Long-Range Forecasting and Climate Research

The climate of the world

VI — Carbon dioxide and climate (with appendix on simple climate models)

by

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(LRFC 6)

THE CLIMATE OF THE WORLD

VI - CARBON DIOXIDE AND CLIMATE
(with Appendix on simple climate models)

by

D E PARKER, C K FOLLAND AND D J CARSON

BASED ON AN ADVANCED LECTURE DELIVERED TO THE SCIENTIFIC OFFICERS' COURSE, METEOROLOGICAL OFFICE COLLEGE, MARCH 1985

The Appendix is based on an Advanced Lecture delivered by D J Carson in March 1982.

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This Memorandum is the last in a series of six on:

THE CLIMATE OF THE WORLD

By

C K Folland and D E Parker

Based on nine Advanced Lectures delivered by C K Folland to the Scientific Officers' Course 1-7 March 1985, and one Advanced Lecture delivered by D J Carson in March 1982.

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ADVANCED LECTURE No. 9

CARBON DIOXIDE AND CLIMATE VARIATION

by D E Parker and C K Folland

9.1 INTRODUCTION

The evidence is reviewed for the possible past and future impact of changing atmospheric carbon dioxide (CO_2) levels on climate. The influence of other radiatively active gases is also considered.

9.2 EVIDENCE FOR INCREASING ATMOSPHERIC CARBON DIOXIDE

Direct and reliable measurements of the amount of CO_2 in the atmosphere are available only from 1958, and show (Figure 9.1) an increase from 315 parts per million (ppm) to over 340 ppm. Until the early 1980's it was generally thought, on the basis of earlier, less accurate measurements, that the pre-industrial concentration was about 290 ppm. However recent measurements of carbon-13 in tree-rings, and of carbon dioxide concentration in air bubbles trapped in polar ice, yield estimates in the range 260 to 280 ppm (see for example Barnola et al (1983)). The basis of the carbon-13 method is that changes in the size of the biosphere, which are likely causes of past changes of atmospheric CO_2 concentration, introduce tiny changes in the atmospheric carbon-13/carbon-12 ratio because organic materials are about 1.8% deficient in carbon-13 relative to the nominal concentration of carbon 13 in atmospheric carbon dioxide. Possible difficulties with these methods include contamination with present day air (which would cause estimates of past CO_2 concentration to be too high), and more diffusion than expected of carbon-13 in the tree rings or of carbon dioxide locked in polar ice layers.

Analysis of air bubbles in polar ice has been used to extend the record of atmospheric CO_2 concentration back for more than 30,000 years. Figure 9.2 shows the results: of particular note is the low concentration in the late Pleistocene and the rapid (possibly sudden) increase when the ice-sheets melted after the Allerod-Younger Dryas oscillation around 10,000 years ago (Siegenthaler and Wenk (1984)). This increase is likely to have accelerated the global warming during the early Holocene (Sarmiento and Toggweiler 1984)). See also lecture 7.

Estimates of CO_2 input to the atmosphere, as a result of man's activities, yield a rate of increase of atmospheric concentration around double that observed. The discrepancy can largely be accounted for by oceanic absorption processes (Broecker et al (1979)), though a small part of the difference may result from increased photosynthesis in the biosphere because of the increased atmospheric carbon dioxide concentration (Schiff (1981)).

9.3 THE RADIATIVE EFFECTS OF INCREASING ATMOSPHERIC CARBON DIOXIDE

9.3.1 DIRECT EFFECTS

In the 3rd Lecture, a very simple explanation of the greenhouse effect was provided. Figure 9.3, along with Table 3.14 of Kondratyev (1969), illustrates the fact that CO_2 absorbs perhaps 2 to 3 times more outgoing long wave terrestrial radiation than radiation reaching the Earth from the sun: some absorption of the longer solar wavelengths occurs. There is not, at present, 100% absorption of long wave radiation in the wavelengths affected by CO_2 (Figure 9.4): therefore, as the concentration of CO_2 increases, the absorption of outgoing long-wave radiation will increase: absorption is given by

$$I\lambda = I_{\lambda o} \exp\left(-\int_o^\infty k\lambda\rho dz\right) \quad (1)$$

at any given wavelength λ, where $I_{\lambda o}$, I_λ are the outgoing beam intensities at the surface (z = 0) and the top of the atmosphere (z = ∞) respectively; k is the absorption coefficient of the gas, ρ is its density, and integration is with respect to height z, from the surface to the top of the atmosphere (see also Lecture 4). Because CO_2 is distributed throughout the depth of the atmosphere, and because there is a temperature lapse with height, the "effective" re-emission of terrestrial radiation to space will be at intensities corresponding to the temperature at the atmosphere's "centre of mass" of emission (roughly half-way up through the mass of the atmosphere) rather than at the higher intensities corresponding to the higher temperature of the surface (Figure 9.4). Lecture 3 shows that if α, the albedo of the earth-atmosphere system remains unchanged, the outgoing radiation to space from the earth-atmosphere system remains unchanged. Thus for e = 1 (perfectly absorbing "glass"), the outgoing radiation B is still equal to the ingoing solar radiation I. However the fluxes within the earth-atmosphere system change, giving a changed vertical temperature profile within the atmosphere (with the aid of convective and dynamical processes) and a net increase in incident radiation at the surface of the earth.

9.3.2 INDIRECT EFFECTS

Lack of radiative equilibrium at the earth's surface caused by increasing atmospheric CO_2 will cause the ocean surface, as well as the land surface, to warm. As a result, the mixing ratio of atmospheric water vapour will, on a global average, increase. This increase will enhance surface warming because of the increased downward flux of long-wave radiation provided by the extra water vapour in the troposphere (Figures 9.3 and 9.5) ie the 'glass' becomes more opaque so its emissivity increases. Radiative-convective models indicate that the effect of the additional water vapour on the long-wave flux is markedly greater than the direct effect of the additional CO_2 on the long-wave flux (see Figure 9.3 and eg Lal and Ramanathan (1984)).

An increase of global surface temperature might be expected to result in a decrease in the area covered by snow and ice. Since both snow and ice have a larger albedo than water or land, the amount of solar radiation reflected back into space will decrease, further enhancing the global surface warming. The temperature rises are likely to affect atmospheric circulation and, therefore, cloud, providing further potential influences on the earth's radiation budget. Clouds both reflect insolation (encouraging cooling) and absorb terrestrial radiation (encouraging warming) and in a given situation the balance between these two depends intricately on cloud level, cloud thickness and water content, though low clouds tend to cause cooling and cirrus clouds warming. Because of the complex factors involved, and the rudimentary treatment of clouds in atmospheric general circulation models, varying results have been obtained by different modellers who have attempted to simulate the effects of changes in cloud amount when atmospheric CO_2 concentration is increased.

Not all the indirect effects of increased CO_2 need enhance surface warming. For example, Choudhury and Kukla (1979) suggest that increased atmospheric absorption of solar radiation resulting from an increase in atmospheric CO_2 concentration will hinder the melting of snow, given that the surface air temperature is low enough (snow slowly melts in bright sunshine even when air temperature is below 0°C). Also the warming of water surfaces eg the ocean, by direct solar heating may also slightly decrease. These effects would tend to offset, to a small extent, the surface warming that would result from an increased flux of long-wave radiation.

9.3.3 A CAUTIONARY TALE

By considering the energy balance at the surface a few authors (eg Newell and Dopplick (1979) (ND), Idso (1982)) have deduced that a doubling of atmospheric CO_2 concentration would result in a global warming of $^1/_4$°C or less, in conflict with the consensus range of 1.5°C to 4°C obtained by general circulation models and vertical profile (radiative-convective) models. Cess and Potter (1984) have demonstrated that, despite the considerable uncertainties that remain with AGCM results, the results of ND and of Idso can unfortunately be shown to be based on violations of the first law of thermodynamics. ND did not take into account the response of the depth of the atmosphere (they considered the surface in isolation), while Idso implicitly regarded the atmospheric feedbacks as part of the initial forcing.

The global annual average surface energy balance may be expressed, as:-

$$F + LH + SH = Q \qquad (2)$$

F and Q denote the net upward terrestrial and net downward solar radiation fluxes at the surface, while LH and SH are the net upward latent and sensible heat fluxes from the surface. Let the global surface energy balance be initially perturbed by an amount ΔG resulting from a change in the radiation fluxes F and Q due to the influence of a change in CO_2 concentration (Figure 9.3), all other terms being kept fixed and no feedbacks due to changes in cloudiness etc being initially considered. When equilibrium is later reached, the total change in the surface energy balance, allowing for the operation of feedbacks etc is:-

$$\Delta F_D + \Delta F_I + \Delta LH + \Delta SH = \Delta Q_D + \Delta Q_I \qquad (3)$$

where subscript D denotes direct effects in the fluxes F and Q and I the indirect effects that result from feedback processes (ΔQ_I, for example, might result from changes in cloudiness or changes in ice cover). Now by definition $\Delta G \equiv \Delta Q_D - \Delta F_D$, the direct change in net radiation and ΔLH and ΔSH are by definition, special cases of indirect effects of the changes in net radiation. Substituting $\Delta G = \Delta Q_D - \Delta F_D$ yields:-

$$\Delta G = \Delta F_I + \Delta LH + \Delta SH - \Delta Q_I \qquad (4)$$

ie the indirect changes in energy flux at the surface are balanced by the direct change in net radiation at the surface when equilibrium is attained. So ΔG is a measure of either change.

Let $\lambda(T_s)$ be a surface temperature function defined by

$$\frac{dT_s}{dG} = \lambda(T_s)$$

So $\lambda(T_s)$ is the rate of change of surface temperature (a) that directly results from the change in the imposed flux of net radiation or (b) resulting from the total of the the net change in indirect radiative and surface energy fluxes.

From (4):-

$$\frac{1}{\lambda(T_s)} = \frac{dF_I}{dT_s} + \frac{d(LH)}{dT_s} + \frac{d(SH)}{dT_s} - \frac{dQ_I}{dT_s} \qquad (5)$$

(T_s is surface temperature, not air temperature). Equation (5) describes the sensitivity of T_s to the terms of the surface energy balance. These terms are forced to change by ΔF_I, ΔLH, ΔSH and $-\Delta Q_I$ respectively when CO_2 increases. If a surface temperature change ΔT_s occurs then from (4)

$$\frac{1}{\lambda(T_s)} = \frac{\Delta F_I}{\Delta T_s} + \frac{\Delta LH}{\Delta T_s} + \frac{\Delta SH}{\Delta T_s} - \frac{\Delta Q_I}{\Delta T_s}$$

or $$\Delta T_s = \lambda(T_s) \times \Delta G \qquad (6)$$

Equations (4), (5) and (6) are statements of the first law of thermodynamics (total change in surface heating = total change in internal energy of the surface layer + total of work done by the surface). Note that the definition of λ refers to the equilibrium response of each part of the climate system including all feedbacks and so is a sum of total derivatives of the fluxes.

Now ND ignored indirect changes in Q, (ΔQ_I) and also assumed a constant atmospheric temperature-height and water vapour content-height profile during the response of the climate system. This latter assumption corresponds to replacing the total derivatives of LH and SH within equation (5) by their partial derivatives. They also, for the same reason, effectively used the direct effect F_D in their balance equation. Thus ND's surface temperature response function, (λ_{ND}), is given by

$$\frac{1}{\lambda_{ND}(T_s)} = \frac{dF_D}{dT_s} + \frac{\delta(LH)}{\delta T_s} + \frac{\delta(SH)}{\delta T_s} \qquad (7)$$

Since the total derivitives $d(LH)/dT_s$ and $d(SH)/dT_s$ include the inhibiting effects on the upward surface energy flux of the moistening and warming of the overlying atmosphere (Fig 9.4) whereas $\delta(LH)/\delta T_s$ and $\delta(SH)/\delta T_s$ do not, the total derivatives of LH and SH imply reduced rates of change of the upward fluxes, compared with the partial derivatives, eg

$$\frac{d(LH)}{dT_s} < \frac{\delta(LH)}{\delta T_s} \qquad (8a)$$

and

$$\frac{d(SH)}{dT_s} < \frac{\delta(SH)}{\delta T_s} \qquad (8b).$$

In fact many models give $d(SH)/dT_s < 0$ whereas the partial derivative is strongly positive. In addition,

$$\frac{dF_I}{dT_S} < \frac{dF_D}{dT_S}$$

because the former includes increased back-radiation from increased atmospheric water-vapour (remember F is positive upwards). Thus equation (7) tends to give a value of λ smaller than does the thermodynamically-consistent equation (5), and ND's calculated surface temperature response would be too small. Of course if ΔQ_I was sufficiently negative eg due to increased cloudiness reducing the solar radiation input to the ground by a sufficient amount and ΔF_I larger (more positive) than expected then small values of λ are possible - but not for the reason stated by ND. The essential point is that one cannot consider the surface in isolation: the whole troposphere-surface system must be taken into account. According to Kiehl and Ramanathan (1982) the doubling of CO_2 concentration will only cause an equilibrium increase of 0.55 Wm^{-2} in downward long wave radiation at the surface, whereas there will be an equilibrium increase of 3.99 Wm^{-2} in downward long wave radiation at the tropopause. The difference results from a modelled reduction in emissivity near the tropopause larger than that near the surface. As a result, the radiative heat balance of the upper troposphere relative to that of the surface will change and the upper troposphere will tend to warm more than the surface; equilibrium will be maintained by changes in convection (here, reduced convection of heat from the surface, and it is this reduction which would play an important role in increasing surface temperature). . ND's equations do not take changes in the vertical distribution of water vapour into account, whereas changes in moist convection are likely to occur and affect surface relative humidity, air-sea temperature difference, surface wind speed etc. Table 9.1 summarises Cess and Potter's analysis of ND's results.

Idso (1982) carried out a very simple calculation: proceeding from one assumed equilibrium state (a 254.4K earth surface with no atmosphere) to a second equilibrium state (the present earth-atmosphere system), he calculated a change of downward infrared surface flux from an initial value of zero to 348 Wm^{-2} for a 33.6°C surface temperature increase. He equated the 348 Wm^{-2} to the forcing ΔG, ie the DIRECT change in net radiation, whereas the downwards flux at the surface of 348 Wm^{-2} in fact includes the net effect of both forcing and response (feedback), ie the sum of the direct and indirect changes. Thus Idso's λ, (about 0.1), calculated using (6) with ΔT_s = 33.6°C, was much too small.

9.4 NUMERICAL MODELLING RESULTS

Numerical models may be used to make detailed studies of the above feedbacks and terms in the energy balance equation. The present discussion will concentrate on the results of Mitchell and Lupton (ML) (1984). We begin however with a brief summary of CO_2-induced changes of temperature predicted by several general circulation models.

Table 9.2 summarises the basic properties of 11 GCM experiments, and their predicted global-mean surface warmings resulting from a doubling of carbon dioxide. Note that the predicted global-mean surface warming is in the region of 2°C to 3°C for a doubling of atmospheric CO_2, and that warming is likely to be greater in polar latitudes, partly because of the ice-albedo feedback effect. A further common feature is a predicted stratospheric cooling (see eg Manabe and Wetherald (1980), Mitchell (1983)), of about 5°C to 10°C. The stratosphere cools because its emissivity, just like that of the troposphere, increases as the concentration of carbon dioxide increases within it but, unlike the situation in the tropopshere, the extra absorption of long-wave radiation above the stratosphere is insufficient to offset the increased emission. It is more difficult to find common changes of atmospheric circulation, regional precipitation etc between the GCM results but there is a general tendency to reduce soil moisture in mid-latitudes in summer largely because of increased evaporation in the warmer climate. So concern over possible effects of an increase in atmospheric CO_2 on crop growth in mid-latitudes has been expressed.

The AGCM experiment of Mitchell and Lupton (1984), where CO_2 is increased by a factor of four above present levels, differs from other work reported in Table 9.2, in that a separate calculation of predicted changes in surface heat fluxes appropriate to a quadrupling of carbon dioxide was used to estimate a time-invariant but latitudinally-varying sea-surface temperature (SST) anomaly relative to a climatology appropriate to today. This perturbed SST distribution was applied as a fixed external boundary condition. Using results from Mitchell (1983), where CO_2 was doubled and SST was increased by a fixed 2°C in one set of experiments (C2S2), and CO_2 was doubled, without an increase of SST, in another set, zonally averaged changes in net surface heat flux due to those changes were estimated and averaged over one or more annual cycles for bands of latitude 15° wide. Figure 9.6 (dashed line) illustrates these flux changes for C2S2. It was then assumed (probably justifiably) that the response of model variables was linear with change in sea surface temperature and logarithm of carbon dioxide concentration. Then separate estimates were made, for each latitude band, of the increase in the flux of heat from the atmosphere to the ocean due to quadrupling carbon dioxide, (F), and the increase in the flux of heat, H, from the ocean to the atmosphere due to a 1°C increase of SST. The SST change in a given latitude band produced by a quadrupling of CO_2 concentration was then assigned the value F/H. So it was assumed effectively that a local response in SST balances local changes in surface heat flux; this is not quite true, of course because of advective effects. The assumption was relaxed at higher latitudes though, because a marked increase in the advection of atmospheric heat and moisture from low latitudes, could not be neglected. Thus poleward of 40° latitude, the estimated SST changes F/H were replaced by annually averaged changes in ocean mixed layer temperature found by Manabe and Stouffer (1980) (MS). A further increase in SST was imposed at some latitudes because of changes in prescribed sea-ice, and the resulting values smoothed (Figure 9.7). One might add that the above method also assumes the unimportance of systematic changes of $^F/_H$ with longitude, such as might result from large-scale longitudinal variations in average SST. These SST variations however reach 6°C-8°C in the S Pacific between 0° and 15°S (Lecture 2). In reality, therefore, the true CO_2-induced changes of SST will almost certainly vary with longitude, with the result that, because of the nonlinearities in the ocean-atmosphere system, the zonally-averaged SST changes are likely to differ from ML's quite appreciably. The solid line in Figure 9.6 illustrates the extent to which the SST imposed in C4S2 minimised the change of zonal-mean heat flux into the ocean relative to the control.

