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Solutions of the integral equation of diffusion and the random walk model for continuous plumes and instantaneous puffs in the atmospheric boundary layer.

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SOLUTIONS OF THE INTEGRAL EQUATION OF DIFFUSION AND THE RANDOM WALK MODEL FOR CONTINUOUS PLUMES AND INSTANTANEOUS PUFFS IN THE ATMOSPHERIC BOUNDARY LAYER

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ABSTRACT

The integral equation method is related to the random walk modelling that has proved so effective and popular in recent years. The I.E. method, by using simple probability techniques, avoids the inefficient determination of thousands of trajectories in order to build up concentration profiles. In fact it is so simple and efficient it can be run on a conventional programmable calculator. The method is applied to passive material being released from an elevated source within a neutrally stable surface layer over a uniform surface, and also to an instantaneous release when the effect of wind shear is examined. The latter scenario is also studied using random walk techniques and a comparison of the solutions obtained. Agreement is very good, although downwind spread is shown to be quite sensitive to gridlength size in the I.E. method.

INTRODUCTION

Most, but not all, experimental studies of how material injected into the atmosphere is dispersed have, for very practical reasons, been within the surface layer (see Pasquill and Smith, 1983). Paralleling these have been the development of theoretical techniques for predicting vertical and crosswind plume growth. The concepts of an eddy diffusivity K(z) and the parabolic differential equation of eddy diffusion are very familiar. Except in neutral stability conditions the definition of K(z) requires a large measure of empiricism in which diffusion experiments have to be carried out in order to define K(z), which when generalised in terms of basic meteorological surface layer parameters can then be inserted back into the diffusion equation to yield plume behaviour in more general conditions. When the source is very close to the ground, the solutions of the diffusion equation are considered to yield a satisfactory description of vertical plume growth. But when the source is elevated or when considering crosswind dispersion the solutions are theoretically dubious.

A second very familiar technique, similarity theory, is based on dimensional considerations, but is strictly applicable only to ground-level sources, and yields only limited information on the distribution of concentration within the plume.

Higher-order closure techniques applied to the basic conservation equations of momentum, heat and injected material are in principle capable of giving considerable detail in a wide variety of conditions, but the method is complex, partially empirical, and laborious.

In recent years the relative simplicity of so-called random walk techniques has been widely exploited very successfully with the ready availability of computers. The basic ideas of the method are presented briefly in the next section. Above all it is a highly versatile method capable of yielding considerable detail of plume behaviour, given the spatial distribution of turbulence and details of the boundaries.

In the third section, we present a development of a rather new technique which is closely related in principle to the random walk method. It has been called the integral equation method (Smith, 1982), since it involves a step by step summing up (or integration) of all possible random motions of the particles in the plume through the definition of probabilities. It is almost as simple to formulate as the solutions have the virtue of being substantially easier and quicker to compute. In some simple situations the method can be programmed and solved using a hand-held programmable calculator.

In the fourth section the results of the integral equation method and the random walk method are applied to diffusion from an elevated crosswind line source in the neutral surface layer. The solutions using the two methods are in close agreement. Solutions are obtained for a continuous release, and for an instantaneous release in which the chief interest is in the particle distribution in the xz plane (i.e. alongwind and in the vertical) at given times. These distributions show the interaction of vertical diffusion and the shear of the horizontal wind.

2. THE RANDOM WALK MODEL.

The random walk method of modelling dispersion consists of simulating numerically the motion of many particles of the pollutant, in order to build up a picture of the concentration distribution. The method has been used previously by many authors, for example Reid (1979) and Ley (1982). The trajectory of each particle is simulated by modelling the evolution of the particle's vertical and downwind velocities over a succession of time-steps. The crosswind motion is not modelled here and so all concentrations from the model must be interpreted as crosswind integrated values. (x_i, z_i) will be used to denote the coordinates of the particle before the ith time step, and (u_i, w_i) will represent the particle's velocity components during the step.

The vertical motion of each particle is modelled by

$$w_{i+1} = w_i (1 - \Delta t/\tau(z_{i+1})) + \mu_{i+1} \qquad (2.1)$$

where Δt is the length of the time-step, $\tau(z)$ is the Lagrangian time-scale for the particle's motion and μ_{i+1} is a Gaussian random number. The physical interpretation of this equation is that over a time Δt the particle loses a small fraction $\Delta t/\tau$ of its momentum to the surrounding air and in return receives a random impulse μ_{i+1}.

