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THE CHARACTERISTICS OF THE FREE ATMOSPHERE.

BY

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THE CHARACTERISTICS OF THE FREE ATMOSPHERE.

§ 1. METHODS AND PLACES OF OBSERVATION.

A PRECISE knowledge of meteorological events could not exist before the invention of the thermometer and barometer, but since that time instrumental observations have been accumulating in constantly increasing numbers. These observations, however, were of necessity confined to the surface of the earth, though they included the results from mountain stations; and, as we now know, observations at the surface do not give much clue as to what is going on above.

The broad fact was established that the temperature of the air fell with increasing elevation at the rate of about 1° F. for 300 feet, or ·6° C. for 100 metres, so that even near the equator the summits of high mountains are above the snow line; and it is amusing to read in old books various conjectures about the explanation of the fact. It is curious that the greater proximity to the sun of the mountain summit, the utterly inappreciable amount of one or two miles out of ninety millions, should have been so often brought into the discussion, for it has long been known that we are some millions of miles nearer the sun in our northern winter than in the summer.

More than a hundred years ago a few tentative attempts were made to send up thermometers attached to a kite, and Glaisher's observations made from the car of a balloon are too well known to need further mention. The credit of initiating systematic observations of the upper air belongs to the late Lawrence Rotch of Blue Hill, near Boston, U.S.A., who, about 1895, started the system of sending up a self-recording instrument attached to the string of a kite. Employing a train of kites and using fine steel pianoforte wire instead of string or small cord, and having steam power for the purpose of winding in, Lawrence Rotch was able to obtain records of the temperature and humidity of the air up to a considerable height. But it is only in exceptionally favourable circumstances that more than three-tenths of the whole atmosphere can be passed through by a kite, so that it is fortunate that, soon after Lawrence Rotch had commenced his observations, the late Teisserenc de Bort and Prof. Assmann inaugurated the system of using small free balloons.

In this system the recording instrument hangs below a small balloon, which is filled with a gas lighter than air, preferably hydrogen, and is then let go. The balloon rises sometimes to ten or more miles and then falls to the earth. If it falls on land it is generally found by someone, and to ensure its return a label is attached offering a reward to the finder if he will return the instrument untampered with. It is astonishing how many are returned; the Continental stations do not lose more than one out of ten, but in England many fall in the sea, and the loss reaches 30 or 40 per cent.

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Usually, but not necessarily, the balloons used are made of india-rubber and are tied up securely so that the gas cannot escape; as the pressure decreases with increasing height the rubber stretches until the balloon finally bursts. This system produces a uniform free lift until the internal gas pressure becomes influenced by the tension of the rubber, and the ascensional velocity is also nearly uniform. On the assumption that there is no escape of the gas, the velocity increases with increasing height; but in practice there is some leakage, with the result that almost uniform velocity is attained over the whole of the ascent. (M.O., No. 202, p. 27.)

It will be readily understood that there is considerable difficulty in getting reliable observations of the temperature and humidity of the air at great heights. In the ordinary way, in the temperate latitudes a barograph has to cover a range of about one-tenth of the pressure of the whole atmosphere, say from 930 mb. to 1050 mb. (27·50 in. to 31·00 in.), and an instrument is not inconveniently large if it allows for this a scale of 6 inches. A thermograph has in England to cover a range of about 50° C. or 90° F., but the common plan is to use two printed sheets, one for the winter and one for the summer, so that a scale of 1 inch to 15° F. can be attained. For the upper air the range of pressure is ten times as great; from 1050 mb. to zero or nearly so must be covered, and temperatures with a range of quite 100° C. (180° F.) from 300 a. to 200 a. may be met with. The conditions imperatively demand a light instrument, and "light" in this case means small; hence a very contracted scale is unavoidable. The chief difficulty occurs with the measurement of pressure, since some 6 inches must serve for the full scale of 1050 mb., and readings correct to the nearest millibar can hardly be expected. The case is better with regard to temperature, since on the record $\frac{1}{20}$ inch goes to about 1° C., and therefore the temperature, in so far as the trace is concerned, can be read to the nearest degree. With regard to the humidity the difficulty is of another kind. The only method yet devised is to measure the expansion and contraction of a small bundle of hairs. This is satisfactory while the temperature is reasonably high, but for the low temperatures that prevail in the upper strata it is of very little use. Perhaps, however, this is not of much consequence, for the actual amount of water vapour that can be present in a cubic metre or other definite volume decreases very rapidly with decreasing temperature, so that the lower part of the atmosphere where the temperature is high contains almost the whole of the water.

Two different systems for making observations are in use, one on the Continent and in the United States, the other in the British Isles and some of the Colonies. On the Continent the records are made on smoked paper, or thin metal, placed on a drum and turned by a clock in the usual way. The three pens write on the same drum and record pressure, temperature, and humidity. The instrument weighs about 2 lbs., and is carried upwards by a balloon which, at starting, has a diameter of about 6 feet. The system adopted in England is to use a very small light instrument giving a very small trace, and to decipher the trace by the aid of a microscope. It is probable that the optical magnification is more accurate than the Continental system of magnification by several levers. The record consists of a pressure-temperature diagram and a pressure-humidity diagram, scratched by a metal point on a small piece of thin metal the size of a postage stamp, electroplated with silver.[1] The weight, including the aluminium cylinder for protection from the sun, is 2 ounces. The accuracy obtainable depends far more upon the skill and care of those who calibrate and use the instru-

[1] See *Computer's Handbook*, M.O., 223, Section II.

ments than upon the system which is adopted, and after nearly ten years' work the two methods have given results that are practically identical, but the English system has the advantage of cheapness. Not only is the English instrument much less expensive, but its smaller weight renders it unnecessary to use so large a balloon or so much gas.

The expense of each observation is so high that it is not practicable to make daily observations. It is obviously of advantage that the observers should all follow some general plan, and this has been secured by the appointment of an international committee which fixes definite times and dates for the ascents, and is responsible for the publication of the results. Before the war the committee met every three years to discuss and settle various details. With regard to dates, the arrangement that has been in force since 1909 has been to have during the year twenty-three days, called international days, on which balloons and kites carrying recording instruments are to be sent up, and observations on the motion of the atmosphere are to be made by means of pilot balloons and cloud observations. In general, the appointed days are the first Thursday in each month; in three months of each year, the adjacent Wednesday and Friday in addition, giving three consecutive days three times in the year, and once a run of six consecutive days, called the international week. The time is invariably 7^h G.M.T.

§ 2. Amount and Reliability of the Material.

Some fifteen stations in Europe have participated more or less regularly in the scheme, and about 150 successful observations are obtained annually. Unfortunately, the mere sending up of a registering instrument attached to a balloon does not necessarily mean a good observation. The instrument may never be found, the clock may stop, the pen may not write, the finder may efface the record; there are many possibilities of failure. Out of Europe, observations have been made in Batavia, East Indies, Canada, the United States, Australia, and elsewhere, but all the results have not yet been published. Also three special expeditions have made observations in Central Africa and over the Tropical Atlantic. The great bulk of the published observations come from Europe. In the British Isles there are 450 records available, which, since one-third of the balloons sent up are not heard of again, means the sending up of about 700 instruments. For the Continent the number exceeds 1000; no one seems to have counted it exactly, but Lt.-Col. Gold (*Geophysical Memoirs*, No. 5, p. 73) enumerates 667 ascents reaching 10 k. height during the period 1904–1909, and subsequent years have no doubt added another 500. The non-European countries add about another 200, so that on the whole there are some 2000 observations, of which Europe supplies about 90 per cent.

These figures refer to observations which give the temperature of the air up to great heights, and which can only be obtained by means of a free balloon. Observations in the first few kilometres that may be obtained by the use of kites and captive balloons are far more numerous. There is a daily record from Lindenberg running without a single break over many years, and possibly still running. A large amount of this material has never been worked up, and the practice of most upper-air observatories is steadily changing towards more registering balloons and less kite-flying, for the chief centre of interest lies in the strata beyond the range of kites.

Before giving tables showing the temperature, pressure, and density of the air, it is necessary to discuss the accuracy of the observations. The results obtained have revolutionised our ideas about the circulation of the atmosphere, and therefore it is desirable to make sure that there can be no mistake. Various sets of evidence all lead to the same conclusion, which is that the probable error of a temperature observation at a definite height does not exceed 1° C. That is to say, that if in the published figures the temperature at 10 k. is given as 230 a. for such and such a station on such and such a day, it is an even chance whether the correct temperature was between 229 a. or 231 a., or lay outside those limits. Also, according to the usual statistical rule, it is almost certain that the true temperature lay between 225 a. and 235 a.

It has been pointed out that the pressure scale on the recording instruments is so contracted that it is not possible to be certain of the pressure within a few millibars. We may take the probable error of pressure made by the person working up the records as 4 mb. Thus an error of 20 mb. is just possible. Now, in the lower strata a pressure difference of 4 mb. means a difference of 40 metres in the height, but at a height of 20 k. 4 mb. pressure change may correspond to 500 metres change of height. When, therefore, a balloon reaches about 20 k. an error of 2·0 k. in the maximum height may very well be made.

The error in the pressure may also lead to an error in the temperature given for some fixed height, for it leads to a wrong estimate of the height, and therefore in places where the temperature is changing with height to a wrong temperature. Above 12 k. in general there is very little change of temperature, and therefore no error occurs from this cause. Below 5 k. an error of 4 mb. will not cause much inaccuracy in the estimated height, so that here also no serious error occurs in the temperature; but at 10 k., where the temperature gradient may be large, and small pressure changes are becoming important in the estimation of height, the wrong estimate of 4 mb. in the pressure may lead to an error of 1° C. in the temperature.

Also, since the height at which a given temperature occurs is calculated by means of the temperature of the underlying air column, any error in this causes an error in the temperature at points above it, but happily not to a serious amount. An incorrect reading of the temperature may be caused in various ways—the lag of the thermograph, the influence of solar radiation, incorrect calibration, etc. Broadly, perhaps the best proof that none of these causes are important is shown by the very close agreement between the British and Continental results. The meteorographs are different, the mode of calibration is different, but, excepting for the magnitude of the annual variation at great heights, the results are practically identical down to quite small details. In accordance with the international scheme, the times of observation at the different stations are identical, and some of the observing stations are near together. Thus it happens that there are many instances of simultaneous records from Ditcham Park, Petersfield, and Pyrton Hill, in S. Oxfordshire, stations about fifty miles apart. If the records were incorrect it would be impossible that these two stations should show the very close agreement that is found between them. Similarly, on the Continent there is good agreement between stations that are near together.

Direct proof that the probable instrumental error does not exceed 1° C. is given in two independent ways. Twice at Manchester a series of hourly ascents over a period of twenty-four hours was carried out for the purpose of ascertaining the diurnal variation of temperature. Now we have no *a priori* knowledge of how the temperature changes from hour to hour in the upper strata; it may follow a smooth curve or it may show sudden changes. The actual change at the surface for one definite day is often far from smooth. But it is obvious that a set of hourly temperature readings taken at each consecutive hour by a different thermometer could not show a smooth curve if the thermometers were seriously incorrect. Correct thermometers might show a rough curve because the curve might really be rough, but a rough curve could not be transformed into a smooth one by the casual errors of the instruments, rather it must be made more rough. A careful examination of the figures at Manchester on both occasions leads to the conclusion that, if the temperature curves at each height were really smooth curves, the probable instrumental error was about 1° C. Doubtless they were not smooth curves, and the error was therefore less than 1° C. This refers to the English type of meteorograph.

