



Met O (APR) Turbulence and Diffusion Note No. 211

The Effect of Coriolis Force on Long Term Lateral
Dispersion

by

J R Kemp

8 February 1994

Headquarters, Bracknell

ORGS UKMO T

National Meteorological Library
FitzRoy Road, Exeter, Devon. EX1 3PB



Met O (APR) Turbulence and Diffusion Note No. 211

The Effect of Coriolis Force on Long Term Lateral Dispersion

by

J R Kemp

8 February 1994

Met O (APR)
(Atmospheric Processes Research)
Meteorological Office
London Road
Bracknell
Berks, RG12 2SZ

Note

This paper has not been published. Permission to quote from it should be obtained from the Assistant Director, Atmospheric Processes Research Division, Met O (APR), Meteorological Office, London Road, Bracknell, Berkshire, RG12 2SZ.

© Crown copyright 1994

The Effect of Coriolis Force on Long Term Lateral Dispersion

J.R.Kemp

8th February 1994

1 Introduction

In 1982, Crabtree reported on a number of long range dispersion experiments, in which plumes from Eggborough power station, 'labelled' with Sulphur Hexafluoride, were tracked across the North sea using the Meteorological Research Flight Hercules aircraft. The measurements reported by Crabtree suggest that the plume does not get much wider than 10 km even after 100,000 seconds of travel time. In this paper we investigate the possibility that this may be the result of Coriolis effects following a suggestion by Hunt (1992). In essence Hunt suggested that the Coriolis parameter, f , might produce a restoring force in the lateral direction inhibiting motion in this direction and thus limiting dispersion, analogous to the way in which the buoyancy force inhibits vertical dispersion in stable boundary layers (e.g. Pearson, Puttock, Hunt (1983)). In order to test this idea a large eddy simulation was run under neutral conditions for around 100,000 seconds (many times the Coriolis time-scale). Dispersion properties were calculated by following a large

number of particles which were released into the flow. It was hoped that any limiting of the lateral dispersion by the Coriolis force would be observed from the statistics of particle displacements.

2 Theory

The basic equations for motion of a fluid parcel in the horizontal plane are:

$$\frac{dU}{dt} - fV = -\frac{1}{\rho} \frac{\partial P}{\partial x} \quad (1)$$

$$\frac{dV}{dt} + fU = -\frac{1}{\rho} \frac{\partial P}{\partial y} \quad (2)$$

Taking $V = \frac{dY}{dt}$ and $U = \frac{dX}{dt}$ we can obtain after various manipulations the equation :

$$\frac{d^2 Y}{dt^2} + f^2 Y = -\frac{1}{\rho} \frac{\partial P}{\partial y} + \int \frac{f}{\rho} \frac{\partial P}{\partial x} dt + c \quad (3)$$

where c is a constant of integration. The right-hand side of this equation represents the effect of local pressure variations on a particular parcel. It is assumed that in a turbulent environment these variations may be represented as a stationary random function $H(t)$ that is independent of f . This is only likely to be true for pressure fluctuations due to motions with time-scales $\ll f^{-1}$ (as is the case in boundary layer turbulence). Assuming this form the equation becomes:

$$\frac{d^2 Y}{dt^2} + f^2 Y = H(t) \quad (4)$$

This equation has an associated time-scale of order $\frac{2\pi}{f}$ i.e. 60,000 seconds at our latitude and hence this is the time-scale over which the Coriolis force is liable to affect dispersion. Now, if H and \dot{Y} are bounded, then it is reasonable to assume that $\frac{d^2 Y}{dt^2}$ must also be bounded, since otherwise the consequential changes in \dot{Y} are likely to mean that \dot{Y} does not remain small. Therefore from (4) it follows that Y is bounded and hence lateral dispersion is limited. This argument can be formalized by first integrating (4) to give