In homogeneous turbulence the mean and mean square vertical velocity of a particle of tracer over an ensemble of realisations is equal to zero and σ_w^2 respectively at all times, where σ_w^2 is the Eulerian mean square vertical velocity. If we force our model particles to satisfy this condition, ie if we force $\overline{w}_i = \overline{w}_{i+1} = 0, \overline{w_i^2} = \overline{w_{i+1}^2} = \sigma_w$, we obtain

$$\overline{\mu} = 0 \qquad (2.2a)$$

$$\overline{\mu^2} = 2\frac{\Delta t}{\tau} \sigma_w^2 + O(\Delta t^2) \qquad (2.2b)$$

for the mean and variance of the random variable μ. In inhomogeneous turbulence this derivation is no longer strictly valid; however if σ_w is constant, as in the neutral surface layer considered in this paper, the result is still correct (for a full explanation of this and of how to proceed when σ_w does vary, see Thomson 1983)

There are two assumptions in (2.1). The first is that w_{i+1} depends linearly on w_i as postulated by Smith (1968), and the second is that w_{i+1} depends only on w_i and is not influenced by μ_i, μ_{i-j}, w_{i-j} ($j \geqslant 1$) except through w_i. Hanna (1979) has presented some exerimental evidence in support of the first assumption. These assumptions connot be exactly true since (2.1) implies a discontinuous acceleration whose size $|-w_i \Delta t/\tau + \mu| / \Delta t$ tends to infinitely as $\Delta t \rightarrow 0$. However atmospheric accelerations are large and are significantly correlated only over very short times. (of the order of the Kolmogorov time-scale - Monin and Yaglom 1975 p 370, pp 548-549).

The horizontal velocity of the particle is taken to be equal to the mean wind at the particle's height:

$$u_i = \overline{u}(z_i) \qquad (2.3)$$

Horizontal turbulence could be easily included in this equation. However, Ley (1982) showed that in models of this type it has only a small effect on the results.

$\overline{u}(z)$, σ_w and τ are taken to be the standard similarity forms appropriate to a neutral surface layer:

$$u(z) = \frac{u_*}{k} \ln \frac{z}{z_o} \qquad (2.4a)$$

$$\sigma_w = 1.3\, u_* \qquad (2.4b)$$

$$\tau = \frac{k\, u_*\, z}{\sigma_w^2} \qquad (2.4c)$$

where u_* is the friction velocity and z_o is the roughness length.

The time-step Δt must be chosen to be short in comparison to so that the particle motions are well resolved. Of course this is not possible at all heights if the profile of τ is as given by (2.4c). To solve this problem a modified profile is used in the model:

$$\tau = \begin{cases} \dfrac{k u_* z}{\sigma_w^2} & z > 0.1\text{m.} \\[2ex] \dfrac{k u_* 0.1}{\sigma_w^2} & 0.1\text{m.} \geqslant z \geqslant 0 \end{cases}$$

By keeping τ constant below 0.1m, the difficulty is avoided. The particles are perfectly reflected at the ground. To avoid using a very small time-step when it is not necessary, Δt is allowed to vary as the particle moves. In the simulations presented in this paper $\frac{\Delta t}{\tau} = 0.05$ was used.

In a real cloud of pollution, the velocities of the fluid elements within the cloud are correlated. In the model however, each particle is assumed to move independently. Hence the concentrations obtained from the model must be regarded as ensemble average values.

The random walk approach has several advantages over the more classical diffusion equation approach. Firstly, and most importantly from a theoretical point of view, the velocities of the particles change smoothly in time, in line with Taylor's (1921) ideas about diffusion. In contrast the diffusion equation assumes very large velocities and very short time-scales and so is only strictly valid for molecular diffusion. Secondly, the method is very flexible and can be adapted to any turbulent flow provided the mean flow and turbulence statistics are known. Finally, because the method does not need an Eulerian grid, there are no problems with artificial diffusion when the cloud width is of the same order as the grid spacing.

A drawback of the technique is that the Lagrangian correlogram is fixed by the model (in homogeneous turbulence it is equal to $\exp(-t/\tau)$ where t is the time-lag between velocity measurements) and does not necessarily agree with the true correlogram. However Pasquill and Smith (1983 §3.5) have shown that dispersion characteristics depend mainly on σ_w and τ and are only weakly dependant on the shape of the correlogram.