Figures 9.8a and 9.8b show ML's calculated changes in zonal average atmospheric temperature for quadrupled CO_2 averaged for June to August and December to February. Stratospheric cooling is evident but its small value is not to be taken as significant, because the model has only 5 layers, each about 200 mb thick. Enhanced surface warming at high latitudes is mainly a winter phenomenon.

ML present a variety of results inducing changes in zonal-mean zonal winds, upper tropospheric transient eddy kinetic energy, poleward moisture fluxes, and precipitation minus evaporation. The latter is reproduced here as Figure 9.9 and shows the expected enhanced evaporation relative to precipitation over land in summer at mid-latitudes.

The radiation scheme in ML's model has recently been found to underestimate downward longwave radiation fluxes in the lower troposphere by a global average of 13 Wm^{-2}, and by 20 Wm^{-2} over the oceans. So, an improved radiation scheme has been devised (Slingo and Wilderspin (1986)), in which absorption and emission spectra are temperature-dependent and the treatment of the water vapour continuum and its overlap with the carbon dioxide 15 μm band is refined. As a result, there may in future be a slight reduction in the sensitivity of the model's global surface temperature to changes in atmospheric CO_2 concentration.

9.5. THE DELAYING ROLE OF THE OCEANS

The necessarily over-rudimentary treatment of the oceans by ML and by most other modellers highlights the need to develop fully interactive ocean-atmosphere-cryosphere models, despite their computational expense. Initial attempts have been made by Spelman and Manabe (1984) and others, which indicate that the oceans should produce a delay of several decades in CO_2-induced warming as a result of their thermal capacity. In reality, the magnitude of the delay depends on the extent to which the deep and abyssal waters are connected with surface waters. Fig 2.5, Lecture 2, shows that in the tropics the regions of cool ocean water that result from large-scale upwelling are quite extensive. So, the effect of upwelling ocean waters is likely to be important in the transient response of the climate system to changes in CO_2 concentration. Further observational studies are required to clarify this important point.

Observations of ocean and atmospheric temperature suggest that the oceans have already substantially delayed any CO_2-induced warming. Theoretically, the (approximately) 25% increase in CO_2 concentration observed since 1870 should have given a mean global warming of SST of about $^1/_2$°C. However Figure 9.10, which is a smoothed update of the results of Folland et al (1984) indicates that present day (1975-84) marine temperatures when averaged worldwide may be only a little (about 0.1°C) higher than in 1860-1880, and that there has been a substantial globally-averaged fluctuation of order 0.5°C between these dates, with coldest conditions around 1910 and warmest conditions around the 1940-1950's. This fluctuation may itself have resulted in part from atmosphere-ocean interactions due to as yet unknown mechanisms. Figure 9.10 clearly indicates the substantial magnitude of the 'noise', in addition to short-term weather and ENSO-related fluctuations, against which the 'signal' of CO_2-induced global warming remains to be detected.

9.6. EFFECTS OF OTHER RADIATIVELY ACTIVE GASES ON ATMOSPHERIC TEMPERATURE

The trace gases of interest include nitrous oxide (N_2O), methane (CH_4), chlorofluorocarbons ($CFCl_3$ and CF_2Cl_2), sulphur dioxide (SO_2), ozone (O_3), and stratospheric water vapour from aircraft. The effects on surface temperature of changes in the concentrations of these gases is summarised by Mitchell (1984): for likely changes, he finds that the total effect of these gases on surface temperature may be considerably less than that resulting from a doubling of carbon dioxide, though not negligible by comparison. Lacis et al (1981), on the other hand, find that, between 1970-1980, the equilibrium warming for the measured increments of CH_4, chlorofluorocarbons and N_2O was probably between 50% and 100% of that for carbon dioxide for the same period. The subject is considered in detail by WMO (1982): their compilation of the climatic effects of increases in trace gas concentration is reproduced as Table 9.3.

REFERENCES

Barnola, J.M., Raynaud, D., Neftel, A. and Oeschger, H.	1983	Comparison of CO_2 measurements by two laboratories on air from bubbles in polar ice. Nature, 303, pp 410-413.
Broecker, W.S., Takahashi, T., Simpson, H.J., and Peng, T.-H.	1979	Fate of fossil fuel carbon dioxide and the global carbon budget. Science, 206, pp 409-418.
Cess, R.D. and Potter, G.L.	1984	A commentary on the recent CO_2 climate controversy. Climatic Change 6, pp 365-376.
Choudhury, B. and Kukla, G.	1979	Impact of CO_2 on cooling of snow and water surfaces. Nature, 280, pp 668-671.
Donner, L. and Ramanathan, V.	1980	Methane and nitrous oxide: their effects on the terrestrial climate. J. Atm. Sci., 37, pp. 119-124.
Fishman, J., Ramanathan, V. Crutzen, P.J. and Liu, S.C.	1979	Tropospheric ozone and climate. Nature, 282, pp. 818-820.
Folland, C.K., Parker, D.E., and Kates, F.E.	1984	Worldwide marine temperature fluctuations 1856-1981. Nature, 310, pp 670-673.
Hameed, S., Cess, R.D. and Hogan, J.	1980	Response of the global climate to changes in atmospheric chemical composition due to fossil fuel burning. J. Geophys. Res., 85, pp 7537-7545.
Hansen, J., Johnson, D., Lacis, A., Lebedeff, S., Lee, P., Rind, D., and Russell, G.	1981	Climate impact of increasing atmospheric carbon dioxide. Science, 213, pp 957-966.
Hansen, J., Lacis, A., Rind, D., Russell, G., Stone, P., Fung, I., Ruedy, R., and Lerner, J.	1984	Climate sensitivity: analysis of feedback mechanisms. In 'Climate Processes and Climate Sensitivity' (J.E. Hansen and T. Takahashi, Eds.), Maurice Ewing Series 5, 368 pp. Amer. Geoph. Union, Washington, DC.
Hansen, J., Russell, G., Rind, D., Stone, P., Lacis, A., Lebedeff, S., Ruedy, R., and Travis, L.	1983	Efficient three-dimensional global models for climate studies: Models I and II. Mon. Weath. Rev., 111, pp 609-662.

Houghton, J.T.	1979	Greenhouse effects of some atmospheric constituents. Phil. Trans. R. Soc. Lond. A.290, pp 515-521.
Hummel, J.R. and Reck, R.A.	1981	The direct thermal effects of $CHClF_2$, CH_3CCl_3 and CH_2Cl_2 on atmospheric surface temperatures. Atmos. Environ., 15, pp 379-382.
ICSU/WMO	1983	Report of the WMO (CAS) meeting of experts on the CO_2 concentrations from pre-industrial times to IGY. WMO Project on Research and Monitoring of Atmospheric CO_2. WCP-53, Geneva.
Idso, S.B.	1982	A surface air temperature response function for Earth's atmosphere. Boundary Layer Met., 22, pp 227-232.
Kiehl, J.T. and Ramanathan, V.	1982	Radiative heating due to increased CO_2: the role of H_2O continuum absorption in the 12-18 μm region. J. Atm. Sci., 39, pp 2923-2926.
Kondratyev, K.Ya	1969	Radiation in the atmosphere. International Geophysics series, Vol. 12. Academic Press, New York and London. 912 pp.
Lacis, A., Hansen, J., Lee, P., Mitchell, T., and Lebedeff, S.	1981	Greenhouse effect of trace gases, 1970-1980. Geoph. Res. Lett., 8, pp 1035-1038.
Lal, M. and Ramanathan, V.	1984	The effects of moist convection and water vapour radiative processes on climate sensitivity. J. Atmos. Sci., 41, pp 2238-2249.
Manabe, S. and Stouffer, R.J.	1979	A CO_2 climate sensitivity study with a mathematical model of the global climate. Nature, 282, pp 491-493.
Manabe, S. and Stouffer, R.J.	1980	Sensitivity of a global climate model to an increase of CO_2 concentration in the atmosphere. J. Geoph. Res., 85, pp 5529-5554.
Manabe, S. and Wetherald, R.T.	1975	The effects of doubling the CO_2 concentration on the climate of a general circulation model. J. Atm. Sci., 32, 3-15.

Manabe, S. and Wetherald, R.T. 1980 On the distribution of climate change resulting from an increase in CO_2 content of the atmosphere. J. Atm. Sci., 37, pp 99-118.

Mitchell, J.F.B. 1983 The seasonal response of a general circulation model to changes in CO_2 and sea temperatures. Quart. J.R. Met. Soc. 109, pp 113-152.

Mitchell, J.F.B. 1984 The effects of pollutants on global climate. Met. Mag. 113, pp 1-16.

Mitchell, J.F.B. and Lupton, G. 1984 A 4 x CO_2 experiment with prescribed changes in sea surface temperatures. Progress in Biometeorology, 3, pp 353-374

National Academy of Sciences (NAS) (Washington) 1979 Carbon dioxide and climate: a scientific assessment. Pp xi + 22.

National Academy of Sciences (NAS) (Washington) 1982 Carbon dioxide and climate: a second assessment. Nat. Acad. Press. Pp xx + 72.

Newell, R.E. and Dopplick, T.G. 1979 Questions concerning the possible influence of anthropogenic CO_2 on atmospheric temperature. J. Appl. Met., 18, pp 822-825.

Newell, R.E., Kidson, J.W., Vincent, D.G. and Boer, G.J. 1974 The General Circulation of the Tropical Atmosphere and Interactions with Extratropical Latitudes. 2 Vols. MIT.

Ramanathan, V. 1975 Greenhouse effect due to chlorofluorocarbons: climatic implications. Science, 190, pp 50-52.

Sarmiento, J L and Toggweiler, J R 1984 A new model for the role of the oceans in determining atmospheric P CO_2. Nature, 308, pp 621-624.

Schiff, H.I. 1981 A review of the carbon dioxide greenhouse problem. Planet. Space Sci., 29, pp 935-950.

Siegenthaler, U and Wenk, Th | 1984 | Rapid atmospheric CO_2 variations and ocean circulation. Nature, 308, pp 624-626.

Slingo, A. and Wilderspin, R.C. | 1986 | Development of a revised long wave radiation scheme for an atmospheric general circulation model. Q.J.Roy. Met. Soc., 112, in press.

Spelman, M.J. and Manabe, S. | 1984 | Influence of oceanic heat transport upon the sensitivity of a model climate. J. Geoph. Res., 89, C1, pp 571-586.

Wang, W.C., Yung, Y.L., Lacis, A.A., Mo, T. and Hansen, J.E. | 1976 | Greenhouse effects due to man-made perturbations of trace gases. Science, 194, pp 685-690.

Wang, W.C., Pinto, J.P. and Yung, Y.L. | 1980 | Climatic effect due to halogenated compounds in the earth's atmosphere. J. Atm. Sci., 37, pp 333-338.

Washington, W.M. and Meehl, G.A. | 1983 | General circulation model experiments on the climatic effects due to a doubling and quadrupling of carbon dioxide concentration. J. Geoph. Res., 88, C11, pp 6600-6610.

Wetherald, R.T. and Manabe, S. | 1981 | Influence of seasonal variation upon the sensitivity of a model climate. J. Geoph. Res., 86, C2, pp 1194-1204.

Williams, W.J., Kosters, J.J., Goldman, A. and Murcray, D.G. | 1976 | Measurements of stratopsheric halocarbon distributions using infrared techniques. Geophys. Res. Lett, 3, pp 379-382.

WMO | 1982 | Report of the Meeting of Experts on Potential Climatic Effects of Ozone and other Minor Trace Gases, Boulder, Colorado, 13-17 September 1982. WMO Global Ozone Research and Monitoring Project, Report No. 14.

WMO/UNEP/ICSU | 1981a | On the assessment of the role of CO_2 on climate variations and their impact. WMO, Geneva 33 pp. Joint WMO/ICSU/UNEP meeting of experts, Villach. Nov. 1980. WCP-3, Geneva.

WMO/UNEP/ICSU	1981b	Papers presented at the WMO/ICSU/UNEP Scientific Conference on Analysis and Interpretation of Atmospheric CO_2 data. (Bern, 14-18 September 1981). WCP-14, Geneva.

Table 9.1. Effects of a doubling of atmospheric CO_2 (from the 2-dimensional model of Cess and Potter (1984))

Quantity		Value
ΔT_s		1.87°C
ΔG		1.69 Wm^{-2}
$\frac{dF_{\downarrow}}{dT_s}$	($F_{\downarrow}$ is downward component of terrestrial radiation at the surface)	5.87 Wm^{-2} °C^{-1}
$\frac{dF_{\uparrow}}{dT_s}$		-0.24 Wm^{-2} °C^{-1}
$\frac{dQ}{dT_s}$		-0.39 Wm^{-2} °C^{-1}
$\frac{d(LH)}{dT_s}$		1.11 Wm^{-2} °C^{-1}
$\frac{d(SH)}{dT_s}$		-0.35 Wm^{-2} °C^{-1}
$\frac{\delta(LH)}{\delta T_s}$		51.64 Wm^{-2} °C^{-1}
$\frac{\delta(SH)}{\delta T_s}$		18.26 Wm^{-2} °C^{-1}

TABLE 9.2

Model Characteristics	MW75	MW80	MS80	$WM81_1$	$WM81_2$
Domain	$0° < \lambda < 120°$ $0° < \phi < 81.7°$	$0° < \lambda < 120°$ $0° < \phi < 90°$	Global	$0° < \lambda < 120°$ $-90° < \phi < 90°$	
Land-ocean distribution	Ocean for $60° < \lambda < 120°$ $0° < \phi < 66.5°$	Ocean for $60° < \lambda < 120°$ $0° < \phi < 90°$	Realistic	Ocean for $60° < \lambda < 120°$ $0° < \phi < 90°$	
Ocean	Swamp$^+$	Swamp$^+$	Mixed layer	Mixed layer	
Seasonal variation	No	No	Yes	Yes	No
Cloud Feedback	No	Yes	No	No	
Horizontal resolution	About 500 km	5° in long 4.5° in lat	Spectral models with max zonal wavenumber 15	Spectral models with max zonal wavenumber 15	
Global mean surface temp rise (K)	2.9	3.0	2.0*	2.4*	3.0*
Surface temp rise at lat 70°	About 7	About 6	About 3*	About 4*	About 6*

Table 9.2 . Summary of characteristics of GCMs. Also given are the global mean temperature rise (K) and the zonal mean temperature rise at 70° latitude (mean of 70°N and 70°S where appropriate) for a doubling of CO_2 concentration. An asterisk implies that the value given is half the value obtained in a CO_2-quadrupling experiment. The models are those of Manabe and Wetherald (1975) -MW75; Manabe and Wetherald (1980) - MW80; Manabe and Stouffer (1979 and 1980) - MS80; Wetherald and Manabe (1981) $WM81_1$ and $WM81_2$; Hansen et al (1983, 1984) - H1 and H2; Washington and Meehl (1983) - WWGM; Spelman and Manabe (1984) - SM; Mitchell (1983) - M83; Mitchell and Lupton (1984) - ML84.

+ A 'swamp' ocean has zero heat capacity but an unlimited supply of moisture for evaporation.

Table 9.2 (continued)

H1	H2	WWGM	SM	M83	ML84
Global	Global	Global	0°<λ<120°	Global	Global
Realistic	Realistic	Realistic	Ocean for 60°<λ<120°	Realistic	Realistic
Mixed layer	Swamp+	Swamp+	Coupled with currents	Climatological SST +2°C	Climatological SST + latitude-dependent anomaly
Yes	No	No	No	Yes	Yes
Yes	Yes	Yes	No	No	No
10° in long. 8° in lat.	10° in long 8° in lat.	Spectral with max. zonal wavenumber 15	Spectral with max. zonal wavenumber 15	330 Km	330 Km
3.5	3.9	1.3	About 2.6*	2.25 but 2.9 over land	2.3*
Unavailable	Unavailable	1.4	About 4.5*	Similar to global value	About 4*

Table 9.3 From WMO (1982)

Compilation of the climatic effects of trace gas increase. The change in net flux at the tropopause is the reduction in the net (up-down) outgoing longwave flux at the tropopause.