$$\dot{Y}(T) - \dot{Y}(t) = \int_t^T (H - f^2 Y) dt \quad (5)$$

Now, suppose $|\dot{Y}| < v$, $|H| < h$ and $Y(t) = \frac{h}{f^2} + y$ with $y > 0$. Then because \dot{Y} is bounded by v , Y remains $> \frac{h}{f^2} + \frac{y}{2}$ for a period $\frac{y}{2v}$. Therefore for a time $T = t + \frac{y}{2v}$, the magnitude of the l.h.s of (5) will be $< 2v$ and the r.h.s will be $> \frac{y^2 f^2}{4v}$. Therefore $y^2 f^2 < 8v^2$ and $Y(t) < \frac{h}{f^2} + \frac{2\sqrt{2}v}{f}$. By symmetry $Y(t) > -\frac{h}{f^2} - \frac{2\sqrt{2}v}{f}$ and Y is bounded.

The underlying assumption here is that $H(t)$ is a stationary random function, which is not obviously the case since the integral term on the right hand side of (3) may not remain small, and the evidence from the simulations suggests this is incorrect.

3 Numerical Simulation

The preceding theory was tested using large eddy simulation and particle tracking routines to obtain dispersion data. The model used in this simulation is identical to that

described in Mason (1992). The equations for the resolved scale velocity and temperature fields are

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla P_0 + \mathbf{u} \wedge \mathbf{k} f + \mathbf{B} + \nabla \cdot \boldsymbol{\tau}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot \mathbf{h}$$

where

$$\mathbf{B} = g(\bar{T} - T)/\bar{T}$$

Here p is pressure divided by density, and $\nabla P_0 \equiv \frac{\partial P_0}{\partial y}$ is the constant mean pressure gradient driving the geostrophic wind in the x -direction. $\boldsymbol{\tau}$ is the sub-grid stress tensor and \mathbf{h} the sub-grid heat flux.

The large eddy simulation used a domain of dimensions 3.2 km by 3.2 km in the horizontal and 1.2 km in the vertical utilizing a 32 x 32 x 24 mesh of gridpoints, non-uniform in the vertical giving enhanced resolution near the surface. A geostrophic wind of 20 ms⁻¹ was imposed under neutral static stability conditions and the simulation run as described in Mason (1992) (it is identical to Mason's R20N run), but for a time period of 100,000 seconds in order to detect any Coriolis dependent dispersion effects. Dispersion statistics were calculated by following the trajectories of particles in the simulated flow. Of course, the distances travelled by the particles over the 100,000 second period, far

exceeds the box size. To overcome this problem, the flow was assumed to be periodic in the x and y directions, consistent with the periodic boundary conditions used in the LES. The dispersion calculations were organised to obtain statistics for the dispersion of an ensemble average puff. To this end, particles were released from points spread throughout the domain in the horizontal and over 2000 seconds in time. At various travel times during the run the particle positions relative to their start positions were stored to allow calculation of statistics of the puff.

Five puffs were simulated corresponding to five different release heights (25, 100, 200, 400 and 700m), and data was stored at nine times up to 100,000 seconds. Each puff involved tracking 5120 particles.

It was later discovered that a damping layer, which relaxed the velocities to geostrophic values, had inadvertently been switched on in the top 200m of the domain. However, apart from introducing an increase in velocity shear at the top of the domain, this is not expected to have affected the major results.

Having run the large eddy simulation it was decided to attempt a numerical solution of (4) itself. Using a simple finite differencing scheme with a 10 second timestep, and taking $H(t)$ as a random function, independent from timestep to timestep, with a Gaussian distribution, values of $\overline{Y^2}$ were calculated out to large times.

4 Dispersion Results

Figures 1 and 2 show \bar{z} and σ_z respectively. The five curves for the various release heights clearly converge to constant values for both the above plots: $\bar{z} = H/2$ and $\sigma_z =$

