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Turbulent Motion and the General Stochastic Process

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Introduction Among the most flexible and useful techniques for modelling the dispersion of atmospheric pollutants over short to medium range, the random walk, or Markov process, takes pride of place. The technique (Hall, 1975, Reid 1979, Ley 1982) has now been extended to diabatic and inhomogeneous conditions (Wilson et al 1981, Legg and Raupach 1982, Ley and Thomson 1983, Thomson 1984a) and to three dimensions (Thomson 1984b). So far, however, no attempt has been made to explore the possibility of using other stochastic models as a basis for simulation - the Markov process is the simplest member of an extensive family. It is the purpose of this paper simply to introduce alternative stochastic processes and to try to identify the areas in which they may be relevant.

1. The general stochastic process Stochastic time series models fall into three main categories:

(a) Autoregressive (AR) processes

$$Z_t = \sum_i \phi_i Z_{t-i} + b_t \qquad (1)$$

where the ϕ_i (<1) are coefficients and b_t an innovation at time t.
The 1st order AR process is the Markov process

$$Z_t = \phi Z_{t-1} + b_t \qquad (2)$$

often called a random walk, although strictly a random walk is a Markov process for which $\phi \equiv 1$.

(b) Moving Average (MA) Processes

$$Z_t = b_t - \sum_i \theta_i b_{t-i} \qquad (3)$$

That is, Z is composed of a current innovation plus a linear combination of earlier innovations, which are partially 'discounted' by the θ_i $(<1.)$ coefficients.

(c) Mixed (ARMA) Processes

Simply a combination of types (a) and (b):

$$Z_t = \sum_i \phi_i Z_{t-i} + b_t - \sum_j \theta_j b_{t-j} \qquad (4)$$

Thus the 1st order mixed process is

$$Z_t = \phi Z_{t-1} + b_t - \theta b_{t-1} \qquad (5)$$

2. Integrated Models (ARIMA)

The above models are properly applied only to stationary data. If a time series is non-stationary it must be differenced to form a new series

$$Y_t = Z_t - Z_{t-1} = \nabla . Z_t \quad \text{for all } t, \qquad (6)$$

which normally produces a close approach to stationarity. If not, the differencing operation can be repeated until stationarity is obtained, e.g.

$$X_t = \nabla Y_t = \nabla^2 Z_t \qquad (7)$$

To restore the Z values, these differenced series must be integrated -hence the name. ∇ is the backward difference operator.

3. Notation

A convenient and conventional representation of a stationary stochastic process is (N,M,P) where N is the order of the AR process

M " " " " " MA "

P " " number of differencings.

Thus we express the Markov process (2) (1,0,0)

and the 1st order mixed process (5) (1,0,1)

For a comprehensive account of the ARIMA processes and their applications see Box and Jenkins (1976).

Do higher order AR, differenced, and MA models have any relevance to the simulation of turbulence? In the following sections the problem is approached heuristically (and later, empirically); the mathematics is practical rather than stringent.

4. Higher order autoregressive models The Markov process implies a single constraint on the evolution of a stationary time series of Lagrangian turbulent velocity measurements. That is, the inertia of the moving particle represented by ϕ in