Trace Gas	Band Center cm^{-1}	Reference Mixing Ratio[1] (ppb)	Perturbed Mixing Ratio in Trace Gas (ppb)	Change in Net Flux at Tropopause (W/m^2)	Change in Surface Temperature[5] (°K)	Source[6]
CO_2	667	330×10^3	660×10^3	4.0	2.0	NAS
N_2O	589, 1168, 1285	300	600	0.65	0.3 - 0.4	DR, W1
CH_4	1306, 1534	1500	3000	0.61	0.3	DR, H
O_3(T)	1041, 1103	F(φ, Z)[2]	2 F(φ, Z)	1.24[8]	0.9[4]	H, F
$CFCl_3$ (CFC11)	846, 1085, 2144	0	1	0.31	0.15[7]	R, W1[9]
CF_2Cl_2 (CFC12)	915, 1095, 1152	0	1	0.26	0.13[7]	R, W1[9]
CF_4	632, 1241, 1261, 1283	0	1		0.07	W2
CF_2HCl (CFC22)	1117, 1311	0	1		0.04	HR
CCl_4	776	0	1		0.14[4]	R, W1[9]
$CHCl_3$	774, 1220	0	1		0.1[4]	R, W1[9]
CH_2Cl_2	714, 736, 1236	0	1		0.05[4]	R, W1[9]
CH_3Cl	732, 1015, 1400	0	1		0.013[4]	R, W1[9]
CH_3CCl_3	707, 1084	0	1		0.02	HR
C_2H_4	949	0.2	0.4		0.01	W1
SO_2	518, 1151, 1361	2	4		0.02	W1
NH_3	950	6	12		0.09	W1
HNO_3	1695, 1333, 850, 459	F(Z)[3]	2 F(Z)		0.06	W1
H_2O	0-2000	3×10^3 [10]	6×10^3		0.6	W1

[1]Unless specified, refers to volume mixing ratio. Many of these mixing ratios are chosen for the convenience of sensitivity studies; for realistic values and trends see the previous section. Whenever possible, we have chosen the reference mixing ratio for minor trace gases to be zero, since the effects of these gases are linearly proportional to the mixing ratio, the change in surface temperature reflects the surface warming per ppb of the gases for which the reference mixing ratio is zero.

[2]Ozone distribution varies with latitude and altitude. The doubling applies primarily to tropospheric O_3.

[3]Profile varies with altitude.

[4]These results are for a fixed cloud-top temperature model. All other results are for a fixed cloud-top altitude model.

[5]Global mean surface temperature change. The problem of uncertainties in these values is analogous to CO_2 uncertainties. WMO CO_2 (1981a).

[6]DR = Donner and Ramanathan (1980); F = Fishman et al. (1979); H = Hameed et al. (1980); HR = Hummel and Reck (1981); NAS = NAS (1979, 1982); R = Ramanathan (1975); W1 = Wang et al. (1976); W2 = Wang et al. (1980).

[7]Temperature changes were obtained with a fixed cloud altitude model as described in Ramanathan (1982). The flux changes were obtained from Ramanathan (1975).

[8]The flux change is from H but the surface temperature change is from F.

[9]These results are from R, which are larger than W1 values by about 30%. However, recent estimates by Wang (private communication) are in very good agreement with R.

[10]Stratospheric mass mixing ratio.

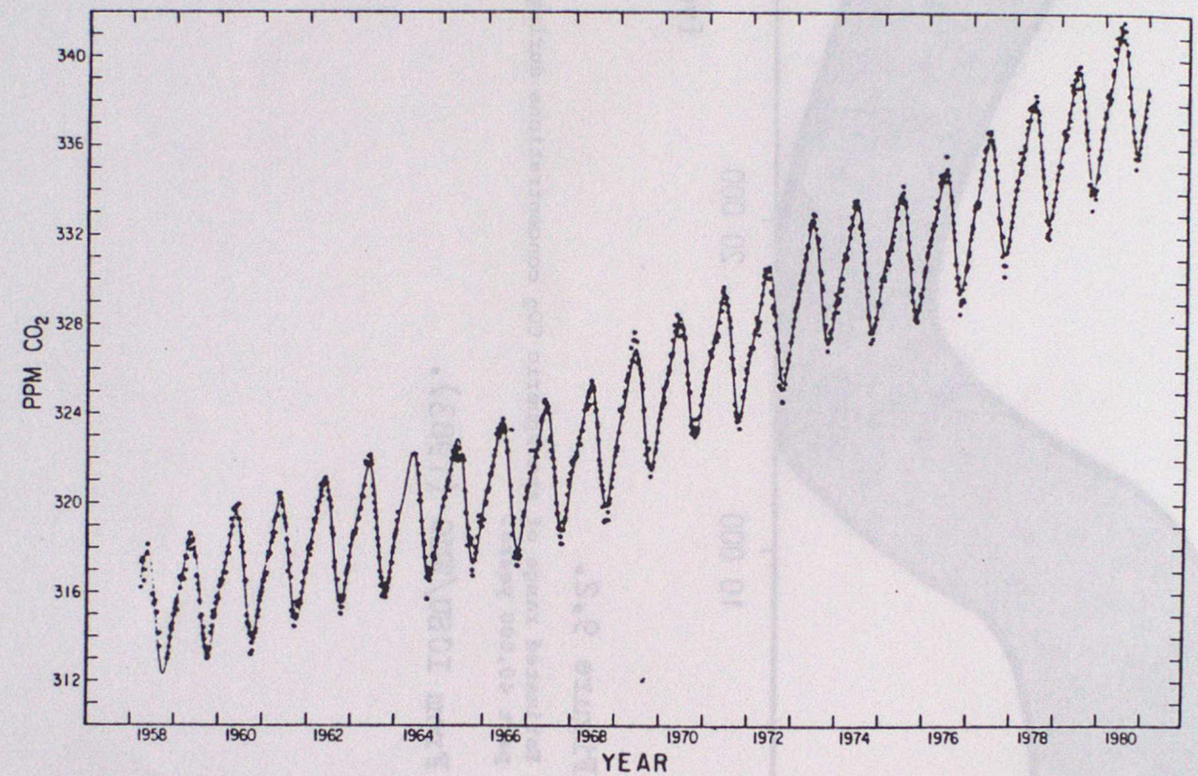


Figure 9.1 Concentration of atmospheric CO_2 at Mauna Loa Observatory, Hawaii, 19.5° N, 155.6° W. Dots are seven day averages of daily averages of blue-lined data (see text). (After WMO/UNEP/ICSU(1981b)).

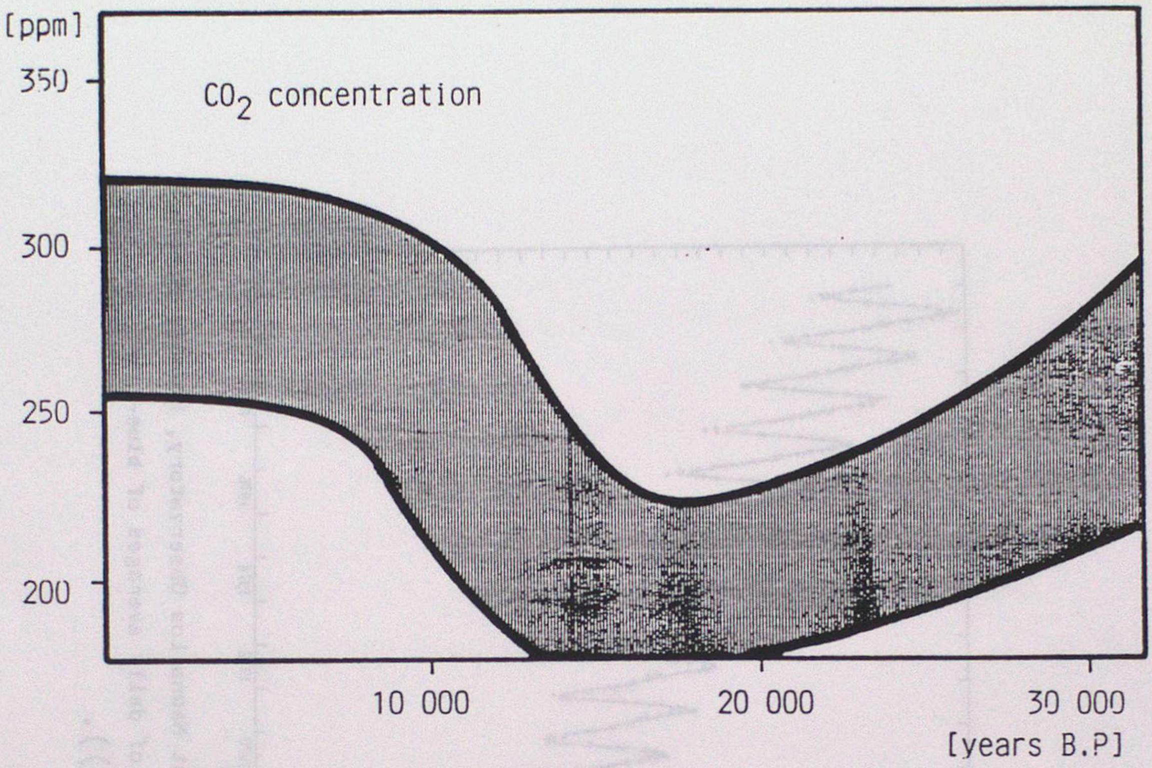
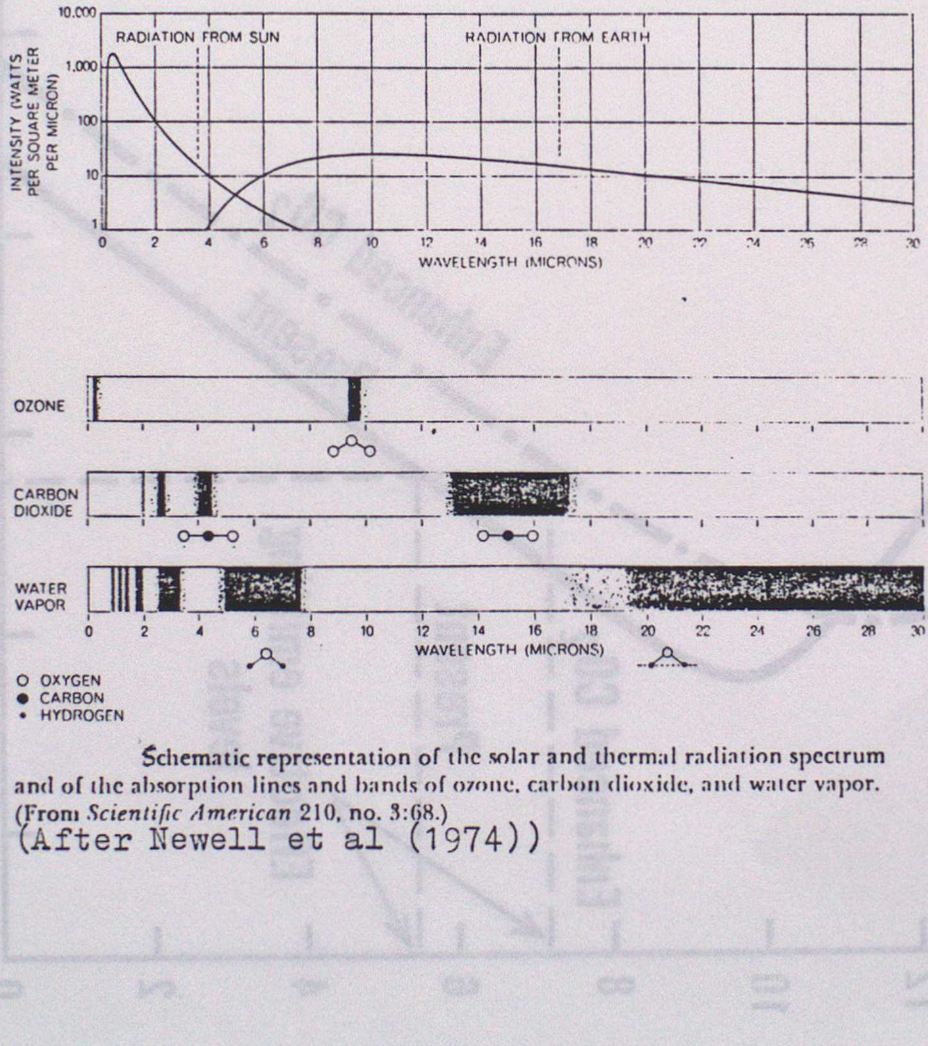


Figure 9.2.

Estimated range of atmospheric CO_2 concentrations during the past 40,000 years.'

From ICSU/WMO (1983).

Figure 9.3.



Schematic representation of the solar and thermal radiation spectrum and of the absorption lines and bands of ozone, carbon dioxide, and water vapor. (From *Scientific American* 210, no. 3:68.)
(After Newell et al (1974))

Figure 9.4

Effect of increasing CO_2 on the vertical profile of temperature (schematic)

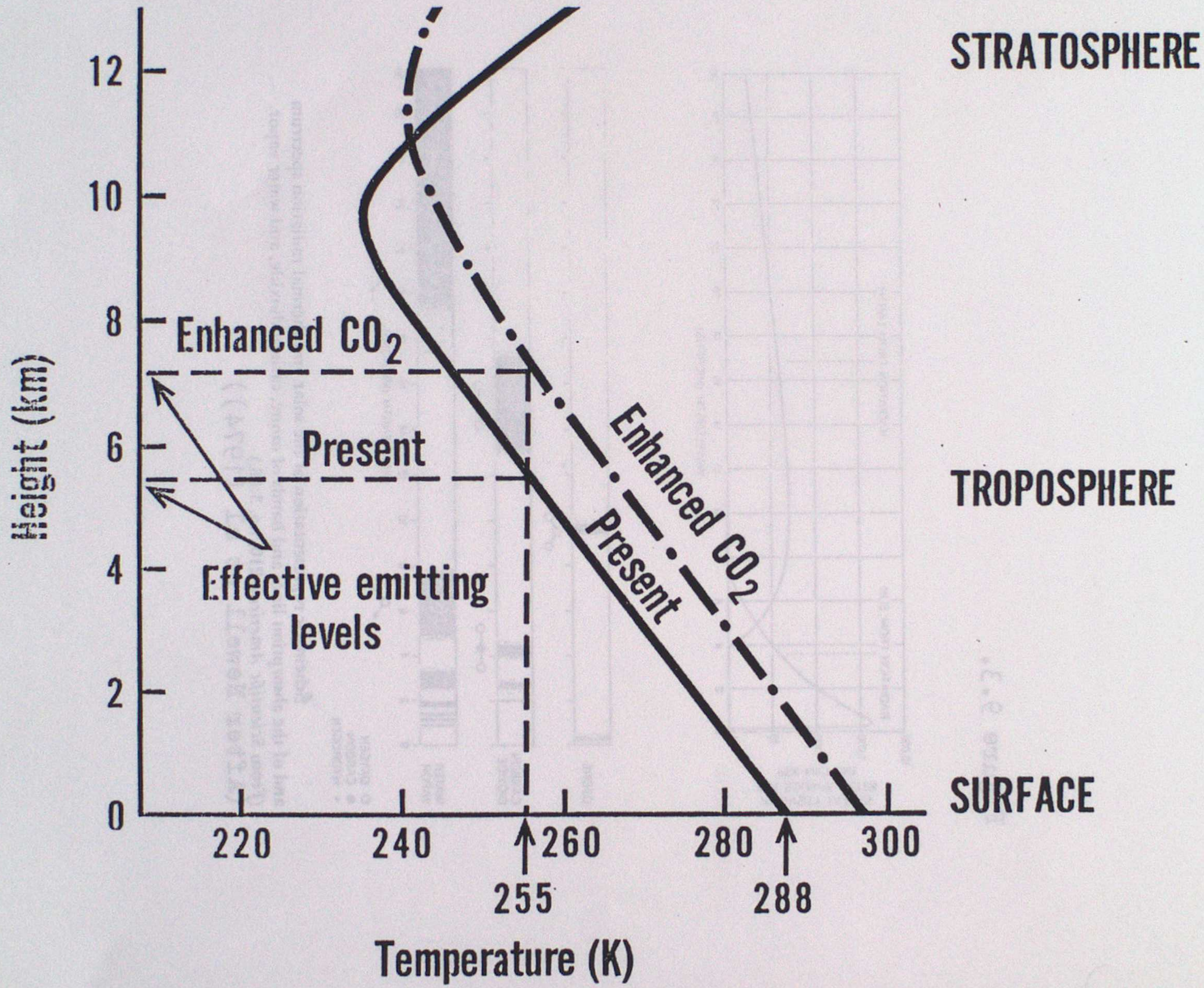


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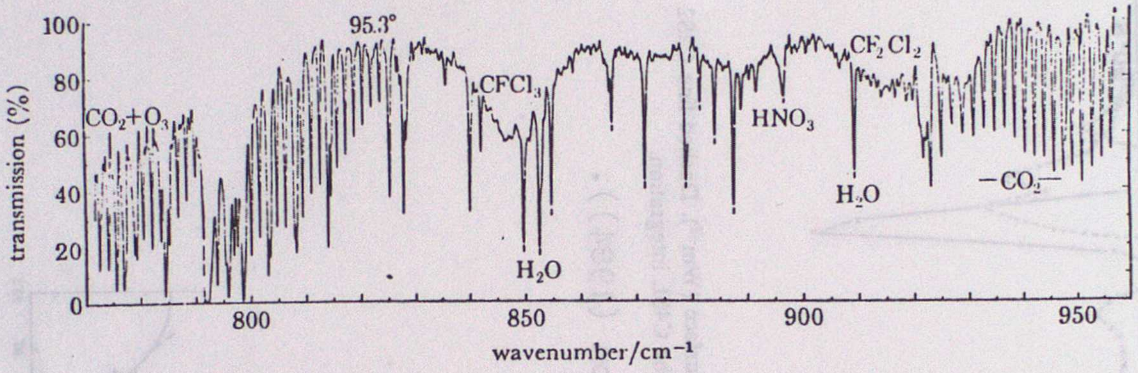
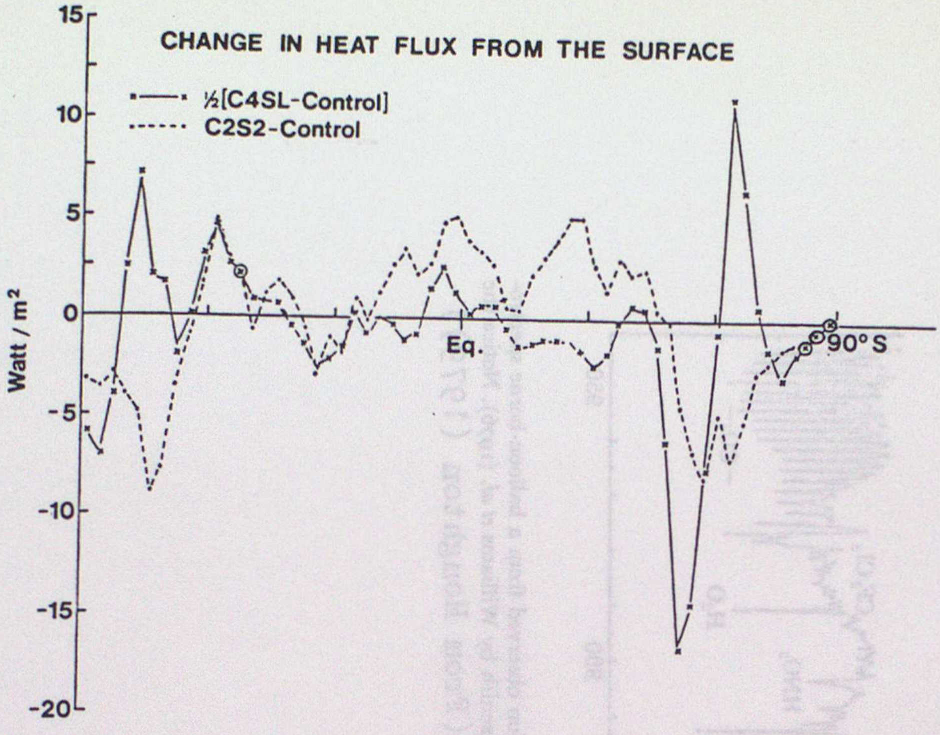
Figure 9.5



Atmospheric transmission in the atmospheric window region observed from a balloon-borne spectrometer from a height of 30 km along a limb path at 95.3° to the zenith by Williams *et al.* (1976). Notice the absorption bands of $CFCl_3$ and CF_2Cl_2. Resolution is 0.2 cm^{-1}. (From Houghton (1979))

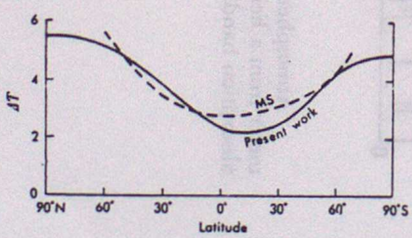
Figure 9.6.