THE INTEGRAL EQUATION OF DIFFUSION

The I.E. method is strongly related to the random walk method that has just been described. By introducing probability functions and summing up (or integrating) over all possibilities, considerable computational saving can be achieved, since it is no longer necessary to simulate dispersion by following thousands of distinct particle trajectories one after the other.

The general idea of the method can be best understood by considering its application to the simple case of dispersion in homogeneous turbulence. Later the technique will be applied to the main study of this paper, dispersion within the neutrally-stratified surface layer of the atmosphere.

The method, in one sense, is more primitive than the random walk method as it has been described above, which strictly could be referred to more accurately (but more pedantically) as a Markov chain simulation technique, since it describes turbulent motion correctly as a continuous process. The I.E. method is more akin to the original random walk, or drunkard's walk, concept. It describes diffusion as a collision process in which elements of marked fluid, originating from the source, undergo collisions with other elements, of unspecified origin, totally lose correlation with their previous motion and then travel with constant velocity until they collide again. In this sense the method is strictly analogous to Brownian diffusion of elemental particles. An element will experience collision at a point with a probability that in general depends on the location of the point. The probability is proportional to the local Lagrangian timescale τ . In stationary homogeneous turbulence τ is, of course, an absolute constant, whereas in other situations τ may vary in time or in space. The inherent difficulty of specifying τ is common to all theories of diffusion (even though the difficulty may be implicit since τ may not appear explicity in the theory), and represents the greatest barrier to specifying dispersion in terms of an Eulerian fixed-point description of turbulence, without recourse to empiricism, no matter what theory or technique is used. In this respect the I.E. method is no harder or easier than any other.

The method can recognise in a rational way the statistical properties of the turbulence at every location; in particular it can reflect the probability distribution of the velocity fluctuations, its variance, skewness and finite range.

Before describing the method, it should be noted that it has been simplified quite significantly from its earliest form which has been described by Smith (1982) and by Pasquill and Smith (1983).

The basic ideas of the method can be summarised as follows:

(i) calculations are performed on a grid of equal squares (or rectangles) of sides Δz and Δx .

(ii) collisions occur only at the mid-line of each square: at $\frac{\Delta x}{2}$ in from the side of the gridsquare.

(iii) some elements (or "particles") will collide whilst passing through a given square, others will not. Greater accuracy is achieved when the squares are sufficiently small that only a few percent undergo collision in a single square.

(iv) between collisions, elements travel with constant velocity.

(v) in the simpler forms described here, the turbulence is considered in the cross-flow z-direction only. In the along-flow x-direction, the velocity is given by the mean windspeed u.

(vi) the probability that an element will undergo a collision in a gridsquare is given by $a = \Delta x / u\tau$

(vii) if w is the turbulent velocity fluctuation in the z-direction, then the probability P(w) can be assumed zero for $|w| \geqslant w_m$, a specified maximum velocity. For $|w| < w_m$, P(w) is supposed given. In many cases it will be close to a Gaussian distribution and may be approximated by the analytical expression:

$$P(w) = \frac{15}{16\, w_m}\left(1 - \frac{w^2}{w_m^2}\right)^2 \qquad |w| \leqslant w_m \, .$$

This satisfies the following:

$$\begin{cases} P(-w) = P(w), \quad P(w_m) = 0, \\ \int_{-w_m}^{w_m} P(w)\, dw = 1 \, , \\ \sigma_w = 0.37796\, w_m . \end{cases}$$

In the homogeneous turbulence case, in which u and w_m are constant, the gridlengths Δz and Δx are chosen so that $u\Delta z \equiv w_m \Delta x$, and elements cannot diffuse further than one gridsquare either way in the z direction in the time taken to advance Δx in the x-direction.

(viii) Consider the gridsquare labelled (M,N) in Figure 1 . M indicates that the gridsquare is M squares up in the z-direction from some arbitrarily selected origin. Similarly N indicates it is N squares along in the x direction from the same origin; which could be the source gridsquare, or could be the square adjacent to the surface immediately below the source in the case of diffusion in the atmospheric surface layer. In general let the source be at $(M_s, 0)$, where M_s might be chosen to be zero.

