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CHAPTER 2

DYNAMICAL IDEAS IN WEATHER FORECASTING

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CHAPTER 2

DYNAMICAL IDEAS IN WEATHER FORECASTING

2.1 INTRODUCTION

During the years when weather forecasting was an entirely subjective activity, a set of synoptic models was developed which served to describe the atmosphere in a moderately satisfactory manner to the minds of human forecasters. Unfortunately, these classical tools of the synoptic analyst - air masses, fronts, surface pressure patterns and the like – were not so valuable for prognostic purposes. For really there was little more that could be done in traditional forecasting practice beyond moving forward the geometrical patterns of isobars and isotherms in a manner which, although based on a qualitative appreciation of physical principles, relied greatly for its quantitative expression on an extrapolation of observed trends, modified as necessary by the skill and experience of the forecaster.

The introduction of computed forecasts and the increasing prominence of such concepts as vorticity, and vorticity advection, have led rather to the reverse state of affairs. We now have mathematical tools which are admirable for prognostic purposes but which lend themselves rather less easily to simple, descriptive diagnostic analysis. It is true that patterns of contour lines or isobars may be interpreted in terms of vorticity quite simply, but it is less easy to see how fronts, or air masses, may be described in such terms. Attempts have been made to do so but the relationship between a front and some appropriate dynamical parameters is not one that is easily and convincingly demonstrated in simple terms. Indeed fronts still continue to be one of the main weapons in the armoury of the human forecaster and yet they are a notable absentee on any computer-produced forecasts.

This chapter is concerned with ideas which are relevant to weather forecasting at a time when this is an occupation pursued partly objectively and partly subjectively. Forecasters are no longer in the position of having to rely exclusively upon their own judgement and experience in order to assess future dynamical developments. Whereas in the past they could only, at best, delineate quite roughly a few of the major development areas, they now receive, routinely, the precisely computed results from dynamical models of the atmosphere which spell out the detailed future development all over the chart, right down to the last millibar. Computer forecasts are covered in more detail in Chapter 3 - Background to computer models, and here it is not necessary to do more than recognize them as a fact of life whose presence in the background must determine the attitude of present-day forecasters to the subject of dynamical meteorology. Forecasters should recognize that numerical models are indeed only 'models'. Every numerical forecast, even the very best, differs from actuality in some respect or other and, to get the most out of a model, forecasters should understand as much as possible about the principles on which it works. A general knowledge of dynamical meteorology is still very much required by forecasters for the double purpose of understanding both the atmosphere itself and also the numerical models of the atmosphere.

So this chapter sets out to be a commentary, in terms which are relevant to forecasters today, on the interrelation between the old synoptic approach to weather forecasting and the newer, dynamical approach which has accompanied the introduction of computers. It is <u>not</u> a textbook on dynamical meteorology. Mathematical results are quoted and discussed, but their detailed derivations must be sought in other texts, of which there are many.[1-4] Nor is this a textbook on synoptic meteorology, of which there is also a plentiful supply.[5,6] It is addressed to practising forecasters for whom a knowledge of synoptic meteorology is their bread and butter and who, though they do not work in an environment where the language and ideas of mathematics are in daily use, do have a basic mathematical training and knowledge. It attempts to bring the older ideas up to date and to present a balanced account of what is best in the older and the newer aspects of present-day synoptic-scale weather forecasting.

Standard notation is used, except where otherwise specified.

2.2 THEORETICAL TOPICS

2.2.1 Large-scale distribution of wind, temperature and moisture

2.2.1.1 The atmospheric energy balance. The energy required to drive atmospheric circulations comes from the radiation received from the sun. This radiation is absorbed by the surface of the earth and by the atmosphere. It is initially recognizable in the form of heat.

It is found that, when the radiation gains and losses at different wavelengths are totalled, there is a net gain of heat in tropical latitudes and a net loss in polar regions. But despite this latitudinal imbalance there is, over the globe as a whole, a long-term equilibrium in the distribution of heat. It follows that atmospheric and oceanic circulations play a vital part in redistributing heat between different latitudes and maintaining the long-term balance that exists.

The part played by ocean currents is smaller than that of the atmosphere, but it is not by any means negligible in the long term. However, for day-to-day forecasting it can be accepted that the pattern of sea-surface temperatures is unchanging and is governed by the steady pattern of the oceanic circulation. Similarly, at very high levels, such as the mid stratosphere and above, the circulations do not appear to be of direct importance in determining heat exchanges which have significant short-term effects on the daily weather near the earth's surface.

It is in the troposphere and, to a lesser extent, the lower stratosphere that the main heat transfers occur. These are the levels with which the practising forecaster has to deal and it is the atmospheric motion systems at these levels with which he is most concerned. Since these 'motion' systems form an integral part of the global mechanism for redistributing the incoming solar energy, it is natural to look at the large-scale distribution of temperature in the horizontal and the vertical as a background to any study of the 'motion' systems themselves.

2.2.1.2 Tropospheric temperatures and winds. The fact that the motion of the atmosphere is, on the broadest scale, approximately geostrophic (see 2.2.3.1, page 10) has one particularly convenient result. The wind flow at any level can be depicted very simply, outside the equatorial regions, by the field of pressure or of geopotential. The winds may be assumed to blow parallel to the contours with low values of geopotential on the left in the northern hemisphere, and with a speed that is inversely proportional to the spacing between adjacent contour lines. Also, patterns of thickness lines (see 2.2.6.1, page 27), representing the geopotential difference between two given pressure levels, may be regarded as identical with the pattern of mean temperature between those two levels. (Strictly, thickness is equivalent to virtual temperature.) Figures 1 and 2 show contours for the 500-mb level, and thickness for the 1000-500-mb layer. These maps show only one particular occasion among the endless variety of daily situations, but they are not at all untypical. It can be seen that the major troughs, ridges and closed centres of the contour pattern bear a marked similarity to corresponding features in the thickness pattern. Concentrations of contours occur in similar places to concentrations of thickness lines throughout much of the troposphere, and closed thickness patterns in one layer lie almost vertically over similar closed patterns in lower layers.

If the patterns are viewed on a broad scale in this manner we see that there exists throughout the troposphere, over large horizontal distances around the globe, an almost vertical zone in which is concentrated a belt of strong winds and large temperature contrast. This zone is located in temperate latitudes. On either side of it there are extensive areas within which the horizontal variation of temperature is small and the winds light. This broad-scale pattern is strikingly similar to the day-to-day patterns of fronts separating different, quasi-homogeneous air masses. In this case, however, the narrow band of frontal activity is replaced by a much broader zone in which there exists a temperature gradient which is fairly uniform, both in magnitude and direction, throughout the troposphere.

This relative simplicity of the broad-scale structure of the troposphere makes it possible for the main features of the wind and temperature distributions to be exhibited on a single chart. It also accounts, incidentally, for the quite considerable success achieved by the simplest single-level numerical-forecasting models of the atmosphere.

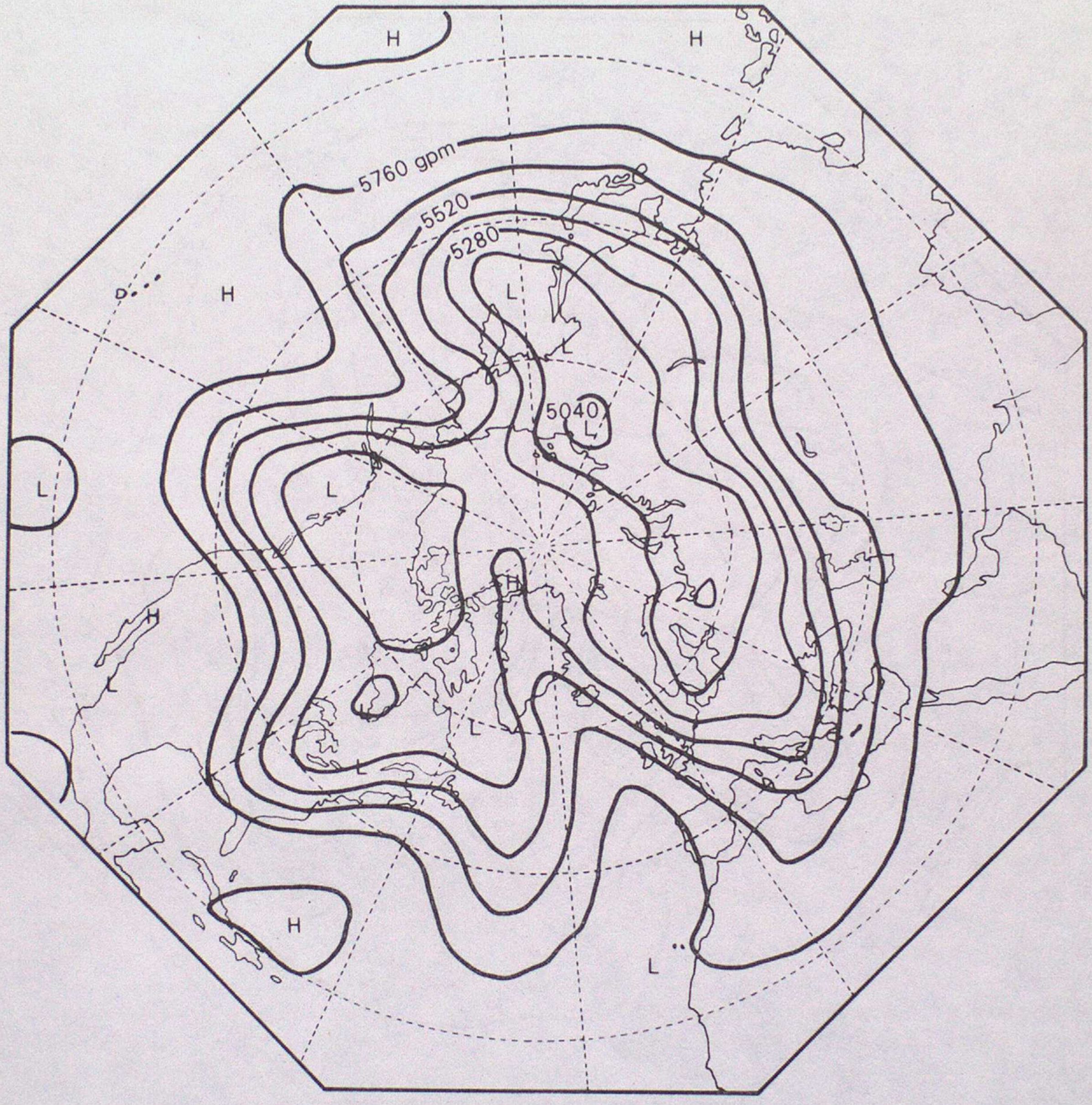


FIGURE 1. 500-mb geopotential chart for 0000 GMT, 29 November 1973

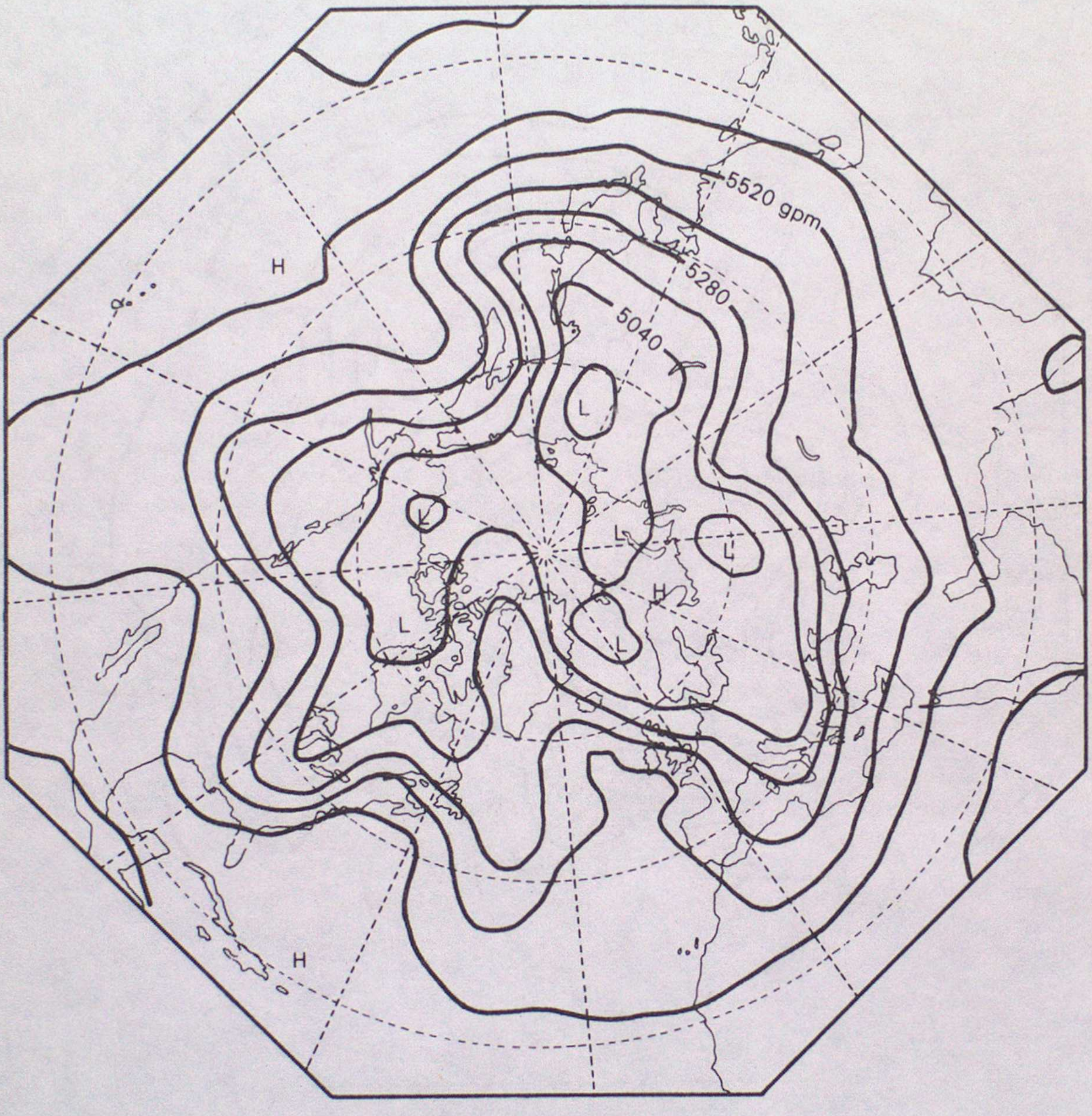


FIGURE 2. 1000-500-mb thickness chart for 0000 GMT, 29 November 1973

As regards temperature, the 1000-500-mb chart has traditionally been the most convenient chart to use. It normally displays all the temperature features which are relevant on any particular day to the synoptic-scale weather systems in middle latitudes. What it does not show is the detailed temperature structure near the ground, or any features which are entirely confined to the upper troposphere above 500 millibars. Similarly, as regards wind, the 500-mb level has always been a major tool. This is a level which more or less divides the atmosphere into two halves, of equal mass; it is also very close on many occasions to the level of non-divergence (see 2.2.5.2, page 21), and it is a natural level to take in conjunction with the 1000-500-mb thickness chart. But for highlighting the distribution of jet streams and their very important effects on the distribution of mass in the atmosphere there is much to be said for using charts near the top of the troposphere. In mid latitudes the 300-mb chart is particularly valuable.

2.2.1.3 The general circulation of the atmosphere. The general circulation of the atmosphere is the name given to the broad-scale pattern of wind flow over the globe. It is normally illustrated in the form of seasonal values which depict a mean flow in which the effects of day-to-day synoptic-scale systems are averaged out.

If the three components of the mean flow are u, v, w, u represents zonal or west/east flow, v represents meridional or south/north flow, and w represents vertical motion. With the normal conventions, positive values are given to westerly winds, southerly winds and (using pressure, p, as a vertical coordinate) subsiding motion. In charts of the mean flow, the zonal component is almost everywhere the dominant one. Even on individual daily charts, such as Figures 1 and 2, this can be seen to be largely true, and the distribution of u can conveniently be displayed in mean meridional cross-sections, for summer and winter, as in Figure 3.

Note that at the surface there are easterly winds in low latitudes, westerly winds in mid latitudes and easterly winds in high latitudes. These features are common to both seasons, but with some change in the details of location and intensity.

In the troposphere, except very near the equator, the vertical gradient of zonal wind is positive ($\partial u/\partial z > 0$). In other words, westerly winds increase with height and easterly winds decrease. This is consistent with the observed tropospheric temperature gradient, for it can be shown (see 2.2.6.1, page 27) that the thermal wind equation implies a relationship between the change of wind with height ($\partial u/\partial z$) and the latitudinal variation of temperature ($\partial T/\partial y$) of the form:

$$\frac{\partial u}{\partial z} \approx - \frac{g}{f}\frac{1}{T}\frac{\partial T}{\partial y} \qquad \ldots \quad (2.1)$$

Thus if temperature decreases towards the north ($\partial T/\partial y > 0$), then the zonal wind speed (u) increases with height ($\partial u/\partial z > 0$) and vice versa.

In the troposphere the increase of u with z is greatest between 20° and 30°N, and the maximum average zonal winds in winter (40-50 m s^{-1}) occur in the upper troposphere at this latitude. This is the 'subtropical jet stream' which is a very persistent climatological feature of the atmosphere and is also readily identifiable on synoptic charts as a westerly jet stream at much the same location and having much the same intensity from one day to the next, during winter months, particularly over North Africa. In contrast, the equally important polar-front jet is so variable in location, orientation and intensity from day to day that it appears as a much weaker feature on mean charts and cross-sections than it does on a synoptic basis.

2.2.1.4 The lower stratosphere. In low latitudes, where the tropopause is high, the lower stratosphere is very cold. In high latitudes the opposite is true – the tropopause is low and consequently the stratospheric temperatures over the poles are warm. Above the tropopause, therefore, there is a reversal of the meridional temperature gradient of lower levels; $\partial u/\partial z$ is negative, westerly winds decrease with height and easterly winds strengthen.

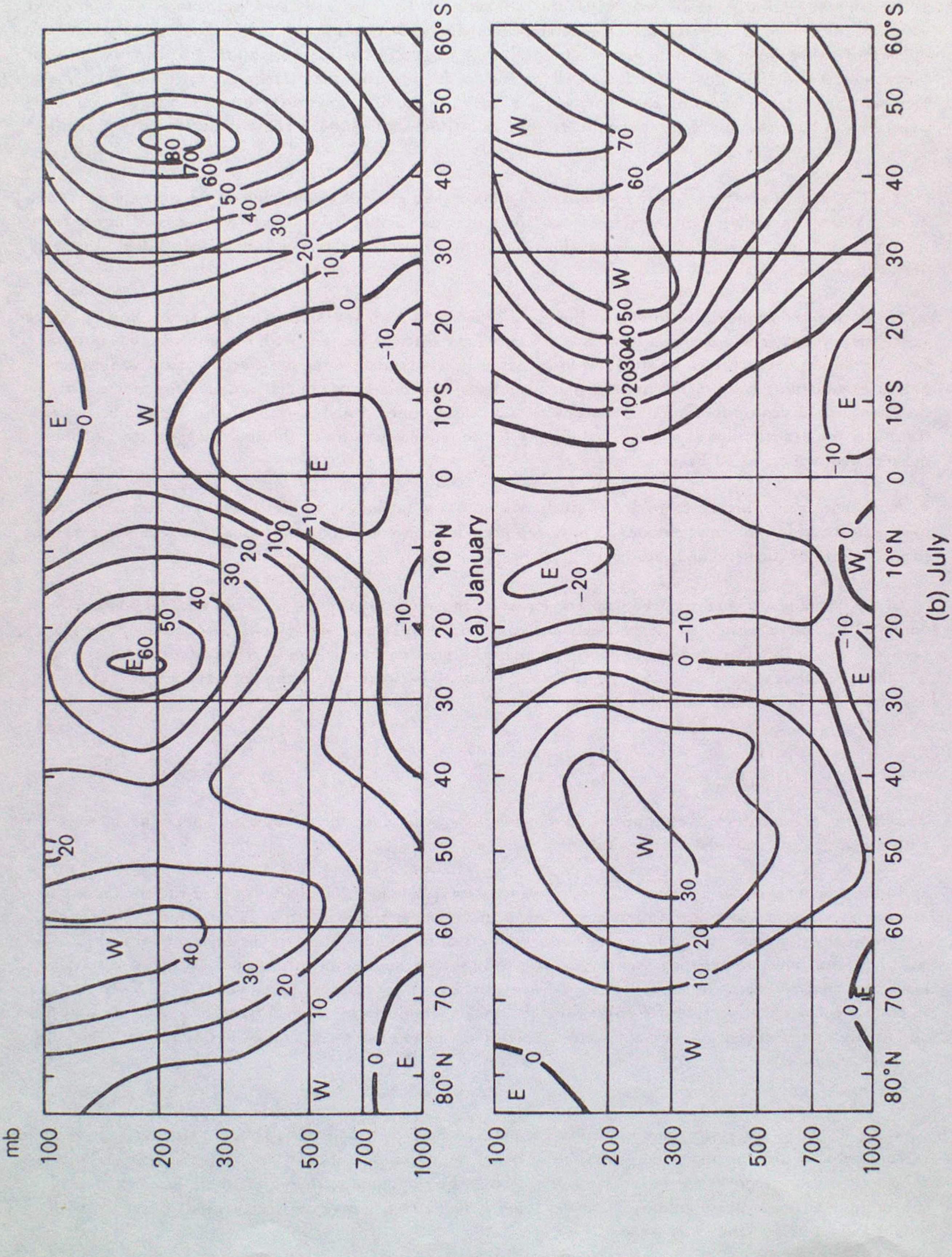


FIGURE 3. Mean meridional cross-sections, for summer and winter, of the zonal flow u (kn) at 10°W

These normal, climatological conditions are frequently advected from day to day by the wind flow in long-wave circulations (see 2.2.8.2, page 38). Thus the warm tropospheric ridges in a long-wave pattern are associated with a high tropopause and cold stratosphere even at high latitudes. Similarly, long-wave troughs penetrating deeply into low latitudes carry with them their cold tropospheric air with its low tropopause and the warm stratospheric air above. Although the interrelation of cause and effect between changes in the troposphere and changes in the lower stratosphere is not fully understood, it is clear that these two regions are closely connected. (See also Chapters 8 and 11.)

2.2.1.5 Atmospheric water vapour. Water vapour is one of the constituents of the atmosphere. In its life cycle it is continually being depleted by precipitation and replenished by evaporation, and in its distribution it is a highly variable quantity, both in space and time. Although it is present in only relatively small quantities, it is of the greatest possible importance to meteorologists.

In total content, the mass of water vapour very rarely exceeds about 2 per cent of the mass of the atmosphere at any time. More usually it is about one-tenth of this amount. Bannon and Steele[7] show the typical distribution of water vapour with height over southern England in summer and winter. Figure 4 is taken from their paper. Throughout the troposphere as a whole there is an average quantity of about 2-3 grams of water vapour per kilogram of air. This is not distributed uniformly, there being much more near the ground than at high levels. About half the available atmospheric moisture is below 850 millibars, and the quantities in the upper troposphere are very small.

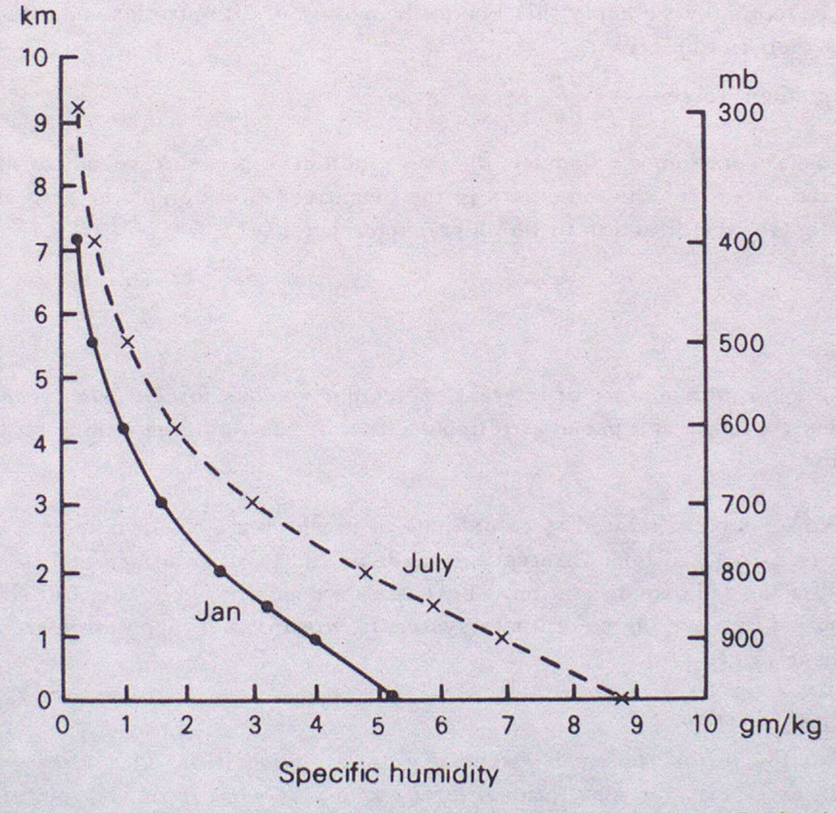


FIGURE 4. Mean distribution of water vapour over southern England in summer and winter

Apart from the direct effect of this moisture on the formation of clouds and rain, there is another reason why it is of great importance in the dynamics of the atmosphere. The latent heats associated with the phase-changes of water are very large – about 2.5 MJ kg^{-1} for the processes of condensation and evaporation. Changes of this sort can clearly exert a great effect on the air temperature (the specific heat of dry air at constant pressure c_p = 1.0031 kJ kg^{-1}K^{-1}) and, through the temperature, on the motion of the air.

2.2.2 The equations of motion

2.2.2.1 Newton's momentum equation. Dynamics is concerned with the movement or flow of air – the wind, not only the horizontal components of the wind (u, v) which can be measured, but also the vertical motion (w) which usually cannot. Since the horizontal dimensions of the atmosphere (which, in this chapter, we take to mean the troposphere and lower stratosphere up to about 100 millibars) are so much larger than its vertical dimension, the large-scale atmospheric motions are, to a high degree of accuracy, horizontal. However, the weather systems responsible for all the detail of the 'good' and 'bad' weather which we experience from hour to hour are usually of limited horizontal extent and certainly have very significant vertical motions. But even in these systems the actual magnitudes of the vertical velocities, for all their relative importance, will normally be much smaller than the horizontal wind speeds associated with the same systems at upper levels.

Dynamical equations concerned with forecasting the future pattern of winds must include terms which describe the changes of wind with time or, in other words, its accelerations. Since wind velocity is a vector quantity, being specified by both direction and speed, an acceleration may indicate a change of either the wind speed, or the wind direction, or both.

The fundamental physical law describing the motion of air (or any other moving body) is Newton's second law of motion – the momentum equation – which states, 'The rate of change of momentum of an object, when measured relative to co-ordinate axes fixed in space, equals the sum of the forces acting on that object.' In meteorology we apply this law to the motion of air particles, and the three fundamental forces which affect their motion are:

(a) Pressure gradient force $-\dfrac{1}{\rho}\dfrac{\partial p}{\partial n}$ x mass

where, in the usual notation, ρ = density, $\partial p/\partial n$ = gradient of pressure normal to the isobars. The negative sign indicates that the force acts in the 'negative' direction of the gradient, i.e. from high to low pressure, the direction in which pressure decreases.

(b) Gravity g.

(c) Friction F,
the latter being a complex mixture of internal, molecular viscous forces, aerodynamic drag due to roughness at the ground, and a pseudo-frictional effect due to turbulent mixing on various scales in the atmosphere.

Now our observations are not related to co-ordinate axes which are fixed in space. The motion of a radiosonde balloon is calculated from observations made on the earth's surface and is referred to co-ordinate axes fixed at the radiosonde station. These axes are moving, in space. But Newton's second law can still be applied in a moving co-ordinate system if certain other 'apparent forces' are included in the equation. These forces are:

(i) Centrifugal force $\Omega^2 R$
where R = distance from the earth's axis, Ω = its rate of rotation. This force can be combined very conveniently with the gravitational force, g, to give what is termed the 'effective gravity' $= g + \Omega^2 R$. In normal usage of the word, and in the rest of this chapter, the term 'gravity' (denoted simply by g) will properly mean 'effective gravity'.

(ii) Coriolis force $f\mathbf{V}$
where $\mathbf{V}$ is the horizontal wind vector and $f = 2\Omega \sin\phi$ is the Coriolis parameter. This force accounts for the apparent deviation of moving objects towards the right in the northern hemisphere. It arises because we observe motions in a co-ordinate reference frame that is moving, with angular rotation Ω. The magnitude of the effect varies with latitude ϕ.

So Newton's second law gives the general form of the equations of motion in the atmosphere:

	Sum of forces acting				
Acceleration =	Pressure gradient force	+ Coriolis force	+ Gravity	+ Friction	. . . (2.2)

which may be written in Cartesian co-ordinates (x, y, z) as:

$\dfrac{du}{dt}$ =	$-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$	$+fv - f'w$		$+F_x$	
$\dfrac{dv}{dt}$ =	$-\dfrac{1}{\rho}\dfrac{\partial p}{\partial y}$	$-fu$		$+F_y$	. . . (2.3)
$\dfrac{dw}{dt}$ =	$-\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}$	$+f'u$	$-g$	$+F_z$	

where $f = 2\,\Omega\,\sin\phi$ and $f' = 2\,\Omega\,\cos\phi$.

2.2.2.2 Pressure co-ordinates. In meteorological dynamics it is found to be more convenient to use pressure (p) as the vertical co-ordinate in place of the more conventional distance (z). It may not seem obvious at first sight that a vertical co-ordinate which varies from day to day both in its surface value (p_0) and in the vertical separation between successive pressure levels (Δp) would be of much practical use. However, our present interest lies not just in simple measurement, but in dynamical understanding. Fundamental to this is a knowledge of the distribution of the mass of the atmosphere at any time, and the mass of air above any point is given by the pressure at that point. Also, the flow of air between two given pressure levels (whose vertical separation will be a varying distance, or thickness, h') is the flow of a known, fixed mass of air. To have upper levels which are related to pressure is therefore of value to theoretical understanding.

But there are practical advantages to the forecaster also; for if we assume that in the large-scale flow the vertical accelerations are negligible, and that in consequence the air is in 'hydrostatic equilibrium' in the vertical (see 2.2.4.2, page 17) under the balance of gravity and the vertical pressure gradient force, then the relationship between the pressure (p) and the height (z) may be written $dp = -g\rho\, dz$. With this relationship, if we change the vertical co-ordinate from z to p, the effect on the form of the equations of motion is to replace terms like $-1/\rho\ \partial p/\partial x$, which represent the pressure gradient at a constant height, by terms like $g\ \partial z/\partial x$, which represent the gradient of geopotential at a constant pressure level.

The practical advantage of this lies in the fact that the variable quantity ρ is eliminated from the equation and is replaced by the almost constant quantity g. Thus it is possible to construct a single 'geostrophic wind scale' (see 2.2.3.1, page 10) relating the wind speed at any level to the gradient of geopotential at that level. If pressure co-ordinates were not used, a separate scale would be required for every level in order to take account of the widely varying values of density with height. The manner in which ρ and g vary with height in mid latitudes is shown in Table 2.1. In pressure co-ordinates, the vertical velocity is written as ω, where

$$\omega = \frac{dp}{dt}$$

(that is, change of pressure with time following the motion). It is related to the vertical velocity, $w(= dz/dt)$, in Cartesian co-ordinates by $\omega = -g\rho w$ under hydrostatic conditions (see 2.2.4.2, page 17).

