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MET.O.15 INTERNAL REPORT

No 19

SOME USEFUL CLOUD PHYSICS FORTRAN STATEMENT FUNCTIONS

by

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Updated by

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March 1980

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PREFACE

The Fortran Statement Functions which follow are updates of those originally produced by M Bader.

As the new versions reduce the percentage error by an order of magnitude, using tabulated values, there is some loss of simplicity, and in one or two cases a slight reduction in the range of validity. For the convenience of the user the original versions have been included in the APPENDIX.

March 1980

CONTENTS

FORTRAN ARITHMETIC STATEMENT FUNCTION

DENSITY (P, T) = Ø.3484*P/(T+273.15)

Purpose

This statement computes the air density, given the air pressure and temperature.

Method of Use

As a normal arithmetic statement function

eg   RHO = DENSITY (PRESS, TEMP)

where RHO will be assigned the calculated value of the density.

Description of parameters

P - Atmospheric pressure in mb.

T - Atmospheric temperature in degrees Celsius

Result is given in kg m^{-3} in the real mode.

Method of Computation

The variation of density with pressure and temperature is expressed by the gas law:

$$\rho = \frac{P}{R(T+273.15)}$$

where ρ is the density, P is the pressure, R is the gas constant for dry air and T is the temperature, in degrees Celsius.

$R = 2.87 \times 10^{2}$ joule kg^{-1} $^{o}C^{-1}$

$$\rho = \frac{0.3484P}{T+273.15} \quad kg\ m^{-3},$$

where P is in mb.

Remarks

The result is correct to within 0.1% of tabulated values for temps. in the range $-40^{o}C$ to $+40^{o}C$ and pressures in the range 200 mb to 1050mb.

Subprograms required

None.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
VISDYN (T) = 1.4963E-Ø6*SQRT((T+273.15)*(T+273.15)*(T+273.15))/(T+293.15)
```

Purpose

This statement computes the dynamic viscosity of air, given in the air temperature.

Method of Use

As a normal arithmetic statement function.

eg   XMU = VISDYN (TEMP),

where XMU will be assigned the calculated value of the dynamic viscosity.

Description of parameters

T - Atmospheric temperature in degrees Celsius.

Result is given in kg $m^{-1} s^{-1}$ in the real mode.

Method of computation

The temperature dependence of dynamic viscosity can be computed from Sutherland's equation which states'

$$\frac{\mu}{\mu_o} = \left(\frac{to+c}{t+c}\right)\left(\frac{t}{to}\right)^{3/2} , \qquad (1)$$

where μ_o is the dynamic viscosity at absolute temperature to, c is Sutherland's constant, assumed to have the value of 120°C and μ is the dynamic viscosity at absolute temperature t. μ is independent of pressure except at very low pressures.

At $to=296.16^\circ K$, $\mu_o = 1.8325 \times 10^{-5}$ kg m^{-1} sec^{-1},

Substituting these values in (1)

$$\mu = \frac{1.4963 \times 10^{-6}}{T+393.15} \; (T+273.15)^{3/2} \; kg \; m^{-1} \; sec^{-1},$$

where T is the temperature in degrees Celsius.

Remarks

The result is correct to within 0.1% of tabulated values for temps. in the range -40°C to $+40^\circ$C.

Subprograms required

None.

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p. 394.

FORTRAN ARITHMETIC STATEMENT FUNCTION

VISKIN (P, T) = VISDYN (T) / DENSITY (P, T)

Purpose

This statement computes the kinematic viscosity of air, given the air pressure and temperature.

Method of Use

As a normal arithmetic statement function.

eg XNU = VISKIN (PRESS, TEMP),

where XNU will be assigned the calculated value of the kinematic viscosity.

Description of parameters

P - atmospheric pressure in mb.

T - atmospheric temperature in degrees Celsius.

Result is given in m^2 sec^{-1} in the real mode.

Method of computation

Kinematic viscosity, ν, of a fluid is defined as

$$\nu = \frac{\mu}{\rho},$$

where μ is the dynamic viscosity and ρ the density of the fluid. The dynamic viscosity of air is given by the arithmetic statement function VISDYN (T) and the density of air by the arithmetic statement function DENSTY (P,T)

Remarks

The result is correct to within 0.1% of calculated values for the temperature range $-40^{\circ}C$ to $+50^{\circ}C$ and pressures in the range 200 mb to 1050 mb.

Other Arithmetic Statement functions required

VISDYN (T)

DENSTY (P, T).

FORTRAN ARITHMETIC STATEMENT FUNCTION

THCOND (T) = 1.4132E+Ø3*VISDYN (T)

Purpose

This statement computes the thermal conductivity of air, given the air temperature.

Method of Use

As a normal arithmetic statement function.

eg XKAPPA = THCOND (TEMP) ,

where XKAPPA will be assigned the calculated value of the thermal conductivity.

Description of parameters

T - air temperature in degrees Celsius.

Result is given in joule m^{-1} sec^{-1} $^{o}C^{-1}$ in the real mode.

Method of computation

Values of thermal conductivity, κ , at different temperatures are assumed to be proportional[1] to the dynamic viscosity μ i.e.

$$\kappa = \frac{\kappa'}{\mu'}\mu$$

where the primes denote values at $0^{o}C$.

$\kappa' = 24.279\times10^{-3}$ joule m^{-1} $^{o}C^{-1}$ at $0^{o}C$

$\mu' = 1.718\times10^{-5}$ kg m^{-1} sec^{-1}. at $0^{o}C$

The dynamic viscosity of air at temperature T is given by the arithmetic statement function VISDYN (T). Substituting for κ' and μ',

$\kappa = 1.4132\times10^{3}$ x VISDYN (T). joule m^{-1} sec^{-1} $^{o}C^{-1}$.

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $+40^{o}C$.

Other arithmetic statement functions required

VISDYN (T).

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p 394.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
DIFFTY (P, T) = 8.8Ø35 E-Ø7 * ((T+273.Ø)**1.81)/P
```