$$V_{t+1} = \phi V_t + \mu_{t+1} \qquad (8)$$

This is a finite difference form of the Langevin equation

$$\frac{dv}{dt} = -\frac{1}{\tau} v + \varepsilon \qquad (9)$$

(τ a time scale) which integrated over a time interval δt gives

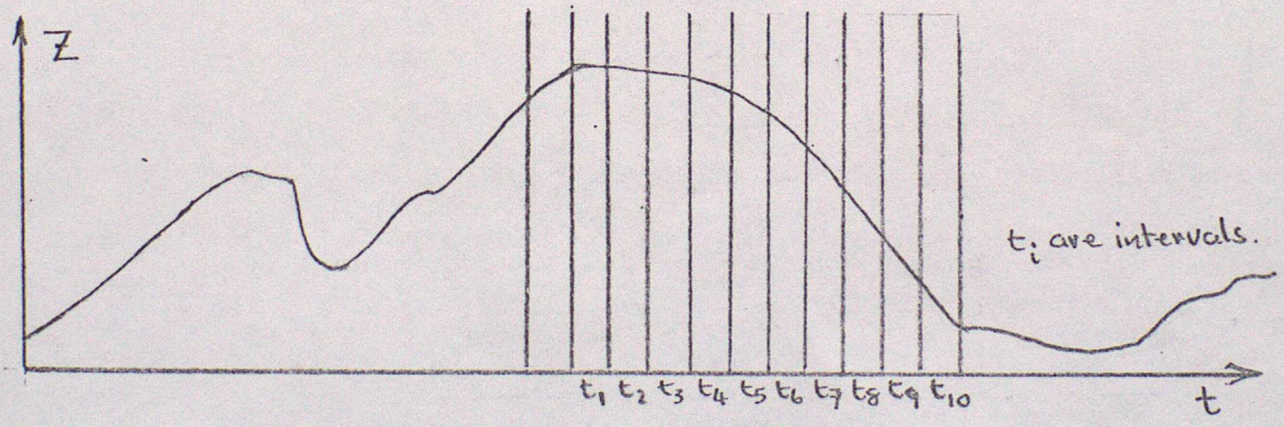
$$v(t+\delta t) - v(t) = -\frac{\delta t}{\tau} v(t) + \int_t^{t+\delta t} \varepsilon \, dt$$

yielding (8) when

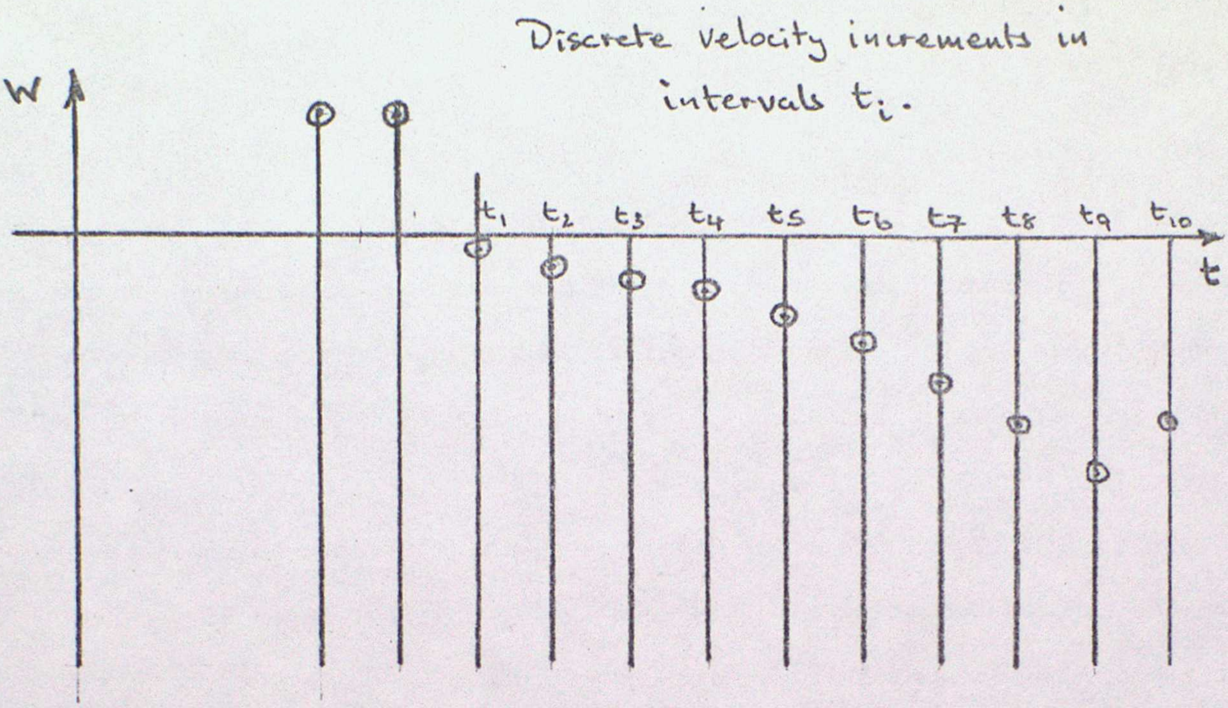
$$\mu_{t+1} = \int_t^{t+\delta t} \varepsilon \, dt \qquad (10)$$

ε is a white noise process, a random acceleration, analogous to molecular theory. Thus $\overline{\varepsilon(t)\,\varepsilon(t+t')} = \sigma^2(t)\,\delta_{t,t'}$ where $\delta_{t,t'}$ is the Kronecher delta; by assumption the successive values of ε are uncorrelated, and the rate of change between successive values is infinite. μ_t is accordingly also a random sequence.

