MW

THE AERODYNAMIC TERM - METHODS OF EVALUATION

by P. B. Wright

NOTE This is an amended version of Memo 6, the difference being an addition to Para 5 and Figure 3.

1. Introduction

This memo is a continuation of Evap Memo 4, which should be read first.

The conclusions should be regarded as preliminary; the work was done only for one station (London Airport) for one year (1965). Similar studies for other station-years should be straightforward as they will need only minor changes in existing computer programs.

2. The effect of evaluation of the term over different periods

Let us assume that the correct value of the aerodynamic term (hereinafter called "the Term") is given by A, namely an hourly evaluation of

$$a_2\,(e_a - e_d)(u + \frac{100}{21}) \qquad (1)$$

where a_2 is a constant to be determined later, e_a is the SVP and e_d is the VP, both in mb, and u is the mean hourly wind speed at 33 ft in kt.

Evap Memo 4 showed that, if we use daily evaluations, an unbiased estimate is given by

$$1.08\,a_2\,(e_a - e_d)(u + \frac{100}{21}) \qquad (2)$$

Let us now consider the appropriate formula to be evaluated monthly; that is, when we obtain the monthly mean temperature, vapour pressure and wind speed, then calculate e_a and thence evaluate the Term. Define the monthly evaluation of (1) to be C. For each month and for the year in our specimen station-year, the ratio of A to C is given in Figure 1. The results show consistency in summer but a wide scatter in winter. They suggest that we should use the following multiplying factors:

summer: $1.13\ a_2$
winter: $1.35\ a_2$ (not reliable)
year: $1.20\ a_2$

Finally, let us consider annual evaluation. The ratio of E, the evaluation of (1) over a 360-day period, to the mean of 12 successive values of C, is 1.17. This means that the formula for annual evaluation is

/1.40

$$1.40\, a_2\, (e_a - e_d)(u + \frac{100}{21})$$

This effect is due to the fact that the Term is not a linear function of the basic variables T, u and e_d.

3. Earlier recognition of the effect

Penman (1950) used expression (1) for the aerodynamic term. In stating " ... it was not to be expected that the sum of twelve monthly means would be exactly equal to the annual mean" (referring to total potential evaporation) he was clearly aware of the effect, although his use of the word "exactly" in the above statement suggests that he was not clear why it arose and treated it simply as if it were an error due to use of less accurate data. He found for 15 stations a correction factor for E_o which varied in the range 1.09 to 1.21. It is difficult to compare our result with his because although his VP and wind speed were annual means, it is not clear whether or not annual means of the other elements were calculated before evaluating E_o.

Penman then goes on to say "Experience - limited almost entirely to Rothamsted data - indicates that no significant change in estimates is produced by working with periods shorter than a month". This certainly suggests that he was not aware of the effect of different evaluation periods. However I get the impression that most of his Rothamsted analyses concerned summer. Our results show that the conversion factor from monthly to daily evaluation of the Term in summer is about 1.05, and in that season the Term accounts for less than half of the total evaporation. Thus it is not surprising that he came to his conclusion, which is probably acceptable for summer although not for winter.

Penman (1962) evaluated evaporation weekly in summer and monthly in winter. This technique may have been accurate enough for the purpose, but is not to be recommended without a full appreciation of the errors involved.

4. The constant factor

To discover what value of a_2 to use, we must look at the history of the aerodynamic term.

Penman (1948) first quotes a formula by Rohwer which, with the units changed to those used in (1), is

$$E = 0.045\, (e_s - e_d)(u + 6.71) \qquad (4)$$

He then obtains his own formula for open water evaporation which, again in our present units, is

$$E_o = 0.055\, (e_s - e_d)(u + \frac{100}{21}) \qquad (5)$$

suitable for open water surrounded by a moist surface.

He then defines the aerodynamic term E_a as the value of E_o obtained by replacing e_s by e_a. Thus

$$E_a = 0.055\, (e_a - e_d)(u + \frac{100}{21}) \qquad (6)$$

He obtained the values of the constants using, as I understand it, daily evaluations of E_o, his data being:

$$\left.\begin{array}{r} \text{09h VP} \\ \dfrac{T_x + T_n}{2} \\ \text{mean daily wind speed (based on continuous readings)} \end{array}\right\} \qquad (7)$$

His results appear to be based mainly on summer observations.