Purpose

This statement computes the diffusivity of water vapour in air, given the air pressure and temperature.

Method of Use

As a normal arithmetic statement function,

eg D = DIFFTY (PRESS, TEMP)

where D will be assigned the calculated value of diffusivity.

Description of parameters

P - atmospheric pressure in mb.

T - atmospheric temperature in degrees Celsius.

Result is given in $m^2 \sec^{-1}$ in the real mode.

Method of computation

The variation of diffusivity, D, with temperature T and pressure P is given by

$$D = Do\left(\frac{(T+273)}{(TO+273)}\right)^{1.81} \frac{Po}{P}$$

where Do is the diffusivity at temperature To (degrees Celsius) and pressure Po

$$Do = 2.26 \times 10^{-5}\ m^2 \sec^{-1} \text{ at } 0^\circ C \text{ and } 1000 \text{ mb.}$$

$$D = 8.8035 \times 10^{-7} \times \frac{(T+273)^{1.81}}{P}$$

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^\circ C$ to $+40^\circ C$ at a pressure of 1000 mb.

Subprograms required

None.

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p 395.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
VLATNT (T) = 2.5E-Ø6-T*(2349.Ø-T*(3.Ø84-Ø.Ø734*T))
```

Purpose

This statement computes the latent heat of vaporization of water, given the temperature.

Method of use

As a normal arithmetic statement function.

eg XLV = VLATNT (TEMP)

where XLV will be assigned the calculated value of the latent heat of vaporization.

Description of parameters

T - temperature in degrees Celsius.

Result is given in joule kg^{-1} in the real mode.

Method of computation

The variation of latent heat of vaporization of water, Lv , with temperature T is determined by integration of the first law of thermodynamics[1] which gives the relation

$$Lv = Lvo + \alpha T$$

where Lv is the latent heat of vaporization at $0^{\circ}C$ and α is a constant.

$$Lvo = 2.5003 \times 10^{6} \text{ joule } kg^{-1}$$

$$\alpha = -2369.3 \text{ joule } kg^{-1}\ {}^{o}C^{-1}$$

$$Lv = 2.5003 \times 10^{6} - 2369.3\ T \text{ joule } kg^{-1}.$$

Updated using empirical cubic equation.

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{\circ}C$ to $+40^{\circ}C$.

Subprograms required

None.

Reference

1. Hess, S L, 1959. Introduction to Theoretical Meteorology, Henry Holt & Co., p 44.

<u>FORTRAN ARITHMETIC STATEMENT FUNCTION</u>

