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VARIATION OF WIND WITH TIME AND DISTANCE

BY

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VARIATION OF WIND WITH TIME AND DISTANCE

SUMMARY

The variation is discussed of upper wind with time and place as calculated from the radio and radar wind ascents made in recent years. Though the results of the observations made over the British Isles are treated in most detail, observations from other areas are also used to enable generalizations to be made.

The treatment is by vector statistics the elements of which are outlined in the Introduction (Part I). In Part II there are set out data concerning the variation of wind with time; from these it appears that the variation obeys an identical and simple statistical law at all heights up to at least 50,000 ft. In Part III data are shown of the variations of wind with distance, and again the same law appears to hold within a little for all heights up to 50,000 ft. In Part IV a brief discussion is given of the degree of accuracy with which various distributions of observing stations can be said to represent the general wind over an air route. In Part V data are given of the more complex statistical problem of the variation of wind with time and place combined, and this leads on to the final Part VI which discusses the use of statistical regression equations in forecasting winds.

PART I—INTRODUCTION

§ 1—MEANING OF VECTOR CORRELATION COEFFICIENTS

This paper arose from the necessity of giving the probable value of wind at one position in the free air when no information was available except an observation at another position. That this could be done if a system of vector correlation coefficients and vector regression equations could be established was suggested by the present writer. The theoretical work of establishing such a system was undertaken by G. H. Gilbert in 1945, and this work will, it is hoped, be published separately. It was subsequently found that the same type of problem had presented itself to C. E. P. Brooks in 1943, and he and N. Carruthers had evolved the correlation of vectors in which the simple stretch was the paramount feature.

The concept of vector correlation coefficients is comparatively simple. Suppose that there are two sets of vectors (e.g. winds) $\mathrm{V_A}$ and $\mathrm{V_B}$ at two places, A and B; and these fluctuate about their vector mean values $\overline{\mathrm{V}}_\mathrm{A}$ and $\overline{\mathrm{V}}_\mathrm{B}$ so as to give departures of which $\mathrm{v_A}$ and $\mathrm{v_B}$ are represented typically in Fig. 1. It is required to know how $\mathrm{V_B}$ is likely to behave when $\mathrm{V_A}$ has any given value, and this leads to the idea of a vector regression equation of the form

$$\mathrm{v_B} = c r_{\mathrm{AB}} \mathrm{v_A}$$

$$\text{or } (\overline{\mathrm{V}}_\mathrm{B} - \mathrm{V_B}) = c r_{\mathrm{AB}} (\overline{\mathrm{V}}_\mathrm{A} - \mathrm{V_A}) \qquad \ldots\ldots(1)$$

where c is some constant dependent on the variability of $\mathrm{v_A}$ and $\mathrm{v_B}$, and r_{AB} is some numerical value between $+ 1$ and $- 1$ of the nature of a correlation coefficient.

One can conceive a relationship between $\mathrm{v_A}$ and $\mathrm{v_B}$ in which $\mathrm{v_B}$ tends to be parallel to $\mathrm{v_A}$ and to be proportional to it in magnitude. The closeness of such a relation can be measured by a form of correlation coefficient which can be called the " stretch correlation coefficient ", and which can be expressed as

$$\frac{\Sigma(|\mathbf{v}_A|\,|\mathbf{v}_B|\cos\theta_{AB})}{\sqrt{\{\Sigma(\mathbf{v}_A{}^2)\,\Sigma(\mathbf{v}_B{}^2)\}}}$$

where θ_{AB} is the angle between $\mathbf{v}_A$ and $\mathbf{v}_B$ (see Fig. 2).

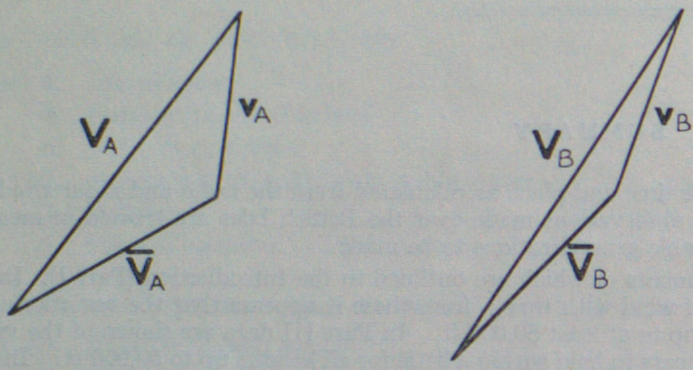

FIG. 1—TYPICAL WIND VECTORS AT TWO PLACES

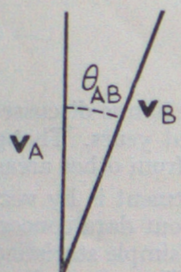

FIG. 2—TYPICAL RELATION OF WIND VECTOR DEPARTURES AT TWO PLACES

Other more complicated relations between $\mathbf{v}_B$ and $\mathbf{v}_A$ can be considered, in particular the case in which $\mathbf{v}_B$ tends to have a direction rotated by an angle, α_{AB}, from $\mathbf{v}_A$ (which is called the angle of turn). The best value of α_{AB} can be expressed as

$$\tan^{-1}\frac{\Sigma(|\mathbf{v}_A|\,|\mathbf{v}_B|\sin\,\theta_{AB})}{\Sigma(|\mathbf{v}_A|\,|\mathbf{v}_B|\cos\,\theta_{AB})},$$

and then the vector correlation coefficients can be written in the form

$$\frac{\Sigma(|\mathbf{v}_A|\,|\mathbf{v}_B|\cos\,\theta_{AB}+|\mathbf{v}_A|\,|\mathbf{v}_B|\sin\,\theta_{AB})}{\sqrt{\{\Sigma(\mathbf{v}_A{}^2)\,\Sigma(\mathbf{v}_B{}^2)\}}}.$$

The inclusion of this additional term increases the correlation coefficient, though in most problems concerning wind the increase is small.

The concept of the standard vector deviation has been used in mapping the upper winds in *Geophysical Memoirs* No. 85 " Upper winds over the world "[1*]. It is seen that this standard vector deviation is the same as $\sqrt{\{(\Sigma(\mathbf{v}^2)/(n-1)\}}$ or σ_A. Moreover, it is found as in scalar statistics that the value c in the regression equation (1) is in fact σ_B/σ_A.

Thus, it appears that the analytic form of vector statistics is very similar to that of scalar statistics, but a word of caution must be said ; in some cases the analogy does not hold in all respects ; if the stretch vector correlation coefficient only is being considered it is safe to extend to vectors the concept of total and partial correlation coefficients in an analogous way to scalar statistics.

By way of illustration the correlation coefficients between the surface winds and those at 1,500 ft. are set out in Table I ; they are based on 70 observations made during the winter of 1939–40 at stations in southern and eastern England and on 100 observations in the summer of 1940. These observations were all made between midnight and dawn.

* The index numbers refer to the bibliography on p. 31.

TABLE I—VARIATION OF WIND WITH HEIGHT IN THE FRICTION LAYER

	Surface wind			Wind at 1,500 ft.			Correlation coefficient		
	Direction	Speed	Standard vector deviation	Direction	Speed	Standard vector deviation	Stretch	Angle	Total
	°	kt.	kt.	°	kt.	kt.		°	
Winter ..	165	1·0	13·6	221	3·7	26·5	+0·725	−23	+0·790
Summer ..	293	4·8	14·2	296	8·0	23·7	+0·903	−6	+0·908

The solution of the appropriate regression equation in winter leads to winds illustrated in Fig. 3 in which westerly winds of 5, 10 and 20 kt. are found to be associated with winds at the surface of 229° 1 kt., 239° 2½ kt. and 246° 7 kt. respectively.

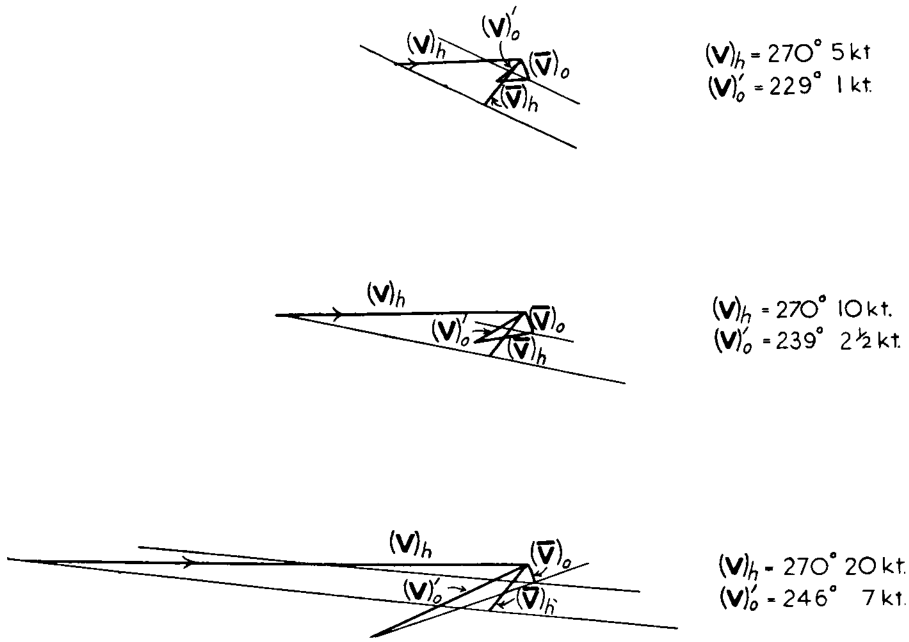

FIG. 3—VARIATION OF WIND WITH HEIGHT IN FRICTION LAYER FOR CERTAIN WINDS IN WINTER

$(\bar{V})_0$ is mean wind at surface $(\bar{V})_h$ is mean wind at 1,500 ft.
$(V)'_0$ is deduced wind at surface $(V)_h$ is assumed wind at 1,500 ft.

§ 2—VECTOR CORRELATION COEFFICIENTS OF SIMPLE STRETCH CALCULATED FROM LINEAR REGRESSION EQUATIONS

Supposing $\mathrm{v_A}$ (or $\mathrm{V_A} - \bar{\mathrm{V}}_\mathrm{A}$) and $\mathrm{v_B}$ (or $\mathrm{V_B} - \bar{\mathrm{V}}_\mathrm{B}$) to be the departures from mean of two vector quantities, it is desired to form a value $\mathrm{v} = k\mathrm{v_B}$ such that Q (or $\Sigma(\mathrm{v} - \mathrm{v_A})^2/n$) is a minimum.

Now
$$Q = \frac{1}{n}\,\Sigma(k\mathrm{v_B} - \mathrm{v_A})^2 \qquad \ldots\ldots(2)$$
$$= \frac{1}{n}\{k^2\Sigma\mathrm{v_B}^2 + \Sigma\mathrm{v_A}^2 - 2k\,\Sigma\,(\mathrm{v_A v_B})\}$$
$$= k^2\sigma_\mathrm{B}^2 + \sigma_\mathrm{A}^2 - 2\sigma_\mathrm{A}\sigma_\mathrm{B}kr_\mathrm{AB}, \qquad \ldots\ldots(3)$$

where $$r_{AB} = \frac{\Sigma\,(\mathrm{v}_A\mathrm{v}_B)}{n\sigma_A\sigma_B} \text{ when } n \text{ is large,}$$

or $$= \frac{\Sigma\,|\mathrm{v}_A|\,|\mathrm{v}_B|\,\cos\theta_{AB}}{n\sigma_A\sigma_B},$$

and θ_{AB} is the angle between v_A and v_B. It is readily seen from equation (3) that the condition that makes Q a minimum is $$\frac{\partial Q}{\partial k} = 0$$

or $$k = \frac{\sigma_A}{\sigma_B} r_{AB}\,, \qquad \ldots\ldots(4)$$

i.e. the regression of v_A on v_B is represented by the equation

$$\mathrm{v}_A = \frac{\sigma_A}{\sigma_B} r_{AB}\,(\mathrm{v}_B)$$

or $$\mathrm{V}_A - \bar{\mathrm{V}}_A = \frac{\sigma_A}{\sigma_B} r_{AB}\,(\mathrm{V}_B - \bar{\mathrm{V}}_B) \qquad \ldots\ldots(5)$$

and c in equation (1) is seen to be σ_A/σ_B.

Similarly, the regression of v_B on v_A is represented by the equation

$$\mathrm{V}_B - \bar{\mathrm{V}}_B = \frac{\sigma_B}{\sigma_A} r_{AB}\,(\mathrm{V}_A - \bar{\mathrm{V}}_A). \qquad \ldots\ldots(6)$$

Now the standard vector error ε_{AB} is equal to Q,

i.e. $$\varepsilon_{AB} = \sqrt{\{k^2\sigma_B{}^2 + \sigma_A{}^2 - 2\sigma_A\sigma_B k r_{AB}\}}$$

or substituting from (4)

$$\varepsilon_{AB} = \sigma_A\sqrt{(1 - r_{AB}{}^2)}. \qquad \ldots\ldots(7)$$

Gilbert has deduced similar forms of correlation coefficients for other types of relationship than simple stretch and has derived the corresponding values of the standard vector error. For these the reader is referred to Gilbert's paper[2].