Change in the net flux of heat from the surface (Wm^{-2}). Dashed line C2S2 integration; solid line, half the change in the C4SL integration.

(From Mitchell and Lupton (1984)).

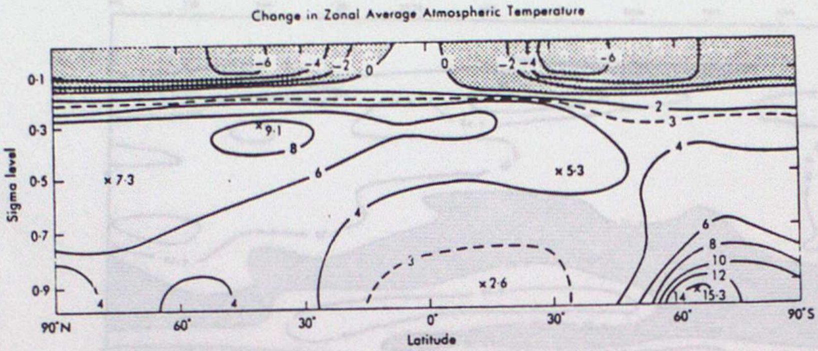
Figure 9.7.



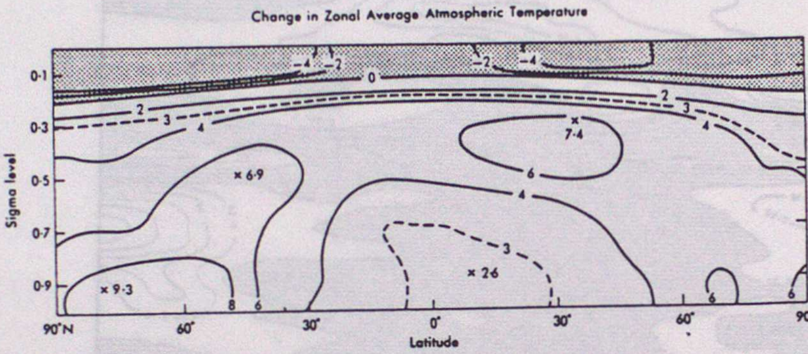
Changes in sea surface temperature. Solid line, prescribed in present work; dashed line, annual mean mixed layer temperature changes estimated from Figure 18 of Manabe and Stouffer, 1980.

(From Mitchell and Lupton (1984)).

Figure 9.8.



(a)



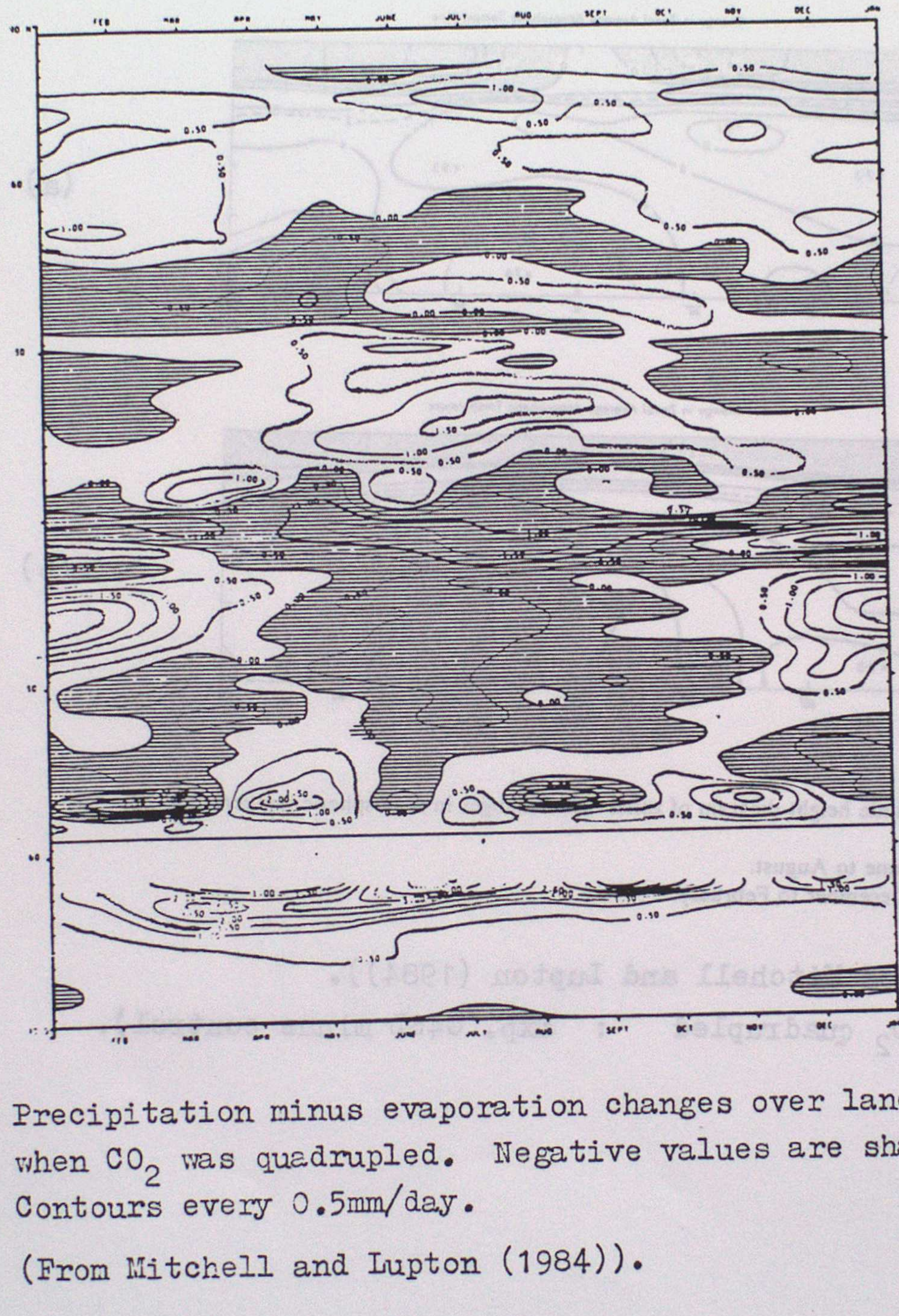
(b)

Latitude height diagram of zonal mean changes in atmospheric temperature (K).
a. June to August.
b. December to February.

(From Mitchell and Lupton (1984)).

(CO_2 quadrupled : Exp. C4SL minus control).

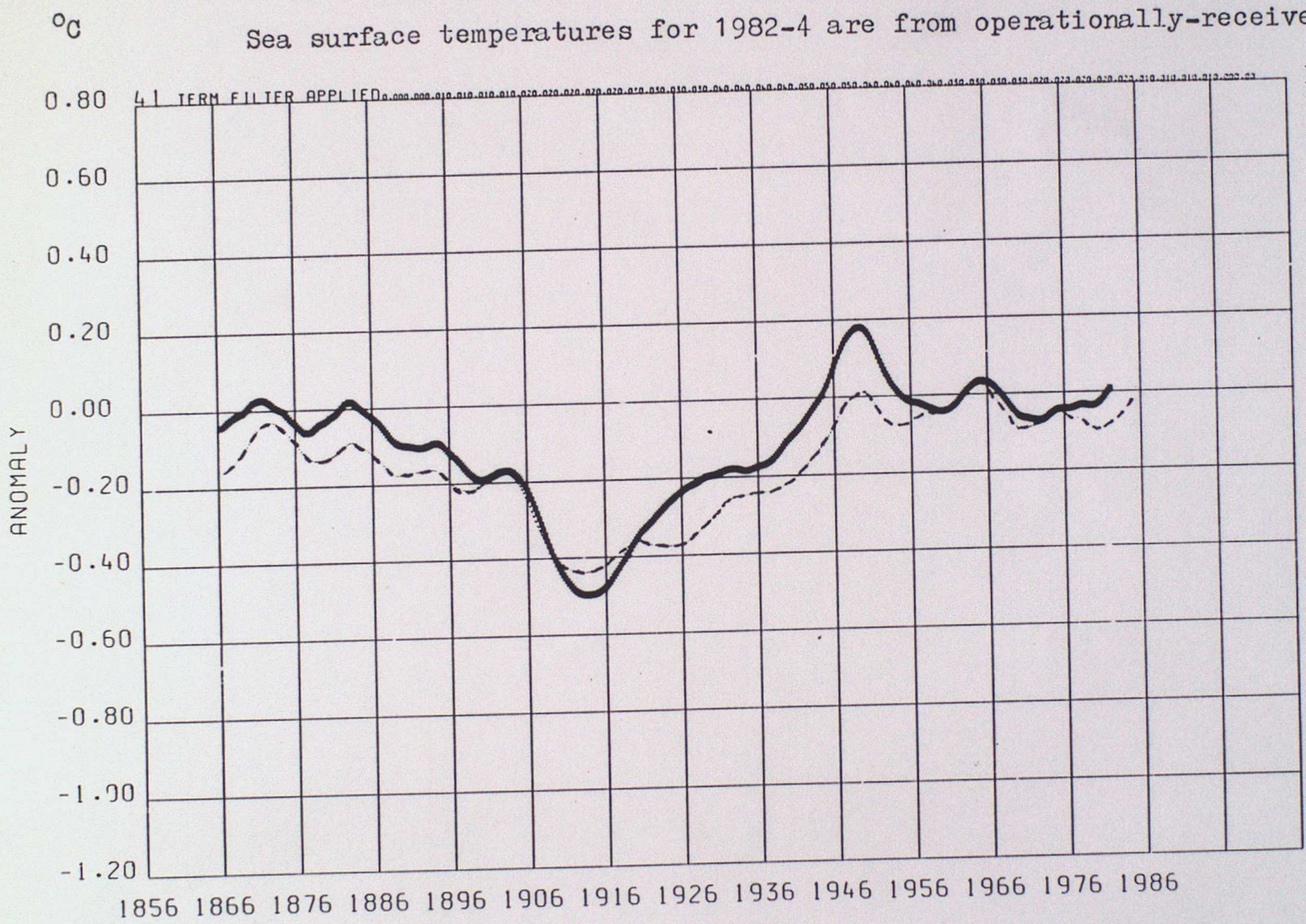
Figure 9.9.



Precipitation minus evaporation changes over land when CO_2 was quadrupled. Negative values are shaded. Contours every 0.5mm/day.

(From Mitchell and Lupton (1984)).

Figure 9.10. Anomalies (w.r.t. 1951-60) of global night marine air temperature (solid) and sea surface temperature (dashed).
Instrumental corrections as in Folland, Parker and Kates (1984), but +0.03, +0.04 for 1971-5, 1976-81 night marine air temperature.
Sea surface temperatures for 1982-4 are from operationally-received reports from ships.



Plotting is at end of filter interval

VS APPENDIX I

TO 1985 ADVANCED LECTURES ON THE CLIMATE OF THE WORLD

METEOROLOGICAL OFFICE - ADVANCED LECTURES 1982

THE CLIMATE OF THE WORLD

Lecture 5

Simple Energy-balance Climate Models

by

D J Carson

Met O 13

March 1982

Simple Energy - balance Climate Models

by

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1. Introduction

An increasing awareness of man's vulnerability to climate variations and a growing belief that his activities may have a significant influence on the climate have led, over the past decade, to the World Climate Conference in 1979 (WMO, 1979) and to the formulation of the World Climate Programme (WMO, 1980), one component of which is the programme for research on climate change and variability, the World Climate Research Programme (WCRP). One of the main elements in the WCRP is the development and evaluation of climate models to be used for the study of climate predictability and climate sensitivity.

There is as yet no comprehensive theory of climate to explain its variability, nor are there, at this juncture physical-dynamical models which can simulate adequately the subtleties and complexities of the full climate system. Indeed there exists an almost continuous spectrum of types of so-called climate model, ranging from simple, one-dimensional, equilibrium representations of the vertical radiative processes in the atmosphere to the full complexity of the three-dimensional, time-dependent, global general circulation models. The latter, in which the effects of large-scale disturbances in the atmosphere are included explicitly, are sometimes referred to as comprehensive models in contrast to the simple models in which many more dynamical and physical processes have to be parametrized. There is of course debate about the advantages and deficiencies of each class of model but at present it is probably fair to claim that each type has a rôle to play in furthering our understanding of the real climate system. It should be obvious, for example, that some of the time scales of interest in climate modelling make it both economically and practically impossible to apply the fully comprehensive models in many studies and so the simpler, usually less realistic, models have to be used.

The Meteorological Office has a major commitment to develop its atmospheric general circulation models to produce, ultimately, a comprehensive, coupled atmosphere - land - ocean - cryosphere climate model. The structure and performance of such general circulation models have been discussed, for example, by Dr A Slingo in the complementary lecture series on The General Circulation of the Atmosphere. The aim of this lecture is not to review the state of the art of simple climate modelling (there exists too large a body of literature!) but rather it is to introduce the reader to the concept of such models and to the assumptions made in the simplest examples. Their application to climate-related problems and the somewhat surprising properties of their solutions will also be discussed.

Before describing the models it is worth introducing some terms which are likely to be encountered, increasingly without being explained, in the literature concerned with the design of climate models and their application to sensitivity and predictability studies.

Predictability of Different Kinds

Consider the two questions:

(1) Is climate predictable in some sense or another and at some time range ?

(2) Are man's activities influencing the climate in any way ?

Different modelling methodologies are required to tackle the two, generally distinct, aspects of climate predictability embodied in these questions. Lorenz (1975) recognised these as questions of predictability of the first and second kind, respectively.

Predictability of the First Kind

This is defined as the prediction of the natural variations of the climate system i.e. it is effectively long-range forecasting using the present state of the climate system as an initial condition.

In principle this includes variations from the synoptic scale limit of predictability of a few weeks up to the epochs of glaciations. The nature of climate predictability of the first kind is not understood anywhere in its range. Indeed, a major open question is whether there are any characteristics of the circulation that retain predictability in the range of several weeks to several years. Even if climate predictability of the first kind is possible in principle it remains to be determined how the problem can be approached. The long-range forecasting for a month to a season ahead, which is being researched in the Synoptic Climatology branch (Met O 13), comes into this category at the short-period end of the range. Because such relatively short periods (in climate terms) of prediction are involved at present in such studies, then fully comprehensive general circulation models can be used.

Predictability of the Second Kind

This is defined as the determination of the response of the climate system to changing external influences which might be prescribed or predicted in terms of other dynamical and physical considerations. Such predictions are not concerned directly with the chronological order of occurrence of atmospheric states, but rather with the long-term average statistical properties of the climate (or model).

Most applications of atmospheric general circulation models to climate problems are necessarily predictability studies of the second kind i.e. the typical set of control and perturbation integrations to determine the sensitivity of a simulated climate to prescribed changes in external forcing. A topical example is the study of the impact on the simulated climate of doubling the amount of CO_2 in the atmosphere (see, for example, Mitchell (1981)). It is still an unresolved question whether predictability of the second kind is a meaningful concept or not.

Transitivity, Intransitivity and Almost Intransitivity

An important issue of climate theory is whether the causes of most climate fluctuations are external to the climate system or internal to it, i.e. have past changes been caused, for example, by fluctuations in solar emissions or changes in the earth's orbital parameters (the so-called Milankovich Theory of climate change), or can these changes be explained to a large extent by the intrinsic internal variability within the system, which needs only an external trigger ? This question has been given a more precise mathematical formulation by Lorenz (1968, 1970, 1975) who introduced the concepts of transitivity, intransitivity and almost intransitivity, taken from ergodic theory.

