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THE ESTIMATION OF HUMIDITY PARAMETERS

by

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Summary

The calculation of various measures of humidity from wet and dry bulb temperatures is made difficult by the need to compute the saturation vapour pressure. Complex semi-empirical equations due to Goff and Gratch are available, but for many applications simpler and less accurate formulations are required. In meteorology, there is a need to calculate the dew point from the wet bulb temperature at synoptic observing stations, and this can be conveniently carried out on a programmable calculator. For this purpose an algebraically simple equation due to Magnus is shown to be most suitable, and procedures for its use in calculating humidity parameters at an observing station are described.

1. Introduction

The moisture content of the atmosphere is generally measured cheaply and conveniently by means of a wet and dry bulb psychrometer. Unfortunately, however, the calculation of alternative measures of humidity such as the dew point (T_d), vapour pressure (e) and relative humidity is complex. The calculations are based on the Regnault equation

$$e = e_s\,(T_W) - Ap\,(T - T_W)$$

where T = dry bulb temperature

T_W = wet bulb temperature

p = atmospheric pressure

A = a ventilation coefficient

and $e_s(T_W)$ = saturation vapour pressure at the wet bulb temperature.

The source of the computational problems is the saturation vapour pressure (e_s). This is related to temperature by an integration of the Clausius Clapeyron equation, but this is made difficult by departures from the ideal gas law and the variation of latent heat with temperature. Consequently semi-empirical formulae were derived by Goff and Gratch (1945) to enable e_s to be calculated to a high degree of accuracy. These equations have been subject to continual refinement and the latest versions are given by Wexler (1976, 1977). The Goff-Gratch equations apply, however, only to the saturation pressure of water vapour in the absence of other gases. When air is added to the water vapour e_s is increased, mainly due to the extra forces of attraction between the molecules of air and water vapour. The magnitude of the effect at sea level pressures, however, is only about 0.5%. A complete account of humidity and moisture in the atmosphere is given by Wexler and Wildhack (1965), while a useful summary is provided by WMO (1966).

The Goff-Gratch equations are relatively costly to evaluate on a regular basis and for many purposes they are unnecessarily accurate. Consequently, many attempts have been made to devise empirical approximations in which simplicity is traded against accuracy. Some workers eg Richards (1971), Lowe (1977), and Rasmussen (1978) have aimed for relatively high accuracy and developed high order polynomials. The efforts of Richards and Lowe, together with some unpublished work by Hooper, are reviewed by Sargent (1980). Algebraically simpler but less accurate formulae were suggested by Tabata (1973), Revfeim and Jordan (1976), and Blackadar (1983), but the longest established of these simpler approximations is due to Magnus (1844). By incorporating an amendment due to Bogel (1979), Buck (1981) is able to recommend the use of the Magnus formula for a wide variety of uses and provides a choice of coefficients according to the temperature range of interest and the accuracy required.

Buck also supplies a choice of equations for taking into account the 'enhancement factor' which represents the difference in saturation pressure between pure water vapour and moist air.

Many of the empirical approximations developed have assumed that the calculations will be performed on a large computer. In this context the simplicity of an equation has been interpreted in terms of its speed of execution. An IBM 3081 computer at the Meteorological Office, for instance, can evaluate a sixth order polynomial in about the same time as a single exponential expression. In operational meteorology, however, calculations have to be made at synoptic observing stations in order to convert from T_w (which is observed) to T_d (which is required by the WMO synoptic codes). These calculations have generally been performed with the aid of either tables or slide rule, but they may be made more conveniently on a microcomputer or programmable calculator. In this context algebraic simplicity is important as it minimises storage requirements and enables an equation to be reversed in order to calculate temperature from saturation vapour pressure as well as vice versa.

For routine meteorological purposes, the accuracy required of e_s is that equivalent to an error of 0.1 C in temperature. This is of the order of 0.5% to 1%, and a number of algebraically simple expressions are capable of achieving this accuracy. A number of such formulae are compared, and the brevity, reversibility and accuracy of the Magnus formula is shown to make it ideal for use with a pocket calculator. It is also perfectly suitable for routine climatological applications on a large computer.