The other proof is this :—As will be seen in the subsequent tables, there are certain very close and intimate relations between the pressures and temperatures in the upper air. Such close relationships could not appear if the recorded values were seriously wrong. The proof is a technical one depending on a knowledge of statistical methods (see M.O., No. 210*b*, p. 36), but it leads to the conclusion that the probable error of the temperatures can hardly exceed 1° C. and is probably less. This refers to both types of instrument. Thus we may certainly accept the values published as being in the great majority of cases correct within a few degrees.

The accuracy of the mean values is of about the same order. So far as the instruments are concerned, we might accept means from thirty or so observations as quite correct; but when dealing, say, with an annual mean which is dependent on records on twenty-three days only, it is obvious that some of those twenty-three days may have been of an exceptional character and so have spoilt the result. From this cause the means given in the following tables have in general a probable error of about 1° C.

§ 3. Mean Temperatures and Gradients.

The following tables are taken from the papers of various authors. Where possible, the number of observations reaching up to 12 k. on which the mean values rest is given; the lower strata naturally depend on more and the higher on less than the given number. A difficulty always occurs in forming a table of this sort. At the greater heights the observations are few and therefore the mean is less reliable, and the stoppage of the figures from some one ascent at its highest point is apt to cause an artificial discontinuity unless that ascent represented average conditions. As an example, suppose that five ascents are available up to 18 k. and six up to 15 k.; let the mean shown by the five from 13 k. up to 18 k. be between 220 a. and 221 a., and let the sixth ascent break off at 15 k. with a temperature of 208 a., both quite likely contingencies; the means will show a sudden but fortuitous break of 2° between 15 k. and 16 k. Where breaks occur in the upper strata in the following tables they are often due to this cause. In nearly all cases in Europe where great heights have been reached and where the recorded temperatures are certainly free from the influence of solar radiation, it has been found that the change of temperature above 14 k. is very small and has no systematic tendency to increase or to decrease; hence the following tables have not been carried beyond 14 k., but mean temperatures between 14 k. and 20 k. may be taken as equal to the temperature at 14 k.

Temperatures are expressed in the absolute scale, where 273 corresponds to 0° C., but to save space the initial 2 is omitted.

Table I. is obtained chiefly from means given by Lt.-Col. Gold (M.O., *Geophysical*

Memoirs, No. 5, p. 72), and refers to the period 1904 to 1909, but the stations have been rearranged in order of their latitude. Where the latitudes are nearly the same, the western station is put first. Also the mean for England has been replaced by the

TABLE I.—MEAN TEMPERATURES.

Height in Kilometres.	Petrograd.	Scotland.	Konchino.	Manchester.	Hamburg.	Berlin.	Limerick.	England S.E.	Brussels.	Paris.	Strassburg.	Munich.	Vienna.	Zurich.	Pavia.	Mean.	Canada.
	200 a. +																
14	23·5	22·0	21·0	20·5	19·4	18·7	19·0	18·9	16·4	19·1	17·9	17·3	19·6	15·9	17·7	19·1	12·5
13	23·4	21·8	21·6	20·6	20·2	19·3	19·2	18·7	15·3	19·3	17·6	18·2	19·6	16·7	16·4	19·2	14·0
12	20·7	21·6	18·7	20·0	18·4	18·3	19·3	18·8	16·4	19·5	16·8	17·2	18·3	16·2	16·1	18·4	16·2
11	20·0	20·5	16·8	20·9	19·2	19·2	19·9	19·6	17·5	20·2	18·1	20·8	18·4	17·1	18·5	19·1	19·3
10	21·3	21·2	19·8	23·2	22·5	21·9	21·7	22·2	21·7	24·3	22·3	23·8	21·8	23·1	22·7	22·2	23·2
9	24·4	24·8	25·7	28·2	27·7	26·8	25·5	27·5	27·7	30·0	27·8	29·1	26·9	27·9	27·3	27·2	29·3
8	29·8	30·2	32·9	33·8	33·5	33·1	30·9	33·6	34·4	36·9	34·8	35·7	33·6	34·5	33·9	33·4	35·9
7	37·1	38·0	40·8	40·2	39·7	40·8	37·7	40·7	41·5	44·3	42·1	42·9	41·2	42·3	41·2	40·7	43·5
6	43·3	45·0	47·7	47·0	46·2	47·9	45·4	47·8	48·5	51·4	49·3	49·9	48·8	49·9	49·4	47·8	50·9
5	49·8	52·0	54·2	53·8	52·6	54·8	52·2	54·8	55·3	58·1	56·1	56·6	55·6	57·2	56·2	54·6	57·7
4	55·7	58·4	60·3	60·4	59·1	61·0	59·4	61·7	62·0	64·3	62·4	62·7	61·9	63·9	62·9	61·1	64·1
3	61·3	64·0	66·2	66·6	64·8	66·9	66·3	67·7	67·8	69·8	68·4	68·8	67·6	69·9	69·2	67·0	69·6
2	66·7	70·3	70·2	71·7	70·4	71·7	71·7	73·2	73·0	74·5	73·8	74·4	73·0	75·9	75·1	72·4	74·8
1	71·0	75·3	74·4	77·0	75·4	76·8	75·8	78·0	77·1	78·5	78·2	79·0	77·6	78·8	80·7	76·8	78·3
No. of Obs.	31	29	25	73	20	74	27	167	36	86	73	59	47	15	42	804	37

means for Scotland, Ireland, Manchester, and England S.E. from 1908 to 1915. The values for Canada are chiefly from latitude 43° 8′, and are due to Mr Patterson (*Upper Air Investigation in Canada*, Part I.). Means for Canada up to 20 k. are given as follows: 15 k. 11°·0, 16 k. 10°·9, 17 k. 11°·4, 18 k. 13°·7, 19 k. 14°·9, 20 k. 14°·0, but these

TABLE II.—MEAN TEMPERATURE GRADIENTS.

Height in Kilometres.	Petrograd.	Scotland.	Konchino.	Manchester.	Hamburg.	Berlin.	Limerick.	England S.E.	Brussels.	Paris.	Strassburg.	Munich.	Vienna.	Zurich.	Pavia.	Mean.	Canada.
	°C.																
13·5	−0·1	−0·2	0·6	0·1	0·8	0·6	0·2	−0·2	−1·1	0·2	−0·3	0·9	0·0	0·8	−1·3	0·1	1·5
12·5	−2·7	−0·2	−2·9	−0·6	−1·8	−1·0	0·1	0·1	1·1	0·2	−0·8	−1·0	−1·3	−0·5	−0·3	−0·7	2·2
11·5	−0·3	−1·1	−1·9	0·9	0·8	0·9	0·6	0·8	1·1	0·7	1·3	3·6	0·1	0·9	2·4	0·7	3·1
10·5	1·3	0·7	3·0	2·3	3·3	2·7	1·8	2·6	4·2	4·1	4·2	3·0	3·4	6·0	4·2	3·0	3·9
9·5	3·1	3·6	5·9	5·0	5·2	4·9	3·8	5·3	6·0	5·7	5·5	5·3	5·1	4·8	4·6	4·9	6·1
8·5	5·4	5·4	7·2	5·6	5·8	6·3	5·4	6·1	6·7	6·9	7·0	6·6	6·7	6·6	6·6	6·3	6·6
7·5	7·3	7·8	7·9	6·4	6·2	7·7	6·8	7·1	7·1	7·4	7·3	7·2	7·6	7·8	7·3	7·4	7·6
6·5	6·2	7·0	6·9	6·8	6·5	7·1	7·7	7·1	7·0	7·1	7·2	7·0	7·6	7·6	8·2	7·2	7·4
5·5	6·5	7·0	6·5	6·8	6·4	6·9	6·8	7·0	6·8	6·7	6·8	6·7	6·8	7·3	6·8	6·8	6·8
4·5	5·9	6·4	6·1	6·6	6·5	6·2	7·2	6·9	6·7	6·2	6·3	6·1	6·3	6·7	6·7	6·5	6·4
3·5	5·6	5·6	5·9	6·2	5·7	5·9	6·9	6·0	5·8	5·5	6·0	6·1	5·7	6·0	5·9	5·9	5·5
2·5	5·4	5·7	4·0	5·1	5·6	4·8	5·4	5·5	4·2	4·7	5·4	5·6	5·4	6·0	6·3	5·3	5·2
1·5	4·3	5·0	4·2	5·3	5·0	5·1	4·1	4·8	4·1	4·0	4·4	4·6	4·6	2·9	5·6	4·5	3·5

figures rest on too small a number of observations to be regarded as final, and the discontinuity at 17 k. is due to the breaking off of five records out of ten at that height. The ascents that have been made in the United States are not distributed over the year sufficiently well to afford a good annual mean, since no observation is available between October 12 and February 8.

The figures in Table I. are interesting. Their general agreement is very good, and they show clearly the lower temperature of the more northern stations up to 8 k. or 10 k., and their higher temperature above that height.

In Table II. the differences between successive kilometres are given, and the figures therefore show the temperature gradient with height in degrees Centigrade per kilometre. The increase of gradient with height up to about 8 k. is apparent at each station, and the falling off above; so also is the smaller height at which the gradient ceases at the northern stations. This is particularly noticeable for Canada, the place in the list which has the lowest latitude. There are discrepancies in the table which are probably fortuitous, but there does not seem to be any systematic difference, save the one already mentioned between north and south.

Additional information is given in Table III. Except for the equatorial values it does not add much to Table I., but it is useful to show the amount of variation that may be reached by different samples of the material, and from year to year.

TABLE III.—MEAN TEMPERATURES FOR DIFFERENT HEIGHTS.

Height in Kilometres.	1. British Isles, 1908–1911.	2. British Isles, 1912.	3. British Isles, 1913.	4. British Isles, 1914.	5. Continent.	6. Continent, 1902–1907.	7. Petrograd.	8. Equatorial.
	a.	a.	a.	a.	a.	a.	a.	a.
20	220	...	...	...	...	...	...	193 ?
19	20	...	...	...	...	...	...	93 ?
18	20	...	...	...	...	...	...	93 ?
17	20	...	...	...	...	...	...	93
16	20	...	...	...	...	...	...	95
15	20	...	...	...	...	...	...	98
14	20	222·0	218·6	220·1	219·1	218·6	. .	203
13	19·8	21·8	18·2	19·5	19·1	18·5	...	11
12	19·8	21·6	17·9	17·9	18·3	18·8	222·2	19
11	19·6	21·4	17·0	18·0	19·0	20·2	21·7	27
10	23·1	21·2	19·8	21·1	22·3	23·4	22·4	35
9	28·9	26·1	24·9	27·1	27·3	28·6	24·9	43
8	35·0	32·1	30·2	33·5	34·1	35·0	29·6	51
7	41·8	39·8	38·1	40·5	41·5	42·2	35·9	58
6	48·9	46·2	46·1	47·9	48·7	49·7	42·6	65
5	55·5	53·9	54·0	55·6	55·6	56·1	49·0	72
4	61·7	60·0	60·3	62·0	61·8	62·3	55·0	79
3	67·7	65·8	66·3	68·6	67·6	68·0	60·9	85
2	72·6	70·9	71·9	74·3	72·8	73·1	66·2	90
1	77·0	75·6	76·9	78·7	78·1	77·6	70·9	95
0	82·6	80·1	81·8	83·6	...	80·9	76·1	300

1 to 4. *Annual Supplement to Geophysical Journal.*
5. *Report to British Association*, 1909, Gold and Harwood. 400 observations.
6. *Die Temperaturverhaltnisse in der freien Atmosphäre*, Wegener. 380 observations.
7. *Meteorologische Zeitschrift*, January 1911, Rykatchew. 90 observations.
8. *Monthly Weather Review*, November 1915.

The equatorial values are very striking. Unfortunately they have a much larger probable error than the others, because they rest upon far fewer observations; but the general result of a high temperature in the lower layers and an extremely low temperature in the higher layers is perfectly certain and not due to instrumental errors. It is here only that any trouble occurs from omitting the first 2 in the absolute temperature. The significance of these low temperatures will be discussed later on.