Transitivity. If the very long-term averaged statistics of a climate system (or climate model) are independent of the initial state, then that climate (or model) is said to be transitive.

Intransitivity. If no unique set of such equilibrium climate statistics exists then the climate (or model) is called intransitive, i.e. there are two or more sets of statistical properties and some initial states lead to one set while others lead to another.

Almost Intransitivity. If the climate system (or model) is transitive for statistics taken over an infinite period and yet contains shorter-period self-fluctuations that might be interpreted as climate changes indicative of an intransitive climate system, then it is called almost intransitive. An almost intransitive system could be mistaken for a truly intransitive one if the statistics over long but finite times were identified as those over an infinite period. For example, the existence of glacial and interglacial periods raises the possibility that the climate system is almost intransitive.

There exists the possibility that for fixed external conditions, very different climate states may be implied by finite-interval time averages owing solely to intransitivity or almost intransitivity. The association of a particular climate with a given external state requires an assumption (as yet unfounded) of transitivity of the climate. With any climate model it is very important to determine the sensitivity of its equilibrium state statistics to variations of internal (e.g. initial) conditions. It will be discussed below that some of the simplest climate models are intransitive and do in fact support two steady state solutions corresponding to glacial and interglacial climates.

2. Radiative-convective Thermal Equilibrium Models (Vertical Column Energy-balance Models)

2.1 Introduction

This class of one-dimensional models computes the annually and horizontally averaged, global, surface and atmospheric temperatures. The temperature profile is determined essentially from a balance between the incoming short-wave (or solar) and the outgoing long-wave (alternatively called terrestrial, infrared or thermal) radiative fluxes, which, in turn, depend upon specified surface properties and vertical distributions and optical properties of clouds, aerosols and of the most important of the optically active gases such as water vapour, carbon dioxide and ozone. A method of convective adjustment is used to ensure that realistic vertical temperature profiles are obtained.

The first radiative-convective models for the earth's atmosphere were developed by Manabe and Strickler (1964) and Manabe and Wetherald (1967). Further variations of the approach have also been developed and are discussed in the comprehensive review by Ramanathan and Coakley (1978). Radiative-convective modelling is a technique commonly employed in studies of stellar and planetary atmospheres.

The paper by Manabe and Wetherald (1967) is to some extent a repeat of the computation of radiative-convective equilibrium of the atmosphere which was discussed in the earlier paper by Manabe and Strickler (1964). The main difference is that Manabe and Wetherald specify a given distribution of relative humidity instead of a given distribution of absolute humidity as used in Manabe and Strickler. The former assumption is thought to be the more realistic. The prime objective in both cases was to formulate and test a method to incorporate radiative transfer into a general circulation model.

Note that with respect to the formulation of the radiative transfer processes in the single column the radiative-convective models are not necessarily simple. Because so much else is parametrized in such a simple way the combination of high vertical resolution and the small demand made on computing resources enables the radiative processes to be modelled in a very detailed fashion. It is not the purpose of this lecture to examine the subtleties and complexities of the actual schemes used by Manabe and his colleagues; for this information the reader is referred to the original papers.

Further objectives of such modelling were:

(1) To determine the influence of various factors such as the solar constant, cloudiness, surface albedo and the various atmospheric absorbers in maintaining the existing thermal structure of the atmosphere.

(2) To determine the rôles of the above factors in maintaining the observed climate at the earth's surface.

2.2 Model assumptions and structure

a. Pure radiative equilibrium

In order to prepare for the study of thermal equilibrium with convective adjustment, calculations were first done for the simple case of purely radiative equilibrium, in which the equilibrium state was approached as an asymptotic steady state of an initial value problem. Equilibrium is considered to have been reached when the temperature change at each level over the time-step used in the model is less than a specified small value.

The following constraints are also satisfied in the equilibrium state:

(1) At the earth's surface, the net upward flux of long-wave radiation is equal to the net downward flux of solar radiation. This guarantees radiative equilibrium at the surface.

(2) At the top of the atmosphere the net upward flux of long-wave radiation is equal to the net downward solar radiation. This guarantees the radiative equilibrium of both the atmosphere and the earth as a whole.

The unrealistic temperature profiles which result when radiation alone is acting have been described by Dr A Slingo in the complementary lecture series on the General Circulation of the Atmosphere and so there is no need to discuss them further here.

b. Radiative-convective equilibrium

The procedure of convective adjustment is to modify the lapse rate to the critical lapse rate whenever that critical lapse rate is exceeded in the course of the integration. The critical lapse rate generally adopted is 6.5 K km^{-1} which was assumed to be a reasonably realistic value for the observed tropospheric lapse rate of temperature. The process of convective adjustment will transfer heat energy from the surface into the atmosphere and thereby permit more realistic temperatures to occur throughout the troposphere.

The requirements for radiative-convective equilibrium are:

(1) At the top of the atmosphere, the net incoming solar radiation should be equal to the net outgoing long-wave radiation.

(2) No temperature discontinuity should exist.

(3) Free and forced convection, and mixing by large-scale eddies, prevent the lapse rate exceeding a critical value of 6.5 K km^{-1}.

(4) Whenever the lapse rate is subcritical, the condition of local radiative equilibrium is satisfied.

(5) The heat capacity of the earth's surface is zero.

(6) The atmosphere maintains the given distribution of relative humidity. (This is a requirement introduced in Manabe and Wetherald (1967)).

Figure 1 gives a simple schematic representation of the iterative procedure used to obtain the state of radiative-convective equilibrium. Both 18 and 9 atmospheric levels were used in the computations such that the pressure thickness of the layer is thin both near the surface and near the top of the atmosphere.

Mathematical Note: r (the humidity mixing ratio) = $\dfrac{0.622\,e}{p-e}$

where e is the water vapour pressure, p is the atmospheric pressure.

h (the relative humidity) = $\dfrac{e}{e_s}$

where e_s is the saturation water vapour pressure.

Therefore, r can be written

$$r = \frac{0.622\,h\,e_s(T)}{p - h\,e_s(T)}$$

where e_s as a function of temperature, T, can be evaluated from the Clausius-Clapeyron relationship,

$$\frac{de_s}{dT} = \frac{0.622\,L\,e_s}{R_d T^2}$$

where L is the latent heat of evaporation and Rd is the specific gas constant for dry air.

c. Details of the radiation scheme

The reader is referred to Manabe and Strickler (1964) for details of:

(1) The computation of the flux of long-wave radiation.

(2) The computation of the depletion of solar radiation.

(3) The determination of mean absorptivity and emissivity.

d. Assumptions made about standard distributions of atmospheric absorbers

The following constituents and their vertical distributions are adopted unless otherwise specified.

Water vapour — The relative humidity is assumed to be a linear function of pressure throughout most of the atmosphere, as illustrated in Figure 2. The line was chosen to fit the data quoted in the Figure legend and a constraint of a minimum value of the humidity mixing ratio of 3×10^{-6} is enforced.

Carbon dioxide — The mixing ratio of carbon dioxide in the atmosphere is assumed constant at 0.0456 % by weight (300 ppm by volume).

Ozone — The adopted vertical distribution of ozone is shown in Figure 3.

The verticaldistributions of gaseous absorbers are those appropriate for 35°N in April. Insolation is assumed to be the annual mean value for the hemisphere.

Clouds — The characteristics of the clouds implied in the model are listed in Table 1. The albedo of the earth's surface is assumed to be 0.102.

Cloud	Height (km)	Amount	Albedo
High	10.0	0.228	0.20
Middle	4.1	0.090	0.48
Low: Top	2.7	0.313	0.69
Bottom	1.7		

Table 1. Cloud characteristics used in the radiative-convective equilibrium model of Manabe and Wetherald (1967).

2.3 Some results from radiative-convective equilibrium models

Figure 4, from Manabe and Wetherald, shows the purely radiative equilibrium profile obtained with the assumption of fixed absolute humidity (as used by Manabe and Strickler) and that obtained with the assumption of fixed relative humidity. Note that with fixed relative humidity there is a very sharp decrease with height near the surface and that the temperature of the troposphere is generally much lower than that obtained with fixed absolute humidity. The radiative-convective equilibrium profile for the case with fixed relative humidity is also shown and demonstrates quite clearly the role of convective adjustment in establishing and maintaining the existing distribution of atmospheric temperature (Cf. Figure 6 of Slingo (1982)).

Figure 5, from Manabe and Strickler, shows the approach to pure radiative equilibrium and radiative-convective equilibrium from both initially warm and initially cold isothermal states, respectively. This figure indicates that the final states may be regarded as steady states.

A wide range of experiments has been done to examine the sensitivity of the radiative-convective equilibrium temperature profiles obtained, for example, with different values of the solar constant and the surface albedo and different combinations and vertical distributions of the main atmospheric absorbers such as water vapour, CO_2, O_3 and clouds. Of particular, topical interest are the experiments of Manabe and Wetherald which examined the response of the surface temperature to changes in the atmospheric CO_2 content.

Figure 6, from Manabe and Wetherald, shows the vertical distributions of equilibrium temperature corresponding to three different CO_2 contents, viz. 150, 300 and 600 ppm. Note:

(1) The larger the mixing ratio of CO_2 the warmer is the equilibrium temperature of the earth's surface and troposphere. This results primarily from an enhanced absorption of the surface long-wave radiation by the tropospheric CO_2 but is also partly due to the effective increase in downward radiation emitted by stratospheric CO_2.

(2) The larger the mixing ratio of CO_2, the colder is the equilibrium temperature of the stratosphere. This results from the increase in long-wave radiation emitted to space by CO_2 in the stratosphere.

(3) Relatively speaking, the dependence of the equilibrium temperature of the stratosphere on CO_2 content is much larger than that of the tropospheric temperature.

Tables 2 and 3 show the equilibrium temperature of the earth's surface corresponding to different CO_2 contents and also the change of the equilibrium surface temperature corresponding to the specified changes in CO_2. Values for both fixed absolute humidity and fixed relative humidity cases are given. Note that the equilibrium temperature of the latter is almost twice as sensitive to the change in CO_2 as that of the former due to the adjustment of water vapour content to the temperature variation of the atmosphere. The changes resulting from doubling the CO_2 content are in reasonable agreement with those obtained in subsequent experiments with general circulation models. Table 4, from Mitchell (1981), gives a comparison of estimates of the change in global mean surface temperature obtained for specified changes in the CO_2 content from single-column, radiative-convective equilibrium models (including the results of Manabe and Wetherald (1967)) with the changes obtained from corresponding general circulation model integrations.

	Average cloudiness		Clear	
CO_2 content (ppm)	Fixed absolute humidity	Fixed relative humidity	Fixed absolute humidity	Fixed relative humidity
150	289.80	286.11	298.75	304.40
300	291.05	288.39	300.05	307.20
600	292.38	290.75	301.41	310.12

Table 2. Equilibrium temperature of the earth's surface (K) for different values of the CO_2 content of the atmosphere, from Manabe and Wetherald (1967).

Change of CO_2 content (ppm)	Fixed absolute humidity		Fixed relative humidity	
	Average cloudiness	Clear	Average cloudiness	Clear
300 ⟶ 150	-1.25	-1.30	-2.28	-2.80
300 ⟶ 600	+1.33	+1.36	+2.36	+2.92

Table 3. Change of equilibrium temperature of the earth's surface corresponding to various changes of CO_2 content of the atmosphere, from Manabe and Wetherald (1967).

2.4 Limitations of the radiative-convective equilibrium models

(1) The most important limitation is that results from these models are mostly of academic interest, since they do not give any information about regional and latitudinal temperature changes.

(2) Many of the model parameters (cloud amounts, surface albedo, relative humidity, critical lapse rates, etc) are prescribed on the basis of present day conditions which may not apply for large departures from present day conditions. Radiative-convective models cannot be applied for large perturbations from present conditions.

(3) The model results depend very critically on the details of the parametrizations. Also like many other models, particularly the simpler models, these models are probably too deterministic.

It is of course very important that the limitations of all the models, especially the simpler, cruder models, be borne in mind when their results are being interpreted.

3. Horizontally Varying Energy-balance Models

3.1 Introduction

In contrast to the radiative-convective equilibrium models of the previous section, this class of one-dimensional, energy-balance models treats the vertical structure with much less detail than the horizontal structure. There exist many types of model of varying degrees of complexity within this class but here we shall concentrate on those which presume to compute the zonally-averaged surface temperature as a function of latitude only, obtained effectively as a balance between incoming solar and outgoing terrestrial radiation. Budyko (1969) and Sellers (1969) are generally acknowledged as having proposed independently the first such models and it is now common practice to refer to the class of derivatives and extensions of their approaches as Budyko - Sellers type models.

Table 4. Estimates of global mean changes in temperature and precipitation due to increasing atmospheric carbon dioxide.

Source	3-dimensional model							
	Topography	Solar Radiation	Cloud	Ocean	CO_2	PPTN	T*	1-D model T*
Manabe and Wetherald, 1975	Idealized	Annual mean	Fixed	Swamp	2	7%	2.93	2.36 †
Manabe and Wetherald, 1980	Idealized	Annual mean	Model Dependent	Swamp	2	7%	3.0	2.0
					4	12%	5.9	4.1
Manabe and Stouffer, 1980	Realistic	Seasonal	Fixed	Fixed depth	4	6.7%	4.1	
Present work								
(a) 2 x CO_2 + 2K	Realistic	Seasonal	Fixed	Present day + 2K	2	5%	2.25	
(b) Estimate assuming no change in global mean net surface heat balance				Present day + 1.65K	2	3.5%	1.9	1.7

† Manabe and Wetherald (1967)

This table is taken from Mitchell (1981).

There is incontrovertible evidence of the existence of large fluctuations in past climate. There is also debate that the impact of man's activities may possibly initiate a significant response in the climate system. Therefore there is concern about and interest in the question of the stability of the global climate to small perturbations of any sort. This led Budyko (1969) and Sellers (1969) to develop their simple models of the earth's climate, based on the mean annual balance of heat entering and leaving latitudinal strips of the earth-atmosphere system, for the purpose of testing the sensitivity of the equilibrium state of the climate to changes in the solar radiative heating.

Schematically, the heat balance for the i-th strip in equilibrium may be written:

$$\left\{\begin{matrix}\text{Absorbed solar}\\ \text{radiation}\end{matrix}\right\}_i - \left\{\begin{matrix}\text{Outgoing infrared}\\ \text{radiation}\end{matrix}\right\}_i = \left\{\begin{matrix}\text{Horizontal divergence of}\\ \text{thermal energy flux}\end{matrix}\right\}_i$$

Each of the above terms is parametrized by some theoretical or semi-empirical formula, usually in terms of the sea-level temperature field. All of the equations are then solved simultaneously to give the surface temperature field. Note that the balance equation assumes that there is no heat or moisture storage in the oceans, land or atmosphere and hence no long-term climatic trend. There is also the gross assumption that the character of the climate of the whole column of the earth-atmosphere system is embodied in a single variable, the surface temperature. Although the various particular studies differ in the exact details of their parametrizations of the above components of the balance it is generally believed that they are at least qualitatively equivalent.

Although these models are highly simplified their solutions to particular problems have a rather complicated structure. Indeed it will be shown that a multiplicity of solutions is obtained for the present value of the solar constant. The interesting and, arguably, unexpected character of such solutions, their mathematical stability properties and the consequent implications for our appreciation of climate are the main reasons why an increasingly large amount of study has been devoted to this class of simple climate model since the appearance of the papers by Budyko (1969) and Sellers (1969).

3.2 The model of Budyko (1969)

In the first instance let us consider the model and results of Budyko (1969). The simplified, integrated thermodynamic energy equation for the whole earth-atmosphere system is represented in terms of annually and zonally averaged variables and equations, thus

$$R + \operatorname{div} \underset{\sim}{F} = 0 \qquad (1)$$

where R is the net radiation for a column of the earth-atmosphere system at a given latitude and $\operatorname{div} \underset{\sim}{F}$ represents the net gain or loss of heat, i.e. it represents the divergence of the net energy flux, as a result of the atmospheric and hydrospheric circulations for that latitude

The net radiation is expressed as

$$R = Q\,(1-\alpha) - I \qquad (2)$$

where Q is the annual mean solar radiation at the outer boundary of the atmosphere, α is the planetary albedo (or reflectivity) and I is the net terrestrial radiation to space, all for a given latitude.

An analysis of radiation and temperature data led Budyko to propose and use the empirical formula

$$I = a + BT_o - (a_1 + B_1 T_o)n \qquad (3)$$

where I is the outgoing radiation in kcal cm^{-2} $month^{-1}$,
T_o is the temperature at the earth's surface in ^oC,
n is the cloudiness in fractions of unit, and
a, B, a_1, B_1 are dimensional coefficients with values
$a = 14.0$, $B = 0.14$, $a_1 = 3.0$ and $B_1 = 0.10$.

For the earth as a whole (i.e. with div $\underset{\sim}{F} \equiv 0$) Budyko found that with $n = 0.50$ and constant $\alpha = 0.33$, a change of solar radiation by 1% results in a temperature change of 1.5K. He cites a corresponding estimate from Manabe and Wetherald (1967) as giving a 1.2K change in surface temperature as a result of a solar radiation change of 1%. Budyko argues that the long time-scales involved in any climate response resulting from radiation variations require that changes in the earth's albedo should be taken into account which are due to the expansion or reduction of the ice-cover at the surface. The parametric coupling that results between α and T_o provides a strong positive feedback between ice-cover and temperature and we shall see later that it is this empiricism which is responsible for many of the surprising results.