$H/\sqrt{12}$ where H is the domain depth. These values are characteristic of an evenly mixed distribution in the vertical, as would be expected under these conditions. Figures 3 and 4 show σ_x and σ_y versus time, on a log-log scale. The x axis is taken to be in the direction of a line drawn from the origin of release to the centroid of the plume. If the Coriolis force had affected the dispersion in the manner predicted by equation (4), then we would expect to see a levelling off of the σ_y graph with time. This does not appear to be the case and even after 100,000 seconds both σ_x and σ_y are still increasing. The gradients of these curves both have values close to the 0.5 (0.49 and 0.56 respectively) expected from a simple Lagrangian analysis which predicts a $t^{0.5}$ relationship at times \gg the Lagrangian time-scale. A further aspect of these graphs worth mentioning is the accelerated growth of σ_y for all release heights and σ_x for the elevated releases. This is neatly accounted for using Smith's theory (1965) of the role of wind shear in horizontal diffusion. As the plume grows vertically the role of wind shear in spreading out the plume grows in importance. For a near surface release, however, this is not the case for σ_x as $\frac{dU}{dz}$ is greatest near the ground.

The above appears to rule out the idea of the Coriolis Force being significant in terms of limiting long range dispersion and our simulation does not confirm the ideas of section 2. The problem appears to lie in the nature of the $H(t)$ term and assumptions made about its behaviour. $H(t)$ is assumed to be a bounded function. This assumption is thrown into question firstly by the fact that it contains the time integral from the right hand side of (3), which does not necessarily remain small for all time. Secondly the presence of significant wind shear implies that σ_y is unlikely to remain small and thus $H(t)$ cannot remain bounded.

If the Coriolis effect on dispersion is real, but is being masked by the complicated wind

profile in a 'real' boundary layer, then a much more idealized situation should show this up. This is effectively what was attempted in the numerical solution of equation (4) using the simple formulation for H . As figure 5 shows, no tendency of σ_y to become constant in time is apparent even after up to 1,000,000 seconds. Now, as noted above, if \dot{Y} and H are bounded then Y will be bounded. In this simulation however, σ_y continues to grow, which implies that \dot{Y} cannot be bounded, and this is found to be the case. Therefore our simple formulation for H must be flawed since it is producing unphysical results. This implies that the properties of H are much more subtle than initially supposed and gives us less reason to trust the intuitive assumption that H may be a stationary random function.

5 Conclusion

The large eddy simulation of the neutral boundary layer has not shown any tendency for the growth of σ_y to level off as a result of the Coriolis force. The theory of section 2 appears to be weakest in its treatment of the 'pressure' term $H(t)$. Firstly there is no guarantee that this term will remain bounded as is assumed in the derivation of (4). Secondly the direct numerical solution of equation (4) has shown that if $H(t)$ is considered to have a simple Gaussian distribution independent from timestep to timestep then, even without any complicating wind shear, no tendency to limit σ_y can be seen and \dot{Y} grows unphysically. Of course, the possibility of this dispersion limiting effect occurring has not been ruled out completely, but it is clear from this study that its existence must depend on some fairly subtle aspects of the time correlation of $H(t)$ and is by no means intuitive.

It should also be noted that the Crabtree data is not wholly conclusive. The limited number of data points involve a reasonably large amount of scatter, up to half a decade

or more, and the existence of a limit to the horizontal spread is not very convincing. Furthermore, even if the rate of spread is reduced at large distances downwind, this could be caused by greater stability over the sea limiting the observed dispersion.

References

- Crabtree J., 1982, 'Studies of plume transport and dispersion over distances of travel up to several hundred kilometres', Turbulence and diffusion note No.139.
- Hunt J.C.R., 1992, Private communications.
- Mason P.J., 1992, 'Large-eddy simulation of dispersion in convective boundary layers with wind shear', *Atmos. Environ.*, **26A**, 1561-1572.
- Pearson H.J., Puttock J.S. and Hunt J.C.R., 1983, 'A statistical model of fluid-element motions and vertical diffusion in a homogeneous stratified turbulent flow', *J. Fluid Mech.*, **120**, 219-249.
- Smith F.B., 1965, 'The role of wind shear in horizontal diffusion of ambient particles', *Quart. J. Roy. Meteor. Soc.*, **389**, 318-329..

Figure Captions

Figure 1 - Mean heights of plumes, \bar{z} .

Figure 2 - Vertical root mean square dispersion of particles, σ_z .