§ 4. THE SEASONAL VARIATION.

In the British Isles the seasonal variation of temperature persists upwards without much change to 10 k. or 11 k., above which height the range is much reduced and the period of maxima and minima is shifted backward. The number of observations at each station is not sufficient to show the finer details of the annual change of temperature; in the south-east of England, the district with the largest number of observations, there are only on the average fourteen observations in each month, and it will readily be seen that if some four or five of those fourteen in some one month happened to refer to days of exceptional temperature, the means for that month would be quite provisional. The other stations are still worse off, and in general the mean temperature of one of the twelve months—January, for example—at a given station will depend upon about four observations. Under such circumstances some sort of smoothing process is necessary, and where it is perfectly certain that there is a variation with a definite period, in this case a year, the best form of smoothing is to find the amplitude and phase angle by the process of harmonic analysis. In non-technical language, we assume that the change follows a certain course known as a sine curve, that equal intervals separate the time of maximum and of minimum, and that the change is most rapid midway between them. The length of daylight, for example, throughout the year very nearly follows such a course. When, therefore, a series of observations is scattered over the twelve months, so that no single month has sufficient observations to make its mean reliable, since we know that the temperatures of the successive months will follow a more or less regular course, we can get the best value for any month, not by considering the observations in that month alone, but by also taking into account the observations in the neighbouring months. This has been done in the following tables.

But before giving the tables the following point must be noted. It may perhaps be that the temperature of the upper air, as it follows its seasonal change, does not conform to the regular course prescribed by a single sine curve; it may not reach its maximum exactly six months after passing its minimum value. If the number of

TABLE IV.—MEAN MONTHLY TEMPERATURES FOR ENGLAND. SMOOTHED.

Height in Kilometres.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.
	200 a. +											
14	16	17	19	21	22	23	22	21	19	17	16	15
13	16	17	19	21	22	23	23	21	19	18	17	16
12	17	18	19	20	21	22	22	21	20	19	18	17
11	17	17	17	19	20	21	22	22	21	20	19	18
10	20	20	20	22	24	25	26	26	26	24	23	21
9	24	23	24	26	29	32	34	33	33	31	28	25
8	30	29	30	32	36	38	41	41	41	38	35	32
7	37	36	37	39	42	45	47	48	47	45	41	38
6	43	43	44	46	49	52	55	55	54	51	49	45
5	50	49	50	52	56	59	61	62	61	58	55	52
4	57	56	57	59	62	65	67	68	67	64	61	58
3	63	62	63	65	68	71	73	74	73	70	67	64
2	67	66	67	70	73	76	78	79	78	75	72	69
1	71	71	73	76	79	82	83	83	81	79	75	72
0	76	76	77	82	85	88	89	89	86	83	80	77

observations were sufficient this could be ascertained, but the number of observations is very far short of that required for this purpose. If we search for irregularities of this sort we are certain to find them, as they would naturally be formed by casual errors in the observations; but there is no possibility, until our observations are greatly multiplied, of knowing whether these irregularities are genuine or fictitious. In technical language, the amplitudes of the second, third, fourth, etc., terms of the harmonic analysis are of the same order of magnitude as the probable errors of these same amplitudes, and therefore it is quite likely that any discovered irregularities may be purely chance ones.

In Table IV. the mean temperature at each height in each month for the British Isles is given. This table is taken from the *Phil. Trans. Roy. Soc.*, series A, vol. 211, p. 256.

In Table V. similar information is given for other places, but it is put into another form. For full information see the *Computer's Handbook* (M.O., 223, Section II. § 3). It will suffice here to say that the amplitudes given in the table represent half the difference between the warmest and coldest time; and the date, which is expressed in days measured from the beginning of the year, is the date of the lowest temperature. Thus at Manchester at 9 k. the amplitude is 5°·4 C. and the date 55, which represents February 24. From Table I., the mean is 28°·2. Hence for February 24, the coldest time, the mean is 22°·8; for August 24, the hottest time, it is 33°·6; and for May 24 and November 24, it has its mean value of 28°·2. Of course these dates are not exact, any of the four may perhaps be a fortnight out. Thus for the S.E. of England the maximum temperature near the ground is given as occurring on February 7. From the long series of daily temperatures that we have, we know that this is three weeks or a month too late. The error is caused by pure chance, and similar errors may be present in other parts of the table.

TABLE V.—SEASONAL RANGE OF TEMPERATURE.

Height in Kilometres.	Amplitudes °C.					Date of Minimum.					Means 200 a. +	
	Manchester.	England, S.E.	B.	C.	Canada.	Manchester.	England, S.E.	B.	C.	Canada.	B.	C.
15	2·7	3·0	4·6	3·1	...	353	23	0	353	...	19·2	18·7
14	2·6	2·8	3·9	3·6	3·0	351	27	2	353	212	18·7	18·8
13	2·5	2·5	3·1	3·5	1·6	345	14	12	360	238	18·9	18·8
12	2·8	2·2	2·9	3·4	1·2	26	29	24	22	4	18·4	17·9
11	3·2	2·9	3·2	3·8	4·8	35	54	42	32	36	19·5	19·0
10	3·7	4·1	4·3	5·3	7·7	56	56	42	36	41	22·8	22·4
9	5·4	5·2	5·6	7·2	9·5	55	44	47	33	34	28·8	27·7
8	5·8	6·0	6·5	8·1	9·9	30	35	43	33	39	35·3	34·3
7	6·7	6·3	6·8	8·1	10·0	27	30	40	32	39	42·4	41·6
6	6·8	6·6	7·0	7·7	10·2	26	31	37	32	39	49·3	49·0
5	6·2	6·6	7·0	7·2	9·9	26	36	39	32	41	56·0	55·7
4	6·2	6·2	6·8	6·7	9·7	32	35	39	32	39	62·5	62·1
3	5·2	5·2	6·2	6·4	9·2	26	37	42	32	39	68·3	68·0
2	5·0	5·4	6·1	6·5	9·3	25	37	41	32	37	73·4	73·2
1	5·2	6·1	6·7	7·9	11·3	14	35	34	23	31	77·8	77·9
0	5·2	6·2	...	...	12·5	15	38	...	...	26	81·2	80·3

B. England, Hamburg, Paris, and Brussels.
C. Berlin, Munich, Strassburg, and Vienna.

The figures for B and C are taken from Lt.-Col. Gold's paper, M.O., No. 210*e*; for Canada the values have been worked out from Mr Patterson's values, for England the figures are from values yet unpublished in the possession of the Meteorological Office.

Some very interesting features are shown in Table V., and since they occur more or less in all five columns, they can hardly be due to chance. As already stated, broadly the annual range extends up to 8 k. or 9 k., and thence falls off to less than half its surface value at 12 k. and above. There is a secondary minimum at 2·5 k., and then an increase to a maximum at about 7 k. With a continental climate the actual maximum occurs at the surface, with a coast climate at about 7 k.

The changes occur distinctly later with increasing height up to 11 k. or so, but above 11 k. they fall back rapidly till the minimum at 13 k. and over in Europe occurs at the end of December or beginning of January. In Canada the minimum at great heights appears to occur in the summer. This is by no means unlikely in itself, since it is colder high up over the equator than at the same height in temperate latitudes, but the number of observations is not sufficient to establish the fact with absolute certainty.

It is probable that the range of temperature on the Continent at the higher levels is not so great as is shown in Table V., and for this reason. Practically all the Continental ascents were at 7^h G.M.T. In the winter the sun is low and its radiant energy small at this time; in the summer, on the contrary, at 7^h on the Continent, where the local time is equivalent to 8^h or 9^h in London, the sun has become quite powerful, and no person acquainted with the difficulty of determining the true air temperature in full sunshine, even at the earth's surface, can doubt the effect at altitudes where the radiation is far more intense. The summer temperatures are therefore systematically high compared with the winter ones, and the apparent range is increased.

In Canada all the observations were made after sunset, at Manchester very nearly all, and in England S.E. about half.

§ 5. The Daily Temperature Range.

The information on this point is very scanty, and this is unfortunate, because if the magnitude of the daily range of the temperature at different heights were available, it would afford most useful information about the radiative properties of the air. The difficulties of obtaining the information are very great. In the lower heights, say up to 3 k., kites and captive balloons have been employed, the procedure being to carry out a series of ascents night and day in rapid succession. Twice at Manchester a succession of registering balloons were sent up at hourly intervals for a period of 24 hours; both series of ascents were very successful in the number of instruments found and the records obtained.

Broadly it may be said that the diurnal change of temperature as we know it at the surface decreases rapidly with height and practically ceases between 1 and 2 k.; very possibly at places where the range at the surface is large it may reach somewhat higher, but in general it has ceased at 2 k. Above that the range is so small that it cannot be stated with any certainty at what time either the maximum or minimum occurs.

The following statements have been made on the subject:—

Helm Clayton, from the kite ascents at Blue Hill, near Boston, U.S., considers that the 24-hour variation vanishes at 1 k. and is reversed above that height; that is to say, that above 1 k. the maximum occurs in the night and the minimum in the day.

Wendt, using observations made in Jutland, gives at 1200 metres an amplitude of 0°·55 C. with a maximum at 1·28 p.m.

Lt.-Col. E. Gold gives at Berlin, at 1 k., an amplitude of 0°·85 C. with the maximum at 6·0 p.m., and at 2 k. an amplitude of 0°·64 with a maximun at noon.

For 1 k. at Petrograd he gives an amplitude of 0°·72 C. with a maximum at 2·28 p.m.

W. R. Blair, from observations at Mount Weather, of Washington, U.S., gives two series of curves from which it appears that the 24-hour variation ceases at 1·5 k. and is reversed above.[1]

There are three papers in the *R. Met. Soc. Journal* dealing with the daily change at greater heights. The policy of the International Committee of fixing 7^h G.M.T. as the time for all ascents has rendered the Continental records useless for the purpose of finding the diurnal range, but a considerable number of ascents have been made in England at about the time of sunset, and a comparison of the temperature in the morning and evening thus becomes possible (*R. Met. Soc. Journal*, vol. 40, No. 169). The results show a rather higher temperature from 3 k. and upward at sunset than at sunrise, but this may perhaps be due to a casual error; the only certain result is that the diurnal change, if it exists, is quite small.

The figures for Manchester showing the temperatures at 1, 5, 10, and 15 k. on the two occasions of June 2–3, 1909, and March 18–19, 1910, are given in Table VI. (see

TABLE VI.

					Midnight.													Noon.										
	19.	20.	21.	22.	23.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.
15 k.	21	22	22	...	...	...	21	...	26	...	25	23	...	...	28	27	28	26	...	28	27	24	24	20	..	...	...	...
	...	...	...	...	22	21	20	19	22	16	...	24	26	...	27	24	...	22	...	..	21	...	...	...	...	...	17	18
10 k.	24	26	22	...	22	...	20	...	26	...	27	24	...	...	28	27	25	23	...	28	26	28	27	16	18	...	...	...
	...	...	...	20	20	17	19	19	19	19	16	17	26	...	24	17	...	19	...	...	14	...	...	...	...	..	21	14
5 k.	57	58	56	...	56	...	54	...	57	...	60	59	...	...	60	60	62	59	...	58	59	62	60	59	56	...	...	...
	40	...	...	44	46	47	48	49	50	52	52	49	...	...	53	54	...	54	...	...	52	...	...	...	...	...	51	51
1 k.	77	77	75	...	75	...	75	...	74	...	75	77	...	...	79	78	78	78	...	77	78	77	77	76	75	..	...	...
	66	...	...	...	63	68	69	63	67	...	67	66	...	66	66	...	...	...	...	...	69	...	...	...	...	...	71	71

R. Met. Soc. Journal, vol. 36, No. 154, and vol. 37, No. 157). The upper rows of figures refer to June 2–3, 1909, the lower rows to March 18–19, 1910. Unfortunately during the critical time of 13^h to 20^h on the second occasion most of the balloons fell into the Bristol Channel and were lost. Higher temperatures are shown during the day, but they are irregular. Mr Harwood writes, referring to June 2–3: "The diagrams indicate a decided diurnal variation at the ground level, which at first sight appears to extend to considerable altitudes. Closer examination, however, shows that already at a height of 3 k. the temperature had begun to rise at 1^h, three hours before sunrise. Moreover, the two maxima of temperature recorded at the higher levels occurred at 9^h and 3^h, while at the extreme heights noon was marked by a minimum of temperature, The diagrams do not determine the height at which the diurnal variation observed at

[1] Since this was written observations with kites at Drexel, U.S.A., have shown an amplitude of about ·7° C. up to 3 k. (see *R. Met. Soc. Journal*, vol. 45, No. 189, p. 41).

low levels disappears, but show that there was no considerable diurnal range at the higher levels." With reference to the series of March 18–19, Miss M. White writes: "Both series indicate a general drift of temperature, but the most noteworthy result is the absence of any considerable effects which can be attributed either to seasonal or diurnal change."