```
SLATNT (T) = 2.834E+Ø6-T*(284.Ø+5.Ø6*T)
```

<u>Purpose</u>

This statement computes the latent heat of sublimation of water, given the temperature.

<u>Method of use</u>

As a normal arithmetic statement function.

eg   XLS = SLATNT (TEMP),

where XLS will be assigned the calculated value of the latent heat of sublimation.

<u>Description of parameters</u>

T - temperature in degrees Celsius.

Result is given in joule kg^{-1} in the real mode.

<u>Method of computation</u>

The variation of latent heat of sublimation of water, Ls, with temperature, T, is determined by integration of the first law of thermodynamics[1] which gives the relation

$$Ls = Lso + \alpha T$$

where Lso is the latent heat of sublimation at $0^{\circ}C$ and α is a constant.

$$Lso = 2.8339 \times 10^{6} \text{ joule kg}^{-1}$$

$$\alpha = -259.53 \text{ joule kg}^{-1}\ {}^{o}C^{-1}.$$

$$Ls = 2.8339 \times 10^{6} - 259.53T \text{ joule kg}^{-1}$$

<u>Updated using empirical equation</u>

<u>Remarks</u>

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{\circ}C$ to $0^{\circ}C$.

<u>Subprograms required</u>

None.

<u>Reference</u>

1. Hess, S L, 1959. Introduction to Theoretical Meteorology.
Henry Holt & Co., p 44.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
FLATNT (T) = 3.337E+Ø5+T*(2159.Ø-T*(Ø.9-Ø.16*T))
```

Purpose

This statement computes the latent heat of fusion of water, given the temperature.

Method of Use

As a normal arithmetic statement function.

eg XLF = FLATNT (TEMP)

where XLF will be assigned the calculated value of the latent heat of fusion.

Description of parameters

T - temperature in degrees Celsius.

Result given in joule kg^{-1} in the real mode.

Method of computation

An equation which fits the currently accepted values[1] of latent heat of fusion at different temperatures is

$$L_f = 3.3614 \times 10^5 - 2473.9\ T \text{ joule } kg^{-1}$$

where Lf is the latent heat and T is the temperature in degrees Celsius.

Updated by extending empirical equation

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range -40°C to 0°C.

Subprograms required

None.

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968 p 343.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SHTICE(T) = 21Ø7.Ø+T*(7.58+T*(Ø.84E-Ø2+Ø.8E-Ø4*T))
```

Purpose

This statement computes the specific heat of ice, given the temperature.

Method of use

As a normal arithmetic statement subprogram

eg SI = SHTICE (TEMP)

where SI will be assigned the calculated value of the specific heat.

Description of parameters

T - temperature in degrees Celsius.

Result is given in joule $kg^{-1}\ {}^{o}C^{-1}$ in the real mode.

Method of computation

An empirical formula relating the specific heat of ice Ci to temperature is[1]

$$Ci = 2.115 + 7.79 \times 10^{-3}T \quad \text{joule } kg^{-1}\ {}^{o}C^{-1},$$

where T is the temperature in degrees Celsius.

Updated by extending emirical formula

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $0^{o}C$.

Subprograms required

None.

Reference

1. Dorsey, N E, 1940. Properties of ordinary water substance.
Rheinhold Publishing Corporation, p 479

FORTRAN ARITHMETIC STATEMENT SUBPROGRAM

