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POTENTIAL VORTICITY AND THE ELECTROSTATICS ANALOGY: QUASI-GEOSTROPHIC THEORY

by

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ABSTRACT

The potential vorticity (PV) is a property of the atmosphere which is characterized by the divergence of a vector field. It is illuminating to draw an analogy between electrical charges and PV anomalies. Here the field induced by quasi-geostrophic PV charges is chosen to be a quantity which does not depend on conditions imposed at boundaries at a finite distance from the anomaly or on the mean static stability or density profiles. This field describes action-at-a-distance. Elementary PV-charges involve circulation about a vertical axis as well as a vertically-oriented temperature dipole. A further consequence of this electrostatics analogy is that the atmosphere is analagous to an anisotropic dielectric material and hence the existence of a "bound" PV charge is implied. The mean static stability measures this dielectric property of the atmosphere and the dielectric tensor for the quasi-geostrophic system is here presented. The bound charge concept provides an elegant physical picture of how vertical gradients in the mean static stability parameter, such as occur at the tropopause, affect the flow produced by PV anomalies.

The problem of attribution, namely attributing parts of the flow to particular PV features, is considered in the light of the electrostatics analogy. Imposing conditions at boundaries is equivalent to including possibly spurious PV anomalies exterior to such boundaries.

1. Introduction

In this paper the analogy is pursued between the particle-based theory of electrostatics and the quasi-geostrophic potential vorticity. This physical model provides a theoretical basis for the concept of "action-at-a-distance" which is a cornerstone of potential vorticity thinking. The model is based on parcels of air having an inherent property, namely their PV. Just as in basic physics to understand a significant subset of fluid dynamical behaviour the atmosphere must be considered *to act as though it were composed of particles, or charges, of PV*. An advantage of imagining that the atmosphere is composed of particles or parcels of super-molecular scale is that conservation laws can be applied easily (c.f. Hockney and Eastwood 1981). Feynman et al 1965 notes that analogies are common in physics and Hoskins et al 1985 (HMR) refer to the electrostatics analogy for PV in their review paper.

The essential aspect of this analogy is that it emphasises the field theoretical aspects of potential vorticity. Charge induces a field and it is this field that implies action-at-a-distance. Here we consider the implications of this analogy for the static aspects of the balanced flow associated with potential vorticity anomalies. Advection is of equal importance but is not dealt with here in any detail. It should be noted that the quasi-geostrophic formulation has a linear superposition principle associated with it which is apparently absent from the non-linear Ertel-Rossby PV.

An important motivation in exploring this analogy is to provide insight into ways of partitioning the effects on the flow due to parts of the PV distribution. We will call this partitioning "attribution". The idea is a simple one and it encapsulates the theoretical and operational viewpoint of atmospheric dynamics. Stated in its simplest form it seeks to attribute to a feature on a weather chart, such as a vorticity anomaly, a unique influence on the rest of the atmosphere. An example is when we speak of an upper-level trough inducing development via ascent and vortex stretching. A causal relationship is imagined between the appearance of the trough and changes, for example, to the surface flow. Without a scheme of attribution all one can say is that everything affects everything else and causal relationships are difficult if not impossible to establish.

Current thinking is indeed rooted in an action-at-a-distance principle associated with potential vorticity anomalies. Here the full implications of the electrostatics model are pursued particularly in the practical application of attribution. A simple example of the problem is as follows. Suppose we wish to find how much of the observed surface wind is due to the existence of a PV anomaly in the troposphere i.e. how much of the wind field can be attributed to the PV anomaly. (This attribution problem relates to distinct anomalies and should be distinguished from inversions including all the PV structure.) An approach is to define a volume of atmosphere including the region of interest and to perform a PV inversion by solving for the geopotential field. This requires knowledge of the PV anomaly and a suitable specification of boundary conditions on the edges of the volume. Often these boundary conditions are taken to be that the potential temperature on the ground and the upper boundary are constant and that the relative vorticity on the lateral boundaries is zero. There are, however, many plausible choices for these boundary conditions. However the result of the attribution inversion is different for every choice. Here we show that any boundary condition implies the existence of PV anomalies exterior to the domain and show how the electrostatics analogy illuminates their role.

This paper will set out in detail the exact correspondence that exists between the quasi-geostrophic potential vorticity and electric charge by first recalling some basic principles from the theory of electrostatics in section 2. In section 3 the implications of this isomorphism are explored and an archetypal problem is solved involving a PV-charge residing beneath the tropopause. Also the consequences for the attribution problem are explored.

2. Theoretical Development

2.1 *Introduction*

An important approximate form of the potential vorticity is given from quasi-geostrophic theory. The definition of the quasi-geostrophic potential vorticity, q, is:

$$q = f + \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma^2}\frac{\partial \psi'}{\partial p}\right) \tag{1}$$

and $\psi' = \phi'/f_0$ is the geostrophic streamfunction, ϕ' the geopotential, subscript zero refers to a constant value of Coriolis parameter, σ^2 is a reference static stability which only depends on pressure (p), and the prime reminds us that we are dealing with a deviation from the reference state. The definition of σ is:

$$\sigma^2 = -\frac{R}{p}\left(\frac{p}{p_0}\right)^{R/c_p}\frac{\partial \theta}{\partial p}$$

The parameter σ is related to the Brunt-Vaisaila frequency, N, in height coordinates via: $\sigma = N/\rho g$ where ρ is the density. It is convenient to scale the pressure coordinate in the following way, $\bar{z} = p\ (\sigma_0/f_0)$, where σ_0 is a constant value of the static stability parameter and $\bar{z}$ has the dimensions of height.

An important aspect of the theory to be developed is that the quasi-geostrophic potential vorticity can be written in divergence form in these coordinates i.e.

$$q = \bar{\nabla} \cdot \underline{D}_q \tag{2}$$

where $\bar{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial \bar{z}}\right)$. Taking flow on a β-plane then the associated vector field is given by:

$$\underline{D}_q = \left(\frac{\partial \Psi}{\partial x}, \frac{\partial \Psi}{\partial y}, \frac{\sigma_0^2}{\sigma^2}\frac{\partial \Psi}{\partial \bar{z}}\right) \tag{3}$$

where $\Psi = \psi' + (x^2 + y^2)\ f_0/4 + \beta\ y^3/6$.

The physical picture that we now have concerning potential vorticity is in terms of the divergence of a vector field. In a real sense we can say that PV is that property of the atmosphere which exhibits a non-zero divergence of $\underline{D}_q$. The visualization of the attribute PV should be in terms of the vector field; a question such as "what exactly is the potential vorticity" would be answered in this way.

The part of the potential function other than ψ' leads to a component of the field and of its divergence, PV, due to the solid body rotation of the Earth in the absolute frame of reference. Thus even if there are zero measured winds, the atmosphere exhibits a potential vorticity which, in this quasi-geostrophic framework, is associated with the Coriolis parameter f. The part related to ψ' is responsible for the wind and thermal structure in the rotating frame of reference. We can therefore write $q = q_a + q'$ where:

$$q' = \bar{\nabla} \cdot \underline{D}'_q$$
$$\underline{D}'_q = \left(\frac{\partial \psi'}{\partial x}, \frac{\partial \psi'}{\partial y}, \frac{\sigma_0^2}{\sigma^2} \frac{\partial \psi'}{\partial \bar{z}} \right) \qquad (4)$$

and $\underline{D}'_q$ is the field associated with q'.

In much of what follows we will focus on the flow associated with an additional localized PV anomaly or charge i.e. the q' part; the PV, q_a, due to the Earth's rotation is assumed to exist but will not be referred to again until section 4.

The invertibility principle described by HMR states that given the complete q distribution and potential temperature on the edge of the domain in question equation (1) can be solved to find ψ' everywhere. Here we propose that in principle one would like to attribute that part of the ψ' field at a given location which is due to a particular local PV anomaly, q', and the properties of the atmosphere between the anomaly and the point in question. In other words it would be an attractive conceptual tool to imagine that a PV anomaly of a certain magnitude and shape should have the same effect on the flow irrespective of where it is currently located relative to real or artificial boundaries. This is in the spirit of electrostatics where one speaks of the field due to each free electric charge and these can be superposed in the case of multiple charges.