§ 3—SOME USEFUL RELATIONSHIPS

Before discussing actual values of the correlation coefficients which are found in natural winds it will be well to put down certain results for ready reference. These refer in the main to the correlation coefficients of simple stretch but it will be pointed out where the same results are applicable to more general coefficients.

Standard vector deviation of a sum of several vector quantities.—

Let $$S\mathrm{v} \equiv (\mathrm{v}_1 + \mathrm{v}_2 + \mathrm{v}_3 + \ldots\ldots \mathrm{v}_p) +.$$

The standard vector deviation of $S\mathrm{v}$ is $\sqrt{\{\sum_1^n (S\mathrm{v})^2/(n-1)\}}$ where n is a large number.

Now $$\frac{\sum_1^n (S\mathrm{v})^2}{n-1} = \frac{1}{n-1}\sum_1^n (\mathrm{v}_1 + \mathrm{v}_2 + \mathrm{v}_3 + \ldots + \mathrm{v}_p)^2$$

$$= \frac{1}{n-1}\left\{\sum_1^n \mathrm{v}_1{}^2 + \sum_1^n \mathrm{v}_2{}^2 + \ldots + \sum_1^n \mathrm{v}_p{}^2 + 2\sum_1^n \mathrm{v}_1\mathrm{v}_2 + \ldots + 2\sum_1^n \mathrm{v}_s\mathrm{v}_q + \ldots\right\}.$$

If σ_1^2 is written for $\sum_1^n \mathsf{v}_1^2/(n-1)$, σ_2^2 for $\sum_1^n \mathsf{v}_2^2/(n-1)$, etc.,

$$\sqrt{\frac{\sum_1^n (S\mathsf{v})^2}{n-1}} = \sqrt{\left\{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_p^2 + 2\sigma_1\sigma_2 r_{12} + 2\sigma_1\sigma_3 r_{13} + \ldots + 2\sigma_s\sigma_q r_{sq} + \ldots\right\}} \quad \ldots(8)$$

where r_{sq} is the stretch vector correlation coefficient between v_s and v_q. In the case where the standard vector deviation is required of the sum of a number of similar vector quantities, i.e. if $\sigma_1, \sigma_2, \sigma_3, \ldots \sigma_p$ are all equal to σ then

$$\sqrt{\frac{\sum_1^n (S\mathsf{v})^2}{n-1}} = \sigma\sqrt{\left\{p + 2(r_{12} + r_{13} + \ldots + r_{sq} + \ldots)\right\}}. \quad \ldots\ldots(9)$$

In the case where the standard vector deviation of a mean of two similar quantities is required the result is

$$\sqrt{\frac{\sum_1^n (S\mathsf{v})^2}{n-1}} = \sigma\sqrt{\left\{2 + 2r_{12}\right\}}. \quad \ldots\ldots(10)$$

The standard vector deviation of the difference of two similar quantities is seen to be given by

$$\sqrt{\frac{\sum_1^n (S\mathsf{v})^2}{n-1}} = \sigma\sqrt{\left\{2 - 2r_{12}\right\}}. \quad \ldots\ldots(11)$$

$\frac{\Sigma (Sv^2)}{(n-1)\sigma^2} = (2 - 2r_{12})$

$2r_{12} = 2 - \frac{\Sigma(Sv^2)}{}$

$r \ldots 1 - \frac{1}{2}$

Correlation of v_1 *with* $(\mathsf{v}_1 - \mathsf{v}_2)$.—Expressions for the vector correlation coefficient $r_{1,1-2}$ between v_1 and $\mathsf{v}_1 - \mathsf{v}_2$ can be readily obtained from the fact that the mean standard vector error in the value of $\mathsf{v}_1 - \mathsf{v}_2$, based upon a linear regression of $\mathsf{v}_1 - \mathsf{v}_2$ on v_1, is the same as the mean standard vector error in v_2, based upon the regression of v_2 on v_1.

i.e. $$\sigma_{1-2}^2(1 - r_{1,1-2}^2) = \sigma_2^2(1 - r_{1,2}^2) \quad \ldots\ldots(12)$$

where $$\sigma_{1-2}^2 = \sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 r_{12} \quad \ldots\ldots(13)$$

so that $$r_{1,1-2} = \frac{\sigma_1 - \sigma_2 r_{12}}{\sigma_{1-2}}. \quad \ldots\ldots(14)$$

This is true of coefficients of simple stretch, since the coefficient involved in σ_{1-2}^2 is that of simple stretch only.

Correlation of v_1 *with the mean of* $\mathsf{v}_1, \mathsf{v}_2, \mathsf{v}_3, \ldots$.—The correlation of v_1 with $(\mathsf{v}_1 + \mathsf{v}_2 + \mathsf{v}_3)/3$ will be considered but the method can be extended to any number of vectors and can be adapted to the consideration of a weighted mean.

The correlation of v_1 with $(\mathsf{v}_1 + \mathsf{v}_2 + \mathsf{v}_3)/3$ is the same as that between v_1 and $(\mathsf{v}_1 + \mathsf{v}_2 + \mathsf{v}_3)$

$$\begin{aligned} r_{1,1+2+3} &= \frac{\overline{\mathsf{v}_1(\mathsf{v}_1 + \mathsf{v}_2 + \mathsf{v}_3)}}{\sigma_1\sigma_{1+2+3}} \\ &= \frac{\overline{\mathsf{v}_1^2} + \overline{\mathsf{v}_1\mathsf{v}_2} + \overline{\mathsf{v}_1\mathsf{v}_3}}{\sigma_1\sigma_{1+2+3}} \\ &= \frac{\sigma_1 + \sigma_2 r_{12} + \sigma_3 r_{13}}{\sigma_{1+2+3}} \end{aligned} \quad \ldots\ldots(15)$$

where $$\sigma_{1+2+3}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 r_{12} + 2\sigma_1\sigma_3 r_{13} + 2\sigma_2\sigma_3 r_{23}.$$

This expression for $r_{1,\,1+2+3}$ can be readily extended to include any number of vectors, but it is to be noted that these expressions for the correlation coefficient of mean values derived in this simple way are only valid for coefficients of simple stretch.

Standard deviation of a vector mean of a continuous function.—A problem which is constantly arising in connexion with navigational winds is to determine the standard deviation of the vector mean wind over a route when the winds at different points on the route are connected by a definite correlation law. The solution of this in the case of an equivalent headwind (a scalar quantity) has been found by Sawyer[3] to be

$$\bar{\sigma} = \frac{2\sigma}{l}\sqrt{\left\{\int_0^l \int_0^s r_x dx ds\right\}}. \qquad \ldots\ldots(16)$$

a formula which is equally applicable to standard vector deviation provided that correlation coefficients are of simple stretch only. In this formula then $\bar{\sigma}$ is the standard vector deviation of the vector mean wind over the route, σ is the average standard vector deviation of winds at all points on the route, l is the length of the route and r_x is the stretch vector correlation coefficient between winds at points distant x miles apart.

PART II—VARIATION OF WIND WITH TIME

§ 4—INTRODUCTION

In what follows are set out various estimates of the correlation of the wind at one time with that at the same place but at a succeeding time. In this type of relationship the principal correlation coefficient is the stretch vector correlation coefficient for which r_t is used, the suffix denoting that it is the coefficient between wind at one time and that t hours later, also σ_t is used for the root-mean-square vector difference between wind at one time and at t hours later. The connexion between σ_t and r_t is readily seen from equation (11) to be

$$\sigma_t = \sigma\sqrt{\{2(1-r_t)\}} \qquad \ldots\ldots(17)$$

where σ is the standard vector deviation of the winds considered.

§ 5—DATA FOR THE BRITISH ISLES

The variation of wind with time has been calculated by various authors in various published and unpublished notes; in what follows these have been assembled together and reduced as far as possible to uniformity of presentation.

In September–October 1946 a series of observations of smoke puffs released at short intervals of 2 to 5 min. over the camera obscura at Orfordness showed that there were short-period fluctuations in the wind at heights between 2,000 and 5,000 ft. The root-mean-square variation of these fluctuations was from 0·5 to 3·5 kt. on different days[4]. One or two sets of observations at about 20,000 ft. have subsequently shown the same sort of variation with time at that level also. It may be noted that the accuracy of the observations on which these results are based is high: the standard vector deviation of the observational errors is less than 0·5 kt.

A further series of observations[5] with smoke puffs were made at Orfordness at heights between 30,000 and 35,000 ft. From these observations the vector change in wind in 10 min. was deduced on 86 occasions. There appeared to be some relationship between greater wind speeds and greater vector change. The occasions fell about equally in the stratosphere, in the troposphere and within 1,500 ft. of the tropopause level. The greatest vector changes occurred in the last group followed by those in the stratosphere, the fewest occasions with big changes being in the troposphere. These observations were distributed throughout 1950 and 1951, and σ_t for 10 min. was 5·5kt.

Observations obtained from double-theodolite pilot balloon ascents to 2,500 ft. at intervals of 6, 18 and 30 min. at Larkhill[6] show that the vector variation of wind is very nearly constant throughout the layer between the surface and 2,500 ft. The values of the probable vector variation (50 per cent. value) in each of these intervals were deduced as shown in Table II. The root-mean-square variation σ_t was calculated on the assumption of a normal distribution.

TABLE II—PROBABLE VECTOR VARIATION OF WIND IN LEVELS UP TO 2,500 FT.

Interval	Probable vector variation		Root-mean-square variation(σ_t)
min.	ft./sec.	kt.	kt.
6	4·2	2·5	3·0
18	4·8	2·8	3·3
30	5·7	3·4	4·1

Photographic records of the motion of smoke puffs at heights of 6,000 or 7,000 ft. in East Anglia[4] gave values, given in Table III, of probable vector variation (50 per cent. value) in different intervals of time. The probable error of a single observation of wind by this method is about 1 ft./sec.; the root-mean-square variation σ_t was calculated on the assumption of a normal distribution.

TABLE III—PROBABLE VECTOR VARIATION OF WIND IN LEVEL OF 6,000 OR 7,000 FT.

Interval	Probable vector variation		Root-mean-square variation(σ_t)
min.	ft./sec.	kt.	kt
10	3·7	2·2	2·6
20	5·0	2·9	3·4
30	6·3	3·7	4·4
40	7·6	4·5	5·3
50	7·0	4·1	4·9
60	7·4	4·4	5·2

The vector variations of wind in intervals of a few hours and at various heights have been obtained by using the following sets of wind measurements.

(*a*) Larkhill, routine measurements (by radio direction-finding) at 6-hourly intervals from August 1944 to December 1944.

(*b*) Rye, Sussex, routine measurements (by GL. III equipment) at 2-hourly intervals from August 19, 1944 to September 21, 1944.

(*c*) 21 Army Group Area (northern France, Belgium, Holland), routine measurements (by GL. III equipment) at 2-, 4- or 6-hourly intervals at 10 observing sites occupied for varying durations during the period October 5, 1944 to January 21, 1945. The number of measurements at 2-hourly intervals in this set was small in comparison with the measurements at 4- and 6-hourly intervals.

These data are summarized in Table IV.

Observations by radar at Wittering at intervals of ½, 1, 2 and 4 hr. were made during the period June 30, 1945 to August 31, 1945 (44 days). The measurements were in these cases made over layers of 500 ft. at every 1,000-ft. interval up to 12,000 ft., and in Table V the values of the root-mean-square variation deduced for each interval of time and at every 2,000-ft. level are set out. There are also added in the last two lines the values of root-mean-square variations of winds for the same intervals of time but measured over deeper layers, 8,000 ft. and 4,000 ft. thick.

TABLE IV—PROBABLE VECTOR VARIATION OF WIND IN VARIOUS TIME INTERVALS

Letters in brackets indicate set of measurements used

	Interval							
	2 hr.		4 hr.		6 hr.		12 hr.	
	Probable vector variation	σ_2	Probable vector variation	σ_4	Probable vector variation	σ_6	Probable vector variation	σ_{12}
ft.	kt.	kt.	kt.	kt.	kt.	kt.	kt.	kt.
30,000	..	..	13(*c*)	16	19(*a*) 20(*c*)	23 24	31(*a*)	37
20,000	9(*c*)	11	11(*c*)	13	15(*c*)	18	..	..
18,000	..	..	..	..	12(*a*)	15	19(*a*)	23
10,000	6(*b*)(*c*)	7	8(*b*)(*c*)	9	9(*b*)(*c*)	11	..	..
8,000	..	..	..	..	9(*a*)	11	13(*a*)	16
5,000	5(*b*)(*c*)	6	7(*b*)(*c*)	9	8(*b*)(*c*)	10	..	..
2,000	5(*b*)(*c*)	6	7(*b*)(*c*)	9	8(*b*)(*c*)	10	..	..

