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A PROPOSAL FOR ASSIMILATING DETAILED AIRCRAFT WIND DATA IN A LOCAL AREA

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ABSTRACT

A method is proposed for assimilating detailed wind observations transmitted from a distribution of aircraft equipped with a datalink. The method is based on optimum interpolation in space and time using background error covariance functions that incorporate some of the effects of dynamical advection. In addition to the usual estimation of the analysis state, a provision is made for the estimation of the statistical covariances themselves using the same aircraft observations. A tentative assessment of the likely improvements in the accuracy of aircraft flight time estimates is made in a variety of simulated conditions using a simple representation of the effects of a small cluster of observations from aircraft on inbound trajectories to the London Terminal Area.

1. INTRODUCTION

During the next decade several commercial aircraft will be equipped with transponders capable of automatically relaying digital information to air traffic control concerning winds and air temperatures at the aircraft positions. It is obviously beneficial, both to controllers and to airlines, to exploit this new source of information once it becomes available. For example, one might maintain a frequently updated "nowcast" of winds, either at regularly spaced grid points of a space-time array or at a selection of points along anticipated future flight paths. In this way it would be possible to translate the enhanced detail in the wind analysis into improved precision of the estimated aircraft arrival times, to the benefit of controllers and operators.

The aim of this project is to provide a satisfactory method of exploiting the very detailed series of wind measurements expected to become available in the early 1990's so that air-traffic control can make more reliable estimates of the times of flight of incoming aircraft at the major control centres. The proposed model is statistical rather than dynamical, combining the information from the new observations and an earlier forecast using "optimum interpolation" (Gandin, 1963) to provide estimates of the wind at points along the anticipated flight tracks. The choice of a statistical framework is favoured by the need to provide the information frequently and in a timely manner - a complete high-resolution dynamical model would be inordinately demanding in computing resources given the frequency (e.g., hourly) at which the numerical product would need to be updated. It should be emphasised that the use of a statistical framework does not preclude the presence of at least some dynamical content, such as the effect of the large scale advection of the detailed wind structure. Deficiencies and omissions in modelled dynamics are expected to be relatively insignificant over the very short periods covered by the proposed scheme.

It is proposed that the framework of optimum interpolation (OI) be augmented by a further statistical step of "likelihood" or "Bayesian" estimation of the statistical covariances themselves. In this process the covariances for use in the OI are themselves parametrised and made the objects of an estimation procedure, relying on the same set of aircraft observations. This procedure involves an iterative match of the actual observations with the covariance parameters that most plausibly explain their distribution of values. In this way the present vagueness concerning the anticipated spatial structure of the background error covariances will not substantially hinder the application of a statistically based method of assimilation when the observations do become available in sufficient quantities for a detailed analysis to be realistically attempted.

The next section surveys the basic theory of OI. Section 3 briefly discusses the relationship between analysis error covariances and the expected timing errors of aircraft in flight within the domain. Section 4 discusses the proposed method used to model the spatial and temporal covariances of background error. The technique of likelihood estimation and its Bayesian generalisation are introduced in section 5. Numerical experiments designed to assess the quality of the analyses under various conditions are presented in section 6 in order to provide, at least crudely, a notion of the kind of gains of accuracy of aircraft timings that should be possible using the new data. Section 7 discusses the conclusions that might be drawn from these experiments regarding the design of a practical analysis system.

2. DATA ASSIMILATION THEORY

It is assumed that at sufficiently frequent intervals a recent numerical forecast of winds is available, a so-called "background" field, and that there are interpolation procedures enabling the corresponding values to be extracted at any given point and time within the domain and time-span of interest. It will be convenient to introduce an index notation to distinguish the points where new observations are taken (whether they originate from aircraft or from the regular, but infrequent, radiosonde ascents) from the points where it is deemed necessary to make a prediction. For the former (observation points) I shall use indices α, β, γ, etc., and for the latter (analysis points) I shall use i,j,k, etc. The components of the background wind field interpolated to the observation positions formally constitutes a vector whose components will be denoted B_α while the values of the background field at the location where an analysis is to be made will be denoted B_i.

It is assumed in general that these values are in error (relative to a hypothetical "true" value that is never known precisely) by amounts B'_α and B'_i respectively. The observations themselves will be denoted O_α and their errors O'_α, while the "optimal analysis" values at either the observation points α or the analysis points i will be denoted A_α or A_i accordingly with corresponding error components A'_α, A'_i. At this point it is as well to note that the values used here are interpreted as being local space and time averages of the wind components. This simplifying assumption allows the quantity of observational data to be reduced to manageable proportions without a significant loss of the information relevant to predicting flight times. Accompanying this averaging is a corresponding augmentation of the presumed measurement error variances to allow for the residual lack of representativity of each averaged observation.

Provided the observations and background are statistically unbiased and independent of each other, the optimal linear combination of these sources of information to form estimates A is given by the matrix equation:

$$A_i = B_i + \sum_{\alpha,\beta} C_{i\beta} (C + E)^{-1}_{\alpha\beta} (O - B)_\beta \quad , \qquad (1)$$

where matrices C and E are the covariances:

$$\left.\begin{aligned} E_{\alpha\beta} &= \langle O'_\alpha \, O'_\beta \rangle \quad , \\ C_{ij} &= \langle B'_i \, B'_j \rangle \quad , \\ C_{i\alpha} &= \langle B'_i \, B'_\alpha \rangle \quad , \\ C_{\alpha\beta} &= \langle B'_\alpha \, B'_\beta \rangle \quad . \end{aligned}\right\} \qquad (2)$$

and the angled brackets denote the expectation operator. The formula (1) is obtained by minimizing the expected squared error, $\langle A'_i\ A'_i \rangle$ at each point of the analysis, and is the standard formula of OI. The covariance model for observation errors is assumed to be very simple - a diagonal matrix E implying zero covariance between the errors of one observation and another (e.g., between u at s_α and u at s_β, $\alpha \neq \beta$, or between any u and any v). The covariance model C for the background errors demands much greater care in its formulation since it is known that such errors are smooth and therefore highly correlated from one point to another (especially when the points are close together). A more detailed discussion of the construction and verification of the covariances C will be presented in sections 4 and 5. Here it is merely noted that the presence of off-diagonal elements in C makes the inversion of the linear system in (1) a nontrivial problem for a large number N of observations (the effort being proportional to N^3 when the inversion is performed by a direct method). The theoretical covariance matrix of errors in the optimal analysis is given by the formula:

$$\hat{C}_{ij} = \langle A'_i\ A'_j \rangle = C_{ij} - \sum_{\alpha,\beta} C_{i\alpha}\ (C + E)^{-1}{}_{\alpha\beta}\ C_{\beta j} \quad . \tag{3}$$

When the observation-minus-background increments are combined linearly in some other (sub-optimal) way to form an analysis, e.g.,

$$A_i = B_i + \sum_{\alpha} W_{i\alpha}(O - B)_\alpha \quad , \tag{4}$$

then the covariance of analysis error takes the form,

$$\hat{C}_{ij} = \langle A_i' A_j' \rangle = C_{ij} - \sum_{\alpha}(C_{i\alpha}W^T{}_{\alpha j} + W_{i\alpha}C_{\alpha j}) + \sum_{\alpha,\beta} W_{i\alpha}(C + E)_{\alpha\beta}W^T{}_{\beta j} \quad . \tag{5}$$

Note that (3) is a special case of (5). It is on the basis of these final covariances that estimates will be made of the impact that the new aircraft data are likely to make on the accuracy of projected arrival times of the subsequent flights.

3. ESTIMATING AIRCRAFT FLIGHT TIMES

From the wind components along each anticipated flight-track it is possible to estimate the normal ground speed of the aircraft flying along it. Let this speed be U(s) at distance s along the track. Assuming the error in the ground speed is simply due to the error in tail wind component U'(s) the time of the entire flight segment,

$$T = \int \frac{1}{U(s)}\, ds \quad , \tag{6}$$

will be in error (to a very good approximation) by amount,

$$T' = -\int \frac{U'(s)}{U^2(s)}\, ds \quad . \tag{7}$$

Provided $|U'| \ll |U|$ the bias in timing error $\langle T' \rangle$ is negligible when the bias in U' is (as I shall continue to assume). The variance of error is then obtained from:

$$\langle T'T' \rangle = \iint \frac{\langle U'(s_1)U'(s_2) \rangle}{U^2(s_1)U^2(s_2)}\, ds_1\, ds_2 \quad , \tag{8}$$

the covariance in the integrand being immediately identified with the relevant components of $\hat{C}$ of the previous section.