2.2 Free PV charges

Imagine a uniform q anomaly (q) which, for simplicity, has a "ball" shape. The solution to equation (2) depends on the variation of the mean static stability parameter with height. Suppose we take the simplest case of constant σ. Inside the charge it is clear from symmetry grounds that a simple quadratic function for ψ' must exist with the form:

$$\tilde{r} < b \qquad q' = q \quad \text{and} \quad \psi' = -q\,(\,b^2 / 2 - \tilde{r}^2 / 6\,)$$

where $\tilde{r} = (x'^2 + y'^2 + \bar{z}'^2\,\sigma^2/\sigma_0^2)^{1/2}$, $(x', y', \bar{z}') = (x-x_0,\ y-y_0,\ \bar{z}-\bar{z}_0)$, $(x_0, y_0, \bar{z}_0)$ is the centre of the anomaly, and b is the radius of the anomaly in these coordinates. Hence the anomaly is circular in the horizontal plane and elliptical in the vertical plane.

In the atmosphere outside the charge then it is clear that the geopotential takes the following form:

$$\tilde{r} > b \qquad q' = 0 \quad \text{and} \quad \psi' = -q\,b^3 / (\,3\,\tilde{r}\,) \tag{5}$$

This solution is such that $\phi' \to 0$ as $\tilde{r} \to \infty$. In figure 1 the solution is shown for $q = 3 \times 10^{-4}\ s^{-1}$ and $b = 250$ km for a flow with constant density and Brunt-Vaisaila frequency. Note that this is equivalent to the solution given in HMR for different coordinates.

A convenient example of such an anomaly is the point PV charge which is a delta-function PV distribution; the mathematical form of such point PV anomalies was first described by Charney 1963. It can be obtained from equation (5) by letting $b \to 0$ and $q \to \infty$ in such a way that $Q = q\,\frac{4}{3}\,\pi\,b^3$, the volume-integrated PV anomaly, remains finite. Then the streamfunction has the following form:

$$\psi' = -\frac{Q}{4\pi\,(x'^2 + y'^2 + \bar{z}'^2\,\sigma^2 / \sigma_0^2)^{1/2}} \tag{6}$$

In figure 2 we show a schematic of such free PV charges in the vertical plane. Positive and negative free PV charges such as shown in figure 2(a) form the building blocks of the theory. Such building blocks are shown schematically in Hoskins and Berrisford 1988; equation (6) gives their mathematical form. For example, a positive charge is an elemental piece of cyclonic circulation accompanied by a warm anomaly above and a cold anomaly below. It is important to note that at the location of the PV-charge there is increased static stability <u>and</u> cyclonic vorticity. However <u>away</u> from the charge itself, in the sector defined by $x'^2 + y'^2 > 2\ \bar{z}'^2\ \sigma^2/\sigma_0^2$, there is increased stability with anticyclonic vorticity and outside this sector there is reduced static stability with cyclonic vorticity; see the schematic vertical section of figure 2(b) for the plane $y' = 0$.

Free PV charges are commonplace on weather charts - they are the upper short-wave troughs, jet-streaks, and tropopause PV anomalies etc. The terminology is new but the time-honoured picture of, say, a trough moving over a lower level baroclinic zone and inducing cyclogenesis implicitly invokes the free charge concept.

2.3 *The electrostatics analogy and bound PV charge*

The isomorphism between the QG equations and those used in electrostatics will now be outlined. It arises from the requirement that potential vorticity be associated with a vector field, like the electric field $\underline{E}$, which is produced by each element of charge *independent of boundary conditions and the properties of the medium.* The medium is a dielectric or insulating electrical material. The electric field is the gradient of the electric potential *V*:

$$\underline{E} = -\nabla V$$

The divergence of the electric field is the total electric charge which, for an insulating material, is the sum of free (q) and polarization, or bound, (q_b) charges:

$$\nabla \cdot \underline{E} = (q + q_b) / \epsilon_0$$

where ϵ_0 permittivity of free space. In electrostatics a "free space" Green's function can be associated with each element of charge. For a single charge this potential has a 1/r dependence, independent of the medium properties, due to the simple ∇^2 form of the operator governing V. The free charge is that part of the total charge which is conserved and is related to the divergence of an electric displacement field, $\underline{D}$:

$$\nabla \cdot \underline{D} = q \qquad (7)$$

Unlike the electric field the displacement field depends on the (dielectric) properties of the medium. We have given it the same symbol as the vector field for potential vorticity from equation (4) to show their exact correspondence. Bound charges are due to the polarization of the dielectric medium by the electric field.The polarization field, $\underline{P}$, is the difference between the displacement and the electric field and its divergence is proportional to the bound charge:

$$\underline{D} = \epsilon_0 \underline{E} + \underline{P}$$
$$\nabla \cdot \underline{P} = - q_b$$

Furthermore the displacement and polarization fields are each proportional to the electric field using the permittivity and susceptibility tensors:

$$\underline{E} = (E_1, E_2, E_3)$$

$$\underline{D} = \left(\sum_{j=1}^{3} \epsilon_{1j} E_j, \sum_{j=1}^{3} \epsilon_{2j} E_j, \sum_{j=1}^{3} \epsilon_{3j} E_j \right) \qquad (8)$$

$$\underline{P} = \epsilon_0 \left(\sum_{j=1}^{3} \chi_{1j} E_j, \sum_{j=1}^{3} \chi_{2j} E_j, \sum_{j=1}^{3} \chi_{3j} E_j \right)$$

where ϵ_{ij} and χ_{ij} are the elements of the relative permittivity and electric susceptibility tensors respectively (cf. Lorrain and Corson 1970).

Equations (7) and (8) are isomorphic to those for the quasi-geostrophic potential vorticity when one sets V to $- \psi'$, $\epsilon_0 = 1$, $\epsilon_{33} = \epsilon_q$, $\chi_{33} = \chi_q$, and all other elements of the permittivity and susceptibility tensors are set to zero. Here χ_q and ϵ_q are defined by:

$$\chi_q = \sigma_0^2 / \sigma^2 - 1 \quad \text{and} \quad \epsilon_q = \sigma_0^2 / \sigma^2$$

So from electrostatics $\epsilon_q = 1 + \chi_q$ may be called the dielectric "constant". For clarity we repeat the equivalent equations for the quasi-geostrophic system using a suffix 'q' to indicate quasi-geostrophic:

$$\underline{E}'_q = \left(\frac{\partial\psi'}{\partial x}, \frac{\partial\psi'}{\partial y}, \frac{\partial\psi'}{\partial \bar{z}} \right)$$

$$\underline{D}'_q = \left(\frac{\partial\psi'}{\partial x}, \frac{\partial\psi'}{\partial y}, \epsilon_q \frac{\partial\psi'}{\partial \bar{z}} \right) \tag{9}$$

$$\underline{P}'_q = \left(0 , 0, \chi_q \frac{\partial\psi'}{\partial \bar{z}} \right)$$

It can be seen that there are consequently three vector fields associated with PV which arise from the requirement to have a field, $\underline{E}'_q$, which is independent of the static stability of the atmosphere. The isomorphism between the QG equations and the electrostatic equations suggests the terminology that $\nabla \cdot \underline{D}'_q$ is the free PV and $-\nabla \cdot \underline{P}'_q$ the bound PV (q'_b). It is clear that from this viewpoint that ψ' is induced by both the free and bound PV and that the dielectric anisotropy is evident from the fact that $\underline{P}'_q$ is only non-zero in the vertical direction. In the electrical case such anisotropic dielectric materials are commonplace in nature.

As in electrostatics, each element of either free or bound PV can be associated with a field, $\underline{E}'_q$, which is the gradient of a geopotential proportional to $1/r$ where $r = (x'^2 + y'^2 + \bar{z}'^2)^{1/2}$. Although this field depends on the constant σ_0 it does not depend on σ or on the existence of boundaries. The streamfunction given in equation (6) for a point charge represents the sum of such field contributions from both free and bound charges.

From the definition of bound PV it can be seen that there are potentially two situations in which there is bound charge:

$$q'_b = -\frac{\partial}{\partial \bar{z}} \left(\chi_q \frac{\partial\psi'}{\partial \bar{z}} \right) = -\frac{\partial\chi_q}{\partial \bar{z}} \frac{\partial\psi'}{\partial \bar{z}} - \chi_q \frac{\partial^2\psi'}{\partial \bar{z}^2} \tag{10}$$

The first is associated with the vertical variation of the mean static stability parameter σ and the second is present even when that parameter is a constant. For the latter case the interpretation of the constant σ_0 is of importance. It would be possible, for an atmospheric flow with a constant σ, to choose σ_0 such that $\chi_q = q'_b = 0$. (In electrostatics the medium which has zero electric susceptibility is free space.) However in practice σ in the atmosphere is always a function of height (or pressure) and so we can deduce that bound charge will always occur.