We shall therefore assume that, at least in summer, formula (6), using approximations (7) and evaluating daily, gives an unbiased estimate of the Term.

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A comparison between daily evaluations (B) using hourly data with daily evaluations (D) using the approximations (7) is given in Figure 2. During the summer months the ratio was fairly consistently close to 1; the slight tendency for it to be less than 1 will be assumed insignificant unless subsequently confirmed by other data. We shall therefore assume that the approximation (7) is a good one in summer, and use the same constant in (2) as Penman chose to make (6) correct. Hence $a_2 = 0.051$. Thus the aerodynamic Term can be written

$$E_a = b\,(e_a - e_d)(u + \frac{100}{21})$$

where b = 0.051 for hourly evaluations
= 0.055 for daily evaluations
= 0.058 for monthly evaluations in summer
= 0.069 for monthly evaluations in winter (not reliable)
= 0.061 for monthly evaluations averaged over a year.
= 0.071 for annual evaluations.

5. Accuracy of Penman's approximation

We can now look more closely into the approximation (7). Figure 2 shows that the relationship between B and D is less good in winter than in summer, and in particular December shows a large discrepancy. The December value appears to have been due to the occurrence during that 30-day period of three days when D had large spurious negative values (below minus 15). Such values are, of course, nonsense; they arose as a result of the arrival during the day of mild, humid, windy weather causing a rapid rise of temperature. On days such as these the approximation (7) is clearly inappropriate; it seems from Figure 2 that the occurrence of only three such days can completely upset the stability of the monthly mean. (Large negative values did not occur in any of the other months studied, although small negatives (above minus 4) occurred on four occasions.) These results suggest that daily evaluations using this approximation are undesirable in winter.

Further information on the accuracy is given by Figure 3, which shows the standard deviation of the daily errors each month. On the right is the standard error in units. (Reminder: the mean daily value of the Term, taking b = 1 as has been done in these calculations, ranges from about 23 units in Dec-Feb to about 68 units in May-Aug). The standard error was in the general range 10 to 20 units, with no clear seasonal variation. In winter the standard error was about half the actual value; I think this implies that percentage errors will be meaningless, particularly since negative values occur on some days. Alternatively, perhaps we ought to calculate percentage error for each day instead of dividing the mean daily difference by the mean value for the month. The graph of percentage standard error is shown on the left. If we ignore the winter months as discussed, it is seen that the error was in the range 19% to 32% of the correct value; not good, but perhaps good enough for some purposes. It may be worth considering whether a much simpler formula (eg a value depending only on the time of year) might be an adequate substitute for Penman's aerodynamic term in winter.

As monthly evaluation is the most common method of calculation, let us see how this compares with hourly evaluation. Figure 4 compares monthly evaluation (F) using approximation (7) with hourly evaluation (A). It shows that, during April to October, the ratio averaged about 1.08, the same as that for daily evaluations. Thus there is an important difference in behaviour with respect to evaluation period between hourly data and Penman's approximate data. Although Penman was justified in assuming that the period of evaluation, if less than a month, made no significant difference, he was not justified in extending this assumption to the whole year, and he would not have been justified if he had been using the (more accurate) hourly data instead of his approximate data. The conclusion is that we can in summer use the same value for b (0.055) as we used for daily evaluations, but in winter we must use a higher value which is not very reliable.

/6. Summary

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1. The most correct value of the aerodynamic Term throughout the year is given by hourly evaluation of

$$b\,(e_a - e_d)(u + \frac{100}{21}) \text{ where } b = 0.051$$

2. If daily evaluations are made using hourly data, an unbiased estimate is obtained by taking b = 0.055, with a standard error of about 7%; this is valid throughout the year.

3. In summer (May-August) an unbiased estimate may be obtained with b = 0.055 by calculating:

   a. daily evaluation using approximation (7) (this has a standard error of 20 to 30%) or:

   b. monthly evaluation using approximation (7).

4. Conclusion (3) may hold for a longer portion of the year; however use of this approximation is not recommended at least during November to January.

5. If hourly data are used, the value of b must be chosen according to Para 4 to suit the evaluation period. Monthly evaluation appears to be unreliable in winter.