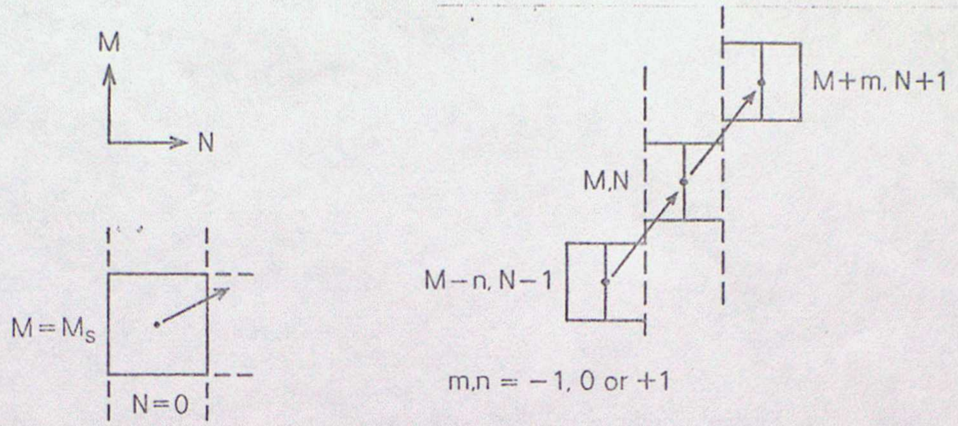

Figure 1. Definition of grid square notation

(ix) Define Q(M,m) as the probability that a "particle" undergoing collision within gridsquare (M,N) will subequently move to grid-square (M+m,N+1), where m can take the value -1, 0 or 1.

Furthermore, define R(M,N,m) as the probability that a marked "particle" originating from the source, arriving at (M,N) will move to (M+m,N+1), regardless of whether or not it undergoes collision in (M,N).

If D(M,N) is the number of marked particles in (M,N), then the basic equations that govern this diffusion process are

$$D(M,N)R(M,N,m)=D(M,N).a.Q(M,N)+\sum_{n=-1,0,1} f\, D(M-n,N-1).(1-a).R(M-n,N-1,n)$$

and $D(M,N+1)=\sum_{n=-1,0,1} R(M+n,N,-n).D(M+n,N)$

where in the first of these equations, the left-hand-side is the number of marked particles moving from (M,N) to (M+m,N+1), the first term on the right-hand-side is the number of particles which undergoing collision in (M,N) then move to (M+m,N+1) whereas the second term represents the sum of those marked particles in (M,N) that do not experience collision there and have come from (M-n,N-1) and proceed to (M+m,N+1). The factor f takes the following values:

$$f = \begin{cases} 0.6 & \text{if } |n-m| = 0 \\ 0.4 & \text{if } |n-m| = 1 \text{ and } |n| = 1 \\ 0.2 & \text{if } |n-m| = 1 \text{ and } |n| = 0 \\ 0 & \text{if } |n-m| = 2 \text{ or if } |n| > 1 \text{ or } |m| > 1. \end{cases}$$

These values can be seen to preserve a uniform concentration distribution. The second equation sums up all the marked particles arriving in (M,N+1) from (M+1,N), (M,N) and (M-1,N).

In homogeneous turbulence, when P(w) given above is a good approximation,

$$Q(M,0) = \int_{-W_m}^{W_m} \left(1 - \frac{|w|}{w_m} \right) \frac{15}{16w_m} \left(1 - \frac{w^2}{w_m^2} \right)^2 dw = \frac{11}{16} = 0.6875$$

and $Q(M,\pm 1) = 0.15625$

At the source $D(M_s,0)$ = number of particles released, say 10^4.

and $R(M_s,0,n) \equiv Q(M_s,m), = Q(m)$ in homogeneous turbulence.

The choice of a, and hence Δz and Δx, is arbitrary, but to achieve reasonable smoothness it should be at most 0.1. Smaller values will give smoother solutions at the expense of more computing time.

Concentration is obtained by dividing D(M,N) by the wind speed u(M). The resulting concentration distribution for homogeneous turbulence is very quickly derived, and has moments in very close agreement with analytically - derived moments (see Smith 1982). Thus the second moment is in virtually complete agreement with Taylor's statistical theory result with an exponential Lagrangian correlogram with the same timescale :

$$\sigma^2(x) = \frac{2\tau\sigma_w^2}{u} \left(x + u\tau(\exp(-x/u\tau) - 1) \right)$$

The method gives more than just the moments of course: the ensemble average concentration distribution results, and in this case the distribution is very close indeed to being Gaussian.