In figure 3 we show a schematic of the various fields associated with a PV charge in a flow with constant σ, as given mathematically by equation (6). If $\sigma_0 > \sigma$ is chosen then ϵ_q, the equivalent to the dielectric constant, is greater than unity so that, as in electrostatics, $\underline{P}'_q \cdot \underline{E}'_q > 0$. Then the bound charge has the opposite sign to the free charge at the location of the free charge. In the sector defined by $x'^2 + y'^2 > 2\,\bar{z}'^2\,\sigma^2/\sigma_0^2$ it also has the opposite sign whereas it has the same sign outside this region.

In electrostatics the presence of bound charge is due to the fact that in the presence of an electric field the insulator acquires a polarization charge which is due to the electrons being attracted and the protons being repelled in the direction of the field. The normally electrically neutral atom thus acquires a polarization charge. This charge is often called a bound charge to distinguish it from the free charge which, in our case, is conserved following the geostrophic motion.

When an electric charge is introduced into a dielectric material, it takes a finite amount of time for electromagnetic waves to establish the electric field associated with such a charge. The electric field induced by the free charge polarises the atoms, molecules, crystal structure, etc. Such polarisation produces bound charges which produce more electric field and hence more bound charge which produce more electric field and hence more bound charge and so on. Eventually this interplay between bound charges reaches a steady state and the electric potential will satisfy (8). All the bound charges of this steady state are attributable to the free charge. For our purposes it will be convenient to speak of such bound charges as *belonging* to the free charge.

Such ideas are easily transferred to the QG system. A convergent iterative process which mimics the way free charge induces bound charge which in turn induces more bound charge, etc is discussed in the appendix. The existence of such a procedure is important as it provides a method whereby the bound charges belonging to a particular free anomaly can be established without explicit reference to any imposed boundary condition. The convergence of such a procedure is important because, *inter alia,* it assures us that this boundary independent view of potential vorticity inversion will not lead to spurious internal free anomalies.[1] It also assures that for a finite PV anomaly the constraint on ϕ' is that $\phi' \rightarrow 0$ as $r \rightarrow \infty$. Given that the original PV inversion problem amounts to the solution of an elliptic equation then it is to be expected that this procedure is convergent. Alternatively one might try to argue that quasi-geostrophic air is sufficiently similar to known dielectric materials that exhibit no infinities then neither must the quasi-geostrophic system.

The electrostatics analogy shows that extra or "bound" PV charges must exist and this allows us to maintain a domain independent interpretation of the field induced by PV anomalies. The convergence of the iteration scheme given in the appendix shows that the view of free charge inducing bound charge is consistent with the boundary conditions that ψ' and its derivatives tend to zero at infinity. Since solutions to Poisson's equation are unique given the boundary conditions then all solution methods must yield the same result.

[1]Such an eventuality may seem unlikely, but if one considers that the polarizability of QG fluid is not constrained by the laws that hold atoms together, it is a serious concern. For example, suppose that the dielectric tensor of the system was such that a free PV charge induced more bound charge in the fluid immediately surrounding it than that contained in the free charge itself. Since the bound charge is assumed to affect ϕ' in exactly the same way as free charge, the bound PV charges could induce bound charges stronger than itself and so on until a discontinuity was reached.

3. Implications of the electrostatics analogy

3.1 *The effect of the tropopause*

The role and importance of bound PV charge is clearly evident when considering the tropopause. The following example is given to show these effects in practice. Consider a free PV charge, at $\bar{z}' = 0$, located beneath the tropopause, at $\bar{z}' = a$: both the troposphere and the stratosphere are taken to have uniform but different mean static stabilities. (Note that as $\bar{z}$ is a pressure coordinate a is negative.)

From equation (4) it is clear that at the tropopause there are free PV charges unless the following boundary condition applies at the tropopause:

$$\frac{\sigma_0^2}{\sigma_s^2}\frac{\partial\psi'_s}{\partial\bar{z}} = \frac{\sigma_0^2}{\sigma_t^2}\frac{\partial\psi'_t}{\partial\bar{z}} \qquad (11)$$

where suffices t and s refer to the troposphere and stratosphere respectively. This follows also from the quasi-geostrophic thermodynamic equation from which it can be shown that this condition makes any vertical motion which might occur continuous across the tropopause. It is therefore the appropriate condition to apply. (Note that the implied temperature jump at the tropopause can, in principle, be removed by deforming the tropopause, Rivest et al 1992). The other interfacial condition which must be satisfied is the matching of ϕ' itself.

This tropopause problem can be most easily solved by the method of images as used in electrostatics where there is a dielectric constant transition between two media. These image charges are superposed so as to satisfy the matching boundary conditions at the tropopause and to produce $\psi' \to 0$ as $r \to \infty$. First an image charge is placed in the stratosphere a certain distance above the tropopause and the field in the troposphere is taken to be the superposition of that due to the charge itself and its image. For that calculation the dielectric constant is taken to be that of the troposphere everywhere. (Note that the image charge is just a simple way of representing the effect of bound charge in the stratosphere on the troposphere.) Equally to calculate the field in the stratosphere one extends the stratospheric dielectric constant throughout the atmosphere and a different image charge is substituted for the actual free PV charge. The total field is then the appropriate one computed for either the troposphere or stratosphere. An important point is that for each of the calculations it is only necessary to know the inherent potential due to a particular point charge in an infinite atmosphere. The principle of superposition allows us to simply sum the fields due to each charge; for this problem there are 3 charges whose fields we superpose. The following is the solution:

$$\psi'_t = -\frac{Q}{4\pi}\left(\frac{1}{(x'^2 + y'^2 + \bar{z}'^2\,\sigma_t^2/\sigma_0^2)^{1/2}} + \frac{n}{(x'^2 + y'^2 + (\bar{z}' - 2a)^2\,\sigma_t^2/\sigma_0^2)^{1/2}}\right) \tag{12}$$

$$\psi'_s = -\frac{Q\,(1 + n)}{4\pi\,(x'^2 + y'^2 + (\bar{z}' - \ell a)^2\,\sigma_s^2/\sigma_0^2)^{1/2}}$$

where $n = \ell / m$, $\ell = (1 - \sigma_t/\sigma_s)$, $m = (1 + \sigma_t/\sigma_s)$, and suffices t and s refer to tropospheric and stratospheric values respectively. Note that the ratio σ_t/σ_s that appears in these expressions is simply the square root of the ratio of the dynamical dielectric constants in the troposphere and stratosphere. In figure 4 the solution is shown for $\sigma_t/\sigma_s = 0.33$ and $a = -0.1$ H where the horizontal scale in the figure is divided by σ_t H / σ_0 and the vertical scale is divided by H (where H is an arbitrary vertical scale). Note that this is a local solution whereby the geopotential becomes small at large distances away from the free charge.

The existence of bound PV-charge is obvious from equation (12) as at the location of the free PV-charge, $\bar{z}' = 0$, the radial gradient of ψ' is not zero. If the only potential-producing charge was the free charge then this gradient would, on elementary symmetry grounds, be zero. The fact that it is not zero is due to the potential produced by the extra or bound charge associated, in this case, with the tropopause. Equation (12) is an example of a Green's function which includes the field from both a free charge and its associated bound charge.

The distance of any given PV anomaly from the stratosphere must be known if the effect of that anomaly is to be calculated. However we have found that the streamfunction can be regarded as a linear superposition of several point charges (three in this case) each of which act as though they are in an infinite uniform medium. Of course the distance of the anomaly to the tropopause, a, enters the superposed solution but the physical interpretation requires nothing more than the individual effects of point charges.

3.2 *PV "atoms" ?*

The existence of a polarization charge in electrostatics can be understood in terms of the structure of the atom. In the absence of an applied electric field the atoms in an insulator are electrically neutral with equal but opposite charges on the bound electrons and protons. With an applied electric field the electrons are attracted and the protons repelled and they move fractionally apart producing an atomic polarization. The sum of these atomic dipoles is the net polarization charge.

It is of interest then to examine the circumstances in which free PV charges are "created" and "destroyed". The mechanisms by which this occurs are processes such as friction and diabatic effects like radiation and latent heat release. From the work of Truesdell 1951, Thorpe and Emanuel 1985, and Haynes and McIntyre 1987 we know that the total mass-weighted PV is unchanged by these processes. This means that, away from the ground free PV charges are created in equal amounts of positive and negative sign. This could be imagined as being a consequence of there being PV "atoms" composed of equal and opposite bound charges. These bound charges can be separated irreversibly to generate free charges. Of course this is only a picture consistent with the effects of irreversible processes and not a proof of the existence of such PV atoms.