TABLE V—STANDARD VECTOR VARIATIONS IN VARIOUS TIME INTERVALS

Wittering, June 30, 1945 — August 31, 1945

	Interval							
	½ hr.		1 hr.		2 hr.		4 hr.	
	Standard vector variation	No. of observations	Standard vector variation	No. of observations	Standard vector variation	No. of observations	Standard vector variation	No. of observations
ft.	kt		kt.		kt.		kt.	
12,000	5·3	247	6·0	223	7·1	158	8·7	55
10,000	5·7	283	6·3	253	7·7	183	9·9	61
8,000	5·0	297	5·6	264	6·7	195	9·5	64
6,000	4·7	318	5·6	283	6·5	210	9·1	68
4,000	4·8	343	5·5	302	6·3	224	8·6	75
2,000	5·0	319	5·8	278	6·9	209	8·9	73
2,000–10,000	1·9	268	2·5	241	3·8	173	6·2	59
4,000–8,000	2·3	297	3·3	263	4·3	195	7·2	65

TABLE VI—WIND VARIATIONS IN PERIODS OF 6 AND 12 HR. AT 30,000 AND 53,000 FT.

Height	Interval	Period 1944	Mean vector variation	Root-mean-square vector variation	Maximum speed	No. of values	Mean wind speed	No. of values
ft.	hr.		kt.	kt.	kt.		kt.	
53,000	6	Jan., Mar., May	8	11	37	207	29	281
		Aug. to Dec.	12	14	48	155	27	298
	12	Jan., Mar., May	10	13	45	210	29	281
		Aug. to Dec.	14	17	56	155	27	298
30,000	6	Jan., Mar., May	19	24	95	363	50	368
		Aug. to Dec.	24	29	110	584	59	598
	12	Jan., Mar., May	29	35	135	355	50	368
		Aug. to Dec.	35	41	120	584	59	598

For greater heights comparisons are available of the variations of wind in 6 and 12 hr. at 30,000 and 53,000 ft. These data are contained in Table VI.

An examination was made of the vector variation of wind with time at Larkhill for the period July 1946 to June 1947, by taking the values published in the *Daily Weather Report*, selecting one observation each day, usually the observation at midnight and calculating the departure at 6, 12, 18, 24 and 48 hr. later. These departures were calculated for heights of 700, 500, 300 and 200 mb. In addition the departures in 12 and 24 hr. were calculated for the height of 900 mb. From these measurements frequency tables were constructed, and the root-mean-square variations σ_t were then calculated. The results for each season and for the year are shown in Table VII.

By using equation (17) it was then possible to calculate the values of r_t provided the standard vector deviations were known. For this purpose the seasonal standard vector deviations for the period November 1939 to May 1945 were used, and the resulting correlation coefficients were entered in Table VII.

TABLE VII—RELATION BETWEEN THE WIND AT LARKHILL AT A GIVEN HOUR AND THE WIND AT 6, 12, 18, 24 AND 48 HR. LATER

July 1946 to June 1947

Height		6 hr.		12 hr.		18 hr.		24 hr.		48 hr.	
		σ_6	r_6	σ_{12}	r_{12}	σ_{18}	r_{18}	σ_{24}	r_{24}	σ_{48}	r_{48}
mb.	ft.	kt.		kt.		kt.		kt.		kt.	
				SPRING (March, April, May)							
200	38,700	19	0·85	29	0·66	31	0·61	34	0·53	39	0·39
300	30,100	26	0·84	36	0·70	47	0·48	52	0·36	59	0·18
500	18,300	18	0·85	25	0·72	32	0·54	33	0·50	44	0·00
700	9,900	13	0·88	18	0·76	23	0·62	24	0·58	31	0·30
900	3,200	..	..	17	0·70	..	..	23	0·45	..	..
				SUMMER (June, July, August)							
200	38,700	19	0·88	27	0·76	32	0·66	35	0·60	46	0·30
300	30,100	28	0·81	39	0·63	41	0·59	51	0·37	57	0·21
500	18,300	15	0·87	21	0·74	25	0·63	27	0·57	34	0·32
700	9,900	12	0·86	16	0·76	18	0·69	20	0·62	24	0·55
900	3,200	..	..	15	0·65	..	..	19	0·45	..	..
				AUTUMN (September, October, November)							
200	38,700	21	0·91	29	0·82	36	0·72	42	0·62	54	0·36
300	30,100	25	0·90	33	0·83	42	0·73	49	0·63	64	0·37
500	18,300	17	0·89	26	0·74	29	0·68	33	0·58	46	0·19
700	9,900	13	0·89	20	0·74	22	0·68	25	0·59	31	0·38
900	3,200	..		16	0·74	..	..	23	0·46	..	..
				WINTER (December, January, February)							
200	38,700	22	0·88	29	0·80	36	0·69	42	0·58	52	0·31
300	30,100	26	0·89	47	0·65	49	0·62	51	0·58	59	0·44
500	18,300	18	0·89	27	0·76	34	0·61	37	0·53	45	0·27
700	9,900	15	0·87	19	0·80	23	0·71	25	0·65	28	0·56
900	3,200	..	..	18	0·76	..	..	23	0·61	..	..
				YEAR							
200	38,700	20	0·89	29	0·76	34	0·67	38	0·59	48	0·35
300	30,100	26	0·87	39	0·71	45	0·61	51	0·51	60	0·32
500	18,300	17	0·88	25	0·73	30	0·62	33	0·54	42	0·26
700	9,900	13	0·88	18	0·77	21	0·68	24	0·58	28	0·43
900	3,200	..	..	17	0·72	..	..	22	0·52	..	..

In addition a calculation was made of the annual value of σ_{96} at 500 mb.; the value was 45 kt.

§ 6—DATA FOR OVERSEAS

In addition to the data for the British Isles stretch vector correlation coefficients have been calculated for certain places overseas, and these are set out in Table VIII. In these cases the correlation coefficients were calculated direct from the wind components by the use of the formula

$$r_t = \frac{\Sigma u_0 u_t + \Sigma v_0 v_t}{n\sigma^2} \qquad \ldots\ldots(18)$$

where $u_0 v_0$ are wind components at time 0 and u_t and v_t wind components at time t. For comparison the coefficients for Larkhill are repeated from Table VII.

TABLE VIII—VARIATION

		WINTER Vector mean wind		σ	No. of obs.	r_{12}	r_{24}	SPRING Vector mean wind		σ	No. of obs.	r_{12}	r_{24}
		°	kt.	kt.				°	kt.	kt.			
							200 mb.						
Larkhill	July 1946–June 1947	308	33	46·0	88	0·80	0·58	290	21	35·0	70	0·66	0·53
Habbaniya	1948	266	75	27·0	123	0·70	0·49	258	75	26·0	125	0·67	0·41
Bahrein	Nov. 1948–Oct. 1949	270	100	28·0	63	..	0·39	258	69	37·0	68	..	0·68
Nairobi	1949	110	20	19·0	52	..	0·34	90	4	18·0	44	..	0·59
							300 mb.						
Larkhill	July 1946–June 1947	309	31	56·0	89	0·65	0·58	288	·22	46·0	82	0·70	0·36
Gibraltar	1948	300	25	39·4	145	0·62	0·37	257	18	40·7	161	0·69	0·54
Malta	1948	297	34	46·6	172	0·67	0·47	294	26	38·4	118	0·70	0·57
Habbaniya	1948	264	63	41·7	174	0·75	0·54	256	60	33·5	181	0·71	0·57
Bahrein	1948	268	72	31·0	44	..	0·47	258	43	26·0	23	..	0·62
San Juan	1947	294	22	27·6	141	0·71	0·53	279	24	26·9	165	0·76	0·63
Nairobi	1948	87	11	14·6	73	..	0·28	93	10	14·2	49	..	0·45
							500 mb.						
Larkhill	July 1946–June 1947	305	20	39·0	90	0·76	0·53	283	15	33·0	90	0·72	0·50
Gibraltar	1948	295	15	32·6	175	0·71	0·47	246	14	29·2	181	0·79	0·60
Malta	1948	300	23	35·8	180	0·62	0·45	296	17	25·3	121	0·79	0·66
Habbaniya	1948	262	35	25·8	179	0·60	0·47	253	33	20·0	182	0·61	0·36
Bahrein	1948	274	39	22·0	61	..	0·69	264	33	20·0	48	..	0·74
San Juan	1947	278	14	18·6	164	0·71	0·53	225	1	14·1	172	0·73	0·59
Nairobi	1948	86	13	11·7	74	..	0·52	90	13	9·7	50	..	0·37

In Table IX there are given some values deduced from observations of radar winds made from August 1943 to July 1944 by the Naval Meteorological Service at Mombasa.

TABLE IX—VARIATION OF WIND WITH TIME AT MOMBASA

	Height	Vector mean wind		σ	σ_{24}	σ_{48}	r_{24}	r_{48}
	ft	°	kt.	kt.	kt.	kt.		
May–October	20,000	75	6	16	19	..	0·30	..
	10,000	186	9	18	19	22	0·46	0·25
November–April	20,000	93	12	13	16	..	0·14	..
	10,000	38	8	13	13	15	0·45	0·30

§ 7—ACCURACY OF STANDARD VECTOR DIFFERENCES

It has to be remembered that the values quoted in these notes are derived from the differences of observations which themselves are liable to observational errors. As is pointed out in § 5 an observation made at any moment of time will differ from that made at an interval of only a few minutes by a quantity which may be $3\frac{1}{2}$ kt. This is due to small-scale eddies. If the error of observation is called ε and the departure due to small-scale eddies η, and if the standard vector deviations of these quantities are σ_ε and σ_η then the root-mean-square variation due to fluctuations over an interval t would be $\sqrt{\{\sigma_t^2 - 2(\sigma_\varepsilon^2 + \sigma_\eta^2)\}}$.

It is well to remember that there is this degree of uncertainty in the measurements. By

OF WIND WITH TIME

SUMMER Vector mean wind		σ	No. of obs.	r_{12}	r_{24}	AUTUMN Vector mean wind		σ	No. of obs.	r_{12}	r_{24}	Mean values r_{12}	r_{24}
°	kt.	kt.				°	kt.	kt.					
						200 mb.							
280	27	39·0	90	0·76	0·60	295	26	48·0	90	0·82	0·62	0·76	0·58
259	26	19·0	81	0·80	0·68	259	53	24·0	48	0·65	0·74	0·71	0·58
98	12	15·0	73	..	0·58	259	30	20·0	86	..	0·67	..	0·58
90	11	18·0	48	..	0·43	250	7	15·0	47	..	0·50	..	0·47
						300 mb.							
278	27	46·0	92	0·63	0·37	293	27	57·0	91	0·83	0·63	0·70	0·49
267	23	27·7	158	0·69	0·49	292	12	34·4	157	0·74	0·61	0·69	0·50
276	33	22·6	90	..	0·41	294	17	29·1	133	0·67	0·46	0·68	0·48
265	26	23·9	136	0·85	0·71	258	44	30·7	122	0·77	0·69	0·77	0·61
13	3	19·0	76	..	0·49	258	28	31·0	86	..	0·79	..	0·59
266	7	16·0	160	0·71	0·55	320	10	24·5	139	0·72	0·53	0·73	0·56
143	5	15·9	66	..	0·32	126	6	13·0	64	..	0·59	..	0·41
						500 mb.							
273	19	29·0	92	0·74	0·57	281	19	36·0	91	0·74	0·58	0·74	0·55
256	21	19·8	169	0·69	0·39	280	10	23·3	170	0·76	0·61	0·74	0·52
291	23	19·3	92	..	0·45	291	13	21·8	137	0·69	0·59	0·66	0·54
263	14	15·9	140	0·72	0·61	260	22	17·6	129	0·63	0·68	0·64	0·53
18	4	13·0	79	..	0·42	268	13	20·0	89	..	0·71	..	0·64
96	7	9·9	174	0·47	0·31	24	4	13·5	153	0·68	0·43	0·65	0·47
27	7	11·0	66	..	0·40	84	12	11·1	66	..	0·63	..	0·48

the comparison of pilot balloons at rather low levels it was estimated[7] that $\sqrt{(\sigma_\varepsilon^2 + \sigma_\eta^2)}$ might be 4·5 kt., most of which was due to observational error.

Bannon[8] concludes that σ_ε, in determining wind in a layer about 1,200 ft. thick by GL. III radar, is not more than 2 kt. up to a height of 10,000 ft., seldom exceeds 3 kt. at 20,000 ft. and is never more than 5 kt. at greater heights. The errors of winds determined by radio are perhaps somewhat larger. Johnson[9] has confirmed that the errors in wind at 100 mb. are in accordance with Bannon's estimates.

