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A THREE DIMENSIONAL CUMULONIMBUS MODEL: WARM MICROPHYSICAL PARAMETRIZATION

by

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A THREE DIMENSIONAL CUMULONIMBUS MODEL: WARM MICROPHYSICAL PARAMETRIZATION

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MET O 15

1. INTRODUCTION

The central problem of cumulonimbus modelling, and of understanding cumulonimbus clouds themselves, is the interaction between the motions of the air and the changes of phase of water that these motions induce. These interactions introduce a wide range of length and time scales, from molecular to mesoscale (10 orders of magnitude), that must be represented in some way. The physics of water vapour, liquid water and ice (the "microphysics") are complex and not fully understood. We cannot hope to explicitly model the molecular interactions governing the microphysics at the same time as the bulk air motions and so we parametrize the microphysics, that is represent the microphysical processes in functional form, the parameters of the functions being determined by the air motion scale or externally (such as the density of CCN). This note describes the parametrization scheme chosen for the CYBER cumulonimbus model (introduced in van den Berghe (1985)) in sufficient detail to enable the computer code to be followed, to delineate the areas of weakness of the parametrization and to indicate where improvements could be made.

The first approximation is to neglect the ice phase of the microphysics and consider only the interactions with and between liquid and gaseous water. This gives a large simplification since, to a good approximation, the effects of liquid water depend only (!) on the droplet size - a single parameter. The complexity of the next step, considering the effects of the different shapes of ice crystals and the increase in the number of interacting species, has meant that almost all Cb models ignore ice. Neglect of ice is valid if the cloud top temperatures are warmer than about -15°C which is reasonably valid in the tropics but not in mid-latitudes. The ice phase will be added to the model later.

The choice of the external parameters is usually made to force a model to fit certain aspects of observed clouds (eg rainfall rates, amounts, cloud height: see Bennetts and Rawlins (1981)). In the early formulations functional forms were postulated with all the external parameters as simple constants of proportionality (eg Kessler (1969)) but later formulations have been in terms of properties, such as dispersion, of the droplet distribution (eg Lin, Farley and Orville (1983)). Since it holds the promise of a physically consistent choice of external parameters we choose the second method; and so a more plausible way of changing parameters in differing situations (since there is no a priori reason for assuming that the same external parameters apply to both tropical and continental Cbs).

The following section describes the basis of the parametrization and the conservation equations used. Section 3 shows how the calculations are made and consistently approximated. Section 4 describes the functional forms used and discusses the choice of external parameters.

2. ANALYTICAL EQUATIONS AND NUMERICAL APPROXIMATIONS

As described in van den Berghe (1985) the model is based on an expansion of the anelastic equations that assumes that the zeroth and first order thermodynamic variables are hydrostatically balanced ie

$$\theta(\underline{x},t) = \Theta + \theta'(z) + \theta''(\underline{x},t) = \Theta(1+\theta^*)$$
$$p(\underline{x},t) = \underline{P}(z) + p'(z) + p''(\underline{x},t)$$
$$\rho(\underline{x},t) = \underline{\rho}(z) + \rho'(z) + \rho''(\underline{x},t)$$
$$T(\underline{x},t) = \underline{T}(z) + T'(z) + T''(\underline{x},t)$$
$$q_v(\underline{x},t) = q_v'(z) + q_v''(\underline{x},t)$$

Where T, ρ, p, θ, q_v are the air's temperature, density, pressure, potential temperature and water vapour mixing ratio. The basic state $(\theta(\underline{x}) = \Theta)$ is dry.

We assume that the continuous spectrum of condensed water drop size can be represented by two discrete classes. The choice of these classes is made by assuming that there are two regimes of drop size with distinct physical properties. The first is small drops which do not fall relative to the air but rapidly convert to water vapour; these we call cloud water (mixing ratio q_c). Larger drops, however, fall relative to the air and take a finite time to evaporate (rain water, mixing ratio q_R).