3.3 *The superposition principle*

The principle of superposition, as used in electrostatics theory, can be used to calculate the field due to many free charges. For example consider the potential in an atmospheric layer containing two equal PV anomalies a certain vertical distance, 2 H, apart: see figure 5. Notice that at the mid-plane between the charges there is zero temperature anomaly. Equally we can take two opposite sign PV charges in a vertical line, see figure 6. Now on the mid-plane the geopotential perturbation, the flow, and consequently its relative vorticity is zero. In this latter case the field lines emanate from the negative PV anomaly and "flow" towards the positive PV anomaly; this is, of course, exactly analagous to an electric dipole and also because the mid-plane is an equipotential surface then that surface is like a conducting sheet.

The dipole solution is of relevance also to the case of diabatic forcing in the atmosphere. A point heat sink will produce a dipole PV anomaly as in figure 6 but where the distance between the free charges is very small (but not zero); this is because the PV changes according to the vertical gradient of the heating. We can see that this fits into our conceptual picture of the bound PV charges in PV atoms being "liberated" by the action of irreversible processes. The diabatic forcing acts to separate the PV charges a small distance in the vertical (or along the vorticity vector in the Ertel PV case). Therefore it is as if a finite period of heating or cooling acts to free a pair of opposite sign PV charges. The balanced response to that forcing can be obtained from the superposition of the effects of each PV charge. This picture incidentally shows that the action of a heat source/sink is not simply to warm/cool the atmosphere local to the diabatic forcing. Due to the ageostrophic circulation induced by the forcing there are regions where there is adiabatic cooling/heating also. These are above and below the dipole as can be seen in figure 6. Similarly the effect of friction creates a dipole oriented normal to the frictional force vector. For a horizontal frictional force this amounts to a horizontal displacement of the two PV charges.

3.4 *The attribution problem*

The term "attribution" has been coined in this paper to describe the process of finding the contribution to the flow and temperature from a particular PV anomaly. Davis and Emanuel 1991 and Davis 1992 use the term piecewise potential vorticity inversion for this process. In PV inversions, such as given by Kleinschmidt 1950 and Thorpe 1985 and 1986, a complete PV distribution is specified in a limited domain bounded below by the Earth's surface. Furthermore conditions are imposed at the boundaries of the domain. It is common in such inversions to specify θ at these boundaries perhaps because this accords well with a PV-θ view of atmospheric dynamics (Hoskins 1991).

The electrostatics analogy suggests that for attribution, which involves a piecewise PV inversion, it is appropriate to assign a Green's function to each free PV charge which includes its associated bound charge. In order to incorporate conditions at the Earth's surface the electrostatics analogy requires a subterranean static stability to be specified. The choice of surface condition will be governed by a hypothesis or model of the relationship between a PV anomaly and the surface flow and temperature associated with it. For example, if it is believed that a lower tropospheric PV anomaly is due to latent heating in clouds then an attribution of its field could take $\theta' = 0$ at the Earth's surface as there is no latent heating at the surface. (Note that the PV anomaly in this case does contribute to the surface flow but not to its temperature.) Clearly such knowledge concerning the origin of PV anomalies may not always be available. Therefore the derivation of this hypothesis is not, in general, straightforward.

It can be seen that equation (11) implies that if the subterranean static stability is infinite then bound charge will ensure that $\theta' = 0$ at the surface. This bound charge is of course due to the implied jump in the "dielectric constant". The suggestion by Bretherton 1966a that potential temperature anomalies are dynamically equivalent to PV anomalies is consistent with this choice. It has proved of considerable insight in interpreting aspects of atmospheric dynamics such as baroclinic instability; c.f. Bretherton 1966b. The $\psi' = 0$ surface condition results when we assume that the surface has the characteristics analagous to a conducting sheet.

As an example imagine a single positive PV anomaly, at $\bar{z}' = 0$, located above the surface ($\bar{z}' = -a$) in an atmosphere with constant σ. (For simplicity we take the point charge solution here although the ball solution of equation (5) may be more appropriate in practice.) The solution is:

$$\psi' = -\frac{Q}{4\pi}\left(\frac{1}{(x'^2 + y'^2 + \bar{z}'^2\,\sigma^2/\sigma_0^2)^{1/2}} \pm \frac{1}{(x'^2 + y'^2 + (\bar{z}' + 2\,a)^2\,\sigma^2/\sigma_0^2)^{1/2}}\right) \qquad (13)$$

where the plus sign allows the $\theta' = 0$ condition to be chosen for the attribution and the minus sign applies to the $\psi' = 0$ condition. A schematic of these two cases can be taken from figures 5 and 6 respectively; the mid-plane between the charges now is taken as the Earth's surface. For the $\theta' = 0$ case the effect of bound charge at the surface can be described, for visualization purposes only, in terms of the field due to the PV anomaly superposed with that of an equal image charge located "beneath" the surface where the subterranean static stability is taken equal to that in the atmosphere. Equally for the $\psi' = 0$ case the solution can be visualized in terms of an opposite sign image charge residing in the subterranean extension to the atmosphere. It should be stressed that the image charges are a device for visualization only.

An option arises to assume that the subterranean static stability is equal to that immediately above the Earth's surface. Then for a constant static stability the Green's function of equation (6) applies for each PV anomaly. This represents perhaps the weakest form of surface condition that can be applied.

Thus different assumptions about the dielectric nature of the hypothetical subterranean QG air will give different surface temperature distributions attributable to interior PV anomalies. Generally these surface patterns will not be equal to the observed thermal pattern. This difference may be attributed to surface charge. This is "free" in the sense that it creates field but it will not necessarily be "free" in the sense that it is conserved following the geostrophic motion. The important case where surface charge is "free" in both these senses arises when the subterranean static stability is infinite, the flow is frictionless and adiabatic. When these conditions do not apply then some source of surface PV charge must be present.

In principle then one can construct surface and subterranean properties such that the field due to bound charge automatically provides the required boundary conditions. With this attribution method there is no need to apply lateral boundary conditions. Consequently the electrostatics analogy makes it possible to infer the effect of PV anomalies exterior to a finite observational domain in an unambiguous way. This is achieved by simply subtracting the field due to the free and associated bound charge from the observed geopotential field. Since the remainder was not induced by the interior free PV, it may be attributed to free PV exterior to the domain. Note that this remaining field will always have zero PV associated with it within the domain. The linear deformation field of simple frontogenesis models is an example of such a field. A practical example of this technique for assessing the effect of sources exterior to a domain (albeit for vorticity and divergence inversions) is discussed in Bishop 1994.

If lateral boundary conditions are imposed then possibly spurious exterior PV is implied. The method of images can be used to illustrate the effect on the field associated with a PV charge of the imposition of such lateral boundary conditions. For example, a commonly used lateral boundary condition is to require that the normal gradient of the wind tangential to the boundary be zero. We can use figure 6 to illustrate how the method of images can account for the exterior PV consistent with this boundary condition. If figure 6 is rotated by 90^0, such that the (x, z) axes become the $(-z, x)$ axes, then there are PV charges side-by-side. The θ' field in figure 6 then becomes the new v field. Hence the normal *gradient* of v is zero half-way between the 2 charges. So the exterior PV needed to produce that lateral boundary condition is an equal magnitude but opposite sign anomaly to the one inside the domain. Equally for a $v = 0$ lateral boundary condition we require an equal magnitude and sign anomaly exterior to the domain; figure 5 rotated by 90^0. The contribution to the internal field from this exterior PV is thus made explicit.

4. Concluding Remarks

It is apparent that the quasi-geostrophic PV, an approximation to the full Ertel PV, has a simple interpretation suggested from the theory of electrostatics in a dielectric medium. A crucial element suggested by this analogy is that a vector field independent of static stability, density, or conditions at boundaries can be assigned to each PV anomaly. The quasi-geostrophic atmosphere, being analagous to a dielectric material, consequently exhibits bound PV and this implication is one of the main results of this paper. The notion that PV is to be understood only in terms of the associated vector field, and in particular its divergence, is fundamental to this view of atmospheric dynamics.

The electrostatics analogy provides a useful interpretation of the attribution problem. Each PV anomaly has a field described by a function which incorporates both the field due to the anomaly and its associated bound charge. The bound charge can include conditions at the Earth's surface. However the relationship between a particular PV anomaly in the atmosphere and the surface boundary conditions is not in general known. This remains as an important arbitrary factor in attribution. The dynamical viewpoint being taken will often suggest a choice of boundary conditions.

It is appropriate to comment now on the part of the quasi-geostrophic PV associated with the Earth's rotation. The simplest model of a Rossby wave is where the conservation of absolute vorticity, in a horizontal plane, leads to a wave disturbance propagating due to the β-effect. The charge interpretation of PV is applicable in this case and it relies on the component of potential vorticity described in section 2.1 as q_a. Suppose that a simple single-cell meridional circulation occurs: this will displace free PV charge to create a local dipole of vorticity anomalies. Each of these charges induces a field which causes further meridional advection around the original dipole. The combined effects of these fields is to displace further PV charges and this process leads to the propagation of a wave-train of alternating PV charges. Such PV charges do not produce any bound PV because a barotropic Rossby wave has no vertical variations.