The flux divergence term in Equation (1) is also specified empirically. Budyko derived a relationship which enabled him to represent this term quite simply in terms of the difference between the local zonal mean and the planetary mean temperature, thus

$$div\ \underset{\sim}{F} = -\beta (T_o - T_p) \qquad (4)$$

where T_p is the globally averaged, annual mean surface temperature and β is an empirically derived coefficient equal to 0.235 kcal cm^{-2} $month^{-1}$ K^{-1}.

From (1) - (4) we obtain the following equations:

$$T_o = \frac{Q(1-\alpha) - a + a_1 n + \beta T_p}{\beta + B - B_1 n} \qquad (5)$$

$$T_p = \frac{Q_p(1-\alpha_p) - a + a_1 n}{B - B_1 n} \qquad (6)$$

where Q_p, α_p are the corresponding planetary values of incident radiation and albedo, respectively. Budyko used these to derive the average latitudinal annual mean temperatures corresponding to present climatic conditions for the Northern Hemisphere. Q and Q_p were computed corresponding to a solar constant of 1.92 cal cm^{-2} min^{-1}. In the calculation, the influence on the temperatures of deviations of cloudiness values from the mean planetary value of 0.50 is neglected.

Based on observational data Budyko chose the following specification of albedo:

$$\alpha = \begin{cases} 0.32 & 0 \leq \varphi \leq 60^\circ \quad (\varphi \text{ is latitude}) \\ 0.50 & \varphi = 70^\circ \\ 0.62 & \varphi = 80^\circ \end{cases}$$

Budyko's calculation of contemporary average latitudinal distribution of temperature is shown in Figure 7. The computed values of T_o are in good agreement with the observed values, T, and this encouraged Budyko to use his model to study the effects of variations in the radiation on the Earth's thermal regime and glaciations.

Budyko asserts that the southern boundary of the Arctic ice corresponds to a mean latitude of 72°N and adopts an ice-albedo-temperature coupling such that the ice-cap expands or contracts to coincide with the surface whose temperature is less than or equal to the temperature observed now at 72°N i.e. about -10°C. The -10°C isotherm therefore determines the extent of the ice-covered area which is given an albedo of 0.62 with a discontinuity and drop to 0.50 at its southern boundary. Budyko then computed the zonal mean temperatures and hence the position of the edge of the ice-sheet for effectively different values of the solar constant. The results are shown in Figure 8 where the lines $T_{1.0}$ and $T_{1.5}$ correspond to the temperature distributions for a 1% and a 1.5% decrease in the incoming solar radiation. It is assumed that the relative decrease in radiation is the same at all latitudes. Figure 9 shows the values obtained for the mean planetary temperature, T_p and the mean latitude to which glaciation extends, φ_o, as a function of the relative radiation changes.

The most striking feature of Figure 9 is the implication that only a small per cent (about 1.6%) drop in the solar constant is required for the earth to ice over completely. When radiation decreases by 1.6% the ice-cover reaches the mean latitude of about 50°N after which it starts shifting towards lower latitudes up to the equator as a result of self-development. Simultaneously, T_p drops sharply to values several tens of degrees below zero.

3.3 The model of Sellers (1969)

Sellers' starting point is also the integrated energy-balance equation for the earth-atmosphere system for a given latitude belt. His dependent variable is the average annual sea-level temperature, T_s, in each 10° latitude belt. Again, as in Budyko (1969), several empirical and relatively crude relationships are involved in the model. Using the same definitions and notation as was used above for Budyko's model, the components of Sellers' model can be expressed as follows:

Net terrestrial radiation to space:

$$I = \sigma T_s^4 \left[1 - m \tanh \left(19 T_s^6 \times 10^{-16} \right) \right] \tag{7}$$

where σ is the Stefan-Boltzmann constant ($1.356 \times 10^{-12}\ ly\ s^{-1} K^{-4}$), and m is an atmospheric attenuation coeffient, taken to be 0.5 for present conditions. Equation (7) is based on an empirical fit to data.

Albedo:

$$\alpha = \begin{cases} b - 0.009 T_o & T_o < 283.16\text{K} \quad \text{(8a)} \\ \alpha_d = b - 2.548 & T_o \geq 283.16\text{K} \quad \text{(8b)} \end{cases}$$

with the constraint that $0.25 \leq \alpha \leq 0.85$ for all T_o,

where $T_o = T_s - 0.0065Z$ is the average surface temperature, Z is the average surface elevation in metres, and b is an empirical coefficient. Values of Z and b are tabulated by Sellers for each latitude belt of 10^o latitudinal span.

As in Budyko's model, it is assumed that variations in α are associated mainly with variations in surface snow cover and Equation (8a) is valid only as long as $T_o < 10^oC$ and $\alpha < 0.85$. The planetary albedo α_d of (8b) is used for higher temperatures i.e. in the absence of snow or ice cover. Equation (8a) implies an increase in α by 0.009 for every 1K drop in T_o. The coefficient b was selected so that the equation would fit observed data. The effects of variations in cloud cover on albedo were ignored.

The flux-divergence term:

Sellers identifies three separate components making up the total net flux of heat out of the belt at a given latitude. They are due to, respectively, the net fluxes of water vapour and sensible heat by the atmospheric circulation and sensible heat by ocean currents.

Let F_o be the meridional sensible heat transport due to ocean currents, F_A the corresponding sensible heat transport due to atmospheric motion, and F_q the meridional transport of latent heat energy (q is specific humidity),

then Sellers adopts a parametrization which relates these fluxes to the horizontal mixing by large-scale eddies and to a vertically integrated mean meridional wind $\langle v \rangle$ which, in turn, is parametrized as an empirically determined function of the North-South temperature gradient, thus

$$F_o \propto -K_o \frac{\partial T_s}{\partial y} \qquad (9a)$$

$$F_A \propto -K_A \frac{\partial T_s}{\partial y} + \langle v \rangle T_s \qquad (9b)$$

$$F_q \propto -K_q \frac{\partial q(T_s)}{\partial y} + \langle v \rangle q(T_s) \qquad (9c)$$

where K_o, K_A and K_q are effective eddy diffusion coefficients.

An implicit important assumption here is that average atmospheric temperatures and specific humidities and ocean temperatures are proportional to their respective sea-level values. Further the actual specific humidity is replaced by the saturation specific humidity which enables it to be related to the temperature through the Clausius-Clapeyron relationship (see, for reference, the Mathematical Note of Section 2.2b). The poleward transport of water vapour and sensible heat (equations (9b) and (9c)) are assumed to consist of two parts, one associated with a mean meridional motion and the other with large-scale eddies or cyclones and anticyclones. The inclusion of the mean meridional motion is necessary in order to avoid having to deal with negative diffusivities. Values of the K's are tabulated by Sellers at 10^o intervals for each latitude circle.

Sellers' system of equations leads to a second-order equation, effectively in T_o , with only one of the roots giving realistic temperatures. The model was first used to reconstruct the present state of the earth-atmosphere system. The reader is referred to Sellers (1969) to verify that the model specifies present conditions very well. Sellers then applied his model to several different problems, two of which are considered here.

a. Modification of the polar ice caps

A number of different combinations of albedo were used, with α at one or both poles (70 - 90°) being set equal to and held fixed at either 0.50 or 0.40 (with the exception of 70 - 80°S where a minimum value of 0.44 was imposed). The results, shown in Figure 10, have a number of interesting features, viz,

(1) Cases 1 and 2. Removal of north polar ice results in an increase in temperature poleward of 70°N by at most about 7K. Temperature would rise by about 1K in the tropics and 1-3K near the South Pole.

(2) Cases 3 and 4. Removal of antarctic ice produces a temperature increase there of 12-15K with a rise in the Arctic of about 4K.

(3) Cases 5 and 6. Removal of both polar ice-caps yields temperature increases of 7-10K in the Arctic and 13-17K in the Antarctic. Tropical warming does not exceed 2K.

Therefore albedo changes at either pole have global repercussions. An interesting result, comparing the curves for Cases 1 and 3, is that reducing the albedo south of 70°S to 0.50 would, according to this model, produce a larger temperature rise in the Arctic than would a reduction of the albedo north of 70°N to 0.50. The greater effects of changes to the albedo in the Antarctic than in the Arctic are simply the result of the Antarctic's higher initial albedo. Others have argued that the extent of sea-ice in the Southern Hemisphere has a much more significant effect on the global circulation than the extent of sea-ice in the Northern Hemisphere.

b. Variations in the solar constant

Sellers examined the sensitivity of his models to variations in the solar constant and his calculations corroborate, qualitatively at least, the implications from Budyko's model that a slight reduction in the solar constant (or, for example, increase in global dust contamination) would be sufficient to initiate another ice age. In Figure 11 the global mean sea-level temperature, T_{sp} , is plotted as a function of the solar constant, S , currently assumed to be 2.00 ly min^{-1} (cf. Budyko used 1.92 ly min^{-1} as the current value!) The present value of T_{sp} is about 14.0°C. Three analyses were carried out, two with α allowed to vary according to $\alpha = b - 0.009\,T_o$ and $\alpha = b - 0.005\,T_o$, respectively, and the other with α held fixed at its assumed current values. There are no very dramatic events when α is kept fixed at the present values and S allowed to vary. However with a realistic albedo-temperature coupling a decrease of about 2% in the solar constant is sufficient to create another ice-age with ice-caps extending equatorward to 50° (i.e. where $T_o = -10$°C). Any further drop in the solar constant is accompanied in the model by a rapid transition to an ice-covered earth with albedo of 0.85 (cf. Equation (8a)) and an equilibrium temperature of -100°C. With the weaker dependence of α on surface temperature and ice-cover (i.e. 0.005 used in place of 0.009 and b adjusted accordingly) a severe ice-age would not occur until the solar constant decreased by about 5 - 6%.

Sellers also reports a few sensitivity studies where he has kept the solar constant fixed but has increased or decreased the effective diffusion coefficients in his model. He concludes that a relatively small change in the ability of the system to transfer heat poleward could conceivably offset any increase or decrease in the intensity of solar radiation. Hence the obvious caution about allowing only one variable to change without being able to allow for consequent changes in the others. Nevertheless Sellers' model implies quite conclusively that a decrease in the solar constant of less than 5% would be sufficient to start another ice-age.

3.4 The sensitivity of time-dependent Budyko-Sellers type models

Schneider and Gal-Chen (1973) examined the sensitivity of time-dependent models of the Budyko-Sellers type to changes in initial as well as external conditions. The problem is then formulated as an initial value problem in which the basic equations are integrated from a given initial state until a final asymptotic equilibrium state is obtained.

The time-dependent version of the zonally averaged, vertically integrated heat balance equation for the earth-atmosphere system is

$$C \frac{\partial T_0}{\partial t} = R + \operatorname{div} \underset{\sim}{F} \qquad (10)$$

where C is a prescribed thermal inertia coefficient for a zonal column through the land-ocean-atmosphere system. R is defined as in Equation (2) with I the net infra-red cooling to space given by

$$I = c(\varphi)\, \sigma T_0^4 \left[1 - m \tanh\left(19 T_0^6 \times 10^{-16}\right)\right] \qquad (11)$$

with $m = 0.5$. Note that with $c(\varphi) = 1$ (and ignoring the distinction between T_0 and T_s) this formula is identical to that used by Sellers (1969). Scheider and Gal-Chen claim to be using the zonally averaged 1000-mb temperature which has been written here as T_0, the notation used previously for the surface temperature. Their numerical scheme requires that the 'consistency factor' $c(\varphi)$ be adjustable over a small range of values close to unity.

Schneider and Gal-Chen examined four different versions of the model, defined by different formulations of $\operatorname{div} \underset{\sim}{F}$ and α.

(1) Labelled (S), is the formulation of Sellers (1969) with respect to $\operatorname{div} \underset{\sim}{F}$ and α.

(2) Labelled (SV), is exactly as (S) but with $\langle v \rangle \equiv 0$.

(3) Labelled (B), uses α as defined in (S) but has the Budyko (1969) formulation for $\operatorname{div} \underset{\sim}{F}$ as in Equation (4).

(4) Labelled (FG), is as (S) with respect to $\operatorname{div} \underset{\sim}{F}$ but uses a formulation for α attributed to Faegre (1972) such that

$$\alpha = 0.4860 - 0.0092\,(T_0 - 273)$$

$$0.25 \leq \alpha \leq 0.85 \quad \text{for all values of } T_0$$

ie. α is a function of temperature everywhere.

The asymptotic equilibrium climate from the time-dependent version was shown to exhibit the same sensitivity to small changes in the solar constant as experienced with the steady-state models. Figure 12 shows the results of Schneider and Gal-Chen for their version (S) of the Sellers (1969) approach. Again with a decrease in solar constant of more than 1.6% a planetary mean temperature of 174.65K was obtained corresponding to an entirely ice-covered earth. Calculations for a 1% decrease in solar constant and Sellers-type energy fluxes with, (S), or without, (SV), mean meridional motion gave quite similar results both in the global average amount of temperature decrease and in the latitudinal distribution of the temperature decrease. The results for the Budyko parametrization, (B), are qualitatively similar to those of (S) and (SV) for the global average. However the corresponding latitudinal distribution of temperature decrease differs significantly from that of Sellers. The particular form used for $\operatorname{div} \underline{F}$ does appear to be significant for determining the details of the latitudinal distribution of temperature.

The stability of the (B) and (S) solutions to various perturbations of the initial temperatures were examined and the results are summarised in Table 5. It was found in both cases that the respective steady-state control climates could be recovered exactly if the temperature perturbation was positive. Perturbations of 2K and 16K were tried and in both energy-balance requirements alone produced a return to the control climate ie. the control climates appear to be unique, fully stable and transitive. For an initial temperature decrease of magnitude not exceeding 18.3K the apparent slight intransitivity in the equilibrium climates has been attributed to computer rounding errors. The models then show that the asymptotic equilibrium climate is extremely insensitive to changes in the initial conditions. However for initial temperature decreases greater than about -18.3K both models lead to an ice-covered earth solution similar to curve D in Figure 12. Thus it takes a very large negative perturbation to the initial temperatures to lead to the highly intransitive alternative ice-covered earth regime of these models. The insensitivity of the model to decreases in the initial temperature conditions of less than 18K follows from an unchanged albedo in equatorial latitudes.

Initial Temperature Perturbation (K)	Mean Planetary Temperature (K)	
	Sellers (S)	Budyko (B)
0	287.06	287.09
$0 < P < 16$ for all φ	287.06	287.09
$-18.3 < P \leq 0$ for all φ	286.67	286.87
$P < -18.3$ for all φ	175.58	175.44
-22 for $\lvert\varphi\rvert \geqslant 20^\circ$; 0 for $\lvert\varphi\rvert < 20^\circ$	286.67	286.87

Table 5. Asymptotic steady-state global average temperatures resulting from various perturbations to the initial temperature distributions for two of Schneider and Gal-Chen's (1973) parametrizations. This table is taken from the review of climate modelling by Schneider and Dickinson (1974).

The implication from these experiments is that even when ice-cover exists down to temperate latitudes, but if the solar constant and the tropical albedos are unaltered then there is still sufficient energy available to the tropics to provide a poleward heat transport to melt the ice and restore the equilibrium climate to near the present-day value. If the equatorial temperature is reduced into the range of the positive feedback temperature-albedo coupling then climate instability is triggered, and an ice-covered earth results. This hypothesis was tested (see Table 5) by introducing a very large negative perturbation in the initial temperatures, viz. -22K, for all $|\varphi| \geqslant 20^\circ$ only. In both models there was a return to near the present-day climate.

The model, (FG), which adopts Faegre's (1972) formulation for α is very sensitive to negative perturbations in both solar constant and initial conditions. The interpretation is that with the temperature-albedo coupling for all temperatures and at all latitudes then a slight negative perturbation in either initial temperature or solar constant can cause a decrease in energy input to the tropics thereby enhancing the albedo-temperature positive feedback at all latitudes. Experiments with such energy-balance models suggest that the stability of the climate may be quite dependent on the functional relationship between the albedo and the surface temperature.

3.5 Multiple solutions and their stability

We have seen from the numerical perturbation studies of Schneider and Gal-Chen (1973) that more than one solution is admissable in the Budyko-Sellers models with the present value of the solar constant ie. the system appears to be intransitive. North (1975) studied the latitude of the ice-line as a function of the solar constant by using a model, described as intermediate between that of Budyko and that of Sellers, which yielded exact analytical solutions. His results, summarised in Figure 13, show a multiplicity of solutions for the present-day value of the solar constant; indeed, there are no less than five solutions for the present value of the solar constant! Other studies with models of this type, but with different formulations for the individual terms, give results qualitatively similar to these. Lowering the solar constant by a few per cent leads to a catastrophic plunge to an ice-covered planet but more than a 30% increase is needed to yield an ice-free earth as the only solution.

North (1975) performed a linear stability analysis about the equilibrium solutions on branches I, II and III and confirmed that the present climate (I) and the ice-covered earth (III) solutions are stable (shown numerically by Schneider and Gal-Chen (1973)) while the intermediate solution (II) with the earth about two-thirds covered with ice is unstable and therefore of no physical significance.