The conservation equations relevant to the microphysical scheme are

$$\frac{d\theta^*}{dt} = \frac{L}{c_p \Pi}(C_{vc} + C_{vR}) + \frac{1}{\rho}\frac{\partial}{\partial x_i}\rho K_H \frac{\partial \theta^*}{\partial x_i} \tag{1}$$

$$\frac{dq_v}{dt} = -C_{vc} - C_{vR} + \frac{1}{\rho}\frac{\partial}{\partial x_i}\rho K_H \frac{\partial q_v}{\partial x_i} \tag{2}$$

$$\frac{dq_c}{dt} = C_{vc} - C_{cR} + \frac{1}{\rho}\frac{\partial}{\partial x_i}\rho K_H \frac{\partial q_c}{\partial x_i} \tag{3}$$

$$\frac{dq_R}{dt} + \frac{1}{\rho}\frac{\partial}{\partial z}(\rho \bar{V}_T q_R) = C_{vR} + C_{cR} + \frac{1}{\rho}\frac{\partial}{\partial x_i}\rho K_H \frac{\partial q_R}{\partial x_i} \tag{4}$$

C_{ij} is a conversion rate from field i to field j, V_T a suitable fall speed for rain water. K_H the eddy mixing coefficient, assumed the same for all fields (but not constant, see van den Berghe (1985)). L is the latent heat of condensation of water vapour. Equations (1) to (4) are simplified versions of the full equations, for example the change in θ^* due to advection of water from different temperatures has been ignored (see eg Lin, Farley and Orville (1983)).

All the **rain** is falling at its terminal velocity and so exerts its full weight on the surrounding air, the vertical momentum equation becomes

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p''}{\partial z} - \frac{g}{\rho}P'' - g(q_v'' + q_c + q_R) + \frac{1}{\rho}\frac{\partial \tau_{i3}}{\partial x_i}$$

The equation of state for moist air is

$$P = \rho R_d \left(1 + q_v \frac{R_v}{R_d}\right) T \tag{5}$$

R_v and R_d are the gas constants for dry air and water vapour. This approximation is valid provided $q_v < q_{vs}(T)$ where $q_{vs}(T)$ is the saturated value of q_v at temperature T. Equation (5) is always valid in the model since we will assume no positive supersaturations.

Equations (1) to (4) are approximated as

$$\delta_t \overline{\theta^*}^t = ADV(\theta^*)^t + \frac{L}{c_p \Pi}\left(C^*_{vc} + C^*_{vR}\right) + DIFF(\theta^*)^{t-1} \tag{6}$$

$$\delta_t \overline{q_T}^t = ADV(q_T)^t - C^*_{vR} - C^*_{cR} + DIFF(q_T)^{t-1} \tag{7}$$

$$\frac{q_R^{t+1} - q_R^t}{\Delta t} = ADV(q_R)^t - FALL(q_R)^t + C_{vR}^{t+1} + C_{cR}^{t+1} + DIFF(q_R)^t \tag{8}$$

Where $q_T = q_v' + q_v'' + q_c$, $C^*_{ij} = \frac{1}{2}(C^t_{ij} + C^{t+1}_{ij})$, $ADV(\phi)$ the numerical form of the advective terms, FALL (q_R) the form of the term with $\overline{V}_T$ and DIFF (ϕ) the form of the diffusive terms. The superscripts denote the time level at which the approximation is made.

Equations (6) to (8) contain a mixture of leapfrog and forward time differencing with implicit backward approximations to the conversion terms. Equation (7) is the approximation to equation (2) plus equation (3). The remainder of this section will explain the reasoning behind this system.

The diffusive terms are approximated by a forward time step (over $2\Delta t$ in equations (6) and (7) and Δt in equation (8)) for stability. The most accurate and economical advective approximation is leapfrog and equations (6) and (7) use this (with ADV written using second order centred space approximation). Rainwater mixing ratio is always positive but approximations to ADV with leapfrog time differencing can produce spurious negative values so equation (8) is written using a forward time step and ADV-FALL can now be approximated by a scheme guaranteed to keep fields positive (a number of approaches to this problem exist, the earlier methods being based on arbitrary apportioning of fluxes while a steady stream of newer more rigorous methods is being developed; the simplest such scheme is upstream differencing, while adequate schemes are the FCT variant of van den Berghe and Nash (1984) or the scheme of Schneider (1984)). The implicit formulation of the conversion rates also arises from the need to ensure that q_c and q_R remain positive.