In the case of the Ertel-Rossby PV the equivalent of the dielectric constant depends on the field itself and so there is an inherent non-linearity. It is known that for large electric fields the dielectric constant depends on the field so there may be an electrostatic analogy for Ertel-Rossby PV. How serious this non-linearity is for the field notions developed here has yet to be established. Without these concepts one might question the usefullness of potential vorticity as a fundamental dynamical variable and so it remains an important unresolved question whether such a field theory can be devised in this case. Using a different formulation Shutts 1991 has described a point charge solution for the semi-geostrophic (SG) equations. The SG formulation, whilst being more accurate than the QG approximation, involves a vorticity vector which differs from the geostrophic vorticity vector. However for the SG system, with the important restriction of constant density and background static stability, a simple analytical form for the Greens function can be found. The relationship of the implied field for this SG PV-charge to the quasi-geostrophic field described in this paper remains as a subject of further research. Another important aspect of quasi-geostrophic PV dynamics is the existence of a linear superposition principle. This does not carry through to the Ertel-Rossby PV case so complicating the conceptual simplicity of the quasi-geostrophic model.

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Appendix: *Convergent iteration scheme*

The aim here is to demonstrate an iterative process which converges whereby the field induced by free charge induces bound charge which in turn induces more field etc. It is important to do this because if such a convergent iterative process exists then (a) the boundary independent view of potential vorticity inversion will not lead to spurious internal free PV anomalies and (b) the ψ' associated with a free PV anomaly and its associated bound PV anomalies will become vanishingly small at some finite distance from the free PV anomaly.

Firstly, suppose, following equation (10), that the bound charge associated with changes in perturbation static stability $(-\chi_q \partial^2\psi'/\partial\bar{z}^2)$, immediately comes into balance with the field induced either by the free PV charge density, q', or by that part of the bound charge density, q'_b, associated with changes in the mean stratification $(-\partial\chi_q/\partial\bar{z}\ \partial\psi'/\partial\bar{z})$. This can be achieved by using functions similar to the point charge solutions described in the text; to be precise, by attributing to each element of charge (q' or q'_b) a geopotential field proportional to $1\ /\ \tilde{r}$ (c.f. section 2.2). With this assumption the iteration procedure reduces to the statement that the streamfunction after n+1 iterations, ψ'^{n+1}, is equal to the field, $\psi'[q']$, induced by the free charge plus the field, $\psi'[q'_b(\psi'^n)]$, induced by the q'_b which has been created at the nth iteration. Furthermore we denote $\psi'^0 = \psi'[q']$. The mathematical details of this procedure are more simply expressed in terms of the coordinates:

$$(x,\ y,\ z) = \left(\ x,\ y,\ \int_{z_0}^{\bar{z}} S\ d\bar{z}\ \right)$$

where $S = \epsilon_q^{-1/2}$ and in these coordinates:

$$q'_b = \alpha \frac{\partial\ \psi'}{\partial\ z}$$

where $\alpha = \dfrac{\partial\ \ln(S)}{\partial\ z}$.

Since any free charge distribution may be constructed from a complete set of elements of free charges, we only need to prove that the iteration procedure will converge for *any* element of free charge initiating the procedure. Let δQ represent such an infinitesimal element of free charge at the point $(x_0,\ y_0,\ z_0)$:

$$\delta Q = q' \, \delta x \, \delta y \, \delta z \qquad \text{(A.1)}$$

and this induces the buoyancy field δb^0, where $b = \partial\psi'/\partial z$ and

$$\delta b^0 = \delta Q \ (z - z_0) \ / \ (4\pi r^3), \qquad \text{(A.2)}$$

where $r = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{-1/2}$. The δb^0 field induces bound charges where-ever $\alpha \neq 0$ i.e.

$$\delta Q_b^0 = \alpha \, \delta b^0 \, \delta x \, \delta y \, \delta z \qquad \text{(A.3)}$$

The sum of the buoyancy fields induced by Q and the Q_b^0 field may be written as δb^1. The δb^1 field induces a new field of bound charges which in turn induces a δb^2 field and so on. Thus, the iteration formula for these buoyancy fields at position (x, y, z) is:

$$\delta b^{n+1} = \delta b^0 + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{(z - z') \ \alpha \ \delta b^n}{4\pi r'^3} \, dx' dy' dz' \qquad \text{(A.4)}$$

where $r' = [(x - x')^2 + (y - y')^2 + (z - z')^2]^{-1/2}$

To investigate the convergence of this iteration scheme, we rewrite (A.4) in matrix form. Then a strict upper bounmd on the largest eigenvalue of the iteration matrix is established. If the bound is less than unity then convergence is assured. First an ordering for (A.4) is made by letting:

$$x' = i' \, \delta s, \ y' = j' \, \delta s, \ z' = k' \, \delta s,$$
$$x = i \, \delta s, \ y = j \, \delta s, \ z = k \, \delta s$$

where δs is an infinitesimal distance. Then for k', j', i', k, j, and i having magnitudes less than a positive integer M, we may define:

$$\left.\begin{aligned} l' &= k' + j'M + i'M^2, \\ l &= k + jM + iM^2, \end{aligned}\right\} \qquad \text{(A.5)}$$

so that as δs tends to zero and M tends to infinity, discrete values of l' and l can uniquely identify every rational point in (x', y', z') and (x, y, z) space, respectively. With this ordering, (A.4) may be written in the form:

$$(\delta b^{n+1})_l = (\delta b^0)_l + \sum_{l'=-\infty}^{l'=+\infty} A_{ll'} \, (\delta b^n)_{l'} \qquad \text{(A.6)}$$

where $A_{ll'} = 0$ for $l = l'$, while for $l \neq l'$:

$$A_{ll'} = \lim_{\delta s \to 0} \frac{\frac{1}{4\pi} (k - k') \ \alpha \ \delta s}{\left[(i-i')^2 + (j-j')^2 + (k-k')^2 \right]^{3/2}} \qquad \text{(A.7)}$$

where l' and l are related to the coordinate indices via (A.5). It is easy to show (cf. Varga 1962) that the iteration procedure defined by (A.6) will converge provided that the largest eigenvalue of the matrix **A** having elements $A_{ll'}$, is less than 1. According to Gerschgorin[2] (1931), the largest eigenvalue of **A** must be less than the sum of the absolute values of any row of **A**. Each row (constant l) of **A** pertains to a single point in (x, y, z) space and the sum of the absolute values of its elements, Λ, is given by

$$\Lambda = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{|(z - z')|\,|\alpha|}{4\pi r'^3}\, dx'\,dy'\,dz' \qquad \text{(A.8)}$$

Transforming to cylindrical polar coordinates,

$$(R,\ \eta,\ z') = \left[\left\{(x - x')^2 + (y - y')^2\right\}^{1/2},\ \tan^{-1}\left(\frac{y-y'}{x-x'}\right),\ z' \right]$$

and letting $\chi = R^2 + (z - z')^2$ so that $\partial(\chi, \eta, z') / \partial(R, \eta, z') = 2R$, (A.9) may be expressed,

$$\Lambda = \int_{-\infty}^{+\infty}\int_{0}^{2\pi}\int_{(z-z')^2}^{+\infty} \frac{|(z - z')|\,|\alpha|}{8\pi\chi^{3/2}}\, d\chi\ \ d\eta\ \ dz'$$

$$= \int_{-\infty}^{+\infty}\frac{1}{4\pi}\int_{0}^{2\pi} |\alpha|\ d\eta\ dz'$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} |\alpha|\ dz'$$

Note that Λ is smaller when the vertical variations in static stability are small. If

$$\tilde{S} = S(0) + \int_{0}^{z'} |\alpha|\ \ dz'.$$

then

[2] The proof is given in English in Varga (1962).

$$\Lambda = \frac{1}{2} \ln\left(\frac{\tilde{S}\ (\infty)}{\tilde{S}\ (-\infty)}\right)$$

Thus, convergence is assured provided that

$$\Lambda = 1.15 \log_{10}\left(\frac{\tilde{S}\ (\infty)}{\tilde{S}\ (-\infty)}\right) < 1 \qquad \text{(A.9)}$$

In the non-Boussinesq QG system, S varies from some finite value at the ground and increases with height into the stratosphere. Allowing for density variations we expect (A.9) to be satisfied across the observed tropopause. If the Boussinesq approximation is made then the condition is satisfied even if 4 such tropopause jumps were present. It should be stressed that a prediction of lack of convergence using this technique does not necessarily mean that the method will not converge. Rather if (A.9) is satisfied then convergence is assured.