4. THE BACKGROUND COVARIANCE MODEL

In formulating a model for the covariances of the background field it is necessary to reduce as far as possible the number of residual parameters. I shall assume that the horizontal wind is described in terms of a velocity potential χ and stream function ψ in a Cartesian coordinate system, $\underset{\sim}{x}$ = (x,y,z). Then

$$\left.\begin{aligned} u &= \frac{\partial\chi}{\partial x} - \frac{\partial\psi}{\partial y} \quad , \\ v &= \frac{\partial\psi}{\partial x} + \frac{\partial\chi}{\partial y} \quad , \end{aligned}\right\} \tag{9}$$

where the errors of χ and ψ are independent and possess horizontally isotropic covariance functions,

$$\left.\begin{aligned} \langle\chi'(\underset{\sim}{x}+\underset{\sim}{X})\chi'(\underset{\sim}{x})\rangle &\equiv C_{\chi\chi}(\underset{\sim}{X}) = f_\chi(|X^2+Y^2|^{1/2},Z) \quad , \\ \langle\psi'(\underset{\sim}{x}+\underset{\sim}{X})\psi'(\underset{\sim}{x})\rangle &\equiv C_{\psi\psi}(\underset{\sim}{X}) = f_\psi(|X^2+Y^2|^{1/2},Z) \quad , \end{aligned}\right\} \tag{10}$$

where $\underset{\sim}{X} = (X,Y,Z)$. For simultaneous observations the corresponding covariances for the components of the wind at points separated by displacement vector $\underset{\sim}{X}$ are,

$$\langle u'(\underset{\sim}{x}+\underset{\sim}{X})u'(\underset{\sim}{x})\rangle \equiv C_{uu}(\underset{\sim}{X}) = -\frac{\partial^2}{\partial x^2}C_{\chi\chi}(\underset{\sim}{X}) - \frac{\partial^2}{\partial y^2}C_{\psi\psi}(\underset{\sim}{X}) \quad , \tag{11a}$$

$$\langle u'(\underset{\sim}{x}+\underset{\sim}{X})v'(\underset{\sim}{x})\rangle \equiv C_{uv}(\underset{\sim}{X}) = -\frac{\partial^2}{\partial x\,\partial y}C_{\chi\chi}(\underset{\sim}{X}) + \frac{\partial^2}{\partial x\,\partial y}C_{\psi\psi}(\underset{\sim}{X}) \quad , \tag{11b}$$

$$\langle v'(\underset{\sim}{x}+\underset{\sim}{X})v'(\underset{\sim}{x})\rangle \equiv C_{vv}(\underset{\sim}{X}) = -\frac{\partial^2}{\partial y^2}C_{\chi\chi}(\underset{\sim}{X}) - \frac{\partial^2}{\partial x^2}C_{\psi\psi}(\underset{\sim}{X}) \quad . \tag{11c}$$

In order to deal with non-simultaneous events it is convenient to assume space-time covariances that can be factored into separate space and time parts. Taking (11a) as an example, then ignoring advective effects the easiest way to accomplish this is to replace it with,

$$C_{uu}(\underset{\sim}{x}_1,t_1;\underset{\sim}{x}_2,t_2) = G(t_1,t_2).H_{uu}(\underset{\sim}{x}_1 - \underset{\sim}{x}_2) \quad , \tag{12}$$

where function H_{uu} denotes the right hand side of (11a). A refinement that attempts to incorporate, at least to the crudest approximation, the effects one might expect for advection of error structures by the ambient wind field, is to substitute for the vector argument $(\underset{\sim}{x}_1-\underset{\sim}{x}_2)$ of the function H the "effective separation" $\underset{\sim}{X}$ computed according to some measure of the closeness of the trajectories that pass through events $(\mathbf{x}_1, t_1)$ and $(\mathbf{x}_2, t_2)$. For a uniform ambient wind field $\mathbf{U}_o$ the effective separation is unequivocally given by,

$$\mathbf{X} = \mathbf{x}_1 - \mathbf{x}_2 - (t_1 - t_2).\mathbf{U}_o \tag{13}$$

For a more general wind field with no single common value $\mathbf{U}_o$ it is not obvious how best to choose the effective displacement $\mathbf{X}$, but experiments using real fields and data should help to determine an adequate choice.

5. VERIFICATION AND TUNING OF THE STATISTICAL MODEL

If we allow that the statistical variation of the errors of the background field and of the observations may be adequately approximated by multivariate normal probability densities then the observational data may be regarded as a sample for verifying, and to some extent, tuning any adjustable parameters of this statistical model. One method that has been applied to meteorological problems is "generalised cross-validation" (e.g., Wahba and Wendelberger, 1980) which seems best suited to the estimation of a very small set (≤ 5) of parameters from observations of similar type and consistent quality. Another technique due to Hollingsworth and Lönnberg (1986) is more empirical but has nevertheless proved very successful in exploiting the large accumulation of statistics available from an operational global model to extract an appropriate partitioning of error between background and observations. The method proposed here is based on the evaluation and maximisation of the "likelihood" function implied by the observations. This function (or more conveniently, its logarithm) may be used to assess the relative degree of "support" (Edwards, 1972) given to a particular statistical model by the actual data whose variability the model attempts to explain. Numerically, the likelihood corresponds to the conditional probability density of the data observed given the statistical parameters, but, since as a function, its arguments are the statistical parameters, not the data, it is better thought of as being distinct from any particular probability function.

For the case presented here with normal statistics assumed, the log-likelihood is, apart from an arbitrary additive constant,

$$\ell(C,E) = \log\{\mathrm{Det}[(C+E)_{\alpha\beta}]^{-1/2} \exp[-\tfrac{1}{2} \sum_{\alpha,\beta} (O-B)_{\alpha}(C+E)^{-1}{}_{\alpha\beta}(O-B)_{\beta}]\} \quad ,$$

i.e.,

$$\ell(C,E) = -\tfrac{1}{2}\log \mathrm{Det}(C+E)_{\alpha\beta} - \tfrac{1}{2} \sum_{\alpha,\beta} (O-B)_{\alpha}(C+E)^{-1}{}_{\alpha\beta}(O-B)_{\beta} \quad . \tag{14}$$

The effort involved in the evaluation of ℓ is comparable with that required to perform the inversion (1) for the analysis. By varying the adjustable parameters of our statistical model, for example, changing the shapes of the functions G or H or changing the magnitudes of the variances $E_{\alpha\alpha}$, the corresponding change in the likelihood provides a criterion by which each model may be compared. In the absence of independent constraints or reasonable criteria for judging the overall merits of alternative models, the model which maximises the likelihood (i.e., enjoys most observational support) is consequently the obvious choice. As a further verification of the statistical assimilation, note that the quadratic form,

$$q = \sum_{\alpha,\beta} (O - B)_{\alpha}(C + E)^{-1}{}_{\alpha\beta}(O - B) \quad , \tag{15}$$

from the second term of (14) is expected to be distributed as a "Chi-squared" variable with N-degrees of freedom if the model chosen is the correct one.

6. SIMULATION STUDIES

In order to obtain some idea of the likely potential for improving the accuracy of timing estimates for aircraft flights this section is devoted to a simple simulation of the effects of a small network of observations assumed to originate from aircraft following flight paths inbound to the London Terminal Area. The geographical locations of the four flight tracks used are displayed in figure 1 and are numbered for easy reference. The small number at the start of each inbound track denotes the commencement time in minutes assumed for the respective aircraft in the model described here. Each flight terminates at one of the four stacks surrounding the airport. It is assumed that the inbound aircraft descend from flight level 300 (nominally 30000') to arrive at the stacks at flight level 150 (15000') with a uniform gradient of descent. The discrete observations closely spaced along these routes are used to predict the wind velocities at locations along future routes. These future locations constitute the "analysis" points and for this study four (1-4) of the eight simulated trajectories of analysis points are assumed to follow the same tracks as the observation flights (but at a later time), two more analysis trajectories (5, 6) are assumed to descend (again from flight level 300 to flight level 150) along radials not previously observed directly and, to complete the comparison, two further trajectories (7, 8) are created to be over-pass flights at flight level 300, all as shown in figure 2. Descending flights are assumed to occur at a true airspeed of 150 ms^{-1}, while the over-pass flights on routes 7 and 8 , distinguished in figure 2 as dashed lines, are assumed to have the true airspeed of 225 ms^{-1} typical of cruising flight.