```
SHTWAT (T) = 4.219E+Ø3+T*(-3.Ø99+T*(Ø.121-T*(2.764E-Ø3-2.417E-Ø5*T)))
```

Purpose

This statement computes the specific heat of water, given the temperature.

Method of use

As a normal arithmetic statement function,

eg   SW = SHTWAT (T)

where SW will be assigned the calculated value of the specific heat of water.

Description of parameters

T - temperature in degrees Celsius.

Result is given in joule $kg^{-1}\ {}^{o}C^{-1}$ in the real mode.

Method of computation

An empirical formula which fits the currently accepted values[1] of the specific heat of water, Cw, at different temperatures is

$$Cw = 4.2153\times10^{3}+T(-2.4229+T(1.622\times10^{-1}-T(3.1395\times10^{-3})))\ \text{joule}\ kg^{-1}\ {}^{o}C^{-1},$$

where T is the temperature in degrees Celsius.

Updated by extending empirical formula

Remarks

The result is correct to within 0.2% of tabulated values for temperatures in the range $-40^{o}C$ to $+40^{o}C$.

Subprograms required

None.

Reference

1. Mason B J, 1957. The Physics of Clouds - Oxford University Press, p 444.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SURTEN (T) = 7.564E-Ø2-T*(1.647E-Ø4-T*(7.637E-Ø7-Ø9815E-Ø8*T))
```

Purpose

This statement computes the surface tension of pure water, given the temperature.

Method of use

As a normal arithmetic statement function

eg SIGMA = SURTEN (TEMP)

where SIGMA will be assigned the calculated value of the surface tension of pure water.

Description of parameters

T - temperature in degrees Celsius.

Result is given in kg sec^{-2} in the real mode.

Method of computation

An empirical formula which fits the currently accepted values[1,2] of the surface tension, , of water at different temperatures is

$$= 6.304 \times 10^{-7}T^2 - 1.755 \times 10^{-4}T + 7.588 \times 10^{-2} \text{ kg sec}^{-2},$$

where T is the temperature in degrees Celsius.

Updated by extending empirical formula

Remarks

The result is correct to within 0.3% of tabulated values for temperatures in the range -40°C to +50°C. But see <u>note</u> below.

Subprograms required

None.

Reference

1. Mason B J 1957. The Physics of Clouds. Oxford University Press, p 444.
2. Dorsey N E 1940. Properties of ordinary water substance, Rheinhold Publishing Corporation. p 514