TABLE 2.1 Approximate values of ρ and g, expressed as percentage of their values at 1000 mb

mb	Air density	Gravity
100	13%	99.9995%
300	38	100
500	57	100
700	75	100
1000	100	100

2.2.3 Special cases of simple horizontal flow

2.2.3.1 Geostrophic wind. One of the simplest expressions describing atmospheric air motion is the well-known geostrophic wind equation, (see also Chapter 16). In conditions where the air flow is horizontal and is not accelerating this equation expresses the instantaneous balance between the two dominant forces acting horizontally on the air particles.

Following the notation of 2.2.2.1 (page 8), geostrophic flow is defined by equating the pressure gradient force and the Coriolis force:

$$\left.\begin{aligned}\frac{1}{\rho}\frac{\partial p}{\partial x} &= f v_g \\ \frac{1}{\rho}\frac{\partial p}{\partial y} &= -f u_g\end{aligned}\right\}$$

where (u_g, v_g) are the components of the geostrophic wind $(\mathbf{V}_g)$.

These may be conveniently expressed as a single equation:

$$\frac{1}{\rho}\frac{\partial p}{\partial n} = f\mathbf{V}_g \qquad \text{. . . . (2.4)}$$

or

$$\left.\mathbf{V}_g = \frac{1}{\rho f}\frac{\partial p}{\partial n}\right\}$$

where $\partial p/\partial n$ is the gradient of pressure in the direction normal to the isobars.

Alternatively, using pressure co-ordinates to describe the motion on quasi-horizontal isobaric surfaces (i.e. surfaces of constant pressure, on which $\partial p/\partial n = 0$), the geostrophic wind equation becomes:

$$\mathbf{V}_g = \frac{g}{f}\frac{\partial h}{\partial n} \qquad \text{. . . . (2.5)}$$

This equation is not a forecasting equation. Since it does not contain any terms involving the time rate of change, $\partial/\partial t$, it cannot be used for prediction. It is purely diagnostic, describing the instantaneous balance of forces at a particular moment.

The general relationship between the distribution of surface pressure and the direction of the actual wind has been known for a long time. In 1857 it was formally expressed in Buys Ballot's law which states that, in the northern hemisphere, the wind blows in such a way that lower pressure lies to the left of the flow and higher pressure to the right. Expressing this as a property of geostrophic flow, it states that streamlines of the geostrophic wind are parallel to the isobars, with the direction of flow being such that lower pressure lies to the left. The relationship between the speed of the geostrophic wind and the spacing of the isobars, or contours, can be seen if we write equation 2.5 in terms of small, measurable quantities Δh, Δn instead of the differential $\partial h/\partial n$. Then we have:

$$V_g = \frac{g}{f}\frac{\Delta h}{\Delta n} \qquad \text{. . . . (2.6)}$$

or, if g = constant (which it is, for all practical purposes),
f = constant (which it is at a given latitude),
Δh = constant (which is the case if isopleths are drawn at regular intervals, say every 60 geopotential metres),

$$V_g \propto \frac{1}{\Delta n}$$ where Δn is the perpendicular distance between two adjacent contours.

Thus the geostrophic wind speed is inversely proportional to the spacing of the contours, or isobars. This result forms the basis for the construction of geostrophic wind scales, which are used for estimating wind speeds from charts with contours, or isobars, drawn on them.

The geostrophic wind equation is so simple that it clearly implies some rather big assumptions if it is to describe any kind of atmospheric motion at all realistically. These assumptions are that the air is moving horizontally and is undergoing no accelerations. Actual air motion can only be geostrophic, therefore, where the flow is:

horizontal	(no vertical motion)
straight	(no direction changes)
steady	(no speed changes).

In addition, since the geostrophic wind equation ignores all frictional terms, it does not give a good representation of the real wind in those regions where friction is significant. The lowest kilometre of the atmosphere, particularly over land, is the region where frictional effects are most important. This is often referred to as the 'friction layer', and is a region where forecasters soon become accustomed to the non-geostrophic character of the actual wind which rarely blows exactly parallel to the surface isobars. This is largely brought about by frictional retardation of the air at the earth's surface.

But in the free atmosphere, above about 1 kilometre, the geostrophic relation is a very useful one, especially if we confine our attention to those large-sized weather systems which are the normal features of synoptic charts. In such systems the pressure gradient force and the Coriolis force are of about the same magnitude, and each is much greater than the frictional forces. As a result, the flow in such systems often achieves an approximately geostrophic balance.

In mesoscale weather systems, however, where the space and time scales of the motion are quite small, the importance of the Coriolis term relative to others is very much reduced. This term is a measure of the apparent deflexion of moving bodies when their motion is observed from the surface of the rotating earth. For a mesoscale system, where the circulation may extend for 50-100 kilometres and last only a few hours, the apparent deflexion will be very much less than that observed in synoptic-scale motion, where the scale of the circulation will be about ten times as great, both in time and space. So, when the Coriolis term becomes unimportant, geostrophic balance is impossible. With motions on this scale the geostrophic relationship is much less frequently of value than it is with large-scale motions.

On the larger, synoptic scale the very important vertical motions associated with significant weather developments are not represented at all by the geostrophic wind. But the regions of active development are quite limited in area at any given time and are separated by extensive areas of quiet, undramatic weather where the geostrophic wind approximation gives a remarkably useful, practical representation of the actual wind.

The geostrophic wind varies with latitude. Since $V_g \propto 1/f$, the geostrophic wind corresponding to a given contour spacing increases as the latitude decreases. Near the equator, f is nearly zero and, as a result, the geostrophic wind becomes quite unrealistic. In equatorial regions there is no simple correspondence between the patterns of streamlines and contours.

Away from equatorial and tropical regions, forecasters should bear in mind that the proper use of standard geostrophic wind scales requires that account be taken of the variation of latitude. Over a limited area, say less than 10° of latitude near the British Isles, the differences that occur in measurements of V_g due to variations in latitude are normally small enough to be ignored, being of the order of 5 per cent or less of the measured wind. But, over wider latitudinal bands, care should be taken to use a scale which is appropriate to the required latitude, or to make a correction if such a scale is not available.

2.2.3.2 Gradient wind. Forecasters working with synoptic charts, who want to apply the concepts of geostrophic flow by using geostrophic wind scales, soon realize that to confine their use to regions of straight isobars is a severe limitation. Surface charts, in particular, have isobaric patterns with very pronounced curvature. Upper-air charts, also, although they lack the many small vortices found at the surface, do invariably show troughs and ridges with significant curvature.

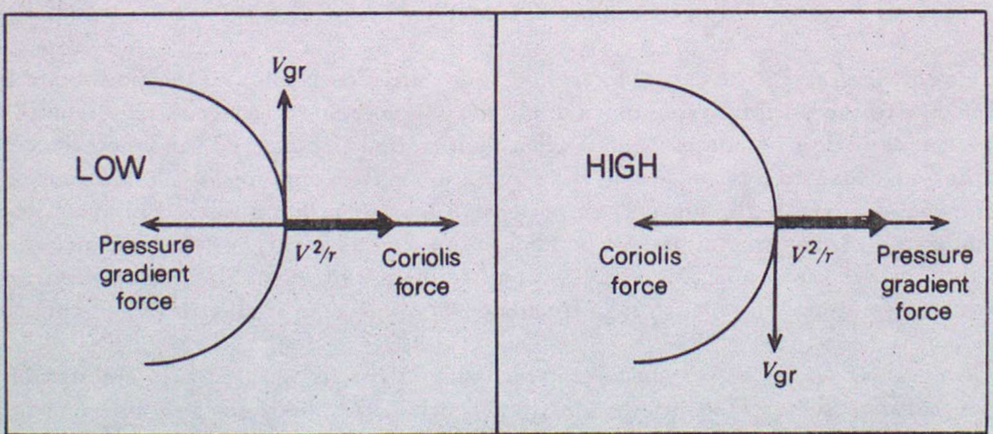
An extension to the concept of geostrophic flow can be derived, which gives a closer approximation to the actual wind in situations of balanced horizontal motion around a curved path. Air flowing along a path which has a radius of curvature, r, has a centripetal acceleration V^2/r. Following the form of the equation of motion, this acceleration may be equated to the resultant effect of the two dominant horizontal forces discussed in 2.2.3.1 (page 10). In this case we do not assume that the pressure gradient force and the Coriolis force are exactly equal in magnitude, but that they are slightly out of balance, which results in the air experiencing an acceleration and following a curved path. The wind that results from these conditions is called the gradient wind ($\mathbf{V}_{gr}$), (see also Chapter 16).

In considering motion of this type it is necessary to distinguish two cases, depending upon whether the flow is cyclonically or anticyclonically curved. In the mathematical treatment, care must be taken to describe the correct sense in which the forces act by proper use of the algebraic signs.

The gradient wind equation may be written:

$$\pm \frac{\mathbf{V}_{gr}^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + f\,\mathbf{V}_{gr} \quad \ldots\ldots \quad (2.7)$$

where the sign on the left side is positive for anticyclonic flow and negative for cyclonic flow, as illustrated by the balance of forces depicted in Figure 5.



(a) Cyclonic motion (b) Anticyclonic motion

FIGURE 5. Gradient wind and balance of forces in the northern hemisphere

There are some important differences between the character of anticyclonic and cyclonic flow. These may be illustrated by simple algebraic manipulations of the gradient wind equation. Thus:

Anticyclonic flow

$$+\frac{\mathbf{V}_{gr}^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + f\mathbf{V}_{gr}$$

Cyclonic flow

$$-\frac{\mathbf{V}_{gr}^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial n} + f\mathbf{V}_{gr}$$

or, rearranging the terms,

$$\mathbf{V}_{gr}^2 - rf\mathbf{V}_{gr} + \frac{r}{\rho}\frac{\partial p}{\partial n} = 0$$

$$\mathbf{V}_{gr}^2 + rf\mathbf{V}_{gr} - \frac{r}{\rho}\frac{\partial p}{\partial n} = 0 \quad . \quad . \quad (2.8)$$

and solving for $\mathbf{V}_{gr}$

$$\mathbf{V}_{gr} = \tfrac{1}{2}rf - \sqrt{\left(\tfrac{1}{4}r^2f^2 - \frac{r}{\rho}\frac{\partial p}{\partial n}\right)}$$

$$\mathbf{V}_{gr} = -\tfrac{1}{2}rf + \sqrt{\left(\tfrac{1}{4}r^2f^2 + \frac{r}{\rho}\frac{\partial p}{\partial n}\right)} \quad (2.9)$$

where the sign in front of the radical is chosen in each case to ensure that $\mathbf{V}_{gr} = 0$ when $\partial p/\partial n = 0$.

If the solution is to have a real physical meaning, the term under the radical must be positive. Hence $\partial p/\partial n <$ constant × r; i.e. as r decreases so $\partial p/\partial n$ decreases. This is observed in nature.

Only weak pressure gradients are found in the centres of highs, where r is small.

The term under the radical is always positive, so there is no comparable restriction on the magnitude of the pressure gradient to that which occurs in anticyclonic motion.

If $\mathbf{V}_g$ is the geostrophic wind corresponding to the given pressure gradient, then equation 2.8 may be rewritten as:

$$\mathbf{V}_{gr} = \mathbf{V}_g + \frac{\mathbf{V}_{gr}^2}{rf}$$

In anticyclones the gradient wind is larger than the geostrophic wind for the same pressure gradient.

The gradient wind in a high has a maximum value, which is determined by the condition that the square-root term in equation 2.9 equals zero. Using the geostrophic-wind relationship this condition is $rf = 4\mathbf{V}_g$, so the maximum value of $\mathbf{V}_{gr}$ in an anticyclone equals $2\mathbf{V}_g$ (twice the corresponding geostrophic wind speed).

$$\mathbf{V}_{gr} = \mathbf{V}_g - \frac{\mathbf{V}_{gr}^2}{rf}$$

In depressions the gradient wind is smaller than the geostrophic wind for the same pressure gradient.

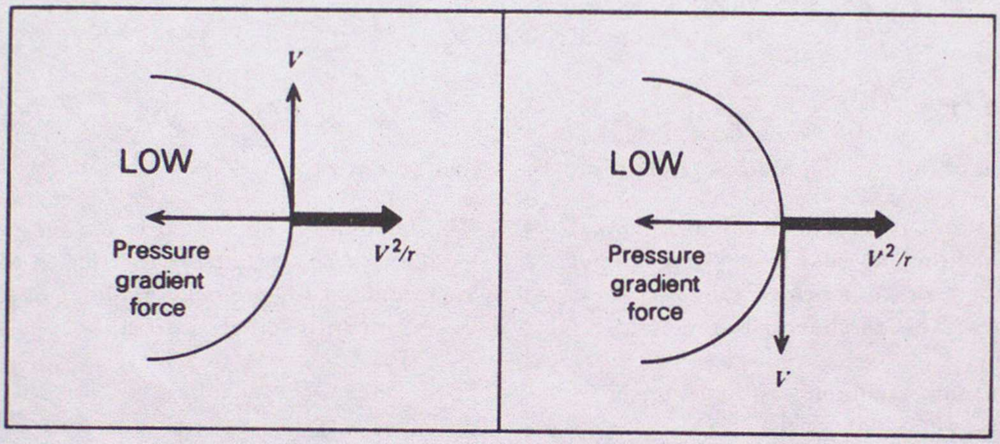
There is no upper limit to the wind speed around a low. $\mathbf{V}_{gr} = 0$, when $\partial p/\partial n = 0$, is a necessary lower limit, but otherwise $\mathbf{V}_{gr}$ can be as large as the pressure gradient allows.

One of the main drawbacks to using the gradient wind operationally is that, strictly, r is the radius of curvature of the trajectory of an air particle, and trajectories are not easy to determine in practice. Only if the flow patterns are very slow-moving do the air trajectories correspond closely with the isobars, or contours. In this case gradient-wind measurements can confidently, though somewhat tediously, be made from the contour pattern. But around any mobile features of the chart, the air trajectories will differ greatly from the instantaneous contour pattern. Gradient-wind estimates made from the contours in these regions will certainly be rather poor approximations to the actual wind. Nevertheless it has been found that, as estimates of the actual wind, gradient winds are generally better than geostrophic winds on upper-air charts. Their superiority is not so evident on surface charts where the normal forecasting practice is to use the geostrophic wind.

2.2.3.3 Cyclostrophic motion. A special case of gradient-wind flow around a low-pressure area occurs when the space and time dimensions of the motion are sufficiently so small that the Coriolis force has a negligible effect. In this case either clockwise or anticlockwise motion is possible around the centre of low pressure, as shown in Figure 6. The motion, which is called cyclostrophic, is described by the simple equation:

Centrifugal acceleration = pressure gradient force.

$$\frac{V^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial n} \qquad \ldots\ldots \quad (2.10)$$



(a) Normal case, of anticlockwise motion around the low centre.

(b) Less common case, which may occur with very small vortices, such as dust devils.

FIGURE 6. Cyclostrophic motion and the balance of forces

Dust devils and waterspouts are typical examples of motion of this kind. These systems last only a few minutes and are very limited in horizontal extent. The direction of the rotation of such systems has been observed in both clockwise and anticlockwise senses.

Somewhat larger-scale systems of a closely related kind are tornadoes. The central pressure in the core of a tornado falls locally to very low values indeed and the wind speeds near the core are normally estimated to be of the order of 100 m/s. These systems are awesomely destructive in their effects, particularly over the central plains of the United States of America. Moderately fierce tornadoes do occur in the British Isles, though usually not more than 10 or 12 each year.[8] Although the motion in these systems is primarily governed by the cyclostrophic terms, the circulation around tornadoes is normally cyclonic. This implies a degree of geostrophic control, with the Coriolis force having some effect, as would be expected in a system having a life-span of the order of an hour.

2.2.3.4 Isallobaric wind. One of the assumptions implied in the derivation of the geostrophic wind equation is that the distribution of pressure is fixed and unchanging. If this is so, a balance can be struck between the (constant) pressure gradient force and the Coriolis force. But in nature it is very obvious that surface-pressure patterns are always changing, and these changes produce departures of the actual wind from simple geostrophic flow.

The rate at which pressure varies is denoted by $\partial p/\partial t$ which, for convenience, we may write $\dot{p}$. On surface synoptic charts reports of 3-hourly pressure tendencies are readily available and these provide an approximation to the instantaneous values of $\dot{p}$. The extent to which this approximation is valid is shown by the pressure 'characteristic'. Lines of equal pressure tendency, drawn on surface charts, are called isallobars.

If the actual wind vector ($\mathbf{V}$) is expressed as the sum of the geostrophic wind ($\mathbf{V}_g$) and another, ageostrophic, wind ($\mathbf{V}_i$) due to the changing pressure field, then

$$\mathbf{V} = \mathbf{V}_g + \mathbf{V}_i ,$$

where $\mathbf{V}_i$ is called the isallobaric wind and can be expressed as

$$\mathbf{V}_i = -\frac{1}{\rho f^2} \frac{\partial \dot{p}}{\partial n} .$$

This expression for the isallobaric component of the wind is very similar in form to the equation for the geostrophic wind (equation 2.4, page 10). In fact, just as $\mathbf{V}_g$ is related to the isobaric gradient, so there is a relationship between $\mathbf{V}_i$ and the isallobaric gradient. The direction of $\mathbf{V}_i$ is, in fact, always from high values of pressure tendency to low. So the isallobaric effect produces outflow from regions of rising pressure (positive isallobars), and inflow to regions of falling pressure (negative isallobars). These effects may make important contributions to the development of weather, on a synoptic scale, in such regions.

The magnitude of the isallobaric-wind component near the ground is frequently about 5-10 m/s. It therefore represents a significant correction to the measured value of $\mathbf{V}_g$ in regions where the surface pressure is changing rapidly. Not only is this of significance when applied to simple wind forecasts, but it is also specially relevant to estimating the movement of fronts. Fronts which have a strong pressure gradient across them are regions where rapid pressure changes are occurring. In such regions the isallobaric wind is likely to be large, and directed in such a way as to make the actual winds differ markedly from the geostrophic winds. The non-geostrophic cross-isobar flow which is normally so pronounced near rapidly moving fronts, occurs largely for this reason. It is illustrated in Figure 7.

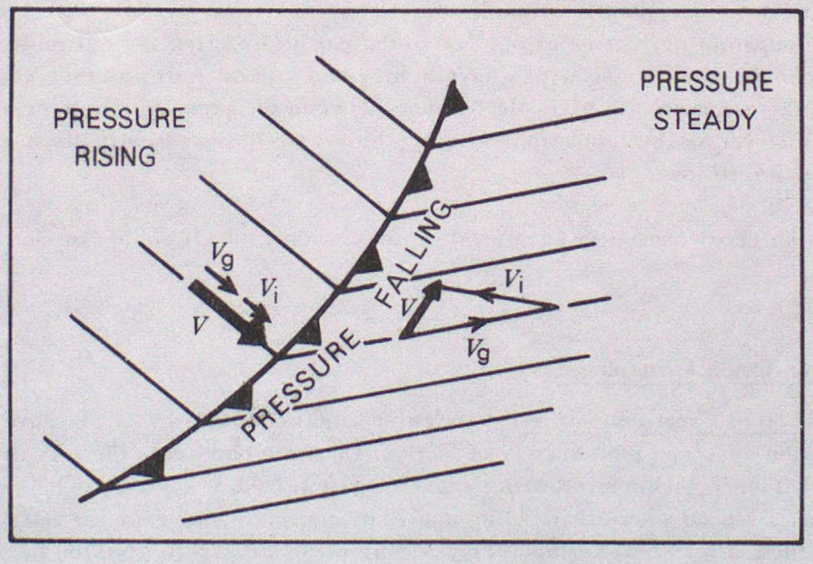


FIGURE 7. Effect of the isallobaric wind leading to markedly ageostrophic effects near a rapidly moving front

V_g = Geostrophic wind (parallel to isobars)
V_i = Isallobaric wind (from high to low values of pressure tendency)
V = Actual wind ($\mathbf{V} = \mathbf{V}_g + \mathbf{V}_i$)

An isallobaric effect may also be very important with quasi-stationary fronts. The movement of these fronts may be very slow, but it is frequently of great significance in detailed local-weather forecasting. Often the only indication a forecaster has of the frontal drift is from the direction of the isallobaric gradient across the front.

A simple practical technique for measuring the magnitude of the isallobaric-wind component does exist, but it suffers from a severe practical limitation in the region of the British Isles. To estimate V_i it is necessary to be able to draw the isallobaric patterns with some degree of accuracy. Over the continents this is easy, but over the oceans it is not. The necessity of having to adjust the pressure tendencies reported by moving ships and their irregular spatial distribution are two factors which make oceanic isallobaric analyses very uncertain. Since British forecasters are primarily concerned with weather developments over the Atlantic, their past experience of routine, quantitative, isallobaric measurements in this region have generally been accompanied by frustration and disappointment. However, there may be times, with continental developments or with mesoscale developments over the British Isles itself, when isallobaric measurements are both possible and profitable. Forecasters should be alert for such occasions.

There is no reason why isallobaric effects should not also be important at upper levels of the atmosphere, especially where there are fast-moving, small-scale features. But as radiosonde measurements are made only every 12 hours it is not possible to assess the effect from routine upper-air charts.

2.2.3.5 Antitriptic winds. A class of winds which, in temperate latitudes, are of more interest on the local mesoscale than on the synoptic scale are those which are sometimes termed antitriptic winds. They are not readily calculable in a forecast office, but since they form another simple special case of the equations of motion, a brief general description is not out of place.

Antitriptic winds are local winds of limited vertical and horizontal extent. Sea-breezes and hill/valley winds are typical examples. They are located in the lowest parts of the atmosphere, largely controlled by the topography and are consequently strongly affected by frictional forces. Since the scale of the motion is small, the apparent deflecting effect due to the earth's rotation is negligible unless the wind persists for several hours. Once these winds have achieved a steady state, as they commonly do, the equation describing the air motion is a simple balance between the pressure gradient force and the frictional retardation. In so far as the topography allows, the air motion is directly down the pressure gradient, from high pressure to low.

Such winds are also very common in equatorial regions where, for all scales of motion, the Coriolis force is nearly zero.

2.2.4 Vertical motion in the atmosphere

2.2.4.1 The character of vertical motion. Vertical motion occurs in the atmosphere, in three-dimensional circulation systems, on a variety of scales. On the broad scale there is the frontal ascent and anticyclonic subsidence, associated with synoptic-scale pressure systems. The most elementary concept of a front, as a region where there is an upgliding motion of the warm air mass, was based on the rather primitive idea of a frontal surface being a more or less discrete division between two uniform, but contrasting, air masses. In recent years, aircraft and radar observations have combined to give more realistic descriptions of fronts in some important details, but the work of Harrold[9] and Browning and Harrold[10,11] still ascribes the major rain-producing mechanism in an oceanic frontal depression to the ascent of a comparatively narrow band of warm moist air. This air originates in the surface convective layers of the subtropics and rises along a sloping path (see Figure 8), rather in the manner of a 'conveyor belt' (which is the name given to it by Harrold[9] and Browning[12]). This conveyor belt is typically about 200 kilometres wide and 2 kilometres deep at a warm front, and the vertical velocities attained within it are of the order of 10 cm/s. The release of potential instability at the top boundary of the conveyor belt gives rise to local variations in the detailed pattern of the vertical motions and results in the mesoscale rain bands which are found within the main frontal rain area.

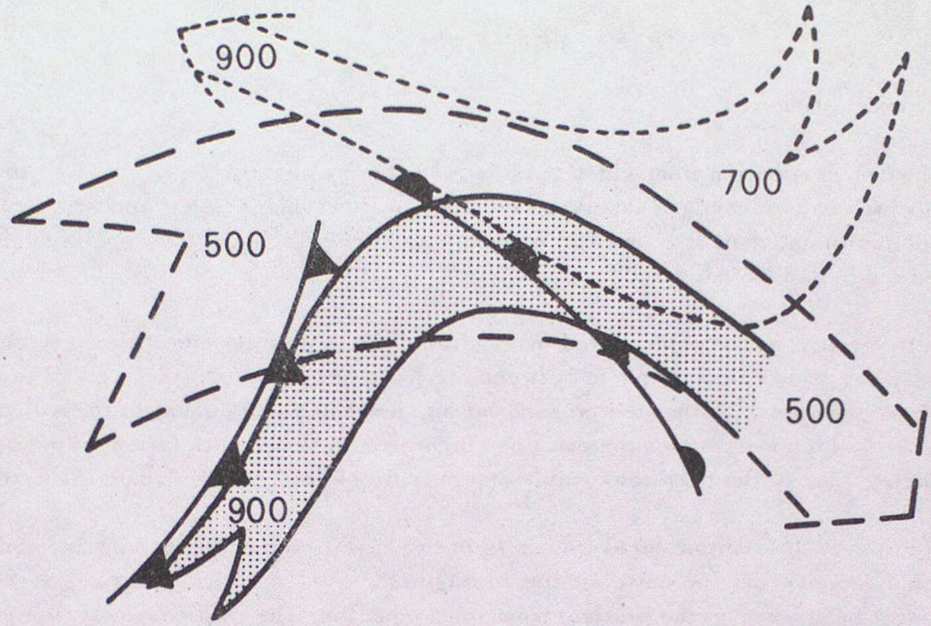
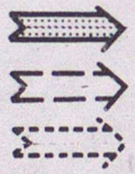


FIGURE 8. The warm-air 'conveyor belt': broad arrows denote airflow relative to a mobile frontal depression



Ascending air moving from warm sector up the warm-front surface.

Mid-tropospheric air having small vertical motion.

Subsiding air from the pre-frontal ridge.

Figures within arrows indicate level of flow in millibars.

The rather narrow tongue of ascending warm, moist air at a front is apparently somewhat smaller in dimension and more concentrated in its effects than the areas of subsidence associated with neighbouring anticyclones. These are normally more extensive in area, but have less-strong vertical velocities.

Mesoscale circulations show a similar pattern of concentrated areas of strong ascent surrounded by more extensive areas of weaker subsidence. For example, sea breezes have a vertical circulation pattern in which there is a narrow zone (only a few hundred metres wide) of quite vigorously ascending air at the sea-breeze 'front', with a much more extensive area of gently subsiding motion in the sea air that follows. Circulations of even smaller scale also have a similar character, but these are outside the domain of synoptic forecasters.

2.2.4.2 The hydrostatic approximation. A brief mention of this topic was made in 2.2.2.2 (page 8). The broad-scale ascent and subsidence associated with synoptic systems are obviously of fundamental importance to forecasters because of their relation to the distribution of cloudy and cloud-free areas. Yet, when the atmosphere is viewed on this scale, it is clear that the vertical velocities ($\omega \approx 0.5$-1.0 knot) are, in general, much smaller than the horizontal wind velocities ($u, v \approx 50$-100 knots). So in the equations of motion (equation 2.3) it can be assumed that ω is small, and consequently that $d\omega/dt$ is negligible.

In the vertical momentum equation, therefore, it is possible to ignore $d\omega/dt$ and F_z because of their small size and also to ignore $2\,\Omega\, u \cos\phi$ in comparison to g. As a result, this equation reduces (for synoptic-scale motion) to

$$0 = -\frac{1}{\rho}\frac{dp}{dz} - g \qquad \text{. . . . (2.11)}$$

or

$$dp = -g\rho\,dz. \qquad \ldots . \quad (2.12)$$

This is the hydrostatic equation.

Unlike the momentum equation from which it is derived, the hydrostatic equation is no longer a prognostic equation which can be used, in theory, to forecast ω. However, it is a diagnostic equation which is a most helpful analytical tool. It is used in order to compute, and check, the upper-air temperatures and heights reported by radiosondes.

The hydrostatic approximation represented by equation 2.11 is a statement of the very close balance which undoubtedly exists in the atmosphere between the force of gravity and the vertical pressure force, and it follows from equation 2.12 that the pressure at any level is simply equal to the weight of air above that level. This is, in fact, only an approximation, which gives a very 'static' view of the atmosphere, but it is nevertheless one of the most reasonable approximations made in all dynamical meteorology.

Where the hydrostatic assumption breaks down is in the description of mesoscale and smaller-scale systems. In such systems ω may be quite similar in magnitude to u, v, and the vertical acceleration $d\omega/dt$ can no longer be ignored in the vertical momentum equation. The hydrostatic approximation therefore becomes inappropriate.

2.2.5 Some dynamical topics

2.2.5.1 Vorticity. One of the most evident features of the motion of fluids is the very frequent occurrence of rotational motion. Swirls, vortices, whirlpools are all common descriptive terms of phenomena which are easily observed in any moving liquid. Similarly, charts depicting the flow of the atmosphere also show patterns which have a rotational or circulatory character. In fact, at all levels in the atmosphere, rotational motion systems are common and are important for both the description and the prediction of the weather. Vigorous low-pressure circulations are associated with strong winds and stormy weather; large high-pressure systems have dry, settled weather. So there is some relationship between the character of the weather and the vigour of the circulatory motion in the atmosphere, and it is desirable that we should have some way of describing this concept of rotation within a moving fluid.

One feels intuitively that a deep depression, for example, with its accompanying strong winds, is a 'more vigorous' circulation, or has a greater 'quantity of rotation' than a low of equal size around which the winds are light. Any measure of this 'quantity of rotation' contained in a fluid motion system must be proportional to the wind speeds in that system. In a similar way it seems reasonable to expect the rotation to be related in some manner to the curvature of the patterns which depict the circulating system.

There is a mathematical concept, termed 'circulation', which agrees well with these simple, intuitive ideas. It is defined formally by

$$C = \oint \mathbf{V}.d\mathbf{l} \qquad \ldots . \quad (2.13)$$

where C is the 'circulation', equalling the integral, or sum, around a closed curve (as in Figure 9) of a succession of small lengths $d\mathbf{l}$ multiplied by the wind $\mathbf{V}$ along each of those lengths. Thus C is proportional to the wind speed around the closed curve and to the size of the curved path itself. This circulation is a measure of the amount of rotation contained in a finite, physically measurable body of fluid. Although it comes close to formally describing the simple ideas of fluid motion which result from casual observation it is not the most convenient concept from the point of view of dynamical understanding. For this, and to keep a uniform mathematical treatment, we require a measure of the quantity of rotation at a point.

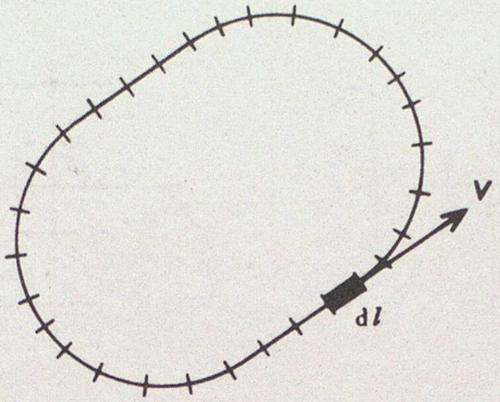


FIGURE 9. The circulation around a closed curve

In general, the rotation at a point is not quite such an easy thing to visualize as the rotation of a finite body of fluid. It is more a mathematical than a physical concept, and the mathematical expression for it is called vorticity. Vorticity is the amount of circulation per unit area, and is a measure of the rotation of a fluid at a point. It is a vector quantity, whose 'direction' is that of the axis about which the fluid is rotating locally.

Sometimes, on a small scale, the presence of vorticity is dramatically illustrated in the atmosphere. Dust devils, waterspouts, and the funnel clouds associated with tornadoes are all examples of vorticity being concentrated into very narrow dimensions and made easily visible. In these systems the air spins violently around the central axis, which is nearly (but not necessarily exactly) vertical. These systems give a clear visual impression of the vorticity being concentrated in a 'vortex tube' extending from near the ground up to some height above. In these particular cases, where the axis of the vortex tube is nearly vertical, the vertical component of the vorticity vector is greater than any horizontal component.

Rotor clouds associated with some types of airflow over mountainous terrain are an example of rotation, or vorticity, about a horizontal axis; so also are the roll clouds associated with line-squalls. In the study of synoptic-scale systems, since the horizontal dimensions of the atmosphere are so much larger than the vertical dimension, the flow is very close to being horizontal; although vertical wind shear can lead to high values of vorticity about a horizontal axis, it is the component of vorticity about a vertical axis which is of overriding importance dynamically.

