LONDON, METEOROLOGICAL OFFICE.

Met.O.19 Branch Memorandum No. 3.

Temperature changes and energy losses in the cell of the TIROS-N pressure modulated radiometer. By STEWART,K.H. and BARWELL,B.R.

London,Met.Off.,Met.O.19 Branch Mem.No.3, [1973],30cm.Pp.7,pls.4.2 Refs.

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Temperature changes and energy losses in the cell of the TIROS-N pressure modulated radiometer

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TEMPERATURE CHANGES AND ENERGY LOSSES IN THE CELL OF THE TIROS-N PRESSURE MODULATED RADIOMETER

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Introduction

One of the most important sources of energy loss in the Pressure Modulated Radiometer is that due to the irreversibility of the thermodynamic cycle which the gas in the detector experiences. The magnitude of this energy loss is dependent on the ease with which temperature changes during each cycle can be smoothed out by heat conduction to the cell walls. If the time taken for the gas temperature in the cell to decay to the temperature of the surroundings is much greater than the time taken for the piston to complete one cycle, there will be very little heat lost by conduction and the cycle will be close to a reversible adiabatic one. If the decay time is much less than the period of the piston, any temperature difference between the gas and the surroundings during the piston cycle will quickly disappear and the cycle will therefore be approximately an isothermal one in equilibrium with the surroundings. In both of these extreme cases the energy losses are very small since, in the first case there is negligible heat loss, while in the second case the heat exchange occurs reversibly as there is negligible temperature difference between the gas in the cell and the surroundings. Between the adiabatic and isothermal cases there are cycles in which heat conduction to the cell walls will occur through a finite temperature difference. Such cycles will be irreversible and will exhibit a net energy loss per cycle, the maximum value of which would be expected to occur when the thermal decay time of the cell is comparable with the period of the piston.

Temperature variations during the cycle

A simple method of estimating the temperature variations during the cycle is to combine the temperature changes in an adiabatic cycle with an exponential decay of temperature in the following way. In an adiabatic cycle the absolute temperature T and volume V of a fixed mass of gas vary in such a way that

$$TV^{\gamma-1} = \text{a constant} \tag{1}$$

where γ is the ratio of specific heats (c_p/c_v). In differential form this may be written

$$\frac{dT}{dt} = -(\gamma-1)\frac{T}{V}\frac{dV}{dt}\;. \tag{2}$$

The rate of change of temperature of a body whose temperature decays exponentially to the surroundings temperature To is

$$\frac{dT}{dt} = -\frac{(T-T_o)}{\tau}\;, \tag{3}$$

where T is the temperature of the body and τ is the time taken for the temperature difference between the body and the surroundings to fall to 1/e of its initial value. τ will be referred to as the 'decay time'.

In the derivation of an expression for the temperature changes in the cell it will be assumed that at any point in the cycle, the temperature of the gas is uniform so that T is a function of time only. The rate of change of temperature of the gas in the cell will be represented by combining (2) and (3) to give

$$\frac{dT}{dt} = -(\gamma-1)\frac{T}{V}\frac{dV}{dt} - \frac{(T-T_o)}{\tau}\;. \tag{4}$$

The sinusoidal motion of the piston can be represented by

$$V = V_0 \, (1 + \delta \sin wt) \tag{5}$$

where V is the volume of gas in the cell and cylinder head, V_0 is the mean value of V, δ is the ratio of maximum piston volume displacement to mean volume, and w is the frequency of the piston multiplied by 2π. Substituting (5) into (4) gives the differential equation for T :-

$$\frac{dT}{dt} + \left[\frac{(\gamma-1) w \delta \cos wt}{1 + \delta \sin wt} + \frac{1}{\tau} \right] T = \frac{T_0}{\tau} \tag{6}$$