Figure Captions:

FIGURE 1: The geopotential (contour interval equivalent to 1 mb) with superimposed contours of the total potential temperature (contour interval 6K), and v wind (contour interval 5 ms^{-1}) field in a vertical plane for a finite positive PV charge. The PV anomaly has a constant value of $3 \times 10^{-4}\ s^{-1}$ with a radius in the horizontal plane of 250 km. Constant density and Brunt-Vaisaila frequency have been assumed such that $N/f_0 = 100$ and the diagram is plotted with height as the vertical coordinate. The vertical scale of the diagram is 15 km and the horizontal is 2000 km.

FIGURE 2: (a) Schematic diagram showing the positive and negative PV charges which form the building blocks of the theory. Each charge has circulation of the *same sign* as its charge and a potential temperature anomaly dipole oriented vertically. The sense of the dipole is such that there is an anomaly in static stability at the charge of the *same sign* as the charge. (b) However note that, as can be deduced from equation (6), outside cones emanating from the charge there is a static stability anomaly, denoted here by SS, which is of the *opposite sign* of the PV-charge. The vorticity anomaly, ζ', has the same sign as the static stability anomaly at the charge but has the opposite sign away from the PV-charge inside the cones. In this schematic we indicate the sign of

these anomalies in the plane $y = 0$ and in cartesian coordinates for a positive PV charge.

FIGURE 3: A schematic of the solution for the geopotential, given by equation (6), in the x-$\bar{z}$ plane showing examples of the field vectors for $\underline{E}$, $\underline{D}$, and $\underline{P}$. (These vector fields are, of course, defined everywhere.) The geopotential is negative and a minimum at the location of the positive free charge which is shown by the central black dot. The bound charge q_b' is zero on the diagonal dashed lines with a negative value (i.e. the opposite sign to the free charge) inside these cones and a positive value outside.

FIGURE 4: The geopotential, perturbation potential temperature, and v wind field for the tropopause problem of a positive point PV charge located beneath the tropopause. The figures are in a vertical plane defined by $y' = 0$. The location of the tropopause is evident from the abrupt change in geopotential gradient. (Here and in figures 5 and 6 the horizontal scale is x' divided by $\sigma_t H/\sigma_0$ and the vertical scale is $\bar{z}'$ divided by H, where H is an arbitrary height scale. Dotted regions indicate increasingly large magnitudes as the point charge is approached; the sign of the field in that region is as indicated.)

FIGURE 5: The geopotential, perturbation potential temperature, and v wind field in a vertical plane for the superposition of a pair of positive PV charges oriented in a vertical line. Note that there is no temperature perturbation on the mid-plane between the two charges.

FIGURE 6: The geopotential, perturbation potential temperature, and v wind field in a vertical plane for the superposition of a pair of equal but opposite PV charges oriented in a vertical line with the positive anomaly above the negative one. Note that there is no geopotential (and therefore wind) perturbation on the mid-plane between the two charges.