A comparison between the temperatures of day and night must take account of the effect of solar radiation. The instruments sent up hanging from the balloons are well protected against direct radiation, but the balloons themselves are greatly heated by the sun and the instruments rise in a wake of heated air. An ascent in strong sunshine does on an average give a higher temperature at great heights than one at night, but not sufficiently so to prove that there is a real variation of the temperature of the air.

§ 6. The Humidity.

The humidity of the air must be carefully distinguished from the total amount of vapour that is in the air. The means of observation is the hair hygrometer, an instrument that is quite satisfactory so long as the temperature is reasonably high, but which is said to be unreliable at low temperatures. The hair hygrometer measures the ratio of the amount of vapour that is present to the amount that would be present if the air were fully saturated, and since at quite low temperatures this latter amount is very small, the unreliability of the hygrometer is of less consequence.

Observations of the humidity are numerous, but very few meteorologists appear to have worked up the material.

The general rule is that the humidity increases from the surface upwards to the lower cloud level, about 1 k. to 2 k., and then decreases above. Inversions of temperature, that is to say, cases where there is a sharp increase of temperature with increasing height, nearly always show a low relative humidity. For great heights a low relative humidity is shown, but as a rule the hygrometers used do not show much variation after the temperature has fallen to about 0° F., 255 a., but remain at the same point, the inference being that they have ceased to act.

The actual amount of water vapour in the atmosphere, neglecting the small amount in the part where the temperature is below 255°, has been calculated for 250 ascents in Europe. The amount when expressed as the depth of rain that would fall if all the vapour were condensed is surprisingly small. In summer the amount is on the average about ·80 in. (20 mm.), in winter about ·40 in. (10 mm.). The extreme limits in summer are from about ·50 in. to 1·50 in. (12·5 mm. to 37·5 mm.), in winter from about ·25 in. to ·80 in. (6 mm. to 20 mm.).

The humidity of the atmosphere acts in two ways: it has an important effect upon the power of radiating and absorbing heat—this will be discussed subsequently; it also alters the density of the air, since the specific gravity of water vapour is less than that of air. But in Europe the weight of the water vapour in the air near the surface is only one six-hundredth part of the weight of the air, so that the effect on the pressure is trifling, and it is customary to treat its effect as negligible. The pressures subsequently given are so calculated, and the result is that they are rather too low; but the error affects them all more or less equally, and is not so large as the probable casual error. This refers to Europe. In tropical countries, owing to the high temperature, the air can hold a much larger proportion of water vapour, and the density is appreciably altered. Thus the density of dry air at

283° and 1000 mb. is ·00123, and of saturated air it is ·45 per cent. less; at 300° and 1000 mb. for dry air it is ·00116, and for saturated air it is 1·37 per cent. less.

§ 7. The Troposphere and Stratosphere.

In the tables showing the mean temperature it will be noticed that the decrease of temperature with height begins to fall off at about 8 k. or 9 k., and ceases entirely at 12 k. or 13 k. This is the result of taking averages, and does not represent the true state of affairs for the individual occasion. What generally happens is that the fall of temperature ceases abruptly, not uncommonly with an inversion, but as the height at which the cessation occurs is variable the mean temperatures do not show the same sudden change. The following are examples from the observations made at Pyrton Hill, Oxfordshire:—

July 1, 1912.		September 20, 1912.		January 3, 1913.		February 6, 1913.	
Height.	Temperature.	Height.	Temperature.	Height.	Temperature.	Height.	Temperature.
k.	a.	k.	a.	k.	a.	k.	a.
12	230	14	211	12·3	215	12·3	208
11	30	13	10	12	15	12	7
10·4	26	12·6	9	11	15	11·4	4
10	24	12·2	5	10	14	11	5
9	30	12	6	9	20	10	13
8	37	11	12	8	28	9	21
7	44	10	21	7	38	8	29

These examples represent the usual type, and it is customary to look upon the atmosphere as divided into two parts, the lower part called by Teisserenc de Bort, who discovered this definite boundary, the troposphere, and the upper part called by him the stratosphere. The upper part used to be called the "isothermal." It is practically isothermal in the vertical, but not in the horizontal direction. The thickness of the troposphere is very commonly denoted by the symbol H_c, but a name would be convenient. The terms "tropopause" and "lapse-limit" have been suggested to denote the plane of cessation of the vertical temperature gradient.

As a rule the boundary is quite definite, but in a few instances the temperature gradient ceases gradually; also, now and again, the gradient may cease for a kilometre or so and then start again. In such cases the value of H_c is indefinite, and hence some sort of definition must be adopted. The following instructions are issued by the Meteorological Office for defining the value of H_c, or the level of the tropopause:—

Type I.—When the stratosphere commences with an inversion, H_c is the height of the first point of zero temperature gradient.

Type II.—When the stratosphere begins with an abrupt transition to a temperature gradient below 2° per kilometre without inversion, H_c is the height of the abrupt transition.

Type III.—Where there is no such abrupt change of temperature gradient, the base of the stratosphere is to be taken at the point where the mean fall of temperature for the kilometre next above is 2° or less, provided that it does not exceed 2° for any subsequent kilometre.

In the troposphere inversions within 2 or 3 kilometres of the ground are quite common, especially on cold clear nights in any type of weather, and during the whole day in winter in anticyclonic weather. In the upper half of the troposphere they do occur, but very rarely. When they occur by day, it is usually at the upper surface of a sheet of low clouds. When once the inversion, if there chances to be one, at the beginning of the stratosphere is passed, very little further change of temperature is met with in a vertical direction; there may be trifling gradients of 1° but hardly more per kilometre, but there is no systematic change either way, and over Europe at least the mean temperature at 14 k. does not differ by one degree from that at 20 k.

The height of the tropopause over Europe varies with cyclonic and anticyclonic weather from about 8 k. to 13 k.; there are a few instances where it is less than 8 k., and a few where it is more than 13 k., but such extremes are exceptional. The general mean is from 10·5 k. to 11·0 k., more in the South and less in the North, for there is a very distinct variation with latitude. There is also a distinct seasonal variation, and a very intimate relationship between the height of the tropopause and that of the barometer, of which particulars will be given later.

Mean values of the height of the tropopause and some particulars of the annual variation are given in Table VII.

There is, in general, very good agreement between the different mean values. The stations with the higher latitudes have the lower values of the tropopause;

TABLE VII.—HEIGHT OF THE TROPOPAUSE (H_c).

	No. of Observations.	H_c in Kilometres.	Amplitude in Kilometres.	Date of Minimum, and Period Covered by Observations.	
Scotland	29	9·8	...	...	1908–1914
Ireland	27	10·1	...	...	1908–1914
Manchester	73	10·3	·56	19	1908–1914
England, S.E.	167	10·7	·59	47	1908–1914
British Isles	150	11·1	·55	60	1908–1910 A
" "	52	10·0	...	...	1912
" "	61	10·2	...	...	1913 B
Continent	400	10·6	..	..	...
"	290	10·5	...	..	1902–1907 C
Berlin	79	10·4	...	...	1904–1909 D
Munich	76	10·7	...	...	1904–1909 D
Strassburg	86	10·7	...	...	1904–1909 D
Vienna	58	10·3	...	...	1904–1909 D
England	150	10·6	...	...	1904–1909 D
Hamburg	35	10·1	...	...	1904–1909 D
Paris	90	10·5	...	...	1904–1909 D
Brussels	33	10·9	...	...	1904–1909 D
Petrograd	41	9·6	...	...	1904–1909 D
Italy	46	11·0	...	...	1904–1909 D
Berlin, Strassburg, Vienna	212	10·5	·78	43	1904–1909 D
Paris, Hamburg, Brussels	158	10·6	·62	70	1904–1909 D
Petrograd	90	9·6	...	...	... E
Canada	47	11·7	2·2	47	... F

A. *Phil. Trans. Roy. Soc.*, series A, vol. 211, p. 253.
B. Gold and Harwood, *Report to British Association*, 1909.
C. Wegener, *Die Temperaturverhaltnisse in der freien Atmosphäre*, III. Band, Heft 2/3, Leipsic, 1909.
D. E. Gold, M.O., No. 210e, *Geophysical Memoirs*, No. 5.
E. Rykatchew, *Meteorologische Zeitschrift*, Jan. 1911.
F. *Upper Air Investigation in Canada*, Part I.

the difference between Scotland and the South of England is quite marked, so also is that between Petrograd and Italy. The Italian stations were in the northern part of Italy. The large range in Canada is perhaps partly fortuitous; 47 observations do not suffice to give an accurate value.

§ 8. The Pressure and Density.

There is no feasible method of measuring directly the pressure of the higher regions of the atmosphere. It is true that the upward course of a balloon can be ascertained by observations with two theodolites at the ends of a long base line, and hence the height at any given time is known; then the pressure at that same time is given by the meteorograph, and the pressure at that height determined. But in practice the various unavoidable observational errors prevent the attainment of much accuracy. The pressures are calculated in the following manner:—The surface pressure is read on a standard barometer. Suppose it to be 1000 mb. Then from the record the mean temperature of the air between the pressures of 1000 mb. and 900 mb. is taken, and the height at which the pressure is 900 mb. is calculated. This forms a starting-point for the next step, and so on. This method assumes that there is no vertical acceleration acting on the air save gravity, and though there may be other trifling forces they must be too small to affect the result appreciably. It follows, therefore, that the mean pressures at each height can be calculated from Tables I. and III. almost as accurately as they can be found by averaging the values given at each height in each ascent, and Tables VIII., IX., and X. are formed in this way.

The densities are also given.

Table VIII. has been calculated from Table I., using for the purpose the mean pressures at sea-level given by the isobaric charts in Hann's *Meteorology.* The calculation has been made graphically on semilogarithmic paper. The values are in millibars and the probable error is about 2 mb., less at small and great heights, but most at about 8 k. or 9 k. This is because the error is chiefly due to instrumental or casual errors in the temperatures of Table I., and the effect of such an error upon the pressure is greatest at about 8 k.

The differences of pressure increase up to 6 k. or 7 k., but are large and do not show much change between 2 k. and 10 k. Above 12 k. they fall off rapidly, until at 20 k. Paris is the only station showing any appreciable departure from the general mean. The change is greatest from North to South, and shows at 8 k. a rate of nearly 1 mb. per latitude degree. No definite variation is shown with change of longitude. The table is very satisfactory, inasmuch as it shows such uniformity at great heights. There is good theoretical ground for postulating such agreement, and had it not occurred it would have shown the existence of systematic errors in the measurement of the temperature. At 20 k. a difference of 1° C. in the temperature of the underlying air column causes, if we assume an invariable sea level pressure, a change of ·6 mb. in the pressure. Hence if there are systematic errors they must be of about the same magnitude and sign at all the stations. There would seem to be such an error of about +1°·8 C. at Paris, and it may be due to the use of large balloons. Larger balloons have been used there, possibly with the result of a greater error from solar radiation. As previously stated, the meteorograph ascends in a wake of more or less

TABLE VIII.—MEAN PRESSURES.