Unless specifically noted otherwise, all references to 'vorticity' in the literature of dynamical meteorology are to the vertical component of the total vorticity vector. It is denoted by the Greek letter zeta, ζ.

In Cartesian co-ordinates, the expression for vorticity is

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} . \qquad \ldots \quad (2.14)$$

This represents the twisting, or rotational, effect at a point due to the variation of v in the x direction and the variation of u in the y direction, as illustrated in Figure 10 The algebraic sign of vorticity is conventionally taken to be positive in an anticlockwise, or cyclonic, sense. Thus with u and v being positive in the directions of increasing x and y, it can be seen from Figure 10 that cyclonic vorticity, for example, has contributions from $(+\,\partial v/\partial x)$ and $(-\partial u/\partial y)$. ζ is called the 'relative vorticity' since it is evaluated from wind components u, v which are measured relative to co-ordinate axes on the moving earth.

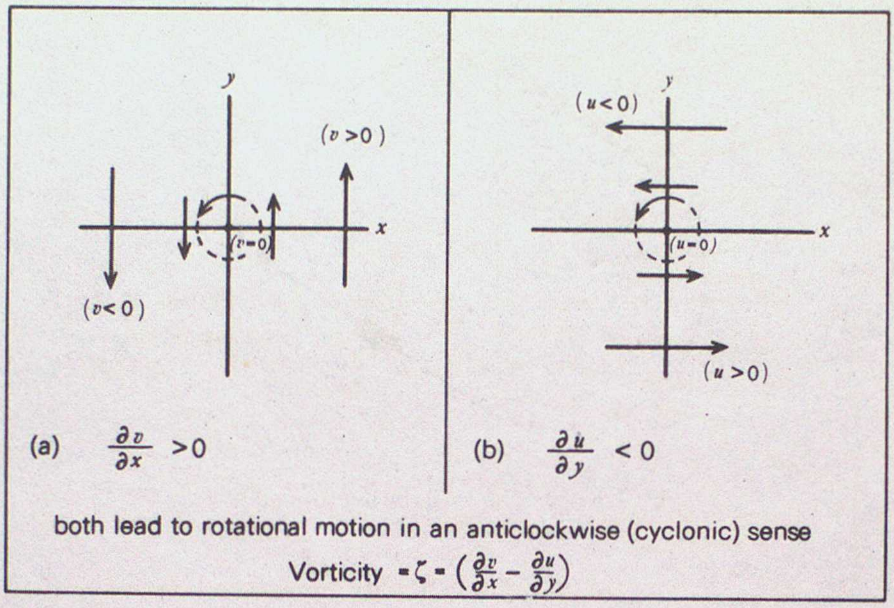


FIGURE 10. The vorticity at a point resulting from the variation of u and v

For a solid body, both ($+\ \partial v/\partial x$) and ($-\ \partial u/\partial y$) are equal to the angular velocity, Ω, of the body. So, for solid-body rotation, $\zeta = 2\Omega$. The most important solid body with which meteorologists are concerned is the earth itself. It spins about its axis and every particle, either on the earth or in its atmosphere, possesses in addition to its relative vorticity a certain amount of vorticity by virtue of the rotation of the earth. The magnitude of the vorticity due to the earth's rotation is $2\Omega \sin\phi$, which is the Coriolis parameter, f. It varies from a maximum value at the poles to zero at the equator. The sum of the relative vorticity of a particle, ζ, plus the vorticity it has from the earth's rotation (f), is called the 'absolute vorticity' ζ_a. So $\zeta_a = \zeta + f$.

In polar co-ordinates, ζ may be expressed in a form which is very useful in the subjective interpretation of vorticity patterns by forecasters.

$$\zeta = \frac{V}{r} - \frac{\partial V}{\partial n} \quad \ldots \quad (2.15)$$

where n is directed towards the left of the flow. Here V/r, the flow with wind speed, V, around a curved path, with radius of curvature r, is the component of vorticity due to 'curvature'. $\partial V/\partial n$, the change of V across the direction of flow, is the component of vorticity due to 'shear'. Thus we may think of the vorticity pattern at any level as being built up from two components. It is partly due to the curvature of the flow and partly due to the variation in wind speed across the direction of flow. The curvature component is in line with intuitive ideas that depressions are regions of cyclonic (positive) vorticity, and anticyclones are regions of anticyclonic (negative) vorticity. But this is not the whole story, because vorticity can exist in motion which exhibits no curvature. In such cases the shear term accounts for the vorticity. Jet streams are an excellent illustration of the importance of the shear term in determining the pattern of vorticity at upper levels. All along the poleward side of a westerly jet, as in Figure 11 for example, there is cyclonic vorticity and along the equatorward side there is anticyclonic vorticity. Note, also, that along the central axis of the jet stream, where the curvature and the shear are near zero, there is almost no vorticity.

There is generally no great difficulty in visually identifying those areas of a particular chart where either the curvature or the shear is making a relatively large contribution to the vorticity. What is not so easy to assess subjectively is the relative magnitudes of the two terms, especially in those areas where they may oppose each other in their effects.

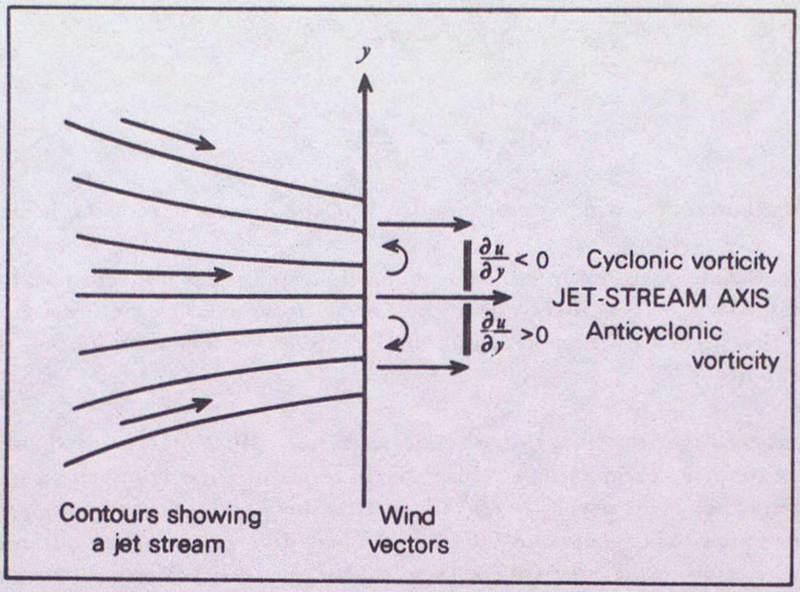


FIGURE 11. Shear vorticity along the flanks of a jet stream

2.2.5.2 Divergence, and the equation of continuity. The term 'divergence' is used in a number of slightly different ways. In the first place it is the name given to a particular, strictly defined, mathematical operator. But also it is used, in conjunction with 'convergence', either as part of the jargon of dynamical meteorology where it still has a precise technical meaning, or as part of everyday language where its usage is somewhat more generalized and imprecise.

In mathematics, the divergence of a vector **A**, which we shall write as div **A**, is defined by

$$\operatorname{div}\mathbf{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z \quad \ldots \quad (2.16)$$

where $\mathbf{A} = (A_x, A_y, A_z)$. So div **A** is a measure of the rate of change of each component of the vector **A** along its appropriate direction. Div **A** itself is not a vector but a simple scalar quantity of a certain magnitude.

In meteorology we are largely concerned with the rate at which the mass of air in a region changes with time, and whether air is accumulating or being depleted. This is connected with the density changes and with the wind flow in that region. So div ρ **V**, the divergence of mass, and div **V**, the divergence of velocity are particularly important concepts in meteorology. Div ρ **V** appears in the equation of continuity, which is one of the basic equations in meteorological theory, being an expression of the principle of conservation of mass in the atmosphere. This is a formal statement of the fact that within a unit volume of air, if we assume that matter is neither created nor destroyed, the rate of change of mass equals the rate at which it is transported into or out of that volume. In mathematical notation this is written:

$$\frac{\partial \rho}{\partial t} = - \left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \right] \quad \ldots \quad (2.17)$$

which is the equation of continuity, expressed in Cartesian co-ordinates.

In fact, in the atmosphere $\partial\rho/\partial t$ is negligible compared with the individual terms on the right-hand side of the equation. Also, when these terms are expanded (e.g. $\partial/\partial x\,(\rho u) = u\,\partial\rho/\partial x + \rho\,\partial u/\partial x$), the terms containing horizontal variations in density may also be taken to be negligibly small. And lastly, if we use pressure as the vertical co-ordinate instead of z, we can eliminate the vertical variation of ρ, (see 2.2.2.2, page 9). All these approximations, concerned with the space and time variations of air density, lead to the simplest form of the equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad \ldots \quad (2.18)$$

which is often written

$$\text{div}_h \mathbf{V} + \frac{\partial \omega}{\partial p} = 0, \qquad \ldots \quad (2.19)$$

where $\mathbf{V} = (u, v)$ is the horizontal wind vector and $\text{div}_h \mathbf{V}$ is the divergence of the horizontal wind.

$\text{Div}_h \mathbf{V}$ is so frequently encountered in both the theoretical and practical computational aspects of dynamical meteorology that the term 'divergence' has come to be used by meteorologists as a shorthand technical jargon for 'divergence of the horizontal wind vector'. When meteorologists refer to 'the divergence', they mean $\text{div}_h \mathbf{V}$.

It should be remembered that the equation of continuity is really a statement about the field of mass in the atmosphere. Its frequent expression exclusively in terms of wind velocities, as in equation 2.18, is a simplification. It is, however, a very useful simplification which serves to highlight the most important association which exists between the convergence/divergence of the horizontal wind and the occurrence of vertical motion in the atmosphere (see 2.2.9.1, page 40). Vertical motion is of the greatest importance, for ascending air expands and cools adiabatically, and this can lead to the formation of clouds and precipitation. On the other hand, subsidence leads to adiabatic warming and the dispersal of existing clouds.

Weather is therefore directly related to the patterns of the $\text{div}_h \mathbf{V}$ and the vertical motion. Both are consequently of fundamental importance. Unfortunately it is very difficult to make direct measurements of either the large-scale convergence/divergence or the vertical velocities. And one of the main problems of theoretical meteorology is to find alternative ways of expressing the dynamical equations, in order that weather developments can be related to easily observed atmospheric variables rather than to variables which, though they may be more fundamental, cannot in practice be adequately observed at all.

As a corollary to the simple two-level model of atmospheric convergence and divergence (illustrated in Figure 22 on page 41), it follows that at some middle level the divergence must be zero. In practice there may often be more than one such level and, except in localized regions, they are unlikely to be horizontal. Nevertheless, the concept of a 'level of non-divergence' existing somewhere in the middle troposphere is a reasonable one. The level is usually taken to be at about 500 millibars in numerical models of the atmosphere and this is frequently borne out in nature.

2.2.5.3 Ageostrophic motion. The ageostrophic wind ($\mathbf{V}_{ag}$) is defined as the vector difference between the actual wind and the geostrophic wind. So $\mathbf{V} = \mathbf{V}_g + \mathbf{V}_{ag}$. An alternative name for $\mathbf{V}_{ag}$ is 'geostrophic departure'.

It was stated in 2.2.3.1 (page 10) that the geostrophic wind is horizontal, non-accelerating and consequently non-developmental in the sense that all aspects of weather development which depend on vertical motions and changes of wind or pressure with time must be associated with the non-geostrophic, or ageostrophic, component of the air motion. Thus geostrophic flow is an acceptable approximation in large areas of the atmosphere at any time in which the broad-scale flow is steady and undramatic. But in those areas where there are active fronts, or large pressure changes, ageostrophic motions must be significant. Such areas may be quite limited in extent, but they are important from every point of view and the magnitude of the dynamic developments will be related in some way to the magnitude of the geostrophic departure.

Since geostrophic flow is a balanced type of motion in which the acceleration is zero, ageostrophic motion is always associated with some resultant acceleration in the equations of motion. It is possible to show, by algebraic manipulation of the equations, that the vector representing the ageostrophic motion ($\mathbf{V}_{ag}$) is directed perpendicularly towards the left of the direction of the resultant acceleration, as in Figure 12. This relationship is a helpful method of assessing the sense of the ageostrophic motion in certain types of flow.

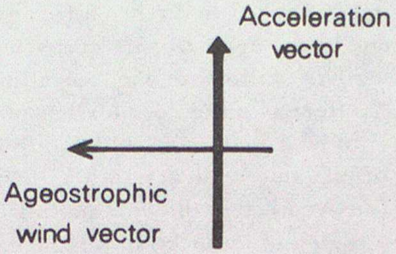


FIGURE 12. Acceleration and ageostrophic wind vectors

As a simple example, consider the case of gradient wind flow around a circular depression. The centripetal acceleration is known to be invariably directed inwards towards the centre of the low — see Figure 5 (page 12). Consequently $\mathbf{V}_{ag}$, directed to the left of the acceleration, is in the opposite sense to that of $\mathbf{V}_g$ which flows anti-clockwise about the centre of low pressure. This is another demonstration of the fact that the gradient wind ($\mathbf{V}_g + \mathbf{V}_{ag}$) is less than the geostrophic wind around a low. Around a high the ageostrophic wind reinforces the geostrophic wind, so the gradient wind is greater than the geostrophic wind in this case.

Two more regions where the mutual relationship between the direction of the acceleration and that of the ageostrophic motion gives a helpful interpretation are at the entrances and exits of jet streams. In Figure 13 the flow of air, given by the geostrophic wind $\mathbf{V}_g$, is from left to right. At the jet entrance, the

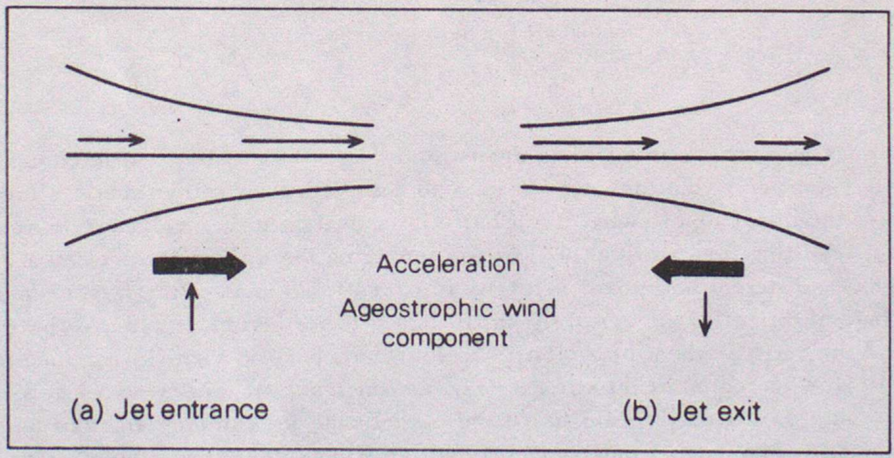


FIGURE 13. Ageostrophic motions at the entrance and exit of a jet stream

air accelerates into the jet, resulting in ageostrophic flow from the warm to the cold side of the jet. There is therefore a physical accumulation of air at high levels on the left of the jet entrance (convergence) and a depletion of air on the right of the entrance (divergence). In the simple development model (illustrated in Figure 22, page 41) this convergence/divergence at high levels is associated with appropriate vertical motions, and compensating low-level divergence/convergence. Thus at the jet entrance the ageostrophic motion leads to an accumulation of air and rising pressure (anticyclonic development) on the left of the entrance, and to falling pressure (cyclonic development) on the right of the entrance. At the exit of the jet, the air decelerates so that the arrow in Figure 13(b), representing the acceleration vector, points in the negative direction which leads to a reversal of the pattern of the ageostrophic motion and consequent surface-pressure developments. The so-called 'C' and 'A' areas, representing cyclonic and anticyclonic developments at the surface, from this and other contour patterns are discussed in 2.2.9.2 (page 42).

Some of the ageostrophic motions associated with particular effects near the surface are comprehensible in general terms but not easy to separate out as individual effects of a particular magnitude on any given occasion. Thus friction near the ground produces one of the most readily apparent ageostrophic wind effects on a surface chart. The actual wind is backed in direction from the geostrophic wind given by the isobars and its speed is less. As in Figure 14 the ageostrophic component is given by $\mathbf{V} = \mathbf{V}_g + \mathbf{V}_{ag}$, but its practical computation is of little value. The isallobaric wind, as illustrated in Figure 7 (page 15), is another ageostrophic wind. In detail its effect is quite complex but in broad terms we may consider an isallobaric high, such as that in Figure 7, to be a region of low-level divergence, since the isallobaric wind everywhere flows from high values of pressure tendency to low. It is therefore associated with subsiding air aloft and a tendency to rather fine weather in the cold north-westerly airflow behind the depression. Conversely, the isallobaric low is a region of surface convergence, ascending air and cloudy wet weather.

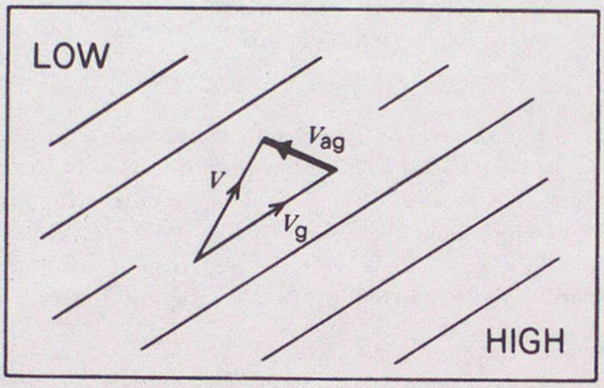


FIGURE 14. Ageostrophic wind component arising from friction at the surface

V = Actual wind (blowing across the isobars)
V_g = Geostrophic wind (parallel to isobars)
V_{ag} = Ageostrophic wind

There are other effects, yet, which give rise to ageostrophic motions and have some consequential effects upon weather developments, but they are not easy to deal with. One rather subtle effect arises from the acceleration of the ageostrophic wind itself but this is thought to be negligible in most cases. There is also an effect resulting from vertical air motions replacing the air at one level by air from a different level which has a different horizontal velocity. It arises from terms of the form $w\,\partial u/\partial z$. Near the ground the magnitude of this effect is certainly small, but at upper levels, in places where both the vertical motion (w) and the vertical shear ($\partial u/\partial z$) are large, it does become significant. Such regions are to be found above and below the cores of jet streams. The normal result of this effect is to diminish the cross-contour, ageostrophic motion on the cold (left-hand) side of the jet entrance and exit but to accentuate that component on the warm (right-hand) side. It is not at all easy to assess the magnitude of the effect subjectively, but there is no difficulty in taking it into account in numerical forecasting procedures.

2.2.5.4 Quasi-geostrophic approximation. The quasi-geostrophic approximation refers to the use of the geostrophic wind in certain terms of the equations of motion but not in others. It is an approximation which was introduced as a device for overcoming some of the theoretical problems which attended the development of the early numerical forecasting techniques.

The equations of motion in their full form (equation 2.3), and using the components of the actual wind (u, v), are extremely difficult to solve numerically because the accelerations on the left-hand side are small quantities which are largely derived from two much larger terms on the other side. These terms (the pressure gradient force and the Coriolis force) are approximately equal in size but opposite in sign.

Given the accuracy to which actual winds are measured, we cannot obtain the required accuracy in the acceleration term from the other terms in the equations. But if observed winds are not suitable for the accurate solution of the equations of motion, it may be best to use the simplest available approximation - the geostrophic wind. Unfortunately, another snag arises, for if geostrophic winds (u_g, v_g) are used instead of actual winds throughout the equations, then it is found that $\mathrm{div}_h \mathbf{V}_g$ is a very bad estimate indeed of $\mathrm{div}_h \mathbf{V}$ and the latter, as was mentioned in 2.2.5.2 (page 21), is a quantity upon which atmospheric development crucially depends. So, while it is very desirable, for numerical computations, to find some way of utilizing the great simplicity of the geostrophic wind, this has at the same time to avoid the explicit use of $\mathrm{div}\, \mathbf{V}_g$ in the equations.

The procedure is to manipulate the horizontal momentum equations (the equations of motion), in the manner described in 2.2.7.1 (page 32), to obtain the vorticity equation. This is a prognostic equation (containing terms with $\partial/\partial t$). Among its terms is one which describes the advection of the existing vorticity field at any level by the wind ($\mathbf{V}$) at that level. Here it is permissible to replace $\mathbf{V}$ by $\mathbf{V}_g$ without incurring any great penalty. But in another term, where the creation/destruction of vorticity is related to the divergence of $\mathbf{V}$, $\mathbf{V}_g$ cannot be substituted for $\mathbf{V}$. The expression in $\mathrm{div}\, \mathbf{V}$ must be retained as it is, and subsequently eliminated algebraically by using the continuity equation.

Thus by a selective use of the actual wind ($\mathbf{V}$) and the geostrophic wind ($\mathbf{V}_g$), the 'quasi-geostrophic' vorticity equation can be derived. This equation is both acceptably realistic as a description of atmospheric motion and also quite easy to solve numerically. It formed the basis of most of the dynamical models used in the first decade or more of operational numerical forecasting. But with the development of more complex models and more powerful computers, the quasi-geostrophic assumptions are not necessary, as other ways have been found for solving the 'primitive equations' - the basic equations of motion in their full form.

2.2.5.5 Barotropic and baroclinic atmospheres. The acceleration of an air particle depends upon its mass and upon the forces acting on it. In the atmosphere, gravity is the only external force and the most important internal force is pressure, which is always of major significance. An understanding of atmospheric motion thus requires a knowledge of the distribution of mass and pressure. For this purpose, the field of pressure is normally represented by maps depicting the height of a given isobaric surface and the field of mass is conveniently represented by a temperature (thickness) chart.

If the pressure forces and gravity are in balance, and there are no buoyancy forces tending to change the motion of the air, then the surfaces of equal mass and surfaces of equal pressure in the atmosphere will coincide. In other words, isobaric surfaces and isothermal (or constant thickness) surfaces will coincide and there will be uniform temperature all over every isobaric surface. Such an atmosphere is termed 'barotropic'. Since it is evident that the real atmosphere normally exhibits some gradient of thickness, even in the most uniform air mass, the concept of barotropy is a somewhat hypothetical one. But, nevertheless, it can be applied very usefully, as a simple approximation, to quite large regions of the real atmosphere, wherever thickness gradients are small.

The contrasting situation in which the surfaces of mass and pressure do not coincide is termed a 'baroclinic' atmosphere. In this case, buoyancy forces tend to adjust the imbalance between the pressure forces and gravity. The resulting accelerations generate circulations which lead to the development of significant weather systems. Baroclinic atmospheres exhibit temperature gradients on the isobaric surfaces and have thickness patterns. The real atmosphere is always, in fact, more or less, baroclinic.

Petterssen[1] showed that the degree of baroclinicity is proportional to the thermal wind speed, so that thickness patterns give a very clear picture of the baroclinicity of the atmosphere. Closely packed thickness lines, such as are associated with frontal regions, indicate a strongly baroclinic zone. Widely spaced thickness lines indicate the weakly baroclinic (or quasi-barotropic) conditions associated with the uniform air masses far from any active frontal region.

2.2.5.6 Baroclinic instability. In the broad, upper-level, westerly flow in the atmosphere, small wave-like perturbations in the wind flow occur from time to time. Sometimes these waves grow into major disturbances and sometimes they do not; but when small waves in a baroclinic zone do amplify with time then the atmosphere is said to be baroclinically unstable. The concept of baroclinic instability is certainly relevant to the development of frontal depressions in the westerlies of temperate latitudes. It is probably also a factor in the development of 'easterly waves' in low latitudes.

Fleagle[13] developed a criterion for the onset of baroclinic instability that can readily be applied in the lower and middle layers of the troposphere. His criterion is the vertical wind shear ($\partial U/\partial z$), which must attain a certain minimum value before baroclinic instability can occur. The relationship between this critical value of $\partial U/\partial z$ and the amplification of developing baroclinic waves is not simple. Parker[14] showed that the critical value of the shear depended on three things:

(a) The static stability of the air

Dry air with a lapse rate approaching the dry-adiabatic lapse rate, or cloudy air with the saturated-adiabatic lapse rate becomes baroclinically unstable with a much smaller vertical wind shear than if the air is statically stable.

(b) The latitude of the perturbation

Air with a given value of static stability and vertical wind shear is more likely to be baroclinically unstable in high latitudes than in low latitudes. However, in the tropics the possibility of baroclinic instability is not ruled out, especially in unstable air.

(c) The wavelength of the perturbation

Perturbations of short wavelength (high wave number) are baroclinically unstable with smaller vertical wind shears than are required by long wavelength features.

These factors are relevant not only to the onset of baroclinic instability but also to the degree of amplification of the disturbance that results. Fleagle's work shown that at 60°N the baroclinic disturbances which amplify most rapidly are those with a wavelength of about 4000 kilometres. This distance covers about 70 degrees of longitude in a west to east direction and is roughly the size of the typical upper-air trough/ridge systems frequently observed in association with North Atlantic frontal depressions and anticyclones on a surface chart.

2.2.5.7 Limiting value of anticyclonic shear. The angular velocity of the earth (Ω) is a constant (2π rad/sidereal day). It is one of the fundamental controls on the types of flow that occur in the atmosphere, and it imparts some vorticity (f) to every particle on the earth or in its atmosphere. The absolute vorticity (ζ_a) of any particle in the atmosphere is given (see 2.2.5.1, page 18) by $\zeta_a = \zeta + f$.

It might be expected that since the earth always rotates in the same sense, and f consequently always has the same sign (positive), this would impose some constraint on the character of possible atmospheric motions. This is indeed so, and although the mathematical demonstration is not easy it turns out that, for airflow on a scale significantly influenced by the earth's rotation, the absolute vorticity of atmospheric particles must have the same sign as that of the rotating earth if the flow is to be dynamically stable, i.e., $\zeta_a > 0$.

Putting it slightly differently, it can be shown that if $\zeta_a > 0$, the flow is dynamically stable, and, if $\zeta_a < 0$, the flow is dynamically unstable. Dynamic instability is a concept which refers to the tendency for any moving fluid not to flow smoothly, but to develop turbulent eddies or ripples.

Since f is always positive, the only situations in which ζ_a can possibly attain negative values are those in which the relative vorticity (ζ) of the flow is negative (anticyclonic). The equatorward side of westerly jet streams, especially those jets which are curved anticyclonically, are the regions where ζ_a most frequently tends to become negative. Indeed, there is no doubt that the anticyclonic shear in these regions does at times build up sufficiently to make $\zeta_a < 0$, but it only occurs for short periods and over limited areas. Where it does occur, the flow develops turbulent ripples, which leads to lateral mixing of the air and reduces the wind shear (and hence the relative vorticity) below the critical value of f. These turbulent ripples, sometimes encountered by aircraft as one form of clear-air turbulence, are therefore a mechanism which prevents the anticyclonic shear from exceeding the critical value of f except for very short periods. Viewing the character of atmospheric flow in a synoptic scale, it can be said that ζ_a is always positive and that there is a limiting value to the possible anticyclonic vorticity which equals the local value of f.

2.2.6 Some thermodynamical topics

2.2.6.1 Geopotential, thickness, and thermal wind. Geopotential (Φ) is the potential energy of a body of unit mass, referred to some arbitrary datum (it has the dimensions L^2T^{-2}). It depends on the geometric height (z) and gravity (g) and, if mean sea level is the selected level where the geopotential is zero,

$$\Phi = \int_0^z g \, dz \qquad . \; . \; . \; . \; (2.20)$$

A geopotential surface is one in which air moves without undergoing any change in potential energy. This is not in general the same as a geometrically level surface (z = constant), because there are small but significant changes of g with latitude.

Because this variation of g is avoided, it is more convenient to describe the distribution of mass in the atmosphere by using Φ rather than the height (z). Thus, the hydrostatic equation $\partial p/\partial z = -g\rho$ can also be written $\partial p/\partial \Phi = -\rho$. From the gas equation $p = R\rho T$, this can be changed to $\partial p/\partial \Phi = -p/R\,T$, whence

$$\Phi = R \int_p^{p_0} T \, d(\log p) \qquad . \; . \; . \; . \; (2.21)$$

and this equation, using radiosonde measurements of p and T, enables the geopotential height of a given isobaric surface to be calculated. The practical unit of geopotential which is most widely used is the 'standard geopotential metre' (Z). It is related to geometric height (z) by $Z = g\,z/9.80665$, and thus has a numerical value approximately equal to z.* It replaced (in 1972) the 'geopotential metre' ($= g\,z/9.8$).

It follows from equation 2.21 that the difference in geopotential between two pressure levels is given by:

$$\Phi_2 - \Phi_1 = R \int_{p_2}^{p_1} T \, d(\log p) = R\,\overline{T} \log \frac{p_1}{p_2} = \overline{T} \times \text{constant} \qquad . \; . \; . \; (2.22)$$

where $\Phi_2 - \Phi_1 = h'$ is the 'thickness' of the layer between the two pressure levels and $\overline{T}$ is the mean (virtual) temperature of that layer. Thus the thickness of a layer is proportional to its mean temperature, and 'thickness charts' depict the horizontal distribution of the thickness, or mean temperature, between two specified pressure levels.

For large-scale motions in the atmosphere, for which the vertical equation of motion reduces to the hydrostatic equation $\partial p/\partial z = -g\rho$, we may write

$$\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial n}\right) = -g\frac{\partial \rho}{\partial n}. \qquad . \; . \; . \; (2.23)$$

This relates the vertical variation of the pressure gradient to the horizontal gradient of the density. And since, from the geostrophic wind equation, the pressure gradient is proportional to the geostrophic wind

* See World Meteorological Organization Technical Regulations, WMO No. 49, Vol. 1, 1971, Appendix C.

and also, from the gas equation, density is related to temperature, equation 2.23 can be rephrased in a form in which it relates $\partial V_g/\partial z$ to $\partial T/\partial n$. So, in a hydrostatic atmosphere, the change of geostrophic wind with height through a layer is proportional to the horizontal temperature gradient in that layer. Because of this dependence on temperature, the vector difference between the geostrophic winds at the top and bottom of the layer is called the 'thermal wind'. Figure 15 illustrates the vector relationship.

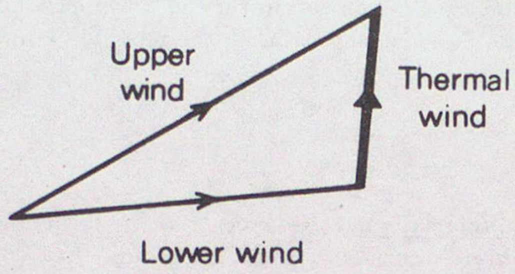


FIGURE 15. Vector diagram illustrating the thermal wind

The geostrophic wind at the top of the layer is the sum of the geostrophic wind at the bottom plus the thermal wind determined by the mean isotherms, or thickness lines, within the layer.

The thermal wind (V_T), given by $V_T = g/f\ \partial h'/\partial n$, is a geostrophic wind, related to the thickness (h') in the normal geostrophic manner. Its direction is parallel to the thickness lines, with low (cold) values on the left, and its speed is inversely proportional to the spacing between the thickness lines.