Equation (9) seems to have been used exclusively (and adequately) as a basis for stochastic models of atmospheric dispersion, despite certain physical shortcomings. Consider a typical parcel motion, in the vertical:



There is continuity or at least sectional continuity. Over short time intervals the velocities are correlated, but the accelerations are by no means always random, especially in the lower frequency motions of large eddies. Thus between t_1 and t_9 –



This is not simply a case of $w(t_7)$ depending upon $w(t_6)$ plus a random shock. There is a trend in the velocity over successive time intervals so that $w(t_8) - w(t_7)$ is related to $w(t_7) - w(t_6)$.

Combining the two phsyical constraints in the form

$$w_{t+1} = \alpha_1 w_t$$

$$\left.\frac{\partial w}{\partial t}\right|_{t+1} = \alpha_2 \left.\frac{\partial w}{\partial t}\right|_{t} + \varepsilon_t$$

corresponds to a second-order differential equation

$$\frac{\partial^2 w}{\partial t^2} = \frac{A}{\tau}\frac{\partial w}{\partial t} + \frac{B}{\tau^2} w + \xi \tag{11}$$

where ξ is an innovation of dimension LT^{-3} (a second order derivative of velocity) or the equivalent stochastic process

$$w_{t+1} = \phi_1 w_t + \phi_2 w_{t-1} + \mu_{t+1} \tag{12}$$

where $\mu_{t+1} = \delta t \int_t^{t+\delta t} \xi \, dt$, δt the time interval between the t_i,

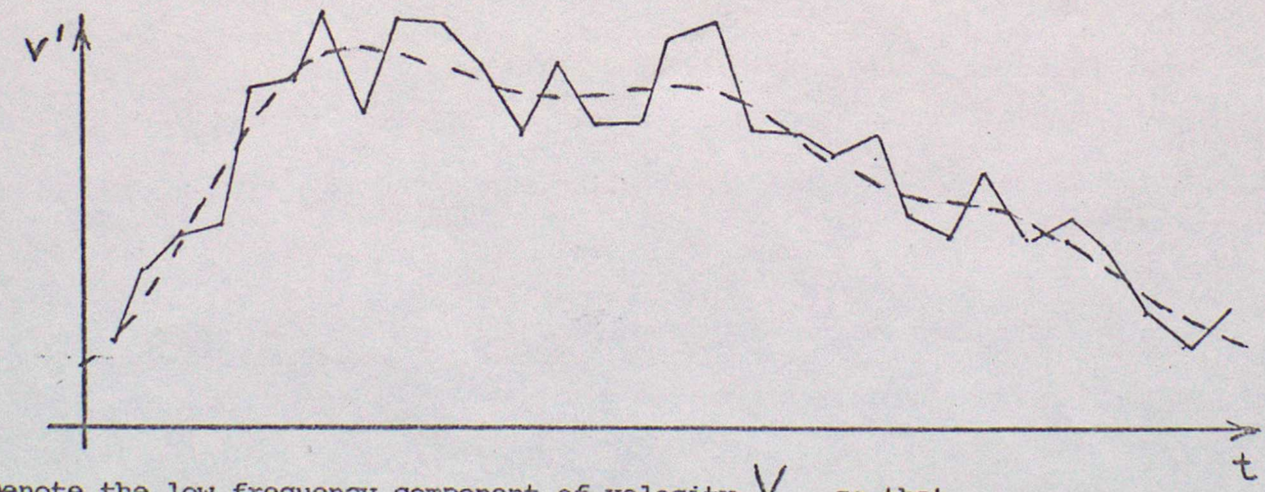
or $\mu_{t+1} = \delta t \, \varepsilon_{t+1}$ if ε_{t+1} is defined $\int_t^{t+\delta t} \xi \, dt$.

This line of reasoning may be applied to higher orders so that, in general, an autoregressive process of order N corresponds to an Nth order linear homogeneous stochastic differential equation: that is, to a turbulent flow in which there are N independent physical constraints on the inertia. The random innovation will be a derivative of velocity of the same order as the equation.