Height in Kilometres.	Petrograd.	Scotland.	Konchino.	Manchester.	Hamburg.	Berlin	Limerick.	England, S. E.	Brussels.	Paris.	Strassburg.	Munich	Vienna.	Zurich.	Pavia.	Means	Canada.	Equator.
	Millibars.																	
20	55·0	55·0	55·0	55·2	54·5	54·8	54·5	54·9	54·5	56·0	54·7	55·0	55·0	54·4	54·8	54·9	?	53
19	64·0	64·2	64·3	64·6	63·8	64·0	63·6	64·1	63·7	65·6	64·0	64·3	64·4	63·7	64·0	64·1	...	63
18	74·5	74·8	75·1	75·4	74·5	74·8	74·3	75·0	74·5	76·6	74·8	75·2	75·2	74·6	75·0	75·0	...	75
17	87·0	87·3	87·6	88·0	87·0	87·4	87·0	87·5	87·2	89·6	87·6	88·0	88·0	87·3	87·6	87·8	...	90
16	101	102	102	103	102	103	102	102	102	105	102	103	103	102	103	102	...	107
15	118	118	119	120	120	120	119	120	120	123	120	121	121	120	121	120	120	128
14	138	138	140	140	140	140	138	140	140	143	141	142	142	141	142	140	142	152
13	161	161	163	164	164	164	162	164	164	167	165	166	165	165	165	164	167	178
12	187	187	191	192	191	192	190	192	193	195	193	194	193	194	194	192	195	209
11	218	219	223	224	223	225	222	224	226	228	226	227	226	227	227	225	228	244
10	255	256	261	261	259	262	259	261	263	266	263	264	263	265	264	262	266	283
9	297	299	304	302	302	305	302	303	305	309	307	307	306	308	307	305	309	327
8	346	348	353	352	351	354	351	352	353	357	355	355	354	357	356	353	358	376
7	400	402	407	407	405	408	406	407	408	412	410	410	409	412	412	408	413	430
6	461	464	469	468	465	470	468	469	470	473	472	472	471	474	474	470	475	491
5	529	532	537	537	534	538	537	538	538	541	540	540	539	542	542	538	543	558
4	606	608	612	613	611	614	613	615	615	617	616	616	615	618	618	614	618	632
3	692	694	698	698	696	699	698	699	699	701	700	701	700	702	703	699	703	713
2	787	787	793	793	791	795	794	795	794	796	794	795	795	796	797	794	798	803
1	896	894	899	898	898	900	899	900	899	900	900	900	900	901	901	899	903	903

dead but heated air behind the balloon, and a larger balloon would naturally lead to a higher temperature at a definite distance below.

Table IX. shows the seasonal variation of pressure at each height over England It is not carried beyond 15 k. for the following reason. Taking the year as a whole, there is no change of temperature between 15 k. and 20 k., but the evidence is hardly sufficient to show that there is no change in the separate months.

The values in the table look very irregular; this is because the variation is produced in two separate ways. In England the barometer is lower in spring and autumn than it is in winter or summer, and this influences the values in the lower strata, but in the higher strata the change is chiefly produced by the seasonal change of temperature.

TABLE IX.—SEASONAL VARIATION OF PRESSURE AT EACH HEIGHT OVER ENGLAND.

Height in Kilometres.	January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Range.
	Millibars.												
15	116	116	117	119	121	123	125	125	123	122	120	117	9
14	136	135	136	138	141	144	146	146	144	143	140	137	11
13	159	158	159	161	165	168	170	170	168	167	165	160	12
12	187	186	187	189	193	196	198	199	197	195	193	188	13
11	218	218	218	221	225	229	232	232	231	228	226	220	14
10	255	254	255	258	262	267	270	271	269	266	263	257	17
9	297	296	298	302	305	310	314	314	313	309	305	300	18
8	345	345	347	350	353	359	362	363	361	357	353	349	18
7	399	399	401	405	409	414	417	418	416	413	408	403	17
6	462	462	463	467	471	476	478	479	477	473	470	466	17
5	530	530	531	535	539	543	546	547	544	541	537	533	17
4	607	607	608	611	615	619	622	623	620	617	613	610	16
3	693	693	693	697	700	704	706	707	703	700	698	696	14
2	789	788	789	792	794	798	799	799	797	794	791	790	11
1	898	897	897	898	899	902	903	903	900	899	898	898	6

The range is shown in the last column: if uniform temperature above 15 k. had been assumed and the table carried on to 20 k., the range at 20 k. would be shown as 5 mb. It cannot be very far from this value. Rather greater variation would be shown on the Continent, because the temperature range is greater there, and nearly double the English variation would appear at intermediate heights over Canada.

TABLE X.—TEMPERATURES, PRESSURES, AND DENSITIES FOR DIFFERENT HEIGHTS UP TO 20 k.

The temperatures are in the absolute scale.
The pressures are in millibars.
The densities are in grammes per cubic metre.
The figures for Canada above 15 k. are somewhat doubtful, and those for the equator very doubtful, owing to the paucity of observations.

Height in Kilometres.	England S.E.			Europe.			Canada.			Equator.		
	T.	P.	D.	T.	P.	D.	T.	P.	D.	T.	P.	D.
20	219	55	87	219	55	87	214	54	88	193	53	91
19	219	64	102	219	64	102	215	63	102	193	63	113
18	·219	75	119	219	75	119	214	74	121	193	75	135
17	219	88	139	219	88	139	211	87	144	193	90	162
16	219	102	162	219	102	162	211	102	169	195	107	191
15	219	120	191	219	120	191	211	120	198	198	128	225
14	219	140	223	219	140	223	212	142	233	203	152	261
13	219	164	261	219	164	261	214	167	268	211	178	294
12	219	192	305	218	192	307	216	195	314	219	209	331
11	220	224	355	219	225	358	219	228	365	227	244	374
10	222	261	409	222	262	411	223	266	415	235	283	419
9	228	303	463	227	305	467	229	309	470	243	327	469
8	234	352	524	233	353	528	236	358	528	251	376	522
7	241	407	589	241	408	590	243	413	592	258	430	581
6	248	469	658	248	470	661	251	475	662	265	491	645
5	255	538	735	255	538	735	258	543	733	272	558	714
4	262	615	819	261	614	819	264	618	815	279	632	789
3	268	699	909	267	699	913	270	703	905	285	713	871
2	273	795	1014	272	794	1017	275	798	1011	290	803	968
1	278	900	1128	277	899	1128	278	903	1134	295	903	1067
0	282	1014	1253	281	1014	1258	282	1017	1258	300	1012	1174

Table X. gives the mean density in grammes per cubic metre for England S.E., Europe, Canada about latitude 43°, and the equator. For convenience the temperatures and pressures are repeated. The probable errors are of the same order as those already stated for the temperatures and pressures, since the density varies directly as the pressure and inversely as the temperature. It does not take account of the water vapour, and is therefore a trifle too high, especially in the lower strata, but it is only up to 1 k. at the equator that the error can reach 1 per cent. Neither has the variation of g with height been considered. The figures have been obtained by the use of a slide rule.

§ 9. THE MOTION OF THE FREE ATMOSPHERE.[1]

The means of getting detailed information about the motion of the atmosphere at any appreciable height above the earth's surface are very limited, and are confined to clear weather. These means consist of observations by kites and pilot balloons and of the motion of clouds, and by noting the distance and the bearing of the falling place of registering balloons.

[1] Since this was written, part iv. of the *Manual of Meteorology*, M.O., 234, by Sir Napier Shaw, has been published. It contains much information on this subject.

The direction and the angular velocity of any cloud with a reasonable altitude that remains visible for a few minutes can be determined easily, but unfortunately the height of the cloud as a rule is not known, and all that can be done is to class the clouds in three groups and assign to each group an arbitrary height. For the motion of the atmosphere as derived from observations of clouds the reader should consult Clement Ley's book (*Cloudland*), and the more recent book of Professor Hildebrandsson (*Rapport sur les observations internationals des nuages, au Comité International Météorologique*).[1]

Kites give more information, as the heights are accurately known and the velocities fairly well known, but when a kite is hidden behind a low sheet of cloud the direction of the wind is more or less uncertain ; also the use of kites is confined to the lower strata.

Pilot balloons afford the best method of observation ; unfortunately their use is confined to clear weather, save for the lowest strata. A pilot balloon is a small rubber balloon of about 2 ft. diameter at the start, filled with hydrogen. It rises with a velocity that is known relatively to the air, and its position is kept in view by one and sometimes two theodolites. If two theodolites placed at the ends of a base line of a mile or so in length are employed, the precise track of the balloon can be charted and the three components, the vertical, the south to north and the west to east, of the air motion are known. If only one theodolite is used, the assumption must be made that the vertical motion of the air is so small that it may be neglected, and then the horizontal components can be calculated. This assumption is justified on the whole, but there are vertical currents in the lower atmosphere on bright sunny days in spring and summer, and underneath the thunder type of cumulus there are often strong updraughts of air reaching velocities of ten or more miles per hour. For details of the method and for results the reader should consult *The Structure of the Atmosphere in Clear Weather*, by C. J. P. Cave, M.A. (Cambridge University Press), and a paper by J. S. Dines on "The Rate of Ascent of Pilot Balloons" (*Quarterly Journal of the R. Met. Soc.*, vol. 39, 1913).

The conditions are too complex to be given in any detail, but the following brief summary is taken chiefly from Captain Cave's book.

The wind on the whole increases with increasing height up to the limit of the troposphere, and it falls off rapidly as the common boundary of the stratosphere and troposphere is passed.

The component of the wind from west to east shows a systematic increase with height, until it begins to fall off in the stratosphere, but the south to north component shows no such increase.

The geostrophic wind is usually reached at a small height, less, that is, than 1 k. ; above that the wind still veers somewhat as a rule, but at great heights the winds cannot be inferred with much certainty from the surface distribution of pressure.

Strong winds from some point between north and south on the western side are occasionally met with at great heights blowing away from the upper part of cyclonic areas.

These results follow from some 200 observations made by Captain Cave in clear weather, mostly with pilot balloons, but some from registering balloons sent up for the purpose of determining the temperature and used also as pilot balloons.

Some further information is available from the falling place of registering balloons.

[1] Translated by R. G. K. Lempfert, M.A., for the *Royal Met. Soc. Quarterly Journal*, vol. 30, p. 317.

Many English balloons fall in the Channel, and the exclusion of these would produce a systematic error, hence all English ascents must be omitted. The result from the Continent is to show a mean drift to S.E. by E. The Continental ascents last as a rule about $1\frac{3}{4}$ hours, and the centre of the falling places is 42 k. east and 30 k. south, with a probable error of 3 k. in position. In general the direction is 18° to the right of the direction of the gradient wind. In 20 per cent. of the cases the motion is reversed and the balloon falls more than 90° away from this direction. The connection between the steepness of the gradient and the distance travelled by the balloon is very slight. In Canada and the United States the general drift is also towards the east. But although the general drift is to the east, occasions are by no means wanting in which no westerly component of the wind is met with.

Fuller details will be found in Lt.-Col. Gold's paper (*Geophysical Memoirs*, M.O., No. 210*e*).

The observations at great heights over the equatorial and tropical regions that have been published are too few to permit of any certain conclusion being drawn as to the upper winds of those regions.