The error covariances reflect the degree of confidence one has in the background state and will therefore presumably depend upon the synoptic situation and the time of day. At the finest scales, where the forecast used to produce the background field provides no information, one should expect the covariance structure to reflect the climatological variability. Again, this could be expected to depend upon the synoptic situation, but published studies of such climatological wind statistics (e.g., Gage and Nastrom, 1986) are not usually stratified comprehensively in terms of the ambient conditions. For practical purposes what is needed is a formulation of covariance functions that enables them to be computed efficiently and adapted easily when a better understanding of the actual quantitative nature of the error fluctuations of the background is gradually built up from experience with real data. For the present tests, the covariances for the stream function and for the velocity potential are created using linear superpositions of Gaussian functions in space. The ambient advecting wind used in each study is assumed for simplicity to be uniform both horizontally and vertically, so (13) is appropriate here. The time modulation of the covariances is assumed to follow the formula (12) with the factor G constant. It is realized that the justification for such a "frozen turbulence" assumption is questionable (e.g., Seaman, 1975), but given the other numerous uncertainties about wind variability and the preliminary nature of the present study, it would seem premature to consider any more sophisticated representation of the temporal variability at this stage. The amplitudes of the background wind variation at any single point are equivalent to a variance for each component of 12 m^2s^{-2}, or a standard deviation of approximately 7kts, which is probably a reasonable estimate of the errors one might see from a 12 hour forecast, although it should be emphasised that such estimates might themselves be subject to substantial fluctuation from day to day in operational practice. The variance of observational error for each component is taken in these experiments to be 9 m^2s^{-2}, or a standard deviation of approximately 6 kts. Since each "observation" is actually taken to represent an average wind measurement along a finite segment of the trajectory, a significant part of this variation is actually the unresolved spatial fluctuations of the true wind rather than the instrument's imprecision. Again, this figure is not to be taken as authoritative. In order to discretise the flights into observation points and analysis points I have simply taken, on each trajectory, points at the centres of the segments formed by dividing the entire track into eight equal portions.

For points with effective separation (X,Y,Z) the background covariances take the form,

$$\left.\begin{aligned} C_{xx} &= a_{x1}\exp\left[-\frac{(X^2+Y^2)}{2b_{x1}^2} - \frac{Z^2}{2b_{z1}^2}\right] + a_{x2}\exp\left[-\frac{(X^2+Y^2)}{2b_{x2}^2} - \frac{Z^2}{2b_{z2}^2}\right], \\ C_{\psi\psi} &= a_{\psi 1}\exp\left[-\frac{(X^2+Y^2)}{2b_{x1}^2} - \frac{Z^2}{2b_{z1}^2}\right] + a_{\psi 2}\exp\left[-\frac{(X^2+Y^2)}{2b_{x2}^2} - \frac{Z^2}{2b_{z2}^2}\right]. \end{aligned}\right\} \quad (16)$$

In each of a total of six experiments the following are defined:

(i) the set of covariance parameters of (16);

(ii) the advecting wind velocity, $\underset{\sim}{U}_o$;

(iii) the times at which each trajectory commences.

For each experiment a study is made of the consequences that follow from a variety of plausible assimilation methods. The most significant diagnostic for the present purposes is the standard deviation of the flight time estimates of each analysis trajectory that would be made on the basis of the particular method of assimilation. Also computed are the root-mean-square (r.m.s.) wind component estimates along analysis tracks for the case of the optimal analysis and for the case of the background alone.

In all, seven distinct methods of employing the available data are investigated for each combination of parameters (i), (ii), (iii) above. The first, method A, is to use the unapproximated OI formula with the correct space-time covariance function [i.e., including the full effects of advection as modelled by (13)]. The resulting standard deviation of timing estimates in seconds, τ_A, is computed from (3) and (8) for each of the eight routes. At the other extreme, method B simply takes the background itself as analysis and completely ignores the observations. The resulting timing standard deviations, τ_B , are clearly expected to be larger than the corresponding τ_A and the comparison of the two values provides a direct measure of the information to be gained about timing estimates from an optimal use of the aircraft data available from the observing flights. Other practical, but possibly sub-optimal, methods of using observational data would normally be expected to lead to values τ intermediate between τ_A and τ_B. In the third method, C, it is assumed that all available observations are utilised but with an OI formula that ignores the advective effects of the ambient wind field $\underset{\sim}{U}_o$, the spatial form of the covariances being otherwise unaltered. Thus, the weights in the analysis are actually sub-optimal and consequently (5) is used to compute the analysis covariance elements and hence the standard deviation, τ_C.

In the next two methods an attempt is made to model the effects of supposed degradation of resolution of observations along each of the four observation routes. In method D successive pairs of observations are averaged, while in method E successive groups of four observations are averaged, then in each case the pooled values are used in the respective "conditional optimal" formulae. The timing errors τ_D, τ_E, are, as expected, generally larger than τ_A but in most cases not substantially so, even for the more drastic coarsening of resolution implied by method E. In the final two methods the effects of degrading the resolution of the analysis points are simulated. In method F this is done by substituting for the analysed wind vector at each of a consecutive pair of the original analysis points along each route their mean wind vector, while in method G the averaging is carried out in groups of four of the original analysis points. As before, standard deviations , τ_F, τ_G, of timing estimates are deduced for these methods. It should be admitted that this way of representing the effects of degrading the resolution of the analysis is somewhat artificial and that, in the case of the particular diagnostic, τ, which is essentially proportional to the average along-track wind variance, this particular way of degrading resolution tends to have very little effect. In addition to the calculations of standard deviations of flight time estimates I include for methods A (the standard analysis) and B (background only) the r.m.s. component wind errors, u_A, u_B, in units of ms^{-1} averaged along each of the analysis routes.

Table 1 lists the values of τ for methods A-G, together with those of u_A and u_B for a set of six numbered experiments that use different combinations of the parameters (i), (ii), (iii) described above. In referring to the results of the individual experiments I shall denote by "table 1.n" the portion of table 1 corresponding to experiment "n". In the first four experiments the covariance parameters of (16) are as listed on the first line of table 2. The form of the covariance functions within a horizontal plane for the three combinations of horizontal wind components are displayed in figure 3 while the vertical profile of the u-u covariance is shown by the solid line of figure 4.. In the first two experiments the ambient wind speed is set to 20 ms^{-1} and westerly in direction. In all of the experiments the sequence of commencement times of the four observing flights was the same: the times in minutes are plotted against the corresponding routes of figure 1. The interval of fifteen minutes between successive observing flights was chosen to represent a plausible rate of reception of the appropriate reports during daytime operation.

In experiment 1 all analysis flights were taken to commence at 60 minutes (i.e., fifteen minutes after the start of the last observing flight). General features to observe are that the standard deviations of timing estimates of the first four inbound flights are reduced to about nine or ten seconds with the optimal formula, as opposed to errors between 20 and 30 seconds for background data alone. Thus, it appears that there is a significant advantage to be gained from exploiting the new observations. The fresh tracks, 5 and 6, show slightly larger values τ_A as expected given that few previous observations lie close to these tracks. Even so, the reduction in error remains quite substantial. The over-pass flights, 7 and 8, while about twice as long as the others, are taken at a higher airspeed which tends to compensate in the evaluation of τ. Here the timing errors are approximately halved using the additional data. It is interesting to observe that, while there is a tendency for timing errors for most routes to be reduced by more than a factor of two, most r.m.s. velocity errors u_A are reduced by less than a factor of two from their background equivalents, u_B. This is perhaps indicative of the fact that the scatter of observations is more effective at reducing large scale error than small scale error and that the timing statistics are more influenced by the coherent large scales than by scales much smaller than the typical flight track lengths. The neglect of advection causes the timing error (τ_C) to increase by less than a second on most tracks, the exceptions being route 1 on which the time lag between corresponding observations and analysis points is a maximum, and routes 6 and 8 which run approximately transverse to the advecting wind. What is perhaps most surprising is how little the effects are of degrading the resolution of the observations (see τ_D, τ_E), amounting to mere fractions of a second additional error even for the four-fold degradation of resolution. Likewise, degrading the resolution of the sequence of values along analysis tracks used to compute the flight times has a very small effect on the expected magnitudes of the timing errors.