In conclusion, higher order models are, strictly, more appropriate than the Langevin equation when $\delta t \ll T_L(z)$, the Lagrangian timescale, but improvements stemming from their use may not compensate for the added complexity.

5. Integrated processes

Consider a Markov process $\frac{dv}{dt} = -\frac{1}{\tau} v + \varepsilon$, which is 'embroidered' on a low frequency (and in general non-stationary) fluctuation - meso-scale, perhaps. Let the Eulerian velocity at time t be v' :



Denote the low frequency component of velocity V , so that

$$v' = V + v \tag{13}$$

and make the assumption that over a short interval

$$\frac{\partial V}{\partial t} = \text{constant}$$

From (13), and the expression for a Markov process,

$$\frac{dv'}{dt} = \frac{\partial V}{\partial t} - \frac{1}{\tau} v + \varepsilon$$

so that

$$\frac{d^2 v'}{dt^2} = \frac{\partial^2 v}{\partial t^2} = -\frac{1}{\tau}\frac{\partial v}{\partial t} + \frac{\partial \varepsilon}{\partial t} \tag{14}$$

assuming ε is differentiable in some sense. (14) is a stochastic differential equation. To obtain a stochastic process it is necessary to integrate (14) over an interval δt. The last term becomes

$$\int_t^{t+\delta t} \partial \varepsilon = \varepsilon_{t+\delta t} - \varepsilon_t .$$

This can represent only 'snapshots' of the random innovation sequence, not its integrated effect. Accordingly (14) cannot be represented by a finite difference formula unless the white noise is assumed at a higher order of velocity and a piecewise ε_{t+1} is defined $\int_t^{t+\delta t} \xi \, dt$ as before. Then the (1,1,0) integrated autoregressive process

$$\nabla v_{t+1} = \phi \nabla v_t + \mu_{t+1}$$

where $\phi = 1 - \frac{\delta t}{\tau}$, $\mu_{t+1} = \delta t \nabla \varepsilon_{t+1}$, which can be expressed

$$\frac{\nabla^2 v}{(\delta t)^2} = -\frac{1}{\tau}\frac{\nabla v}{\delta t} + \frac{\nabla \varepsilon}{\delta t}$$

is analogous to (but does not precisely represent) the differential equation (14). Differencing to produce stationarity in a time series is equivalent to differentiating to remove steady (constant) low order accelerations, such as $\frac{\partial V}{\partial t}$. The diagram implies a large spectral gap which need be nothing like as marked as represented, for these considerations to apply.

Extension to higher orders is straightforward: if the rate of change of $\frac{\partial V}{\partial t}$ is constant in an interval the series can be differenced twice to derive the stationary stochastic process (1,2,0)

$$\nabla^2 V_{t+1} = \phi \nabla^2 V_t + \mu_{t+1}$$
$$\phi = 1 - \frac{\delta t}{\tau}, \quad \nabla^2 V = \nabla(\nabla V)$$
$$\mu_{t+1} = \delta t \nabla^2 \varepsilon_{t+1}$$

analogous to the stochastic differential equation

$$\frac{\partial^3 V}{\partial t^3} = -\frac{1}{\tau}\frac{\partial^2 V}{\partial t^2} + \frac{\partial^2 \varepsilon}{\partial t^2}.$$

Thus an integrated autoregressive process (N,M,0) is a difference equation (which is analogous to a stochastic differential equation) of order N+M which is truncated of the last M terms in the dependent variable, while the innovation (equivalently) is differenced to order M. The difference equation can only be used to represent the differential equation in modified form, that is when the ε_t are assumed piecewise averages of higher order white noise.

For integrated processes, there is no way around this restriction. Thus replacing $\partial\varepsilon/\partial t$ in (14) with a high frequency second order derivative of velocity ς gives

$$\frac{\partial^2 V}{\partial t^2} = -\frac{1}{\tau}\frac{\partial V}{\partial t} + \varsigma$$

which on the face of it integrates easily to give a stochastic process. However, the process is a non-physical one in which the time series velocities are differenced but not the random component of the velocities.