Diffusion in the atmospheric neutral surface layer

(i) A continuous source

The formulation is very similar to that outlined above, the only basic difference is that both u and τ are now functions of z (or M).

A log-law is assumed for u: $u = \frac{u_*}{k} \ln \frac{z}{z_o}$

and a linear variation for τ : $\tau = ku_*z / \sigma_w^2$

Since $a = \Delta x/u\tau$ it follows that a also varies with z:

$$a = \Delta x\, \sigma_w^2/(u_*^2 z \ln z/z_o)$$

It is assumed that $\sigma_w = 1.3u_*$ following the evidence from surface layer studies in the field (Pasquill & Smith, 1983). Furthermore it will be assumed that $a = 0.1$ at $z = z_* \equiv 1000\, z_o$ (where z_o is the surface roughness).

Consequently $\Delta x = \quad a u \tau = \left(\frac{1000\ z_0 \ln 1000 \times 0.1}{1.3^2}\right) = 408.743\ z_0$

and $\Delta z = \left[\frac{w_m \Delta x}{\bar{u}(z_*)}\right] = a\, w_m \left[\frac{k\, u_*\, z_*}{\sigma_w^{\,2}}\right] \qquad = 81.4083\ z_0.$

The midpoint of the first gridsquare (M=O) above the surface is at a height $\frac{\Delta z}{2}$ and that of the gridsquare M is at a height $(M+\frac{1}{2})\Delta z$.

Thus $a = \left[(M+\frac{1}{2}) x .11785056 \times \ln(81.4083(M+\frac{1}{2})) \right]^{-1}$. In practise if a exceeds 1 in the very lowest layers, then a is put equal to 1.

$$Q(M,m) = \frac{15}{16}\left(c_m - Ac_m^3 + Bc_m^5\right) \text{ for } M \geqslant 1 \text{ and } m = -1,0,1$$

where $c_m = 0.144765 \ln (81.4083(M+\frac{1}{2}))$,

$B = \frac{1}{80}$ and $A = \frac{1}{6}$ if $m = 0$

and if $m = \pm 1$: $\quad Q(M,m) = \frac{1}{2}(1-Q(M,0))$

For $M = 0$ put $Q(0,1) = 0.2603$
$Q(0,0) = 1-Q(0,1) = 0.7397$
and $Q(0,-1) = 0$, so that particles are fully reflected at the surface.

Note that except for $Q(0,m)$: $\quad Q(M,-1) \equiv Q(M,1)$

The solution can now proceed using the two basic equations given in the previous section. Results are given later.

(ii) An instantaneous release (or puff)

The method can also be applied to an instantaneous release from a source near the ground in the neutral surface-stress layer. The aim is to learn something about the spatial distribution of particles in such a puff, as a function of time, resulting from the interaction of vertical turbulent spreading and the logarithmic wind shear.

It is suggested that because the diffusive spread is governed by small-scale eddies which are largely controlled and influenced by the proximity of the ground, there is relatively very little difference between the particle distribution for any single realisation of a release and that for the ensemble mean distribution. However if horizontal turbulence were to be taken into account, (which is no problem in principle), then the assumption would be less acceptable since variable larger-scale eddies would create significant differences from one realisation to the next.

Let Δz remain as before. Since we are concerned with spread at a given time, rather than a given distance, Δx will be replaced by Δt. The probability of collision $a = \Delta t/\tau$ is again put equal to 0.1 at $z = z_*$.

Thus $$\Delta t = 81.4087 \frac{z_o}{w_m}$$

and $$a(M) = \frac{1.22837}{(M + \frac{1}{2})} \qquad M \geq 1, \quad a(0) = 1.$$

$Q(M,m)$ takes the same two values found for homogeneous turbulence

i.e. $Q(M,0) = 0.6875$ and $Q(M, \pm 1) = 0.15625$

Let the suffix k denote the k^{th} package which contains particles that after a specified time $N \Delta t$ have travelled the same distance x_k. Suppose this package has moved from $(M-m,N-1)$ to (M,N). Part of this package (ie a sub-package) now moves from (M,N) to $(M+m,N+1)$. The laws governing this are:

$$D_k(M,N).R_k(M,N,m)=D_k(M,N).a(M).Q(m)+D_k(M-n,N-1).(1-a(M)).R(M-n,N-1,n).f$$

with meanings the same as those described earlier. $f(m,n)$ values are also identical to their previous values.