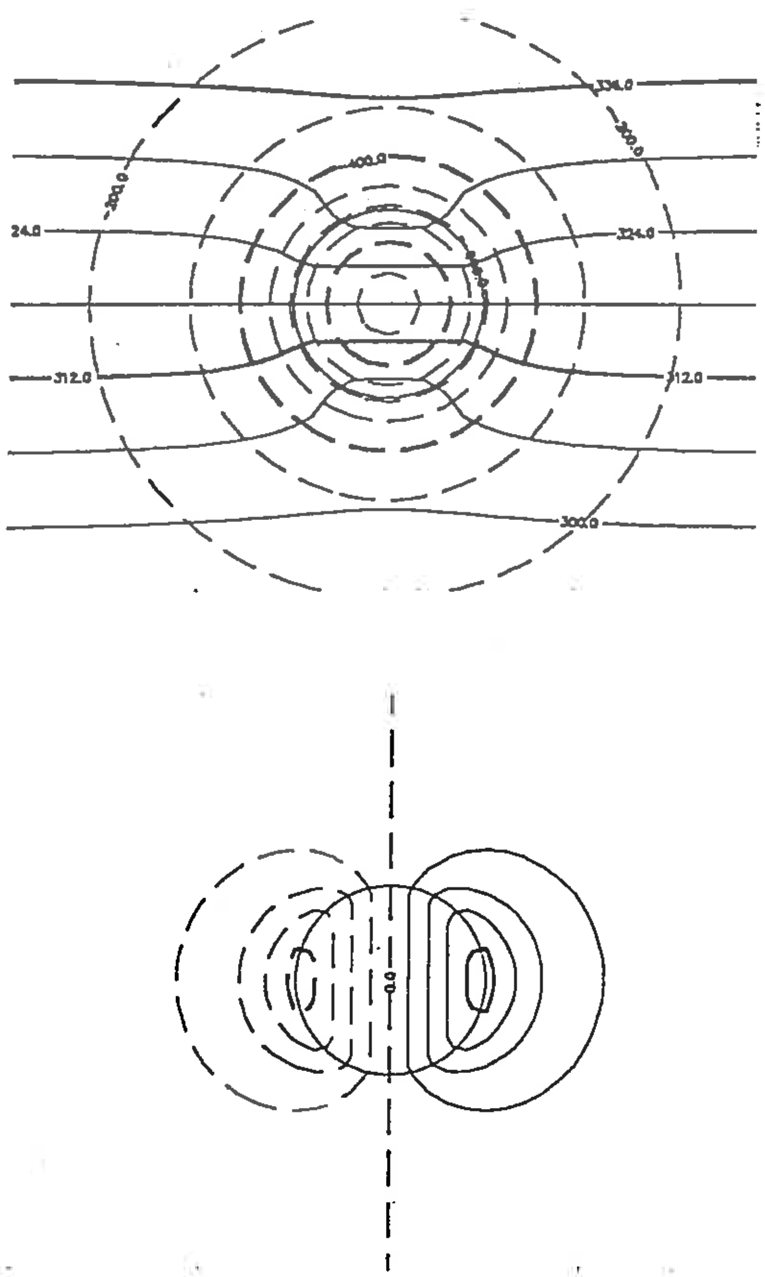

FIG. 1

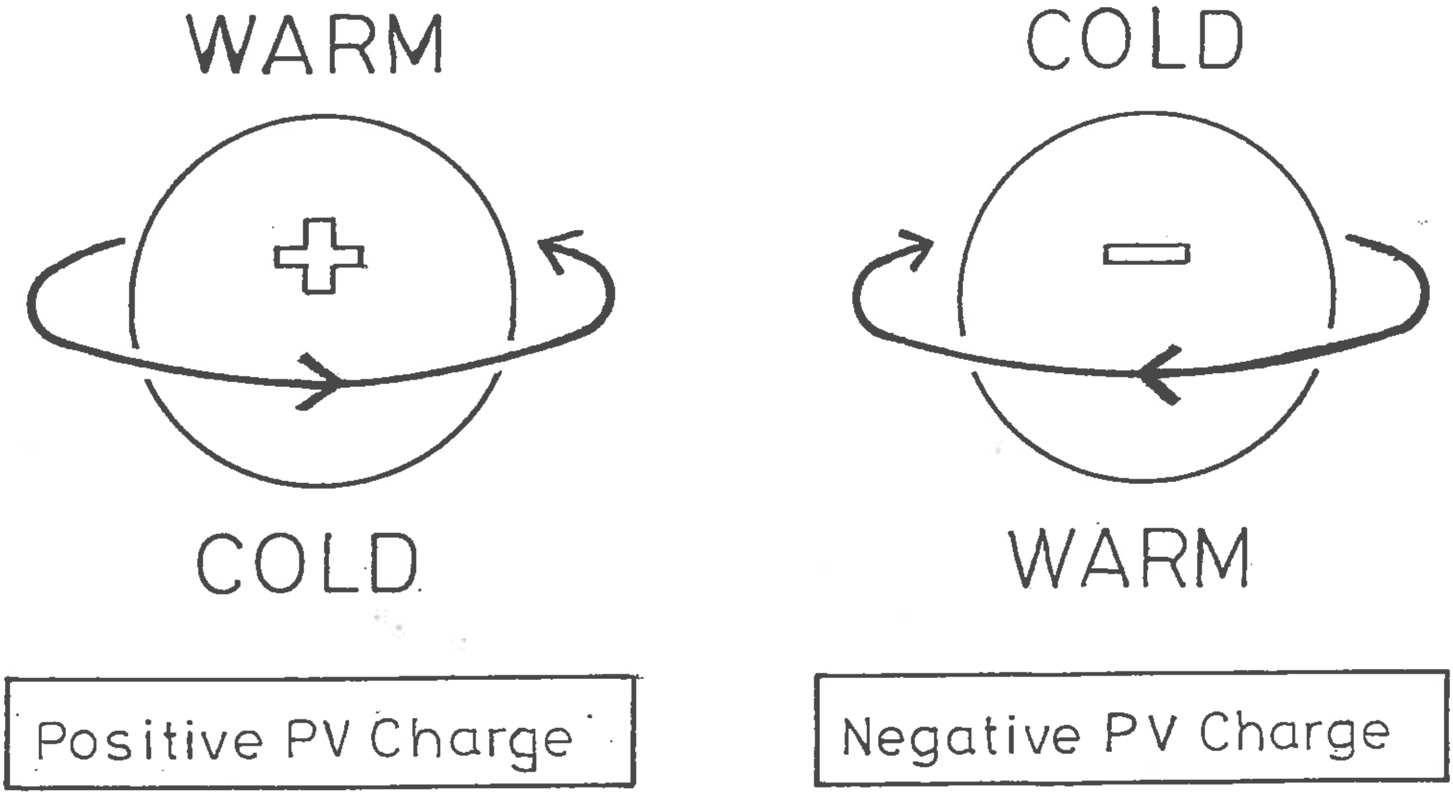

FIG 2a

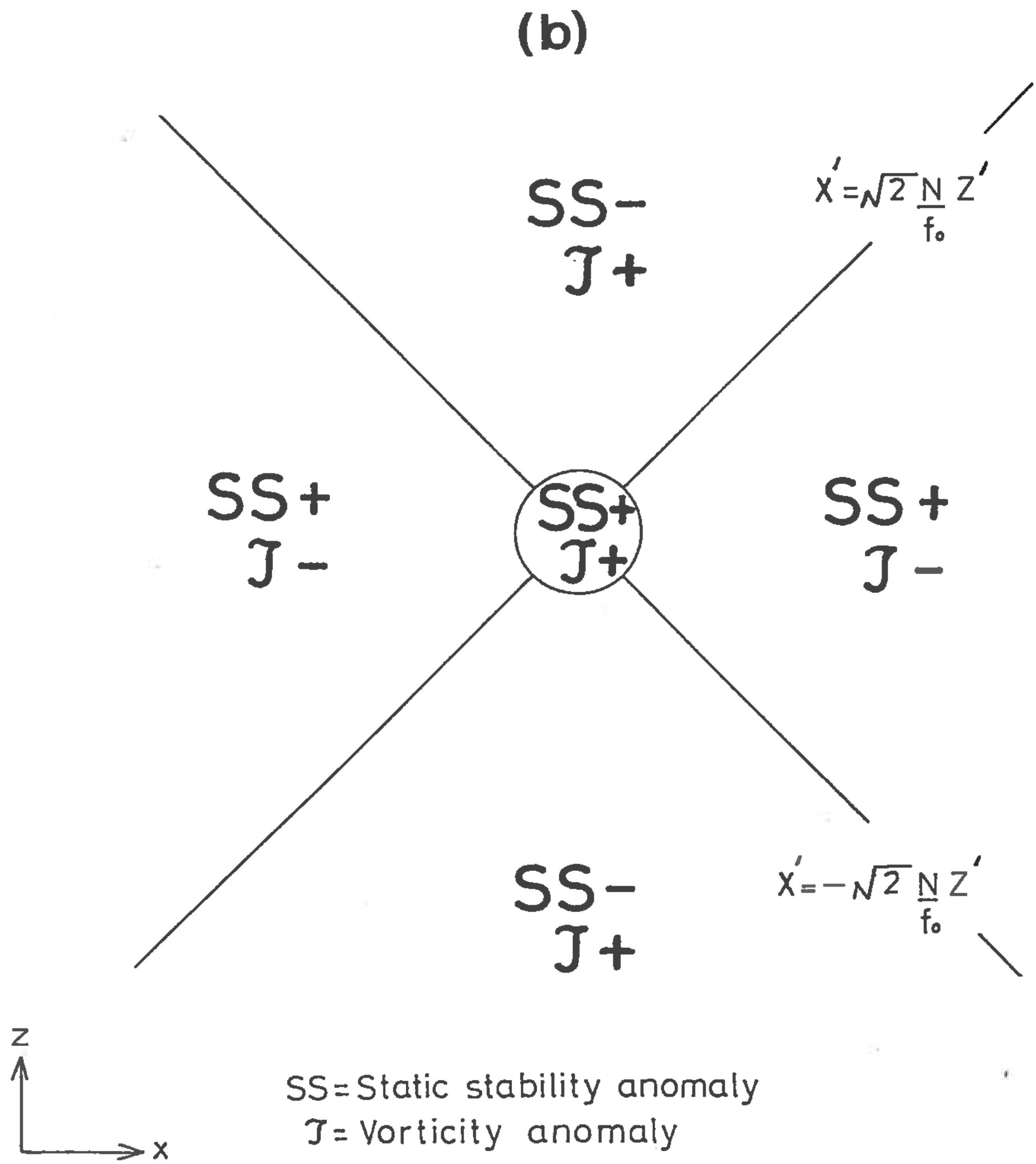
(b)
SS−
J+
X' = √2 N/f₀ Z'
SS+
J−
SS+
J+
SS+
J−
SS−
J+
X' = −√2 N/f₀ Z'
z
x
SS = Static stability anomaly
J = Vorticity anomaly