The solution to this equation consists of temperature variations which settle down to a steady-state cycle after initial transients have died out. The steady-state solution can be written as the infinite series

$$\frac{T}{T_0} = (1 + \delta \sin wt)^{1-\gamma} \left(1 + \sum_{n=1}^{\infty} \frac{(\gamma-1)(\gamma-2)\ldots\ldots(\gamma-n)}{n!} \delta^n I_n \right), \tag{7}$$

where I_1 and I_2 are given by

$$I_1 = \frac{1}{1 + w^2\tau^2} (\sin wt - w\tau \cos wt), \tag{8a}$$

$$I_2 = \frac{1}{1 + 4w^2\tau^2} (\sin^2 wt - w\tau \sin 2wt + 2w^2\tau^2), \tag{8b}$$

and other values of I_n follow from the recurrence relation

$$I_n = \frac{1}{1 + n^2w^2\tau^2} \left(\sin^{n-1} wt \, [\sin wt - nw\tau \cos wt] + w^2\tau^2 n(n-1) I_{n-2} \right) \tag{8c}$$

Figure 1 shows the temperature cycle (7) plotted against wt for $T_0 = 0^\circ C$ ($273.2^\circ K$) and for six different values of $w\tau$. For each curve δ takes the value 0.2 which is approximately the value calculated from experimental results. (See ref. 1). The curves of figure 1 show that when $w\tau$ is small, the amplitude of the temperature variation is small and the cycle is close to an isothermal one, while as $w\tau$ increases, the amplitude increases approaching an adiabatic cycle in the limit as $w\tau \to \infty$. The adiabatic cycle is found from equations (7) and (8) to be

$$\frac{T}{T_0} = (1 + \delta \sin wt)^{1-\gamma} \left(1 + \sum_{n=1}^{\infty} \frac{(\gamma-1)(\gamma-2)\ldots\ldots(\gamma-2n)}{2^{2n} (n!)^2} \delta^{2n} \right), \tag{9}$$

and writing this as

$$T V^{\gamma-1} = T_0 V_0^{\gamma-1} \left[1 - \tfrac{1}{4}(\gamma-1)(2-\gamma)\delta^2 + \ldots\ldots \right], \tag{10}$$

the adiabatic character can be seen on comparison with equation (1). If the cell was perfectly insulated the temperature cycle would be given by

$$T V^{\gamma-1} = T_0 V_0^{\gamma-1}. \tag{11}$$

Equation (10) represents a cycle in which the temperature is a little less than that given by (11) indicating that there was a small net heat loss during the period when the transient components of the temperature cycle had not died out. The cycle (10) represents an adiabatic cycle in equilibrium with the surroundings, whereas the cycle (11) for the perfectly insulated cell cannot be in the same state as the gas then has no contact with its surroundings.

Figure 1 also shows that as the amplitude of the temperature variations decreases, the cycles tend to become 90° out of phase with the piston. This is to be expected as, when $w\tau$ is very small and the cycle is therefore nearly isothermal, heat is conducted away so quickly that the maximum temperature of the cycle will occur close to the point where the internal energy of the gas is increasing most rapidly, and this will be when the piston is at its mean position during the compression stroke. At each end of the stroke, the temperature passes through T_o since the piston is momentarily at rest and the gas temperature quickly decays to the surroundings temperature. As the detector of the Pressure Modulated Radiometer is phase sensitive, the phase shift of the temperature cycle will have the effect of reducing the unwanted signal due to emission of radiation from gas in the cell.

Estimate of energy losses

The work done on the gas in the cell during one complete piston cycle (which, in the steady state must balance the heat loss) is given by

$$W = -\int_{1\,cycle} p\,dV = -P_o \int_{1\,cycle} \left(\frac{V_o}{V}\right)\left(\frac{T}{T_o}\right) dV \qquad (12)$$

where p is the pressure of the gas at any instant and p_o is its pressure at temperature T_o and volume V_o. The integrand may be evaluated using (5) and (7), and if the amplitude of the piston is small enough for δ^2 to be neglected compared with unity, truncation of the series in (7) and application of the binomial theorem lead to

$$W = -p_o V_o \int_{1\,cycle} (1-\gamma\delta \sin wt)\left(1 + \frac{(\gamma-1)\delta}{1+w^2\tau^2}\left[\sin wt - w\tau \cos wt\right]\right) w\delta \cos wt \, dt \; . \qquad (13)$$

The integration may be carried out over any complete cycle such as t=0 to $t=2\pi/w$ leading to the result

$$W = \frac{(\gamma-1)\pi w\tau \delta^2}{1+w^2\tau^2} P_o V_o \qquad (14)$$

This result can only be regarded as neglecting δ rather than δ^2 compared with unity because the major term of (7) vanishes in the integration in (13).