The parametrization of the microphysical processes reduces q_c to a diagnostic quantity (ie given T, p and q_T there is a unique partitioning of q_T into q_v'' and q_c). We can, therefore, sum equations (2) and (3) to give (7), allowing q_c to be advected using the more accurate second order centred/leapfrog combination. The non-positive definiteness of this scheme does not matter since initial conditions with $q_c(\underline{x}, t) = 0$ ensure that $q_T - q_v'$ is a small deviation about q_v'. Another advantage of this choice is that, in the absence of rain, q_T is conserved making the turbulence closure more valid.

The mixture of time differencing schemes means that care must be taken to ensure that total water substance is conserved during the conversion process ((Clark (1973)). Consider the approximation of the coupled equations

$$\frac{\partial \phi}{\partial t} = -C \qquad \frac{\partial \chi}{\partial t} = C$$

by

$$\frac{\phi^{t+1} - \phi^{t-1}}{\Delta t} = -C^* \qquad \frac{\chi^{t+1} - \chi^{t}}{\Delta t} = C^{t+1}$$

then since $\phi^{t+1} = \phi^{t-1} - 2\Delta t C^*$

and $\chi^{t+1} = \chi^{t} + \Delta t C^{t+1} = \chi^{t-1} + \Delta t (C^{t} + C^{t+1})$

χ and ϕ are conserved if $C^* = \frac{1}{2}(C^{t} + C^{t+1})$

Since q_c is diagnosed from q_T we write

$$C_{vc}^{t+1} = \frac{\bar{q}_c^{t+1} - q_c^{t}}{\Delta t}$$

Where $\bar{q}_c$ is q_c before any conversion to rainwater has taken place.

3. CALCULATION OF CONVERSION RATES

The scheme for describing the rates of conversion between the three water classes and the consequent transfers of heat has three main assumptions (and several lesser) which produce an (almost) explicit calculation from the implicit formulation. To help in assessing any limitations of the model microphysics we list these assumptions before describing the scheme in detail. The primary assumption is that the advected and diffused values of θ^*, q_T and q_R are good approximations to the $t+1$ values and so can be used instead of the $t+1$ values in calculating the conversion rates. The "bulk physical" assumption (eg Clark 1979)) is that the relative humidity is never greater than 100% which is justified by the small dynamical effects of supersaturated updraughts in cumulonimbus (see eg Ludlam (1980), p 252). If we also assume that cloud water evaporates and condenses instantaneously then cloud water becomes a diagnostic quantity. Conversely we assume that the evaporation of rain proceeds on a long time scale: so that it does not produce significant supersaturations over a single timestep (or produce sufficient cooling to change q_{vs}), that p'' is small compared to p' and that ΔT is sufficiently small so that $q_{vs}(T+\Delta T)$ can be linearly approximated in ΔT. These approximations have the effect of allowing either q_c or q_R to interact with q_v (but not both) and allowing a single iteration to adjust for the effect of condensation on q_{vs}.

We may be adding the ice phase to the model microphysics at a later date so it will be most convenient to work in terms of temperature rather than potential temperature. The conversion between the two is effected by linearizing $\Theta = T(P/P_0)$ about the basic state to give

$$T(\underline{x},t) = \mathrm{T} + T' + \frac{\mathrm{T}}{\Theta} T''$$

where we have assumed $p'' \ll p$. This approximation overcomes the implicit relationship between p, Θ and T in the anelastic equations (Ogura and Phillips (1962)) and has also been used by Clark (1979) and Dutton and Fichtl (1969) (it is also necessary since the Θ to T calculation takes place at $t+1$ where p'' is unknown).

To find the time stepped values of the fields we firstly advect and diffuse them with the conversion rates set to zero and then use these advected and diffused values to calculate the conversion rates needed to produce the final values. This two step procedure is usually used in bulk microphysical calculations and, as implemented here (and discussed by Klemp and Wilhelmson (1978)), in effect solves equations (1) to (4) in terms of conservation equations for non-precipitating water and equivalent potential temperature, both quantities being more conservative than the original individual fields q_v, q_c and Θ.