The precision to which T and T_d are reported in the WMO synoptic code messages was increased in 1982 from 1 C to 0.1 C. This increase in precision made it possible for climatological collecting centres to calculate the other humidity parameters direct from the messages, rather than through internal reporting of T_W. This procedure therefore offers the advantages of increased automation, but requires the recovery of T_W from T and T_d and this is not straight-forward. The purpose of this paper is, therefore, threefold:-

(i) to compare some of the algebraically simple formulae for calculating e_s.

(ii) to describe and recommend a procedure based on the Magnus formula for calculating humidity parameters from T and T_W.

(iii) to describe a method for recovering T_W from T and T_d.

The recommended procedures can then be implemented either on a large computer at a collecting centre, or on a programmable calculator at the observing site.

2. Integration of the Clausius Clapeyron equation

The saturation vapour pressure is expressed as a function of absolute temperature by the Clausius Clapeyron equation

$$\frac{des}{es} = \frac{E.L.dT}{RT^2}$$

where L = latent heat of vaporisation of water = 2.50084×10^{-3} $J.Kg^{-1}$ at 0°C.

R = gas constant for dry air = 287.05 $J.Kg^{-1}$

and E = ratio of molecular weight of water vapour to that of dry air = 0.62198.

Difficulties in integrating this equation are caused by the fact that L is not constant, but varies with temperature. If this variation is ignored, and L is assumed to be constant then

$$e_s = \exp\left(21.4 - \frac{5351}{T}\right) \qquad (1)$$

where e_s is given in mb if T is expressed in degrees Kelvin. This is the equation used by Blackader (1983) to illustrate the calculation of humidity parameters on a home computer.

A better assumption is clearly to make L a linear function of T,

eg $$L = [2500.84 - 2.34\,(T - 273.15)] \times 10^{-3}\ J.Kg^{-1}$$

when $$e_s = \exp\left(55.17 - \frac{6803}{T} - 5.07\ \ln T\right) \qquad (2)$$

The performance of equations (1) and (2), when assessed against the solution of the Goff-Gratch equations as given by WMO (1966), is illustrated in fig 1. In this diagram, the dotted lines indicate the error in e_s which is equivalent to an error in temperature of 0.1 C. The assumption that L is constant is seen to produce errors within the required limits for temperatures between -10 C and +35 C. The assumption that L is a linear function of temperature, however, results in a considerable improvement and produces errors well within the required limits over the entire range of temperature examined.

3. <u>Empirical expressions for calculating the saturation vapour pressure</u>

The Magnus formula already referred to takes the form

$$e_s = 6.1070 \exp\left(\frac{aT}{b+T}\right) \qquad (3)$$

In this and all subsequent equations, vapour pressure is given in mb if temperature is expressed in degrees Celsius. The most commonly quoted values of the coefficients for evaporation over water are a = 17.3 and b = 237.3. Minimum RMS errors in mb with respect to the Goff-Gratch values over the temperature range -40°C to +40°C, however, are obtained by putting a = 17.38 and b = 239.0 for evaporation over water and a = 22.44, b = 272.4 for sublimation from ice. With these coefficients, fig 1 shows that the Magnus formula produces errors which are within the required range for all temperatures examined above -30 C.

Revfeim and Jordan (1976) used the relation

$$e_s = \exp\,(7.076 - 2.47\,(1.46 - 0.01T)^2) \qquad (4)$$

and fig 1 shows that this formulation achieves the required accuracy over a temperature range of -6 C to +43 C. Better results may be obtained by using a quadratic in the inverse of absolute temperature,

ie $$e_s = \exp\left[\,19.163 - \frac{4063.2}{T+273.15} - \frac{184089}{(T+273.15)^2}\,\right] \qquad (5)$$

This form of equation was suggested by Tabata (1973), although the coefficients used are those which minimise RMS errors (in mb) between ±40 C. The accuracy of this equation is demonstrated in fig 1, where the errors produced can be seen to lie within the prescribed limits over the entire temperature range examined.

Of the 5 equations considered, fig 1 shows that those valid over the narrowest range of temperature are those suggested by Blackader (1983) and Revfeim and Jordan (1976). The widest range of applicability is attained by the equation due to Tabata (1973) and that obtained from an integration of the Clausius-Clapeyron equation with L a linear function of T. Unless very low temperatures are to be regularly dealt with, however, it is the

Magnus equation which is recommended for use with pocket calculators. It has the advantage of being algebraically very simple and hence reversible, and, for temperatures between 0 C and 40 C, is the most accurate of all the equations examined.