6. Moving average models The moving average processes (N,M,P) include a 'history' of random innovations which are weighted and integrated to form a current 'shock': $\sum_{n=0}^{P} \theta_{t-n} b_{t-n}$, $\theta_t \equiv 1$. Many geophysical time series are modelled well by the IMA process (0,M,P) particularly where there is substantial inertia in the system. The involvement of earlier innovations even in (N,0,P) process has a similar effect to the differencing of the preceding section - it restricts the extent to which the stochastic process can be used to represent a stochastic differential equation.

The standard moving average differential equation

$$\frac{\partial v}{\partial t} = -\frac{1}{\tau} v + \int_0^\infty \theta(t') \xi(t-t') dt'$$

cannot satisfactorily be converted to (1,0,P) stochastic form, and it is not obvious that in turbulent atmospheric motion, P can much exceed N. For present purposes it will be assessed that N $\geqslant$ P. The (1,0,1) process can be expressed in finite difference form

$$\frac{\nabla v_{t+1}}{\delta t} = -\frac{1}{\tau_1} v_t + \frac{\nabla \mu_{t+1}}{\delta t} - \frac{1}{\tau_2} \mu_t \qquad (16)$$

where $\mu_{t+1} = \int_t^{t+\delta t} \varepsilon \, dt$ the τ_i appropriate time scales.

Expression (16) is attractive as it reflects both the (1,0,1) symmetry and the physics, and, generalised to any order, allows the simple rule:

A stochastic process (N,M,P) in velocity, $N \geqslant P$, is a difference equation of order N+M for velocity, order P+M for innovation (the impulse being a derivative of velocity of order N-P), each of which is truncated of the last M terms.

$$\left.\begin{array}{ll} \text{Thus for N=P use} & \mu_{t+1} = \displaystyle\int_t^{t+\delta t} \varepsilon \, dt \\ \text{N-P=1 use} & \varepsilon_{t+1} = \displaystyle\int_t^{t+\delta t} \xi \, dt \end{array}\right\}$$

μ a velocity,

ε an acceleration

ξ a rate of change of acceleration

7. <u>Further Considerations</u> The situations discussed so far are those in which the random innovation is piecewise averaged over a finite interval δt. An alternative, if unorthodox approach, is to employ a running mean of the innovations so that at time t the 'random' component of velocity is expressed as the linear average of a number of adjacent (past and future) values: write

$$\hat{\mu}_t^{n+1} = \frac{1}{n+1} \sum_{p=t-n/2}^{t+n/2} \mu_p$$

Although resultant functions would no longer be 'non-anticipating' (see Durbin, 1983 for a precise definition) in that future values are required, this would not matter for purposes of simulation, and the method has the advantage that the random component is now quasi-continuous, so that any process could be expressed as a linear homogeneous differential equation in two smooth and differentiable variables. Such equations would have analogous finite difference forms, which could not be regarded as pure stochastic processes as the latter, by definition, require independent random innovations.

In support of this suggestion recall that air is a weakly viscous continuum, and it is as realistic to assume that many forcings vary in a quasi-continuous manner as to assume that they are all instantaneous shocks.

There are other interesting facets to the suggestion. If a broad-band moving average is applied to all the terms of the Markov process,

$$\hat{V}^{n+1}_{t+1} - \hat{V}^{n+1}_{t} = -\frac{\delta t}{\tau}\hat{V}^{n+1}_{t} + \hat{\mu}^{n+1}_{t+1}$$

(allowing (t+1)-t = δt, the interval between successive shocks), and if a value of n exists sufficiently small for $\hat{\mu}^{n+1}_{t+1} \not\simeq 0$ but sufficiently large such that $\hat{V}^{n+1}_{t+1} \simeq \hat{V}^{n+1}_{t}$, the resultant series reduces to the approximation

$$\hat{V}^{n+1} \simeq \frac{\tau}{\delta t}\hat{\mu}^{n+1}. \qquad (17)$$

The likelihood of a suitable n existing depends, presumably, upon the nature of the process generating the time series. For an nth-order AR process (17) becomes $\hat{V}^{n+1} \simeq \left(\tau_n/\delta t\right)^n \hat{\mu}^{n+1}$. Similar simple approximations can be obtained for equations analogous to other ARMA (but not ARIMA) processes.