The first step is to produce $\tilde{\Theta}^{*\prime}$, $\tilde{q}_T^{\prime}$, $\tilde{q}_R^{\prime}$ and $\tilde{q}_c^{\prime}$

where $\tilde{\Theta}^{*\prime} = \Theta^{*\,t-1} + 2\Delta t \{ ADV(\Theta^*)^t + DIFF(\Theta^*)^{t-1} \} + \Delta t/\textcircled{4}\; C_\Theta^t$

$$\tilde{q}_T^{\prime} = q_T^{t-1} + 2\Delta t \{ ADV(q_T)^t + DIFF(q_T)^{t-1} \} + \Delta t/\textcircled{4}\; C_T^t$$

$$\tilde{q}_R^{\prime} = q_R^t + \Delta t \{ ADV(q_R)^t - FALL(q_R)^t + DIFF(q_R)^t \}$$

$\tilde{q}_c^{\prime} = q_c^t +$ (advection and diffusion); and is used to calculate the amount of heating tnat results from condensation between t and $t+1$.

We then convert $\tilde{\Theta}^{*\prime}$ to $\tilde{T}^{\prime}$, diagnose an initial estimate of q_{vs} $(= \tilde{q}_{vs}^{\prime})$ and adjust the fields as follows:

<u>IF $\tilde{q}_T^{\prime} < \tilde{q}_{vs}^{\prime}$ AND $\tilde{q}_c^{\prime} \neq 0$</u>

$$\tilde{T} = \tilde{T}^{\prime} - \frac{L}{c_p}\tilde{q}_c^{\prime}\,, \quad C_\Theta^{t+1\,\prime} = -\frac{1}{\Delta t}\frac{L}{c_p}\frac{\textcircled{4}}{I}\tilde{q}_c^{\prime}\,,$$

$$\tilde{q}_{vs} = q_{vs}(\tilde{T})\,, \quad \tilde{q}_c = 0$$

<u>otherwise</u> $\tilde{T} = \tilde{T}^{\prime}$, $\tilde{q}_{vs} = \tilde{q}_{vs}^{\prime}$ ETC.

(ie if the air is subsaturated at $t+1$ but some cloud water has moved in then evaporate all this cloud water into the air at $t+1$)

<u>IF $\tilde{q}_T < \tilde{q}_{vs}$ AND $q_R^t = 0$</u>

$$q_T^{t+1} = \tilde{q}_T\,, \quad \Theta^{*\,t+1} = \tilde{\Theta}^{*\prime} + \frac{\Delta t}{\textcircled{4}} C_\Theta^{t+1\,\prime}\,, \quad q_c^{t+1} = 0,\ q_R^{t+1} = 0\ C_\Theta^{t+1} = C_\Theta^{t+1\,\prime},\ C_T^{t+1} = 0$$

(ie the air is subsaturated and there is no rainwater so no conversions)

<u>OR IF $\tilde{q}_T < \tilde{q}_{vs}$ AND $q_R^t \neq 0$</u>

$$q_T^{t+1} = \tilde{q}_T - \Delta t\, C_{VR}\,, \quad q_R^{t+1} = \tilde{q}_R + \Delta t\, C_{VR}\,, \quad q_c^{t+1} = 0$$

$$C_\Theta^{t+1\,\prime} = C_\Theta^{t+1\,\prime} + \frac{L}{c_p}\frac{\textcircled{4}}{I} C_{VR}\,, \quad C_T^{t+1\,\prime} = -C_{VR}\,,$$

$$\Theta^{*\,t+1} = \tilde{\Theta}^{*\prime} + \frac{\Delta t}{\textcircled{4}} C_\Theta^{t+1\,\prime}$$

(ie the air is subsaturated but there is rain so evaporate the rain, without considering possible supersaturations or changes to q_{vs})