Before numerical forecasts of upper-air patterns were introduced, the thickness chart was a key tool in the forecasting technique. Starting with a carefully analysed surface chart, the geostrophic winds at 1000 millibars were derived and then combined vectorially with the thermal wind field by means of a graphical technique known as 'gridding'. Figure 16 illustrates the manner in which the intersections of the low-level contours and the thickness lines can be utilized to obtain values of the upper-level contour

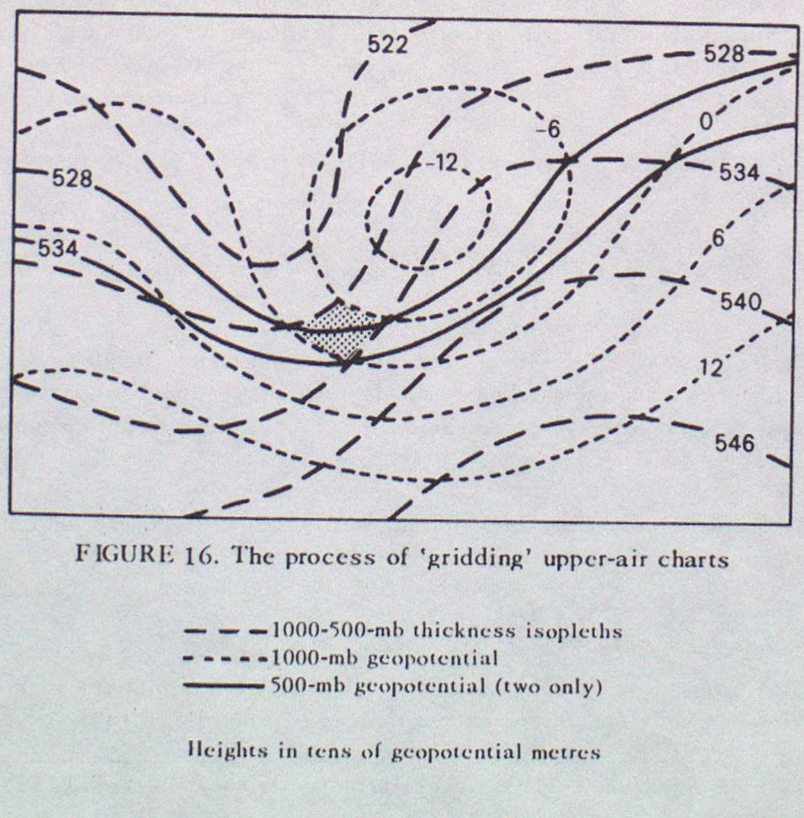


FIGURE 16. The process of 'gridding' upper-air charts

— — — 1000-500-mb thickness isopleths
- - - - - 1000-mb geopotential
——— 500-mb geopotential (two only)

Heights in tens of geopotential metres

height. This gridding technique produced a set of upper-level contour lines and hence, upper-level geostrophic winds. It was used to analyse the conditions at any required level, in a manner which took the fullest possible advantage of the dense observational coverage near the surface, and maintained the vertical consistency of the charts at all levels so that a good three-dimensional analysis was obtained. Furthermore, since thickness is a measure of temperature, it was possible to attempt forecasts of thickness changes based on a knowledge of the physical processes which bring about temperature changes.

These were well understood in a qualitative sense, even though there were big gaps in the knowledge of the exact magnitudes of the various effects. However, with gradually accumulated experience and empirical knowledge it was possible to produce forecast surface charts and forecast thickness charts which, together, formed the basis for a set of mutually consistent forecast upper-level charts, again by using the gridding technique.

Thickness charts no longer have a central place in operational forecasting procedures. However, from the point of view of subjective understanding, their usefulness as pictures of the broad atmospheric temperature distribution has in no way lessened. And in numerical forecasting models, it is still the case that thickness is used both as a representation of temperature and also as a constraint on the vertical consistency of the computations.

2.2.6.2 Thermodynamic equations: entropy and potential temperature. This section is mostly just a brief statement of standard physical equations. They form part of the necessary background of all thermodynamic theory, rather than being of specific utility to forecasters. They are included here for completeness and reference.

The gas equation can be written

$$p\alpha = RT \quad \text{or} \quad p = R\rho T.$$

This is an equation of state for a gas, relating the pressure, temperature and specific volume or density, A gas which obeys this relation exactly is called a 'perfect gas'. For all practical purposes it is applicable to unsaturated air in the atmosphere.

The thermodynamic energy equation, embodying the principle of conservation of energy, is

$$\underset{\substack{\text{gain of heat}\\\text{from surroundings}}}{dQ} = \underset{\substack{\text{change in}\\\text{internal energy}}}{c_v\,dT} + \underset{\substack{\text{work done on}\\\text{surroundings}}}{p\,d\alpha}$$

or, using the differentiated form of the gas equation ($p\,d\alpha + \alpha\,dp = R\,dT$) and $c_p - c_v = R$, then

$$dQ = c_p\,dT - \alpha\,dp\,. \qquad \ldots \quad (2.24)$$

This is the first law of thermodynamics.

From the second law of thermodynamics is derived the concept of entropy (ϕ), which is a fundamental but difficult concept. Entropy is a function of p, α and T and, like potential energy, any actual value assigned to it is by reference to some arbitrary zero. Meteorologically, changes of entropy are of more interest than any absolute value, and a sample of air which absorbs a certain quantity of heat (dQ) at a temperature T (K) undergoes a change of entropy, $d\phi = dQ/T$.

In the atmosphere, adiabatic temperature changes are very important. In these no heat is gained from, or given to, the surroundings. So $dQ = 0$ and consequently there are no entropy changes during adiabatic processes. Such processes are called 'isentropic'.

Potential temperature, θ, is defined as the temperature which a sample of air would attain if it were transferred adiabatically (that is to say, at the dry-adiabatic lapse rate) to the 1000-millibar level. Throughout such a transfer the entropy of the air sample remains unchanged, since it is undergoing an adiabatic process. Consequently an isentropic process is characterized by having a constant potential temperature. ϕ and θ are related by

$$\phi = c_p \log\theta + \text{constant.} \qquad \ldots \quad (2.25)$$

It is fortunate for meteorologists that this equivalence of isentropic and adiabatic processes exists. It means that there is no need to worry too much about the rather obscure concept of entropy, but to think instead of more readily conceived ideas which have direct practical relevance. For example, the basic diagram used in the Meteorological Office to display upper-air temperatures is the tephigram, or T– ϕ gram. The basic axes of this diagram are, in theory, temperature (T) and entropy (ϕ), but it is in fact much more usefully considered to be a diagram of T against $c_p \log \theta$. The isentropic lines, being lines of constant potential temperature, indicate the dry-adiabatic lapse rate.

By using a tephigram it is easy to demonstrate that unstable air is characterized by a decrease of potential temperature with height ($\partial\theta/\partial z < 0$), whereas the more usual stable air shows the potential temperature increasing with height ($\partial\theta/\partial z > 0$).

Potential temperature (or, even more, the wet-bulb potential temperature, θ_w) is a useful parameter for displaying the vertical structure of the air in a cross-section diagram. Inversions of temperature (whether frontal, or anticyclonic, or at the tropopause) show up very clearly by the concentration of isopleths of θ; and areas of instability are typified by near zero, or negative, values of $\partial\theta/\partial z$ which leads to the isopleths of θ being very widely spaced in the vertical.

2.2.6.3 The theory of thickness changes. If h' is the thickness of the layer bounded by the pressure levels p_1 and p_2, equation 2.22 (page 27) gives

$$h' = R\int_{p_1}^{p_2} T \, \mathrm{d}(\log p)$$

so that the change of thickness is given by :

$$\frac{\partial h'}{\partial t} = R\int_{p_1}^{p_2} \frac{\partial T}{\partial t} \, \mathrm{d}(\ln p) \qquad \ldots \quad (2.26)$$

Strictly, T is the virtual temperature, which is the temperature of dry air having the same density and pressure as the given mass of moist air. Virtual temperature, T_v, is always greater than the actual temperature, T, of the air and is given by $T_v = (1 + 0.6r)\, T$, where r is the moisture content (humidity mixing ratio) of the air in g/kg. For convenience, and because the differences are relatively small in temperate latitudes, T_v will simply be referred to as the temperature, T, in the rest of this section.

In equation 2.26 we can make use of the expression for a total derivative

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \omega\frac{\partial T}{\partial p} \qquad \ldots \quad (2.27)$$

and an expression for the rate of change of the temperature of an individual air particle ($\mathrm{d}T/\mathrm{d}t$) derived from the thermodynamic energy equation (equation 2.24, page 29) which leads to

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{c_p}\frac{\mathrm{d}q}{\mathrm{d}t} + \Gamma\omega \qquad \ldots \quad (2.28)$$

where q = heat energy supplied (per unit mass of gas) by effects such as radiation, conduction, condensation of water vapour and generation of heat by friction;

c_p = specific heat at constant pressure; and

Γ = adiabatic lapse rate in pressure co-ordinates.

Eliminating dT/dt between equations 2.27 and 2.28 and then substituting in equation 2.26 for $\partial T/\partial t$, we get

$$\frac{\partial h'}{\partial t} = R\int_{p_1}^{p_2}\left[-\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)+\left(\Gamma-\frac{\partial T}{\partial p}\right)\omega+\frac{1}{c_p}\frac{dq}{dt}\right]d(\ln p) \quad . \; . \; . \quad (2.29)$$

Symbolically, this equation may be written

$$\frac{\partial h'}{\partial t} = \mathrm{H} + \mathrm{V} + \mathrm{N}$$

which equates the thickness change at a point ($\partial h'/\partial t$) with three physical processes which can cause the temperature to vary:

H = Horizontal advection of the existing thickness field.

V = Vertical motion, leading to adiabatic temperature changes.

N = Non-adiabatic (or diabatic) temperature changes due to heat exchanges with regions external to the atmosphere.

The horizontal advection term, H

$$\mathbf{H} = -R\int_{p_1}^{p_2}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)d(\log p)$$

may be written in approximate form

$$\mathbf{H} = -R\left(\bar{u}\frac{\partial \bar{T}}{\partial x}+\bar{v}\frac{\partial \bar{T}}{\partial y}\right) = -R\left(\bar{u}\frac{\partial h'}{\partial x}+\bar{v}\frac{\partial h'}{\partial y}\right) \quad . \; . \; . \quad (2.30)$$

which describes the advection of the mean temperature field ($\bar{T}$), or thickness (h'), between the two pressure levels p_1, p_2 by some suitably defined mean wind ($\bar{u}$, $\bar{v}$) between the same levels. The thickness changes due to horizontal advection are proportional to the strength of the wind and to the gradient of the thickness field. In the days before computer forecasts became available it was often a practical necessity to take the 1000-millibar geostrophic wind as a first approximation to the 'mean wind' when evaluating the effects of this term. Nowadays this practice is unnecessary as the computer can solve equation 2.29 with much better accuracy. But, for subjective appreciation, forecasters may still find it operationally convenient to assess the general effects of advection by considering the thickness lines to be moved by the 1000-millibar geostrophic winds. This is, however, no more than a first approximation.

The vertical motion term, V

$$\mathrm{V} = R\int_{p_1}^{p_2}\left(\Gamma-\frac{\partial T}{\partial p}\right)\omega\, d(\log p) \quad . \; . \; . \quad (2.31)$$

The term ($\Gamma - \partial T/\partial p$) represents the difference between the adiabatic lapse rate and the actual lapse rate of the air (using the appropriate value according as the air is saturated or unsaturated). It is therefore a measure of the static stability of the air, which can be written simply as S, and so $\mathrm{V} = R(\bar{S}\,\bar{\omega})$ where $\bar{S}$ and $\bar{\omega}$ have mean values for the layer p_1, p_2.

If the actual lapse rate is stable, $\bar{S} > 0$.

If the actual lapse rate is unstable, $\bar{S} < 0$.

Now, it is only in rather limited areas that the actual lapse rate does, in fact, exceed the adiabatic lapse rate. The situation certainly does arise: for example, in a shallow layer above the earth's surface on a sunny day the lapse rate becomes superadiabatic; or within the saturated, cloud-filled air of a convective storm cloud the lapse rate may well be greater than the saturated adiabatic lapse rate. However, when considering a substantial depth of the atmosphere on the broad scale, the mean stability of the air is such that $\bar{S} > 0$. As a consequence, the sign of the contribution which the vertical-motion term makes to the rate of thickness change $(\partial h'/\partial t)$ is given by the sign of $\bar{\omega}$. So:

Subsidence ($\bar{\omega} > 0$) leads to warming and an increase of thickness ($\partial h'/\partial t > 0$).

Ascent ($\bar{\omega} < 0$) leads to cooling and a decrease of thickness ($\partial h'/\partial t < 0$).

The less important cases mentioned above, characterized by $\bar{S} < 0$, are only significant over small time intervals and in mesoscale weather systems. Thus, within the vigorous convective downdraught ($\bar{\omega} > 0$) of a cumulonimbus, the rapid subsidence of conditionally unstable air into which rain is evaporated ($\bar{S} < 0$, for saturated air) can lead to a decrease in the mean temperature of the local air column; and, conversely, the ascent of such air may lead to an increase of thickness. This happens in the central areas of tropical storms, where the effect is important in the formulation of the 'warm core' of such storms. In temperate latitudes none of these less-common effects is normally found to be significant on the synoptic scale.

The non-adiabatic heating term, N

$$N = R\int_{p_1}^{p_2} \left(\frac{1}{c_p}\ \frac{dq}{dt}\right) d(\log p) \qquad \ldots\ . \quad (2.32)$$

This term represents the local change in thickness due to the supply or removal of heat (dq/dt) and it covers many physical processes. There are the heat exchanges between the air and the earth's surface, by which cold air may gain heat from a warmer surface or warm air may lose heat to a colder surface. There is radiation, from the sun and from the earth and from all the constituent parts of the atmosphere itself, all forming a complex total effect. These various effects are not yet so completely understood in theory that it is possible to model them with full confidence in numerical work, and certainly the subjective assessment of these effects is almost impossible. Fortunately they normally seem to have only a small direct effect on day-to-day synoptic-scale developments, but they are likely to be of increasing importance as the forecast period extends further ahead to periods of several days or more.

Since the introduction of numerical forecasting methods, forecasters no longer need to attempt a precise estimate of future thickness changes. The effects of horizontal advection and of vertical motion are easily handled by the computer. The non-adiabatic effects are less fully understood, but as the experimental knowledge and theoretical understanding of particular physical processes improve so it becomes possible to simulate these processes in an increasingly satisfactory manner in the computer models.

2.2.7 The vorticity equation and its applications

2.2.7.1 The vorticity equation. Circulation patterns are among the most obvious features of daily synoptic charts. Sometimes there are closed circulations (lows and highs) or, more often, there are curved patterns (troughs and ridges) without any completely closed vortices. These patterns and their predicted development form a large part of the business of weather forecasting. Vorticity, which is a measure of the fluid rotation at a point, is related in a simple way to the concept of circulation and forms the link between the commonplace synoptic patterns of contours or isobars, and their representation in a form that is suitable for precise numerical forecasting. Vorticity is therefore a basic concept in any mathematical forecasting system.

The vorticity equation expresses the reasons why the vorticity of an air particle changes, and it provides the means whereby the magnitude of those changes can be calculated. It is derived quite simply from the equations of motion.

Noting that $\zeta = \partial v/\partial x - \partial u/\partial y$, we proceed by differentiating the v component in the equations of motion by x, and the u component by y, and then subtracting the result. After a little algebraic manipulation this leads to:

$$\frac{d}{dt}\left(\zeta + f\right) = -\left(\zeta + f\right)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \left(\frac{\partial u}{\partial p}\frac{\partial \omega}{\partial y} - \frac{\partial v}{\partial p}\frac{\partial \omega}{\partial x}\right) + \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}\right) \quad . \quad . \quad (2.33)$$

or

$$\frac{d}{dt}\zeta_a = -\zeta_a \operatorname{div}_h \mathbf{V} + \text{TILT} + \text{FRICTION.}$$

This is the vorticity equation.

On the left-hand side of the equation is the expression for the total rate of change of the absolute vorticity of an air particle. This is not the rate of change of ζ_a at a fixed place, which would be written $\partial\zeta_a/\partial t$, but the rate of change of ζ_a of an individual air particle as it moves in the atmosphere.

On the right-hand side are three terms, each of which describes a different physical process which can change the absolute vorticity of a moving air particle. These are, briefly,

(a) the divergence of the horizontal wind field, which can increase or decrease vorticity (but, if $\zeta_a = 0$, the divergence has no effect);

(b) spatial variations in the vertical motion field, which can alter the tilt of the three-dimensional vortex tubes and thus alter the vertical component of the vorticity; and

(c) friction, which opposes motion, and changes relative vorticity as a result.

The frictional term is of fundamental importance in the long-term climatology of the atmosphere but it is quite negligible when considering day-to-day changes.

The tilting term also is small. For, if we are considering large-scale synoptic motions, ω is small and so are $\partial\omega/\partial x$ and $\partial\omega/\partial y$. In practice, therefore, these two terms produce no more than marginally significant refinements to the vorticity forecasts deduced from the vorticity equation.

By far the most important effect is that due to the divergence term and, if we ignore the effects of the frictional and tilting terms altogether, we can write down a simplified vorticity equation:

$$\frac{d}{dt}\zeta_a = -\zeta_a \operatorname{div}_h \mathbf{V} \quad . \quad . \quad . \quad . \quad (2.34)$$

and this equation accounts for nearly all the major changes in vorticity that occur in synoptic-scale weather systems in the atmosphere.

Examining the right-hand side of equation 2.34 we note that $\zeta_a (= \zeta + f)$ can be taken to be positive for synoptic-scale circulations. This does not rule out the possibility that it may attain negative values for short periods in certain localized areas, but such values will be only temporary. For f is certainly positive and ζ can only be less than $-f$ in regions where the anticyclonic wind shear exceeds a certain maximum stable value (see 2.2.5.7, page 26). When this happens the motion tends to break down into turbulent eddies which smooth out the velocity field and restores the anticyclonic vorticity to values less than f. For broad-scale conditions, therefore, the minimum value of ζ_a is zero, and in equation 2.34 the sign of $d\zeta_a/dt$ is opposite to the sign of $\operatorname{div}_h \mathbf{V}$.

So ζ_a increases (becomes more cyclonic) with convergence, and decreases (becomes more anticyclonic) with divergence. Now, changes in ζ_a can arise in meridional flow through changes in f but generally, where the flow is largely zonal, changes in ζ_a are not very different from the changes in ζ. When studying synoptic charts it is permissible to consider the relative vorticity, ζ, and to associate cyclogenesis (or increasing cyclonic ζ) with horizontal convergence, and anticyclogenesis with horizontal divergence.

The simplified vorticity equation is really the analogue in fluid motion of the principle of conservation of angular momentum in solid-body rotation. The familiar example of a skater who dramatically increases his angular velocity by drawing in his outstretched limbs during the execution of a spin is the solid counterpart to the process of convergence in a fluid leading to an increase in cyclonic vorticity.

It is useful to expand the total derivative on the left-hand side of equation 2.34 in order to express the equation in terms of changes occurring at a point. This is frequently of interest to a forecaster. Thus we get:

$$\frac{\partial}{\partial t}\left(\zeta + f\right) + u\frac{\partial}{\partial x}\left(\zeta + f\right) + v\frac{\partial}{\partial y}\left(\zeta + f\right) = -\left(\zeta + f\right)\,\mathrm{div_h}\,\mathbf{V}$$

and, since f varies only with latitude, $\partial f/\partial t = \partial f/\partial x = 0$, so

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + v\frac{\partial f}{\partial y} = -\left(\zeta + f\right)\,\mathrm{div_h}\,\mathbf{V}$$

or

$$\frac{\partial \zeta}{\partial t} = -\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\zeta - v\frac{\partial f}{\partial y} - \left(\zeta + f\right)\,\mathrm{div_h}\,\mathbf{V} \quad . \; . \; . \; . \quad (2.35)$$

which shows that $\partial\zeta/\partial t$, the variation of relative vorticity at a fixed point, is made up of contributions from three sources:

(a) $-(u\,\partial/\partial x + v\,\partial/\partial y)\zeta$ which represents the advection of the existing field of ζ by the horizontal wind (u, v);

(b) $-(v\,\partial/\partial y)f$ which represents the advection of different values of f when there is any meridional wind flow; and

(c) $-(\zeta + f)\mathrm{div_h}\mathbf{V}$ which represents the creation or destruction of vorticity by the divergence.

The importance of the vorticity equation stems largely from the fact that, with only a few very reasonable assumptions, it reduces to a comparatively simple form in which it is suitable for both computational work and subjective understanding. Yet, despite its simplicity, it turns out to be a very good forecasting tool for synoptic-scale weather systems.

2.2.7.2 Geostrophic vorticity. Geostrophic vorticity, ζ_g, is simply the vorticity of the geostrophic wind (u_g, v_g).

When the vorticity equation is used in computational work, it is permissible to substitute the geostrophic wind (u_g, v_g) for the actual wind (u, v) in certain parts of the equation (see 2.2.5.4, page 24) and, similarly, ζ may be replaced by ζ_g.

Since the geostrophic wind (u_g, v_g) can be expressed as $(-\,g/f\;\partial h/\partial y,\; g/f\;\partial h/\partial x)$ (by splitting equation 2.5 (page 10) into x and y components) we have, if we neglect any variations in f,

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{g}{f}\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) = \frac{g}{f}\nabla^2 h \quad . \; . \; . \; . \quad (2.36)$$

where $\nabla^2 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$ is the two-dimensional Laplacian operator. (See part 3 of the Appendix.) So, knowing the values of g/f, ζ_g can be expressed in terms of the spatial variations of the contour height h.

For example, in Figure 17, given the values of h at the five equally spaced points shown by dots, values of $\partial h/\partial x$ can be computed at the two points shown by crosses and $\partial h/\partial y$ at the two points shown by circles (each of these mid-way between two of the dots).

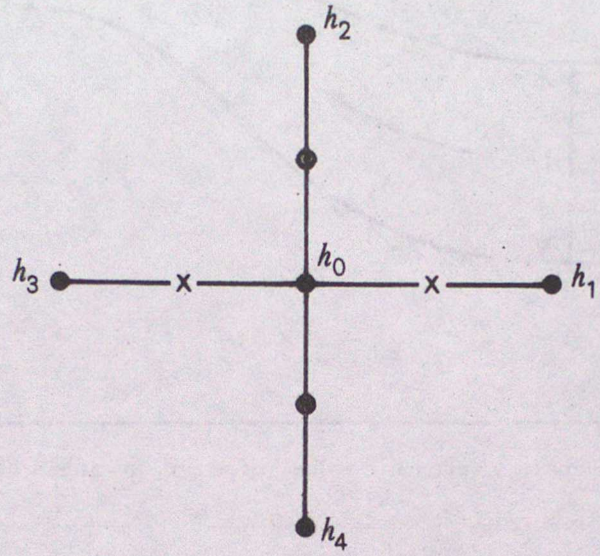


FIGURE 17. Computation of geostrophic vorticity

Thus the two values of $\partial h/\partial x$ are $(h_1 - h_0)/d$ and $(h_0 - h_3)/d$. Consequently the value of $\partial^2 h/\partial x^2$ at h_0 is given by

$$\left(\frac{h_1 - h_0}{d} - \frac{h_0 - h_3}{d}\right) / d$$

In a similar way, the value of $\partial^2 h/\partial y^2$ at h_0 can be obtained, so that

$$\zeta_g = \frac{g}{f}\nabla^2 h = \frac{g}{fd^2}(h_1 + h_2 + h_3 + h_4 - 4h_0) = \frac{4g}{fd^2}(\bar{h} - h_0) \quad . \; . \; . \; . \quad (2.37)$$

which is a very simple means for computing ζ_g at a point, from the values of the contour heights at surrounding points.

In the simpler numerical-forecasting models upper winds are represented, using the geostrophic assumption, by the gradient of the contour height h. In these models the vorticity equation, making use of this simple relationship between ζ_g and the distribution of h, is a central tool in the forecasting procedure. In the more complex numerical models, where the winds (u, v) are represented explicitly in addition to the contour heights, the concept of geostrophic vorticity is not required in the forecasting procedure. Nevertheless, it still remains a useful simple means of calculating and displaying the instantaneous pattern of vorticity whenever this may be required.

2.2.7.3 Potential vorticity. In 2.2.5.2 (page 21) we wrote the equation of continuity in the form $-\mathrm{div}_h \mathbf{V} = \partial\omega/\partial p$. It may also be expressed in terms of the proportional changes in the dimensions of a small volume of air. If, as in Figure 18, the volume has an initial horizontal cross-section of area A, and a vertical dimension expressed as a small pressure difference Δp, then the equation of continuity can be written:

$$-\frac{1}{A}\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{1}{\Delta p}\frac{\mathrm{d}}{\mathrm{d}t}(\Delta p) \quad . \; . \; . \; . \quad (2.38)$$

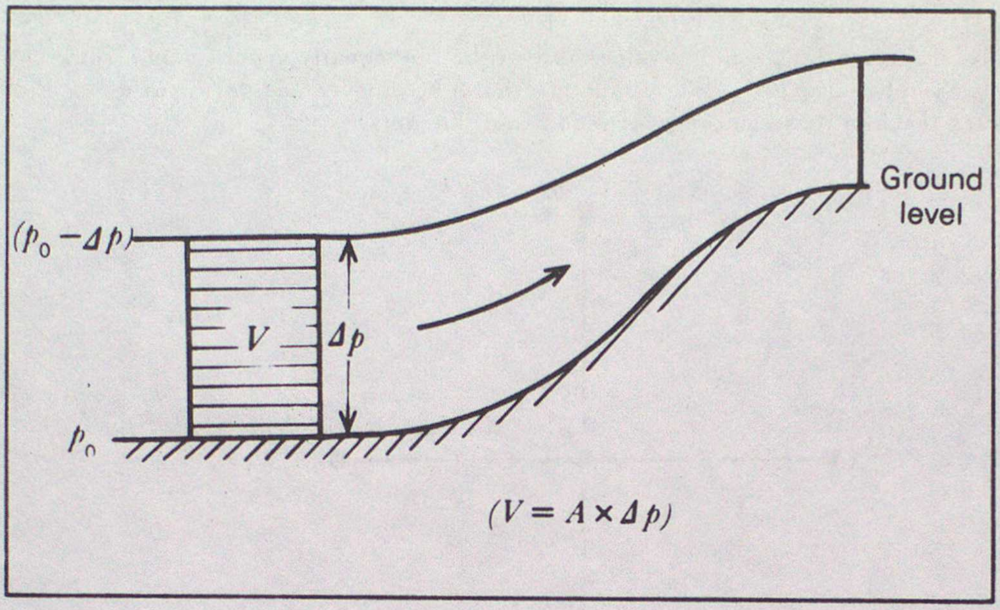


FIGURE 18. Movement of a schematic volume of air, V, towards a hill

reflecting the fact that convergence, for example, is associated with a decrease in the horizontal dimension and a proportional increase in the vertical dimension of the original volume.

But the simple form of the vorticity equation, which is applicable to motions in which the heat exchanges are adiabatic, is

$$-\operatorname{div}_h \mathbf{V} = \frac{1}{\zeta_a}\frac{d\zeta_a}{dt} \qquad \ldots\ldots \quad (2.39)$$

From equations 2.38 and 2.39 it follows that:

$$\frac{1}{\zeta_a}\frac{d\zeta_a}{dt} = \frac{1}{\Delta p}\frac{d}{dt}(\Delta p)$$

or

$$\frac{d}{dt}(\ln \zeta_a) = \frac{d}{dt}(\ln \Delta p)$$

whence

$$\frac{\zeta_a}{\Delta p} = \frac{\zeta + f}{\Delta p} = \text{constant.} \qquad \ldots\ldots \quad (2.40)$$

The quantity $(\zeta + f)/\Delta p$ has been given the name 'potential vorticity'. It is a conservative property of the air so long as the heat exchanges are adiabatic, and serves to illuminate some features of the air motion in the vicinity of hills.

In Figure 19(a) a straight westerly airstream approaches a north/south hill barrier. Consider, as in Figure 18 above, a volume of air, V, extending from the surface up to some initial level, Δp. V has a value of potential vorticity $(\zeta + f)/\Delta p$. As it flows up the windward slope of the hill, the streamlines of the flow at the top of V will be lifted somewhat less than those at the bottom of V where the flow necessarily follows the contours of the ground. Consequently Δp decreases and, to keep the potential vorticity constant, $(\zeta + f)$ must decrease. To achieve this, ζ must become more anticyclonic which will result in the .

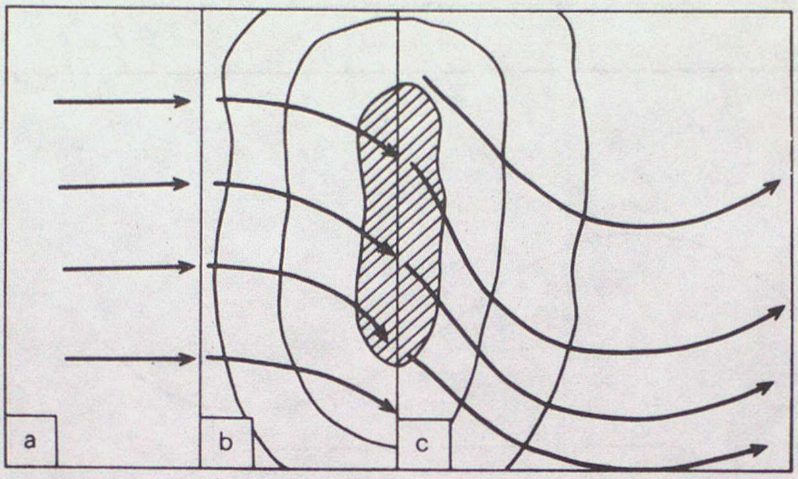


FIGURE 19. Plan view of airflow over a hill: (a) W'ly winds; (b) increasing anticyclonic curvature of streamlines on upslope side; (c) increasing cyclonic curvature and formation of lee trough

flow on the windward slope curving anticyclonically towards the south, as in Figure 19(b). In the flow from the lee slope, the downward flow leads to a vertical stretching of the volume V, accompanied by a horizontal contraction. This counteracts the anticyclonic curvature which was most pronounced at the crest of the hill. But to the lee of the hill (Figure 19(c)), the northerly component of flow, induced by the original upslope motion, leads to a gradual decrease in f. To counter this there must be an increase in ζ, which becomes more cyclonic and leads to the formation of a lee trough, or lee depression.

The formation of lee depressions is most marked in cases of northerly flow across east/west hills. The decrease of f in such situations gives an added emphasis to the increasingly cyclonic ζ on the southern side of the hills.

2.2.7.4 The conservation of absolute vorticity. If the simplified vorticity equation is applied at the level of non-divergence, where $\mathbf{div}_h \mathbf{V} = 0$, it reduces to

$$\frac{\mathrm{d}}{\mathrm{d}t}(\zeta + f) = 0 \qquad . \quad . \quad . \quad . \quad (2.41)$$

or

$$(\zeta + f) = \text{constant}.$$

So, at the level of non-divergence, the air moves (approximately) in such a way that the absolute vorticity of individual air particles is conserved.

If this concept is applied to the broad westerly currents of the troposphere at about the 500-millibar level (more or less the level of non-divergence), it is a straightforward deduction to see that the existence of any meridional component in the flow induces a wave-like pattern to develop. Taking $(\zeta + f)$ to be constant, and the trajectory of an air particle having initially a certain component towards the north (as in Figure 20), the gradual increase in f due to the increase in latitude leads to a gradual decrease in ζ. Consequently the curvature of the trajectory becomes increasingly anticyclonic. Eventually it curves sufficiently for the flow to lose its northward path and bend towards the south. When this occurs, f starts to decrease and ζ to increase. The anticyclonic curvature is gradually counteracted and the trajectory takes on an increasingly cyclonic curvature. Eventually this leads to the formation of a trough around which the trajectory gradually curves until it has a northward component once more, and then the whole wave cycle starts again.

Constant absolute vorticity trajectories (CAVT) are therefore wave-like patterns which are strikingly similar to the actual waves observed in the atmosphere.