Figure 3 - Horizontal root mean square spread of particles in the x-direction, σ_x .

Figure 4 - Lateral root mean square spread of particles in the y-direction, σ_y .

Figure 5 - Lateral root mean square spread of particles in the y-direction, σ_y , by solving equation (4) directly by finite difference methods with H chosen to have a Gaussian distribution.

MEAN HT VS TIME

Figure 1

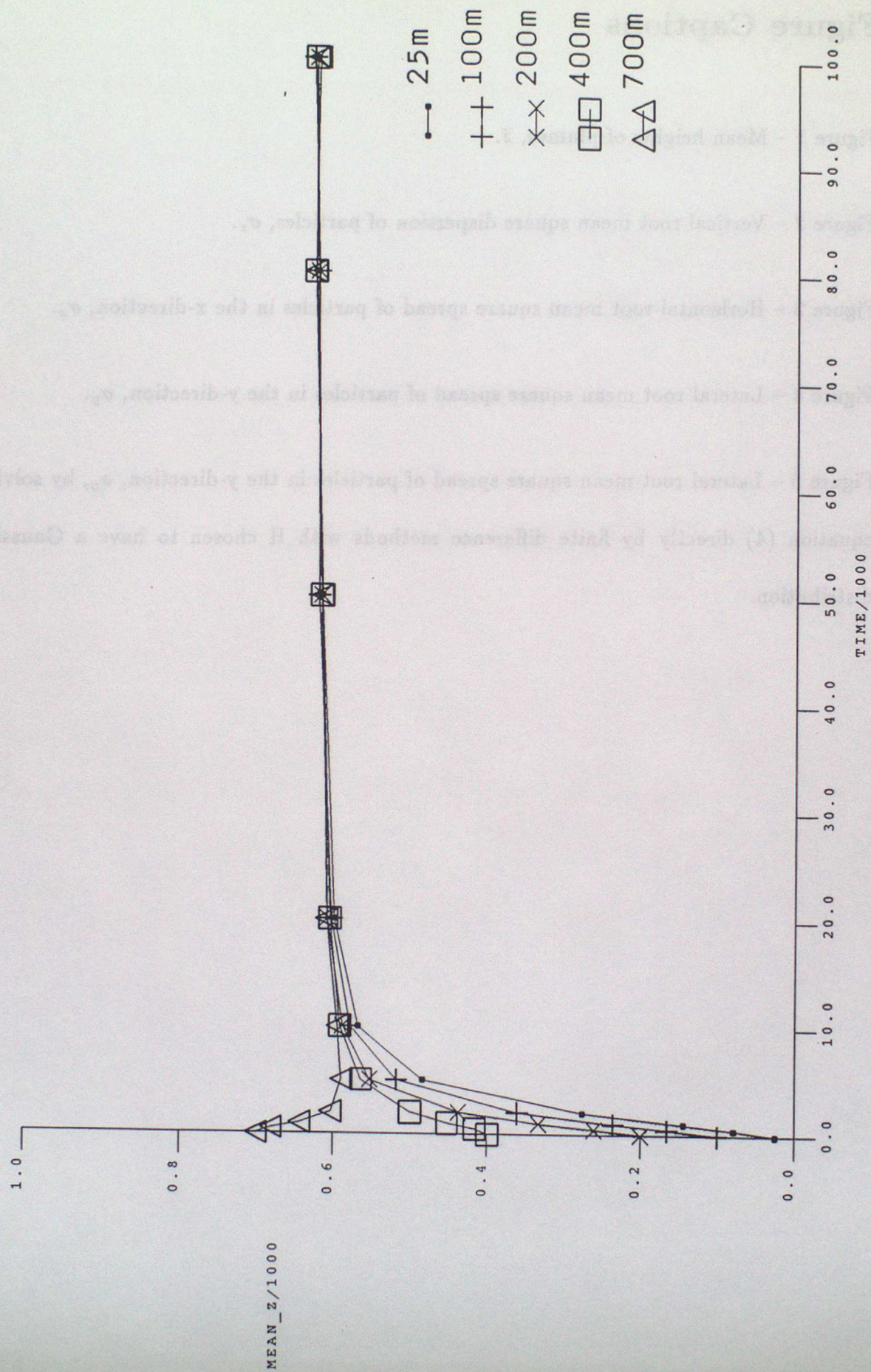


Figure 2