<u>OR IF $\tilde{q}_T > \tilde{q}_{vs}$</u>

The air is above saturation and we convert sufficient vapour to cloud water to just reach saturation, taking account of the release of latent heat which will change q_{vs}. We then convert cloud to rain water. We assume that the adjustment procedure will change a field ϕ by an amount $\Delta\phi$. The equilibrium values (with $q_{vs}^{t+1'} = q_{vs}(\theta^{*t+1})$ NOT $q_{vs}(\tilde{\theta}^*)$) will be

$$\theta^{t+1} = \tilde{\theta} + \frac{L}{c_p}\frac{\Theta}{T}(\bar{q}_c - \tilde{q}_c) + \Delta\theta \quad , \quad \bar{q}_c = \tilde{q}_T - \tilde{q}_{vs}$$

$$q_c' = \bar{q}_c + \Delta q_c \quad , \quad q_{vs}^{t+1'} = \tilde{q}_{vs} + \Delta q_{vs} \quad , \quad q_v^{t+1} = \tilde{q}_v + \Delta q_v$$

$$c_\theta^{t+1} = c_\theta^{t+1'} + \frac{1}{\Delta t}\frac{L}{c_p}\frac{\Theta}{T}(\bar{q}_c - \tilde{q}_c) + \Delta\theta$$

(the final amount of cloud water will be q_c' after any conversion to rain water).

We linearise q_{vs} about $\tilde{\theta}^*$ (Miller and Pearce (1974)) to find Δq_{vs} ie

$$\Delta q_{vs} = (\alpha + \Delta\theta)\left.\frac{\partial q_{vs}}{\partial\theta}\right|_{\theta=\tilde{\theta}} \quad , \quad \alpha = \frac{L}{c_p}\frac{\Theta}{T}(\bar{q}_c - \tilde{q}_c)$$

The bulk physical assumption states that $\Delta q_v = \Delta q_{vs}$

The change in temperature is related to the change in q_v needed to reach equilibrium, as is the change in q_c ie

$$\Delta\theta = -\frac{L}{c_p}\frac{\Theta}{T}\Delta q_v \quad \text{and} \quad \Delta q_c = -\Delta q_v$$

hence

$$\Delta\theta = \frac{\beta\alpha}{1-\beta} \quad \text{where} \quad -\frac{L}{c_p}\frac{\Theta}{T}\left.\frac{\partial q_{vs}}{\partial\theta}\right|_{\theta=\tilde{\theta}}$$

finally

$$q_c^{t+1} = q_c' - C_{CR}$$

$$q_R^{t+1} = \tilde{q}_R + C_{CR}$$

$$q_T^{t+1} = \tilde{q}_T - C_{CR}$$

$$C_T^{t+1} = -C_{CR}$$

Computationally the most efficient way to calculate saturation vapour pressure (and THUS q_{vs}) is to use the polynomial expansions of Lowe and Ficke (1974). These polynomials have sufficient accuracy only in the range of fit (-50°C to +50°C) and must not be used outside this range.

We use
$$q_{vs}(\theta) = \frac{.625}{(p+p')} E_s(T)$$

$$\frac{\partial q_{vs}}{\partial \theta} = \frac{L}{(I+T')^2 R_v} q_{vs} \frac{\partial T}{\partial \theta} = \frac{L}{R_v (I+T')^2} q_{vs} \frac{I}{\Theta} \frac{1}{Z}$$

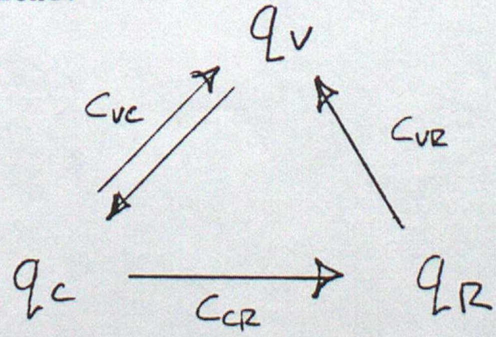
where we have neglected the p'' contribution to the denominator (Clark 1979)).

When E_s is in mb and T is in °C the Lowe and Ficke polynomials are $E_s = a_0 + T(a_1 + T(a_2 + T(a_3 + T(a_4 + T(a_5 + Ta_6)))))$

	E_s
$a_0 =$	6.107799961
$a_1 =$	4.436518521 (-1)
$a_2 =$	1.428945805 (-2)
$a_3 =$	2.650658471 (-4)
$a_4 =$	3.031240396 (-6)
$a_5 =$	2.034080998 (-8)
$a_6 =$	6.136820929 (-11)

4. PARAMETRIZATION OF CONVERSION RATES

The figure below shows the interactions allowed between the water classes. Each of the interactions needs to be parametrized and this section describes some possible formulations.