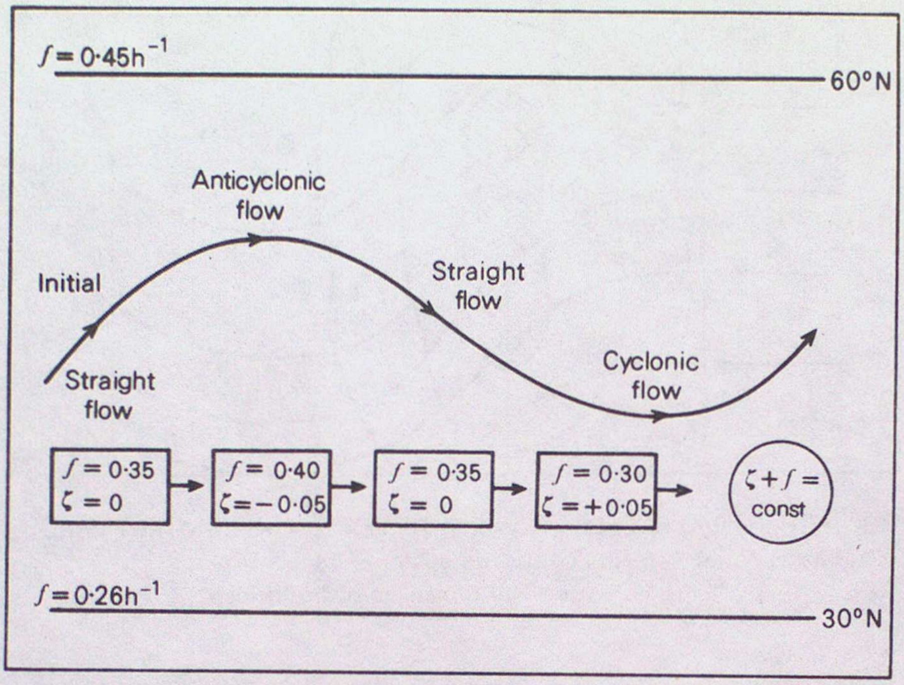


FIGURE 20. The conservation of absolute vorticity along the trajectory of an air particle

2.2.8 Wave motion in the atmosphere

2.2.8.1 Gravity waves. Many different types of wave motion occur in the atmosphere. Some are relevant to meteorology and others, like sound waves or shock waves, generally are not.

Gravity waves, in which the oscillatory motion is in a vertical plane, have some effects on a small scale in the atmosphere but have negligible effects on synoptic-scale systems. Lee waves, associated with some types of airflow over mountains, are gravity waves. They are examples of 'internal' gravity waves; they have a very short wavelength and their speed of propagation is almost zero. Waves of this type are certainly important to weather forecasters concerned with the mesoscale distribution of weather. The vertical motions associated with such waves may have a controlling influence on the detailed distribution of cloudy and clear areas; they may also produce turbulent conditions which are relevant in aviation forecasting (see Chapter 23 — Bumpiness in aircraft). Another kind of gravity wave, the so-called 'external gravity waves', can also occur in the atmosphere. They have a much longer wavelength than the internal gravity waves and are fast moving, like the ripples on the surface of a liquid. They are of no practical concern to synoptic meteorologists, but they are, however, much more noticeable and troublesome in numerical models of the atmosphere than in the atmosphere itself.

2.2.8.2 Long waves and short waves. The waves which are of most interest to synoptic meteorologists are those in which the oscillatory motion is in a horizontal plane. These waves are very common in the middle and upper troposphere and it is the normal practice to divide them into two broad classes called, quite simply, long waves and short waves.

Long waves are slow-moving, wave-like patterns in the major belt of global westerlies. They are characterized by long wavelengths and considerable amplitude. Short waves are mobile waves, with dimensions comparable with those of the synoptic-scale highs and lows on a surface-pressure chart. They move in the same direction as the basic current in the troposphere.

To clarify the nomenclature which is used in this chapter, and generally accepted, see Figure 21.

WAVELENGTH (deg of long.)	12°	18°	24°	30°	36°	40°	45°	51°	60°	72°	90°	120°	180°	360°
DESCRIPTION OF TROPOSPHERIC WAVE PATTERN	Short waves									Long waves				
WAVE NUMBER	30	20	15	12	10	9	8	7	6	5	4	3	2	1
ASSOCIATED SURFACE FEATURES	Secondary depressions Mobile troughs and ridges		Mature frontal depressions			Large slow-moving non-frontal vortices				Very large complex areas of low or high pressure			—	

FIGURE 21. Long waves and short waves in the atmosphere

The normal association of the smaller upper-level feature (short waves) with the larger surface features can be confusing. Long waves in the upper air are not so much associated with individual surface-weather systems as with a whole succession of similar systems which together tend to determine the character of the 'weather type' over an extended period. And at the other end of the scale, the mesoscale surface systems generally have no similarly sized counterpart aloft that can be readily observed.

The scale of these horizontal waves in relation to the size of the earth is frequently expressed in terms of the 'wave number'. If the wavelength of a wave, or a succession of waves, is expressed in degress of longitude,

$$\text{wave number} = \frac{360}{\text{wavelength}} .$$

Thus a wave system whose wavelength is 90° of longitude has a wave number equal to 4. In other words, four waves of this size would completely circle the globe. Such a wave would certainly be called a long wave, which is normally taken to be one with a wave number from 1 to 5 (i.e. a wave system which is rather bigger than the Atlantic in extent). Short waves associated with major synoptic systems have wave numbers 6 to 10, so that they have wavelengths equal to about 30-60 degrees of longitude.

These horizontal-wave systems are a peculiarly atmospheric phenomenon. Waves of this type do not occur in any other moving fluid systems that are easily observed in nature. Their particular character, and the forces which control the motion, arise from the particular physical conditions that distinguish the earth's atmosphere from other moving fluids, including the atmospheres of other planets. Important amongst these conditions are the angular velocity of the earth and the variation of the Coriolis parameter with latitude, and the broad-scale temperature and mass structure of the atmosphere.

2.2.8.3 Rossby waves. The theoretical attack on the dynamics of tropospheric wave patterns started in 1939 with the early work of C.-G. Rossby.[15] Strictly, his theory applied only at the level of non-divergence in a broad westerly current of air moving with a uniform velocity. On the basis of this very simplified treatment of the broad-scale atmospheric flow, and before the introduction of computed numerical forecasts, it was possible for forecasters to make simple, quantitative estimates of the future movement of tropospheric wave patterns. The results were not completely reliable but, like most subjective forecasting techniques, in skilled hands this was a very acceptable forecasting tool in the pre-computer days.

Rossby considered the effect of superimposing a small perturbation, with velocity components (u', v'), on a given zonal flow (U). Applying the simplified vorticity equation (equation 2.34, page 33) at the level of non-divergence,

$$\frac{\mathrm{d}}{\mathrm{d}t}(\zeta + f) = 0 \,,$$

where ζ is the vorticity of the resultant velocity $(U + u', v')$, he showed that this perturbation in the westerly flow could generate waves, whose eastward motion c (degrees of longitude/24 hours) is given approximately by

$$c = U\left(1 - \frac{L^2}{L_s^2}\right) \qquad \ldots\ldots \quad (2.42)$$

where L (in degrees of longitude) is the wavelength of the actual system and $L_s = 2\pi\sqrt{U/\beta}$ is known as the 'stationary wavelength', where $\beta = \partial f/\partial y$, the variation of Coriolis parameter with latitude. These waves are called Rossby waves. They are really only theoretical concepts appropriate to the particular simple conditions which were assumed in Rossby's work. However, as they bear a sufficiently close resemblance, both in scale and in movement, to the atmospheric long waves, the name 'Rossby wave' is nowadays often used as a synonym for them.

General considerations of the conservation of absolute vorticity (see 2.2.7.4, page 37) show that short-wavelength, small-amplitude waves tend to move eastwards in the direction of the broad zonal flow. But long-wavelength, large-amplitude waves, in which there is a considerable variation of Coriolis parameter between the extremities of the troughs and ridges, are very slow moving and may even move towards the west. The motion of an individual wave is the result of the relative effects of the zonal flow (U) and the latitudinal variation of f, (β). When the two effects exactly balance, then a stationary wave pattern results $(c = 0)$ whose wavelength is given by L_s. At latitude 50°N, with $U = 20$ m/s, $L_s \approx 5000$-6000 km, which is very much in line with the size of stationary wave systems observed in the atmosphere.

In the practical application of Rossby's work it is not always easy to assign values to U and L. The wave patterns encountered in real life are not often simple and clear-cut, but are more frequently a complex combination of several waves of different lengths and amplitudes. Consequently the calculation of individual wave speeds, using equation 2.42, is likely to have only a limited success. However, it has been found that the comparison of the actual wavelength (L) with the stationary wavelength (L_s) does serve as a useful indicator of whether or not a ridge/trough system is likely to progress eastwards more or less quickly, or whether it is likely to be quasi-stationary or, possibly, retrogress towards the west.

If $L_s - L \geqslant 15°$ longitude, a wave will almost certainly progress eastwards fairly quickly. If $L_s \approx L$ the wave will be very slow moving, and perhaps stationary. If $L_s < L$, the movement will normally be very complex. Very large wavelength troughs are frequently distorted by the formation of a new trough on their upstream side. This may change the one original long-wave trough into what is effectively two short-wave troughs, which then progress eastwards. Or the new development may grow at the expense of the old and, by a process of development rather than translation, transfer the effective trough axis westwards for a time.

2.2.9 Forecasting atmospheric development

2.2.9.1 Simple concepts of atmospheric development. The term 'development' is used in synoptic meteorology to refer to changes in the patterns of circulation observed on weather charts. A 'developing low' is one in which the central pressure is falling and the cyclonic circulation is becoming more vigorous.

In other words, it is an area where cyclonic (positive) vorticity is increasing, ($d\zeta/dt > 0$). Similarly a 'developing high' is one in which anticyclonic (negative) vorticity is increasing, ($d\zeta/dt < 0$). So we may take the rate of change of vorticity, $d\zeta/dt$, to be a measure of the development which is occurring and, from the vorticity equation, this index of development is seen to be closely related to the convergence/divergence patterns at any level.

The simplified vorticity equation, written as

$$\frac{1}{(\zeta + f)} \frac{d}{dt} (\zeta + f) = -\mathrm{div_h}\,\mathbf{V}$$

shows that an increase in cyclonic absolute vorticity, which is given on the left-hand side by $d/dt\ (\zeta + f) > 0$ or, since $f > 0$, by $d\zeta/dt > 0$, must be associated on the right-hand side with $(-\mathrm{div_h}\,\mathbf{V}) > 0$, or convergence. Similarly an increase in anticyclonic vorticity is associated with divergence.

These associations, of increasing cyclonicity with convergence and increasing anticyclonicity with divergence, apply at any level in the atmosphere. But, when they are applied at the surface, they force us to adopt a model of the atmosphere's structure similar to that in Figure 22. In this case, cyclogenesis at the surface is associated with convergence in the lowest levels, which implies an accumulation of air in a region in which the surface pressure is falling. This paradox can only be resolved if there is ascent of the excess air accumulating at low levels and a slightly larger divergence at high levels.

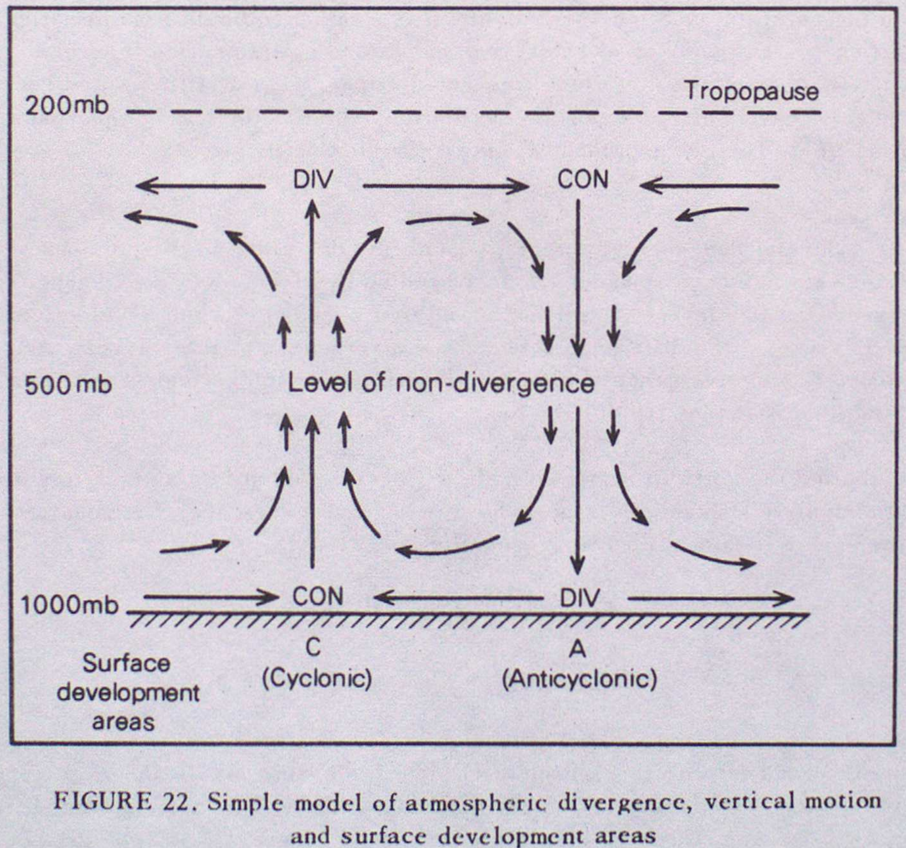


FIGURE 22. Simple model of atmospheric divergence, vertical motion and surface development areas

In practice, of course, there may be several layers in which horizontal convergence and divergence alternate, but the simple two-layer model has proved to be quite acceptable for a general understanding of surface-pressure developments. Regions of surface cyclonic development, known as 'C' areas, are associated with low-level convergence; ascending air in mid troposphere, attaining its maximum upward velocity at about the level of non-divergence; and upper-level divergence which has a slightly greater effect than the convergence lower down. Regions of surface anticyclonic development are known as 'A' areas and have low-level divergence, subsidence, and convergence aloft.

One of the main problems of weather forecasting has always been the prediction of surface-pressure developments. The way in which this problem was tackled in the past depended on the particular tools which were available at the time. Nowadays, it is all calculated routinely, using a computer, but before the introduction of numerical forecasting it was only possible for forecasters to make a very rough assessment of where the most important developmental areas were likely to be. It was done on the basis of Sutcliffe's theoretical work,[16,17] which related 'C' and 'A' areas on the surface chart to the different divergence patterns at 1000 mb and 500 mb. The practical technique made much use of the 1000-500-mb thickness chart, which was a major synoptic forecasting tool of the time. The ideas behind Sutcliffe's work are described in 2.2.9.2.

With the introduction of computed forecasts, based on numerical models of the atmosphere which have several levels in the vertical, the emphasis has changed. There is no longer any operational requirement specifically to identify 'C' and 'A' areas, though this still remains a valuable subjective exercise. On the other hand, there is a need, in the course of computing the numerical forecasts, to evaluate the vertical velocities. This is done by using the omega equation (ω = vertical velocity in pressure co-ordinates) which is described in 2.2.9.3 (page 47). It will be seen that Sutcliffe's development equation and the omega equation are very close relations. Sutcliffe's work was presented in a rather simple form in order that its practical applications could be easily carried out and understood. Because of this, it inevitably contained a number of assumptions and simplifications, the practical consequences of which could not always be easily assessed. The omega equation is a tool designed for computer work and is more generalized and comprehensive than the Sutcliffe equation. As a result, it is a rather forbidding mathematical expression, but it can be simplified for the purposes of subjective understanding into something which is very similar to the Sutcliffe equation. Essentially, returning to Figure 22 (page 41), Sutcliffe related surface developments to the divergence/convergence arms of the circulation, whereas the omega equation relates them to the vertical-motion field. The two approaches lead to similar conclusions.

2.2.9.2 Sutcliffe's development theory. Using the concepts described in the previous section, where cyclonic (anticyclonic) surface developments are accompanied by divergence (convergence) aloft and convergence (divergence) at low levels, Sutcliffe showed that a measure of the development occurring between two levels is given by the difference between the divergences at those levels. And he went on to show that this difference in divergence could be derived from the subjectively analysed charts which were routinely available at that time (1950-65).

Essentially, he applied the vorticity equation both at 1000 mb, which is a satisfactory approximation to the surface, and also at 500 mb, which is near the level of non-divergence. The equations which result are, using the simple form of the vorticity equation,

$$\frac{\partial \zeta_5}{\partial t} + \mathbf{V}_5 \frac{\partial}{\partial s}(\zeta_5 + f) = 0 \qquad \dots . \quad (2.43)$$

$$\frac{\partial \zeta_{10}}{\partial t} + \mathbf{V}_{10} \frac{\partial}{\partial s}(\zeta_{10} + f) = -(\zeta_{10} + f)\,\mathrm{div_h}\,\mathbf{V}_{10}, \qquad \dots . \quad (2.44)$$

where the numeral subscripts refer to the 500-mb and 1000-mb pressure levels and $\partial/\partial s$ represents the rate of change in the direction of the air motion. If equations 2.43 and 2.44 are subtracted, terms like $(\zeta_5 - \zeta_{10})$ and $(V_5 - V_{10})$ appear. These may be written ζ' and V' respectively, where V' is the 1000-500-mb thermal wind and ζ' is the vorticity of this thermal wind. Also, from equation 2.36 (page 34), the vorticity of the (geostrophic) thermal wind is related to the thickness field (h') by

$$\zeta' = \frac{g}{f} \nabla^2 h'$$

so that thermodynamic processes can be introduced into the development equations by expressing changes of ζ' in terms of changes in thickness. Using these devices, and some considerable algebraic manipulation, the full Sutcliffe development equation is obtained, which may be written:

$$(\zeta_{10}+f)\ \mathrm{div_h}\,\boldsymbol{V}_{10} = \boldsymbol{V}'\frac{\partial}{\partial s}(2\,\zeta_{10}+\zeta'+f) + \mathrm{HEAT} + \mathrm{STAB} + \mathrm{DEFM} \quad . \quad . \quad . \quad . \quad (2.45)$$

In this symbolic form:

HEAT is a term representing the effects of non-adiabatic heat exchanges on the thickness. This does not include latent heat exchanges.

STAB is a term which is proportional to the static stability of the air and to the vertical velocity. It is a measure of the adiabatic heating effects due to vertical motion.

DEFM is a term which is related to the mutual interaction of

(a) the surface wind on the thickness pattern, and
(b) the thermal wind on the surface isobaric pattern. It is a measure of the 'deformation' of these patterns and is particularly relevant to the occurrence of frontogenesis and frontolysis.

Any of these three terms may at times be important, but their effect is hard to assess except in a most general way, such as in the discussion in 2.2.6.3 (page 30). In the pre-computer days they were normally ignored in the practical application of Sutcliffe's theory. This was unsatisfactory but inevitable. So the basic Sutcliffe development equation becomes

$$(\zeta_{10}+f)\ \mathrm{div_h}\,\boldsymbol{V}_{10} = \boldsymbol{V}'\frac{\partial}{\partial s}(2\,\zeta_{10}+\zeta'+f), \quad . \quad . \quad . \quad . \quad (2.46)$$

where $\partial/\partial s$ represents the gradient of a quantity in the direction of the thermal wind (V') along the thickness lines. In this equation it is possible to examine the separate effects of the three constituents in the expression on the right-hand side, at least to the extent of evaluating the sign of the various contributions. This gives the sign of div $\boldsymbol{V}_{10}$ on the left-hand side, since $(\zeta_{10}+f) > 0$. It follows from the schematic model of Figure 22 that:

div $\boldsymbol{V}_{10} < 0$ implies low-level cyclonic development ('C').
div $\boldsymbol{V}_{10} > 0$ implies low-level anticyclonic development ('A').

To ascertain the sign of the complete term on the right-hand side of equation 2.46 we split it up into its three parts:

(a) $2\boldsymbol{V}'\dfrac{\partial\zeta_{10}}{\partial s}$ which represents advection of surface vorticity (ζ_{10}) by the thermal wind.

(b) $\boldsymbol{V}'\dfrac{\partial\zeta'}{\partial s}$ which represents advection of thermal vorticity (ζ') by the thermal wind.

(c) $\boldsymbol{V}'\dfrac{\partial f}{\partial s}$ which represents advection of the earth's vorticity (f) in the direction of the thermal wind.

Regarding the magnitude of these terms, it is clear that since each of them is proportional to $\boldsymbol{V}'$, the effect of each of them is greatest where the thermal wind is strongest (that is, in regions where thickness lines are closely concentrated).

The term (c) is a measure of the change in the Coriolis parameter in the direction of the thermal wind. It is often called the 'latitude term' since f depends directly on latitude (see Figure 23). Where the thermal wind is either westerly or easterly, then $\partial f/\partial s = \partial f/\partial x = 0$ and the term makes no contribution to any low-level development, no matter how large is the value of $\boldsymbol{V}'$. The greatest effects occur where the thermal wind has a strong northerly or southerly component. With northerly thermal winds, $\partial f/\partial s < 0$, so $\mathbf{V}'\partial f/\partial s < 0$, which contributes to cyclonic developments at the surface.

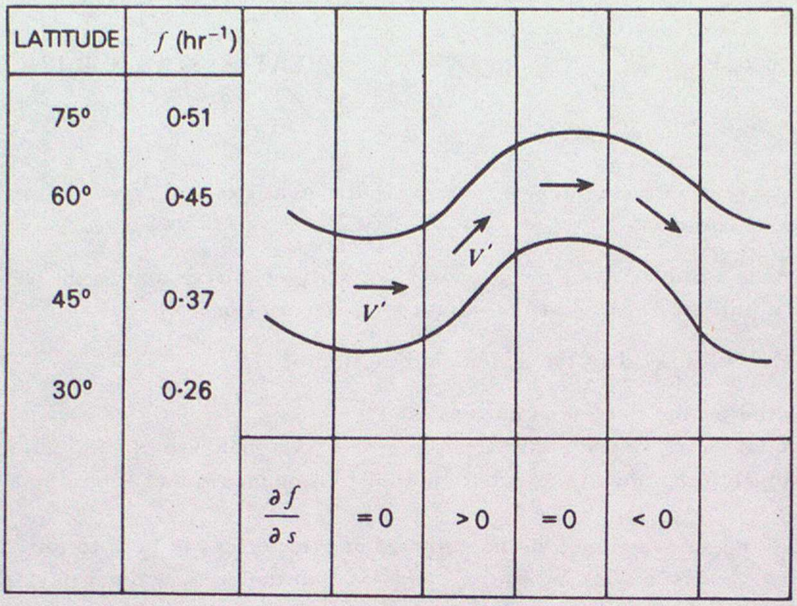


FIGURE 23. The latitude term in the Sutcliffe development equation

Similarly, southerly thermal winds contribute towards anticyclonic development. Except in the rather extreme cases of strong meridional flow, the effect of this term is small. It is very rarely the dominant term in the development process.

The term (a), $2\mathbf{V}'\partial\zeta_{10}/\partial s$, is known as the 'thermal steering' term. Its effect is illustrated in Figure 24. Figure 24(a) shows a schematic pattern of 1000-mb geopotentials and 1000-500-mb thickness lines. Figure 24(b) repeats the thickness lines and interprets the 1000-mb pattern in terms of vorticity, ζ_{10}. In this diagram the curvature effect is assumed to be more important than the shear effect in determining the vorticity of the 1000-mb field, so the lows are maxima of cyclonic vorticity and the highs are maxima of anticyclonic vorticity. Between these centres there are regions such as P and Q.

At P, $\partial\zeta_{10}/\partial s > 0$, so $2\mathbf{V}'\partial\zeta_{10}/\partial s$ contributes to 'A' development.

At Q, $\partial\zeta_{10}/\partial s < 0$, so $2\mathbf{V}'\partial\zeta_{10}/\partial s$ contributes to 'C' development.

With cyclonic development at Q and anticyclonic development at P, the effect of this term is therefore similar to the translation of the existing highs and lows in the direction of the thermal wind. In practice, this is equivalent to a simple bodily movement and gives rise to the concept of the 'thermal steering' of the existing features of the surface-pressure chart. In fact, however, it is a process of continuous redevelopment of a constantly changing fluid system rather than the simple motion of a more or less self-contained volume of the atmosphere.

Trough (and ridges) are also areas of relatively large positive (and negative) vorticity, and the same steering principle applies to them. The thermal steering term contributes to the displacement of such systems in the direction of the thermal wind, by a process of development.

Where $\partial\zeta_{10}/\partial s = 0$, this term does not contribute anything to the development. The centres of depressions and anticyclones are normally maxima, or minima, of vorticity and consequently they are regions where $\partial\zeta_{10}/\partial s = 0$. The thermal steering term does not therefore contribute anything to the intensification or weakening of the vorticity at the centre of these systems, but only to their displacement.

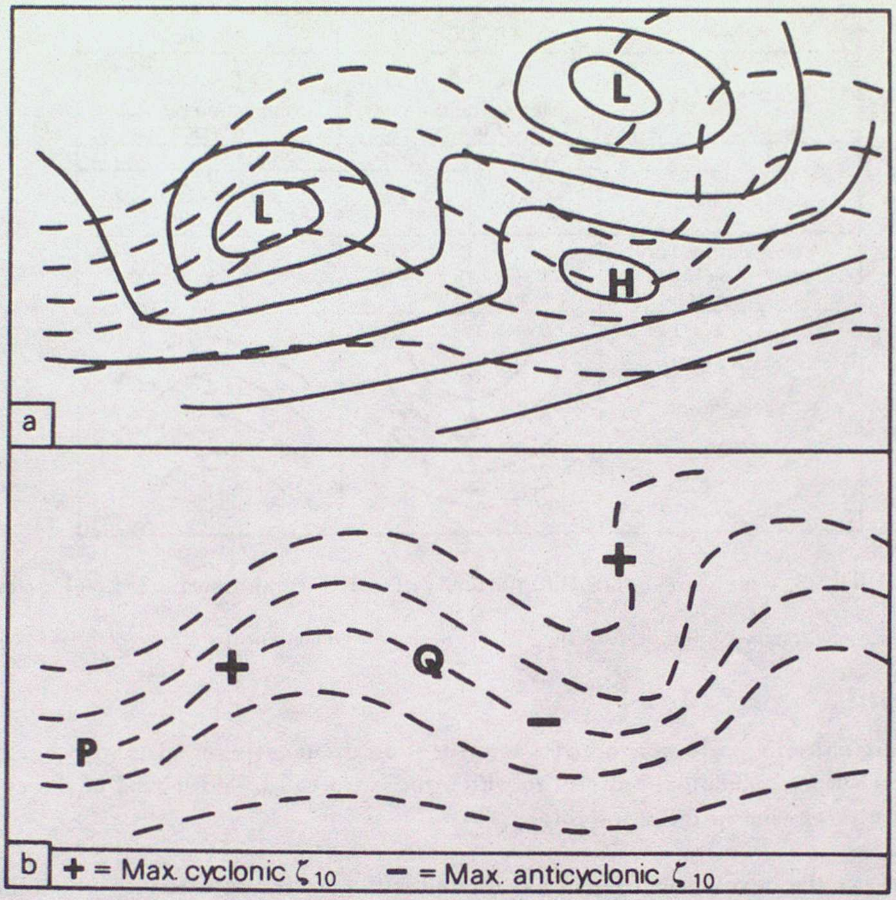


FIGURE 24. The thermal steering term in the Sutcliffe development equation

(a) 1000-mb geopotentials (———) and 1000-500-mb thickness lines (— — —)
(b) 1000-500-mb thickness lines (— — —)

The thermal steering term is frequently of great importance and is the predominant effect in those areas where the vorticity advection term $\mathbf{V}' \,\partial\zeta'/\partial s$ is near zero. A typical area where this is so is under a long straight jetstream. Here ζ' shows little change in the direction of V' and $\mathbf{V}' \,\partial\zeta'/\partial s \approx 0$. In such an area, the rapid thermal steering of existing pressure features is likely to be the development of greatest importance.

The term (b), $\mathbf{V}' \,\partial\zeta/\partial s$, is the thermal vorticity advection term. It can be elucidated purely from the study of the 1000-500-mb thickness chart. The term represents the advection of thermal vorticity by the thermal wind and, in order to gauge its effect, it is necessary to know the distribution of vorticity on the thickness chart. To make a subjective assessment of the vorticity it is best to consider the effects of curvature and shear separately, and then try to combine them so that at least the most important areas can be identified in a general way. It is not possible for a forecaster to do more than this.

The surface development areas ('C' and 'A') associated with typical thickness patterns can be deduced from a study of the basic types. Figure 25 shows the development areas associated with patterns in which the vorticity is due solely to the curvature of the streamlines. There is uniform flow around the cold troughs and warm ridges. ζ' is determined largely by curvature and there is little or no shear vorticity.

Pattern	TROUGH		RIDGE	
Vorticity (ζ')	$\zeta' > 0$ Max. value on axis of trough		$\zeta' < 0$ Min. value on axis of ridge	
	West	East	West	East
Gradient of ζ' in direction of V'	$\frac{\partial \zeta'}{\partial s} > 0$	$\frac{\partial \zeta'}{\partial s} < 0$	$\frac{\partial \zeta'}{\partial s} < 0$	$\frac{\partial \zeta'}{\partial s} > 0$
Vorticity advection $\left(V' \frac{\partial \zeta'}{\partial s}\right)$	+ ve	− ve	− ve	+ ve

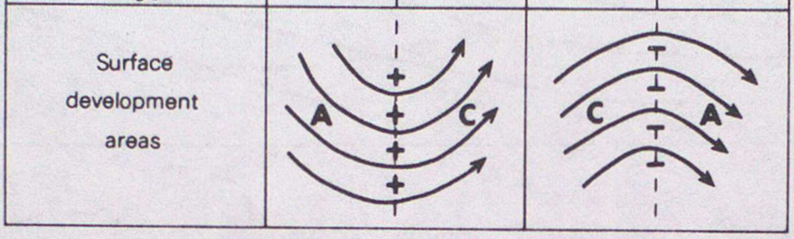


FIGURE 25. Curvature-controlled patterns of vorticity and surface development

+ = maximum cyclonic vorticity. − = maximum anticyclonic vorticity.

The two basic patterns, cold trough and warm ridge, are frequently found joined together in a long series forming a simple sinusoidal pattern, in which the 'C' areas lie downwind of the cold troughs, and the 'A' areas lie downwind of the warm ridges.

Figure 26 shows the development areas associated with patterns of vorticity which are due to shear. These are confluent and diffluent patterns, having the strongest flow along some central axis. ζ' is determined largely by shear and there is little or no curvature vorticity.

Pattern		CONFLUENCE	DIFFLUENCE
Gradient of ζ' in direction of V'	N	$\frac{\partial \zeta'}{\partial s} > 0$	$\frac{\partial \zeta'}{\partial s} < 0$
	S	$\frac{\partial \zeta'}{\partial s} < 0$	$\frac{\partial \zeta'}{\partial s} > 0$
Vorticity advection $\left(V' \frac{\partial \zeta'}{\partial s}\right)$	N	+ ve	− ve
	S	− ve	+ ve

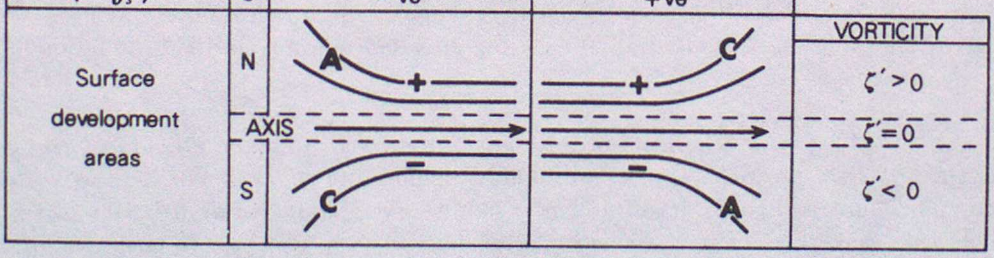


FIGURE 26. Shear-controlled patterns of vorticity and surface development

+ = maximum cyclonic vorticity. − = maximum anticyclonic vorticity

As in the bottom section of this diagram, the basic confluence and diffluence patterns are frequently found joined together, forming the standard 'jetstream' pattern.

The simple trough and ridge patterns of Figure 25 can each be combined with the confluent and diffluent patterns of Figure 26, and the vorticity advection of the resultant combinations is easily deduced. The locations of the principal development area associated with these combined patterns are illustrated in Figure 27.