§ 8—DEDUCTIONS

From Table VII it seems that the temporal stretch vector correlation coefficients vary little with height over the British Isles, though perhaps they are on the whole rather lower at 300 mb.

than at other heights. As a broad generalization it can be said that to a first approximation they vary linearly with the time interval as shown in Table X. It may, moreover, be surmized that with shorter and shorter time intervals the correlation coefficient continues to approach unity linearly.

TABLE X—STRETCH VECTOR CORRELATION COEFFICIENTS AT VARIOUS TIME INTERVALS

Time interval	6 hr.	12 hr.	18 hr.	24 hr.	48 hr.
Stretch vector correlation coefficient (r_t) ..	0·88	0·75	0·65	0·55	0·35

Possibly it may be more exact to expect the correlation coefficient to follow the law $r = e^{-at}$ where a is $6{\cdot}9 \times 10^{-6}$ when t is measured in seconds. In Fig. 4 curves have been plotted for σ_t/σ based on this law for r since $\sigma_t^2 = 2\sigma^2(1 - r_t)$; values of σ_t/σ from the various measurements over the British Isles have been plotted for comparison; two curves are shown, one extending to 24 hr., the other to 90 min. so as to show that the curve agrees fairly closely even at the very short intervals of time.

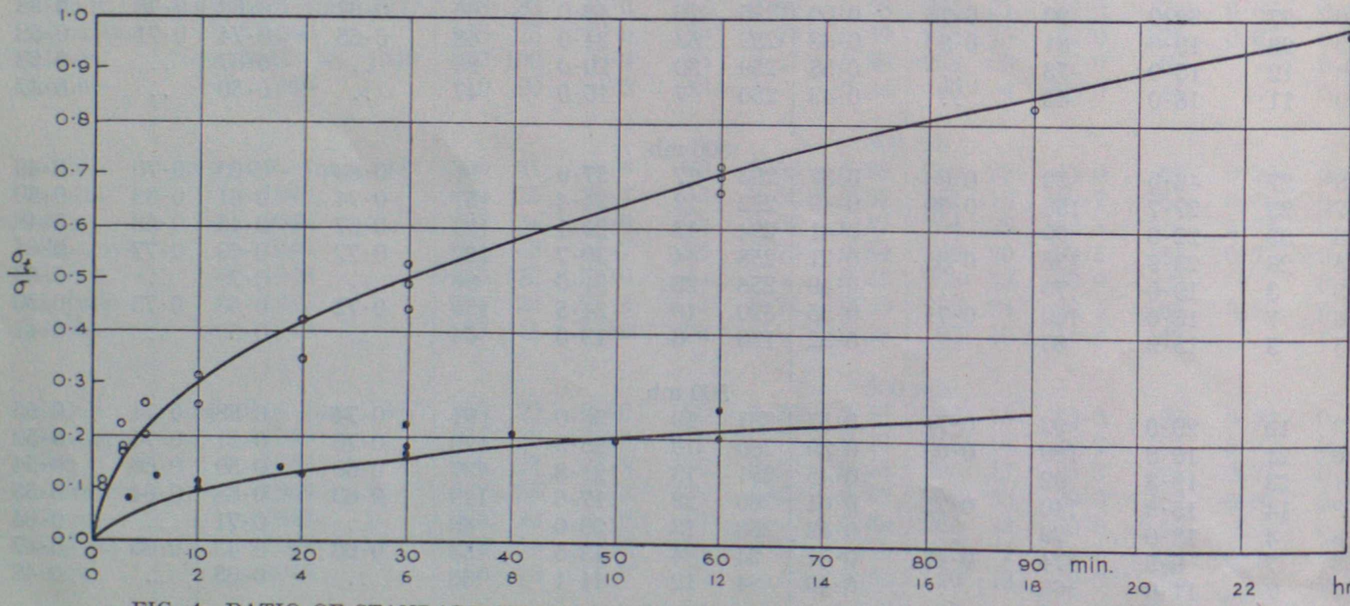

FIG. 4—RATIO OF STANDARD VECTOR VARIATION OF WIND IN VARIOUS TIME INTERVALS TO STANDARD VECTOR DEVIATION

In the lower curve the time scale has been exaggerated to show the detail at the short-time intervals

From Table VIII it may be concluded that at 500 mb. at overseas stations the coefficients of the correlation of wind with time in 12 and 24 hr. do not vary greatly from place to place, but at 300 mb. there is an indication that the coefficients are higher over the Mediterranean and the subtropics than over the British Isles. This is not unnatural since the tropopause is more frequently below the 300-mb. level in the latitude of the British Isles than further to the south, and when the tropopause is in the neighbourhood of this level the variations of wind with time are most likely to be great.

However, the essential feature of the table is that there does not seem to be any radical and fundamental difference in the statistical law of variation of wind with time between the latitudes of the British Isles and elsewhere, except that the coefficients at Nairobi and Mombasa are smaller than in extratropical latitudes.

The magnitudes of the 12- and 24-hr. standard vector differences of wind are dependent on the standard deviations. These standard deviations have been mapped in *Geophysical Memoirs* No. 85, and hence those maps show at once where winds are most liable to considerable changes in 12 or 24 hr. The maps of standard vector deviation were largely extrapolated, but the figures in Table VIII are in substantial agreement with those maps except that in the tropics the values on the maps, particularly at 200 mb., are decidedly too big.

From the apparent regularity of the correlation coefficients it is possible by the use of equation (17) to give an approximate value of the standard vector differences in various time intervals for various places in the world. It is found, moreover, that the standard error of forecasts is very nearly proportional to the standard vector difference of wind appropriate to the interval between the time of the forecast and the time of the maps on which it is based. So these statistics of wind changes provide a basis for the assessment of probable errors in wind forecasts, even when no comparisons have been made between the forecast and the actual winds.

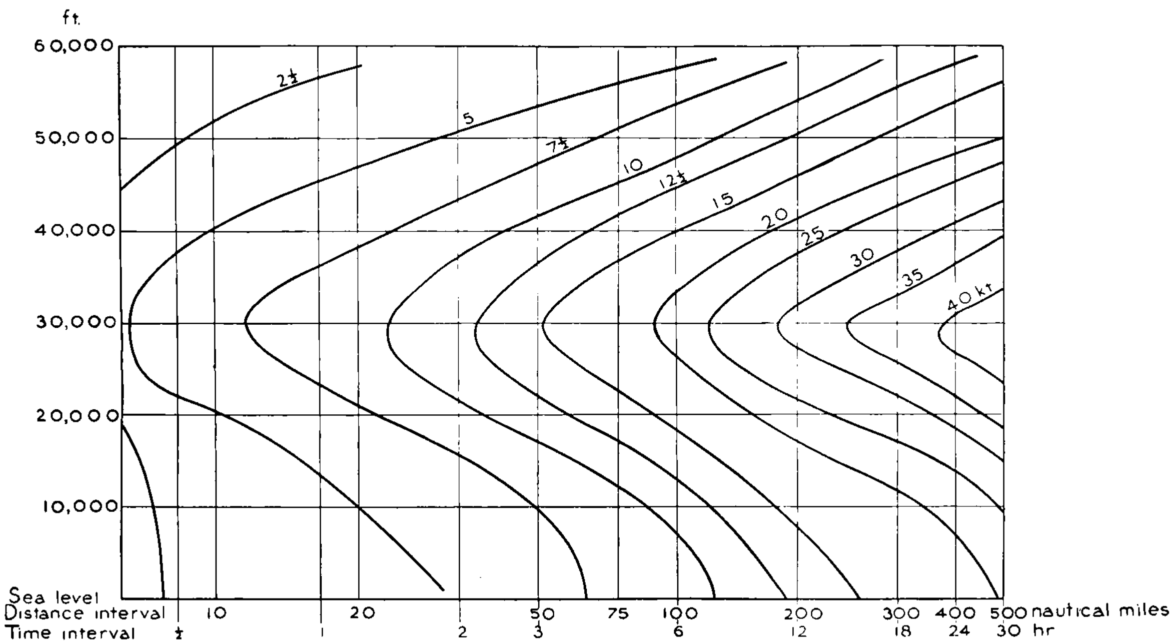

FIG. 5—PROBABLE VECTOR VARIATION OF WIND WITH TIME AND PLACE OVER ENGLAND

In Fig. 5 there are given the probable vector variations of wind over England for time intervals up to 30 hr. and for heights from surface to 60,000 ft. In compiling this diagram the data given in § 5 have been used, and it has been assumed that the vector correlation coefficients approached unity by a linear law at short time intervals. No very high degree of accuracy can be claimed for this diagram, but it certainly shows the general trend of variation of the 50-per-cent. errors which will arise by the assumption that a wind measured at one time is a forecast of a wind at a later time.

If the vector mean wind of the season is assumed to be the forecast of the wind at any time the standard error will be the standard deviation σ (or the probable error $0{\cdot}83\ \sigma$). It is then seen from equation (17) that if r_t is less than $0{\cdot}5$ the error caused by using a wind observation t hr.

old as a forecast is greater than the error made by using the normal wind. The extreme value of the standard variation of wind from one time to another very much later time is $\sqrt{2}\sigma$. Now it is seen from Table X that the value of $r_t = 0 \cdot 5$ is at 27 hr., so in round figures it may be said that any wind observation made 24 hr. before an operation is worse than useless as a forecast unless it has been modified by a meteorologist.

For other parts of the world than England (but excluding the tropics) a corresponding diagram to Fig. 5 could be constructed by using the correlation coefficients in Table X and the appropriate standard vector deviations from *Geophysical Memoirs* No. 85. These values could then be substituted in equation (17) to give the standard vector differences in any given time interval. It would be found that the probable vector variation of wind with time is in general considerably less than over England, for instance over the western half of North America the standard vector differences are only about two-thirds those shown in Fig. 5. Over the Mediterranean the height at which the probable vector variation reaches a maximum almost certainly rises, probably to about 40,000 ft. in winter.

§ 9—DIURNAL VARIATION OF WIND CHANGES

During the calculation of the variation of wind with time, values of the root-mean-square variation were calculated for the height of 500 mb. at Larkhill for the period from noon to midnight as well as for midnight to noon. These showed, surprisingly, that the afternoon values were consistently greater than the morning values. The variation was calculated in the first case for July 1946 to June 1947, and then, as a confirmation, for December 1945 to February 1946. The results are set out in Table XI.

TABLE XI—VARIATION OF WIND IN 12 HR. AT 500 MB. AT LARKHILL

	Root-mean-square variation (σ_{12})		Standard vector deviation (σ)		Correlation coefficient (r_{12})	
	morning	afternoon	0000	1200	morning	afternoon
	kt.	kt.	kt.	kt.		
Dec. 1945–Feb. 1946 ..	29	32	43	43	0·77	0·72
July–Aug. 1946, and June 1947..	20	23	26	31	0·76	0·68
Sept.–Nov. 1946	26	29	37	34	0·73	0·67
Dec. 1946–Feb. 1947	26	29	37	34	0·73	0·67
Mar.–May 1947	26	27	36	32	0·71	0·68

In this table the correlation coefficient is obtained by the use of equation (17).

TABLE XII—VARIATION OF PRESSURE GRADIENT (GEOSTROPHIC WIND) IN 12 HR. AT 500 MB. AT LAT. 50° N. LONG. 15° E.

	Root-mean-square variation (σ_{12})		Standard vector deviation (σ)		Correlation coefficient (r_{12})	
	day	night	0600	1800	day	night
1946	kt.	kt.	kt.	kt.		
Jan., Feb., Dec.	25	26	36	35	0·75	0·73
Mar., Apr., May	19	20	25	23	0·69	0·65
June, July, Aug.	15	17	22	21	0·76	0·69
Sept., Oct., Nov.	19	23	28	28	0·77	0·66

Subsequently, some data of the pressure gradients (geostrophic wind) at 500 mb. became available for 0600 and 1800 G.M.T. for the year 1946. The values of the root mean squares of differences during 12 hr. of day and night were calculated as is shown in Table XII. A similar type of feature appears since the variation between 0600 and 1800 is less in each season than the variation between 1800 and 0600.

The physical explanation of such a diurnal variation is not obvious. It could have been produced by the observation at one of the hours being consistently late or early. But this is not so in the case of the Larkhill observations used in Table XI, and the charts on which Table XII is based are equally spaced in time.

That the same type of diurnal variation appears both in pressure and wind makes it extremely improbable that it is accidental or due to a consistent instrumental error.

Such a diurnal variation is not the effect of frequent increases in pressure gradients and wind during the early part of the night followed by decreases in the subsequent 12 hr., for that would not produce the effect. Rather it is that when there is a change of wind during the early part of the night the change in the subsequent 12 hr. is comparatively small so that a change in the early part of the night is not recovered during the rest of the 24 hr.