(i) C_{VC} This conversion rate is derived from the difference between the diagnosed q_c at $t+1$ and that after advection and diffusion ie it follows from the bulk physical assumption.

(ii) C_{VR} if the air is subsaturated and there is rain it evaporates (at a rate CEVAP). If the air is above saturation no condensation takes place (since all excess water vapour is assumed to condense on to cloud droplets)

(iii) C_{CR} this conversion rate consists of two parts:

CAUT; "autoconversion" of cloud to rain, a parametrization of the effects of cloud drop collisions in producing rain sized drops. Since, initia it is the only way of producing rain it is important in the early part of the cloud (in mid-latitudes pure water-water collisions are thought not to be important and the ice phase is crucial)

CACW the accretion of cloud water by rain water falling relative to the air and sweeping up droplets.

To specify CEVAP and CACW we take account of the variation of these rates with the size of the rain drops and integrate over an assumed drop size distribution and so relate these rates to the rain water mixing ratio. If D is the diameter of a raindrop and n(D) is the number of raindrops between D and D+dD then

$$n(D) = N_0 \exp(-\lambda D) \tag{9}$$

No is given by Marshall and Palmer (1948) as $8 \times 10^6\ m^{-4}$. The slope parameter, λ, is obtained by multiplying equation (9) by the particle mass, integrating over D=0 to ∞ and equating to q_R, thus

$$\lambda = \left[\frac{\pi \rho_w N_0}{\rho q_R} \right]^{1/4} m^{-1} \tag{10}$$

(ρ_w = density of liquid water).

Most cumulonimbus models start with equations (9) and (10) but then diverge in their parametrizations with large variations in complexity. Most of this complexity is superfluous so a scheme that is computationally efficient but based on sound physical principles is probably best. To enable this proposition to be tested we use the parametrization of Lin, Farley and Orville (1983) which can easily be changed from an accurate but expensive scheme to a cheap scheme. At the end of this section we give the most computationally efficient version of this scheme.

Rain reaches its terminal velocity on formation which, for a drop of diameter D, is

$$V_D = a D^b \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

(the best choice of b is .8 but plausible arguments can be made for the use of $\frac{1}{2}$ or 1, which are computationally more efficient).

The appropriate mass weighted terminal velocity is

$$\bar{V}_T = \frac{a\,\Gamma(4+b)}{6\lambda^b}\left(\frac{\rho_0}{\rho}\right)^{1/2}$$

Assuming that raindrops sweep up the cloud water in their path with efficiency Ec, the accretion rate is

$$CACW = \frac{\pi E_c N_0\, a\,\Gamma(3+b)}{4\lambda^{3+b}}\left(\frac{\rho_0}{\rho}\right)^{1/2} q_c$$

The evaporation of rain is assumed to obey a diffusional growth equation modified by ventilation from the drop fall (eg Orville and Kopp (1977))

$$CEVAP = \frac{2\pi (q_v - q_{vs}) N_0}{\rho A}\left\{\frac{.78}{\lambda^2} + .31\, S_c^{1/3}\,\Gamma\left(\frac{b+5}{2}\right) a^{1/2}\nu^{-1/2}\left(\frac{\rho_0}{\rho}\right)^{1/2}\lambda_R^{-\left[b+\frac{5}{2}\right]}\right.$$

$$A = \frac{L^2}{K_a R_v T^2} q_{vs} + \frac{1}{\rho\psi}$$

(Other symbols are defined at the end of this section.)

Since autoconversion results from the statistical nature of the collision process and also depends on the details of the cloud droplet size distribution parametrizations of it are less well theoretically based. We repeat three forms here with their justification (if any).

The most commonly used, but unjustified, form is (Kessler (1969)),

$$CAUT = \alpha (q_c - q_{CRIT} / \rho) \tag{11}$$

Where q_{CRIT} is a threshold mixing ratio below which no autoconversion takes place.