The maximum value of W occurs when $w\tau = 1$ confirming that the maximum energy loss occurs when the decay time and piston period are comparable. When this condition is satisfied, the value of W is given by

$$W_{max} = \frac{\pi}{2}(\gamma-1)\delta^2 P_o V_o \qquad (15)$$

which is a factor $\pi/8$ times the energy loss in a cycle with the same small piston displacement but with adiabatic compression and expansion alternating with constant volume cooling and heating to the temperature of the surroundings.

Estimates of stored energy and 'Q'

Assuming that the space behind the piston is large enough for the pressure variations it experiences during each cycle to be neglected, the maximum stored energy per cycle E, is given by

$$E = -\int_{V_o}^{V_o(1-\delta)} (p-P_o)\, dV \qquad (16)$$

where (p-po) represents the pressure difference across the piston. For small piston displacements, an isothermal cycle gives

$$(p-P_o) = \frac{P}{V}(V_o - V) \qquad (17)$$

which, when substituted into (16) leads to

$$E_{isothermal} = \frac{1}{2}\delta^2 p_o V_o \tag{18}$$

if terms in higher powers of δ are neglected. Similarly, the adiabatic cycle gives

$$E_{adiabatic} = \frac{1}{2}\gamma\delta^2 p_o V_o = \gamma E_{isothermal} \ . \tag{19}$$

For the general case, (16) may be written

$$E = -p_o V_o \int_1^{1-\delta} \left(\frac{T}{T_o}\frac{V_o}{V} - 1\right) d\left(\frac{V}{V_o}\right) \ , \tag{20}$$

which may be evaluated using (5) and (7). If the binomial theorem is used to simplify the integrand and terms in δ^3 and higher powers of δ are neglected, the result is

$$E = \frac{1}{2}\delta^2 p_o V_o \left(\gamma + \frac{\gamma-1}{1+w^2\tau^2}\left[\frac{1}{2}\pi w\tau - 1\right]\right) \tag{21}$$

which reduces to (18) for the isothermal cycle ($w\tau \to 0$) and to (15) for the adiabatic case ($w\tau \to \infty$).

Defining the 'Q' of the cycle by

$$Q = \frac{2\pi E}{W} \ , \tag{22}$$

(14)and (21) give

$$Q = \frac{\pi}{2} + \frac{1+\gamma w^2\tau^2}{(\gamma-1)w\tau} \ . \tag{23}$$

The minimum value of Q given by this formula occurs when $w\tau = \gamma^{-1/2}$ and is

$$Q_{min} = \frac{\pi}{2} + \frac{2\gamma^{1/2}}{\gamma - 1} \ , \tag{23a}$$

from which values of Q_{min} for air (γ = 1.402) and carbon dioxide (γ = 1.3) are found to be 7.52 and 9.17 respectively.

Estimates of decay time

In the Pressure Modulated Radiometer, neglecting the gas behind the piston in which pressure changes are much smaller, the gas occupies the detector cell, the cylinder head (between the piston and the detector end of the cylinder), and the connecting channel between these two areas. The first two regions may be considered to be circular cylinders while the connecting channel may be neglected as its volume is very small although the transfer of gas along it will have some effect on the energy losses. This effect is assumed to be small and will not be taken into account. The dimensions of the two cylindrical regions are as follows:

	Detector cell	Cylinder head
Radius	0.625 cm	1.5 cm
Height	1.0 cm	0.25 cm
Volume	1.2 cm^3	1.8 cm^3

These values are taken from ref. (1) or are measured from engineering drawings. The volume of the cylinder head is a little less than that given in ref. (1), but as the size of this region varies during the cycle, and the mean volume has been shown experimentally to decrease when the piston frequency is increased, the value given above should be representative.