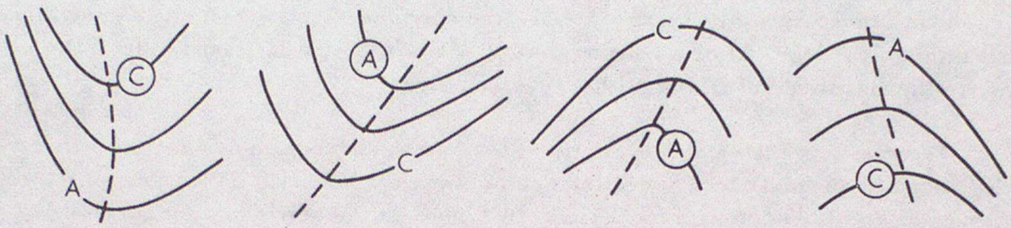


FIGURE 27. Development areas in relation to 1000-500-mb thickness patterns in which vorticity results from both curvature and shear

A Anticyclonic development C Cyclonic development

It can be seen that the 'C' and 'A' areas are located on the same side of the trough/ridge axes as they are in the simple diagrams of Figure 25. But they are displaced towards one side or other of the flow in accordance with the confluent or diffluent effects shown in Figure 26. The most important development areas are those with circled letters, which are all associated with the regions of most-pronounced curvature, it being assumed in these schematic diagrams that the shear vorticity is much the same in both the cyclonic and the anticyclonic areas.

Vorticity advection is a most important factor in all atmospheric developments. In many cases it is certainly the most important single effect, particularly in the formation and deepening of oceanic depressions in mid latitudes.

In the practical application of Sutcliffe's theory the thermal vorticity advection, as illustrated in this section, was taken to be an acceptable indicator of the broad-scale developments occurring throughout the depth of the troposphere. But surface-pressure developments are the result of varying advective effects at different levels. It is the net vorticity advection at all levels, including the highest levels of the troposphere around 300 mb, where the very strong winds produce considerable advection, that determines the final outcome. The 1000-500-mb thermal vorticity advection was only an approximation, albeit a very useful one, developed in the period before the introduction of numerical forecasting, when the 1000-500-mb thickness chart was a major working tool.

2.2.9.3 The omega equation. Starting from the vorticity equation and the thermodynamic energy equation, and using some mathematical manipulations which are beyond the scope of this chapter, it is possible to derive an equation which describes the instantaneous values of the vertical velocity, ω, in terms of more easily measurable quantities, such as h, u, v. This equation is known as the omega equation and in its full form it is rather frightening to behold. Since the equation is designed for use in numerical forecasting and is only capable of solution by using computers, this superficial complexity is not of real importance. Merely for the purpose of glimpsing the essential character of the equation, it may be written in a much simplified form as follows:

$$\nabla^2 \omega \approx \underset{\text{(A)}}{\frac{\partial}{\partial p}\left(u \frac{\partial \zeta_a}{\partial x}\right.} + \underset{\text{(B)}}{\left. v \frac{\partial \zeta_a}{\partial y}\right)} + \underset{\text{(C)}}{\nabla^2\left(u \frac{\partial h'}{\partial x} + v \frac{\partial h'}{\partial y}\right)} \quad \ldots \quad (2.47)$$

The three terms in this equation can be examined separately.

Using the relationship developed in 2.2.7.2 (page 34), we may write $\nabla^2\omega \propto (\overline{\omega} - \omega_0)$ where ω_0 is the value of ω at a grid-point and $\overline{\omega}$ is the mean value of the ω field at the surrounding points. Now ω is a parameter which may have positive or negative values according as the motion is one of subsidence or ascent. At a point where there is a local maximum within an area of subsidence, for example, ω_0 and $\overline{\omega}$ are both positive and $\omega_0 > \overline{\omega}$. Consequently $\nabla^2\omega$ is negative at such a point. Similarly, at a point where there is a local maximum of ascent, ω_0 and $\overline{\omega}$ are both negative and, since $|\omega_0| > |\overline{\omega}|$, the value of $\nabla^2\omega$ is positive. In both these special cases, $\nabla^2\omega$ has the opposite sign to ω. And, in general, we can assign a simple meaning to the term (A) by assuming that $\nabla^2\omega \propto -\omega$. So the left-hand side of the equation is proportional to the magnitude of ω, but with a reversed sign.

The term (B) represents the vertical variation $(\partial/\partial p)$ of the advection of absolute vorticity. Simplifying this a little, it can be expressed as the difference between the vorticity advections at the top and bottom of a layer, such as the 1000-500-mb layer. Thus in Figure 28 the vorticity advection at 500 mb over Newfoundland, north-east of the low centre L, is near zero in the crest of the upper ridge. But at 1000 mb, between the low-level ridge and depression, the vorticity advection is large and positive. This change of vorticity advection, from zero at 500 mb to a positive value at 1000 mb gives the term (B) a positive value. And in the omega equation, positive values on the right-hand side of the equation are equated to $-\omega$, or ascent, on the left.

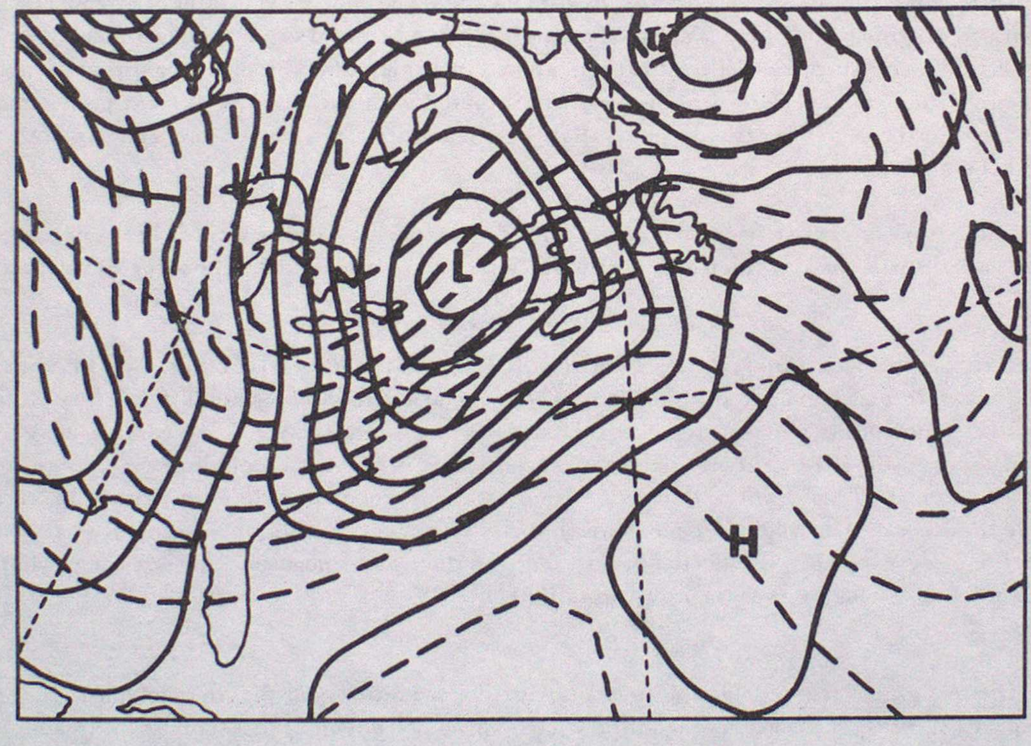


FIGURE 28. Surface isobars (———) and 500-mb geopotentials (– – –) at 0000 GMT, 29 November 1973, illustrating the variation in vorticity advection with height.

Similarly, from Figure 28, in the region of the Great Lakes to the south-west of low L, the 500-mb vorticity advection is again near zero in the upper trough axis, while the 1000-mb vorticity advection, midway between a low and a ridge, is broadly negative. The differential vorticity advection in this case gives the term (B) a negative value. Thus the omega equation gives an indication of ascent in the region ahead of, or north-east of, the depression L, and subsidence in its rear, to the south-west. Term (B) is thus a dynamical term, depending on the advection of vorticity, whereas term (C) is a thermodynamical term. It is written as another Laplacian, which for simple understanding can be rewritten in a similar manner to term (B), namely $-(u\, \partial h'/\partial x + v\, \partial h'/\partial y)$. This is now just the advection of thickness by some suitable wind, with a negative sign attached. Warm advection, which implies that the field of thickness increases in the upwind (negative) direction, makes the gradient terms negative in the advection expression, which, with the negative sign at the front, makes the whole of term (C) positive. This, as with positive values of term (B), contributes to ascending motion on the left side of the equation. Cold advection, in a similar manner, contributes to subsidence.

Thus the omega equation is a diagnostic equation used in numerical forecasting for the purpose of computing values of the vertical velocity, ω. A qualitative appreciation of the various terms in the equation shows that the rate of ascent/subsidence is proportional to the rate of increase with height of cyclonic/anticyclonic vorticity advection, together with the rate of warm/cold temperature advection. If these results are interpreted in the light of the simple development model shown in Figure 22 (page 41), it can be seen that the omega equation and Sutcliffe development equation are closely associated, and that each,in its own generation, has performed essentially the same task – of forecasting areas of surface development. For example, the Sutcliffe 'C' area on the forward side of a cold trough clearly corresponds with the region of ascent implied by term (B) of the omega equation ahead of an upper trough. Similar parallels exist for the other important development areas.

2.2.10 Dynamical aspects of fronts

A front is a narrow zone of transition between two masses of air with differing properties. The gradient of a property across the frontal zone may be about two orders of magnitude greater than the gradients within either air mass well away from the front. It is not easy to see at first how such gradients can be created purely by the horizontal confluence of two air masses, but Sawyer[18] has shown that the confluence is necessarily accompanied by a vertical circulation. This circulation is set up because the intensifying temperature gradient caused by the confluence leads, according to the thermal wind equation, to increasing winds aloft, and possibly to a retardation of the surface winds. The winds are no longer in geostrophic balance and cross-isobaric flow occurs: in the region aloft where the flow is accelerating, the cross-isobaric component is from the warm air towards the cold air, while the opposite holds true near the surface when there is retardation. If continuity is to be maintained, the circulation must be completed by two vertical branches, with warm air ascending and cold air sinking. The process leads to a further intensification of the gradients across the frontal zone, and to a sloping frontal surface with the warm air overlying the cold air. Sawyer also discussed the effects of friction and of the release of latent heat by condensation in the rising warm air, the latter leading to an intensification of the circulation. He found that the circulation formed in this way was not sufficient to account for more than light frontal rainfall. However, active fronts usually occur in regions of marked cyclonic development, where ascending motion, caused by different mechanisms from the frontogenetic one, can account for the greater rates of rainfall.

Kirk[19] has shown that a useful dynamical representation of the concentration of isotherms (or thickness lines) along a front is the parameter $\nabla^2\alpha$. α is the specific volume which, from the gas equation $p\alpha = RT$, is proportional to T on an isobaric surface. ∇^2 is the Laplacian operator, $\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, which, being a second differential, measures the rate of change of the gradient of a quantity. The Laplacian is zero in regions where either the temperature is constant or the gradient of temperature is constant. This would be the normal case within air masses, where a small, uniform, latitudinal variation of temperature may occur. But where the gradient of temperature changes sharply, such as at the boundaries of frontal zones, then $\nabla^2 T$ (or $\nabla^2\alpha$) has significantly large values. $\nabla^2\alpha$ is therefore a convenient way of depicting regions where there are concentrations of isotherms.

The importance of the vertical circulation at a front, emphasized by Sawyer, has been related by Kirk to his parameter $\nabla^2 \alpha$ by considering two of the terms which occur in the complete vorticity equation. These two terms are usually ignored because, although they are individually not insignificant, when taken together they normally cancel each other out in atmospheric motions. The two terms are (a) the 'twisting' term, which describes the change in ζ at any level arising from changes in the slope of the three-dimensional vortex tubes, which thus leads to a variation in the vertical component of the vorticity ζ; and (b) a term describing the vertical advection of ζ. A detailed consideration of these terms, and their near-equality in size, indicates that there must be a strong interdependence in the atmosphere between horizontal gradients of temperature and of vertical velocity. Thus, fronts are to be thought of not only as regions of significant horizontal temperature gradient, but as regions of vertical velocity gradient as well.

Traditional practice among forecasters has been to look for fronts where rapid changes occur in certain commonly observed weather elements. These practices may be translated into dynamical terms, by making use of ζ and other parameters.

Synoptic indicators of a front		Equivalent dynamical parameter
(a) Wind veer on a frontal pressure trough:	$\dfrac{\partial \zeta}{\partial n}$	The trough is a zone of maximum cyclonic ζ. The horizontal gradient of ζ identifies the front.
(b) Discontinuity in the pressure tendency:	$\dfrac{\partial \zeta}{\partial t}$	The passage of a trough gives a variation of ζ with time at any place.
(c) Maximum concentration of isotherms:	$\dfrac{\partial \zeta}{\partial p}$	The vertical gradient of ζ. See below.
(d) Distribution of rainfall and clouds (which are visible manifestations of the vertical motions):	$\int \mathrm{div}_h \mathbf{V}\, dp$	Vertical motion and $\mathrm{div}_h \mathbf{V}$ are related through the continuity equation. This is the integrated $\mathrm{div}_h \mathbf{V}$ through the whole depth of the troposphere.

Thus it is possible to reinterpret the traditional forecasting practices in terms of different aspects of the distribution of ζ, or of the horizontal divergence. The third of these, involving $\partial\zeta/\partial p$, is the least easy to comprehend but is in some ways the most significant; for, by using the expression for geostrophic vorticity $\zeta = g/f\, \nabla^2 h$, we can derive the expression

$$\frac{\partial \zeta}{\partial p} = \frac{g}{f} \nabla^2 \left(\frac{\partial h}{\partial p} \right)$$

which, using the hydrostatic relation, leads to

$$-\nabla^2 \alpha = f \frac{\partial \zeta}{\partial p}.$$

Now, if a front can be defined by the distribution of $\nabla^2 \alpha$, then frontogenesis may be expressed as $\partial/\partial t\, (\nabla^2 \alpha)$. So $\partial\zeta/\partial p$ becomes an important quantity in frontogenesis, which is the dynamical process by which a front is initially formed and subsequently maintained. The frontogenetic term $\partial/\partial t\, (\nabla^2 \alpha)$ can be expanded, in a way that is very similar to the thickness-change equation 2.29 (page 31), into three parts. Each part describes the contribution to the frontogenetic process of particular physical effects: (i) different horizontal advection, (ii) different vertical motions, and (iii) different heating (including radiation, and latent-heat transfers associated with condensation and evaporation) on either side of the frontal zone.

The net result of this work is to indicate that frontogenetic processes in nature are complex and three-dimensional in character, and that a simple index of the combined effects is not readily apparent, even in dynamical terms. The process described by Sawyer has been modelled by Hoskins,[20,21] using the primitive equations of motion. The results show realistic temperature and motion fields, and one particularly interesting feature is the intrusion into the frontal region of a tongue of dry stratospheric air (see Chapter 5 – Fronts and frontal weather). Although the problem is very complex, and simplifying assumptions have to be made in the model, there is hope that this approach will lead eventually to a more realistic representation of fronts in numerical-forecasting models.

2.2.11 The energy of the atmosphere

The atmosphere possesses energy in a number of different forms. It has kinetic energy (KE) by virtue of its motion, which results from the action of pressure forces. It has potential energy (PE), resulting from the action of gravity forces on neighbouring warm and cold air masses. It has internal energy (IE), which is a measure of the vigour of the molecular-scale motions and which, for an ideal gas, is proportional to the temperature. It has latent energy (LE), resulting from the heat exchanges associated with the phase changes of water. There are also some other forms of energy, such as electrical and magnetic energy, which may become very apparent locally from time to time but which are normally quite insignificant in the total energy budget.

The basic energy cycle of the atmosphere is:

(1) generation of PE, by heating in tropical latitudes and cooling in polar regions;

(2) conversion of PE to KE in the various wind systems of the atmosphere; and

(3) dissipation of KE by friction.

The main significance of this cycle lies in its relevance to the long-term general circulation of the atmosphere rather than to individual developments on the time scale of day-to-day forecasting. However, the concepts associated with the creation and destruction of energy in all its forms are important for a full understanding of atmospheric dynamics. The subject is generally known as the energetics of the atmosphere.

Of the energy conversions that occur in the atmosphere only radiation, which adds or removes IE, appreciably alters the total energy of the atmosphere. The remaining processes are internal, and are illustrated in Figure 29. Work done by (or against) the pressure forces converts IE to KE (or vice versa) Work done by (or against) gravity converts PE to KE (or vice versa). Friction converts KE to IE and this is an irreversible process. There is no direct conversion of IE to PE (except through the intermediate creation of KE) or of LE to KE (except through the intermediate creation of IE, for evaporation converts IE to LE and condensation converts LE to IE). In regard to the last-named processes, it is relevant to remark that the transfer of heat to or from a substance, either by convection, conduction or radiation, results in either a change of temperature or a change of state of that substance. So a heated body either (a) acquires a higher temperature, which is known as 'sensible heat' or 'enthalpy', or (b) changes to a higher state and acquires 'latent heat'.

LE is, by its very nature, a store of atmospheric energy which is hidden from view. But, though it is hidden, its importance must not be overlooked. LE released in the hydrological cycle contributes about 16 per cent of the net heat gains of the atmosphere. Even on the broadest scale this is a significant contribution, but, when it is remembered that in day-to-day synoptic events the latent-heat exchanges are concentrated in quite localized areas, its relative importance is seen to be greater still.

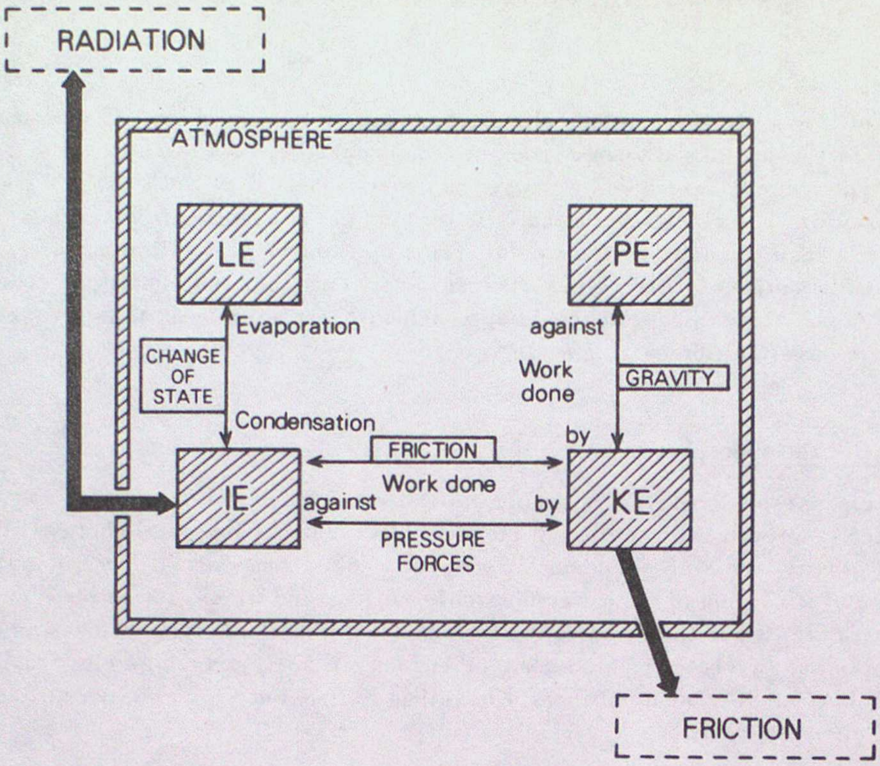


FIGURE 29. Conversion of energy between different forms in the atmosphere

Mathematical expressions for the various forms of energy may be written down, in terms of integrals summed over the whole mass of the atmosphere ($\int dM$):

KE	=	$\int \frac{1}{2} V^2\, dM$	where	V	=	wind speed
IE	=	$\int c_v T\, dM$	where	c_v	=	specific heat at constant volume
PE	=	$\int g z\, dM$	where	L	=	latent heat
LE	=	$\int L q\, dM$	where	q	=	specific humidity.

Under hydrostatic conditions, the expression for PE may be written $\int RT\, dM$. It is then convenient to combine PE and IE into one expression, called the total potential energy (TPE), where

$$\text{TPE} = \int c_v T\, dM + \int RT\, dM = \int (c_v + R) T\, dM = \int c_p T\, dM$$

which is useful for considering the simplified, broad-scale energy changes in the atmosphere. For these, since the horizontal motions are much greater than the vertical motion, we neglect the effects of work done against the vertically acting forces. Work may therefore be done either by or against the horizontal pressure forces. It is a reversible process but, on balance, there has to be a net conversion of TPE to KE in order to offset the continual conversion of KE to the IE part of TPE (which is an irreversible process).

In the atmosphere the presence of air masses having different densities existing side by side calls into play gravitational and buoyancy forces, which tend to rearrange the density contrasts towards an ultimate state in which there is a horizontal stratification. Such an ultimate state, if it were ever achieved, would represent a state of minimum total potential energy (TPE) of the atmosphere. It would be impossible to reach a state which had a smaller potential energy, and the greater the departure from this state the greater is the PE of the atmosphere. Thus it is that continued tropical heating and polar cooling, by increasing the horizontal temperature (density) gradients in the atmosphere, increase its PE. And the consequence of having two neighbouring volumes of fluid with different densities is for the denser of the two to sink, and the other to rise. Thus it is that some of the PE that has been built up is converted to energy of motion (KE).

In the atmosphere, circulations which create motion (or convert PE into KE), through cold air subsiding and warm air rising, are called direct circulations. One such circulation cell of this type is the Hadley cell of the tropics. A longitudinal section through the tropics shows, in the mean, warm air ascending in the equatorial rain belt, poleward flow aloft, subsidence in the latitudes of the dry tropical deserts and the equatorward trade-wind flow at lower levels. Land- and sea-breezes are another example of direct circulations on a much smaller scale.

Indirect circulations, in which the PE of the system increases as the KE decreases, are not common and are generally of small importance.

In the destruction or dissipation of atmospheric motion systems, frictional forces are of fundamental importance. KE is always being dissipated by friction, but the detailed description of the action of friction in the atmosphere is incompletely known. We normally distinguish between the molecular viscous forces, which operate so as to even out the different velocities in neighbouring parts of the fluid, and a virtual friction associated with turbulent eddies. These eddies are much more important in effecting the transfer, and mixing, of momentum in the atmosphere than is molecular viscosity.

Near the surface, where the effects of friction are most marked, it is normal to express the frictional stress τ as being proportional to the square of the wind speed:

$$\tau = \rho C_D V^2.$$

This is an empirical formula that closely describes the frictional effects in the rather special region near the earth's surface. C_D is called a 'drag coefficient', which is a conventional parameter whose value (derived from practical experiments) is a measure of the degree of roughness of the underlying surface.

2.3 PRACTICAL APPLICATIONS

A brief account is given of the characteristics and evolution of synoptic systems; for further details the reader should consult Chapters 4 to 8 in which the weather associated with such systems is also described.

2.3.1 Some characteristics of common synoptic-weather systems

2.3.1.1. Synoptic-scale systems.

Air masses

Air masses are large volumes of air which are (a) extensive, both horizontally (10^3 km) and vertically (10 km), and (b) have a quasi-uniform distribution of physical parameters such as temperature, moisture content and vertical motion. This implies a uniformity of weather type also. Air masses can be categorized by reference to some conservative property of the temperature and moisture fields, and the wet-bulb potential temperature (θ_w) has generally been found to be the best parameter for the purpose. Some schemes of categorization are quite complex, but for north-west Europe the simple division into warm 'tropical' air and cold 'polar' air, lying on either side of the polar front, is the simplest and best. A subdivision is made of each of these two types into 'maritime' and 'continental', according to the nature of the surface over which the air travels towards the British Isles.

At any level within an air mass, the variation of temperature is typically such that $\partial T/\partial x = 0$, $\partial T/\partial y$ = small constant, and $\partial T/\partial z$ = constant × adiabatic lapse rate. Consequently $\nabla^2 T = 0$ within most parts of a uniform air mass, and such systems are almost barotropic in character.

Within the air mass also, ϖ is typically near zero, but close to a frontal zone at the periphery of the air mass there is generally ascent in warm air and subsidence in cold air.

Fronts

Fronts are baroclinic zones, separating two different quasi-barotropic air masses. The normal textbook descriptions are of the typical conditions experienced in fronts (warm fronts, cold fronts, occlusions) which approach the British Isles from the North Atlantic. This is a region in which there are no topographic influences, nor any very rapid changes in the character of the underlying surface temperature. Over continental areas these surface irregularities exert a considerable modifying influence on the standard textbook patterns of oceanic fronts.

Across a frontal zone, there are variations in (a) the surface wind speed and direction (and hence also the divergence and vorticity), (b) the pattern of isobars (or geostrophic wind) and of pressure tendencies, (c) the temperature, or thickness, and (d) the vertical motion and moisture content, which give rise to the typical cloud and weather distributions. In vertical cross-sections, the sloping region of a frontal zone may be displayed by using a simple parameter such as temperature, which usually shows a stable layer or inversion. But a better dynamical parameter is θ (or θ_w) which is related to the adiabatic lapse rate and is a better indicator of the static stability.

Another subdivision of fronts, into ana-fronts and kata-fronts, describes the vertical motion occurring in the warm moist air mass at a front. At an ana-front (whether it is a warm front or a cold front) the warm air is ascending and the front is consequently an active, precipitating system such as is typically described in elementary textbooks. At a kata-front the warm air is subsiding (or at least, not ascending), and the front is consequently a weak system with little precipitation and an atypical cloud structure. Forecasters normally distinguish ana-fronts from kata-fronts either by studying the surface observations or, if a sufficiently dense network is not available, as is often the case over the sea, by plotting the upper winds from a suitable radiosonde stationson a hodograph. Some reasonable deductions about the vertical motion implied by the observed horizontal wind structure can then be made.

Pressure systems

On a surface chart the main isobaric patterns are depressions (lows), anticyclones (highs), troughs and ridges. There has long been known to be an association of high pressure with fine weather, and of low pressure with stormy weather. This can be related to the vertical motions shown in Figure 22 (page 41) for a simple model of atmospheric development.

A vertical cross-section through a pressure system shows that the location of vortex centres at different heights, and the axes of contour troughs and ridges, do not normally lie vertically above the surface position. As shown in Figure 30, they slope towards the upstream direction. Also the location of the warmest and coldest air lies on a sloping line, but this axis is out of phase with the location of the sloping contour axis. This slope and phase lag between contour and thickness trough (ridge) axes is necessary if baroclinic instability is to occur, and the pressure systems are to develop. In mature systems, which are quasi-barotropic and have reached the limit of their development, the motions are largely horizontal and the temperature uniformly distributed. There are then no significant thickness troughs and ridges, and the vortex centres and trough (ridge) axes in the contour patterns at different levels are aligned almost vertically.

Lows can be categorized in a variety of ways. Their main formative processes may be distinguished: for example, frontal lows develop as a result of baroclinic instability, and non-frontal lows develop either through the effects of surface heating and convection (polar lows and heat lows), or through the effects of topography (orographic lows) (see 2.2.7.3 page 35). Alternatively, their characteristic temperature structure may be described, giving rise to the terms 'cold-core' and 'warm-core' lows. Frontal depressions in mid latitudes are normally cold-core systems. The occlusion process gradually lifts the warm air away from the surface low, and cold air covers the whole area. In a cold-core low, the thermal structure is such that the thermal winds enhance the low-level winds so that the vortex is strong at upper levels. On the other hand there are some types of low, of which tropical storms are the best example, which are warm-cored. The warm core results from the release of latent heat of condensation in a region of deep and vigorous convection. And the superposition of a warm core (thickness high) on to a surface low leads to a weakening of the cyclonic circulation with height. The upper circulation over a warm-cored low is very weak.

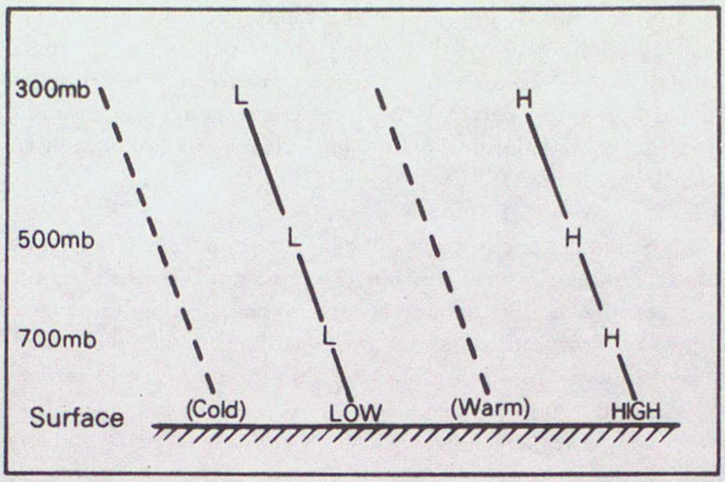


FIGURE 30. The slope of trough and ridge axes with height

In a similar way, warm and cold highs are distinguished by whether they are formed in warm or cold air to the south or north of the polar front. With a cold high, the anticyclonic circulation decreases with height, since cold air or low thickness implies a cyclonic thermal wind pattern. With a warm high, the anticyclonic circulation increases with height, since the thermal wind pattern is also anticyclonic.

Jet streams

Upper-air analyses are based on fewer observations than surface analyses, and the contour patterns at upper levels are generally simpler and smoother than those lower down. Unlike the situation at the surface, where closed vortices are common features which form at a very early stage in the life cycle of a weather system, closed upper-level vortices are less common and are typical of mature systems towards the end of their evolution. The most important feature of charts of the upper troposphere is the belt of broadly westerly winds flowing around each hemisphere in mid latitudes. This is formed as a result of the thermal contrast between cold air over the poles and warmer air at the same level over the equator. This belt of zonal winds may be fairly straight, or may oscillate in a series of troughs and ridges. Sometimes these features amplify so much that the flow becomes almost meridional around the flanks of the troughs and ridges, whose extremities then penetrate to abnormal latitudes and cut-off vortices are formed. But although these features may form part of a 'blocking pattern' which persists for a long time, the block is essentially the final stage in that particular development. More important from the point of view of new developments is the jet stream.

Jet streams are regions of strong wind ($V \approx$ 40-80 m/s) concentrated in a narrow, shallow band at the top of the troposphere. They are also characterized by strong horizontal and vertical shears of the wind ($\partial V/\partial x$, $\partial V/\partial y$ and $\partial V/\partial z$), and by typical patterns at the jet entrance and jet exit of acceleration-ageostrophic motion, vertical motion and surface development areas, as shown in Figures 13 (page 23) and 26 (page 46). These aspects have been described earlier in 2.2.5.3 (page 22) and 2.2.9.2 (page 42).

Jets are very important dynamical systems. They transport large masses of air at high speed, and this movement of mass aloft has a great effect upon the pressure at lower levels. There are compensating motions at lower levels, connected with the high-level developments through the three-dimensional vertical circulation, which counteract much of the effect of the high-level motions (see Figure 22, page 41), but when this compensation is weak a considerable imbalance may arise between the divergence of mass at upper and lower levels, which can result in very rapid surface developments.

2.3.1.2 Smaller-scale systems. There are other significant weather systems with which a forecaster has to deal and which do not readily fall into one of the classical synoptic-scale categories listed in 2.3.1.1 above. However, many of them do not differ very significantly except in size and can conveniently be considered here as small-scale (or mesoscale) versions of the more familiar synoptic systems. Thus we can distinguish mesoscale pressure patterns (such as convergence zones and divergence zones), mesoscale air masses (restricted areas of untypical, but uniform, weather), and mesoscale fronts (shallow, anticyclonic, fronts or sea-breeze fronts).

Convergence zones, in particular zones of low-level convergence, are regions of enhanced upward motion and intensification of cloud and precipitation. They may arise as the result of dynamical processes, or through surface heating, or for topographical reasons. In temperate latitudes their analysis generally requires isobars to be drawn at a smaller interval than usual (every millibar), or for streamlines to be drawn in place of isobars in situations where the pressure field is very uniform. For mesoscale analysis and forecasting, divergence zones (characterized by unusually fine weather) are just as important as convergence zones. They are somewhat less easy to identify and analyse than the more commonly recognized convergence zones.