6. The values of b and the periods of applicability need checking by analyses of other stations and years.

NOTE

This memo is circulated for discussion purposes only. Any comments or suggestions should be sent to the author.

Met O 8
Met Office
Bracknell
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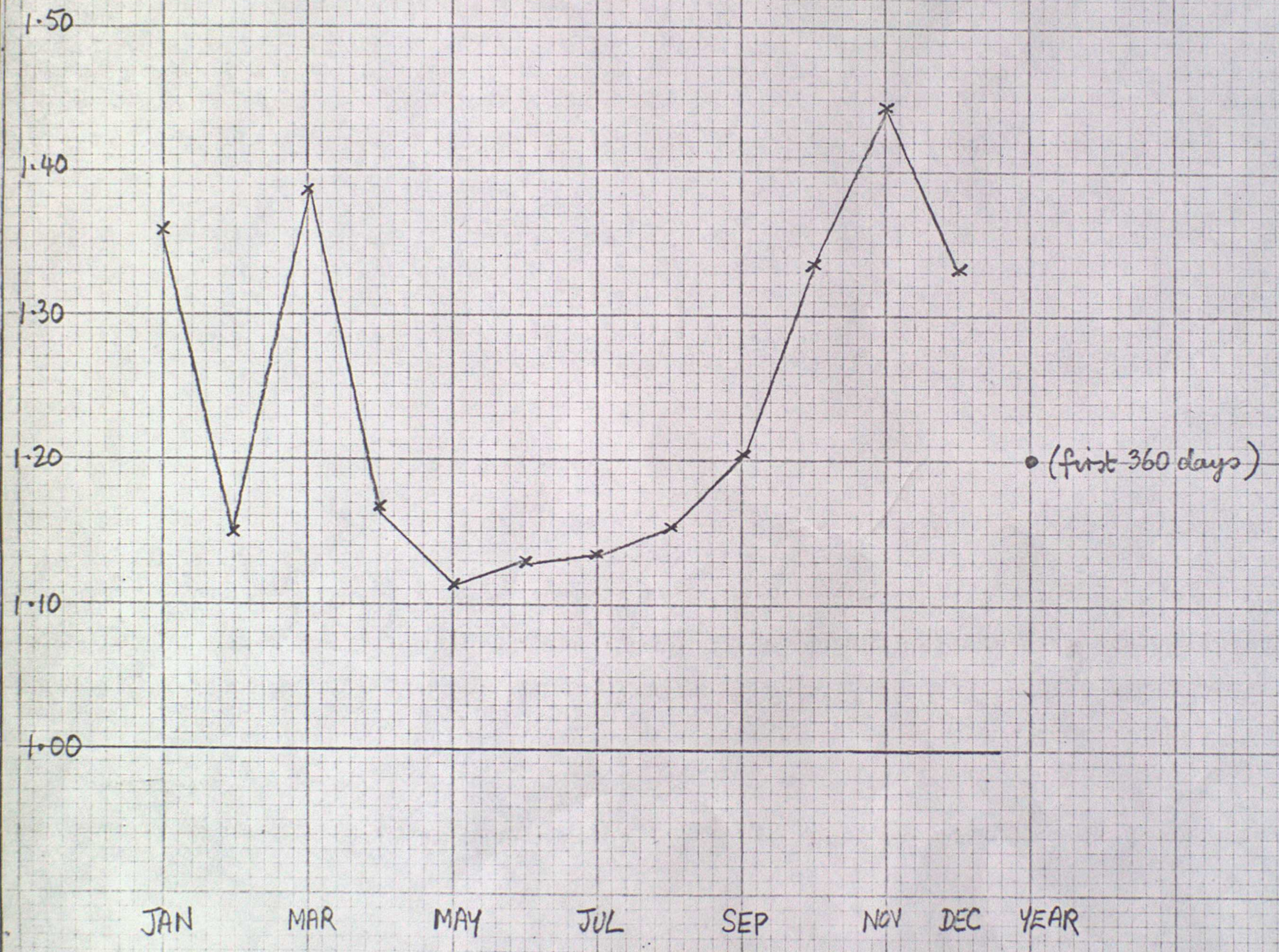
FIGURE 1 LONDON AIRPORT 1965