PART III—VARIATION OF WIND WITH DISTANCE

§ 10—DATA FOR VARIOUS REGIONS

In a Meteorological Office Memorandum[6] the variation of wind with distance is discussed with the aid of pilot-balloon observations ; for a height of 3,000 ft. it is deduced that the 50-per-cent. zones at various distances are as shown in Table XIII. From the 50-per-cent. values the root-mean-square difference has been calculated on the assumption that the distribution is normal. In the last column is given the vector correlation coefficient of stretch (r_x), calculated on the assumption that the standard vector deviation at 3,000 ft. is 23 kt. which is the mean value over southern England for a height of 900 mb.

TABLE XIII—VARIATIONS OF WIND WITH DISTANCE AT 3,000 FT. BETWEEN SHOEBURYNESS AND OTHER PLACES

	Distance from Shoeburyness	Value of 50-per-cent. zone		Standard vector variation (σ_x)	Correlation coefficient (r_x)
	miles	m.p.h.	kt.	kt.	
South Farnborough	70	6·5	5·5	6·5	0·96
Cranwell	112	8·5	7·5	9·0	0·92
Cologne	300	15·5	13·5	16·0	0·74
Leuchars	374	16·5	14·5	17·0	0·72
Hamburg	450	18·5	16·0	19·0	0·66
Berlin	570	22·0	19·0	23·0	0·50

In Table XIV there is given a collection of stretch vector correlation coefficients for various places and at various heights, which have been calculated at different times from the original observations as given in the *Daily upper air bulletin*[10] and in the manuscript records of the Meteorological Office, London. In addition the manuscript observations from Whelus Field, Tripolitania were supplied by the United States Air Force in order to obtain a comparison over a fairly short distance in the Mediterranean region.

TABLE XIV—STRETCH VECTOR CORRELATION COEFFICIENTS FOR VARIOUS DISTANCES

Period	Height	Track	No. of observations	Distance	Stretch vector correlation coefficient
	mb.			n. miles	
July–Oct. 1946, and Jan.–Apr. 1947	200	Larkhill–Liverpool	99	140	+0·86
		Larkhill–Lerwick	129	520	+0·37
	300	Larkhill–Liverpool	136	140	+0·78
		Larkhill–Lerwick	175	520	+0·35
	500	Larkhill–Liverpool	183	140	+0·86
		Larkhill–Lerwick	209	520	+0·37
Jan.–Apr. and Nov.–Dec. 1947	850	British Isles	759	c. 130	+0·88
May–Oct. 1947			847		+0·86
June–July 1948	300	Liverpool–Aldergrove	122	150	+0·80
		Aldergrove—station J	122	500	+0·17
		Liverpool—station J	122	600	+0·16
Dec. 1948–Jan. 1949	300	Liverpool–Aldergrove	124	150	+0·78
		Aldergrove—station J	124	500	+0·33
		Liverpool—station J	124	600	+0·13
Summer and Winter 1949	500	Miami–Tampa	20	160	+0·48
Winter 1948–49			56		+0·71
Summer 1949			74		+0·62
		Weighted mean	150		+0·63
Winter 1948–49	200	New Orleans–Brownsville	22	460	+0·53
Summer 1949			106		+0·39
		Weighted mean	128		+0·41
Winter 1948–49	200	Miami–Havana	68	220	+0·68
Summer 1949		Brownsville–San Antonio	114	200	+0·63
		Weighted mean	182	210	+0·65
Summer 1948	300	Bahrein–Habbaniya	71	620	+0·26
Autumn 1948			81		+0·22
Dec. 1948			27		+0·10
		Weighted mean	189		+0·23
June–Dec. 1949	200	Malta–Whelus Field	67	190	+0·60
	300		133		+0·57
		Weighted mean	200		+0·57
Dec. 1950–Feb. 1951	200	Larkhill–Malta	182	1,120	−0·22
Mar.–May 1951			184		−0·30
June–Aug. 1951			184		−0·20
Sept.–Nov. 1951			149		−0·29
1951	200	Larkhill–Rome	140	750	−0·15

Simultaneous pilot-balloon observations were analysed for a height of 6,000 ft. at Kuala Lumpur and Bayan Lepas, Penang (distance 175 nautical miles) in Malaya for the year 1938, the results are shown in Table XV.

TABLE XV—VARIATION OF WIND WITH DISTANCE AT 6,000 FT. BETWEEN KUALA LUMPUR AND BAYAN LEPAS

	Vector mean wind at Kuala Lumpur		Standard vector deviation (σ)	Standard vector difference (σ_x)	Stretch correlation coefficient (r_x)
	°	kt.	kt.	kt.	
			OCTOBER–MARCH		
0000	58	5·7	19	18	0·55
0500	368	4·5	20	19	0·55
1000	27	7·5	22	19	0·63
			APRIL–SEPTEMBER		
0000	246	10·2	20	15	0·72
0500	254	7·6	18	15	0·65
1000	262	8·4	21	17	0·67

Summary of data.—These data may be generalized as in Table XVI.

TABLE XVI—CORRELATION OF WIND WITH DISTANCE

	British Isles and western Europe					Mediterranean and Middle East		Southern United States and West Indies			Malaya
Distance (n. miles) ..	*c.* 130	140	500–600	750	1,120	190	620	160	210	460	175
Mean stretch vector correlation coefficient ..	+0·87	+0·81	+0·27	−0·15	−0·25	+0·57	+0·23	+0·63	+0·65	+0·41	+0·62

In the case of the correlation between Larkhill and Rome and between Larkhill and Malta there is a decided change of latitude. The points in Table XVI have been plotted in Fig. 6 and curves have been drawn representing temperate latitudes (curves B and C) and subtropical latitudes (including the Mediterranean region) (curve A).

There is a difference between the three curves which may be attributed to the magnitudes of those eddies that have the most influence in the different regions. Clearly in regions where the eddies (depressions) are of the order of 500 or 1,000 miles the correlation coefficients at 200 or 300 miles are higher than in those regions where the eddies are 100 or 200 miles, for in the latter case the winds at points 200 or 300 miles apart are unlikely to be affected in the same sense.

In Fig. 6 a curve (curve D) has also been added which represents a generalization of that given by Sawyer[3]. This curve lies considerably above both those curves now derived. The reason for this may be that Sawyer's curves were all derived from geostrophic winds, and in all measurements of geostrophic winds a considerable element of smoothing is introduced, in fact the geostrophic wind represents an average of all winds which occur during perhaps 6 hr. in time and 200 miles in space. Therefore, it is natural that geostrophic winds at distances of the order of 500 miles should be more highly correlated than winds measured during 1 or 2 min. at a spot defined in dimensions of a mile or so. It may be also that the wind components along a route are more closely correlated than the cross-route components are, and in consequence than the vectors.

At distances beyond 600 miles the information is very uncertain. It is to be expected that at 2,000 miles the correlation coefficient becomes zero, and it is probable that even at 1,000 miles it will be very small in tropical latitudes. Curve A of Fig. 6 has been drawn on this basis, curve B has been drawn to represent the first 5 correlation coefficients in Table XVI and may

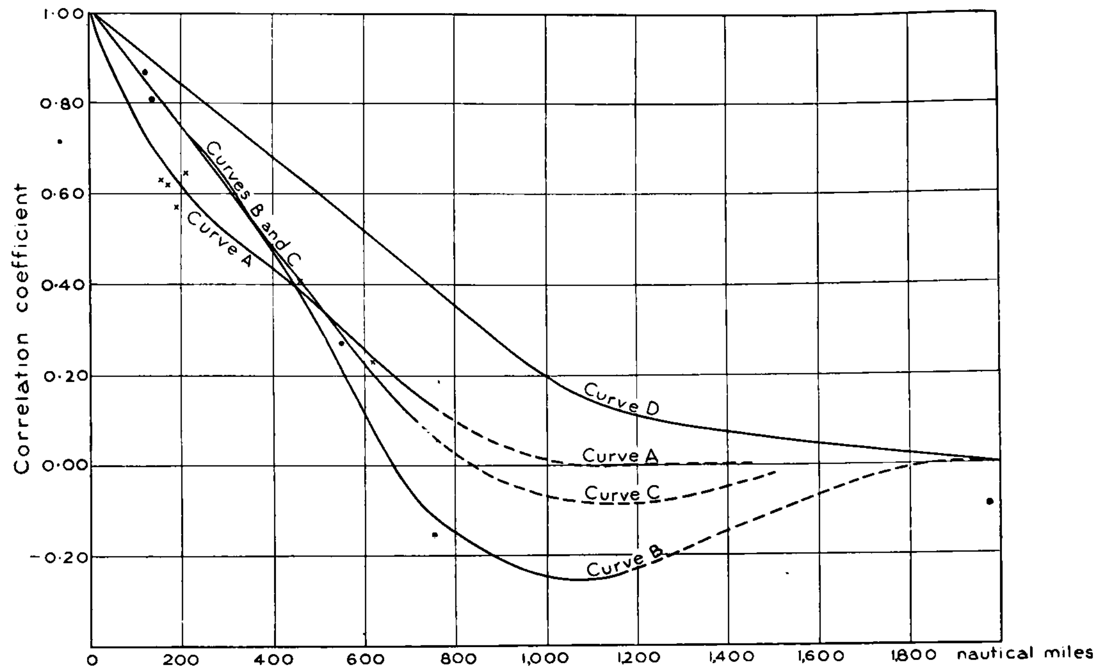

FIG. 6—VARIATION OF WIND WITH DISTANCE

Curve A applies in tropics and subtropics
Curve B applies in temperate latitudes
Curve C applies in temperate latitudes for an east-west route
Curve D represents a generalization of relation of geostrophic winds shown by Sawyer[3]

represent a route in temperate latitudes, but curve C has been modified from this to be more nearly akin to what is believed to be appropriate to an east–west route.

The data in Tables XIV, XV and XVI are in terms of stretch vector correlation coefficients. If, however, components are taken along and across the line joining the two places correlated there is some evidence that the correlation coefficients of the components at right angles to this line is less (algebraically) than those for components along this line. This again can readily be understood by considering the form of the eddies producing these wind components.

It should also be remembered that there is ground for believing the temporal as well as the spatial correlation coefficients are smaller in the subtropics than in extratropical latitudes. On both these accounts an observation of wind in the tropics is less representative of the wind field both in space and time than it would be in extratropical latitudes, and forecasts made on scattered observations are likely to be more widely in error in the tropics than over Europe.

§ II—DEDUCTIONS

In addition to the deduction made above that the correlation of wind with distance is less in the tropics than in temperate latitudes, it would also seem that with decreasing distance between wind stations the correlation coefficients had a linear approach to unity. Moreover, a table can be drawn up for interpolating the correlation coefficients for short distances (or for brief time intervals since those coefficients also appear to approach linearly). Such a table is given in Table XVII, and in it a column is added to give the factor by which σ has to be multiplied to produce σ_x (or σ_t). The last column shows how rapidly the standard vector difference in wind increases as the distance between wind observing points increases. In the tropics the variation of σ_x with distance is even greater so that the ratio σ_x/σ becomes as high as 0·55 for a distance of 50 miles and 0·70 for a distance of 100 miles.

TABLE XVII—ESTIMATED VARIATION OF WIND WITH DISTANCE AND TIME OVER NORTH-WEST EUROPE

Distance		Time	r_x	σ_x/σ
miles	n. miles	hr.		
0	0	0	1·00	..
25	22	$1\frac{1}{2}$	0·97	0·24
50	43	3	0·94	0·35
75	65	$4\frac{1}{2}$	0·91	0·42
100	87	6	0·88	0·49
150	130	9	0·81	0·62
200	175	12	0·75	0·71

PART IV—REPRESENTATIVE CHARACTER OF UPPER WIND OBSERVATIONS

In giving information for aviation purposes it is usual for the data to be required in the form of a wind meaned over a route. This mean wind may be derived from a contour chart (and then is subject to all the errors in drawing and measuring the contours), or it may be determined by taking a mean of wind observations made at a number of stations spaced out along the route. In this part it is proposed to examine the accuracy of the latter representation.

§ 12—REPRESENTATION OF ROUTE WINDS

Let v_0 be the vector departure from normal of the wind at one end of the air route of length l, let v be the vector departure at any point on the route, and in particular let v_d be the departure from normal of the wind at an observing station on the route at distance d from the starting point. If there are a number of observing stations $(N+1)$ spaced equidistantly along the route, $N = l/d$ and the mean departure of wind as derived from these stations is $\sum\limits_0^N \mathsf{v}_d/(N+1)$ which will be called $\bar{\mathsf{v}}'$. Moreover, the true mean departure of wind along the route is $(1/l)\int_0^l \mathsf{v}dx$ which will be called $\bar{\mathsf{v}}$.