Tripoli and Cotton (1980) note that the autoconversion rate will depend on the collision frequency of cloud drops (f_c), the efficiency of the collisions E_{CW} and the amount of cloud water, ie

$$CAUT = E_{CW} f_c q_c$$

and estimating as proportional to the mean droplet concentration (N_c), fall speed of the cloud drops (!) and area swept out (assuming a constant mean radius)

$$CAUT = \frac{.104\, g\, E_{CW}\, \rho^{4/3}}{\nu (N_c \rho_w)^{1/3}} q_c^{7/3} \tag{12}$$

$$\approx \frac{.104\, g\, E_{CW}}{\nu (N_c \rho_w)^{1/3}} (5\times10^{-4})^{1/3} \rho q_c^2$$

They also assume that, if the drops are on average smaller than r_{CRIT}; $E_{CW} = 0$ and this implies a threshold q_{CRIT} of

$$q_{CRIT} = \frac{4}{3}\pi \rho_w r_{CRIT}^3 \frac{N_c}{\rho} \tag{13}$$

Berry (1968) has considered detailed integrations of droplet growth and proposed a form of $CAUT$ that can be simplified to

$$CAUT = 3.76\times10^{13} \frac{D_o}{N_c} q_c^3 \tag{14}$$

Where D_o is the relative dispersion of the cloud drop size distribution.

A version of the microphysical parametrization is given below but before any particular form is fixed for the model experiments should be performed to determine the trade off (if any) between accuracy and efficiency and also the best form of $CAUT$. It should be remembered that different air masses may have different properties that are reflected in the choice of parameters (eg Nc will be vastly different between continental and tropical maritime air). A good scheme will allow a consistent way of taking these variations into account; if they are important. The coded scheme uses a linear fall speed law; $U_D = \left(\frac{\rho_0}{\rho}\right)^{1/2} a D$

(valid for 8×10^{-5} m $\leq D \leq 1. \times 10^{-3}$ m)

so $\bar{V}_T = \alpha_{VT}(z)\, q_R^{1/4}$

$$\alpha_{VT} = \frac{a\Gamma(5)}{6(\pi\rho_w N_0)^{1/4}} \frac{\sqrt{\rho_0}}{\rho^{1/4}}$$

and $CACW = \alpha_{ACW}\, q_C q_R$

$$\alpha_{ACW} = \frac{a\,\Gamma(4) E_c}{4\rho_w} \sqrt{\rho\rho_0}$$

also $CEVAP = \dfrac{\alpha_1 (q_v' + q_v'' - q_{vs})}{(\alpha_2(z) q_{vs} + \alpha_3(z))} \left(\alpha_4(z) q_R^{1/2} + \alpha_5(z) q_R^{3/4}\right)$

$$\alpha_1 = 2\pi N_0$$

$$\alpha_2 = \frac{L^2 \rho}{k_a R_v (\bar{T} + T')^2}$$

$$\alpha_3 = \frac{1}{\rho \psi}$$

$$\alpha_4 = .78\, \rho^{1/2} (\pi \rho_w N_0)^{-1/2}$$

$$\alpha_5 = .31\, S_c^{1/3}\, \Gamma(3)\, a^{1/2} \nu^{-1/2} \left(\frac{\rho_0}{\rho}\right)^{1/4} \left(\frac{\rho}{\pi\rho_w N_0}\right)^{3/4}$$

we use the Tripoli and Cotton autoconversion

$$CAUT = \alpha_{AUT}\, q_c^2 \qquad q_c > q_{CRIT}$$

$$\alpha_{AUT} = \frac{.104\, g\, E_{CW}}{\nu (N_c \rho_w)^{1/3}} (5 \times 10^{-4})^{1/3} \rho \qquad q_{CRIT} = \frac{\frac{4}{3}\pi \rho_w r_{CRIT}^3 N_c}{\rho}$$

The parameters of the microphysical scheme are,

N_0 = intercept of drop size distribution = 8×10^6 m^{-4}

a, b = parameters of terminal velocity

EG SINCE $b = 1$ $a = 4 \times 10^3\, s^{-1}$

E_c = collection efficiency for rain/cloud collision = 1

E_{CW} = collection efficiency for cloud/cloud collisions = .55

N_c = mean concentration of cloud droplets = 3×10^8 m^{-3}

r_{CRIT} = critical radius of cloud drops for efficient coalescence = 10^{-5} m

ρ_w = density of water = 10^3 kg m^{-3}

ν = kinematic viscosity of air = 1.36×10^{-5} m^2 o^{-1}

ψ = diffusivity of water vapour air = 2.26×10^{-5} m^2 s^{-1}

Sc = Schmidt number = ν / ψ = 0.6

K_a = thermal conductivity of air = 2.43×10^{-2} Sm^{-1} o^{-1} K^{-1}

L = latent heat of condensation of water = $3.121 \times 10^6 - 2.274 \times 10^3$

REFERENCES

BENNETTS, D. A. and RAWLINS, F., (1981)

Parametrization of the ice phase in a model of mid-latitude cumulonimbus convection and its influence on the simulation of cloud development.