Furthermore $D_k(M,N) = D_k(M-n,N-1).R(M-n,N-1,n)$

so that $R_k(M,N,m) = a(M)Q(m) + (1-a(M))f(m,n)$

Now $x_k(N+1) = x_k(N) + u((M+\frac{1}{2}(m+1))\Delta z) \times \Delta t.$

Since $u = \frac{u_*}{k} \ln \frac{z}{z_o}$, if X is defined as x/z_o, then:

$$X_k(N+1) = X_k(N) + 59.1719 \ln [81.4087 (M+\tfrac{1}{2}(m+1))]$$

At each new timestep the number of packages trebles and the number would soon become unmanageable if some sort of grouping was not imposed. Various options are available. The following was selected for its relative simplicity:

At N, find the maximum $X(N) = X_{max}$ and the minimum $X(N) = X_{min}$, over <u>all</u> packages regardless of M. Then define $\delta = (X_{max} - X_{min})/25$.

At each level (each M) group together all packages which have X_k lying between $(X_{min} + r\delta)$ and $(X_{min} + (r+1)\delta)$ where r runs from 0 to 24.

In this grouping together, the following rules are to be observed:

(i) if more than one package lies in an interval then $D(M,r) = \sum D_k(M,r)$, ie the new package conserves the number of particles from all the contributing packages, and

(ii) $D(M,r)\ R(M,N,m) = \sum D_k(M,r)\ R_k(M,N,m)$, and

(iii) $X(M,r) = \dfrac{\sum D_k X_k}{D(M,r)}$, averages the values of X_k.

If the packages are kept in order of X at each level, then in marching forward one timestep, the order will be preserved for those with m = 0, and those additional subpackages entering the level from levels immediately above and below (corresponding to m = -1 and +1) can be inserted quickly if their X's are tested against the X's (for m = o) with the same r. Results will now be presented.

Results

(a) The continuous source

Solutions of both the integral equation (I.E.) and random walk (R.W.) methods have been obtained for an elevated source S_1 at a height 2.5 Δz and for a near ground level source S_o at 0.5Δz. The gridsizes, Δz and Δx, are as given in the previous section. Concentration profiles for a continuous release at S_1 are given in Figure 2 in non-dimensional form and in equivalent dimensional form for the following particular values of the parameters:

z_o = 0.02m, Δz = 1.628m, Δx = 8.175m, h_s = height of S_1 = 4.07m, u_* = 1ms^{-1} and $u(h_s)$ = 13.29ms^{-1}

The profiles are given at four downwind distances; N = 2 corresponds to x = 2Δx, N = 6 to x = 6 Δx, and so on. By N = 19 the profile is virtually identical to that from the near-groundlevel source S_o under the same conditions but the latter taken at a greater distance downwind (at approximately N = 30). The shape of the vertical profile is by then closely consistent with a simple form:

$$C(z) = C(o) \exp \left[- az^s \right]$$

where the value of s is found to be 5/4.

In general the solutions from the two methods are in very close agreement, although small but observable differences are often evident very close to the ground where the I.E. method, because it obtains its solutions on the basis of a grid, reflects the details of the rapid u(z) and τ (z) variations there rather too simplistically. The two