If r is the stretch vector correlation coefficient between the mean wind derived from the observing stations and the true mean wind then by definition

$$r = \frac{\sum\limits_1^n |\bar{\mathsf{v}}|\,|\bar{\mathsf{v}}'|\cos\theta}{\sqrt{\left\{\sum\limits_1^n \overline{\mathsf{v}^2}\sum\limits_1^n \overline{\mathsf{v}'^2}\right\}}} \qquad \ldots\ldots(19)$$

where n is a large number of individual occasions.

That is $$r = \frac{1}{n-1}\frac{1}{\bar{\sigma}\,\bar{\sigma}'}\sum\limits_1^n |\bar{\mathsf{v}}|\,|\bar{\mathsf{v}}'|\cos\theta$$

where $$\overline{\sigma^2} \equiv \frac{1}{n-1}\sum\limits_1^n \overline{\mathsf{v}^2}$$

and $$\overline{\sigma'^2} \equiv \frac{1}{n-1}\sum\limits_1^n \overline{\mathsf{v}'^2}.$$

From equation (19) values can be derived of the degree of inaccuracy engendered by having observing stations at finite distances apart.

From equation (16)

$$\bar{\sigma}^2 = \frac{2\sigma^2}{l^2}\left\{\int_0^l \int_0^s r_x dx ds\right\}, \qquad \ldots\ldots(20)$$

where r_x is the correlation coefficient between winds distant x apart and σ is the mean of the normal standard vector deviations at points on the route (or very nearly the average of the standard vector deviations at all points along the route).

The values of $\int_0^s r_x dx$ are shown in Fig. 7 for temperate and tropical latitudes and those of $\bar{\sigma}/\sigma$ in Fig. 8.

Now
$$\overline{\sigma'^2} = \frac{1}{n-1}\sum_1^n \frac{1}{(N+1)^2}\left(\sum_0^N \mathbf{v}_d\right)^2 \qquad \ldots\ldots(21)$$

$$= \frac{\sigma^2}{N+1} + \frac{2N\sigma^2}{(N+1)^2} r_d + \frac{2(N-1)}{(N+1)^2}\sigma^2 r_{2d} + \ldots + \frac{2}{(N+1)^2}\sigma^2 r_{Nd} \qquad \ldots\ldots(22)$$

where r_d is the stretch vector correlation coefficient between winds at two stations distant d apart, r_{2d} between two stations distant $2d$ apart and so on.

Moreover

$$\frac{1}{n-1}\sum_1^n \frac{|\mathbf{v}_d|\,|\bar{\mathbf{v}}|\cos\theta}{\sigma^2} = \frac{1}{n-1}\frac{1}{l}\int_0^l \sum_1^n \frac{|\mathbf{v}|\,|\mathbf{v}_d|\cos\theta}{\sigma^2}\,dx \qquad \ldots\ldots(23)$$

or
$$= \frac{1}{l}\int_0^{l-d} r_x dx + \frac{1}{l}\int_{l-d}^{l} r_x dx$$

or
$$= \frac{1}{l}\int_0^{l-d} r_x dx + \frac{1}{l}\int_0^{d} r_x dx. \qquad \ldots\ldots(24)$$

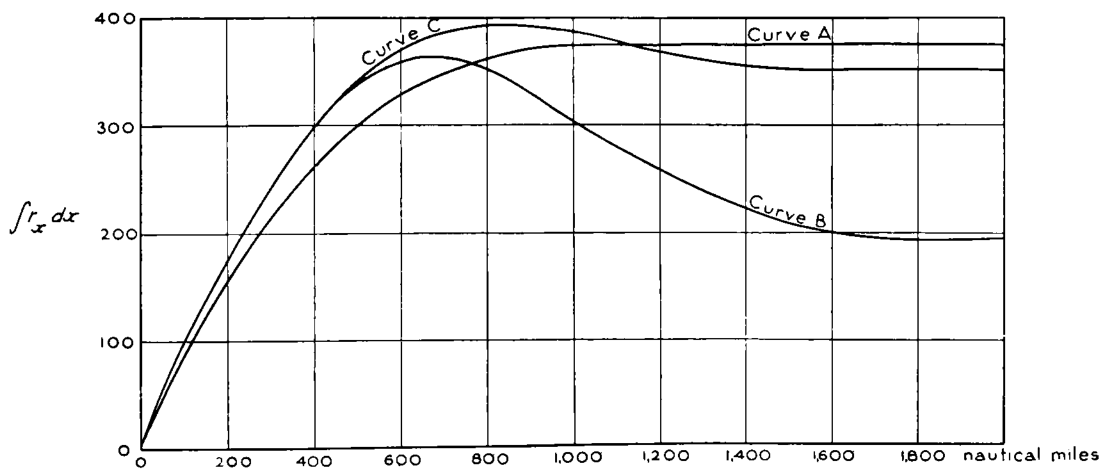

FIG. 7—VARIATION OF $\int_0^s r_x dx$ WITH DISTANCE
Curve A applies in tropics and subtropics
Curve B applies in temperate latitudes
Curve C applies in temperate latitudes for an east-west route

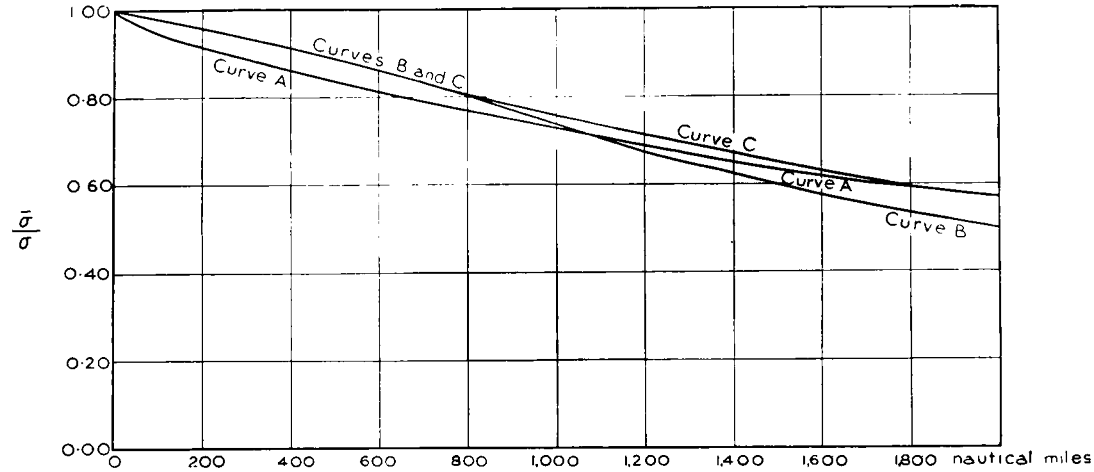

FIG. 8—VARIATION OF $\bar{\sigma}/\sigma$ WITH DISTANCE

Curve A applies in tropics and subtropics
Curve B applies in temperate latitudes
Curve C applies in temperate latitudes for an east-west route

So that

$$\frac{1}{n-1}\sum_1^n |\bar{\mathbf{v}}|\,|\bar{\mathbf{v}}'|\cos\theta \equiv \frac{1}{n-1}\sum_1^n\left\{\frac{1}{N+1}\sum_0^N |\mathbf{v}_d|\,|\bar{\mathbf{v}}|\cos\theta\right\}$$

$$= \frac{2}{l}\frac{\sigma^2}{N+1}\sum_0^N\int_0^{ml/N} r_x dx. \qquad \ldots\ldots(25)$$

Hence it is possible to write

$$r = \frac{2}{l}\frac{\sigma}{\bar{\sigma}}\frac{\sum_0^N\int_0^{ml/N} r_x dx}{\sqrt{(N+1+2Nr_d+2(N-1)r_{2d}+\ .\ .\ .\ +2r_{Nd})}}\,. \qquad \ldots\ldots(26)$$

Moreover, it can be seen in a similar manner that the correlation coefficient of the wind at one end of a route length l with the true mean wind over the route is given by

$$r = \frac{1}{l}\frac{\sigma}{\bar{\sigma}}\int_0^l r_x dx,$$

TABLE XVIII—VARIATION OF CORRELATION COEFFICIENT OF WIND WITH DISTANCE AND CERTAIN DERIVED VALUES

Distance	Curve A			Curve B			Curve C		
	r_x	$\int_0^l r_x dx$	$\frac{\bar{\sigma}}{\sigma}$	r_x	$\int_0^l r_x dx$	$\frac{\bar{\sigma}}{\sigma}$	r_x	$\int_0^l r_x dx$	$\frac{\bar{\sigma}}{\sigma}$
n. miles									
0	+1·000	0	1·000	+1·000	0	1·000	+1·000	0	1·000
250	+0·570	187	0·900	+0·680	210	0·944	+0·680	210	0·944
500	+0·350	301	0·837	+0·310	340	0·886	+0·360	340	0·886
750	+0·135	359	0·781	−0·010	357	0·815	+0·065	389	0·823
1,000	+0·010	377	0·727	−0·250	311	0·737	−0·070	387	0·759
1,250	0·000	377	0·660	−0·215	252	0·660	−0·075	368	0·700
1,500	0·000	377	0·635	−0·110	211	0·595	−0·030	355	0·648
1,750	0·000	377	0·599	−0·020	195	0·541	0·000	353	0·606
2,000	0·000	377	0·566	0·000	194	0·500	0·000	353	0·569

and the correlation coefficient of the wind at the mid point of a route length l with the true mean wind over the route by

$$r = \frac{2}{l}\frac{\sigma}{\bar{\sigma}}\int_0^{l/2} r_x dx. \qquad \ldots\ldots(27)$$

In Table XVIII there are set out the values of r_x, $\int_0^l r_x dx$ and $\bar{\sigma}/\sigma$ for the three curves A, B and C shown in Fig. 6, i.e. for tropical and temperate latitudes.

From these data it is possible to obtain correlation coefficients between the observed wind at a point on the route and the wind over the route as a whole, or by using partial correlation coefficients the total correlation coefficient between the observed winds at a selection of stations suitably distributed along the route and the wind over the route as a whole.

§ 13—ACCURACY OF ROUTE WINDS

In this way it is possible to form regression equations of the route wind on the winds observed at points on the route, and to calculate the errors that would arise from the use of these equations. These errors are shown in Table XIX where they are all expressed as fractions of the average standard vector deviation over the route (derived from *Geophysical Memoirs* No. 85[1]).

TABLE XIX—STANDARD VECTOR ERRORS TO BE EXPECTED IF THE WIND OVER A ROUTE IS CALCULATED FROM A REGRESSION EQUATION RELATING THE ROUTE WIND TO THE OBSERVED WINDS

Length of route	No observations at all	One station at one end	One station at midpoint	One station at each end	One station at one end and one in middle	Three stations
n. miles						
			TROPICAL LATITUDES "CURVE A"			
250	0·90σ	0·50σ	..	0·31σ	..	..
500	0·84σ	0·58σ	0·37σ	0·40σ	0·31σ	0·23σ
1,000	0·73σ	0·62σ	0·41σ	0·50σ	0·37σ	0·31σ
2,000	0·57σ	0·53σ	0·42σ	0·50σ	0·39σ	0·35σ
			TEMPERATE LATITUDES "CURVE B"			
250	0·94σ	0·43σ	..	0·23σ	..	..
500	0·89σ	0·57σ	0·28σ	0·28σ	0·24σ	0·14σ
1,000	0·74σ	0·67σ	0·28σ	0·53σ	0·26σ	0·21σ
2,000	0·50σ	0·49σ	0·39σ	0·48σ	0·35σ	0·29σ
			TEMPERATE LATITUDES "CURVE C"			
250	0·94σ	0·43σ	..	0·23σ	..	..
500	0·89σ	0·57σ	0·28σ	0·33σ	0·24σ	0·16σ
1,000	0·76σ	0·65σ	0·34σ	0·50σ	0·30σ	0·23σ
2,000	0·57σ	0·54σ	0·42σ	0·51σ	0·36σ	0·30σ

§ 14—CONCLUSIONS

The lessons regarding the representation of route winds at the time of observation to be drawn from this table are :—

(i) Unless winds are available from accurate contour charts, the route wind for high-altitude flights is best obtained from actual winds by the use of regression equations, and in the tropics this is always so, since geostrophic winds have no real significance.

(ii) As would be expected, for short routes (up to say 500 miles long) a single station at one end gives a material improvement on using the normal wind, but for a long route (of more than 1,200 miles) it gives little help.

(iii) In general one station in the middle of a route is more valuable than two stations one at each end, and is much more valuable than a single station at one end.