8. Sample turbulence time series In 1982 the Boundary Layer Branch of the Meteorological Office mounted a field experiment at Blashaval, North Uist, to study the mean flow and turbulence over a fairly smooth, conical hill (Mason and King, 1984). Measurements were also made on an adjacent level site (upstream) and some of these data are here fitted with (N,M,P) stochastic processes. Two immediate difficulties arise - the measurements

are Eulerian, not Lagrangian, and it proved feasible to fit only relatively short lengths of the data. Nonetheless, the results are not without interest, and are reproduced in Tables 2 and 3.

The measurements used were 17HZ vertical (w') and crosswind (v') turbulence components recorded at 3m and 14m above ground on 29th Sept 1982. Sections of 500 values, either of the unprocessed 17HZ readings, or of piecewise averages of the same, were utilised. The broader the average, the more data is required to yield 500 values, so direct intercomparisons cannot easily be made. For one or two of the fits, 1000 values of 17HZ data were used.

The (N,M,P) stochastic models fitted were restricted to those for which $N \leq 2$, $P \leq 2$, $M \leq 1$. Lon-Mu Liu's Box-Jenkins algorithms in the Biomedical statistical computer package (1981) were used: these did not converge in all cases so that every possibility could not be covered. Two tests were applied to obtain a best fit:

(i) that the mean square error of the residuals after fitting should be as small as possible, and

(ii) that there should be minimal lag correlation in the residual time series: strong residual correlation is a sure diagnosis of an incorrect or inadequate model.

Allowance must be made for the fact that high order processes will inevitably provide better fits than low order (the Markov process is of the lowest order). No attempt has been made to establish the statistical significance of the fitted parameters - the exercise is simply one of intercomparison.

The w-component, Table 2 Little improvement can be obtained over the Markov process for the 17HZ data, at either level. For piecewise averaged data (up to about 3 secs time-average) the (1,1,1) process (see Table 1) becomes more appropriate, and shows striking improvements over the Markov process at 14m. Evidently the piecewise meaned time series may well be non-stationary, and the innovations at consecutive intervals are not independent.

The v-component, Table 3 This table is not so easily summarised. The Markov process is occasionally adequate but many of the residual lag correlations are rather large. An improvement of over 5% in the residual mean square error can be made quite often, and the correlation structure greatly improved, at the expense of using slightly more complicated models. Note that occasionally very different stochastic models can give not only similar residual errors, but even a parallel correlation structure in the residuals. This may be due in part to the 'real' process being properly of an even higher order, so that large residual correlations occur in the lag 3-5 range.

At 3m there is a tendency for higher order inertia terms in the 17HZ data to be replaced by higher order innovation terms when the data are averaged. At 14m it can only be said that the 17HZ models tend to have parameters of lower order. The (1,1,1) process again scores well at both levels for time-averaged data.

9. Summary The general autoregressive process (N,0,0) corresponds to an Nth order linear, homogeneous, stochastic differential equation; that is, to a turbulent flow with N independent physical constraints upon the inertia. It is in principle suited to simulations where the timestep is considerably smaller than the turbulence time scale.

Exponents of Langevin random walk models argue that velocity fluctuations are correlated over much longer periods than fluctuations in acceleration (which are assumed uncorrelated). For non-stationary series, however, a slowly fluctuating acceleration is superimposed. Integrated processes are non-stationary, and may have some potential for simulating atmospheric motions with irregular low-frequency oscillations, such as often occur with the horizontal turbulence components. The differencing of time series is equivalent to removing constant accelerations by differentiation.

Moving average processes allow representation of a history of accumulated innovation, which may constitute an input to the evolving eddy dynamics.

The general ARIMA process, (N,M,P), yields difference equations according to the rule in Section 6. Unless M=P=0, the difference equations can only model stochastic differential equations, in a non-rigorous fashion, when the white noise input is integrated over a timestep to produce a piecewise average of appropriate dimensionality. Difference equations are best considered as having potential for use in their own right, to approximate atmospheric turbulence in a finite difference form which is not necessarily stringently reducible to a differential equation. Some examples of the difference/differential equations corresponding to stochastic process are given in Table 1, in which the notation follows that of earlier sections.