Experiment 2 differed from experiment 1 only to the extent that the commencement times of the analysing flights were all 120 minutes instead of 60 minutes. The results for this case compared to those of experiment 1 provide a direct way to assess the effects of the additional lag of an hour between observing and analysing for each of the seven methods of assimilation. The standard deviation τ_A increases most significantly for flights following the four tracks previously observed, as one should expect. Also, the neglect of advection reveals itself more strikingly in the higher values of τ_C there. Note that for route 8 the OI timing is now better than it was with a one hour time lag. The effect is presumably due to the advection of more information by the ambient wind from locations upstream. As before, effects of the simulated degradation of resolution remain very small.

It is inevitable that the tabulation of timings from the experiments 1 and 2 will depend upon the orientation of each analysis track relative to the ambient wind, since the expected timing errors are significantly influenced by the true ground speeds, as is clear from (8) and from the advection of information as discussed earlier. In order to observe the extent of this dependence the parameters of experiments 1 and 2 were used again in experiments 3 and 4 respectively except for the substitution of a southerly wind of 20 ms^{-1} for the original westerly. Again the results are presented in tables 1.3 and 1.4. The individual timing errors have clearly changed, in some cases dramatically, with the change in wind direction. Now it appears that routes 5 and 6 benefit from the advection of information from upstream with the longer lag, provided the unapproximated optimal formula (method A) is used. In general, however, the results remain much as before.

In view of the admittedly large uncertainty about the appropriateness of the covariance parameters it was decided to perform experiments directly comparable to experiment 1 (the same ambient wind and commencement times for analysis flights) but with altered covariances. Experiment 5 used the covariance parametrisation in line 2 of table 2 . For velocity-velocity combinations this covariance differs from the original in being horizontally compressed by a factor of two. Shrinking the covariance functions is equivalent to increasing the effective total number of degrees of freedom in the analysis field and also to shifting some power from the large scales to the small scales. It appears that in this case the net effect is to increase the OI errors τ_A and u_A. The effects of the neglect of advection are more pronounced now that the smaller horizontal scales of error have become important.

Finally, experiment 6 tests the effect of retaining the original horizontal scale of the covariances but shrinking the vertical scale by a factor of two, as indicated by the dashed profile of figure 4. Table 1.6 contains the results in this case. It is curious that the effect on the OI timing errors τ_A with vertical scales of error contracted is opposite to the effect of the horizontal contraction of experiment 5. Now a general decrease in τ_A is seen, except for the two level flights, 7 and 8. It would appear that the descending flights are seen as being nearer vertical than horizontal in a frame of reference scaled according to the aspect ratio of background errors, so the additional error structures of small scales do not affect the time errors as much as the general reduction of error amplitudes at large scales does. Note the the r.m.s. wind errors u_A still <u>increase</u> in going from experiment 1 to experiment 6.

For quick reference a descriptive summary of the salient features of the six experiments is given in table 3.

7. SUMMARY

A statistical method of updating the prior (background) estimates of flight track winds is proposed, based on the method of "optimum interpolation" applied in a four-dimensional sense. It is envisaged that the model will be updated throughout the day very frequently (e.g., hourly) so that the lack of any explicit dynamics remains unimportant. The background is assumed to be available by interpolation from a recent fine-mesh or mesoscale model forecast, while the new observations are presumed to be predominantly from aircraft within the area considered. In order to mitigate the present rather poor knowledge of the statistical characteristics of the wind at very small scales consideration has been given to a method whereby the statistical models used may be progressively refined using the observations themselves.

The preliminary studies described in section 6 indicate that flight time estimates for in-bound aircraft descending from cruise level to 15000' at the stacks may be made accurate to within about ten seconds for statistical parameters in the model that are judged to be realistic. A consideration of the effects of advection of error in the model is shown to be worth a few seconds of timing precision, provided there is some validity to the assumption that the errors are largely advected with the ambient flow. Surprisingly, it seems that a low resolution along the flight tracks is not necessarily a serious handicap in the estimation of these timings. The effect of track length on the flight time errors has not been investigated here. However, it can be shown by a simple statistical argument that for tracks much shorter than the scale of the covariances the standard deviations of timing errors should be nearly proportional to the track lengths, while for tracks much longer than the covariance scale the variances of timing errors should be proportional to track lengths.

Future work will be directed to the development of a prototype assimilation system and to a careful examination of the spatial statistics of typical forecast errors. At present, in the absence of a suitable distribution of aircraft data, tests of the proposed assimilation method are being conducted using dropsonde data collected during January 1988 as part of the "Fronts 87" field experiment (Clough, 1987) over the South West Approaches. Through these tests it should be possible to ascertain the computing resources that are likely to be required in order to run such an assimilation model routinely throughout each day.

It is clearly important to gather statistics on the spatial patterns of wind variability at scales presently unresolved by operational forecasting models, and to understand better the characteristics of the forecast wind errors at the scales that are properly resolved. In view of the manifest day-to-day variability in synoptic scale weather regimes experienced over the British Isles it is likely that optimal results with a statistical assimilation model will only be obtained when the covariances fed to the scheme are adapted explicitly to the particular conditions of the day. Thus, the experience needed to build up a reliable "archive" of relevant covariance statistics at these relatively small spatial scales cannot be acquired by a cursory survey of a few randomly chosen cases but rather through the methodical accumulation of statistics representing all seasons and distinct weather types. While there is scope for representing the covariance structures in numerous different ways, the method of likelihood validation outlined in section 5 provides a logical, objective, framework in which a critical appraisal and tuning of possible covariance structures may be carried out according to the evidence supplied by the actual measurements.

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Table 1. Standard deviations of errors of timing estimates, τ, in seconds for analysis methods A to G and r.m.s. wind component errors, u, in ms^{-1} for methods A and B.

	Route	τ_A	τ_B	τ_C	τ_D	τ_E	τ_F	τ_G	u_A	u_B
Experiment 1.	1	8.85	21.07	10.26	8.86	8.90	8.85	9.13	1.97	3.46
	2	8.73	20.62	9.08	8.74	8.78	8.74	8.90	1.95	3.46
	3	10.10	29.81	10.92	10.11	10.17	10.10	10.45	1.68	3.46
	4	8.48	30.22	9.02	8.49	8.57	8.48	8.50	1.51	3.46
	5	11.19	22.37	11.93	11.20	11.26	11.19	11.43	2.22	3.46
	6	10.48	27.11	11.89	10.51	10.56	10.48	10.95	1.88	3.46
	7	8.58	17.88	8.94	8.64	8.90	8.58	8.58	2.06	3.46
	8	11.37	21.12	12.66	11.42	11.63	11.37	11.63	2.20	3.46
Experiment 2:	1	11.31	21.07	15.15	11.32	11.38	11.31	11.51	2.28	3.46
	2	11.63	20.62	12.92	11.64	11.69	11.63	11.73	2.36	3.46
	3	13.43	29.81	18.00	13.45	13.50	13.43	13.67	2.01	3.46
	4	9.93	30.22	14.36	10.00	10.10	9.93	9.94	1.77	3.46
	5	12.69	22.37	16.31	12.71	12.79	12.69	12.88	2.43	3.46
	6	10.54	27.11	17.22	10.57	10.76	10.54	11.02	1.90	3.46
	7	8.79	17.88	10.36	8.87	9.17	8.79	8.79	2.12	3.46
	8	11.21	21.12	16.27	11.28	11.58	11.21	11.48	2.24	3.46
Experiment 3:	1	9.74	30.44	12.47	9.76	9.83	9.74	10.37	1.67	3.46
	2	12.42	28.23	13.89	12.43	12.46	12.43	12.65	1.94	3.46
	3	8.01	20.52	8.31	8.02	8.05	8.01	8.21	1.82	3.46
	4	7.00	24.19	7.64	7.01	7.04	7.00	7.01	1.53	3.46
	5	10.60	19.71	10.87	10.61	10.66	10.60	10.78	2.33	3.46
	6	12.74	31.28	13.40	12.76	12.82	12.74	13.25	1.89	3.46
	7	11.09	21.12	12.06	11.14	11.35	11.09	11.09	2.20	3.46
	8	10.88	17.88	11.08	10.90	11.03	10.88	11.05	2.33	3.46