(iv) On a route of 2,000 nautical miles, such as London to Gander on which the winds at 40,000 ft. have a standard vector deviation about 46 kt. in winter, flying on the normal wind a standard vector error of 26 kt. would be expected; if one took one terminal observation into account the standard vector error would be 25 kt. If there were a single observation from a weather ship but no observation at either end it would be reduced to 19 kt., and if there were a weather ship and terminal observations at both ends it would be reduced to 14 kt.

(v) On a route of 790 nautical miles, such as London to Rome on which the winds at 40,000 ft. have a standard vector deviation of about 43 kt., if one took, say, Larkhill winds only into account one would expect a standard error of about 27 kt., if one took Payerne only into account one would expect a standard error of about 13 kt., and if Larkhill, Rome and Payerne were available the expected standard error would be about 10 kt.

(vi) In all cases the calculation of wind by the use of a regression equation is superior to the use of a mean wind derived from a number of individual observations. This is particularly so when observations are widely separated. The use of a wind derived from one observation only on the route is worse than useless over a 1,000-mile route unless some modification is made either by a regression equation or with the help of a synoptic map. The same applies to a 2,000-mile route even if there are observations at both ends.

PART V—VARIATION OF WIND WITH TIME AND DISTANCE COMBINED

§ 15—VARIATION OF WIND AT A POINT

With travelling systems of perturbations in a general westerly current it is to be expected that the pattern of the wind field should be repeated at a later time at a position further east. Thus, it may be expected that some merit might be found in correlating the wind at one place with that at another to westward at a previous time. For example, if the winds at 300 mb. at Aldergrove and Liverpool in June and July 1948 are considered and the wind at Liverpool at one hour denoted by suffix $_{A12}$, that at Liverpool 12 hr. earlier (zero hour) by suffix $_A$ and that at Aldergrove at this earlier hour by suffix $_B$ the calculated values of the stretch vector correlation coefficients r_{A12A}, r_{AB} and r_{A12B} are $0{\cdot}63$, $0{\cdot}80$ and $0{\cdot}72$ respectively. The partial correlation coefficient $r_{A12A,B}$ is found to be $0{\cdot}09$ and $r_{A12B,A}$ to be $0{\cdot}47$. From these it is seen that if $\varepsilon_{A12,AB}$ is the standard error in an estimate of the wind at Liverpool 12 hr. ahead made statistically on the basis of observations at Liverpool and Aldergrove, $\varepsilon_{A12,AB}$ is $0{\cdot}68\sigma$, where σ is the standard vector deviation of wind at Liverpool; this contrasts with a standard error of $0{\cdot}77\sigma$ if account is only taken of the wind at Liverpool, and there is a gain of over 10 per cent.

§ 16—VARIATION OF WIND OVER A ROUTE

In the navigation of aircraft the winds that are required are not those at a point but over the route traversed by the aircraft. Owing to the correlation of wind with distance the standard deviation of the route winds is less than that of winds at a point, the longer the route the smaller the standard deviation as is shown in Part IV. But in addition to this decrease in the standard deviation there appears to be an increase in the temporal correlation coefficient of route winds in comparison with point winds. This is brought out by some calculations made from the German contour charts for 225 mb.*, and from those at 200 mb. constructed in the British Meterological Office.

The German observations in 1942–43 were most complete over north Germany, and a route of 1,000 nautical miles from England eastwards was used as well as one 850 miles long from London to Rome and Sicily; the latter was investigated as it represented the first stage of the air route to Egypt which is now being used by the Comets of the British Overseas Airways Corporation. The mean pressure gradients along and across the routes were measured once each day, and the resultant geostrophic winds were assumed to be the actual winds which would affect an aircraft.

In the case of the British 200-mb. contour maps two routes were also taken, one from Aldergrove to Berlin which covered the area with the most reliable observations, the other from London Airport to Tripoli. For this latter route two measurements of route winds were made each day, one at 0300 G.M.T., the other at 1500 G.M.T., but only the mean headwind component over the route was measured.

Finally, a complete year (1951) was calculated for the route from London to Rome with two contour charts each day at 200 mb. The mean headwind and the mean beamwind were calculated for each chart and these were correlated for intervals of 12 and 24 hr.

The results are shown in Table XX.

TABLE XX—CORRELATION OF ROUTE WINDS WITH TIME AT 225 AND 200 MB. DERIVED FROM PRESSURE GRADIENTS

Standard deviations derived from the German charts are vectors, those from the British charts are scalar.

Charts used	Period	Route	Height	Length of route	No. of observations	Standard deviation of route winds	Correlation coefficients 12 hr.	24 hr.	36 hr.
			mb.	n. miles		kt.			
German	Winter 1942–43	England eastwards	225	1,000	90	27	..	0·69	..
	Summer 1943		225	1,000	92	30	..	0·79	..
British	Winter 1950–51	Aldergrove–Berlin	200	700	180	19	..	0·66	0·52
German	Winter 1942–43	London–Sicily	225	850	90	31	..	0·72	..
	Summer 1943		225	850	92	29	..	0·60	..
British	Winter 1950–51	London–Tripoli	200	1,250	180	23	..	0·65	0·53
British	Mar.–May 1951	London–Rome	200	860	184	along route 27	0·85	0·70	..
						across route 24	0·77	0·64	..
	June–Aug. 1951		200	860	184	along route 20	0·69	0·50	..
						across route 25	0·85	0·67	..
	Sept.–Nov. 1951		200	860	182	along route 25	0·76	0·61	..
						across route 29	0·84	0·63	..
	Dec. 1950–Feb. 1951		200	860	180	along route 27	0·80	0·68	..
						across route 23	0·85	0·68	..

When it is remembered that the correlation coefficient of wind at a point with that 24 hr. later is about 0·55, it is clear that the fluctuations of wind over a route are very decidedly slower than those at a point; this is not unexpected when account is taken of the likelihood that small eddies will pass over a series of successive points more quickly than large eddies.

* Published in the German *Täglicher Wetterbericht* during the Second World War 1939–45.

It must, however, be noted that in all these cases the route wind is derived from the geostrophic component. There are considerable errors in the drawing of the contours and the measurement of the winds from them, and, too, the contours take no account of the ageostrophic components. These discrepancies are bound to affect the correlation coefficients to some extent.

PART VI—SOME APPLICATIONS

§ 17—CALCULATION OF PROBABLE WINDS AT A SUBSEQUENT TIME

It is clear that if a regression equation of the form

$$\mathsf{V}_{\mathrm{A}t} - \overline{\mathsf{V}}_{\mathrm{A}} = r_t(\mathsf{V}_{\mathrm{A}} - \overline{\mathsf{V}}_{\mathrm{A}}) \qquad \ldots\ldots(29)$$

is used in which $\mathsf{V}_{\mathrm{A}t}$ is the wind at any point t hr. after the wind V_{A} has been observed and r_t is the appropriate temporal vector correlation, an estimate of the probable wind at that time t, deduced from the persistence of wind, can be obtained. Moreover, if a regression equation is formed in which account is taken of the general translation of the wind pattern by some form of circulation index, and if, in addition to the wind observation at the point for which the future wind is required, a wind at some other point is also used, advection as well as persistence can be taken into account. Such a regression equation is of the form :—

$$\mathsf{V}_{\mathrm{A}t} - \overline{\mathsf{V}}_{\mathrm{A}} = a(\mathsf{V}_{\mathrm{A}} - \overline{\mathsf{V}}_{\mathrm{A}}) + b(W\mathsf{V}_{\mathrm{B}} - \overline{W\mathsf{V}}_{\mathrm{B}}) \qquad \ldots\ldots(30)$$

where V_{A} is the wind observed at the point, $\mathsf{V}_{\mathrm{A}t}$ is the wind at that same point but t hr. later, V_{B} the wind at some other fixed point at the same time as V_{A} and W is a suitable figure to denote the circulation index ; a and b are then constants depending on the correlation coefficients between $\mathsf{V}_{\mathrm{A}t}$ and $(W\mathsf{V}_{\mathrm{B}})$, V_{A} and $(W\mathsf{V}_{\mathrm{B}})$, $\mathsf{V}_{\mathrm{A}t}$ and V_{A} as well as the standard vector deviation of V_{A} and $(W\mathsf{V}_{\mathrm{B}})$. Further, if winds over a route are being considered, an equation of the type of equation (30) can be used to obtain a probable wind over a route at any time subsequent to the observation.

The degree of accuracy which would result from this procedure has been tested in comparison with the accuracy of actual forecasts made at the Central Forecasting Office, Dunstable. The comparison has been rendered rather difficult by the fact that the surface maps which are used in the forecasts are drawn for 0000, 0600, 1200 and 1800 G.M.T., whereas the upper air charts are drawn for 0300, 0900, 1500 and 2100 G.M.T. In consequence a forecast which appears to be made for an epoch 12 hr. after the observation is more nearly for 15 hr. after the observation, and similarly an apparent 24-hr. forecast is more nearly a 27-hr. forecast.

The first test made was for the 12 months June 1948 to May 1949 using winds at one point only ; two forecasts were made on each day, one at 0300 and the other at 1500 G.M.T.

The standard vector deviation of the winds at Larkhill was calculated and the 24-hr. correlation coefficients of the actual winds. In Table XXI there are set out for each season the normal vector mean wind and that reported from the observations (sample), the normal standard vector deviation and that of the sample, the standard vector difference of wind in 24 hr. and the standard vector error of the forecasts. In the last column is given the standard vector error which would be expected to have been made had an equation such as equation (29) been used with the value of r_t of $0\cdot55$ shown in Table X.

The standard vector error of the probable wind has been calculated from the formula

$$\varepsilon^2 = r_{24}\sigma_{24}^{\,2} + (1 - r)^2\sigma_n^{\,2} \qquad \ldots\ldots(31)$$

where ε is the standard vector error, r_{24} is the true 24-hr. vector correlation coefficient of wind (in this case taken to be $0\cdot55$), σ_n is the standard vector deviation for the sample examined, and σ_{24} is the standard vector difference in 24 hr. of the sample examined.

TABLE XXI—ERRORS IN 24-HR. FORECASTS, IN COMPARISON WITH STANDARD ERRORS OF PROBABLE WINDS FOR A SINGLE POINT

June 1948–May 1949

Season	Pressure level	Vector mean wind Normal		Vector mean wind Sample		Standard vector deviation Normal	Standard vector deviation Sample	Standard vector difference in 24 hr.	Standard vector error of forecasts	Standard vector error of probable wind
	mb.	°	kt.	°	kt.	kt.	kt.	kt.	kt.	kt.
Spring	300	275	21	294	24	48	51	48	36	42
	500	275	18	293	18	34	36	33	23	29
	700	270	11	290	12	26	27	23	19	21
Summer	300	270	28	278	34	45	46	46	33	40
	500	265	22	274	24	30	30	29	20	25
	700	265	15	268	17	22	21	19	14	17
Autumn	300	275	27	276	35	56	43	49	35	41
	500	265	22	264	27	36	28	33	22	27
	700	270	15	262	20	26	21	22	16	19
Winter	300	300	30	278	27	57	58	62	42	53
	500	300	22	277	21	39	41	40	28	35
	700	275	15	265	17	30	31	28	20	25

In order to see how far the introduction of an advection term as in equation (30) would reduce the standard error of the probable winds, the months June and July 1948 were examined at 300 mb. Stations used were Downham Market at which the probable wind 24 hr. later was deduced and Aldergrove. The advection of wind from Aldergrove to Downham Market was represented by a weighting factor *W* proportional to the difference in height of the 500-mb. contours in latitudes 60° and 45° N. summed for longitudes 0°, 5°, 10°, 15° and 20° W. The unit for *W* was for a thousand feet. Partial correlation coefficients were formed between the wind at Downham Market 24 hr. after zero, V_{A24}, at Downham Market at zero hour V_A, and at Aldergrove at zero hour V_B multiplied by *W*. The resulting standard vector deviations and stretch vector correlation coefficients were as shown in Table XXII.

TABLE XXII—STANDARD VECTOR DEVIATIONS AND CORRELATION COEFFICIENTS BETWEEN WIND OBSERVATIONS AT DOWNHAM MARKET AND WEIGHTED WIND OBSERVATIONS AT ALDERGROVE

300 mb. June and July 1948

Standard vector deviation	Correlation coefficient
kt.	
$\mathrm{V}_{A24} = 46$	$r_{A24,A} = 0{\cdot}540$
$\mathrm{V}_A = 46$	$r_{A24,B} = 0{\cdot}486$
$\mathrm{V}_B = 39$	$r_{AB} = 0{\cdot}574$

The partial correlation coefficients are then $r_{A24,A.B} = 0{\cdot}367$ and $r_{A24,B.A} = 0{\cdot}255$ with the resulting regression equation

$$\overline{\mathrm{V}}_A - \mathrm{V}_{A24} = 0{\cdot}39(\overline{\mathrm{V}}_A - \mathrm{V}_A) + 0{\cdot}064\, W(\overline{\mathrm{V}}_B - \mathrm{V}_B).$$

The standard vector error of V_{A24} which was $38\frac{1}{2}$ kt. when persistence only was used, becomes $37\frac{1}{2}$ kt. when the advection is also taken into account. The regression equation was then used to make estimates of the winds during the month of July 1951 with the results shown in Table XXIII.