Fitting the observed turbulence data at Blashaval with stochastic time series models suggested that (for the very restricted range of conditions studied) the Markov process provides a reasonable first approximation. It can, in principle, be improved upon - any stochastic model can be used to simulate a turbulent process in the same way as the Markov process, and integrated (ARIMA) models clearly have relevance to the non-stationary situations experienced in reality. The feasibility of practical application, however, is not explored here.

Acknowledgement Thanks are due to D J Thomson for discussion and criticism.

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Process	Difference or differential equation
(1,0,0) (Markov)	$\frac{\partial v}{\partial t} = -\frac{1}{\tau} v + \varepsilon$ (Langevin)
(1,1,0)	$\frac{\nabla^2 v}{(\delta t)^2} = -\frac{1}{\tau}\frac{\nabla v}{\delta t} + \frac{\nabla \varepsilon}{\delta t}$
(1,0,1)	$\frac{\nabla v}{\delta t} = -\frac{1}{\tau_1} v + \frac{\nabla \mu}{\delta t} - \frac{1}{\tau_2}\mu.$
(1,1,1)	$\frac{\nabla^2 v}{(\delta t)^2} = -\frac{1}{\tau_1}\frac{\nabla v}{\delta t} + \frac{\nabla^2 \mu}{(\delta t)^2} - \frac{1}{\tau_2}\frac{\nabla \mu}{\delta t}$
(2,0,0)	$\frac{\partial^2 v}{\partial t^2} = -\frac{1}{\tau_1}\frac{\partial v}{\partial t} - \frac{1}{\tau_2^2} v + \xi$
(2,0,1)	$\frac{\nabla^2 v}{(\delta t)^2} = -\frac{1}{\tau_1}\frac{\nabla v}{\delta t} - \frac{1}{\tau_2^2} v + \frac{\nabla \varepsilon}{\delta t} - \frac{1}{\tau_3}\varepsilon$
(1,2,0)	$\frac{\nabla^3 v}{(\delta t)^3} = -\frac{1}{\tau}\frac{\nabla^2 v}{(\delta t)^2} + \frac{\nabla^2 \varepsilon}{(\delta t)^2}$
(2,1,1)	$\frac{\nabla^3 v}{(\delta t)^3} = -\frac{1}{\tau_1}\frac{\nabla^2 v}{(\delta t)^2} - \frac{1}{\tau_2^2}\frac{\nabla v}{\delta t} + \frac{\nabla^2 \varepsilon}{(\delta t)^2} - \frac{1}{\tau_3}\frac{\nabla \varepsilon}{\delta t}$
(2,1,0)	$\frac{\nabla^3 v}{(\delta t)^3} = -\frac{1}{\tau_1}\frac{\nabla^2 v}{(\delta t)^2} - \frac{1}{\tau_2^2}\frac{\nabla v}{\delta t} + \frac{\nabla \xi}{\delta t}$
(2,0,2)	$\frac{\nabla^2 v}{(\delta t)^2} = -\frac{1}{\tau_1}\frac{\nabla v}{\delta t} - \frac{1}{\tau_2^2} v + \frac{\nabla^2 \mu}{(\delta t)^2} - \frac{1}{\tau_3}\frac{\nabla \mu}{\delta t} - \frac{1}{\tau_4^2}\mu$

Table 1. The correspondence between some simple stochastic processes (N,M,P) and difference or differential equations. The difference equations are left in a form which is analogous to differential equations. The τ_i are timescales.