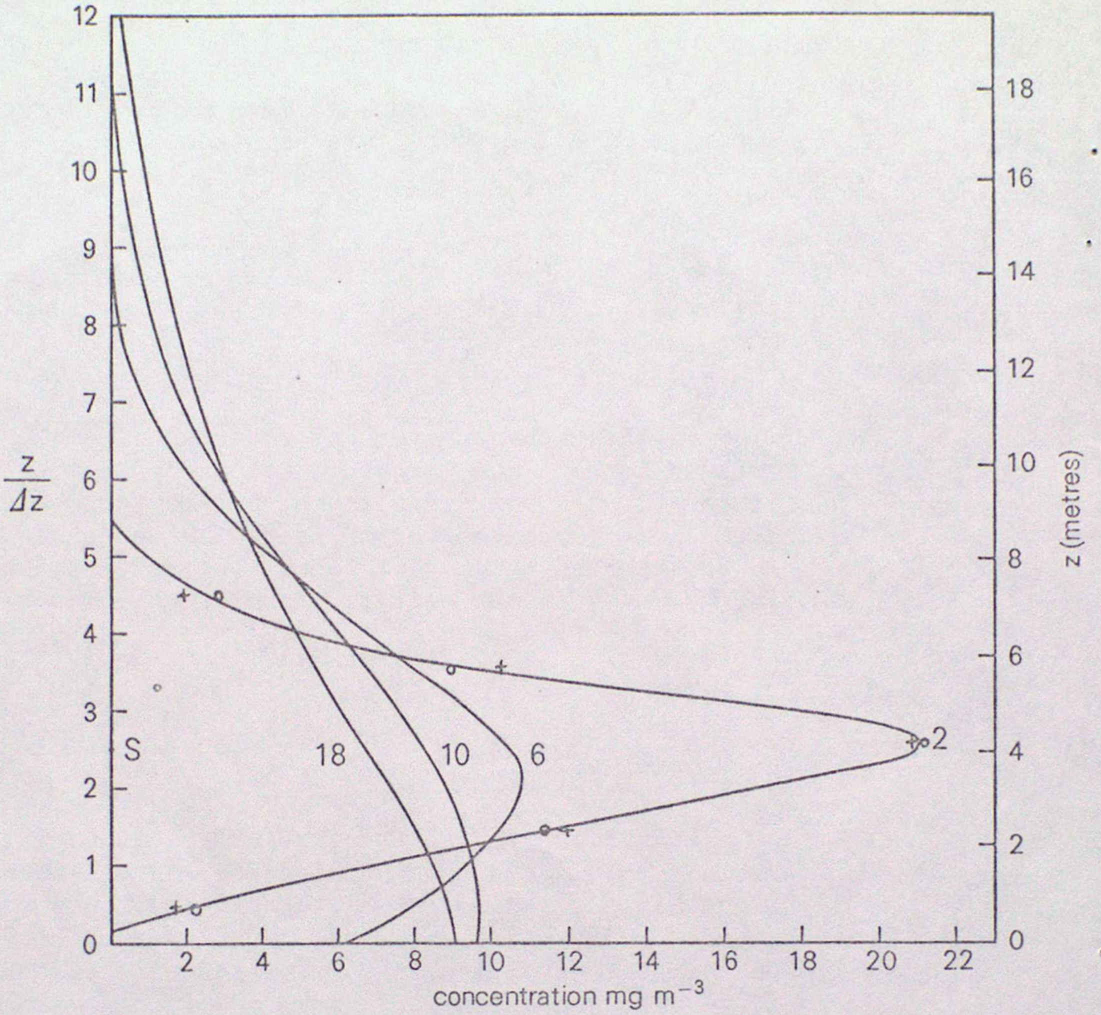

Figure 2. Concentration profiles at four values of N downwind from a source at $z = 2{\cdot}5\,\Delta z$ for a release rate of $1\ gs^{-1}m^{-1}$

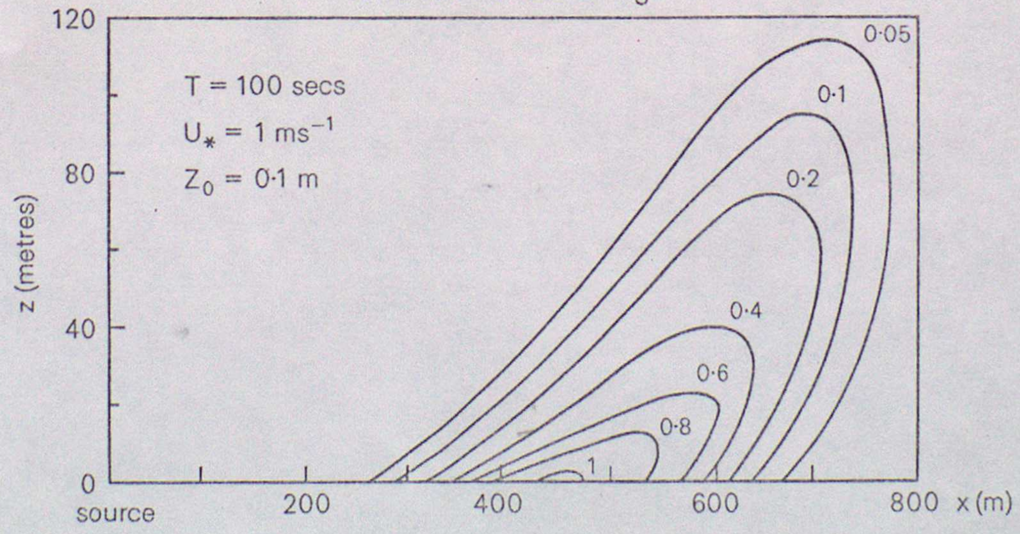

Figure 3. Contours of concentration c/c max within a cloud released instantaneously from a groundlevel source.

solutions are shown at N = 2 by a series of point values. Away from the lowest gridsquares the differences are very small, especially for larger values of N.