Table 1 (continued)

	Route	τ_A	τ_B	τ_C	τ_D	τ_E	τ_F	τ_G	u_A	u_B
Experiment 4:	1	12.63	30.44	18.66	12.64	12.72	12.63	13.07	1.92	3.46
	2	16.84	28.23	21.85	16.85	16.89	16.85	16.99	2.30	3.46
	3	11.96	20.52	13.64	11.96	12.00	11.96	12.06	2.33	3.46
	4	9.10	24.19	14.52	9.12	9.25	9.10	9.11	1.83	3.46
	5	12.53	19.71	13.87	12.54	12.59	12.53	12.64	2.62	3.46
	6	11.76	31.28	15.70	11.80	11.90	11.76	12.34	1.85	3.46
	7	11.92	21.12	16.02	11.98	12.25	11.92	11.92	2.28	3.46
	8	11.50	17.88	12.38	11.52	11.64	11.50	11.65	2.45	3.46
Experiment 5:	1	10.66	20.27	14.51	10.69	10.78	10.66	10.87	2.28	3.46
	2	10.19	19.80	11.19	10.21	10.28	10.20	10.32	2.27	3.46
	3	12.37	28.28	14.15	12.41	12.55	12.37	12.62	1.99	3.46
	4	9.64	28.73	10.51	9.68	9.80	9.64	9.65	1.73	3.46
	5	14.78	21.44	16.37	14.82	15.03	14.78	14.89	2.72	3.46
	6	13.80	25.81	17.37	13.90	14.25	13.80	14.11	2.29	3.46
	7	9.38	15.74	10.01	9.43	9.66	9.38	9.38	2.41	3.46
	8	12.95	18.33	15.81	13.04	13.61	12.95	13.12	2.64	3.46
Experiment 6:	1	7.85	18.03	8.83	7.86	7.93	7.85	8.14	2.08	3.46
	2	7.86	17.61	8.10	7.88	7.94	7.87	8.04	2.07	3.46
	3	9.32	25.51	9.91	9.34	9.40	9.32	9.68	1.81	3.46
	4	8.06	25.84	8.47	8.08	8.14	8.06	8.08	1.64	3.46
	5	9.89	19.09	10.45	9.90	9.97	9.89	10.14	2.31	3.46
	6	9.47	23.17	10.52	9.49	9.55	9.47	9.96	1.98	3.46
	7	10.16	17.88	10.43	10.37	12.11	10.16	10.16	2.30	3.46
	8	13.29	21.12	14.33	13.46	14.97	13.29	13.49	2.42	3.46

Table 2. Covariance parameters used for each experiment.

Expt(s)	$a_{\chi 1}$	$a_{\psi 1}$	$b_{\chi 1}$	$b_{z 1}$	$a_{\chi 2}$	$a_{\psi 2}$	$b_{\chi 2}$	$b_{z 2}$
(Units)	($Km^2m^2s^{-2}$)		(Km)	(feet)	($Km^2m^2s^{-2}$)		(Km)	(feet)
1,2,3,4	90000	810000	300	10000	2500	2500	50	1600
5	22500	202500	150	10000	625	625	25	1600
6	90000	810000	300	5000	2500	2500	50	800

Table 3. Summarising description of parameters used for each experiment.

Expt.	analysis times (minutes)	covariance model	wind direction
1	60	standard	westerly
2	120	standard	westerly
3	60	standard	southerly
4	120	standard	southerly
5	60	narrow (horizontally)	westerly
6	60	thin (vertically)	westerly

FIGURE CAPTIONS

Figure 1: Tracks of observing flights descending from 30000' to 15000'. Commencement times for each track are shown in minutes at the beginning of each track.

Figure 2: Analysis flight tracks. Routes 1-6 descend from 30000' to 15000', routes 7 and 8 are level at 30000'.

Figure 3: Background covariance functions, contoured at intervals of 2 m^2s^{-2}, for experiments 1 to 6 and for combinations: (a) u-u; (b) u-v; (c) v-v; each plotted as a function of horizontal displacement.

Figure 4: Vertical profiles of background error covariances. Solid curve shows the form of the standard covariances used for experiments 1 to 5, the dashed curve shows the thin profile used for experiment 6.