Values of τ can be estimated for the two cylindrical regions by solving the heat conduction equation for the cooling of a circular cylinder initially at uniform temperature T_1 in surroundings at constant temperature T_0, and computing the time taken for the difference between the mean temperature of the cylinder and T_0 to fall to 1/e of its initial value. This decay time is proportional to the heat capacity of the cylinder and hence, in the present case of a cylinder of gas, to the density (or pressure) of the gas and to its specific heat. It is not clear whether the specific heat at constant pressure or at constant volume, or some other value should be used as neither the pressure nor the volume remain constant during the cycle. Decay times have therefore been calculated for both cases, and using suffixes '1' and '2' for the cylinder head and detector cell respectively, the results for air as the cell gas are:-

Using $c_v = 0.718$ c.g.s. units,

$$\left.\begin{aligned} \tau_1 &= 2.274 \times 10^{-5}\ p \text{ secs} \\ \tau_2 &= 1.023 \times 10^{-4}\ p \text{ secs} \end{aligned}\right\} \quad p \text{ in torr.} \tag{24a}$$

Using $c_p = 1.006$ c.g.s. units,

$$\left.\begin{aligned} \tau_1 &= 3.186 \times 10^{-5}\ p \text{ secs.} \\ \tau_2 &= 1.433 \times 10^{-4}\ p \text{ secs.} \end{aligned}\right\} \quad p \text{ in torr.} \tag{24b}$$

The variation of piston frequency with air pressure is known from experiments on a breadboard model (see ref. 1), and using the above values of τ, a 'Q' for the system can be calculated from

$$Q = 2\pi \left(\frac{E_1 + E_2}{W_1 + W_2}\right) \tag{25}$$

where the suffixes mean the same as those used for τ, and E's and W's are computed from (21) and (14) respectively using data for air and the appropriate volumes for V_0. The value of δ^2 is variable, but assuming it is the same for the cylinder head and the cell, it cancels out in the calculation of Q. This assumption is not strictly true since the detector does not change its volume, but the quantity of gas it contains varies during the cycle and this should produce something like the same effect.

Comparison with experiment

The calculated values of Q for several different mean cell pressures are given in table 1 together with the piston frequency, and the results are shown graphically in figure 2 where they are compared with experimental values taken from the electrical response of the system (circled dots and continuous curve). Two sets of values are given and two theoretical curves are plotted; one using values of τ calculated from (24a) with c_v for the specific heat, and the other using (24b) which takes c_p rather than c_v. The former is seen to give better agreement with theory for most pressures.

At low pressures the decay times are very short and the gas cycle is almost isothermal with negligible energy loss. Thus the theoretical curves rise to infinity while the experimentally measured Q, which includes losses due to leakage past the piston, eddy current losses etc remains below these curves. At very high pressures the situation is similar as the cycle becomes practically adiabatic and Q values again rise rapidly, (see table 1) although in practice the actual values of Q would not be expected to rise so quickly because of increased losses associated with the high piston frequencies and high cell pressures (such as the loss due to the 'nozzle' effect of the connecting channel between cylinder head and cell). Except at very low pressures however, the agreement between theory and experiment is fairly good for the range of figure 2, especially considering the number of approximations made in the theory, and the results suggest that the irreversibility of the thermodynamic cycle is the main cause of energy loss in the Pressure Modulated Radiometer.

Conclusions

A theoretical estimation has been made of the thermodynamic energy losses in a Pressure Modulated Radiometer. The theory assumes:-

(i) The deviation from adiabatic flow can be represented by a 'decay time' in the form of equation (4).

(ii) The cell temperature is a function of time only.

(iii) The amplitude of the piston motion is small.

(iv) The gas behind the piston remains at constant pressure.

As the volume of gas behind the piston is greater than the volume in either the cylinder head or the detector cell by a factor of at least eight, equation (14) shows that energy losses in this region will be somewhat smaller than those calculated for the regions in front of the piston.

The results show that over most of the range of cell pressures studied, agreement with experiment is good considering the approximations made, and they indicate that thermodynamic irreversibility is the major source of energy loss in the system except at very low and very high pressures when the thermodynamic losses are small enough for other sources of energy loss to be significant. Such sources include:-

(a) Eddy current and other electrical losses.

(b) Viscous friction and air leakage past the piston.

(c) Gas flow through the connecting channel.

(d) Turbulence and convection in the cell.