Note

```
SURTEN (T) = 7.57E-Ø2-T*(1.517E-Ø4-T*(6.233E-Ø7-1.967E-Ø8*T))
```

is correct to within ±0.1% for temp. range -20°C to +30°C.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SVPWAT (T) = 6.1078*EXP(T*(19.846-T*(9.4027E-03-T*(3.442E-05+3.0E-08*T)))/(T+273.15))
```

Purpose

This statement computes the saturation vapour pressure of water vapour over pure liquid water at any particular temperature.

Method of use

As a normal arithmetic statement function

eg S=SVPWAT (TEMP)

where S will be assigned the calculated value of the saturation vapour pressure.

Description of Parameters

T - temperature in degrees Celsius.

Result is given in millibars in the real mode.

Method of computation

The equation for the variation of saturation vapour pressure over water, ew, with temperature was obtained by integrating the Clausius Claperyon equation assuming that the latent heat of vaporization varies linearly with the temperature. (The equation is not sufficiently accurate if the variation of latent heat with temperature is neglected).

The Clausius Claperyon equation states.

$$\frac{1}{ew}\frac{dew}{dt} = \frac{LvM}{R^* t^2} \qquad (1)$$

where M is the molecular weight of water, R* is the Universal gas constant and t is the temperature in degrees K.

The variation of latent heat of vaporization, Lv, with temperature may be expressed as

$$Lv = Lvo + \alpha (t-273) \qquad (2)$$

where Lvo is the latent heat at 273° K and α is a constant.

Substituting for Lv in (1) and integrating gives

$$ew = ewo \exp\left(\frac{M}{R^*}\left[(273\alpha - Lvo)\left(\frac{1}{t} - \frac{1}{to}\right) + Ln\left(\frac{t}{to}\right)\right]\right) \qquad (3)$$

If $to=273^{\circ}$K, ewo. = 6.1078 mb.

continued

Also,

$$M = 18.016 \times 10^{-3} \text{ kg mol}^{-1}$$

$$R^* = 8.3144 \text{ joule mol}^{-1} \, {}^{o}K^{-1}$$

$$= -2369.28 \text{ joule kg}^{-1} \, {}^{o}C^{-1}.$$

Substituting these values in equation (3) and expanding the logarithm into a series,

$$ew = 6.1078 \exp \left[\frac{T}{T+273} \left(19.846 - 9.4027 \times 10^{-3} T + 3.4442 \times 10^{-5} T^2\right) \right] \text{ mb}$$

where T is the temperature in degrees Celsius.

Updated by including additional term

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $+20^{o}C$., and to within 0.2% for range $-40^{o}C$ to $+30^{o}C$.

Subprograms required.

None.

FORTRAN ARITHMETIC STATEMENT FUNCTION

SVPICE (T) = 6.1Ø7*EXP(((22.493-2.ØE-Ø4*T)*T)/(T+273.Ø)) Note error!

Purpose

This statement computes the saturation vapour pressure of water vapour over pure ice at any temperature.

Method of use

As a normal arithmetic statement function

eg S=SVPICE (TEMP)

where S will be assigned the calculated value of saturation vapour pressure.

Description of parameters

T - temperature in degrees Celsius.

Result is given in millibars in the real mode.

Method of computation

An equation relating the saturation vapour pressure over ice, ei, and temperature is obtained by integrating the Clausius Claperyon equation, neglecting the variation of latent heat of sublimation with temperature.

The Clausius Claperyon equation states

$$\frac{1}{ei}\frac{dei}{dt} = \frac{LsM}{R^* t^2} \qquad (1)$$

where Ls is the latent heat of sublimation, M is the molecular weight of water, R* is the universal gas constant and t is the absolute temperature.

Assuming Ls is independent of temperature,

$$\ln\left(\frac{e_i}{e_{io}}\right) = \frac{L_s M}{R^*}\left[\frac{1}{t_o} - \frac{1}{t}\right] \qquad (2)$$

If $t=273^{o}K$, eio=6.107 mb

Also,

Ls = 2.8339 x 10^{6} joule kg^{-1}

M = 18.016 x 10^{-3} kg mol^{-1}

R = 8.3144 joule mol^{-1} $^{o}K^{-1}$

Rearranging (2) and substituting the above values,

$$ei = 6.107 \exp\left(\frac{22.493\ T}{T+273}\right)$$

where T is the temperature in degrees Celsius.

Updated empirically.

continued

Remarks

This result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $0^{o}C$

Subprograms required

Nonc.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SDENWT (T) = Ø.21668 * SVPWAT (T) / (T + 273.Ø)
```

Purpose

This statement computes the density of saturated water vapour over water at any particular temperature.

Method of use

As a normal arithmetic statement function

eg RHO = SDENWT (TEMP),

where RHO will be assigned the calculated value of the saturated water vapour density.

Description of parameters

T - temperature in degrees Celsius.

Result is given in kg m^{-3} in the real mode.

Method of computation

The density of pure water vapour, ρsw, at saturation over a plane surface of water is given by[1]

$$\rho sw = \frac{ew}{CRwt}$$

where ew is the saturation vapour pressure over water at $t^{o}K$, R_w is the gas constant for water vapour and C is the compressibility factor for water vapour introduced to correct for the deviations of water vapour from the ideal gas laws and is neglected.

If ew is expressed in mb, ρsw is given by

$$\rho sw = \frac{0.21668}{T+273} \times ew \qquad kg\ m^{-3}$$

The saturation vapour pressure, ew, is given by the arithmetic statement function SVPWAT (T).

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $20^{o}C$., and to within 0.2% in the range $-40^{o}C$ to $+30^{o}C$.