Often, within the circulation of large, stationary anticyclones, there are shallow areas of air, of restricted horizontal extent, having uniform weather characteristics which are rather different from those found nearby. A 'mini' air mass of this kind may, for example, give a coherent area of stratus of stratocumulus cloud within the circulation of a high which is otherwise cloud-free. Or there may be similar areas of very moist (and potentially foggy) air, or very dry (and non-foggy) air, that are very important for detailed forecasting, and which may be unusual in the context of the normal descriptions of anticyclonic weather.

Mesoscale frontal systems can occur in some situations. Sea-breeze fronts are probably the best example of a genuine frontal phenomenon, separating two masses of air having different temperature and moisture characteristics and having a distinctive vertical motion and weather distribution. Again, in an anticyclonic circulation it may happen that quite genuine frontal conditions can be observed over a limited depth of the atmosphere. Anticyclonic subsidence may obliterate all traces of typical frontal structure at upper levels, but near the ground some significant discontinuities may remain. Such shallow fronts are, typically, kata-fronts and, like all these smaller-scale systems, though they do not spread their effects over a wide area or have a significant effect upon the overall dynamical development of the atmosphere, they are nevertheless important aspects of the weather for local forecasting.

2.3.1.3 Three-dimensional representations of weather systems. All weather systems are three-dimensional, in space, and they are continuously evolving in time. It is important for a proper dynamical understanding of the atmosphere that this should be recognized. Unfortunately, neither the descriptions found in many textbooks, nor indeed a forecaster's normal working charts, really help to achieve this. Forecasters are, of necessity, trained to take an 'Eulerian' rather than a 'Lagrangian' view of the atmosphere (see the Appendix, part 2). They observe the weather at fixed places and they work with charts representing instantaneous snapshots of the weather at fixed times. Their knowledge of the atmosphere is therefore rather strongly angled towards the changes occurring at fixed places, rather than the changes undergone by individual air particles which move around the atmosphere. They deal with 'local changes' ($\partial/\partial t$), which are required for operational purposes, rather than with 'changes following the motion' (d/dt) which are more important for dynamical understanding.

It is not easy to display a three-dimensional structure on flat sheets of paper. The simplest technique, which is quick but does not reveal very much about the vertical motions occurring in a system, is to display isopleths from two different levels on the same chart. A more comprehensive technique, but one which takes much longer to complete and cannot therefore be used operationally to any great extent, is to use isentropic analysis. In regions where the flow is adiabatic this technique allows one to infer the trajectories of air particles, or the streamlines of the flow relative to a moving system. Some examples of these techniques follow.

The normal location of a jet stream relative to surface fronts

The axis of a jet stream is normally located over the tip of the warm sector and in this region is orientated nearly parallel to the warm-sector isobars, as at A in Figure 31. It lies roughly parallel to the surface fronts in the regions ahead of the warm front and to the rear of the cold front.

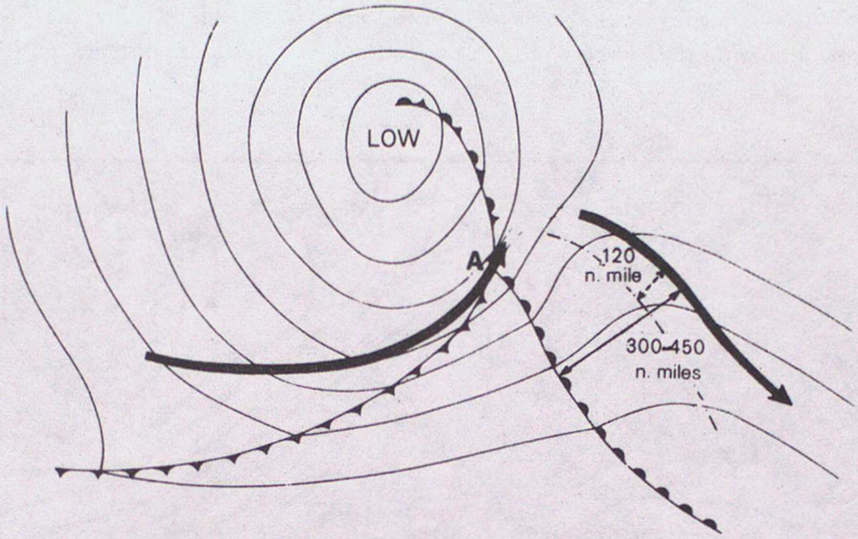


FIGURE 31. Jet stream in relation to surface fronts

⟶ Jet axis
-·-·-· Axis of surface ridge

The normal relation of surface fronts and 1000-500-mb thickness lines

In Figure 32 note the strong gradient of thickness on the cold side of the surface fronts, also the near-uniform thickness in the warm air mass. The thickness line associated with the tip of the warm sector has the same value in each of the four diagrams.

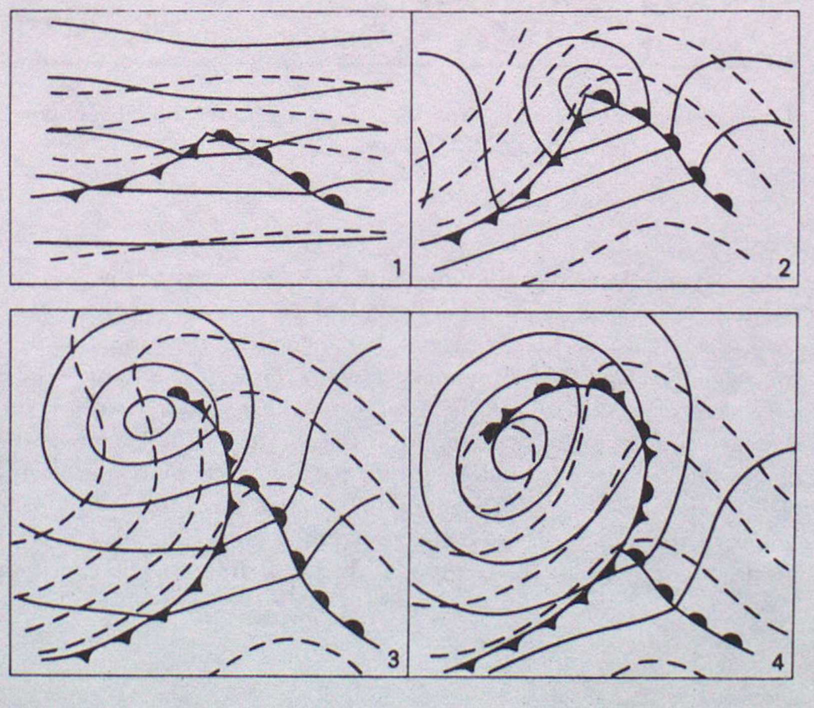


FIGURE 32. 1000-500-mb thickness patterns (– – – –) over a frontal depression at various stages of development

The normal relation of surface-pressure features to the upper-air pattern

Figure 33 shows a particular situation, with the location of surface centres shown in relation to the features of a 300-mb chart. This example displays some normal features: (a) mature surface lows (and troughs) lie beneath or on the eastern flank of the upper troughs (see eastern U.S.A.), (b) surface highs (and ridges) lie beneath or on the eastern flank of upper ridges (see western Atlantic), and (c) incipient, breakaway wave depressions are embedded in the main stream of the upper flow and move rapidly, without as yet distorting the shape of the upper contour pattern (see southern England).

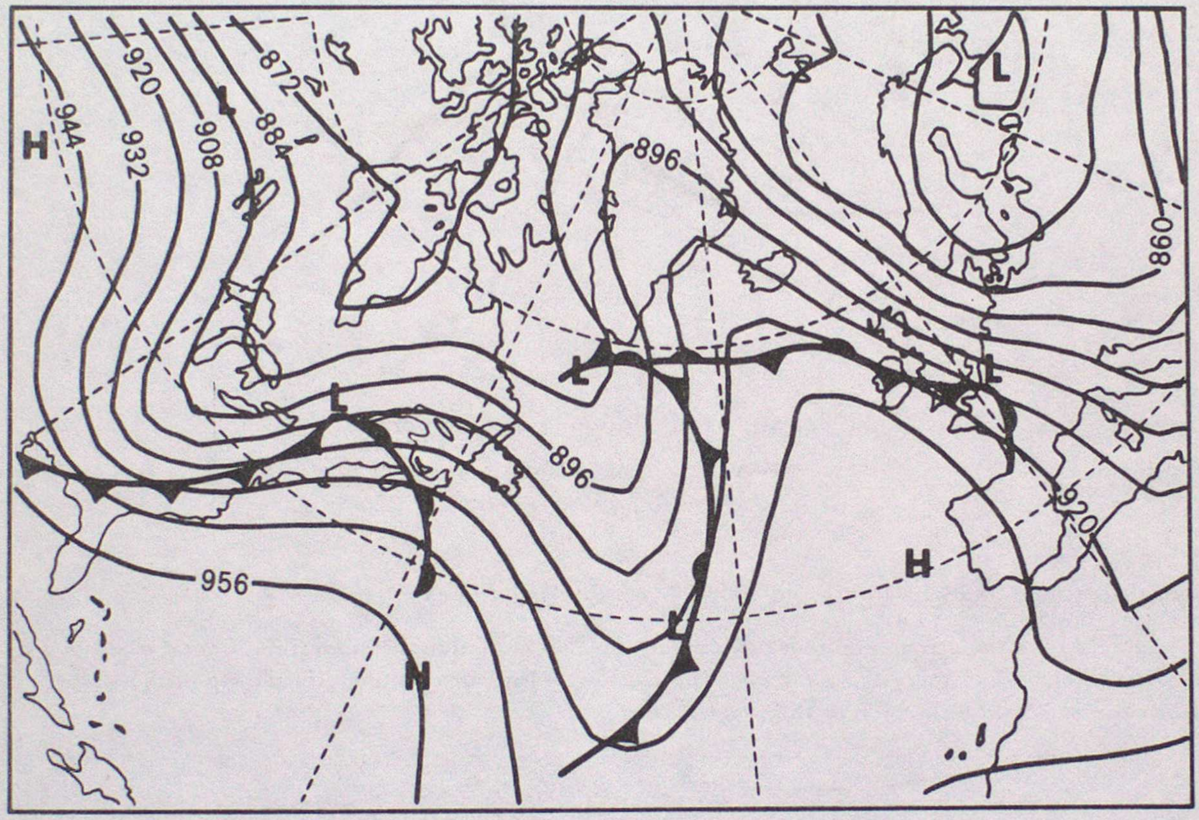


FIGURE 33. Surface-pressure centres and fronts superimposed on 300-mb geopotentials for 0000 GMT, 29 November 1973

Isentropic analysis is a valuable technique in those situations where winds are quite strong and the atmospheric developments are strongly dependent on dynamical processes. In such situations the dominant temperature changes are adiabatic in character and, consequently, isentropic analysis is permissible. When the winds are very light and the air stagnates over a large region, non-adiabatic temperature changes become relatively more important so that isentropic processes are not so relevant. When isentropic (or adiabatic) processes are occurring, the potential temperature (θ) of an air particle can be treated as conservative, and the movement of individual air particles can be assumed to lie always in a surface of constant θ. These surfaces are not horizontal and, where they slope, the air motion has a vertical component. So maps of the height of an isentropic surface (specified by a given value of θ), coupled with maps of the horizontal wind flow, give a direct indication of the vertical motion.

These maps can be formalized into diagrams such as Figure 8 (repeated on page 60) and Figure 36 (page 62) which depict the typical three-dimensional motions in such systems as fronts and severe convective storms.

The broad-scale circulation in trough/ridge systems

Figure 34 is a very simplified sketch, by Green, Ludlam and McIlveen[22] of the general character of the broad-scale circulation in a system of synoptic-scale troughs and ridges.

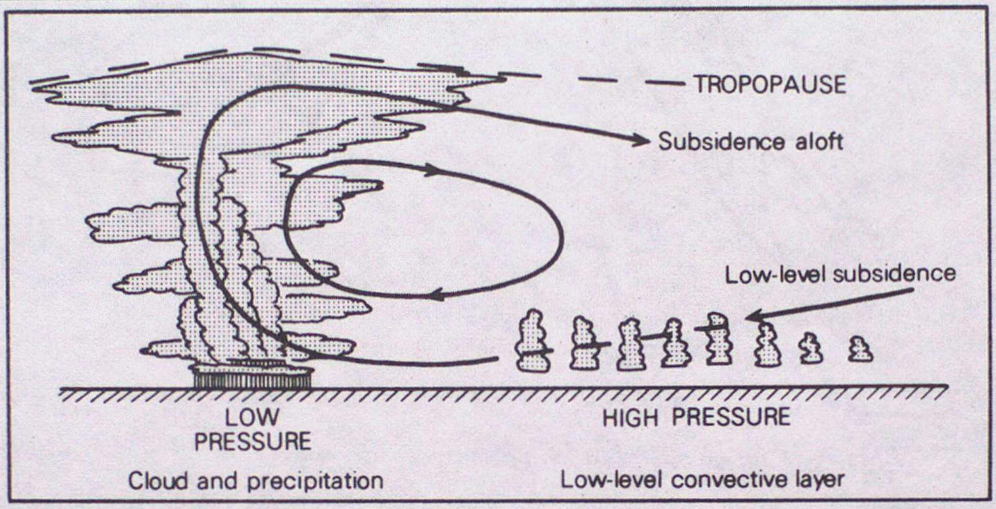


FIGURE 34. Broad-scale circulation pattern in synoptic systems (after Green, Ludlam and McIlveen[22])

The airflow is delineated by arrows which are generalized trajectories derived to a great extent from isentropic-flow computations. They show that there is a marked asymmetry in the vertical motion field. The region of ascent is concentrated into a rather narrow area around the surface low pressure where there is rapid adiabatic inflow at low levels and ascent. The ascending arm is sharpened and concentrated by the latent heat of condensation released in the ascent. Much of the condensed water falls out as rain, and the subsiding arm of the circulation is one of widespread, gentle sinking of cloud-free air of reduced potential temperature whose rate of subsidence is largely controlled by the radiative heat loss (a non-adiabatic process) in mid levels. Eventually, at low levels in the shallow surface convective layer near the high-pressure regions, the air gains heat and moisture once more (increasing its potential temperature in further non-adiabatic processes) before re-entering the moist ascending arm of the circulation.

Three-dimensional motion near fronts

The broad features of the large-scale flow of air in partly occluded frontal systems in mid latitudes have recently been described by Harrold[9] on the basis of detailed isentropic analyses. His model was shown in Figure 8 on page 17 but is repeated here for ease of reference. It indicates that the production of frontal precipitation occurs mainly within a tongue of warm air (stippled in the diagram, and named the 'warm conveyor belt'). This flows ahead of the cold front before ascending above the warm front. The extent of this warm ascending current is well defined, being bounded to the west by the cold front, at the top by air of different origin flowing over the cold front, and to the east by a decrease in the northward component of flow. The conveyor belt is really the ascending arm of the flow described in the previous paragraph and illustrated in Figure 34. The warm air starts to ascend within the warm sector so that the area of precipitation is often not closely related to the position of the surface warm front.

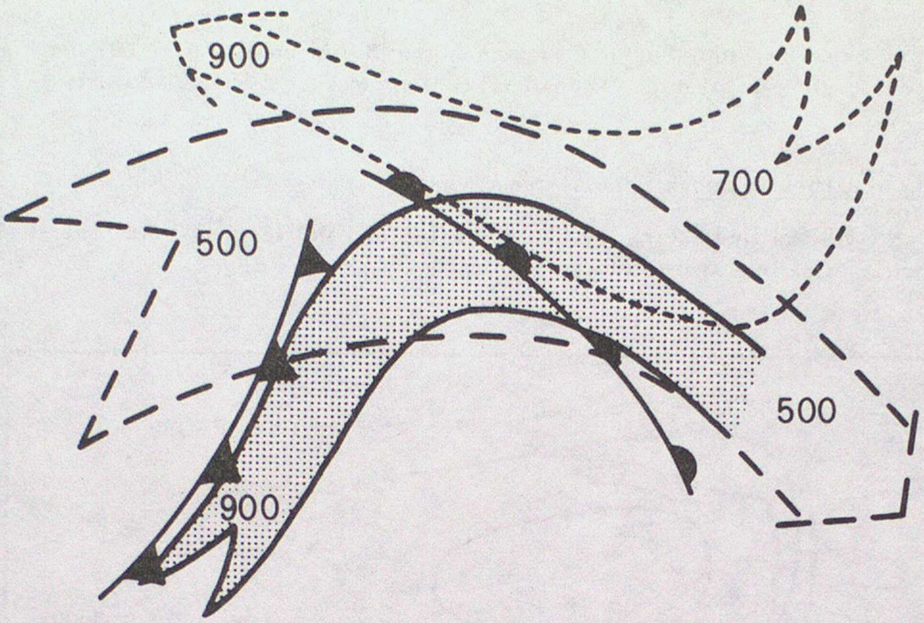


FIGURE 8. The warm-air 'conveyor belt': broad arrows denote airflow relative to a mobile frontal depression

Ascending air moving from warm sector up the warm-front surface.

Mid-tropospheric air having small vertical motion.

Subsiding air from the pre-frontal ridge.

Figures within arrows indicate level of flow in millibars.

Since the air in this system is largely saturated, it is θ_w (the wet-bulb potential temperature) which is the relevant indicator of adiabatic processes. Within the warm, moist conveyor belt, values of θ_w are high: higher, in fact, than the values of θ_w in the air just above the conveyor belt, which is a situation implying potential instability. It is the release of this instability in the upper part of the warm conveyor belt which gives rise to small-scale variations in the distribution of the frontal rainfall, particularly in the warm sector and near the surface warm front. Well ahead of the warm front the potential instability becomes negligible, and the rainfall in this region has an almost uniform character.

The cold front represented schematically in Figure 8 (see above) is a kata cold front. A diagram of the flow at a more active, ana cold front is shown in Figure 35. This is based on the work of Browning and Harrold,[11] which has later been extended by Browning and Pardoe[23] to include observations of low-level wind maxima (often 30-40 knots at a height of about 1 kilometre). Once again the flow *relative to the system* is depicted and the very sharp, almost vertical structure of the cold front near the surface is remarkable. The vertical circulation is a thermally direct one, with rapid ascent of the warm air overlying a less-rapid subsiding motion in the cold air.

Three-dimensional motion in severe convective storms

Most thunderstorms consist of a group of cloud cells, each of which has a life cycle lasting less than an hour. Each cell goes through three stages of evolution: first, the cumulus phase in which the cells develop and there is ascending air; second, the mature phase, when the first precipitation reaches the ground and there are both updraughts and downdraughts coexisting in different parts of the cloud; third, the dissipating stage when there are downdraughts throughout the entire cell. But, occasionally, an ordinary mature thunderstorm of this type is able to evolve into a totally different kind of organization, and these are called 'severe local convective storms'. The required conditions that produce these storms are a strong, vertical wind shear, combined with a stable layer that has trapped warm, humid air beneath cold, dry air aloft.

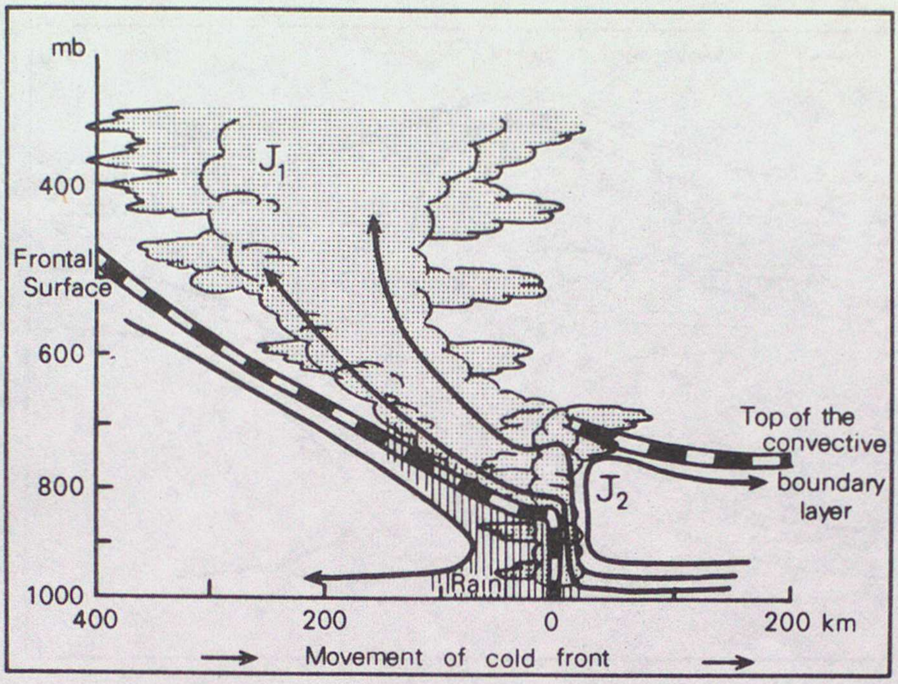


FIGURE 35. The airflow at an active cold front (after Browning and Harrold,[11] and Browning and Pardoe[23])

Broad arrows indicate the airflow relative to the front.
J_1, J_2 are the polar-front jet stream and low-level jet stream.

In a severe local convective storm, the mature stage of the cumulus-cell evolution takes on a new lease of life in which individual cloud cells continually evolve, not to dissipate, but to become part of a large super-cell. The downdraught from one cell reacts upon, and intensifies, the updraught of the succeeding cell which, in turn, gives rise to a stronger downdraught, and so the process repeats and is self-maintaining.

The basic character of the flow in these severe storms was first described by Ludlam.[24] His treatment was largely two-dimensional, for simplicity, and Figure 36 represents the essential character of the flow he described. Once again the flow is represented by means of streamlines relative to the system. The ascending warm air and descending cold air are features which establish the quasi-frontal nature of the system. The narrow region of very rapid ascent, contrasting with the wider areas of gentler subsidence, are familiar features from the systems described earlier.

The role of latent-heat exchanges in this circulation is very important. The release of latent heat of condensation increases the buoyancy of the updraught air. At the same time, the absorption of latent heat, through the partial evaporation of rain falling beneath the inclined updraught, is responsible for the creation of negative buoyancy within dry high-level air undercutting the updraught at the rear of the storm. This reaches the ground as a cold downdraught, the leading edge of which is apparent as the thunderstorm gust-front.

Later work by Browning[25] has shown that the development of 'severe local storms' is characterized not only by a strong wind shear, but also by a pronounced veer in the winds with height. As a result, the updraught-downdraught couplet consists of a rather complex interlocking circulation, in which warm air from low levels enters the updraught from the storm's right forward flank, while cold air from middle levels feeds into the downdraught from the right flank. Air approaching the rear of the storm at high levels is thought to flow around, and to some extent over, the updraught, almost as though it were a solid barrier to the environmental flow.

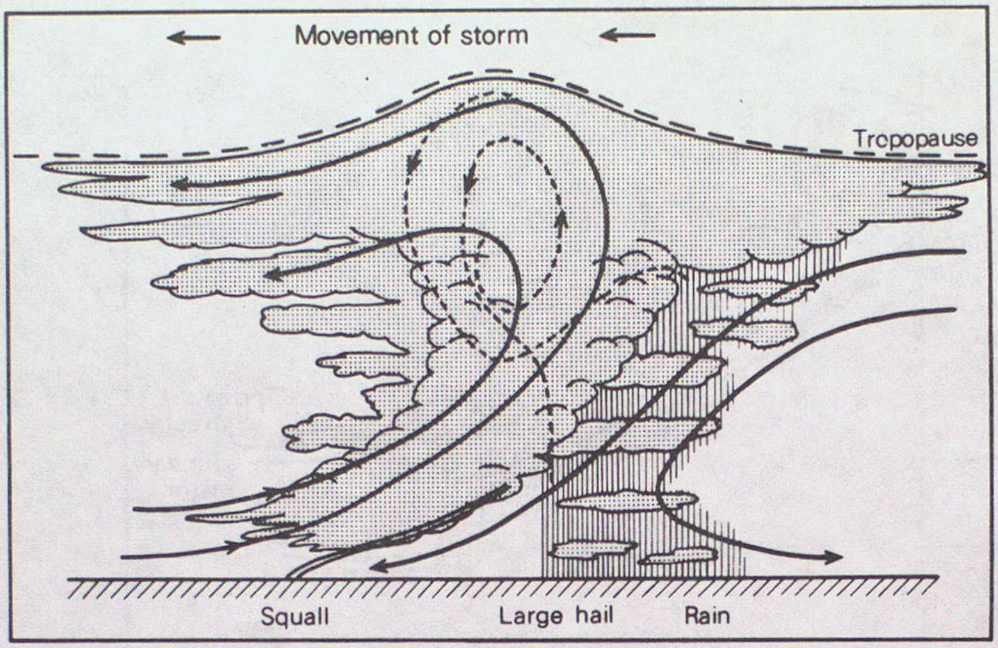


FIGURE 36. Model of a hailstorm (after Ludlam[24])

Broad arrows show the airflow relative to the storm. Dotted lines are typical paths of growing hailstones. Falling precipitation is shown by vertical hatching.

Most 'severe local storms' move either slightly to the right of, and rather more slowly than, the mid-tropospheric winds, or (somewhat more rarely) slightly to the left of, and rather quicker than, the mid-tropospheric winds. This character of the horizontal movement of these storms is apparently an important feature which determines their organization, but it is by no means clear what causes them to travel in such a manner in the first place.

2.3.2 The evolution of surface synoptic systems

2.3.2.1 The movement of existing systems

Frontal depressions

(a) *Formative stage:* 'Thermal steering' ($\mathbf{V}'.\,\partial\zeta_0/\partial s$) is the main developmental effect, with the small low-level circulation moving in the direction of an undistorted upper flow. Shallow frontal waves move along the front (warm or cold) on which they form, the orientation of the front, in depth, being generally very close to the orientation of the 1000-500-mb thickness lines. The speed of small lows is usually less than the wind speed at any level in the warm sector, and traditional forecasting practice used the value of 4/5 of the surface geostrophic wind speed in the warm air mass as a convenient and useful working rule.

(b) *Mature stage:* The initial thermal steering principle becomes increasingly modified as cyclonic vorticity develops to higher and higher levels in the low-pressure system. As the depth of the cyclonic circulation deepens so the upper flow pattern is further distorted, and 'thermal development' ($\mathbf{V}'.\,\partial\zeta'/\partial s$) becomes the dominant term in the development and movement of the system. Reliable empirical rules are lacking for this stage in the evolution of a frontal low. There are only the Sutcliffe development areas (Figure 27, page 47) and the old observation that deepening depressions tend to swing towards the left of their original straight-line track.

(c) *Dissipating stage:* At this stage there is little dynamic control on the movement of the system, which becomes very slow. The system becomes almost barotropic as a stationary vortex with a similar circulation pattern at all levels. Any movement is consistent with its past history and, for an asymmetric vortex, may be determined by the direction of the strongest flow around its periphery. If other old vortex centres are present in the vicinity, they may all revolve slowly and cyclonically around some common 'centre of gravity' - a process referred to as 'dumb-belling'. Alternatively the old low may just stagnate and fill without moving.

Non-frontal depressions

(a) *Orographic lows:* These form as the result of the deflection of an airstream crossing a mountain barrier and their motion is, at least initially, very slow. They remain tied to their parent mountain, especially when they form in a fairly homogeneous air mass. There are times, however, when a mature and significantly large orographic low becomes associated with a baroclinic zone. It then develops a frontal structure and moves away from its original position as a frontal depression. Over the British Isles, the formation and movement of orographic lows is not, in general, a very pressing problem.

(b) *Polar lows:* These systems do affect the British Isles from time to time in winter, approaching from the regions around Iceland where observational data is normally rather sparse. Although they are frequently thought of as being largely convective phenomena, they are also mobile, dynamic, circulation systems which retain their coherence for a far greater length of time than any disorganized cluster of convective cells is capable of doing. They are, therefore, systems in which vorticity advection is carried out in an organized way, probably in the manner of thermal steering at low levels - similar to the way that developing frontal depressions move. Although polar lows are not strictly the same as polar-front depressions, there is increasing documentation[26-28] of the fact that they frequently form in association with baroclinic zones and therefore have some occasional features resembling a normal frontal structure, and their movement is normally along the thickness lines at something like 20-25 knots.

(c) *Heat lows:* These are shallow features, forming inland on hot summer days, generally in conditions of very weak pressure gradients and light winds. The advection of the vorticity is very weak, and these lows show very little movement.

Troughs

(a) *Frontal troughs:* Whether or not the isobaric trough is coincident with the surface front (or, 'change of air mass', however this be defined), the 'trough' and the 'front' must normally be forecast to move together, maintaining their existing relationship. The movement of a frontal trough is therefore identical with that of the front itself — see Fronts on page 54.

As a general rule, the speed of a surface front and a frontal trough is close to the speed of the cold air mass involved in the frontal discontinuity. Thus the speed of a warm front is given by the speed at which the cold air moves away, and the speed of a cold front is governed by the rate at which the cold air advances. This speed should be measured at low levels, but above the friction layer, at a height of about 600 metres.

(b) *Non-frontal troughs:* These are very varied in character, and not easy to classify. Their most distinctive features are that they are generally regions of enhanced vertical motion, with the air particles moving through the pattern on three-dimensional trajectories. Troughs in unstable air probably have the most significant weather, in the form of increased showery activity. They move at a speed which is rather less than the mean wind over the depth of air comprising the 'trough' circulation system. A useful practical approximation is to take the 700-mb wind as an estimate of the advecting wind, if no other information is available. But, whenever possible, continuity from its past movement is the best guide to the future motion of a trough.

Anticyclones

(a) *Cold highs:* These can be separated into two broad types. The first are somewhat baroclinic systems. Although mainly located in cold air on the surface, they are mobile, oceanic systems separating the successive depressions that form on the polar front. In depth, therefore, they are more or less embedded in a broad baroclinic current, and thermal development ($V' \cdot \partial\zeta'/\partial s$) has an important effect in determining their movement. The advection of anticyclonic vorticity in this case is complementary to the advection of cyclonic vorticity which determines the low-pressure developments mentioned in (b) of Frontal depressions (page 62). Both processes are part of the overall development of disturbances in the baroclinic zone.

A second type of cold high is the quasi-barotropic system which forms over very cold continental land masses in winter. The very cold and dense surface air contributes a lot to the high pressure. In these systems the pressure decreases more rapidly with height, in accordance with the hydrostatic equation, than it does in neighbouring warmer air. Thus the cold high is transformed into an area of relatively low pressure in the upper troposphere. Generally the winds over the centre of the high are light and the advection of anticyclonic vorticity is small. Consequently these systems are very slow moving.

(b) *Warm highs:* These are large quasi-barotropic systems, but formed in warm air over the tropical oceans. Their circulation extends, with little variation, through most of the troposphere and their movement is very slow.

Ridges

Like anticyclones, these may be either barotropic or baroclinic in character. The former are usually rather large and slow moving, whereas the latter are small and mobile. The movement of the baroclinic systems is normally in sympathy with, and governed by, that of the surrounding lows and troughs.

Fronts

(a) *Warm fronts:* The speed of a surface warm front is determined by the speed of the cold air near the surface (just above the friction layer at 900 mb), just ahead of the front. This speed will often be different from that of the warm air owing to the ageostrophic motions ahead of the moving low. With a deepening low, the effect of the ageostrophic motion is to slow down the rate at which the cold air moves away, and this has the effect of enhancing the rate at which the warm air ascends in the frontal zone, thus increasing the frontal activity. However, ageostrophic effects are very difficult to assess subjectively and, in traditional forecasting practice, the speed of warm fronts is estimated by taking some proportion of the component of geostrophic wind perpendicular to the front. The usual proportion is 2/3 over land and 5/6 over the sea; but these values can be varied in individual situations if it is clearly apparent from the observed movement of the front that significant ageostrophic motions are affecting the speed of the front.