The theory and experiments described so far have applied for an air-filled instrument whereas, in operation, carbon dioxide will be the gas used. The decay times for carbon dioxide should be greater than those for air by a factor of about 2.5 to 3.0 resulting in a more nearly adiabatic cycle for a given cell pressure. Because γ for carbon dioxide is 1.3 compared with 1.4 for air, both the energy loss per cycle and the maximum stored energy should be reduced, particularly the former which should be about 75% of the value for air with the same value of $\omega\tau$. (See equation (14)). According to equation (23) values of Q for a given $\omega\tau$ should be increased by about 30% for cycles which are nearly isothermal, the increase falling to 20% for adiabatic cycles.

The effect of a light gas in the cell

The temperature changes of the gas in the cell, besides giving rise to energy losses, produce an unwanted component in the radiation reaching the detector of the radiometer, as explained in ref. 1. It is therefore desirable to minimize the temperature changes, for example by reducing the decay time τ in the cell. One way of doing this is by adding a light gas (hydrogen or helium) to the carbon dioxide in the radiometer, so as to increase the thermal conductivity of the mixture and hence reduce τ. Because the added gas will also increase the heat capacity of the mixture, there is an optimum amount of added gas which produces a maximum value of thermal diffusity ($K/\rho c_v$) and minimum τ. This is shown in figure 3, where the variation of thermal conductivity (K) and heat capacity (ρc_v) of gas mixtures of carbon dioxide with hydrogen or helium are plotted against partial pressure of the added gas. The partial pressure of the pure carbon dioxide has been taken as 100 mb. but it has been reduced in the mixtures so as to keep $\sqrt{p \cdot p_{co_2}}$ constant, where p is the total pressure and p_{co_2} the partial pressure of carbon dioxide. This ensures that the optical transmission of all the mixtures will be approximately the same.

Thermal conductivity of the mixtures has been calculated from a formula quoted by Reid and Sherwood (1958):-

$$K = 0.5\ (K_{sm} + K_{rm}) \tag{26}$$

where

$$K_{sm} = x_1 K_1 + x_2 K_2, \tag{27a}$$

$$1/K_{rm} = x_1/K_1 + x_2/K_2, \tag{27b}$$

x_1 and x_2 being the mole fractions (fractional pressures) of the components and K, K_1 and K_2 being the thermal conductivities of the mixture and the components.

Values of 3.07, 31.8 and 33.9 x 10^{-5} cal. cm^{-1}. sec^{-1}. $^oC^{-1}$ have been used for the thermal conductivities of carbon dioxide, hydrogen and helium respectively.

The heat capacities of the mixtures ρc were calculated from

$$\rho c = (p_1 m_1 c_1 + p_2 m_2 c_2)/(p_0 \times 22.4 \text{ litres}) \tag{28}$$

where p_i, m_i and c_i are the partial pressure, molecular weight and specific heat at constant volume (and standard temperature and pressure) of the i^{th} component, p_0 is the standard pressure (1012 mb), and 22.4 litres is the volume occupied by one gram molecule of gas at S.T.P. Values of 0.165, 2.40 and 0.76 cal.gm^{-1}. $^oC^{-1}$ were used for the specific heats of carbon dioxide, hydrogen and helium respectively.

It can be seen from figure 3 that the maximum value of $K/\rho c$ occurs when the pressure of the added gas (150 mb) is about three times that of the carbon dioxide (50 mb). The maximum value of $K/\rho c$ is about four times that for pure carbon dioxide when hydrogen is added and five times when helium is added. Although considerable reductions in τ are thus possible, it must be noted that $w\tau$ will not be reduced in proportion, because the increase in total pressure will result in an increase of oscillating frequency - by a factor of $\sqrt{2}$ in the optimum case just quoted. The maximum reduction in $w\tau$, in fact, occurs with roughly equal pressures (70 mb) of carbon dioxide and the added gas, and amounts to a factor of about 2.8 for hydrogen and 3.6 for helium.

References

1. Preliminary Design Report for the TIROS-N Stratospheric Sounding Unit. The Marconi Company Ltd. February 1973.

2. Reid, R C and Sherwood, T K. The Properties of Gases and Liquids. McGraw-Hill, 1958.

TABLE 1. Values of Q for air in cylinder head and cell.