The relation between the velocity and direction of the wind and the distribution of pressure at the time must be very close at all heights. It is very marked even at the surface, as we know by Buys Ballott's law, where the friction of a plain surface and still more the local configuration due to hills introduce large disturbances. It must be still closer at all points above the surface, where the motion of the air is perfectly free. Indeed, it is obvious that pressure differences between air at the same level could not exist if the air were free to pass without resistance save from its own inertia from regions of high to regions of low pressure. But owing to the earth's rotation air moving along a great circle exerts a pressure towards its right hand (in the northern hemisphere) which is proportional to its velocity and to the sine of the latitude; there is also a further centrifugal action depending on the square of the velocity if the path is curved. It is fairly certain that these forces in general just balance the pressure gradient (Sir Napier Shaw, *Proceedings of the Royal Society of Edinburgh*, vol. 32, part i. p. 78).

In the table of pressures a gradient of about 1 mb. per degree of latitude is shown over Europe at 10 k. To balance this a westerly wind is required at that height, so that the observations on both wind and pressure agree. As stated above, the winds fall off rapidly above 11 k., and this agrees with the equality of pressure that is found at 18–20 k. At these heights the density of the air is slight, and if pressure differences equal to those found near the surface existed, they would lead to winds of twelve times the velocity, so that winds of 600 miles an hour would be common. There is no doubt that the winds are really light, for balloons that reach a great height do not seem to travel further than those that reach a moderate height, and there is therefore additional ground for asserting that the pressure does not vary much from place to place at 18 k. What happens above 20 k. we do not know.

The actual relation between the wind and the pressure distribution is given by the equation

$$\frac{\gamma}{D} = 2\omega V \sin\phi + \frac{V^2}{R}\cot\rho,$$

where γ is the barometric gradient, D the density of the air, ω the angular velocity of the earth's rotation, V the velocity of the wind, R the radius of the earth, and ρ the angle subtended at the earth's centre by the radius of the small circle in which the wind is moving (see *The Weather Map and Glossary*, "Gradient Wind").

The equation is obtained by neglecting the squares and higher powers of $V/R\omega$ and is applicable to all ordinary wind velocities. It does not hold close to the earth's surface on account of the resistance due to friction, but observations show that within the limits of observational errors it holds as soon as half a kilometre height is reached. The first term is the important one in temperate latitudes, except when the curvature is small or the velocity very great. Near the equator, since sin ϕ is small, the first term is of little consequence, and the V^2 term is the important one, as is shown in the tropical hurricanes. The equation presupposes a state of steady motion.

§ 10. STATISTICAL DATA.

In the preceding pages and tables some information has been given about the pressure, the temperature, the motion, and the humidity of the free air; but the most remarkable fact that has been brought to light by the observations is the very close relationship which exists between some of these quantities and the absence of any apparent relationship in other cases where it might be expected.

The close connections are between the pressures and temperatures at various heights; they are very complicated and difficult to disentangle, because some are plainly the result of the simple laws of physics and dynamics, while others, at first sight, would seem to contradict those laws. Thus warm air is lighter, bulk for bulk, than cold, but a high barometer is found in places where the overlying air is mostly warm. This is difficult to explain. There is a law that if air is compressed it becomes warm, and from this point of view it is natural that the high-pressure area should be warm. In practice, however, apart from any vertical motion, it is warmer than the law of dynamic heating should make it.

Broadly, the following rules hold in the free atmosphere over Europe, the free atmosphere meaning from 1 to 20 k. In high-pressure areas the lower strata are warm, the fall of temperature with height continues to about 12 k. (the mean is 10·6 k.), and the strata above 12 k. are cold. In cyclonic areas the lower strata are cold, the temperature gradient ceases at a lower height than usual, and above 10 k. the strata are warm. The temperature of the air depends closely upon the pressure and upon the date; but, save quite close to the earth, it does not depend upon the direction of the wind.

These facts will be expressed most simply in a numerical sense by the use of symbols and correlation coefficients.

A correlation coefficient is a number lying between +1 and −1 and is a measure of the closeness of the connection between two quantities. If the two quantities are such that knowing one the other is known with absolute certainty because their deviations are strictly proportional, the correlation coefficient will be either plus or minus 1; the case, for example, between the volume and the weight of the same kind of substance. A small correlation coefficient means that there is no connection between the quantities. Suppose A is the cause and B the effect, a correlation coefficient between A and B which is numerically greater than ·71 (the value of $1/\sqrt{2}$) means that A is responsible for more than half the variation of B; and in general, if the coefficient is $1/\sqrt{n}$, then A is responsible for $1/n$ of the variation of B. Thus, in rectangles that are approximately alike, the correlation between the area and the length of one side is ·71. For right-angled solids that are approximately cubes the correlation between the volume and the length of one edge is $1/\sqrt{3} = {\cdot}58$.

Correlation coefficients between the following quantities have been found with more or less accuracy and are given in the following tables—

P_0 the barometric pressure at M.S.L.
P_1 ,, ,, 1 k. height.
P_n ,, ,, n k.
T_0 the temperature at the surface.
T_1 ,, ,, 1 k.
T_n ,, ,, n k.
H_c the thickness of the troposphere, or the height of the tropopause.
T_c the temperature at the boundary between the stratosphere and troposphere.
W the west to east component of the drift of the balloon (*i.e.* a west wind is positive).
S the south to north component of the drift of the balloon(*i.e.* a south wind is counted positive).
G_w the west to east component of the gradient wind at the surface.
G_s the south to north component of the gradient wind at the surface.
T_m the mean temperature from 1 to 9 k.
V the total water vapour contents of the atmosphere.
T_{0-4} the mean temperature up to 4 k.

TABLE XI.—CORRELATION COEFFICIENTS.

	P_0.	P_9.	T_m.	H_c.	T_c.	V.	T_0.	T_{0-4}.	T_4.	T_8.	W.	S.	G_w	G_s
P_0 . .	...	·68	·47	·68	−·52	·08	·16	·34	...	...	...	...	...	...
P_9 . .	·68	...	·95	·84	−·47	...	·28	...	·82	...	...	...	...	...
T_m . .	·47	·95	...	·79	−·37	...	...	...	...	...	...	...	...	...
H_c . .	·68	·84	·79	...	−·68	·39	·30	·66	·64	·74	−·09	−·10	·11	−·02
T_c . .	−·52	−·47	−·37	−·68	...	...	...	...	...	...	...	...	...	..
V . .	·08	...	...	·39	...	...	...	·73	...	...	...	...	...	...
T_0 . .	·16	·28	...	·30	...	...	...	...	...	...	·06	·06	·37	·19
T_{0-4} . .	·34	...	...	·66	...	·73	...	...	...	...	...	...	...	...
T_4 . .	...	·82		·64	...	...	...	...	...	...	−·08	−·02	·13	·06
T_8 . .	...	...	...	·74	...	...	...	...	...	...	−·03	·03	·10	−·01
W . .	...	...	...	−·09	...	...	·06	...	−·08	−·03	...	...	...	...
S . .	...	...	...	−·10	...	...	·06	...	−·02	·03	...	...	...	...
G_w . .	...	...	...	·11	...	...	·37	...	·13	·10	...	...	...	...
G_s . .	...	...	...	−·02	...	...	·19	...	·06	−·01	...	...	...	...

In addition to the above, the following values which cannot be conveniently placed in the table are given :—

P_9. and	T_0.	$T_{\cdot 5}$	T_1.	$T_{1\cdot 5}$.	T_2.	T_3.	T_5.
	·28	·60	·68	·73	·74	·82	·82

	T_0 and P_0.	T_1 and P_1.	T_2 and P_2.	T_3 and P_3.	T_4 and P_4.	T_5 and P_5.	T_6 and P_6.	T_7 and P_7.	T_8 and P_8.	T_9 and P_9.	T_{10} and P_{10}.	T_{11} and P_{11}.	T_{12} and P_{12}.	T_{13} and P_{13}.
Jan.–March .	−·02	·54	·82	·79	·86	·85	·84	·87	·91	·81	·35	−·32	−·38	−·37
April–June .	·14	·28	·49	·79	·89	·89	·92	·87	·81	·45	·20	−·12	−·24	−·01
July–Sept. .	−·02	·31	·56	·72	·75	·81	·83	·87	·87	·88	·43	−·08	−·41	−·19
Oct.–Dec. . .	·33	·56	·76	·77	·83	·87	·85	·85	·86	·78	·29	−·24	−·34	−·50
Means . .	·11	·42	·66	·77	·84	·85	·86	·86	·86	·71	·32	−·19	−·36	−·28

Also : Steepness of barometric gradient and H_c . −·22
Barometric rise in past 12 hours and H_c −·17

Full details about most of the correlation coefficients are given in the *Beiträge zur Physik der freien Atmosphäre*, v. Band, Heft 4, in a publication of the Meteorological Office, M.O. No. 210*b*, and in the *Quarterly Journal of the Roy. Met. Soc.* for January 1914, vol. 40, No. 169; but a few are from unpublished data in possession of the Meteorological Office. The probable errors are all small, since in nearly every case more than 100 observations have been used, and in many instances the values are the means of several separate determinations of the same coefficient.

The variability of the temperature of the air is rather less at the ground level than it is above. It reaches a maximum at about 6 k., it falls off at about 10 k., and then increases again at greater heights. The standard deviation varies somewhat with the locality, but 7° C. is about the usual value.

The variations of the pressure are of about the same actual magnitude from the surface up to 8 k. or 9 k. where the S.D. is from 10 to 12 mb.; but above this the range falls off rapidly. Since the actual pressure at 9 k. is only ·3 of that at the surface, the percentage variation at 9 k. is very much larger than at the surface, and it reaches its maximum at about this height.

The standard deviation of H_c (the height of the tropopause) is about 1·5 k.

The standard deviation of a quantity is in general about five-fourths of the average value of the departures from the mean value, taken without regard to sign. Such large departures as three times the standard deviation seldom occur; twice may occur in about one out of twenty instances. Thus 10·5 k. is the mean for H_c, 7·5 k. and 13·5 k. may perhaps be found once each in forty instances; that is to say, the chances are about equal that out of forty records one may or may not show as low a value as 7·5 k. Of course it makes a difference in calculating the standard deviations of the various quantities whether the seasonal variations are taken into account or not. If temperatures for July and January are taken together, the deviations of both from their common mean will be considerable, and the standard deviation large compared with the stardard deviation for one season only.

The standard deviations that have been given include the seasonal variation, and for a single season they require modification as follows. The S.D. of the temperature near the ground for one season is about 4°·5 C. The S.D. of the pressure near the ground is unaffected since the annual pressure variation is small, but higher up, where the amplitude of the seasonal range may reach 9 mb., the S.D. will be reduced from 11 mb. to 9 mb. The variations of H_c under changes of pressure are so large that for Europe, at least, the seasonal variation is without much influence upon the S.D.

It may be of interest to state the rule. If a be the amplitude of a periodic variation, the S.D. due to this variation alone is $a/\sqrt{2}$. If σ be the standard deviation due to casual and all other variations, then the standard deviation corrected so as to exclude the periodic variation is $\sqrt{\sigma^2 - a^2/2}$ (see *Computer's Handbook*, M.O., No. 223, Section V., pp. 27 and 37).

It has been necessary to enter somewhat into the question of the standard deviation before discussing the results given in Table XI., for the correlation coefficients and the standard deviations render it possible, when one quantity is known, to express the most probable value of any other. Thus the correlation between the surface pressure and the mean temperature from 1 to 9 k., that is between P_0 and T_m in the table, is given as ·47. This means that, if P_0 is any definite fraction of its standard deviation above its average value, T_m on that occasion is most likely to be ·47 times the same fraction of its standard deviation above its mean.