DATA	MARKOV PROCESS (1,0,0)						'BEST' PROCESSES							Mean square: % improvement	
	Residual Mean square:	Residual Lag Correlation 1	2	3	4	5	Type:-	Residual Mean Square	Residual Lag correlation 1	2	3	4	5	over (1,0,0)	
3 metres															
17HZ, values 1-500	.0150	.09	-.21	-.12	-.07	.01	(2,0,0)	.0144	.07	-.05	-.13	-.14	-.03	4.0	(0,1,2) equivalent
							(1,0,1)	.0146	-.07	-.15	-.05	-.05	.04	2.7	
17HZ, values 500-1000	.0250	.13	-.16	-.10	-.05	-.01	(1,0,1)	.0241	-.05	-.10	-.05	-.02	.01	3.6	
Means of 10 values (500)	.0482	-.12	.02	-.01	-.01	.01	(1,1,1)	.0464	-.04	.06	.01	.01	.02	3.7	
Means of 30 values (500)	.0420	-.10	.0	.01	-.03	.03	(1,1,1)	.0399	-.02	.04	.02	-.03	.03	5.0	
Means of 50 values (500)	.0375	-.11	-.06	-.05	.0	-.01	(1,1,1)	.0346	.01	-.02	-.03	.01	.0	7.7	
14 Metres															
17HZ, Values 1-500	.0079	.07	-.13	.0	.02	-.02	(2,0,0)	.0078	.08	-.01	.0	.01	-.02	1.3	
							(2,1,0)	.0078	.0	.0	.0	.02	-.03	1.3	(1,2,0) equivalent
Means of 10 values (500)	.0279	-.15	-.12	-.03	-.14	.06	(1,1,1)	.0243	-.02	-.03	.03	-.08	.08	12.9	
Means of 30 values (500)	.0460	-.24	-.14	-.13	-.04	.11	(1,1,1)	.0338	.02	-.03	-.08	-.02	.10	26.5	
Means of 50 values (500)	.0459	-.32	-.15	.0	.06	-.09	(1,1,1)	.0328	-.02	-.02	.06	.08	-.05	28.5	

Table 2. The stochastic processes best fitting the Blashaval turbulence data with diagnostic statistics: w-component. Residual lag correlations $\geqslant$ 0.1 are underlined.

DATA	MARKOV PROCESS (1,0,0) Residual Mean Square	Residual Lag Correlation 1	2	3	4	5	BEST PROCESS Type:-	Residual Mean Square	Residual Lag Correlation 1	2	3	4	5	Mean Square: % improvement over (1,0,0)	
3 metres															
17HZ, values 1-500	.0126	.08	-.18	-.06	.06	-.02	(2,0,0)	.0122	.08	-.06	-.06	.03	-.03	3.2	
							(2,1,0)	.0121	.0	.01	-.06	.03	-.05	4.0	(0,1,2) almost
17HZ, values 500-1000	.0273	.15	-.14	-.15	.06	.04	(2,1,0)	.0263	.0	.01	-.14	.05	.02	3.7	equivalent
							(0,1,2)	.0262	.0	-.01	-.14	.06	.02	4.0	
17HZ, values 500-1500	.0238	.16	-.12	-.07	.0	.0	(1,0,1)	.0230	-.03	-.10	-.05	.01	.01	3.4	
Means of 10 values (500)	.1097	-.23	-.03	.0	-.03	-.06	(1,0,1)	.1033	.0	-.04	-.02	-.05	-.07	5.8	
							(0,1,1)	.1035	.01	-.04	-.02	-.06	-.07	5.7	
Means of 30 values (500)	.1345	-.15	-.12	-.10	-.02	-.04	(1,1,1)	.1175	.0	-.01	-.02	.03	.0	12.6	
Means of 50 values (500)	.1850	-.13	-.15	-.11	.01	-.05	(0,1,2)	.1758	.0	.01	-.16	-.04	-.08	5.0	
14 metres															
17HZ, values 1-500	.0089	.28	.02	-.03	-.01	-.10	(1,1,0)	.0082	.02	-.05	-.04	.03	-.08	7.9	
							(0,1,1)	.0082	.01	.03	-.05	.03	-.08	7.9	
17HZ, values 500-1500	.0058	.23	-.03	-.04	-.04	-.09	(0,1,1)	.0054	.0	-.02	-.04	-.02	-.07	6.9	
							(1,0,1)	.0054	-.03	-.01	-.04	-.01	-.06	6.9	
Means of 10 values (500)	.0419	-.07	-.16	-.04	-.05	-.04	(2,1,0)	.0402	.0	-.02	-.07	-.09	-.05	4.1	
							(2,0,0)	.0407	-.09	-.02	-.06	-.07	-.04	2.9	
Means of 30 values (500)	.0834	-.13	-.06	-.09	-.11	-.01	(1,1,1)	.0746	.0	.03	-.02	-.05	.02	10.6	
Means of 50 values (500)	.1122	-.03	-.19	-.02	-.14	-.04	(2,0,0)	.1089	-.04	-.04	-.03	-.18	-.04	2.9	
							(0,1,2)	.1067	.0	.05	-.05	-.14	-.05	4.9	

Table 3: As Table 2: v-component.