Several other authors, including Held and Suarez (1974), Su and Hsieh (1976), Ghil (1976), Drazin and Griffel (1977), North (1977) and Cahalan and North (1978), have established a general stability theorem applicable to a wide variety of specific models within the general class. The slope stability theorem states that

$$\frac{dX_s}{dS} > 0 \Longrightarrow \text{Stability},$$

$$\frac{dX_s}{dS} < 0 \Longrightarrow \text{Instability},$$

where X_s is the sine of the latitude of the ice-line.
The validity of this theorem for a large class of models is important, since it has implications for those branches of the solutions nearest to that corresponding to the present climate, viz. they are unstable and therefore unphysical. Their instability also explains why they were not located by the earlier approaches of Budyko, Sellers and Schneider and Gal-Chen.

One interesting point is that the slope stability theorem implies that if a cusp near $X_s = 1$ exists in a model solution (as, for example, in Figure 13) then small ice-caps are unstable. However the existence of such a cusp appears to depend very crucially on the details of the parametrizations adopted. Budyko-Sellers type models may be too simple to examine such a point. This is unfortunate since whether small ice-caps are stable or not is an important issue if one recalls the concern over rising levels of CO_2 in the atmosphere and the possible effects of melting ice on the sea-level.

3.6 Further studies with Budyko-Sellers type models

The basic concept of the Budyko-Sellers type models has been extended and made more realistic and much more complicated in a great variety of ways. The comprehensive review by Schneider and Dickinson (1974) and the more recent reviews by North (1979), North et al (1980) and Hunt (1980) are recommended as sources for further study.

REFERENCES TO APPENDIX

Budyko, M I 1969 'The effect of solar radiation variations on the climate of the Earth', Tellus, 21, pp 611-619.

Cahalan, R F and North, G R 1978 'A stability theorem for energy-balance climate models', J. Atmo. Sci., 36, pp 1178-1188.

Drazin, P G and Griffel, D H 1977 'On the branching structure of diffusive climatological models', J. Atm. Sci., 34, pp 1696-1706.

Faegre, A 1972 'An intransitive model of the earth-atmosphere-ocean system', J. Appl. Meteorol., 11, pp 4-6.

Ghil, M 1976 'Climate stability for a Sellers-type model', J. Atm. Sci., 33, pp 3-20.

Held, I M and Suarez, M J 1974 'Simple albedo feedback models of the ice caps;, Tellus, 26, pp 613-629.

Herring, W S and Borden, T R Jr. 1965 Mean distributions of ozone density over North America, 1963-1964. Environmental Research Papers, No. 162, AFCRL-65-913, Air Force Cambridge Research Laboratories, Bedford, Mass., 19 pp.

Hunt, B G 1980 'Numerical modelling of climate', Aust. J. Phys., 33, pp 897-910.

London, J 1962 Mesospheric dynamics, Part III. Final Report, Contract No. AF19(604)-5492, Dept. of Meteorology and Oceanography, New York Univ., 99 pp.

Lorenz, E N 1968 'Climatic determinism', Meteorol, Monogr., 5, pp 1-3.

Lorenz, E N 1970 'Climatic change as a mathematical problem', J. Appl. Meteorol., 9, pp 325-329.

Lorenz, E N 1975 'Climate predictability', Report of the Intl. Study. Conf. on the Physical Basis of Climate and Climate Modelling, App. 2.1, pp 132-136, GARP Publications Series No. 16, WMO-ICSU.

Manabe, S and Stouffer, R J 1980 Sensitivity of a global climate model to an increase of CO_2 concentration in the atmosphere. J. Geoph. Res., 85, pp 5529-5554.

Manabe, S and Strickler, R F 1964 'Thermal equilibrium of the atmosphere with a convective adjustment', J. Atm. Sci., 21, pp 361-385.

Manabe, S and Wetherald, R T 1967 'Thermal equilibrium of the atmosphere with a given distribution of relative humidity, J. Atm. Sci., 24, pp 241-259.

Manabe, S and Wetherald, R T 1975 The effects of doubling the CO_2 concentration on the climate of a general circulation model. J. Atm. Sci., 32, pp 3-15.

Manabe, S and Wetherald, R T 1980 On the distribution of climate change resulting from an increase in CO_2 content of the atmsophere. J. Atm. Sci., 37, pp 99-118.

Mastenbrook, H J 1963 Frost-point hygrometer measurement in the stratosphere and the problem of moistue contamination. Humidity and Moisture, Vol. 2, New York, Reinhold Publ. Co., pp 480-485.

Mitchell, J F B 1981 'The seasonal response of a general circulation model to changes in CO_2 and sea temperatures', Unpublished Met Office Report, Met O 20 Tech. Note No. II/171.

Murgatroyd, R J 1960 Some recent measurements by aircraft of humidity up to 50,000 ft in the tropics and their relationship to meridional circulation. Proc. Symp. Atmos. Ozone, Oxford, 20-25 July 1959, IUGG Monogr. No. 3, Paris, p.30.

North, G R 1975 'Analytical solution to a simple climate model with diffusive heat transport', J. Atm. Sci., 32, pp 1301-1307.

North, G R 1977 'Analytical models of the earth's macroclimate', Proc. First. Intl. Conf. on Mathematical Modelling, Vol. IV, Univ. of Missouri, Rolla, Missouri, pp 2291-2302.

North, G R 1979 'Recent work on heat balance models', Report on the JOC Study Conference on Climate Models: Performance, Intercomparison and Sensitivity Studies, Vol II, 752-765, GARP Publications Series No. 22, WMO-ICSU.

North, G R, Cahalan, R F and Coakley, J A — 1980 — 'Energy-balance climate models', Washington, NASA, Tech. Memo. 82047.

Ramanathan, V and Coakley, J A — 1978 — 'Climate modelling through radiative-convective models', Rev. Geophys. Space Phys., 16, pp 465-489.

Schneider, S H and Dickinson, R E — 1974 — 'Climate modelling', Ibid., 12, pp 447-493.

Schneider, S H and Gal-Chen, T — 1973 — 'Numerical experiments in climate J. Geophys. Res., 78, pp 6182-6194.

Sellers, W D — 1969 — 'A global climatic model based on the nergy balance of the earth-atmosphere system', J. Appl. Meteorol., 8, pp 392-400.

Slingo, A — 1982 — 'Introduction and radiative heat balance', Lecture on The General Circulation of the Atmosphere, Lecture Notes, Met O 20.

Su, C H and Hsieh, D Y — 1976 — 'Stability of the Budyko climate model', J. Atm. Sci., 33, pp 2273-2275.

Telegadas, K and London, J — 1954 — A physical model of Northern Hemisphere troposphere for winter and summer. Sci. Rep. No. 1, Contract AF19(122)-165, Research, Div., College of Engineering, New York Univ., 55 pp.

WMO — 1979 — World Climate Conference, A Conference of Experts on Climate and Mankind, Geneva, Feb 1979, WMO.

WMO — 1980 — Outline Plan and Basis for the World Climate Programme, 1980-1983, WMO-No. 540.

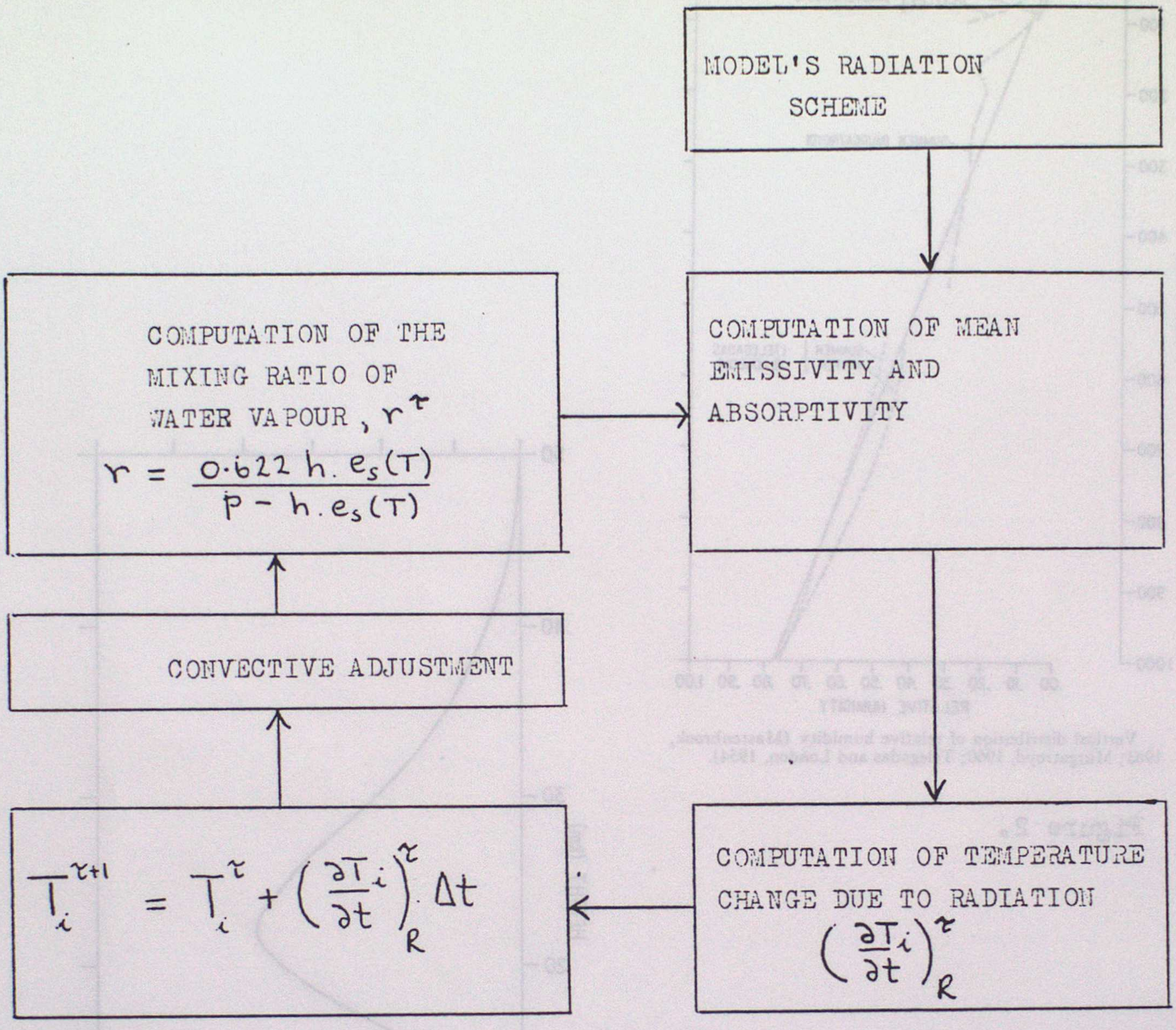
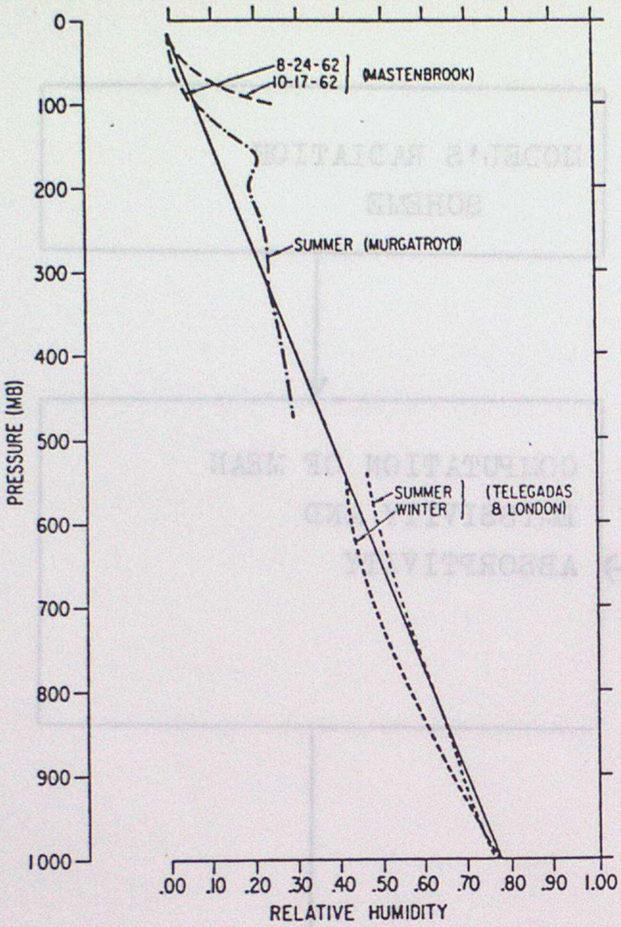
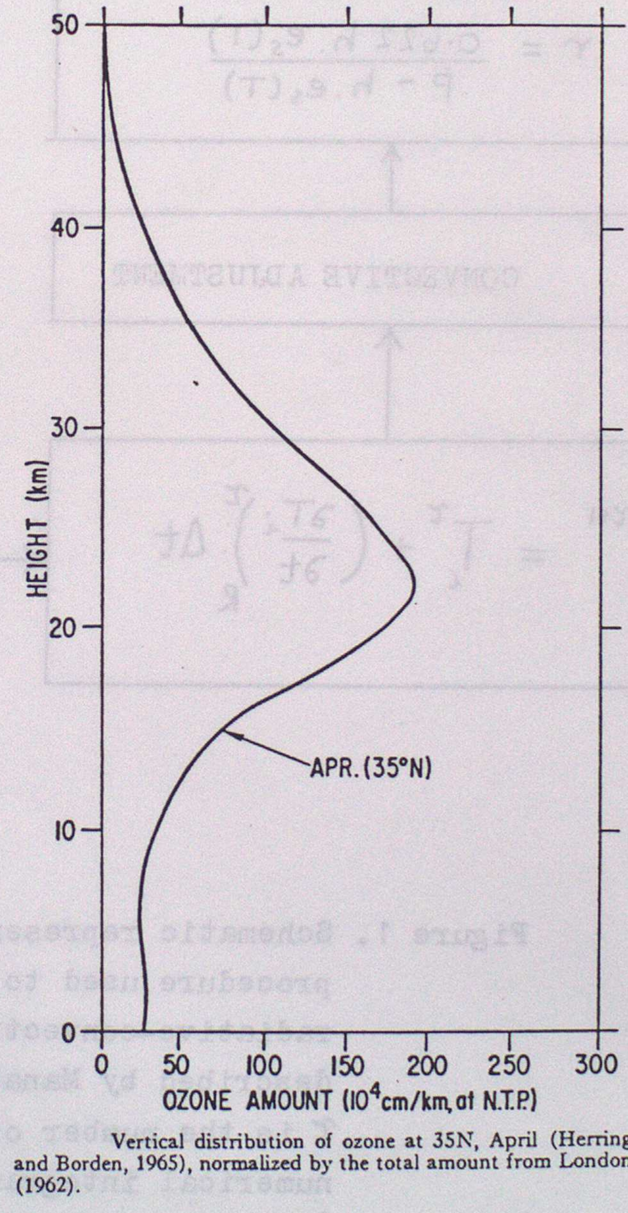


Figure 1. Schematic representation of the iterative procedure used to obtain the state of radiative-convective equilibrium, as described by Manabe and Wetherald (1967).
τ is the number of the time-step of the numerical integration.
i is the index for the finite-differencing in the vertical.



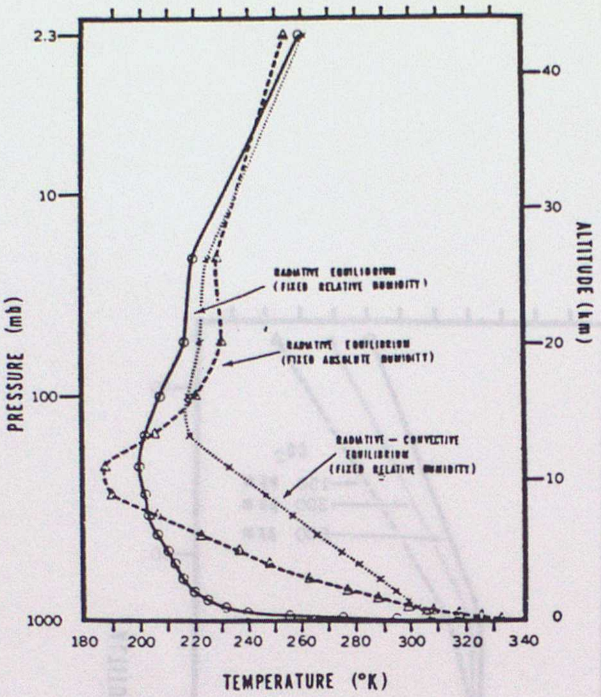
Vertical distribution of relative humidity (Mastenbrook, 1963; Murgatroyd, 1960; Telegadas and London, 1954).

Figure 2.