The enhancement factor for moist air has not been used in the above calculations since the aim was to reproduce the solutions of the Goff-Gratch equations. It is also ignored in the following sections in order to maintain continuity with past and current Meteorological Office practice. Its omission does not lead to any errors in T_d or the relative humidity, while that in e_s is equivalent to an error in temperature measurement of just less than 0.1°C. If it is wished to include the effect, however, this can be achieved simply by increasing e_s by 0.46%, ie by replacing the constant 6.1070 in the Magnus equation by 6.1351.

4. Recommended procedure for calculating humidity parameters from wet and dry bulb temperatures

The starting point is the calculation of e from the Regnault equation, which requires a knowledge not only of T and T_w but also of e_s (T_w). This can be obtained from the Magnus equation by replacing T by T_w in equation (3) to give

$$e_s(T_w) = 6.1070 \exp\left[\frac{17.38\ T_w}{239.0+T_w}\right]$$

for $T_w \geqslant 0$°C and

$$e_s(T_w) = 6.1070 \exp\left[\frac{22.44\ T_w}{272.4+T_w}\right]$$

for $T_w < 0$°C.

Since $e = e_s\ (T_d)$, the Regnault equation may be written as

$$e_s\ (T_d) = e_s\ (T_w) - Ap\ (T-T_w)$$

For measurements in a Stevenson screen the ventilation coefficient A is 0.000799, except when the wet bulb is frozen when it takes on the value 0.000720. It is important that accurate values of p are used otherwise relatively large errors may ensue. At relative humidities of 10%, for example, the assumption of p = 1000 mb when the correct value is 950 mb gives rise to errors of about 20% in e, 2% in relative humidity and 2°C in T_d.

The second variable to be calculated is T_d, which can be obtained from the calculated value of $e = e_s(T_d)$ by using the transposed Magnus equation for evaporation over water:

$$T_d = \frac{239.0\ K}{17.38 - K}$$

where K = lne - ln 6.1070.

The next step is the use of the Magnus formula for water to calculate the saturation vapour pressure at the dry bulb temperature [$e_s(T)$]. This enables the relative humidity to be calculated as $e/e_s(T)$.

These steps may be rationalized and summarized as follows:-

(i) Use the Magnus formula to calculate the saturation vapour pressure at T and T_w.

(ii) Use the Regnault equation to calculate $e = e_s\ (T_d)$

(iii) Use the transposed Magnus equation to calculate T_d.

(iv) Calculate the relative humidity as $e/e_s(T)$.

5. Estimation of the wet bulb from the dry bulb and dew point

The estimation of T_w from T and T_d is a more complex procedure than obtaining T_d from T and T_w. This is essentially because the Regnault equation is expressed in terms of the depression of the wet bulb rather than the dew point, and so can only be solved for the former by using an iterative procedure.

Less accurate estimates can, however, be obtained using an empirical approach which takes advantage of the fact that T_w lies between T and T_d. At low temperatures, when little water is available for evaporation, T_w is not much less than T, but at high temperatures, T_w is closer to T_d. In other words, the ratio of the wet bulb depression to the dewpoint depression always lies between zero and unity and increases with temperature. A convenient linear representation of this change is given by

$$\frac{T - T_w}{T - T_d} = 0.34 + 0.006\ (T + T_d)$$

For values of T from $-10°C$ to $50°C$ and dewpoint depressions up to $15°C$, errors in T_w obtained from this equation are always less than $0.3°C$. Improved accuracy could be obtained by introducing a non-linear dependence on $(T+T_d)$ to the right hand side of the equation – the quoted relation produces a wet bulb depression which exceeds the dewpoint depression at temperatures above about $55°C$ and is negative at temperatures below about $-25°C$. It may be more expedient, however, to solve the Regnault equation iteratively.