Quart J. R. Met. Soc. 107, 477-502.

BERRY, E. K., (1968)

Modification of the warm rain process.

Proc. 1st Nat. Conf. Weather Modif. Albany, Amer. Met. Soc., 81-88.

CLARK, T. L., (1973)

Numerical modelling of the dynamics and microphysics of warm cumulus convection.

J. Atmos. Sci., 30, 857-878.

CLARK, T. L., (1979)

Numerical simulations with a three dimensional cloud model: lateral boundary condition experiments and multicellular severe storm simulations.

J. Atmos. Sci., 36, 2191-2215.

DUTTON, J. A. and FICHTL, G. H., (1969)

Approximate equations of motion for gases and liquids.

J. Atmos. Sci., 26, 241-254.

KESSLER, E., (1969)

On the distribution and continuity of water substance in atmospheric circulations.

Met. Mon. Amer. Met. Soc., 10, No 32.

KLEMP, J. B. and WILHELMSON, R. B., (1978)

The simulation of three-dimensional convective storm dynamics.

J. Atmos. Sci., 35, 1070-1096.

LIN, Y., FARLEY, R. P. and ORVILLE, H. D., (1983)

Bulk parametrization of the snow field in a cloud model.

J. Clim. Appl. Met., 22, 1065-1092.

LOWE, P. R. and FICKE, J. M., (1974)

The computation of saturation vapour pressure.

ENU. Prep. Rsch. Fac., Tech. Paper No 4-74.

LUDLAM, F. H., (1980)

Clouds and storms.

Pennsylvania State Univ. Press.

MARSHALL, J. S. and PALMER, W. MCK., (1948)

The distribution of raindrops with size.

J. Meteor., 5, 165-166.

MILLER, M. J., (1978)

The Hampstead storm: A numerical simulation of a quasi-stationary cumulonimbus system.

Quart. J. Roy. Met. Soc., 104, 413-427.

MILLER, M. J. and PEARCE, R. P., (1974)

A three-dimensional primitive equation model of cumulonimbus convection.

Quart. J. Roy. Met. Soc., 100, 133-143.

OGURA, Y. and PHILLIPS, N. A., (1962)

Scale analysis of deep and shallow convection in the atmosphere.

J. Atmos. Sci., 19, 173-179.

ORVILLE, H. D. and KOPP. F. J., (1977)

Numerical simulation of the history of a hailstorm.

J. Atmos. Sci., 34, 1596-1618.

PIACSEK, S. A. and WILLIAMS, G. P., (1970)

Conservation properties of convection difference schemes.

J. Comp. Phys. 6, 392-405.

RUTLEDGE, S. A. and HOBBS, P. V., (1983)

The mesoscale and microscale structure and organisation of clouds and precipitation in mid-latitude cyclones. VIII: A model for the "Seeder-Feeder" process in warm frontal rainbands.

J. Atmos. Sci., 40, 1185-1206.

SCHNEIDER, H. (1984)

A numerical transport scheme which avoids negative mixing ratios.

Mon. Wea. Rev., 112, 1206-1217.

TRIPOLI, G. J. and COTTON, W. R., (1980)

A numerical investigation of several factors contributing to the observed variable intensity of deep convection over South Florida.

J. Appl. Meteor., 19, 1037-1063.

VAN DEN BERGHE, C. S. and NASH, C. A., (1984)

On the choice of high order scheme for a flux-corrected transport algorithm.

Met O 15 Int. Rpt. No 55.

VAN DEN BERGHE, C. S. (1985)

A three dimensional cumulonimbus model: formulation, numerical approximation, boundary conditions and cooling considerations.

Met O 15 Internal Report, in prep.