Let a δ placed before a quantity denote its departure, either positive or negative, from its mean value expressed in terms of its standard deviation, then the relationship is given by $\delta T_m = \cdot 47 \delta P_0$, and similarly for any other pair of variables between which the correlation has been determined. Thus, suppose the barometric pressure in London to be 1000 mb.; this is 15/12 of its standard deviation below its mean value. The most probable value of T_m over London on that occasion is $-\cdot 47 \times 15/12$ or ·59 of its standard deviation below its mean. The standard deviation is 6° C. and the mean 254°, so that 250°·5 is the most probable value.

§ 11. The Connection between Pressure and Temperature.

The very close connection that exists will be seen by the values of the correlation coefficients that are given in Table XI., especially when it is remembered that the value of H_c is a part of the distribution of temperature. The closest connection is with P_9, the pressure at 9 k., but P_0 the surface pressure shows a high correlation with all the Ts excepting T_0, and from 4 to 8 k. the correlation between pressure and temperature at the same point is very high.

This connection has revolutionised our ideas of the local circulation, because it has shown that the lower strata in a cyclone are cold, whereas it used to be the custom to explain the low pressure of the cyclone by saying that it consisted of a core of warm light air.

The difference of temperature at each height between a cyclone and anticyclone is shown by the following figures (Table XII.), which are taken from the *Journal of the Scottish Meteorological Society*, vol. 16, No. 31, p. 306 :—

Table XII.

Height.	Cyclone 29·20 in. 989 mb.	Pressure.	Anticyclone 30·30 in. 1026 mb.	Pressure.	Temperature Difference.	Density.	
						C.	A.
14	224°	135	215°	143	+ 9°	210	232
13	25	158	15	168	+10	245	272
12	25	184	17	197	+ 8	285	316
11	24	214	20	231	+ 4	333	350
10	25	249	25	269	0	389	427
9	26	289	31	313	− 5	446	472
8	28	337	38	362	−10	516	530
7	34	390	46	417	−12	582	589
6	42	451	53	478	−11	648	658
5	49	519	59	547	−10	724	734
4	56	594	65	623	− 9	808	818
3	63	678	71	708	− 8	898	906
2	70	772	76	802	− 6	997	1010
1	76	875	79	908	− 3	1105	1135
Value of H_c in k.	8·7	...	12·3	...	3·6	...	...

The sequence of events in the upper air as a low-pressure area passes across the British Isles or Europe is perfectly definite. As the barometer falls the temperature of the air column from 1 k. to 9 k. falls also, the value of H_c decreases, and the temperature of the upper air from 11 k. to 20 k. rises. As the depression moves away and the barometer rises the lower air column rises in temperature, H_c increases, and the upper air column falls in temperature. The changes reach their turning-point when the barometer is lowest, though there is some evidence to show that H_c is less after the barometric minimum has passed rather than before, for H_c is found to be less with a northerly than with a southerly gradient wind. If the pressure at 9 k. could be charted instead of that at mean sea-level, the proportionality between the various quantities would be still more precise. A low-pressure area at the surface remains a low, and a high-pressure area remains a high pressure up to about 20 k., but the pressure differences fall off from the ground upward, slightly from 2 k. to 9 k. and rapidly above 10 k.

Of course the figures given are not exact for every cyclone—the ordinary casual error which dogs the steps of all meteorological work prevents that; but there is hardly an instance of a registering balloon affording a record in well-marked cyclonic conditions in which the characteristics of temperature stated above do not occur. There must be some reason for this, and it is of interest to try and trace it.

The same mass of the upper air can only change its temperature in two ways, by radiation or by dynamic heating or cooling. It is hardly conceivable that the cold air of the centre of a cyclone can be air that was originally cold and has brought its coldness with it. There are three objections to this idea. The winds over the area about to be occupied by a cyclone are mostly from some southerly point and are not cold; secondly, the fall of temperature, 12° C. or 22° F. nearly, is equivalent to much too great a change of latitude; thirdly, the correlation between the wind and T_4 shown in Table XI. is so small that it proves that the wind direction at that height cannot have much effect upon the temperature. It follows that the air, in virtue of its position, must just recently either have lost heat by radiation, or have cooled by dynamic cooling.

It is difficult to see why radiation from the air should be particularly active in the lower strata in the centre of a cyclone; and radiation is ruled out for another reason. The whole solar heat received per day at the outer boundary of the atmosphere would not suffice to raise the temperature of the atmosphere more than three degrees, and really, as we know from the small daily range, much less is received and therefore much less radiated away each day. But, under the changing conditions of pressure, variations in temperature of 10° C. or more occur per day, and this is too large an amount to be due to radiation; hence we are driven to ascribe it chiefly to dynamic heating and cooling, which will readily account for the amount of the variation.

The law for small changes under which dynamic heating acts on a portion of air is given by the equation $\delta T/T = \cdot 29 \delta P/P$, where T is the absolute temperature; and substituting the special mean values of P and T for each kilometre height and calling δP one millibar, the corresponding change of temperature is :

Height.	0.	2	4.	6.	8.	10.	12.	14.	16.	18.	20 k.
δT	·08	·10	·12	·15	·19	·25	·33	·45	·62	·85	1°·15 C.

Using the difference of pressure shown by Table XII., we get the following changes of temperature due simply to the adiabatic change from the low-pressure to the high-pressure area at the same level, and the observed differences that are given in

0.	2.	4.	6.	8.	10.	12.	14 k.
3°·0	3°·0	3°·5	4°·0	4°·7	5°·0	4°·3	3°·6
1°.0	6°·0	9°·0	11°·0	10°·0	0°·0	−8°·0	−9°·0

Table XII. are repeated in the third row. Part of the low temperature in the lower strata of the cyclone is thus accounted for, but the high temperatures in the upper strata are accentuated.

The difference must be due chiefly to the vertical component of the air motion, and it is easy to calculate, on the assumption that the whole change is due to dynamic cooling, how far the air must have risen or fallen to acquire the observed temperature.

Assuming that no water vapour is condensed, for small distances the change of level causes a difference of nearly 10° C. per kilometre, and using the gradients of Table II. the difference between 10 and the given gradient is the number of degrees that air rising 1 k. will find itself below the mean temperature of its new position. The following values have been calculated to account for the differences of temperature in Table XII., *i.e.* they show for various levels the height that air leaving the anticyclone would have to rise or fall to adjust itself to the temperature of the cyclone :—

2 k.	4 k.	6 k.	8 k.	10 k.	12 k.	14 k.
Rise.				Fall.		
·6 k.	1·6	2·4	1·6	−·8	−1·3	−1·3 k.

In reality a considerable amount of water vapour is condensed in the first few kilometres of a cyclone, and this will render the ·6 k. at 2 k. and the 1·6 at 4 k. too small. How much too small it is not easy to say. Much importance cannot be ascribed to the value at 10 k., which is sometimes above and sometimes below the tropopause.

About this general rise of the air in the lower part of a cyclone there can be no doubt whatever ; it is proved by the indraught of the surface winds across the isobars. Its velocity may be one or two hundred feet an hour, an amount quite insignificant in comparison with the velocities of the horizontal currents. The difficulty is to see by what process this cold heavy air, which one would expect to be sinking, is forced upwards.

At about 9 k. there is no vertical component, the winds blow horizontally (see Sir Napier Shaw, *Journal of the Scottish Met. Soc.*, 3rd series, vol. 16, No. 30, pp. 176, 177). In Table XI. it will be seen that the only large negative correlation coefficients are those involving T_c, which is virtually the temperature of the upper strata. The rule is general that where the upper strata are cold the lower are warm, and conversely. It is probably due to this rule that at great heights in Canada the summer is colder than the winter, for the range in the lower strata in Canada is very large.

Whatever the cause may be, this rule is a corollary that follows from the uniformity of pressure at 18–20 k. The pressure at 20 k. depends very greatly on the mean temperature of the air from 0 k. to 20 k., and but little on the surface pressure, for a change of one degree in the temperature is operative to the same extent as a change of 15 mb. in the surface pressure. The equality of pressure at 20 k. therefore requires a practical uniformity in the mean temperature of the air column from 0 k. to 20 k. in all parts of the earth and under all conditions of weather, and this uniformity is shown in the tables. It is easy too to see the means of adjustment, for if one column were warmer than another, in view of the actual distribution of temperature in a vertical direction that prevails, the rise of air in the one column and the fall in the other that would naturally occur from convection would tend, as shown above, to restore the balance of temperature. But it is not easy to see why the equalisation should occur in the form it does.

The very low temperature over the equator at great heights is very striking, but is more readily explained. The observations over the equator are too scanty to show the precise value, but there is certainly a difference of 20° C. and probably of 25° C. between the mean temperatures at 18 k. over the equator and over Europe. This is the difference between the very coldest month in the most severe winter in England and the very hottest summer month on record. It is greater than the difference between England in winter and the tropics. But, admitting that the lower strata in the tropics must be warm, this low temperature is necessary to produce equality of pressure at great heights. It is found too where one would expect to find it, at the top of the rising air column, whereas in the cyclone it is in the lower half of the column 1 k. to 20 k. that the low temperature occurs.

With reference to the equality of pressure at 18–20 k. this is an observational fact. If it be a necessary condition, then the low temperature over the tropics is explained. It seems to be necessary, for any appreciable differences of pressure at that height could only be balanced by winds of excessive velocity, since in the formula $\gamma/D = 2v\omega \sin \phi$ already quoted, the D (the density) has only $\frac{1}{12}$ of its surface value, and near the equator $\sin \phi$ is small, so that v would be very large. For values of γ that are common on European weather charts the velocity would reach that of sound, and the formula $\gamma/D = 2v\omega \sin \phi$ would require modification. It may well be that the viscosity of the air, including Major Taylor's eddy viscosity, renders such velocities impossible.

§ 12. The Vertical Temperature Gradient and the Value of H_c.

In the early days of meteorology, fifty years or so back, the cold of mountains used to be explained thus. It was said that the sun warmed the earth's surface, and the summit of the mountain being further away from the surface was further away from the source of heat and therefore colder. Later, when the warming of air by compression and the cooling by expansion became more generally known, the temperature gradient seems to have been generally accepted as the result of the adiabatic cooling of rising air.

The discovery that the gradient ceased at a definite height came as a surprise, and the result was disbelieved and ascribed to observational error; but, as we can see now, there was no good reason why the gradient should have been supposed to continue to the outer limit of the atmosphere.

There are two ways in which the gradient may be explained, and probably both have some part in the result, but in what proportions is very uncertain.

It is quite certain that if the whole atmosphere could be thoroughly mixed up from top to bottom, as water in a bath is mixed, it would (immediately after the process) have the adiabatic gradient of just under 10° C. per kilometre throughout. It would then begin to settle back into its normal condition, which must be the stable condition, by mutual radiation between its parts, but how long the process would take there is no means of knowing. The mixing must be carried on constantly by the winds to just a sufficient extent to produce the actual gradient and no further, that is to say, the effect of mixing must exactly balance the opposed effect of radiation, but it is not easy to see why the lower part only should be affected.

The other explanation that has been put forward may be called the greenhouse effect. A greenhouse under strong sunshine becomes very hot; the glass allows the solar radiation to pass through it unimpeded, but does not allow the long wave radiation into which the solar radiant energy is changed to go back; thus it forms a sort of heat trap. The air is said to perform the same office for the earth that the glass does for the greenhouse, with the result that the lower air strata where the trap is most effective are the hottest.

Some such effect no doubt there is, for earth thermometers show that the earth is in general a degree or so warmer than the air, and therefore heat must pass from the earth to the air and provide a source of solar heat supplied to the air at the bottom, and thus give rise to the temperature gradient. But here again there seems to be no reason why the effect should cease suddenly, so as to give a sudden cessation of the temperature gradient. Also it may be said that the heat of the greenhouse is almost entirely caused by the glass preventing the convective effect of the air rather than by its checking out radiation.