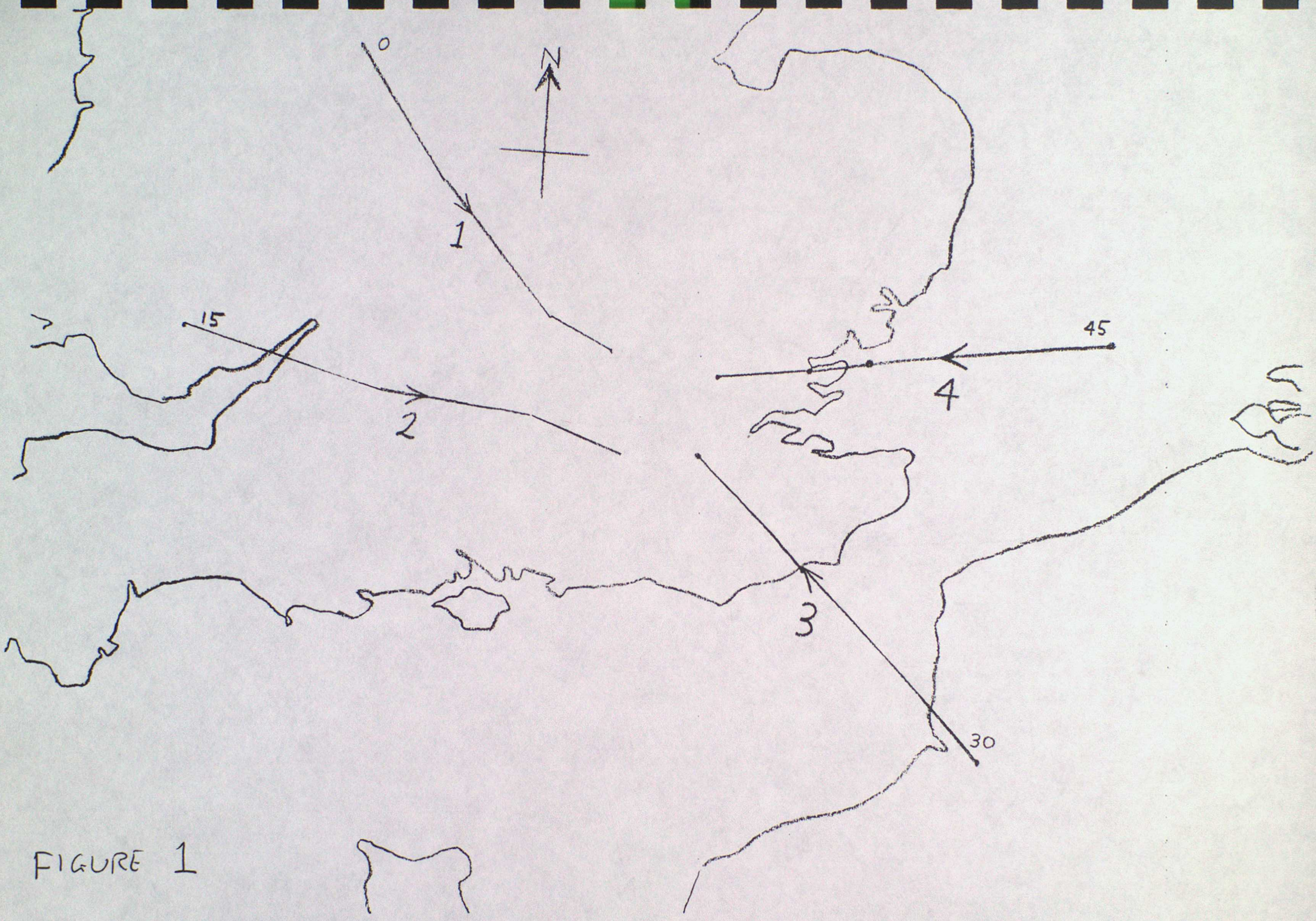


FIGURE 1

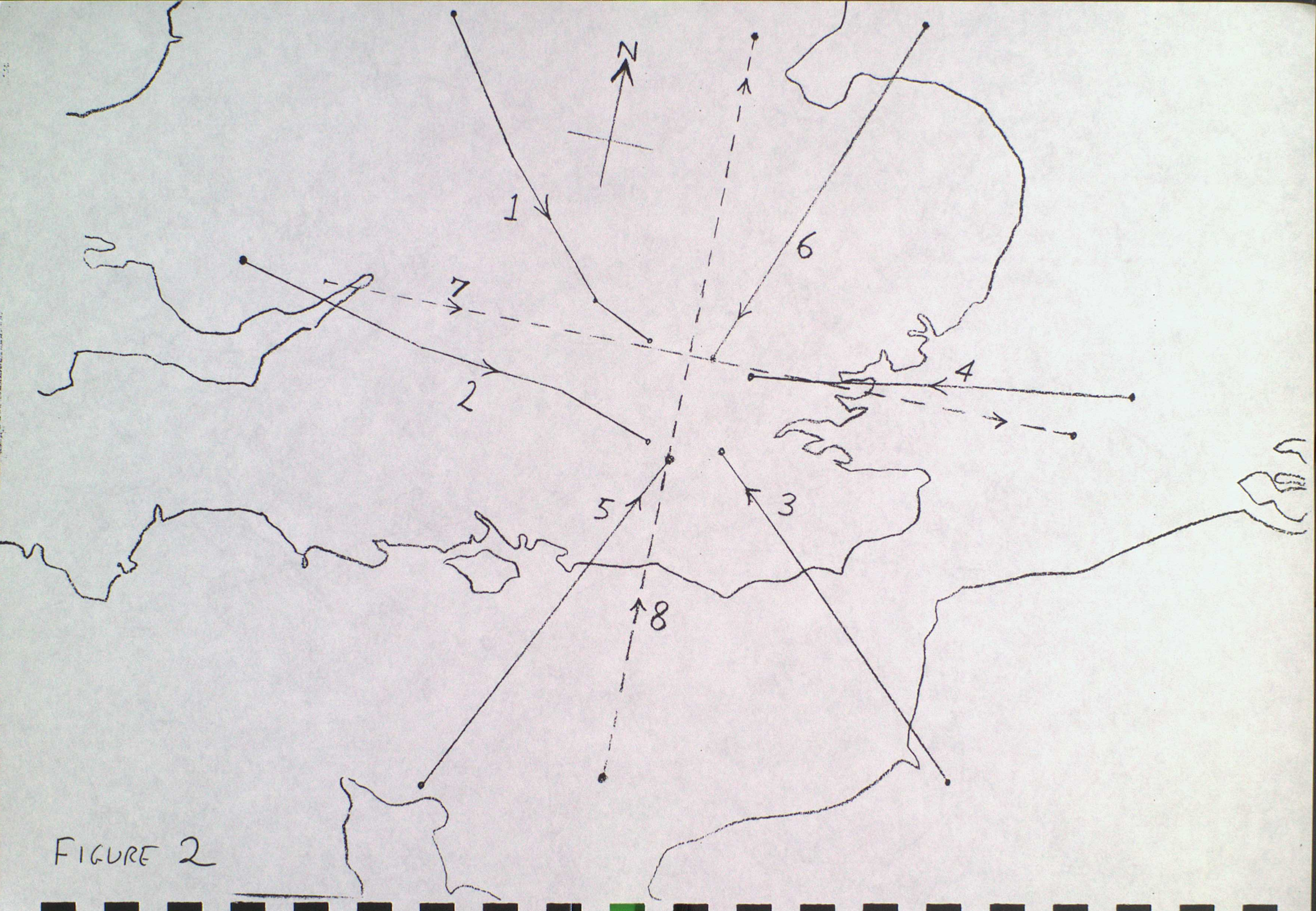


FIGURE 2

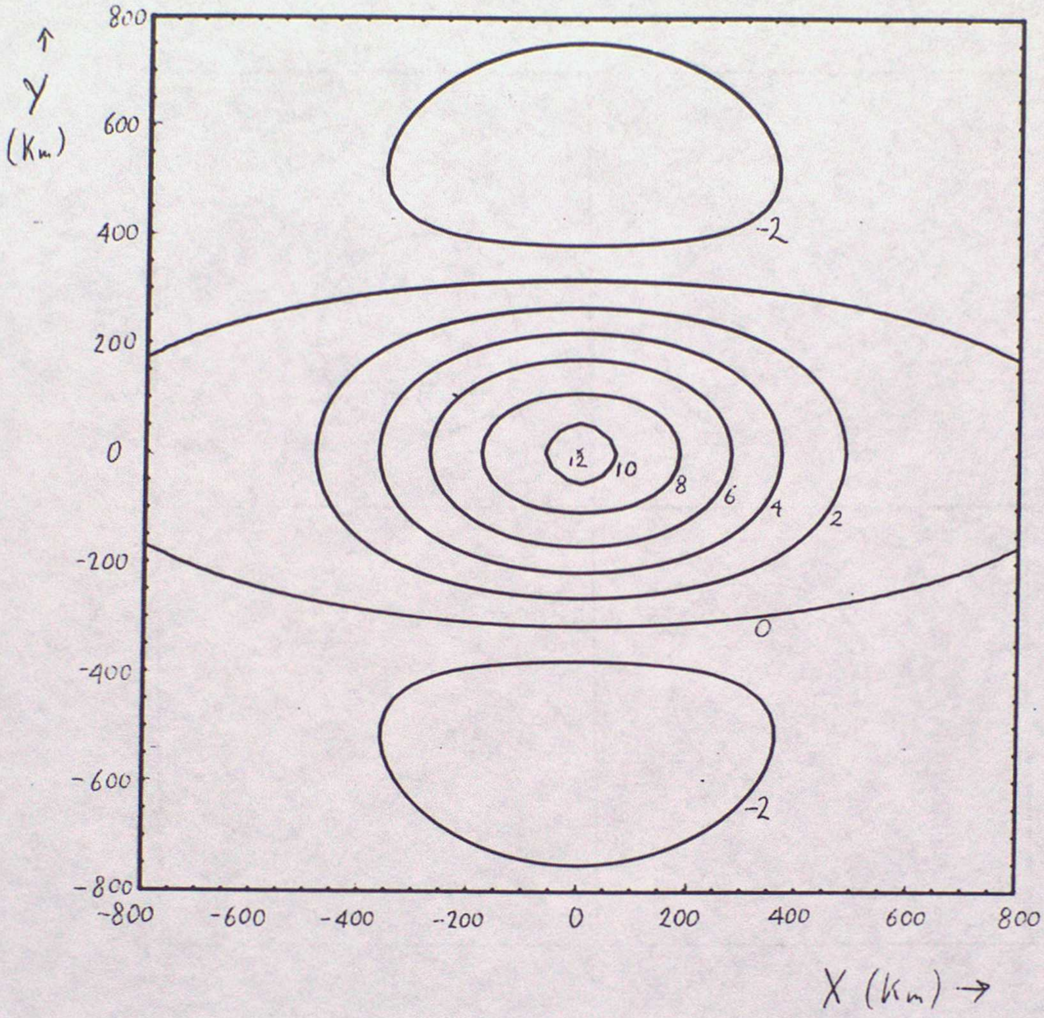

Y (Km)
800
600
400
200
0
-200
-400
-600
-800
-2
0
2
4
6
8
10
12
-2
-800
-600
-400
-200
0
200
400
600
800
X (Km)