TABLE XXIII—FREQUENCY OF ERRORS IN WIND ESTIMATES FOR 24 HR. AHEAD IN COMPARISON WITH FREQUENCY OF ERRORS IN FORECASTS

July 1951

	Vector error								
	>69 kt.	>59 kt.	>49 kt.	>39 kt.	>29 kt.	>19 kt.	>9 kt.	<10 kt.	Total
24-hr. estimates ..	2	4	5	13	25	40	58	4	62
Orthodox forecasts ..	1	3	10	16	29	42	58	4	62

The standard vector error in the estimates is 31 kt. and in the orthodox forecasts is 33 kt.

§ 18—USE OF REGRESSION EQUATIONS IN FORECASTING ROUTE WINDS

The results of the experiments which are recorded above encouraged the idea that forecasts for aviation purposes could be made by the use of regression equations suitably chosen. The merit of regression equations is that the laborious work is done beforehand and the forecast can be made very rapidly. This advantage becomes increasingly great as the complexity of the orthodox forecast charts becomes greater and the time spent on their production becomes longer. For this reason regression equations for aircraft flying at high altitudes were first examined, and as a practical example the case for Comet aircraft flying between London and Rome was taken.

The forecasts for the Comets made by orthodox methods are arranged to give the information of wind and temperature for the following levels :—

(i) 5,000 ft., 10,000 ft., 15,000 ft., 20,000 ft., 30,000 ft. and 40,000 ft. for the climb between London Airport and latitude 49° N.

(ii) 20,000 ft., 30,000 ft. and 40,000 ft. for the cruise between latitudes 49° N. and 46½° N. and between latitudes 46½° N. and 44° N.

(iii) 5,000 ft., 10,000 ft., 15,000 ft., 20,000 ft., 30,000 ft. and 40,000 ft. for the descent between latitude 44° N. and Rome.

Where, as at London and Rome, these are busy airports and aircraft cannot always be accepted at once and may be stacked, it is essential that wind should be given for individual layers for climb and descent. In less frequented places it may be possible to give a mean wind and temperature for the climb and descent, and this can be very conveniently and rapidly evaluated by a regression equation based on the most recent wind observation at a nearby radio-sonde station, for the mean wind up to any height is given directly from the position of the balloon in relation to its point of release. It should be noted that the forecasts are supplied to the Comets about 2 hr. before take-off, and are based on prontour charts derived from the latest surface prebaratic and latest prontour chart. Now it takes some hours to construct a prontour chart so that the forecast is based primarily on observations about 12 hr. stale, though a correction may be made to the forecast by the examination of actual observations of wind and temperature 2 to 8 hr. old.

Regression equations were calculated for the cruise wind at a height of 200 mb., by using the contour charts 24 hr. before the supposed flight in combination with the wind at Larkhill 12 hr. before the flight, and also contour charts 12 hr. before flight in combination with the wind at Larkhill 6 hr. before flight.

The correlation coefficients and standard deviations were as shown in Table XXIV.

TABLE XXIV—STANDARD VECTOR DEVIATIONS AND CORRELATION COEFFICIENTS FOR ESTIMATING WIND AT 40,000 FT. ON THE ROUTE LONDON AIRPORT TO ROME

For estimates made from charts 12 and 24 hr. before flight refreshed with observations at Larkhill 6 and 12 hr. before flight.

	σ_l	σ_A	$r_{l,l12}$	$r_{l,l24}$	$r_{l,A12}$	$r_{l12,A}$	$r_{l,A}$	$r_{l,A6}$	$r_{l6,A}$
	kt.	kt.							
			COMPONENTS ALONG ROUTE						
Winter ..	27	27	+0·80	+0·68	+0·55	+0·72	+0·67	+0·61	+0·69
Spring ..	27	27	+0·85	+0·70	+0·59	+0·72	+0·70	+0·69	+0·71
Summer ..	20	24	+0·69	+0·50	+0·43	+0·61	+0·58	+0·51	+0·59
Autumn ..	25	32	+0·76	+0·61	+0·24	+0·62	+0·60	+0·42	+0·61
			COMPONENTS ACROSS ROUTE						
Winter ..	23	28	+0·85	+0·68	+0·12	+0·54	+0·33	+0·23	+0·44
Spring ..	24	28	+0·77	+0·64	+0·31	+0·54	+0·41	+0·36	+0·47
Summer ..	26	28	+0·85	+0·67	−0·14	−0·30	−0·11	−0·13	−0·21
Autumn ..	29	34	+0·84	+0·63	+0·14	+0·46	+0·28	+0·21	+0·37

σ_l represents the standard deviation of wind over the route. σ_A the standard deviation of wind over Larkhill. The correlation coefficients are between winds over the route at different intervals of time and also between wind over the route and the wind at Larkhill at different time intervals.

From Table XXIV the regression equations given in Table XXV can be deduced.

TABLE XXV—REGRESSION EQUATIONS FOR DEDUCING THE PROBABLE WIND BETWEEN ROME AND LONDON

	ALONG ROUTE	ACROSS ROUTE
Winter	$u_{e24} - 27 = 0\cdot41(u_0 - 27) + 0\cdot46(u_{A12} - 29)$	$v_{e24} - 7 = 0\cdot62(v_0 - 7) + 0\cdot45(v_{A12} - 13)$
	$u_{e12} - 27 = 0\cdot61(u_0 - 27) + 0\cdot30(u_{A6} - 28)$	$v_{e12} - 7 = 0\cdot79(v_0 - 7) + 0\cdot17(v_{A6} - 13)$
Spring	$u_{e24} - 28 = 0\cdot41(u_0 - 28) + 0\cdot46(u_{A12} - 16)$	$v_{e24} - 15 = 0\cdot53(v_0 - 15) + 0\cdot32(v_{A12} - 12)$
	$u_{e12} - 28 = 0\cdot70(u_0 - 28) + 0\cdot22(u_{A6} - 16)$	$v_{e12} - 15 = 0\cdot69(v_0 - 15) + 0\cdot19(v_{A6} - 12)$
Summer ..	$u_{e24} - 21 = 0\cdot29(u_0 - 21) + 0\cdot37(u_{A12} - 18)$	$v_{e24} - 15 = 0\cdot64(v_0 - 15) - 0\cdot15(v_{A12} - 21)$
	$u_{e12} - 21 = 0\cdot53(u_0 - 21) + 0\cdot25(u_{A6} - 18)$	$v_{e12} - 15 = 0\cdot83(v_0 - 15) - 0\cdot08(v_{A6} - 21)$
Autumn	$u_{e24} - 22 = 0\cdot49(u_0 - 22) + 0\cdot38(u_{A12} - 23)$	$v_{e24} - 17 = 0\cdot57(v_0 - 17) + 0\cdot33(v_{A12} - 17)$
	$u_{e12} - 22 = 0\cdot61(u_0 - 22) + 0\cdot27(u_{A6} - 23)$	$v_{e12} - 17 = 0\cdot80(v_0 - 17) + 0\cdot18(v_{A6} - 17)$

u_{e24}, u_{e12}, v_{e24}, v_{e12} are the route wind components expected to occur 24 hr. and 12 hr. after the measured route winds u_0 v_0. u_{A12}, u_{A6}, v_{A12}, v_{A6} are the wind components observed at Larkhill 12 hr. and 6 hr. after the measured route winds u_0 v_0.

When the equivalent headwinds from these expected values of components were calculated for 25 occasions when Comet aircraft were flying from London to Rome or from Rome to London, and the results compared with winds estimated from charts drawn from the subsequent observations at the Central Forecasting Office, Dunstable, they showed a root-mean-square error of 10·5 kt. The forecasts actually issued to the Comet aircraft gave a root-mean-square error of 14·3 kt. A further check was made of this method by comparing the forecasts issued for Comet flights on a further 40 occasions between London and Rome with the estimates made subsequently of the true route winds. Comparison was also made between the probable wind derived from the regression equations of Table XXV and the estimates of the true winds. The traditional type of forecast gave a root-mean-square difference of 12·8 kt., the probable winds derived from a 12-hr. regression equation gave 13·4 kt.

The results of the foregoing comparisons are not to be interpreted as advocating the replacement of orthodox methods of forecasting by regression equation procedures. The more normal methods involving synoptic techniques can provide answers, however imperfect, to questions which regression methods cannot at present tackle ; and even more important, the synoptic methods are sufficiently elastic to allow of modification and development in the light of increased knowledge of the physical processes taking part. Until the synoptic methods can be further improved and developed, however, and particularly where the requirement is for forecasts of winds in the high atmosphere where the observations are scanty and irregular it seems that regression methods can provide valuable help, and serve as a standard to be improved upon in the process of perfecting the more normal methods.

In the intertropical regions of the globe where the response to geostrophic control is slow and the use of synoptic techniques in the development of wind is very imperfectly understood, it is probable that the estimates afforded by the regression technique may have important practical value for aviation, but it must not be forgotten that winds so derived will not assist in the comprehension of the physical processes which govern tropical weather, and so will in no way advance our knowledge of what is the fundamental problem for the tropical meteorologist.

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APPENDIX

NOTATION

a, b, c = constants
d = distance along air route from one end
k = constant
l = total length of air route
m = integer
N = number of observing stations along route
n = number of occasions, observations
$Q = \Sigma(\mathrm{v} - \mathrm{v}_A)^2/n$
r = stretch vector correlation coefficient between the mean wind derived from observing stations and the true mean wind along a route

r_{AB} = stretch vector correlation coefficient between winds at A and B
r_d = stretch vector correlation coefficient between winds at two places distant d apart
r_{2d} = stretch vector correlation coefficient between winds at places distant $2d$ apart
r_{st} = stretch vector correlation coefficient between vectors $\mathbf{v}_s$ and $\mathbf{v}_t$
r_t = temporal stretch vector correlation coefficient
r_x = stretch vector correlation coefficient between winds at places distant x apart
r_{24} = 24-hr. stretch vector correlation coefficient
s = length of section of air route
u = actual wind component along route
u_e = expected wind component along route
u_t = wind component along x-axis t hours after zero hour
u_0 = wind component along x-axis at zero hour
$\mathbf{V}_A$ = vector wind at point A (at a certain time–zero hour)
$\overline{\mathbf{V}}_A$ = mean vector wind at a point A
$\mathbf{V}_{At}$ = vector wind at point A t hours after zero hour
$\mathbf{V}_B$ = vector wind at point B (at zero hour)
$\overline{\mathbf{V}}_B$ = mean vector wind at point B
$\mathbf{V}_{Bt}$ = vector wind at B t hours after zero hour
$\mathbf{V}_t$ = vector wind at point t hours after zero hour
$(\mathbf{V})'_0$ = deduced vector wind at surface
$(\overline{\mathbf{V}})_0$ = mean vector wind at surface
$(\mathbf{V})_h$ = assumed vector wind at h ft.
$(\overline{\mathbf{V}})_h$ = mean vector wind at h ft.
$S\mathbf{v}$ = sum of a number of vectors
$\mathbf{v}$ = vector departure from mean wind at any point along an air route
$\mathbf{v}_A$ = vector departure from mean wind at point A
$\mathbf{v}_B$ = vector departure from mean wind at point B
$\mathbf{v}_d$ = vector departure from mean wind distant d from one end of air route
$\mathbf{v}_s, \mathbf{v}_q, \mathbf{v}_p$ = s^{th} q^{th} p^{th} vectors in a series
$\bar{\mathbf{v}}$ = true mean vector departure of wind along route
$\bar{\mathbf{v}}'$ = mean vector departure of wind along route deduced from values at individual points along route
$\mathbf{v}_0$ = vector departure from mean wind at one end of air route
$\mathbf{v}_1$ = first vector in a series
v = actual wind component across route
v_e = expected wind component across route
v_t = wind component along y-axis t hours after zero hour
v_0 = wind component along y-axis at zero hour
W = weighting factor
α_{AB} = angle of turn of $\mathbf{v}_B$ from $\mathbf{v}_A$
ε = error of observation
η = departure due to small eddies
σ = standard vector deviation
$\bar{\sigma}$ = standard vector deviation of vector mean wind over the route
σ_A = standard vector deviation at point A
σ_B = standard vector deviation at point B
σ_l = standard vector deviation of component of route wind along (or across) a route of length l
σ_n = standard vector deviation for a given sample of a number of vectors
σ_t = root-mean-square vector difference between winds separated by a time interval t
σ_ε = standard vector deviation of ε
σ_η = standard vector deviation of η
θ_{AB} = angle between $\mathbf{v}_A$ and $\mathbf{v}_B$.

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