Table 1 gives values of σ_z/z_0 for continuous releases from S_0 and S_1 for different values of $\bar{x}/z_0$, and it can be seen that σ_z grows almost parabolically for small x, where wind shear is dominant, but approaches a more linear variation at greater x when the linear form of τ predominates over the wind shear.

N	0	1	2	4	8	12	16	19
x/z_0	0	409	817	1635	3270	4905	6540	7766
σ_z/z_0 (S_0)	40.7	67.1	85.0	115	168	216	259	289
σ_z/z_0 (S_1)	203.5	207	216	238	281	320	357	384

Table 1. Plume growth from sources S_0 at $\frac{1}{2}\Delta z$ and S_0 at $2.5\,\Delta z$ in a neutrally-stratified surface layer. σ_z is the root mean square particle elevation above ground.

(b) An instantaneous release

Solutions have been obtained using both methods for an instantaneous release from S_0 , at a height $h_s = 0.5\,\Delta z$. They show the marked interaction of vertical diffusion and the shear of the horizontal wind u(z). An example is given in Figure 3 which shows contours of concentration after 100 seconds, obtained from the R.W. model. Table 2 compares the basic statistics at three different times from the two methods. The differences are small except for σ_x, the overall alongwind root-mean-square spread of all the particles in the cloud. As before the reason for the difference appears to be the relatively limited resolution in the lowest grid layer used in the I.E. method. A particle diffusing into this layer in the random walk model can closely approach the surface, move downwind very slowly (since u(z) is very small there) and and remain there for some time since τ is also very small. In contrast in the I.E. method the values of u and τ are defined by the values at $z = 0.5\Delta z$ and therefore may under-estimate the alongwind shearing effect in the single layer, which the R.W. method captures.

The error is largely a matter of gridsize, of course, and reducing Δz and Δx to a fifth of their previous values gives the value of σ_x in column 6 in Table 2, which is in much better agreement with the R.W. value, albeit at the expense of an increase in computational effort. These differences in σ_x are much less apparent if σ_x is calculated, not for the whole plume, but for a given value of z away from the lowest grid layer. For example at N = 19, M = 10, σ_x (R.W.) = 33 metres and σ_x (I.E.) = 32.4m with the original grid sizes, for $z_0 = 0.1$, $u_* = 1\text{ms}^{-1}$ and $h_s = 4.07$m

N	Method	$\overline{z}$	σ_z (taken about $\overline{z}$)	$\overline{x}$	σ_x	$\sigma_x/\overline{x}$	Plume tilt
	Units : metres: z_o=0.1, u_*=1.0ms^{-1}, h_s=4.07m						
3	R.W.	6.0	4.35	65.3	10.4	0.16	~1
	I.E.(orig.Δz)	7.7	5.56	70.3	6.7	0.095	
	I.E.($\frac{1}{5}$x origΔz)	6.0	4.68	-	10.7	-	
6	R.W.	8.5	7.16	134	25.7	0.19	~1
	I.E.(origΔz)	10.9	8.79	147	16.1	0.11	
19	R.W.	20	19	474	92	0.19	~1
	I.E.(origΔz)	22	19	510	64	0.13	

Table 2. Values of the main statistical parameters describing the cloud originating from an instantaneous release at N = 0, z = $\frac{1}{2}\Delta z$. Plume tilt is defined as the tangent of the best eye-fit linear regression line through the cloud.

The variation of $\overline{x}$ with time is rather faster than linear, as shown in Table 2, due to the increase in average windspeed as the cloud deepens. The variation of $\sigma_x/\overline{x}$, shown in the same Table, is almost constant beyond N = 6 at approximately 0.2, but below 6 increases with increasing N. Similarly the plume tilt above the very lowest grid-layers appears to be almost constant at 45° (tangent = 1) out to N = 19.

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