The Regnault equation may be re-arranged as

$$e_s\ (T_w) + ApT_w = e_s(T_d) + ApT$$

Using the Magnus equation, this becomes

$$6.107 \exp\left[\frac{aT_w}{b+T_w}\right] + ApT_w = 6.107 \exp\left[\frac{aT_d}{b+T_d}\right] + ApT$$

All the terms involving T_w are now on the left hand side. The right hand side, involving terms in T and T_d, can be evaluated and set equal to a value C. We can then define

$$F(T_w) = 6.107\exp\left[\frac{aT_w}{b+T_w}\right] + ApT_w - C \qquad (6)$$

which takes on the value O for a correct solution of Tw. This can be obtained using the Newton-Raphson iterative approximation:

$${}_1T_w = {}_0T_w - F\,(T_w)/F^1(T_w)$$

where ${}_0T_w$ and ${}_1T_w$ are the initial and improved estiates of Tw and F^1(Tw) is the first derivative of F(Tw).

ie $$F^1(T_w) = 6.107 \frac{ab}{(b+T_w)^2} \exp\left[\frac{aT_w}{b+T_w}\right] + Ap \qquad (7)$$

The values of a, b, and A to be used in equations (6) and (7) are determined by the value of ${}_0T_w$.

The calculation of T_w from T and T_d may therefore be summarized as follows:

(i) Make an initial estimate ${}_0T_w = 0.5\ (T+T_d)$

(ii) Select the appropriate values of a, b, and A, ie.

if ${}_0T_w \geq 0°C$	A= 0.000799	a = 17.38	b = 239.0
if ${}_0T_w < 0°C$	A = 0.000720	a = 22.44	b = 272.4

(ii) Calculate $C = 6.107 \exp\left[\frac{17.38T_d}{239 + T_d}\right] + ApT$

(iv) Evaluate F(Tw) and F^1(Tw) from equations (6) and (7).

(v) Obtain an improved estimate ${}_1T_w = {}_0T_w - F(T_w)/F^1(T_w)$.

(vi) Test for convergence of the procedure, ie $|{}_1T_w - {}_0T_w| < 0.005$

If this condition is not met, set ${}_0T_w = {}_1T_w$ and repeat the procedure from step (ii). The criterion is usually satisfied in 2 or 3 cycles.

6. Meteorological Office procedures

The introduction of new WMO synoptic codes containing dry bulb and dew point temperatures to 0.1°C created the possibility of increased automation by allowing the calculation of humidity parameters direct from the reported values of T and T_d. This opportunity has been taken by the UK Meteorological Office, whose archives of observations from synoptic stations are now based on those contained in the coded messages, instead of data keyed from manuscript tabulated returns. The wet bulb temperatures archived are no longer those measured, but are now obtained from an iterative solution of the Regnault equation. This peculiar circumstance could only be avoided by the introduction, as a National practice, of the reporting of T_w as well as T_d in the synoptic codes. The vapour pressure and relative humidity are then obtained from a simple application of the Magnus formula. For voluntary climatological stations, for which the wet bulb temperatures are still received in manuscript form, humidity parameters are calculated using the procedures described in section 4.

The correct pressure to supply to the Regnault equation is that observed at the station, uncorrected for altitude. At voluntary climatological stations the pressure is seldom recorded and so a value of 1000 mb is used. At synoptic stations, users of humidity slide rules are instructed to assume a value of 1000 mb unless the pressure is less than 950 mb. Since the recovery of T_w from T_d is a reversal of a calculation made with a slide rule, the value of the pressure used in this procedure is also set to 1000 mb. For consistency, therefore, all algorithms used to calculate humidity parameters on pocket calculators should use a pressure of 1000 mb. If and when the preparation of synoptic messages is fully automated at all stations, then this could be replaced by the station level pressure.

7. Conclusions

The problems in calculating the various measures of humidity are caused by the difficulty in evaluating the saturation vapour pressure. This is related to departures from the ideal gas law and the variation of latent heat with temperature; these make the Clausius Clapeyron equation difficult to integrate analytically. Semi-empirical equations due to Goff and Gratch (1945) are available for the accurate computation of saturation vapour pressure, but these are relatively costly to evaluate on a regular basis and for many purposes their accuracy is superfluous. Consequently many attempts have been made to devise simpler approximations providing only the accuracy required. In these attempts, simplicity has been interpreted as the speed and cost of evaluation on a large computer. In operational meteorology, however, there is the need to calculate dew point from the wet bulb temperature at synoptic observing stations, and this can be conveniently carried out on a programmable calculator. In this context, algebraic simplicity is important and the brevity, reversibility and accuracy of the Magnus formula make it ideal for such a purpose. The Magnus formula is also perfectly suitable for implementation on a large computer and has been used for routine climatological calculations at the Meteorological Office for many years.

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FIG 1 - ERROR IN SATURATION VAPOUR PRESSURE OBTAINED FROM VARIOUS FORMULAE