Other arithmetic statement functions required

SVPWAT (T)

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p 381.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SDENIC (T) = Ø.21668 * SVPICE (T) / (T + 273.Ø)
```

Purpose

This statement computes the density of saturated water vapour over ice at any particular temperature.

Method of use

As a normal arithmetic statement function.

eg RHO = SDENIC (TEMP)

where RHO will be assigned the calculated value of the saturated water vapour density.

Description of parameters

T - temperature in degrees Celsius.

Result is in kg m^{-3} in the ream mode.

Method of computation

The density of pure water vapour at saturation over a plane surface of ice is given by[1]

$$\rho si = \frac{ei}{CRwt}$$

where ei is the saturation vapour pressure over ice at $t^{o}K$, Rw is the gas constant for water vapour and C is the Compressibility factor for water vapour introduced to correct for the deviations of water vapour from the ideal gas laws and is neglected.

If ei is expressed in mb.

$$\rho si = \frac{.21668\ ei}{T+273} \qquad kg\ m^{-3}$$

The saturation vapour pressure, ei, is given by the arithmetic statement function SVPICE(T).

Remarks

The result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{o}C$ to $0^{o}C$.

Other arithmatic statement functions required

SVPICE (T)

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p 381.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SHMRWT (P,T) = 622.Ø * SVPWAT (T) * (1.Ø + 5.Ø E-Ø6 * P) / (P-SVPWAT(T))
```

Purpose

The statement computes the saturation humidity mixing ratio of air over water, given the pressure and temperature.

Method of use

As a normal arithmetic statement function

eg XW = SHMRWT (PRESS, TEMP)

where XW will be assigned the calculated value of the saturation humidity mixing ratio.

Description of parameters

P - atmospheric pressure in mb.

T - air temperature in degrees Celsius.

Result is given in gm kg^{-1} in the real mode.

Method of computation

The saturation humidity mixing ratio over water, ew, is defined as[1]

$$r_w = \frac{622xC \times ew}{p - Cew} \qquad gm\ kg^{-1} \qquad (1)$$

where ew is the saturation vapour pressure over water in mb.

P is the total pressure in mb.

and C is the correction factor for the departure of the mixture of air and water vapour from ideal gas laws.

C may be expressed as

$C = 1 + 5x10^{-6}P$ and may be neglected in the denominator of equation (1)

Thus (1) becomes

$$r_w = \frac{622\ ew\ (1 + 5x10^{-6}P)}{P-ew}$$

The saturation vapour pressure, ew, is given by the arithmetic statement function SVPWAT (T).

Remarks

This result is correct to within 0.1% of tabulated values for temperatures in the range $-40^{\circ}C$ to $+30^{\circ}C$ and pressures in the range 600 mb to 1050 mb., and to within 1% in the temp. range $-40^{\circ}C$ to $+40^{\circ}C$ and pressures of 200 mb to 1050 mb.

continued

Other arithmetic statement functions required

SVPWAT (T)

Reference

1. Smithsonian Meteorological Tables, 6th Revised edition, 1968, p 302.

FORTRAN ARITHMETIC STATEMENT FUNCTION

```
SHMRIC (P,T) = 622.Ø*SVPICE(T)*1.ØØ5/(P-SVPICE(T))
```

Purpose

This statement computes the saturation humidity mixing ratio of air over ice, given the pressure and temperature.

Method of use

As a normal arithmetic statement function

eg XI = SHMRIC (PRESS, TEMP)

where XI will be assigned the calculated value of the saturation humidity mixing ratio.

Description of parameters.

P - atmospheric pressure in mb.

T - air temperature in degrees Celsius.

Result is given in gm kg^{-1} in the real mode.

Method of computation

The saturation humidity mixing ratio over ice, ri, is defined as[1]

$$r_i = \frac{622 x c x ei}{P-Cei} \quad gm\ Kg^{-1} \qquad (1)$$

where ei is the saturation vapour pressure over ice in mb.

P is the total pressure in mb,

and C is the correction factor for the departure of the mixture of air and water vapour from ideal gas laws and may be neglected.

Thus,

$$r_i = \frac{622ei}{P-ei} \quad gm\ kg^{-1}$$

The saturation vapour pressure, ei, is given by the arithmetic statement function SVPICE (T).

Updated empirically.

Remarks

This result is correct to within 0.1% of tabulated values for temperatures in the range -40°C to +0°C and pressures in the range 200 mb to 1050 mb.

Other arithmetic statement functions required

SVPICE (T).