There must be a certain vertical distribution of temperature which we may call A, and which would be produced by the effect of radiation alone were all air currents strictly horizontal. But just as the horizontal currents make the earth's surface more isothermal than it would otherwise be by carrying heat from the warmer parts to the colder, so the perpetual vertical interchange must make the actual vertical distribution more nearly adiabatic than A. It does not matter if the vertical interchange of air be due to convection currents or to forced mixing by the wind, though the latter is probably the more important agent, because convection currents imply an adiabatic gradient, and that is seldom found except near the surface on bright sunny days in spring and summer.

The value of H_c and its variation are very closely connected with the temperature gradient, because H_c is the height to which the gradient extends. In Table XI., which shows the connection between the various quantities, the two correlation coefficients between the height of the tropopause and the pressure at 9 k. (H_c and P_9) and between the height of the tropopause and the temperature of the air column between 1 k. and 9 k. (H_c and T_m) are the most striking. They are ·84 and ·79, and it is very seldom in dealing with statistics that such high coefficients are found in cases where the form of the connection is not apparent. That the value of H_c depends to some extent on these quantities seems certain : the point is, which of them is it, or is it both?

By the ordinary method of partial correlation the following regression equation is obtained,

$$\delta H_c = -\cdot 09\delta T_m + \cdot 96\delta P_9,$$

the standard deviations being used as units. This equation means that for any definite values of T_m and P_9 at some given date and place the most probable value of δH_c is $-\cdot 09\delta T_m + \cdot 96\delta P_9$. So long then as P_9 remains unaltered, variations of T have little effect, because the coefficient ·09 is small; but since ·96 is very nearly unity, H_c follows the changes of P_9 with great accuracy, no matter what the variation of T_m may be. This equation rests upon some 400 observations, so that the probable errors of the coefficients are quite small. Whatever the reason for the curious division of the atmosphere into two parts may be, it is the value of the air pressure at about 9 k. height which regulates the position of the boundary. If the air pressure at 9 k. is high then H_c is large, and if the pressure is low then H_c is small. It is certainly this pressure that matters ; other things are of trifling or of no importance in comparison.

One other point must be touched upon, and that is the effect of the water vapour, the V of Table XI. Perfectly dry air is very pervious to all kinds of radiation, whereas a mixture of air and water vapour is capable of absorbing and radiating heat. It has been suggested that this radiative property of moist air is the cause of the variation of H_c, and to examine this point the following regression equations connecting together the quantities P_0, T_{0-4}, V, and H_c have been formed. These quantities have already been defined, but it is necessary to say more about V. The water that the air can hold as vapour increases very rapidly with increasing temperature, being nine times as great at 50° F. as at 0° F. In consequence, nearly all the water is held in the first few kilometres, and the percentage of the total that is found in that part of the air that is below 0° F. (255 a.) is very small. That percentage is neglected in forming V. Since the humidity recorded at lower temperatures is unreliable there is no option, but the amount is too small to affect the correlation.

The following are the equations, the standard deviations being units, and T being written for T_{0-4}:

$$\begin{aligned}
\delta P_o &= \cdot 24\delta H_c + \cdot 41\delta T - \cdot 27\delta V \\
\delta H_c &= \cdot 67\delta T - \cdot 08\delta V + \cdot 17\delta P_s \\
\delta T &= \cdot 55\delta V + \cdot 16\delta P_s + \cdot 37\delta H_c \\
\delta V &= - \cdot 19\delta P_s - \cdot 07\delta H_c + \cdot 84\delta T .
\end{aligned}$$

These equations, like the one already quoted, show the most probable value of the departure from the mean of the left-hand quantity when the departures of the right-hand quantities are known. The right-hand quantities are entirely independent of each other, so that the numerical magnitude of each coefficient is a good index of the importance of the variable to which it is attached in causing changes in the left-hand quantity. The equations are based on 200 observations in four groups of 50 each, and all the groups are consistent with each other. The probable errors are about ·04.

The second equation proves that there is very little connection, if any, between the value of H_c and the water vapour. The fourth equation shows that the water contents of the atmosphere are chiefly dependent on the temperature, no doubt on account of the greater capacity of warm air for moisture, but the general result is to show that, so far as the Continent is concerned, the water vapour plays a very small part in the mechanics of the atmosphere, and has but a trifling effect, if any, on the value of H_c.

APPENDIX.

The Standard Deviations of the Density of the Air from 1 to 13 Kilometres, and the Frequency of Occurrence of Deviations of Given Magnitude.

(Prepared as a reply to an inquiry addressed to the Meteorological Office in 1918.)

The values given in the tables have been obtained as follows :—

All available values from Pyrton Hill, Ditcham Park, and Benson have been collected, and after being arranged in four groups, have been treated by the usual statistical method to give the standard deviations of temperature and pressure for each group. The observations have been arranged in the civil rather than in the meteorological quarters, because the maxima and minima from 1 to 10 k. inclusive occur in February and August rather than in January and July, and also because most of the available observations refer to the first few days of the month.

There are two systematic errors which must be mentioned. The spring and autumn quarters are times of rapid change ; the standard deviations are, therefore, increased; the error so caused is about 1° for the temperature and 2 mb. for the pressure, but no correction has been applied.

Also a fair number of the observations were made purposely on days of exceptional barometric conditions, and this will largely increase the standard deviations found, but by how much it is impossible to say. The error will not occur in the summer quarter, because at that season exceptional barometric readings, either high or low, occur very seldom.

The means are not given, because means for each month that are more accurate on account of the elimination of some of the casual errors have already been obtained.

The standard deviations of temperature and pressure[1] have been deduced directly from the crude observations, and they are given in the Table on page 76.

The standard deviations of the density have been obtained from the Table; thus

we have $$D = k\frac{P}{T} \text{ where } D \text{ is the destiny,}$$

hence $$\frac{\delta D}{D} = \frac{\delta P}{P} - \frac{\delta T}{T}.$$

If we write d, p, and t for the percentage variations of the quantities, we have $d_1 = p_1 - t_1$, since the percentage variations are small, so that their squares and products may be neglected.

Hence $$\Sigma(d^2) = \Sigma(p^2) + \Sigma(t^2) - 2\Sigma(pt)$$

or $$\sigma_d{}^2 = \sigma_p{}^2 + \sigma_t{}^2 - 2r\sigma_p\sigma_t$$

where σ denotes the standard deviation and r is the correlation coefficient.

By this means the standard deviations of the density at each height are first obtained as percentage variations, and then, using the mean values, they are expressed in grammes per cubic metre.

The high value of the deviations for the last quarter is to some extent fortuitous, since many of the international days in that quarter happened to be days of large disturbance, and the low values in the summer are perhaps also fortuitous for the opposite reason. But the deviations are undoubtedly less in the summer than in the winter.

[1] The correlation between temperature and pressure is given on page 67.

The variations of temperature are distinctly less at about 10 k. than either above or below ; the variations of the actual pressure are about uniform with a value of 11 mb. from the ground to 9 k., above which they fall off rapidly.

The correlation from 4 to 8 k. is very high indeed (see p. 67), and the real values should exceed those given, because no correction has been made for the observational error. The value must reach ·90, with a standard error not exceeding ·02 for the year's mean. The percentage variation of the density is least from 3 to 6 k., where it hardly exceeds 1 per cent. It increases rapidly at 10 k., but probably falls off above 14 k. The actual variations are least from 5 to 8 k. They increase from 8 to 11 or 12 k., and then fall off again.

There are sufficient observations to show that the distribution is normal or approximately so. The formula for the standard error of the standard deviation, viz. $\sigma/\sqrt{2n}$, is not, however, applicable, because in many cases the observations are too near each other, both in time or place, to be independent, but even if we use n instead of $2n$, the error for the year as a whole is under 10 per cent. of the standard deviation.

Assuming the distribution to be normal and the standard deviation correct, the following are the odds against the given range of values being passed :—

Range.	Odds.
Mean $+ \cdot5\sigma$ to mean $- \cdot5\sigma$	1 to 1·6
,, $+1\cdot0\sigma$,, ,, $-1\cdot0\sigma$	1 ,, 3·2
,, $+1\cdot5\sigma$,, ,, $-1\cdot5\sigma$	1 ,, 7·5
,, $+2\cdot0\sigma$,, ,, $-2\cdot0\sigma$	1 ,, 22
,, $+2\cdot5\sigma$,, ,, $-2\cdot5\sigma$	1 ,, 80
,, $+3\cdot0\sigma$,, ,, $-3\cdot0\sigma$	1 ,, 372
,, $+3\cdot5\sigma$,, ,, $-3\cdot5\sigma$	1 ,, 2270
,, $+4\cdot0\sigma$,, ,, $-4\cdot0\sigma$	1 ,, 25000

or this may be put another way :—

The range $M \pm 1\cdot65\sigma$	includes	9	out of	10 cases
$M \pm 2\cdot58\sigma$	,,	99	,,	100 ,,
$M \pm 3\cdot33\sigma$	,,	999	,,	1000 ,,
$M \pm 3\cdot90\sigma$	,,	9999	,,	10000 ,,

	Standard Deviations in England, SE., of Temperature and Pressure at Different Levels.								Standard Deviations of the Density of Air at Different Levels.							
	Jan.–Mar.	Apr.–June.	July–Sept.	Oct.–Dec.	Jan.–Mar.	Apr.–June.	July–Sept.	Oct.–Dec.	Jan.–Mar.	Apr.–June.	July–Sept.	Oct.–Dec.	Jan.–Mar.	Apr.–June.	July–Sept.	Oct.–Dec.
k.	Temperature in °C.				Pressure in Millibars.				%.	%.	%.	%.	g/m³.	g/m³.	g/m³.	g/m³.
13	5·5	5·0	5·1	6·9	4·7	6·0	5·2	6·5	4·5	4·3	4·1	6·2	12	11	11	16
12	5·2	5·0	5·7	5·9	5·9	6·8	5·3	8·6	4·6	4·6	4·1	6·0	14	14	12	18
11	4·5	4·5	4·0	4·5	7·1	8·0	6·1	10·8	4·1	4·3	3·3	5·6	15	15	12	20
10	3·3	5·1	3·5	3·9	9·0	9·6	6·7	13·1	3·3	4·1	2·4	4·8	13	17	10	20
9	3·2	5·7	4·2	5·2	10·0	10·7	6·8	14·2	2·4	3·3	1·3	3·2	11	15	6	15
8	4·1	6·7	4·9	7·0	11·5	10·6	7·1	14·9	1·8	1·8	1·3	2·2	9	9	7	12
7	4·6	7·0	4·7	8·4	13·1	10·3	7·2	15·0	1·8	1·1	1·2	2·0	11	7	7	11
6	5·5	6·9	4·7	8·7	13·1	10·3	7·5	14·9	1·5	1·2	1·1	1·7	10	8	7	12
5	5·5	6·8	4·4	7·8	13·5	10·1	6·9	14·9	1·3	1·3	1·1	1·5	10	10	8	11
4	5·4	6·6	3·9	6·8	12·7	10·1	6·3	14·5	1·1	1·3	1·0	1·6	9	11	8	15
3	5·2	6·4	3·7	6·6	12·6	9·4	6·6	14·5	1·2	1·6	1·0	1·6	11	14	9	15
2	5·3	6·3	4·2	6·0	12·2	9·2	6·1	14·5	1·1	2·0	1·3	1·3	12	20	14	13
1	4·3	6·3	4·3	4·2	11·5	10·3	5·7	14·3	1·4	2·0	1·5	1·6	16	21	17	18
0	3·0	4·7	3·6	4·5	11·1	10·9	5·8	15·2	1·6	1·8	1·4	1·8	19	23	17	23

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