PRESSURE (torr)	FREQUENCY (hertz)	'Q' FOR THE SYSTEM (c_v)	(c_p)
4	28.0	66.25	47.96
13	32.0	20.05	15.53
22	36.0	12.78	10.89
42	43.0	9.59	9.27
60	48.0	9.21	9.18
80	53.5	9.20	9.54
100	59.0	9.51	10.38
192	82.0	14.60	18.78
392	116.0	35.29	48.43
592	145.0	64.47	89.50
760	160.0	90.47	125.98

Data for air

Thermal conductivity = 2.41×10^{-4} j.cm.$^{-1}$sec.$^{-1}$°K.$^{-1}$

Specific heat at constant volume = 0.718 j.gm.$^{-1}$°K.$^{-1}$

Specific heat at constant pressure = 1.006 j.gm.$^{-1}$°K.$^{-1}$

Ratio of specific heats = 1.40

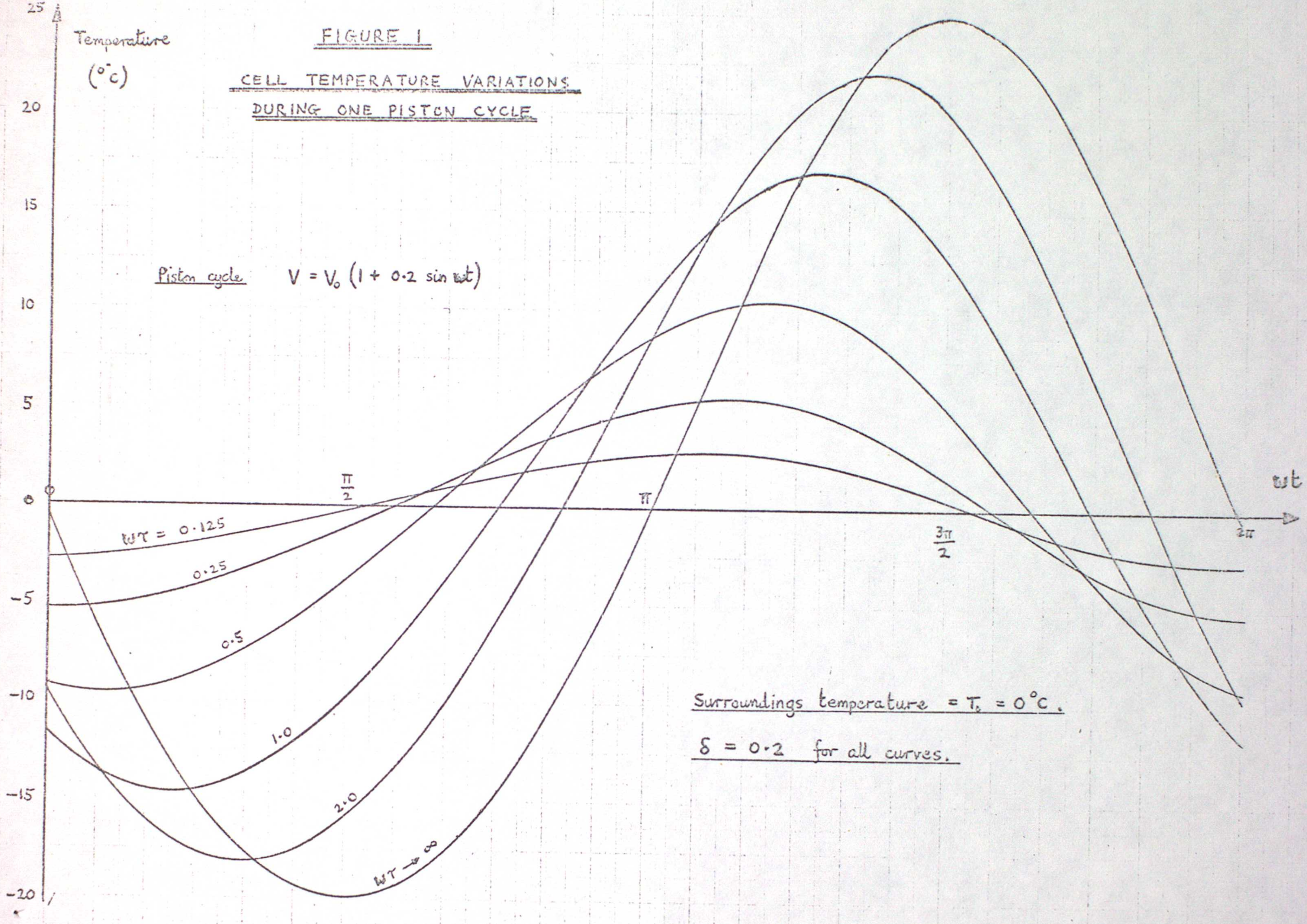


FIGURE 1

CELL TEMPERATURE VARIATIONS DURING ONE PISTON CYCLE

FIGURE 2

Q for P. M. C. against PRESSURE

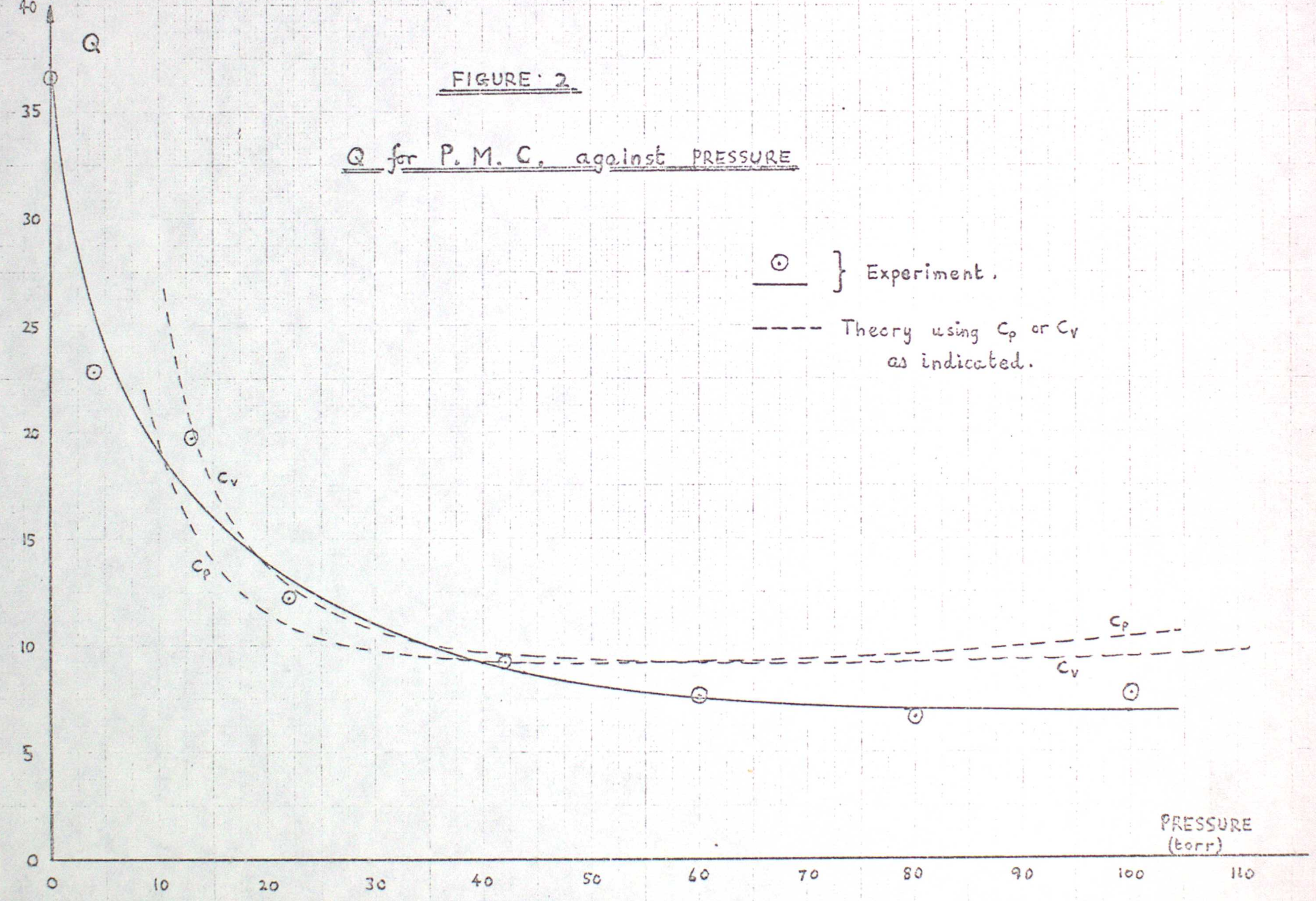
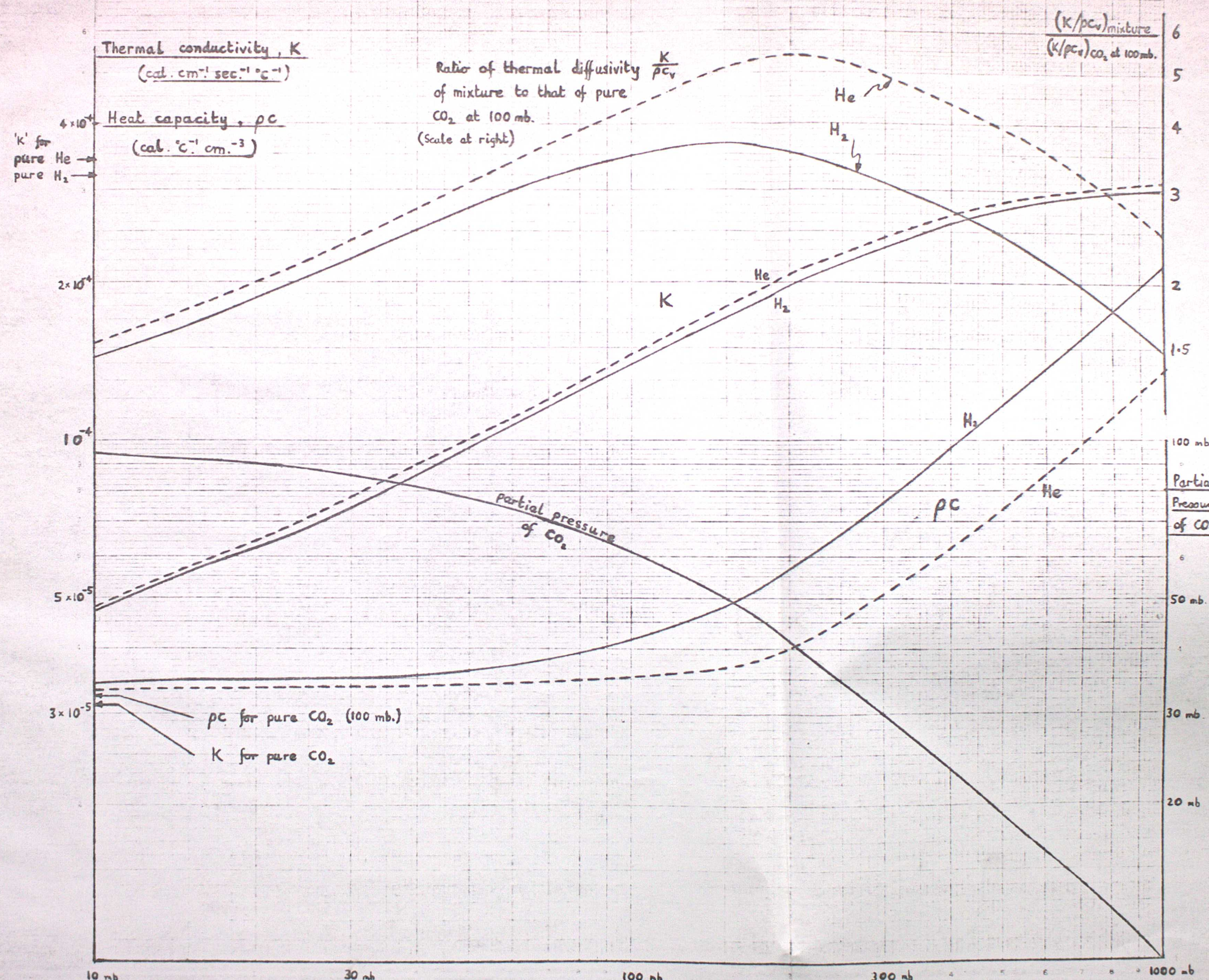


FIGURE 3.



Partial pressure of H_2 or He.