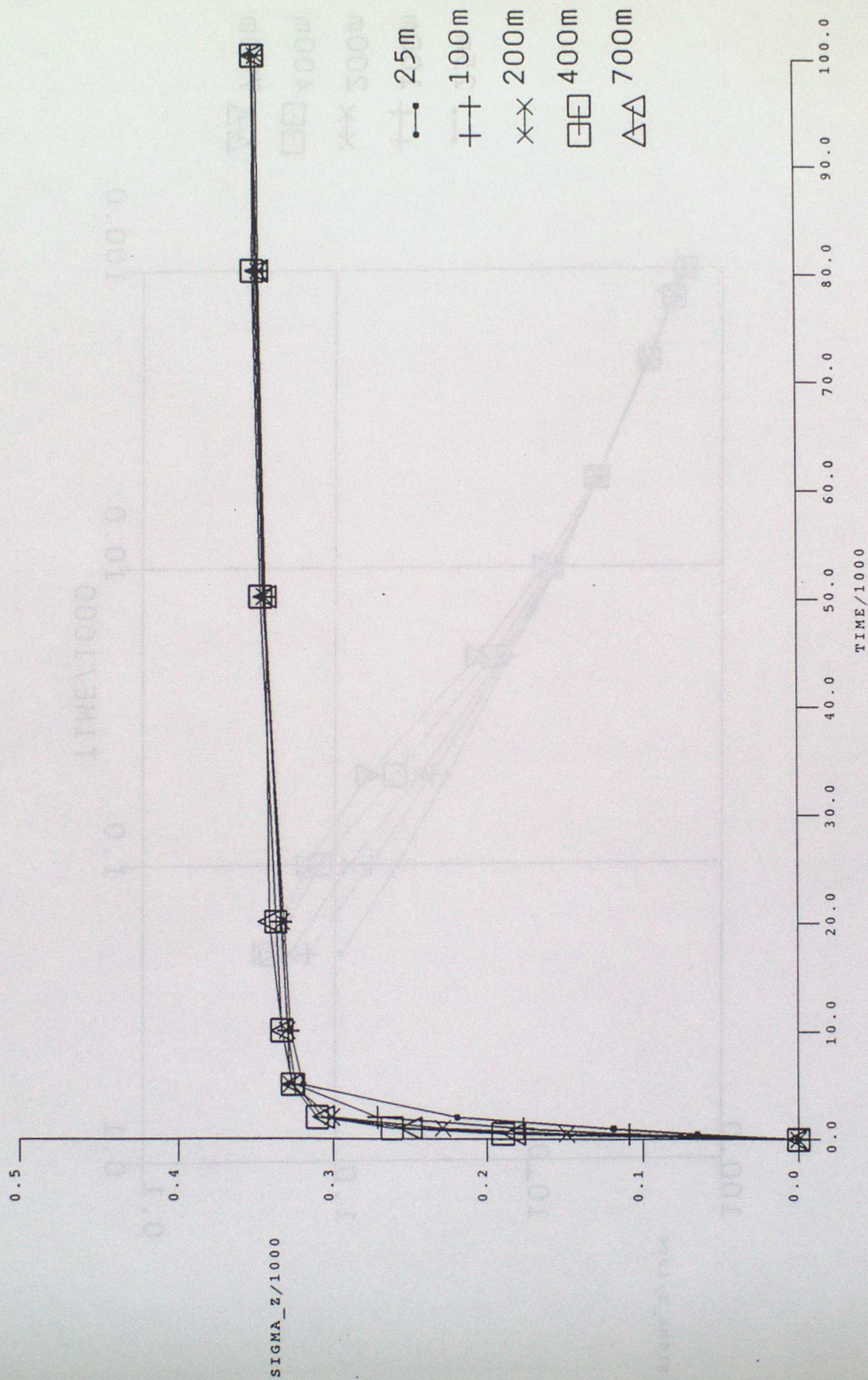


Figure 3

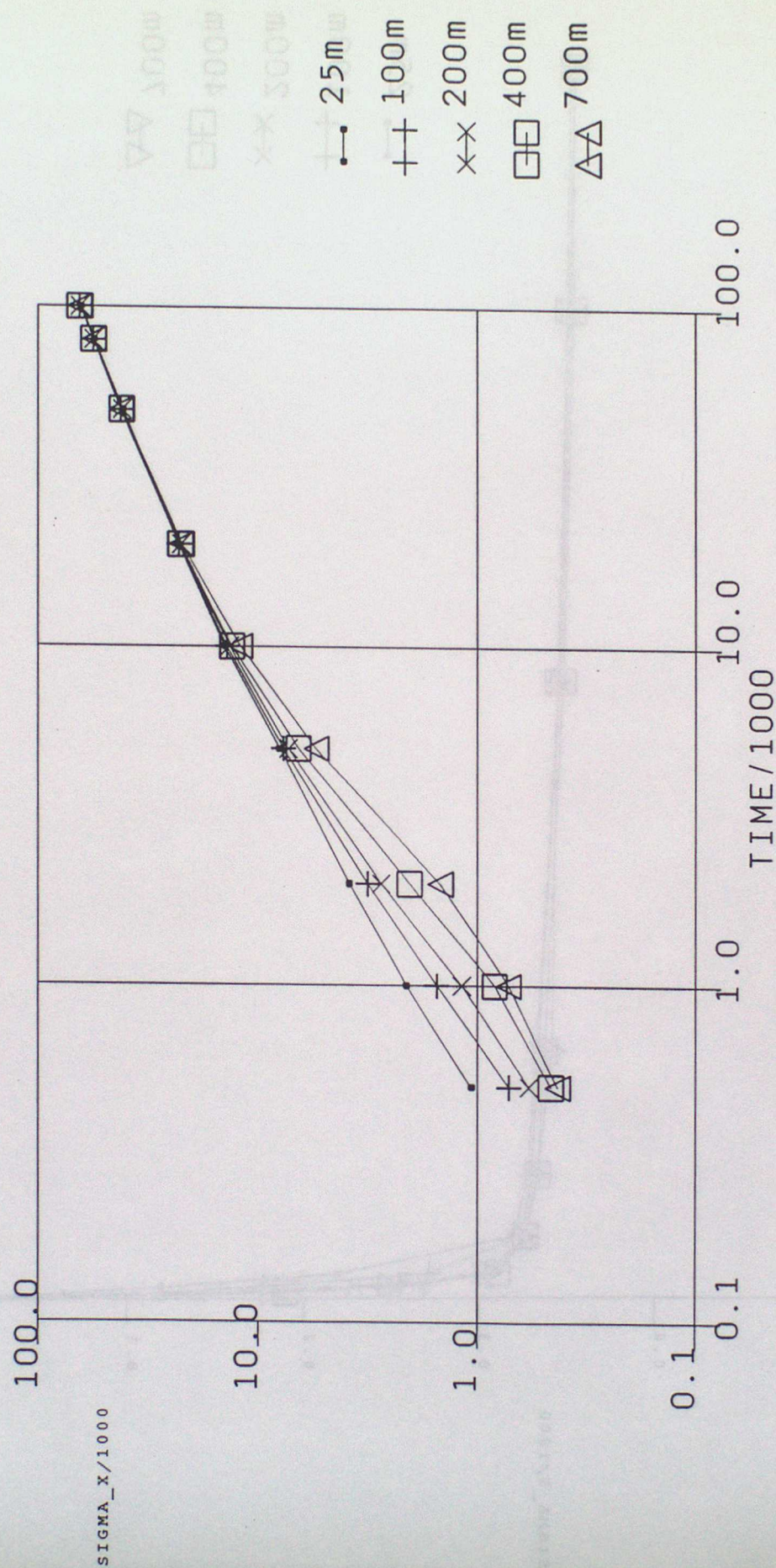


Figure 4

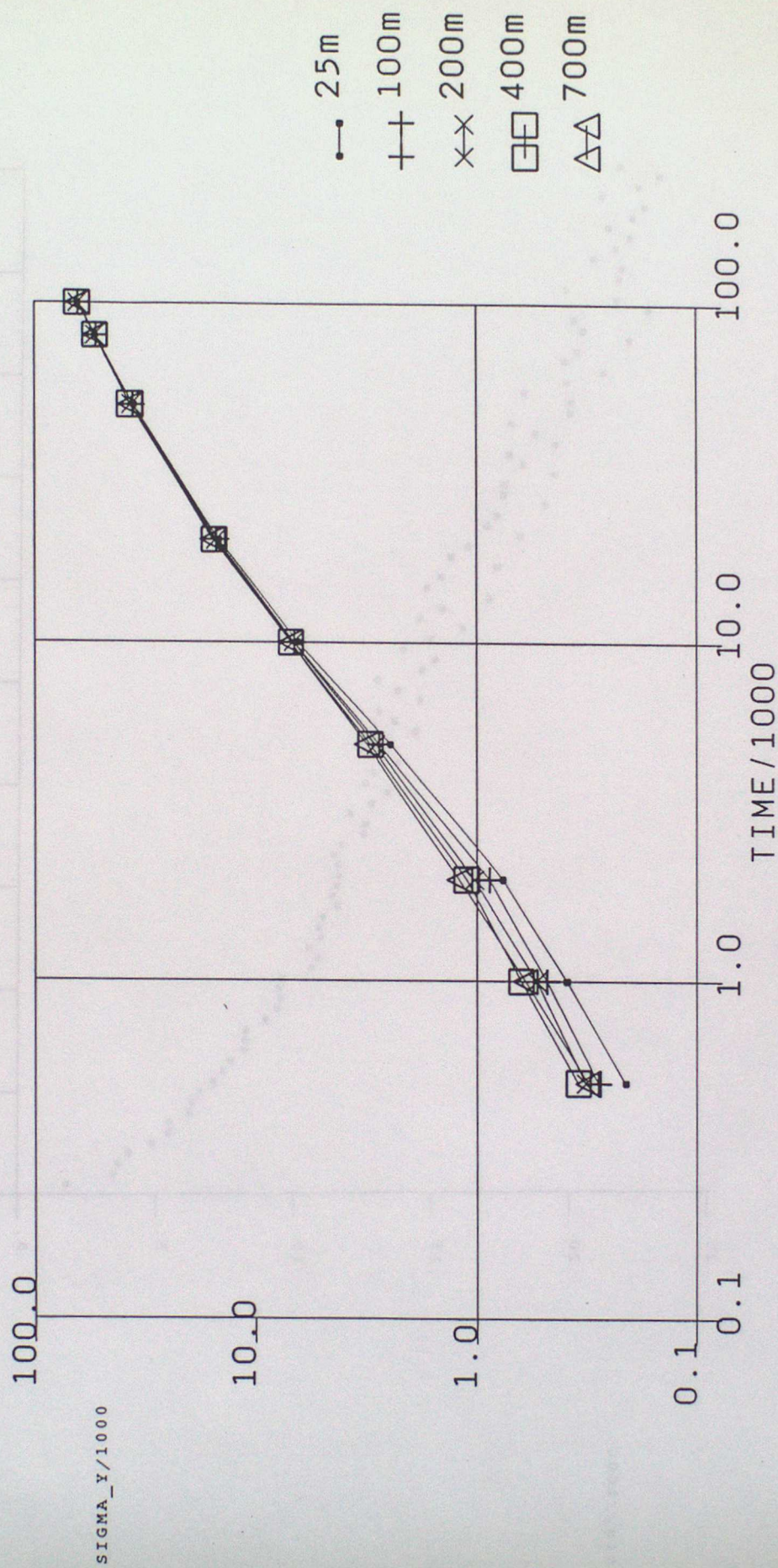


Figure 5