Memo 6 —

monthly value of A (hourly evals)

" " " C (monthly evals using hourly data)



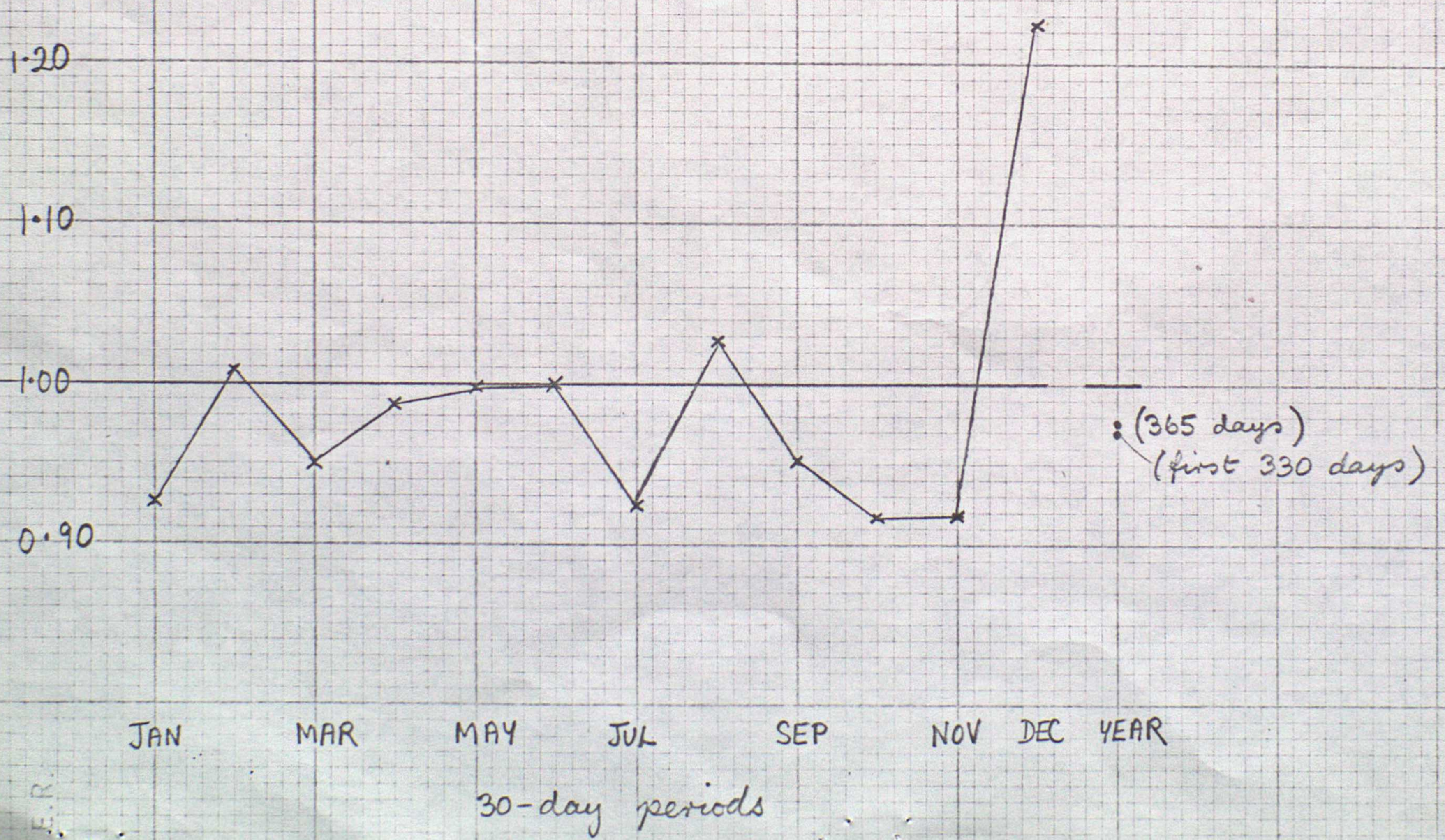
LONDON AIRPORT 1965

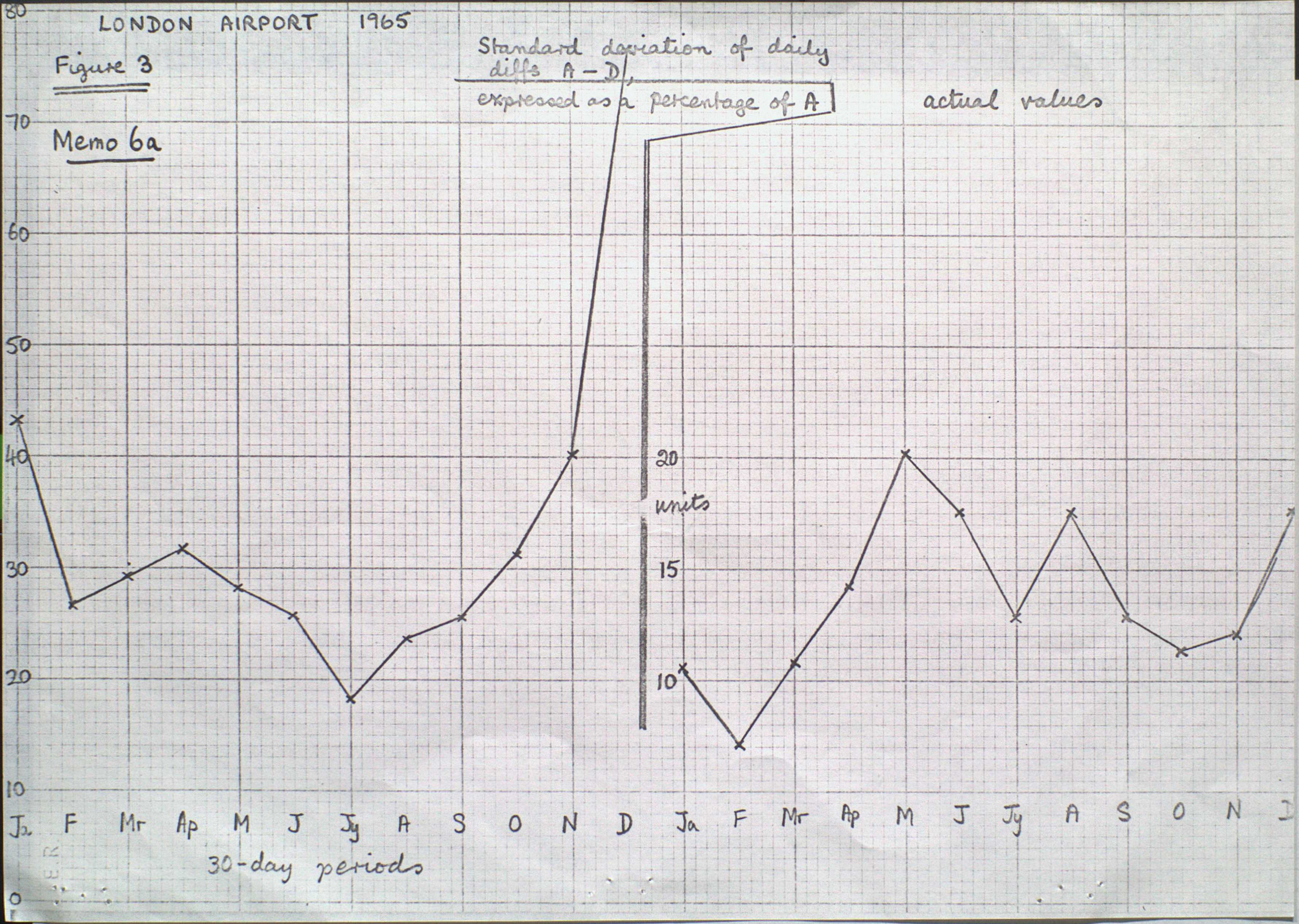
Figure 2

Memo 6 –

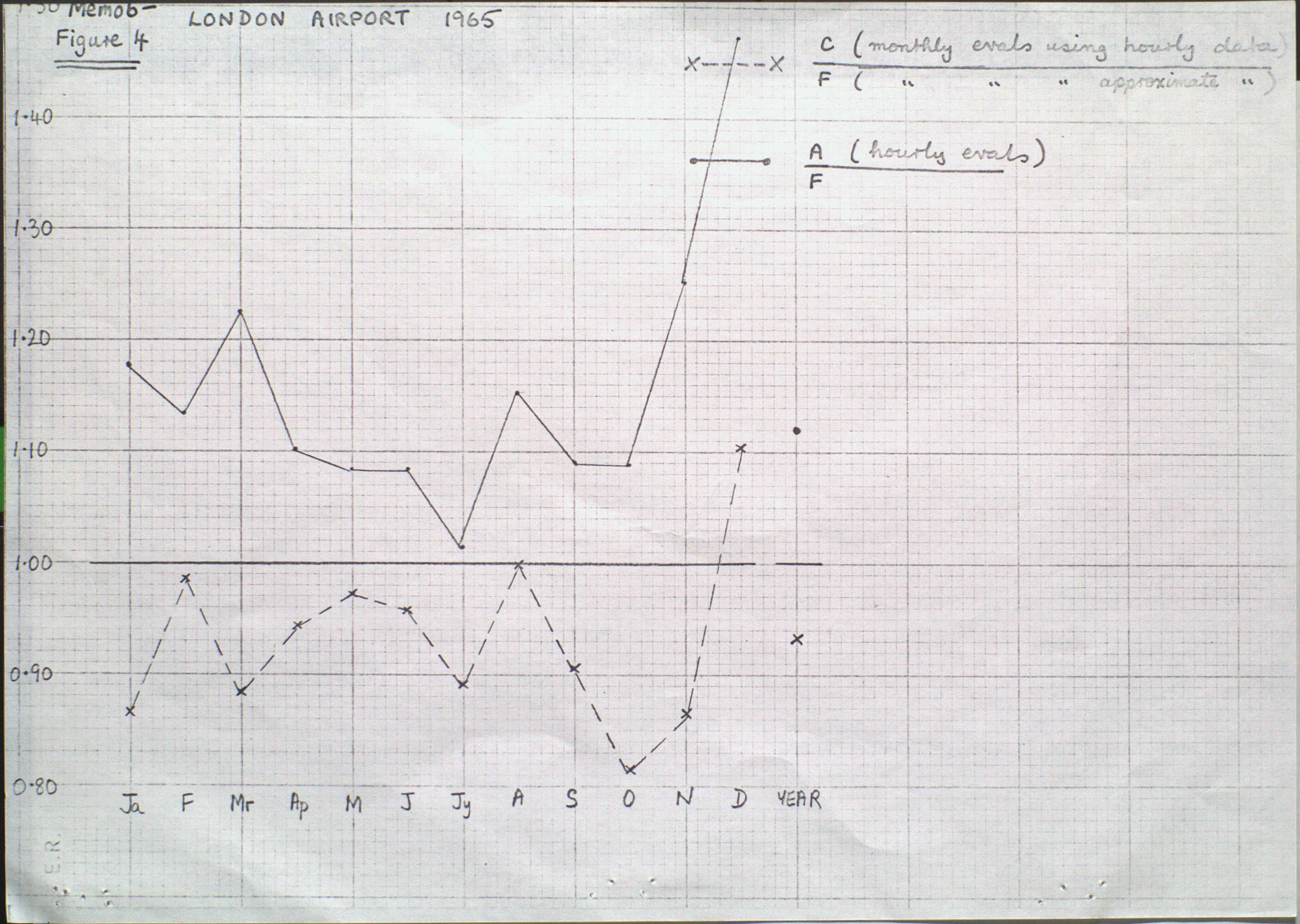
monthly value of B (daily evaluations using ~~approximate~~ hourly data)

" " " ED (" " " ~~hourly~~ approximate ")





LONDON AIRPORT 1965
Figure 3
Memo 6a
Standard deviation of daily diffs A – D, expressed as a percentage of A
actual values
units
80
70
60
50
40
30
20
10
0
20
15
10
Ja F Mr Ap M J Jy A S O N D Ja F Mr Ap M J Jy A S O N D
30-day periods



Memo6 - Figure 4
LONDON AIRPORT 1965
C/F (monthly evals using hourly data)
F (" " " approximate ")
A/F (hourly evals)
1·40
1·30
1·20
1·10
1·00
0·90
0·80
Ja
F
Mr
Ap
M
J
Jy
A
S
O
N
D
YEAR