Reference

1. Smithsonian Meteorological Tables, 6th Revised Edition, 1968, p 306.

FORTRAN FUNCTION SUBPROGRAM

Name of subprogram

COLEFF

Purpose

This subroutine computes the droplet collision efficiency, given the radii of the droplets.

Method of Use

As a normal function subprogram,

eg EFF = COLEFF (RAD1, RAD2)

where EFF will be assigned the calculated value of the normalized collision efficiency for the drops of radius RAD1 and RAD2.

Description of Parameters

On input, the radii are in metres
The result is given in the real mode.

Method of Computation

Use is made of approximate formulae which fit the Davis-Sartor-Shafrir-Neiburger droplet collision efficiency calculations[1]. The collision efficiency Y is calculated using equations 3, 5, 6 and 9 of the paper[1]. (but see later papers - Hocking, Jonas etc).

The normalized collision efficiency Y_n is then given by

$$Y_n = Y \times \left(\frac{R_1}{R_1 + R_s} \right)^2 ,$$

where R_1 and R_s are the large and small droplet radii respectively.

Remarks

1. This computation is valid for $10 \leqslant R_1 \leqslant 136$

2. Note the error in equation (9) of the paper[1]. The equation should read

$$B = \left\{ 1.587R_1 + 32.73 + 344 (20/R_1)^{1.56} \right.$$
$$\left. \times \exp[(R_1 - 10)/15] \sin [(R_1 - 10)/63] \right\} /R_1^2$$

Subprograms required

None.

Reference

1. Scott, W.T., Chong-Yuan Chen 1970

Approximate Formulas Fitted to the Davis-Sartor-Schafrir-Neiburger Droplet Collision Efficiency Calculations. J Atm Sci 27 698.

APPENDIX

The Fortran Statement Functions listed below are the originals produced by M Bader, correct to within ± 1% of tabulated values.

1 to 6, 10 to 12, 14, and 16 are valid for temp range $-40^{o}C$ to $+40^{o}C$
7 to 9, 13, 15, and 17 are valid for temp range $-40^{o}C$ to $0^{o}C$
1,3,5, 16 and 17 are valid for pressure range 200 mb to 1050 mb.

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1.  DENSTY  (P, T) = Ø.3484*P/(T+273.Ø)

2.  VISDYN (T) = 1.4963E-Ø6*SQRT ((T+273.Ø)*(T+273.Ø)*(T+273.0))/(T+393.Ø)

3.  VISKIN (P, T) = VISDYN (T) / DENSTY (P, T)

4.  THCOND (T) = 1.4132E+Ø3*VISDYN (T)

5.  DIFFTY (P, T) = 8.8Ø35 E-Ø7 * ((T+273.Ø)**1.81)/P

6.  VLATNT (T) = 2.5ØØ3 E+Ø6 - 2369.3 * T

7.  SLATNT (T) = 2.8339 E+Ø6 - 259.53*T

8.  FLATNT (T) = 3.3614E+Ø5+2473.9*T

9.  SHTICE (T) = 2115.Ø + 7.79*T

10. SHTWAT (T) = 4.2153 E+Ø3+T*(-2.4229+T*(1.6221E-Ø1-Tx3.13 95E-Ø3))

11. SURTEN (T) = T*(T*6.3Ø4E-Ø7 - 1.755E-Ø4)+7.588 E-Ø2

12. SVPWAT (T) = 6.1Ø78*EXP (T*(19.846 - T*(9.4Ø27 E-Ø3 - 3.4442 E - Ø5*T)) /
                 (T+273.Ø))

13. SVPICE (T) = 6.1Ø7 * EXP (22.493 * T/(T+273.Ø))

14. SDENWT (T) = Ø21668 * SVPWAT (T) / (T + 273.Ø)

15. SDENIC (T) = Ø.21668 * SVPICE (T) / (T + 273.Ø)

16. SHMRWT (P,T) = 622.Ø * SVPWAT (T) * (1.Ø + 5.Ø E-Ø6 * P) / (P-SVPWAT(T))

17. SHMRIC (P, T) = 622.Ø * SVPICE (T) / (P-SVPICE(T))
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