(b) *Cold fronts:* Their movement is determined by the speed of the cold air, undercutting the warm air mass, to the rear of the frontal zone. Ageostrophic motions in this case are usually such as to increase the speed of the front. With a cold front, the speed is nearly always at least equal to the geostrophic wind component perpendicular to the front and, especially when pressure is rising in the cold air and a ridge is building, it may be quite significantly supergeostrophic.

(c) *Occlusions:* Warm occlusions and cold occlusions behave in much the same manner as warm fronts and cold fronts, respectively. Warm occlusions are somewhat rare, but are slow-moving systems and can bring prolonged wet, or snowy, weather in some circumstances in winter-time. Cold occlusions are the normal development resulting from the ascending motion of the warm air mass within the circulation of a polar-front depression. Initially, the occlusion is related to the low-level circulation and can be satisfactorily analysed by assuming the tip of the occlusion to be coincident with the surface low-pressure centre, and moving along with it. Gradually, however, as the occlusion process continues and the warm air is lifted to higher levels, the significant cloud and weather are more closely allied to the location and movement of the upper vortex rather than that at the surface. These may not be vertically aligned, though, as the system evolves from maturity to its final decaying stages, they may become more nearly so. However, by this time it is clear from satellite cloud photographs that the typical cloud formation of the cold occlusion is a complex array of cloud bands spiralling around the upper vortex. The movement of these bands, with vertical motions becoming insignificant, is given by the observed horizontal wind at the cloud level.

(d) *Quasi-stationary fronts:* These, by their very definition, are very slow moving. Nevertheless, it may often make a considerable difference to the correctness of a weather forecast in particular localities if their slight, but significant, movement is not accurately forecast. These fronts have a negligible isobaric gradient across them and the main control on their movement is usually given by the non-geostrophic isallobaric effects. The small speeds involved are best estimated from a series of accurate analyses of successive frontal positions, related to the isallobaric gradients on each chart.

(e) *Small-scale fronts:* These may be fronts which are essentially large-scale systems which have become restricted in their vertical dimensions, or they may be genuine mesoscale circulations (like sea-breeze fronts) whose vertical and horizontal extents are comparable.

Shallow fronts usually attain their characteristic shallowness through the effects of subsidence drying out the mid-tropospheric cloud layers and minimizing the original temperature contrast at these levels. The low-level frontal characteristics that remain may still be very significant in terms of their effect on the weather of local areas. But, dynamically, fronts with small vertical dimensions must have quite small vertical motions. They are therefore mainly non-developmental and the frontal cloud moves with the speed of the horizontal wind at the level concerned.

Sea-breeze fronts are mesoscale systems with a short life and their movement is greatly affected by local topography. During the middle of a summer afternoon an inland movement at 8-10 knots with a maximum penetration of about 90 kilometres (50 n. mile) is typical.

Non-frontal weather systems

(a) *Non-frontal rain:* This is usually associated with convergence and enhanced ascent in a trough (possibly an upper-level trough). Movement of the rain is at a speed which is somewhat less than the wind speed at the main trough level. $2/3\ V$ is a typical value, but there are wide variations.

(b) *Showers and thunderstorms:* Whether in the form of individual cells, which have a very short life ($\leqslant 1$ hour), or organized clusters or lines which last rather longer, or as 'severe local storms' whose coherent life may be 10 hours or so, all convective systems extend through a great depth of the troposphere and are characterized by very significant, localized, vertical currents. Their horizontal motion is therefore a complex result of three-dimensional circulations through a great depth of the atmosphere. For practical forecasting it is only possible to estimate the motion by taking it equal to some mid-level wind in the convective layer. For deep, active, convective storms in warm summer air it has been found that a movement with the 700-millibar wind is a useful working approximation.

(c) *Non-frontal cloud layers:* Where the cloud layer concerned is in the middle of the troposphere (that is to say, an altocumulus or stratocumulus type), its movement is generally given by the horizontal wind at the cloud level. The most significant departure from this rule is in the case of cloud layers which are formed through orographic effects; in this case the cloud (often, but not always, lenticular) may be forming and dissipating in the vertical currents of a lee-wave system and consequently be almost stationary. Non-frontal cirrus cloud is not generally thick enough to affect the weather at the ground, and forecasts of its movement are rarely required.

The movement of low-cloud sheets, especially stratus, are very important for local forecasting. These clouds form and decay in response to variations in the surface temperature and details of the local topography. So, although an existing stratus-cloud sheet will often move at approximately the speed of the wind at its level, it may at times move faster (or slower) than this if the cloud sheet is at the same time growing (or shrinking) at its leading edge (see Chapter 19 – Clouds and precipitation).

(d) *Other significant weather areas:* Radiation fog forms in conditions of very light wind so that its general movement is nil. Locally, small-scale movements of the radiation fog patches do occur. These are largely controlled by the topography and local winds.

Advection fog is very similar in character and behaviour to stratus cloud, mentioned in the preceding subsection.

Haze, if it is thick enough to be troublesome in the British Isles, is usually due to the concentration of industrial smokes in the lowest levels, beneath an anticyclonic subsidence inversion. Areas likely to be affected can be estimated from the mean wind in the layer beneath the inversion. Detailed forecasting may be possible on a very local scale, having regard to any known sources of pollution and the disposition of the local topography, but it is rare for this to be very profitable or of more than local interest.

2.3.2.2 The formation of new systems

Frontal depressions

(a) *Secondary lows on a cold front:* These systems are amongst the most frequent of new low-pressure developments. The characteristic isobaric and thickness pattern is shown in Figure 37.

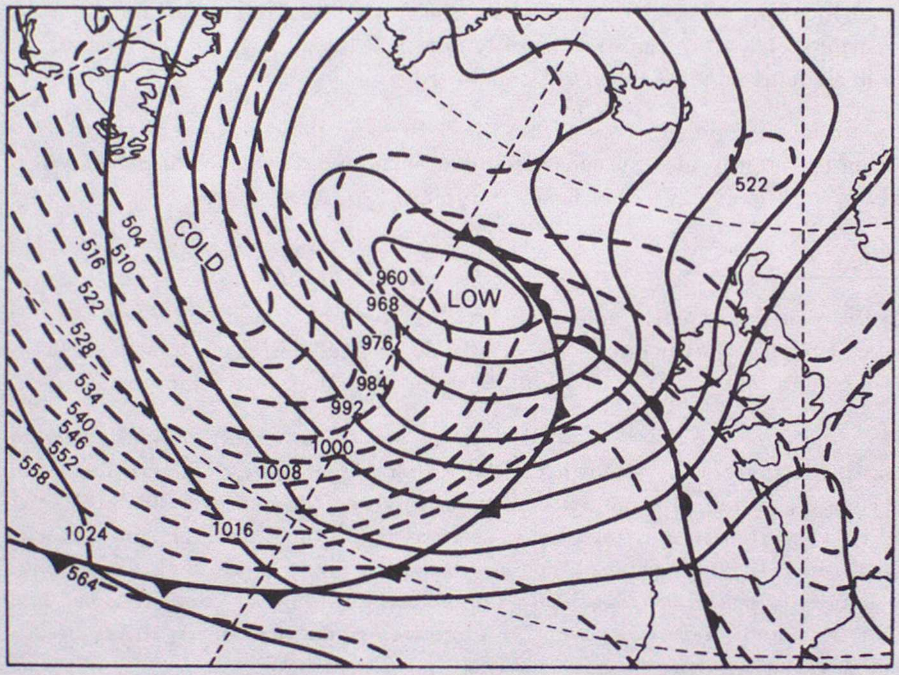


FIGURE 37. Formation of cold-front secondaries

—— Isobars – – – Thickness lines

Typical pattern, occurring on 10 January 1974. A secondary depression formed on the cold front near the Azores, and 24 hours later this new low was approaching south-west Ireland.

Sawyer[29] specified some of the conditions which are favourable for their information. They occur: (a) in locations which are beyond the area of influence of the primary low; usually 2000 km ($\approx$ 1000n. mile) (25-30 degrees of longitude) or more away; (b) on cold fronts which are at least moderately well defined ($V' \geqslant 25$ knots all along the front); (c) in regions where the shape of the front is distorted either by orographic constraints or by the disposition of adjacent pressure systems; (d) where the advection of cyclonic thermal vorticity ($\mathbf{V}'\ \partial\zeta'/\partial s$) is greatest.

The new wave depression frequently forms in the 'C' area ahead of an upper thermal trough (see Figure 27, page 47), the 'C' area being, typically, some 10 degrees of longitude east of the thermal trough line. New waves generally appear first near the upwind edge of the positive thermal vorticity area towards which the thermal wind is directed.

An old-established rule is that wave depressions, once formed, deepen in accordance with the barometric tendency in the warm sector, and the careful observation of the tendency field remains a very valuable subjective forecasting tool. But vorticity advection is the basic dynamical principle, for which the development areas shown in Figure 27 are a rough-and-ready guide.

Some waves move with little change of intensity for some time, and then suddenly deepen when they move through a favourable development area. It has been stated[30] that: 'A wave may first appear in the cyclogenetic regions ahead of the thermal trough and then run away from it without becoming a major feature, at least till much later, after it has moved a long way. Rapid development into a large and deep depression is most likely to occur when an already existing wave runs into the cyclogenetic region from the rear, though considerable deepening can occur in a nearly straight, steep thermal gradient.'

Some warm-sector depressions fail to develop, and may even fill although they have an open warm sector. Such a process may be expected when the low moves into an anticyclogenetic development field – that is, a mutually destroying combination.

(b) *Secondary lows on a warm front:* These are rather unusual formations. In addition, although they may have a short-lived phase of rapid deepening, they seldom, if ever, develop into large systems, so that even when well developed they do not appear as major features on an isobaric chart. However, when they do form they can be responsible for very pronounced deteriorations in the weather, which is accentuated by the rapid speed at which these systems normally travel. The characteristic isobaric and thickness pattern is shown in Figure 38.

Sawyer[31] noted the following conditions as being favourable, but not sufficient, for their information. They occur: (i) in association with a slow-moving primary low and a strongly marked confluent ridge in the thickness field; (ii) on warm fronts having very strong thermal contrast ($V' \approx$ 40-80 knots). Jones[32] has also noted the formation of warm-front secondaries in association with the 'C' development area near the right-hand entrance of a north-westerly jet, which is located on the east side of a 300-mb ridge.

The existence of a suitable thickness pattern is no guarantee that a warm-front wave will inevitably form. Nor, if one does form, is it usually succeeded by any further developments of a similar nature. Those warm-front waves which affect the British Isles usually form over the North Atlantic in the vicinity of Iceland.

(c) *Secondary lows on a warm occlusion:* The characteristics of this type, illustrated in Figure 39, are similar to those of the warm-front secondaries mentioned in the previous paragraphs.

(d) *Secondary lows on a cold occlusion:* A typical pattern of isobars and thickness lines is shown in Figure 40. This class of development probably occurs in a wider variety of situations than any of those mentioned previously in this section, but Sawyer[31] has noted the following conditions as being generally required for their formation:

(i) a slow-moving primary low, with the triple point of the occlusion several hundred miles distant from the low centres;

(ii) a well-marked thermal diffluence ahead of the triple point of the occlusion.

The development of these secondaries is therefore frequently associated with the 'C' development area at the left-hand exit of a jet stream.

The behaviour of these secondaries is less regular than those on warm occlusions and seems to be dependent upon the detail of the flow pattern around them. Although these secondaries often go through a short-lived phase of rapid deepening, very few become major features of the synoptic chart.

Non-frontal depressions

The depressions to be discussed under this heading are termed 'non-frontal' primarily to distinguish them from the classical polar-front depressions of the North Atlantic. Most of these other lows may at times exhibit frontal characteristics of one sort or another once they have developed, but their initial formation can normally be ascribed to some other process than baroclinic instability in a pre-existing frontal zone.

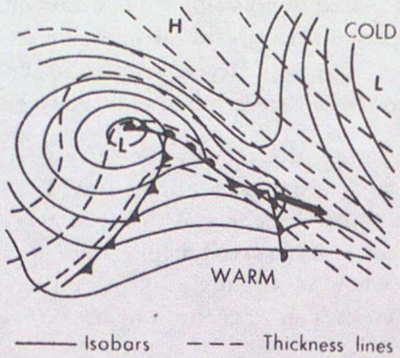


FIGURE 38. Characteristic isobaric and thickness patterns for formation of a warm-front wave

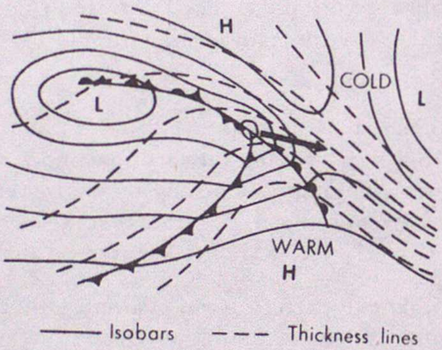


FIGURE 39. Characteristic isobaric and thickness patterns for formation of a warm-occlusion secondary

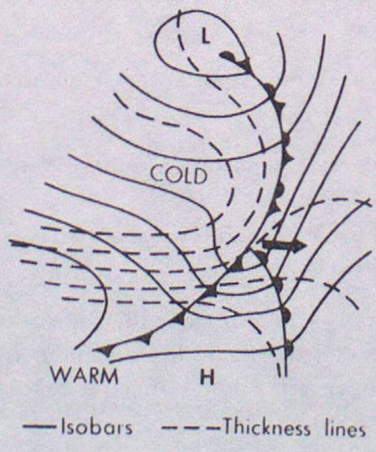


FIGURE 40. Characteristic isobaric and thickness patterns for formation of a secondary on a cold occlusion

(a) *Reinvigoration of old vortices:* Sometimes an old, decayed depression may move across (say) the eastern coast of North America, into the Atlantic, and become rejuvenated in a 'C' area where there is, at least initially, no clearly defined pre-existing frontal system. As the old low deepens again, fronts may be formed within its circulation, but the process is a complex one.

The region ahead of an upper trough is a very favourable cyclogenetic area, and when an old low becomes suitably placed in this area, so that it receives a flow of cold air on its western flank, deepening and development can occur both suddenly and rapidly. It has been stated [30] that, 'when there is simultaneous development . . . of a polar-air and a frontal depression, often amalgamating later to one system, it takes place ahead of the upper thermal trough. Some of the deepest systems are of this type, and . . . the frontal system usually absorbs the other.'

(b) *Polar lows:* The formation of polar lows, usually in the Icelandic region or similar latitudes where detailed observations are sparse, is rather poorly understood. They form in cold, unstable, polar air and Sumner [33] has shown theoretically that merely a very small degree of instability in depth is sufficient to permit small-scale cyclonic development to proceed rapidly. The location of Iceland, to the lee of the huge Greenland plateau, suggests that orographic influences may be important at times and Harley [26] noted one such occasion when this appeared to be the case. Recently, Harrold and Browning [27] emphasized the probable importance of a low-level baroclinic zone as a factor in their development. All of which suggests that, with an increase in observational data, it might be possible to distinguish several different types of polar lows. Meanwhile the only synoptic practice that is currently feasible is to be alert to the possibility of polar-low formation whenever a cold north-westerly current flows across the Greenland-Iceland area.

(c) *Heat lows:* These are shallow depressions forming at times over large land masses in summer. The initial appearance of this type of depression may be associated with a weak 'C' development area, though normally the thermal gradients over hot, largely cloudless, continents in summer are very light. The initial shape of a heat low is usually ill defined. Thunderstorms may develop in parts of this area as a result of convergence into the developing depression, and these subsequently amalgamate into rain areas which do not necessarily coincide with the area of greatest cyclonic development on the surface, or suggested in the thickness pattern.

Depressions of this type which are of particular concern to forecasters in the British Isles are those which form over France in the summer months. They usually form as the hot, cloudless air of a continental high slowly recedes, and thunderstorms are particularly triggered off by the advection of a cold upper trough across the Bay of Biscay above the hot surface layers. These thunderstorms are then advected northwards ahead of a thermal trough, or weak surface cold front, with which they are only loosely associated in their detailed distribution.

Anticyclones and ridges

The characteristics of the anticyclonic developments observed in the atmosphere are rather different from the cyclogenetic effects. The processes which lead to the formation of depressions often occur at a rapid rate and are concentrated into relatively small areas, whereas anticyclones generally build at a slower rate but which is often sustained over longer periods and spread over more extensive areas. Cyclogenesis, therefore, soon leads to the formation of closed isobaric systems at low levels, but anticyclogenesis often only gives rise to an intensified ridge development.

One reason for this difference is that vorticity advection is a smaller effect in anticyclogenesis than in cyclogenesis. ζ varies from 0 to $3f$ in regions of cyclonic development, whereas it only varies from 0 to $-\frac{3}{4}f$ in regions of anticyclogenesis.

Another factor is the different effect of the latent-heat exchanges in the two cases. In areas where cyclonic developments are favoured there is ascent of air, resulting in condensation of water vapour and the release of latent heat. This gives added buoyancy to the air and enhances the existing vertical motion, effectively stretching the vertical vortex tubes, and thus concentrating the developing cyclonic vorticity

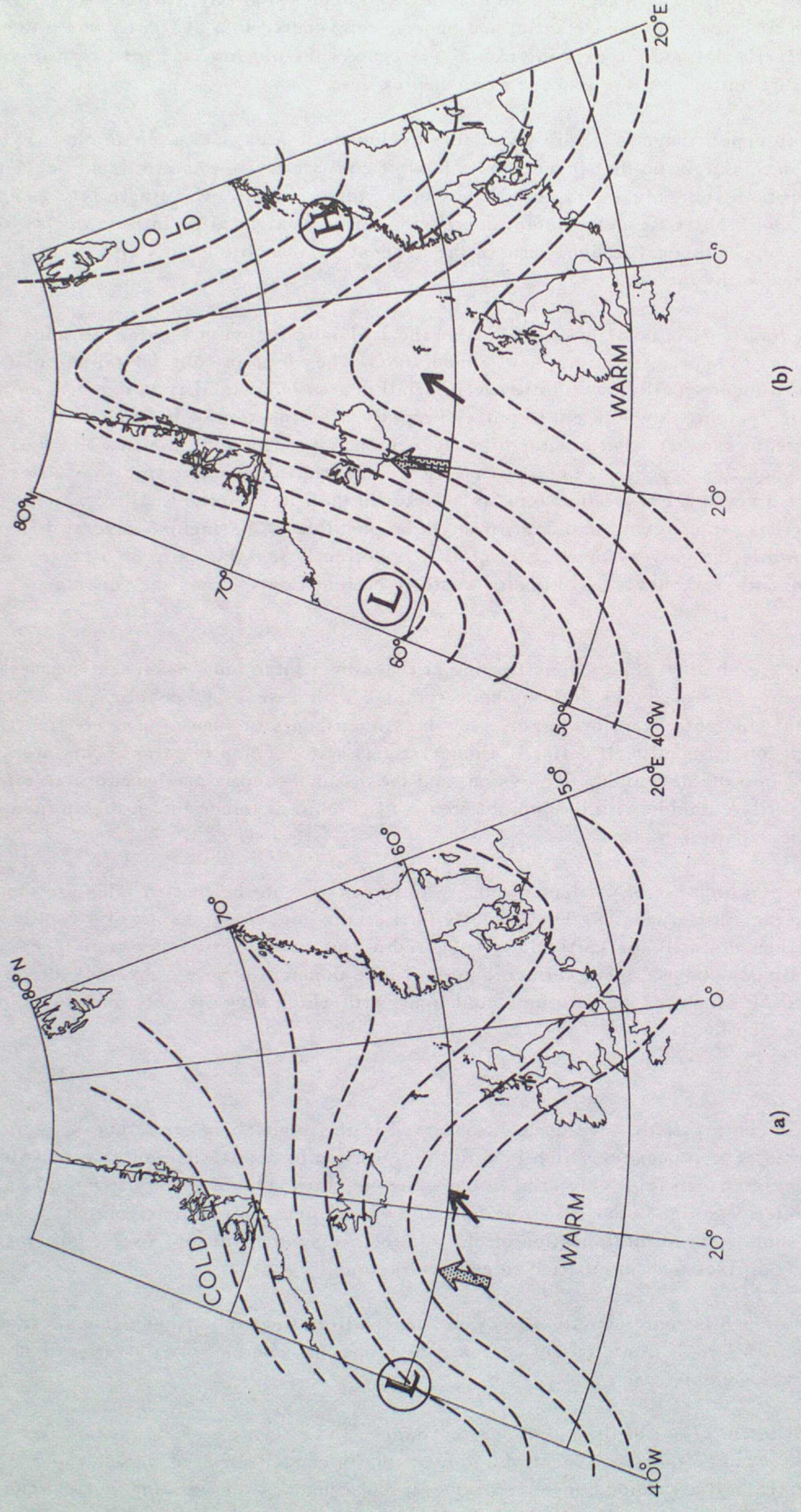


FIGURE 41. Schematic synoptic situation at (a) day D-1 and (b) day D

- - - 1000-500-mb thickness isopleths
⇧ Centre of 500-mb southerly current
↑ Centre of surface warm current

L = Surface low   H = Developing high

into a restricted area, and this release of latent heat continues for as long as the ascending motion continues. On the other hand, the subsidence associated with anticyclonic development, although it promotes desaturation of the air and the absorption of latent heat by the evaporating cloud particles, produces no further latent-heat effects once the clouds have been evaporated by continued subsidence. So, in the subsiding air masses associated with anticyclogenesis, the non-adiabatic temperature changes result from slowly operating radiation processes. These contrast with the far more potent latent-heat exchanges which occur in rising air.

In synoptic terms, three important types of situation have been identified which lead to the formation of new highs or ridges.

(a) *The amplification of a trough/ridge pattern in the upper westerlies:* This mechanism has been discussed by Sutcliffe,[34] Smith,[35] Miles[36] and others. Figure 41 is from Miles's paper, and illustrates the general situation when waves in the barotropic westerly current have started to increase in amplitude. Warm air is increasingly advected northwards, and cold air southwards, on either side of the developing troughs and ridges; and just as a pronounced plunge of cold air southwards favours cyclogenesis to the east of the cold trough, so a thrust of warm air northwards gives rise to anticyclonic building to the east of the warm ridge. Anticyclogenesis is therefore initiated by the development of a jet stream from a southerly point, with the anticyclone building in the right-exit region of the jet. Usually this also coincides with the 'A' development area ahead of the diffluent upper ridge which is the basis of the amplifying pattern.

Sutcliffe[34] noted that the anticyclonic developments occurring in association with an amplifying wave pattern in the upper westerly flow fell into two broad classes. On the one hand, if the baroclinic westerlies are bounded on the south by the subtropical Azores high, then the anticyclonic developments in the baroclinic zone were usually restricted to the formation of mobile ridges, of the type found between successive frontal depressions on the polar front. On the other hand, if there is low pressure to the south of the baroclinic westerlies, then the formation of a slow-moving, closed, anticyclonic vortex was a more frequent occurrence.

(b) *Anticyclonic disruption of a thermal trough:* This has been discussed by Sutcliffe,[34] Smith,[35] and others. It is illustrated in Figure 42, which is taken from Sutcliffe's work. The initial thickness pattern is a confluent trough.

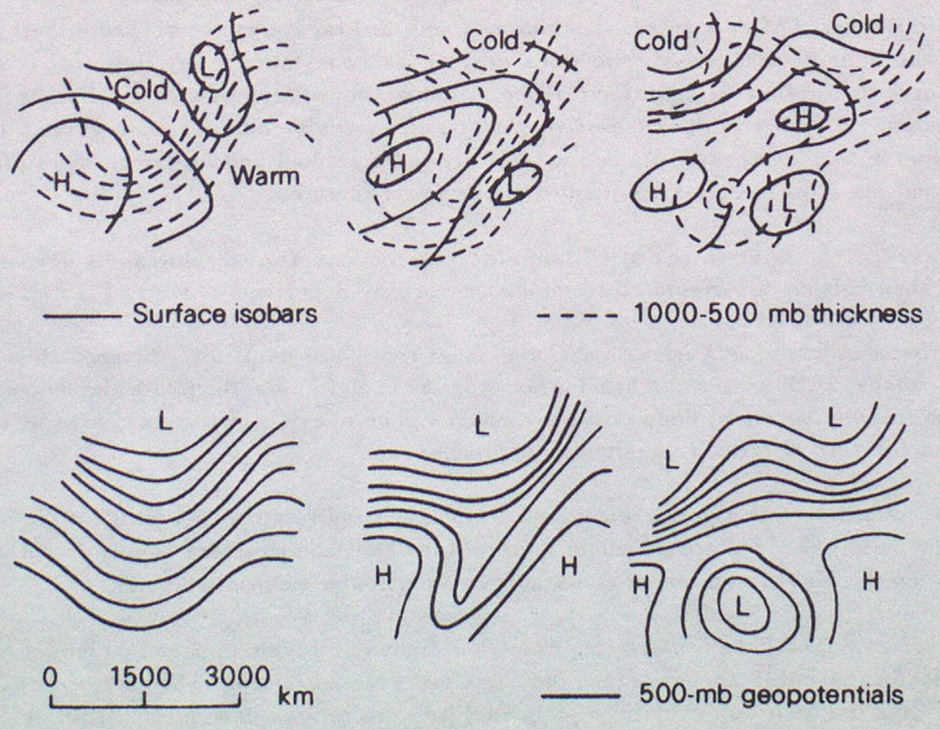


FIGURE 42. Anticyclonic disruption of a thermal trough

From time to time the oscillation of the baroclinic westerlies builds up to large amplitude and the current and the wave pattern disrupt. The stream divides, and two distinct wave-like distortions at different latitudes move at different speeds and rapidly become out of phase. The more rapid movement of the northern part of the trough is the usual occurrence, and this is termed 'anticyclonic disruption' from the sense of the relative shear between the two parts of the trough system. This process leads to anticyclonic building at the surface across the neck of the disrupting trough between the high- and low-latitude systems.

(c) *Diffluent ridge/confluent trough combination:* Haworth and Houseman[37] noted that this particular combination of standard thickness patterns, illustrated in Figure 43, produces a particularly favourable pattern for anticyclonic development at the points labelled A. It is important that the pattern should be more than just a transitory feature of the charts if it is to be effective. The pattern must be

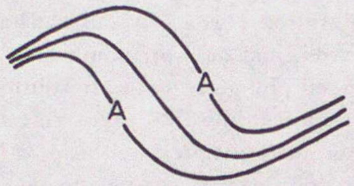


FIGURE 43. Combined diffluent ridge and confluent trough in thickness pattern

accompanied by a pattern of flow that makes the whole system self-maintaining and able to persist for at least a day or so. Haworth and Houseman argue that depressions approaching the diffluence tend to slow down, whilst those forming in the confluence accelerate Thus the separation between the depressions increases and sufficient space is created to allow a disruption of the mobile westerlies.

2.3.2.3 Decay of old systems.

(a) *Frontal depressions:* Depressions which develop from waves on fronts usually lose most of their frontal characteristics during the later stages of their evolution, when they are filling and decaying. In these closing stages of their life they approximate to deep vortices, often extending through much, or all, of the troposphere. After a depression has attained its lowest surface pressure there is often a period during which the central pressure remains more or less constant, before the process of filling commences. There then follows a sustained filling, although not necessarily at a constant rate, for many hours and sometimes for days in the case of very deep and extensive depressions. Where a cold pool is located right over a deep depression its decay is likely to be gradual and, although the surface low may fill, the cold pool sometimes retains its identity as an upper feature.

In some cases in the later stages of filling, a depression may start moving and carry its cold core with it. It may then become reinvigorated in the manner briefly described above in 2.3.2.2.

(b) *Non-frontal depressions:* The formation of most true non-frontal lows is more closely associated with physical causes, such as surface heating, or orographic deflexion than with the dynamical motions that result from the interaction of contrasting air masses. The decay of non-frontal lows is therefore closely associated with the cutting off of these formative processes.

Thus, heat lows decay at night, when inland surface heating is stopped. Polar lows gradually disappear when they move over land areas, where their surface heat and moisture supply is cut off. Orographic lows die out when the air flow no longer flows across a particular mountain barrier.

(c) *Anticyclones:* The decay of a large, well-established anticyclone usually takes a long time. Two or three days is probably about average but there are wide variations. The rate of weakening is not usually steady, and the high may even be rejuvenated for a while as a new anticyclonic cell develops and effectively absorbs the old one.

On a few occasions a high decays quite rapidly. Sawyer[38] indicated one type of situation when the central pressure of a high can fall very quickly. In this situation a thermal trough advances towards the anticyclone, which is initially associated with a diffluent thermal ridge. Douglas[39] also commented on the fact that anticyclones usually weaken rather slowly unless some external influence, usually an upper trough, comes into the picture.

It is pertinent to remark that ridges sometimes decay much more rapidly than anticyclones. This is particularly true of the more mobile, smaller ridges. These features may be of some importance, as they can bring short periods of relief during long spells of generally unsettled weather. The intensity of the ridge is rarely related very closely to its ability to persist, for some quite feeble patterns can be traced on charts covering a fairly long period. Broad ridges of a quasi-barotropic character are, however, often persistent features of a situation over several days.

(d) *Fronts:* Frontolysis, or the decay of fronts, is a complex result of interacting dynamical and physical processes. In so far as it is possible to pick out one feature of decaying fronts that is of general importance, this would probably be the effect of subsidence. Once the warm air mass involved at a front starts to subside, then the front becomes a kata-front and its activity diminishes, and a general subsidence in both air masses involved can reduce the upper-level temperature contrast across the frontal zone very quickly. Synoptic assessments of the vertical velocity field are very difficult and the indirect indications which other parameters give are not very reliable. For example, the mere commencement of building pressure on both sides of a front is not a sufficient condition for the onset of frontolysis. For between a warm and a cold high, the upper flow may remain parallel to the front, which retains its identity and can become an active feature when the next depression approaches. In such a case, even though pressure may be slowly rising, the thermal contrast across the front is maintained. However, persistent and prolonged rising pressure will normally indicate the presence of subsidence sufficiently strong to destroy the front. Further, if a front moves into an anticyclonic flow pattern and is orientated at a large angle to the upper flow there is often little thermal contrast in depth across the front and subsidence will, again, usually destroy such weak frontal characteristics as may still exist.

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APPENDIX

MATHEMATICAL OPERATIONS

1. Advection

The rate at which a quantity A is being advected at a point depends on both (a) the speed of the advecting wind, and (b) the distribution of A; in particular, the spatial gradient of A in the direction from which the wind is blowing. The rate of advection is then,

$$\text{in 1 dimension,} \quad u\frac{\partial A}{\partial x}$$

$$\text{or, in 2 dimensions,} \quad u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y}$$

$$\text{or, in 3 dimensions,} \quad u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z} .$$

2. Time derivatives

$\partial A/\partial t$ represents the rate of change of the quantity A measured at a fixed point. It is known as the 'local change', or 'Eulerian change'. For example, the rate of change of pressure at an observing station on land would be $\partial p/\partial t$.

$\mathrm{d}A/\mathrm{d}t$ represents the rate of change of the quantity A measured at successive positions of a moving air particle. It is known as the 'change following the motion', or 'Lagrangian change'. For example, the rate of change of pressure observed on a ship that happened to be moving with the same velocity as the air would be $\mathrm{d}p/\mathrm{d}t$.

The relationship between these two derivatives is:

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\partial A}{\partial t} + u\frac{\partial A}{\partial x} + v\frac{\partial A}{\partial y} + w\frac{\partial A}{\partial z} ,$$

which shows that the change of A following the motion is the sum of the local change in the field of A together with the rate at which the field is being advected.

3. The Laplacian

This mathematical operator, which is designated by the symbol ∇^2, is defined in three dimensions by

$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} .$$

The nature of the terms, such as $\partial^2 A/\partial x^2 = \partial/\partial x\,(\partial A/\partial x)$, shows that the Laplacian represents the 'gradient of the gradient of A'. Therefore $\nabla^2 A = 0$ if either the field of A is uniform (A = constant), or if its gradient is constant (∇A = constant).