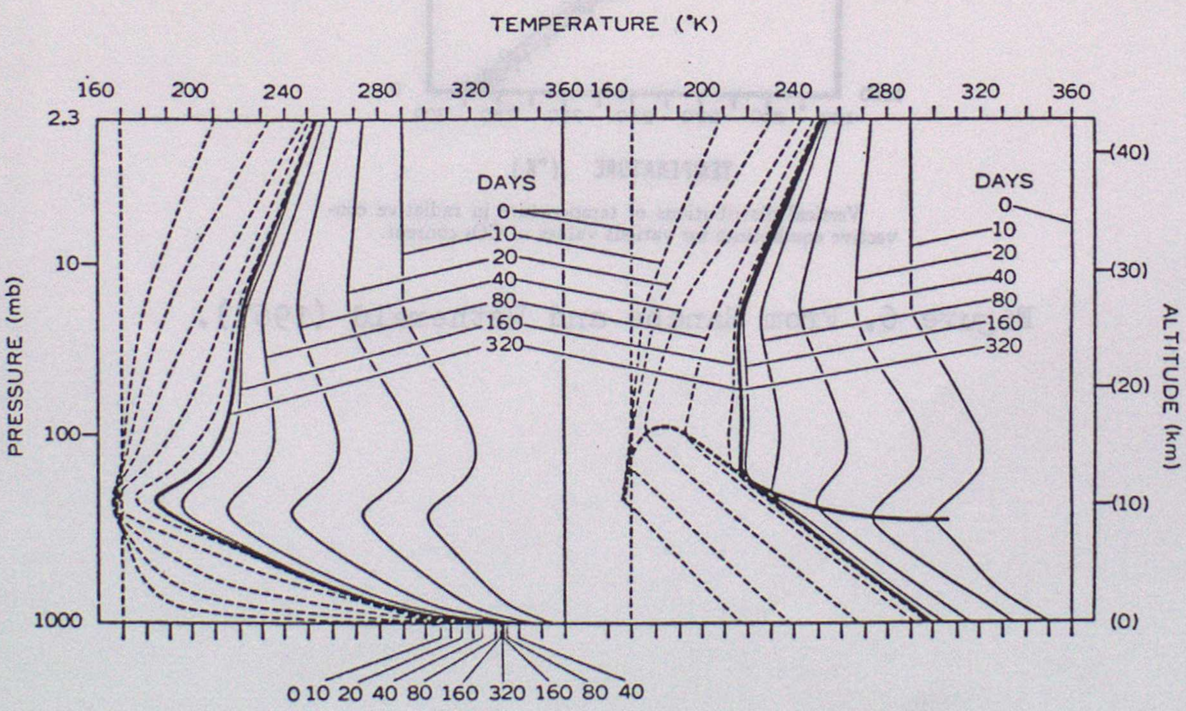
Vertical distribution of ozone at 35N, April (Herring and Borden, 1965), normalized by the total amount from London (1962).

Figure 3.



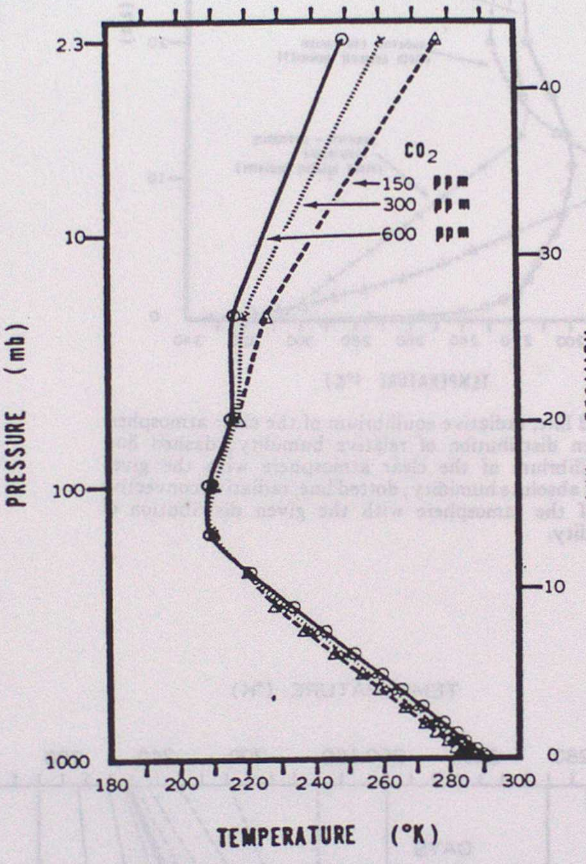
Solid line, radiative equilibrium of the clear atmosphere with the given distribution of relative humidity; dashed line, radiative equilibrium of the clear atmosphere with the given distribution of absolute humidity; dotted line, radiative convective equilibrium of the atmosphere with the given distribution of relative humidity.

Figure 4.
From Manabe and Wetherald(1967).



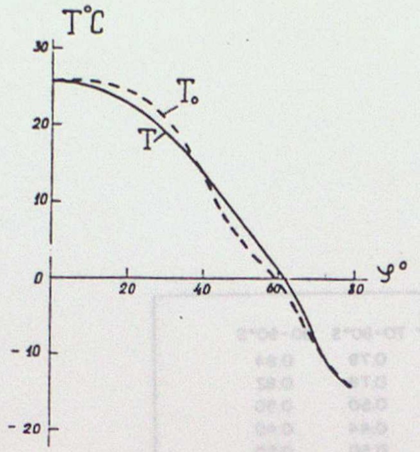
The left and right hand sides of the figure, respectively, show the approach to states of pure radiative and thermal equilibrium. The solid and dashed lines show the approach from a warm and cold isothermal atmosphere.

Figure 5. From Manabe and Strickler (1964).



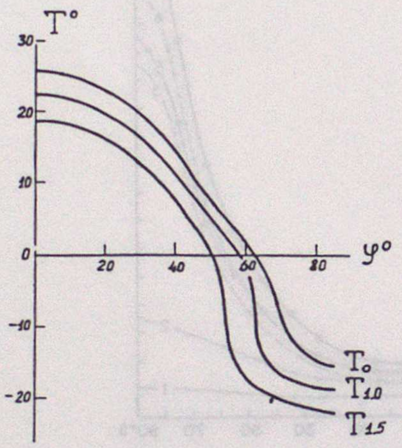
Vertical distributions of temperature in radiative convective equilibrium for various values of CO_2 content.

Figure 6. From Manabe and Wetherald (1967).



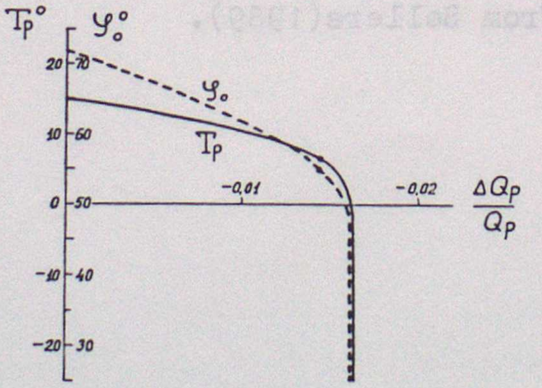
The average latitudinal temperature distribution.

Figure 7.
From Budyko(1969).



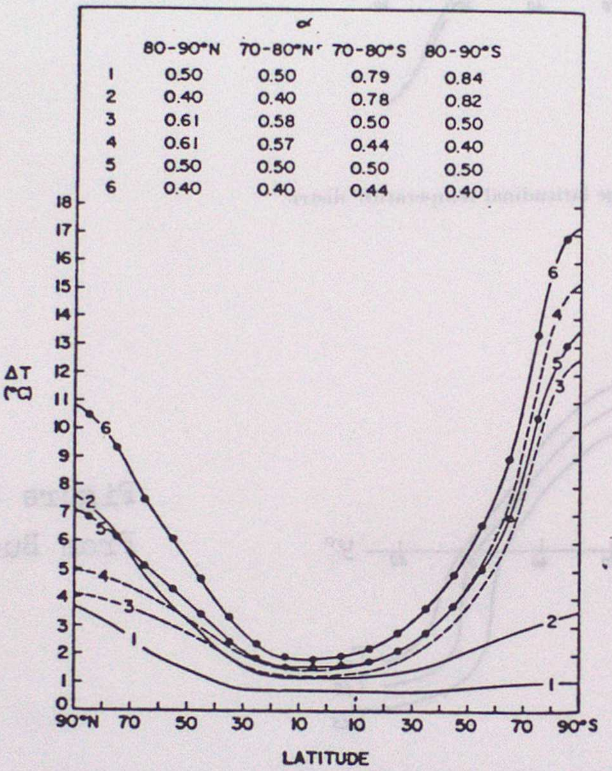
The dependence of temperature distribution on radiation amount.

Figure 8.
From Budyko(1969).



The dependence of the Earth's temperature and ice cover boundary on radiation variations.

Figure 9.
From Budyko(1969).



Predicted latitudinal distribution of the mean annual temperature rise associated with albedo manipulation at one or both poles.

Figure 10. From Sellers(1969).

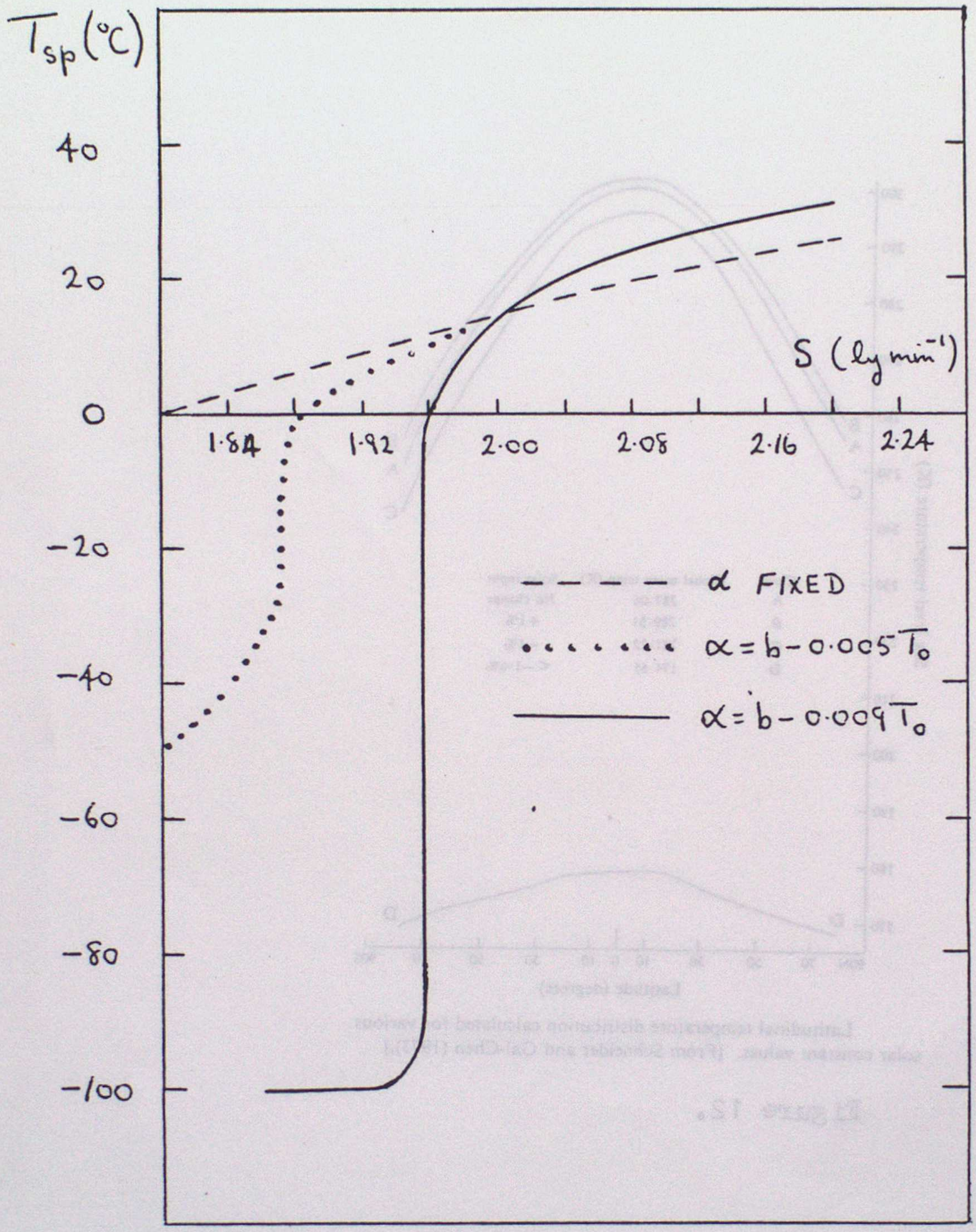
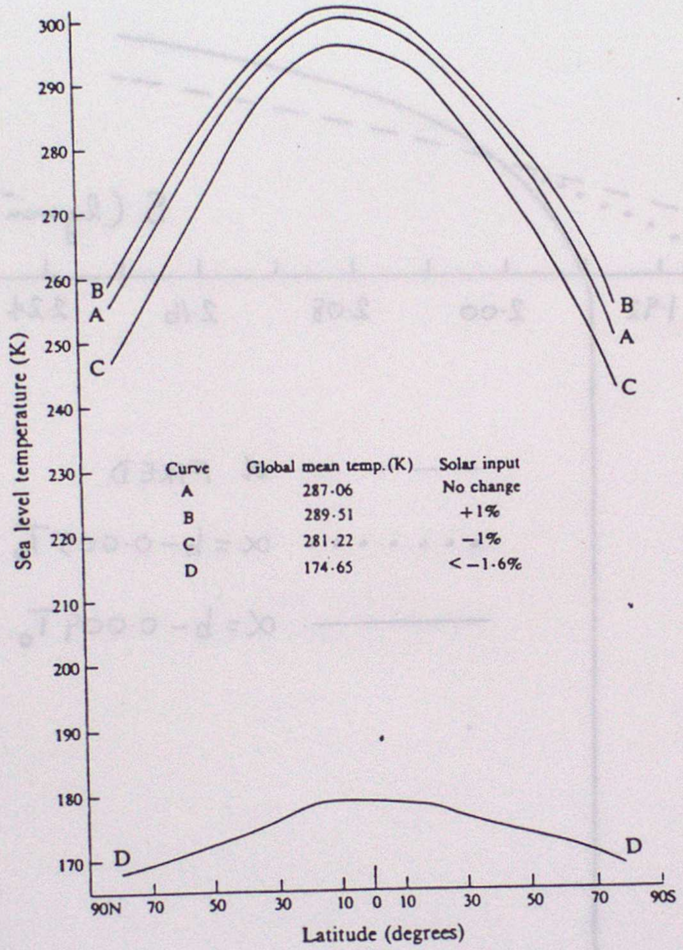


Fig. 11. The mean global sea level temperature, T_{sp}, as a function of the solar constant, S, for different specifications of the planetary albedo, α. This figure is based on the results of Sellers(1969).



Latitudinal temperature distribution calculated for various solar constant values. [From Schneider and Gal-Chen (1973).]

Figure 12.

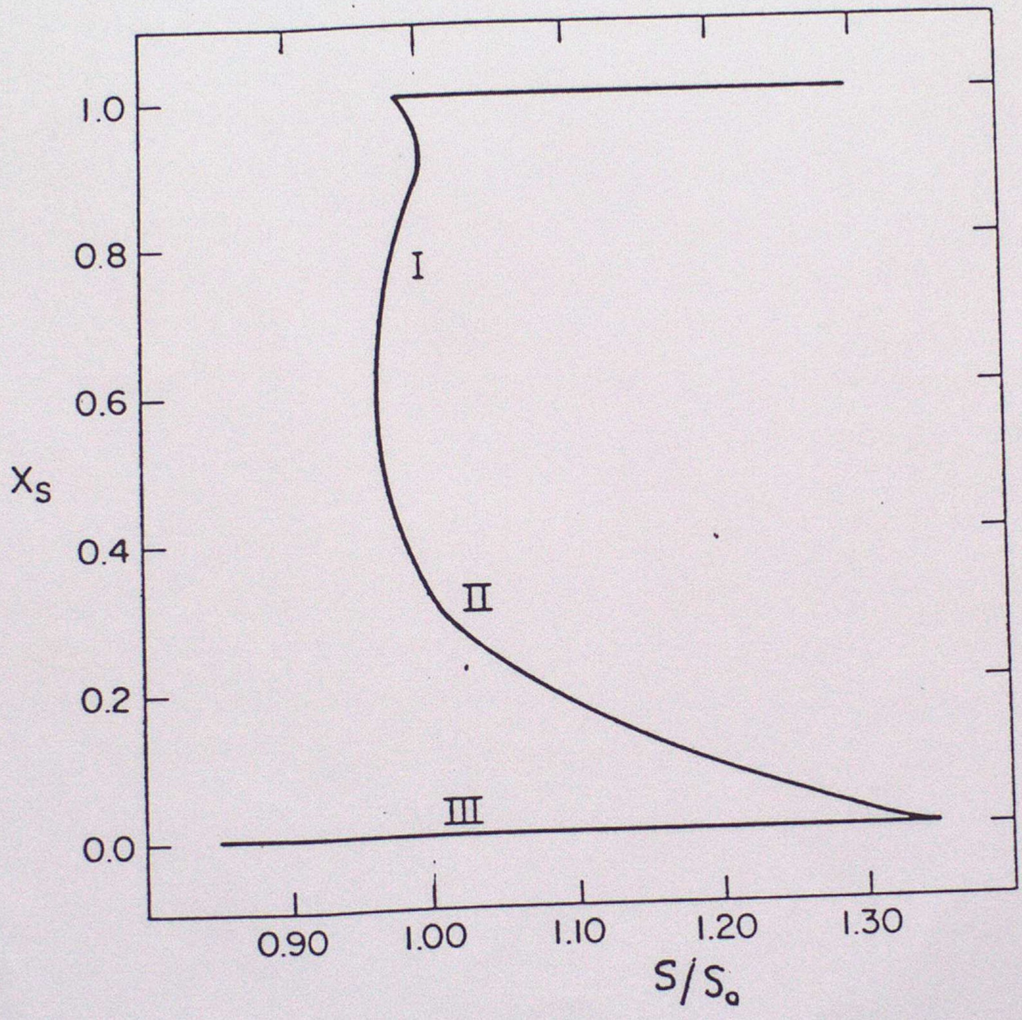


Figure 13. A typical plot of the sine of the latitude of the ice line (X_S) versus the solar constant S/S_o normalized to its present value. This curve is for an exact solution of a model with constant diffusion coefficient and discontinuous albedo at the ice line.

This figure is taken from North(1975).