FIGURE 3(a)

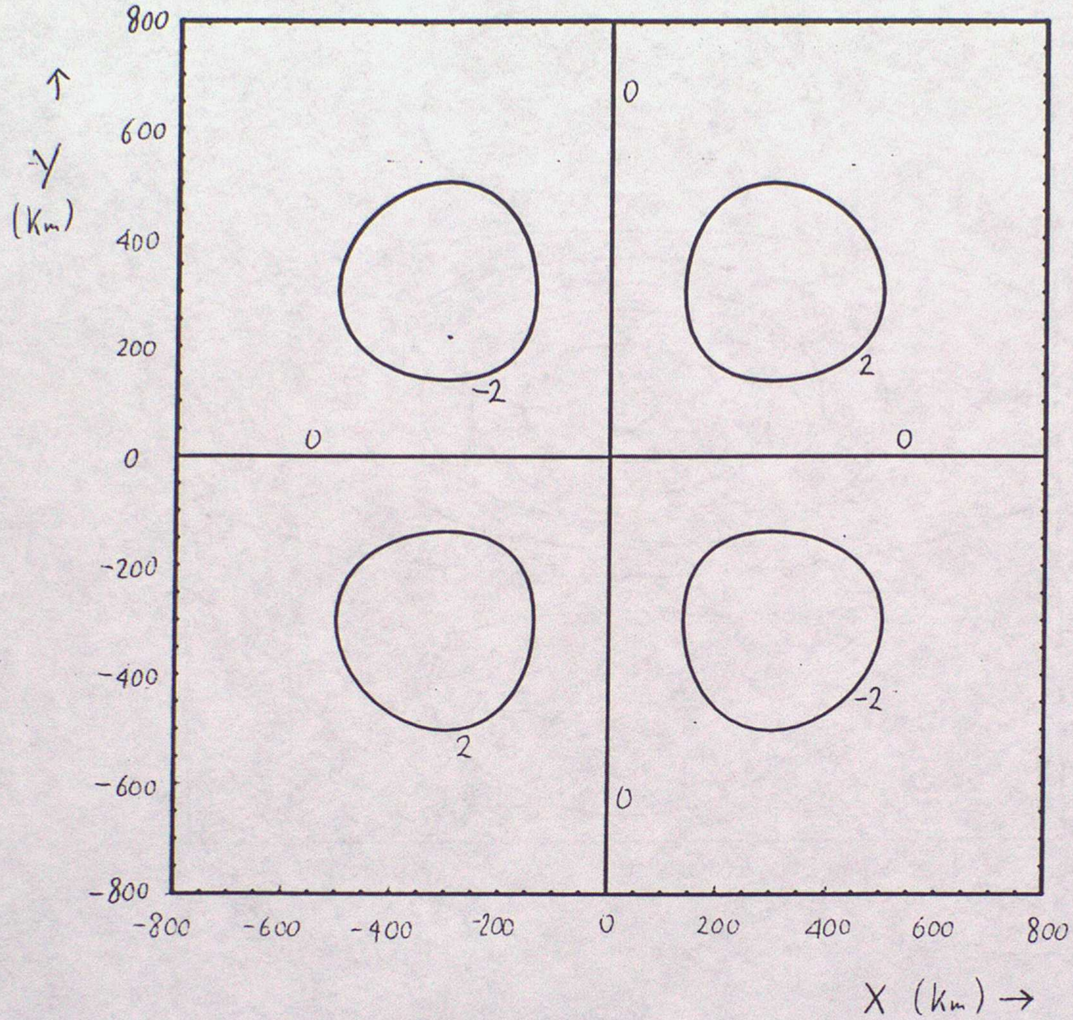
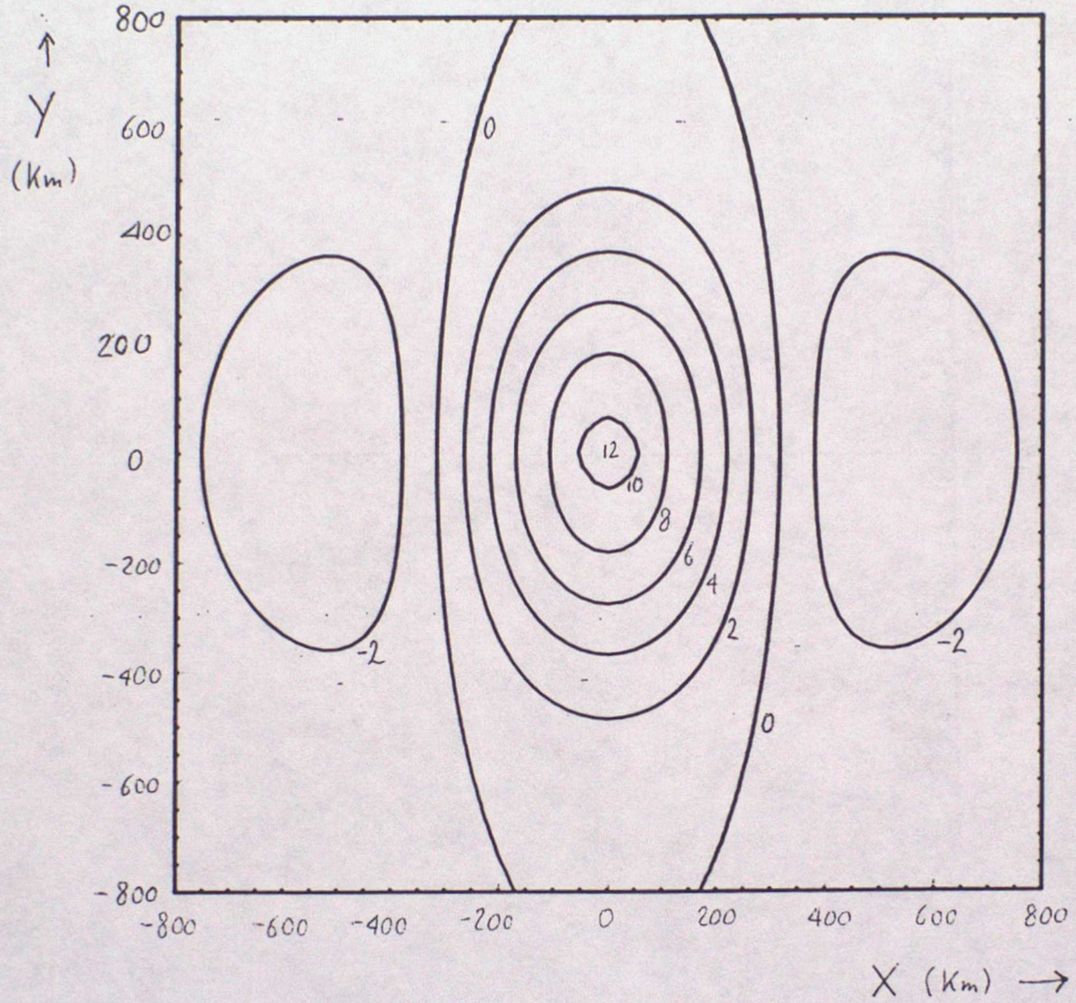

800
600
400
200
0
-200
-400
-600
-800
Y (Km)
0
-2
2
0
0
2
-2
0
-800
-600
-400
-200
0
200
400
600
800
X (Km)

FIGURE 3(b)



Y (Km)
X (Km)
800
600
400
200
0
-200
-400
-600
-800
-800
-600
-400
-200
0
200
400
600
800
0
12
10
8
6
4
2
-2
-2
0

FIGURE 3(c)

