10^{-7}$ $- 0.8 \times 10^{-7}$
17/6/70 1749-1922	45 140	4.7 -	+ 3.7 - 0.6	54 8.8	18.1 10.3	18.1 (a) 9.6	$+ 2.5 \times 10^5$ $+ 3.6 \times 10^5$	3.3×10^{-6}	5.7 1.6	$+ 1.5 \times 10^{-2}$ $- 23 \times 10^{-2}$	6.2×10^{-11} -	7.7 3.2	$+ 5.9 \times 10^{-7}$ $+ 4.3 \times 10^{-7}$
17/6/70 2143-0005	90 185	(1.0) (b) -	+ 2.8 + 1.4	14.9 6.1	14.1 8.1	14.5 7.1	$- 0.5 \times 10^5$ $+ 1.7 \times 10^5$	(0.8×10^{-6}) (b)	3.8 0.8	$+ 3.7 \times 10^{-2}$ $+ 0.4 \times 10^{-2}$	(6.1×10^{-12}) (b) -	2.5 1.2	$+ 2.8 \times 10^{-7}$ $+ 2.1 \times 10^{-7}$

Notes (a) Assuming $\bar{\epsilon} = \epsilon_w$

(b) Unreliable estimate because of height above surface

Table 3. Revised energy balance

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1610-1738	45 140	7.4 -	+ 1.3 - 1.8	4.6 5.5	3.9 4.5	4.3 5.0	$+1.0(1.5) \times 10^5$ $+2.2(2.7) \times 10^5$	1×10^{-7} -	2.3 1.2	$-0.8(-1.7) \times 10^{-2}$ $+4.7(6.8) \times 10^{-2}$	2.5×10^{-11} -	2.0 2.3	$+2.0(2.4) \times 10^{-7}$ $-0.8(-1.3) \times 10^{-7}$
1749-1922	45 140	4.7 -	+ 3.7 - 0.6	5.7 5.4	7.4 3.5	6.5 4.5	$+2.5(2.9) \times 10^5$ $+3.6(5.0) \times 10^5$	3.3×10^{-6} -	4.0 1.2	$+1.5(1.9) \times 10^{-2}$ $-23(-34) \times 10^{-2}$	6.2×10^{-11} -	5.6 2.6	$5.9(6.4) \times 10^{-7}$ $4.3(6.6) \times 10^{-7}$
2143-0005	90 185	(1.0)	+ 2.8 + 1.4	5.9 3.7	3.6 4.4	4.7 4.1	$-0.2(-0.1) \times 10^5$ $+1.7(2.2) \times 10^5$	(0.8×10^{-6})	2.6 0.6	$+3.7(4.7) \times 10^{-2}$ $+0.4(0.4) \times 10^{-2}$	(6.1×10^{-12}) -	1.7 1.0	$2.8(3.8) \times 10^{-7}$ $2.1(2.8) \times 10^{-7}$

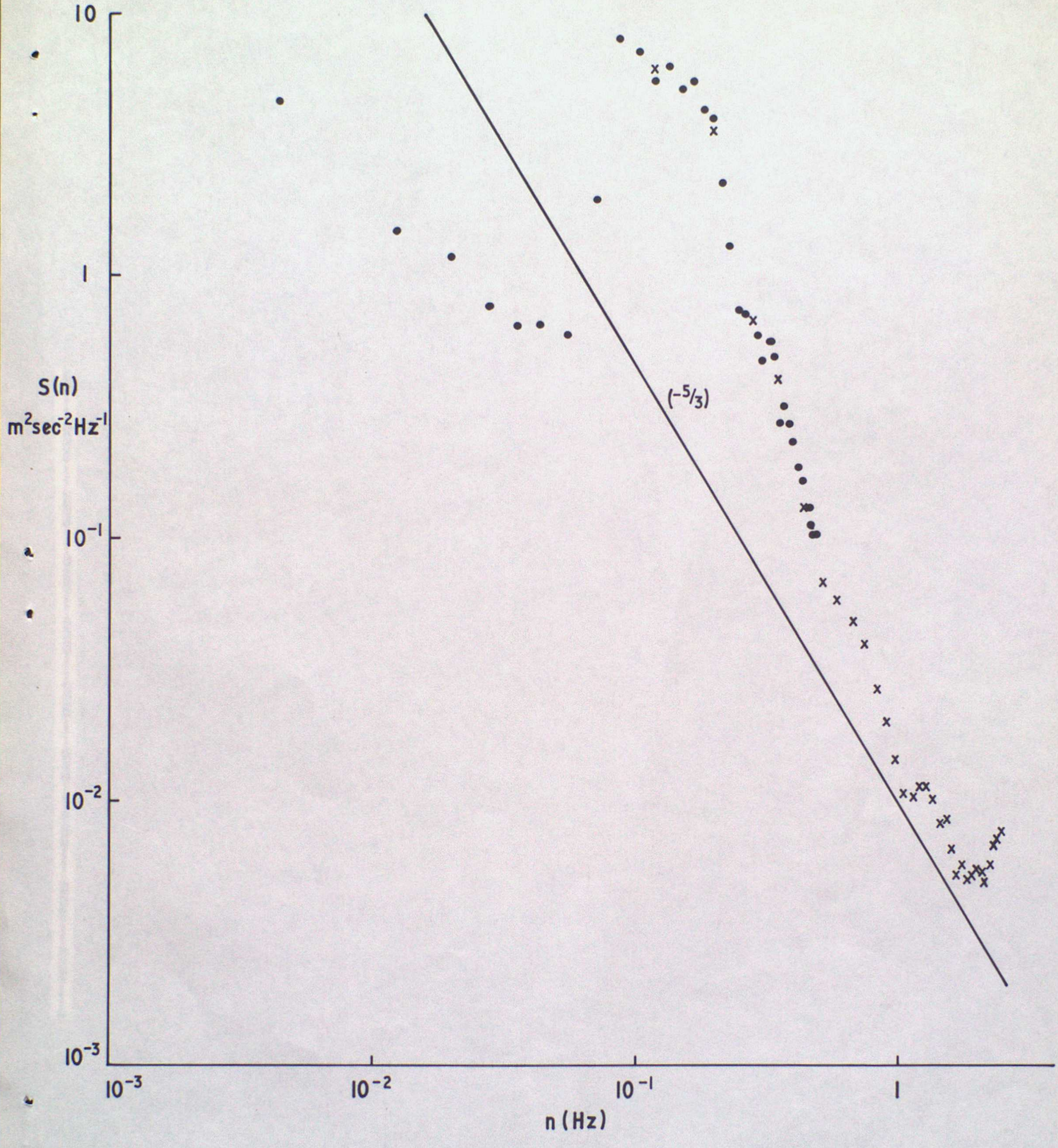

FIGURE I. COMPOSITE W-SPECTRUM, 140m, 1610-1738Z 17/6/70

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