FIG. 2 b

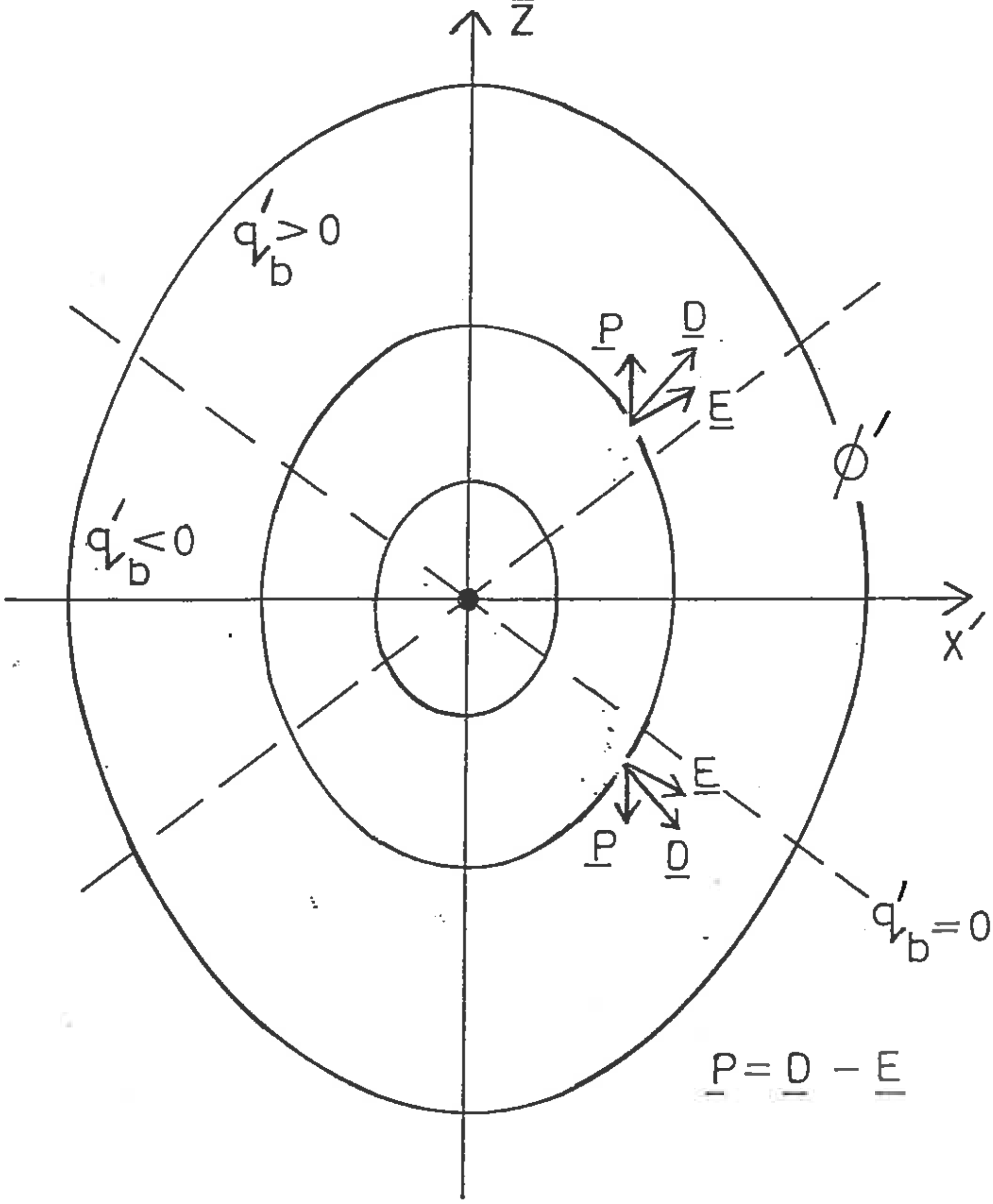

FIG. 3

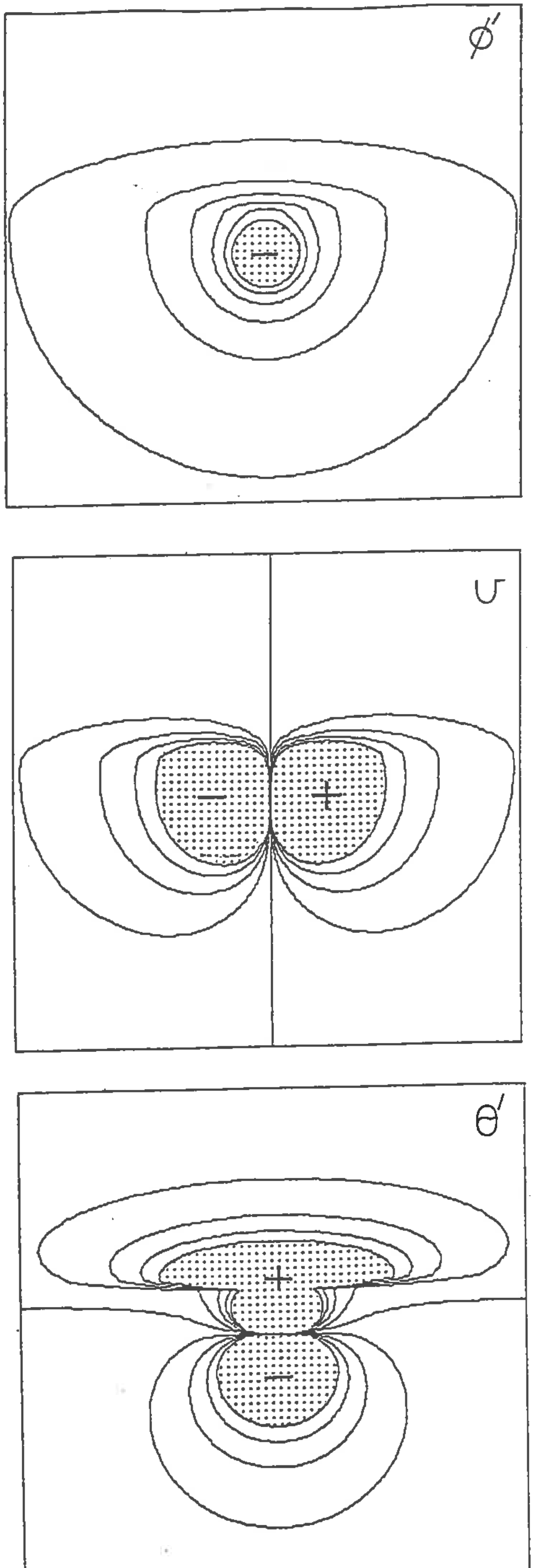

FIG. 4

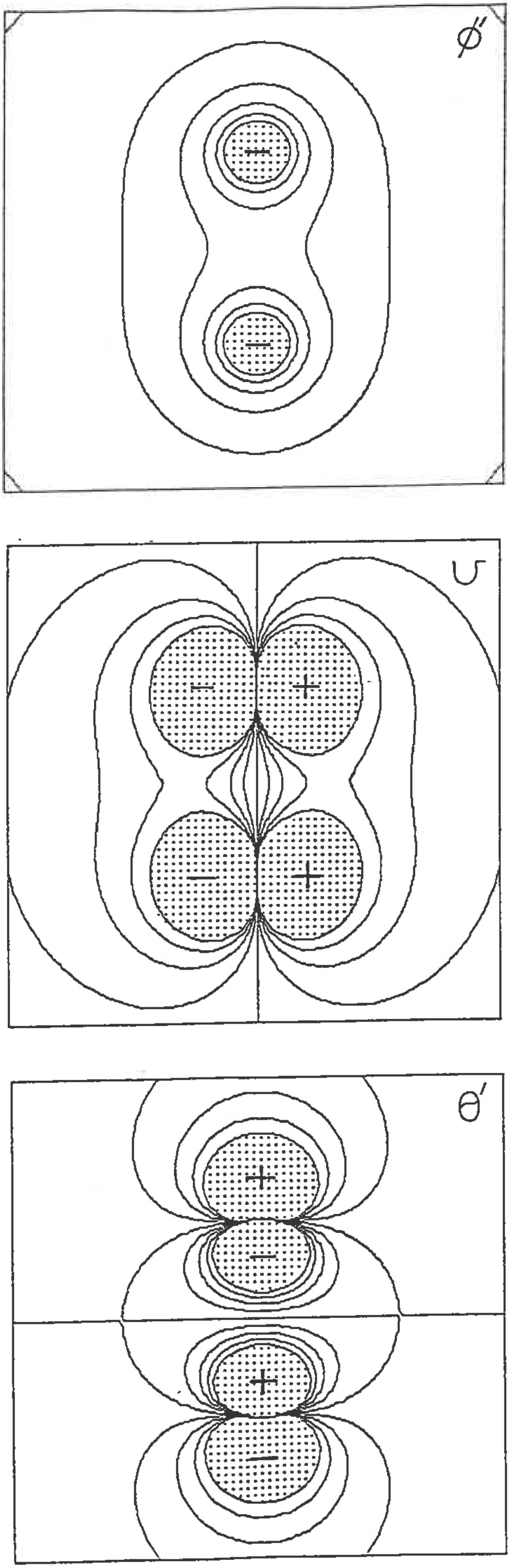

FIG. 5

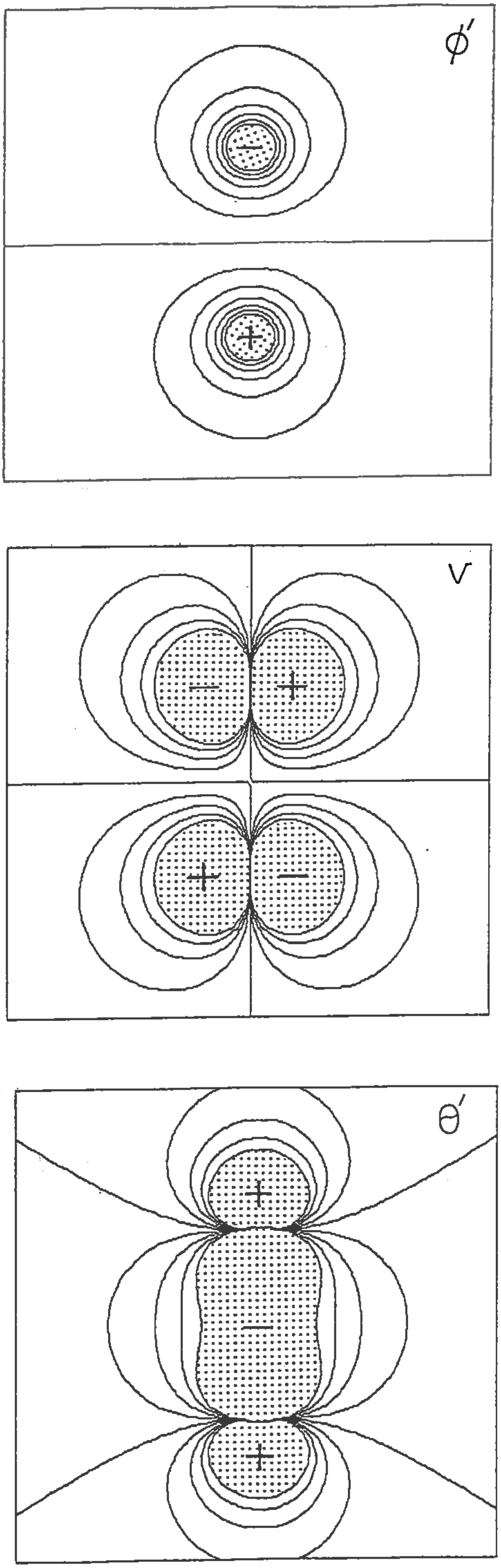

FIG. 6

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