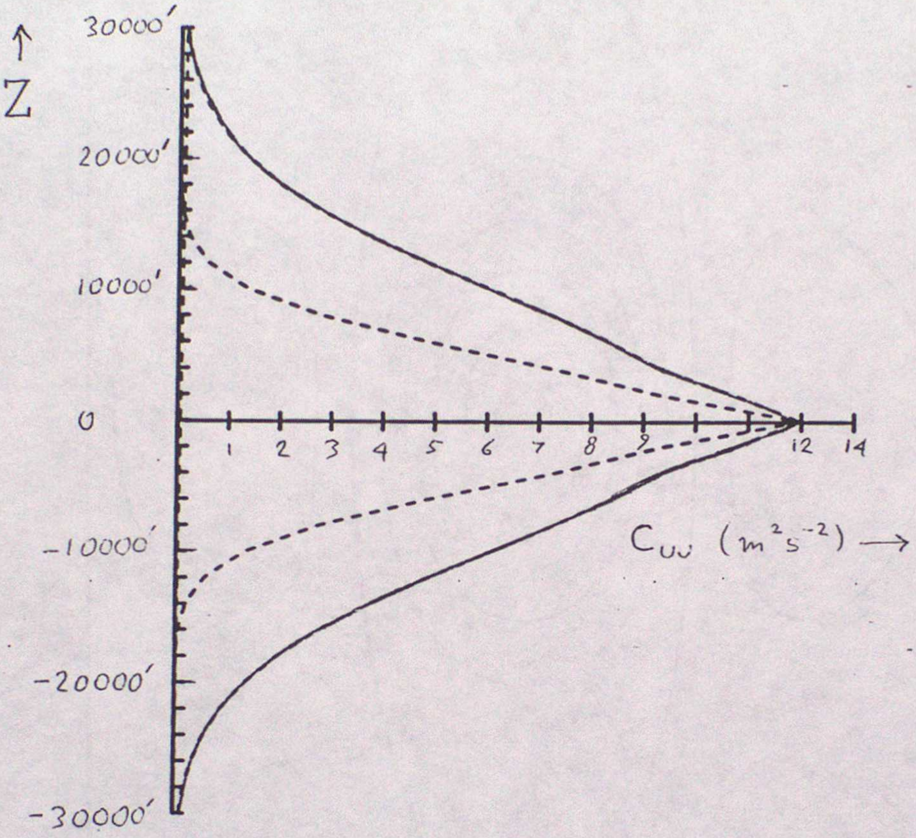

Z
30000′
20000′
10000′
0
-10000′
-20000′
-30000′
1
2
3
4
5
6
7
8
9
12
14
Cuu (m²s⁻²)

FIGURE 4

CURRENT MET O 11 TECHNICAL NOTES (JANUARY 1989)

The Met O 11 Technical Notes which contain information of current use and *which have not been published* elsewhere, are listed below. The complete set of Technical Notes is available from the National Meteorological Library on loan, if required.

186. **The representation of boundary layer turbulence in the mesoscale model. Part 1. The scheme without changes of state.**
R.N.B. Smith
April 1984

187. **The representation of boundary layer turbulence in the mesoscale model. Part 2. The scheme with changes of state.**
R.N.B. Smith
April 1984

195. **Assessment of HERMES data: a case study comparison with the operational analysis for 2nd March 1984.**
W. Adams
August 1984

196. **Solutions in flow over topography using a geometric Lagrangian flow.**
S. Chynoweth
November 1984

197. **An investigation into the likely causes of spurious rain in anticyclones in the fine mesh model.**
W. Hand
October 1984

199. **The impact of data from the HERMES system on the fine mesh data assimilation scheme - a case study.**
R.S. Bell and O.M. Hammon
February 1985

203. **Using an interactive radiation scheme in the fine mesh model.**
A.D. Darlington
March 1985

204. **Snow forecasts from NWP models during the winter of 1984/85.**
O.M. Hammon
March 1985

205. **Results of a trial of a parametrization of gravity wave drag in the operational forecast model.**
J.E. Kitchen, M.J. Carter and A.P. Day
April 1985

229. **A parametrization of deep convection for use in a non-hydrostatic mesoscale model.**
R.T.H. Barnes and B.W. Golding
March 1986

230. **Boundary layer structures and surface variables in operational forecasts.**
R.S. Bell
April 1986

231. **Meteorological Office mesoscale model: an overview, version 1.**
B.W. Golding
December 1986

235. **Snow forecasts from the fine mesh model and mesoscale model during the winter 1985/86.**
O.M. Hammon
June 1986

236. **Vertically-propagating quasi-inertia waves: simulated and observed.**
M.M. Booth and G.J. Shutts
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239. Mesoscale case study - Project Haar.
W.R.P. Taylor
February 1987

240. **A trial of modified diffusion in the coarse mesh model.**
R.S. Bell and R.A. Downton
September 1986

243. The global impact of the recent developments of the physical parameterisation schemes.
R.S. Bell
November 1986

247. **Some experiments with two-dimensional semi-geostrophic and primitive equation models, with sigma as the vertical coordinate.**
C.A. Parrett
February 1987

248. **Moist frontogenesis in the geometric model.**
M.W. Holt
March 1987

249. **Mesoscale model trial of a revised convection scheme and cloud modifications.**
O.M. Hammon
May 1987

250. **Results from the fine mesh trial of a modified physics package.**
O.M. Hammon
July 1987

251. **Verification of mesoscale model forecasts during the winter, November 1986 - February 1987.**
O.M. Hammon
March 1987

252. **Mountain wave generation by models of flow over synoptic-scale orography.**
M.J.P. Cullen and C.A. Parrett
March 1987

253. **Development of the analysis correction scheme, I. The observational weights.**
B. Macpherson
September 1987

256. **Experiments with divergence damping and reduced diffusion in the mesoscale model.**
S.P. Ballard
May 1987

258. **Trials of the interactive radiation scheme in the global model.**
M.D. Gange
May 1987

261. **Modifications to the automatic quality control of ship data and an assessment using case studies.**
B.R. Barwell and C.A. Parrett
1987

263. **Results from a fine mesh model trial using a modified evaporation scheme.**
O.M. Hammon and C.A. Wilson
August 1987

NEW SERIES (Commenced October 1987)

1. **An assessment of the results of trials of a new analysis scheme for the operational global model.**
R.S. Bell
October 1987

2. **A case study showing the impact of analysis differences on medium range forecasts.**
R.A. Downton and R.S. Bell
January 1988

3. **Development of the analysis correction scheme. II. Inclusion of an observation density analysis.**
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4. An assessment of a trial to test small changes to the Convection scheme in the mesoscale model.
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5. Trial of proposed changes to the Mesoscale model for November 1987.
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December 1987

6. Assessment of HERMES soundings processed using the new cloud-clearing scheme.
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7. An assessment of the impact of a correction to the Mesoscale model turbulence/vertical diffusion scheme implemented in March 1988.
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8. Comparison of algorithms for the solution of cyclic, block, tridiagonal systems.
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9. A comparison of alternating direction implicit methods for solving the 3-D semi-geostrophic equations.
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May 1988

10. The automatic quality control of surface observations from ships: the final trial, latest statistics, operational implementation and future work.
C.A. Parrett
May 1988

11. "Panel-beater": a proposed fast algorithm for semi-geostrophic finite-element codes.
R.J. Purser
June 1988

12. The 5-day forecast trial of the AC scheme.
R.A. Downton, R.A. Bromley and M.A. Ayles
September 1988

13. A theoretical study of the information content of the ERS-1 scatterometer data.
R.J. Purser
August 1988

14. A further global trial of the analysis correction scheme - Christmas 1987.
R.S. Bell
August 1988

15. **The sensitivity of a medium range forecast with the analysis correction scheme to data selection in the horizontal.**
B. Macpherson and R.A. Downton
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16. **The sensitivity of fine-mesh rainfall forecasts to changes in the initial moisture fields.**
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17. **Conservative finite difference schemes for a unified forecast/climate model.**
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18. **Interpreting results from numerical models.**
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19. **A comparison of the OWSE assimilation scheme with the operational global assimilation scheme.**
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20. **Improvements to low cloud forecasts from the mesoscale and fine mesh models.**
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21. **The effect of route choice on aircraft wind observations over the North Atlantic.**
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22. **Maximum liklihood de-aliasing of simulated scatterometer wind fields using adaptive descent algorithms.**
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January 1989

23. **A proposal for assimilating detailed aircraft wind data in a local area.**
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January 1989

24. **Basic formulation and boundary conditions of the mesoscale model.**
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Not yet issued.

25. **Development of a new physics package for the global forecast model.**
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