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The reduction of along-track errors in the predicted position of the NOAA satellites.

by

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THE REDUCTION OF ALONG-TRACK ERRORS IN THE PREDICTED POSITION OF THE NOAA SATELLITES

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Abstract:-

It is demonstrated that accumulated along-track errors in the predicted position of the NOAA satellites after ten days of over 60 km can occur if the SDC values of the mean motion $\overset{\prime}{M}$ and its acceleration $\overset{\prime\prime}{M}$ are used in the LOCAT computer subroutine. These errors reduce to less than 10 km if the values of $\overset{\prime\prime}{M}$ are discarded and replaced by zero while still using the SDC values of $\overset{\prime}{M}$. A further reduction in the error to 2 km in the along-track motion can be obtained by using improved values of $\overset{\prime}{M}$ derived from the SDC values of the time of crossing the ascending node.

1. Introduction

The NOAA satellites carry three instruments for making observations of the earth's atmosphere and surface. These are the Vertical Temperature Profile Radiometer (VTPR), the Very High Resolution Radiometer (VHRR) and the Scanning Radiometer (SR). Before the observations made by these instruments can be used they must be located geographically. For the VTPR a knowledge of the latitude and longitude of the observation enables the retrieved temperature sounding to be incorporated in objective analyses, but for the VHRR and SR the main requirement is to enable change of projection of cloud images to be made and coastal outlines to be added.

Errors can occur in the location of observations for several reasons which can be grouped into two classes:-

i) The satellite is not positioned correctly because of an along-track error or a track longitude error.

ii) The satellite is not orientated correctly due to unknown yaw, pitch or roll motions.

Obviously it is desirable to locate the satellite observations to a precision approaching that of their size on the ground (VTPR ~ 60 km, SR ~ 5 km, VHRR ~ 1 km) though this condition can be relaxed if the observations are grouped together for processing.

The positions of satellite observations are predicted using the LOCAT subroutine (see Appendix 2) with orbital elements (ie numerical values of parameters which describe the orbit) obtained from the United States Air Force Space Defence Centre (SDC); these elements are usually about seven days old on arrival and are used for a further six days before being replaced by the next set. However Met O 22 have recently reported that coastal outlines calculated from these elements are frequently displaced from the true coastline as observed by the SR. This displacement appears to be at least partly due to along-track errors in the predicted satellite position arising from errors in the SDC values of the mean motion and its acceleration. In this memorandum a description is given of the results of applying a simple technique for obtaining more accurate values of the mean motion from the SDC ascending node crossing times.

2. <u>SDC orbital elements</u>

Each SDC orbital data set consists of the values of the following elements at a specified epoch t_0 which is the ascending node (south to north equatorial crossing) time for orbit n. The elements are

i) mean anomaly, M_0

ii) mean motion, M' ($'$ denotes $^{d}/dt$)

iii) acceleration of the mean motion, M'' ($''$ denotes d^2/dt^2)

iv) eccentricity, e)

v) inclination, i)

vi) argument of perigee ω_0

vii) right ascension of the ascending node Ω_0

(this last element is included for completeness but does not enter into the work described in this memorandum). The elements M', M'', e and i can be regarded as being constant over intervals of a few days.

At any instant t the satellite travels in a plane defined by i its inclination to the earth's equatorial plane. The satellite moves in an elliptical orbit of eccentricity e (with the earth at one focus) orientated so that the angle from the ascending node to the perigee of the orbit measured positively in the direction of motion is ω, the argument of perigee. Because of the non-spherical gravitational field of the earth the perigee moves around the orbit secularly at a rate ω' so that we may write

$$\omega_t = \omega_0 + (t-t_0).\omega'$$

The expression for ω' in terms of the other orbital elements and the constants defining the gravitational field is given in Appendix 1.

The position of the satellite within the orbit at time t is given by its mean anomaly M_t where

$$M_t = M_0 + (t-t_0)M' + \tfrac{1}{2}(t-t_0)^2 M'' \quad \ldots \quad (1)$$

M increases uniformly from 0 to 2π in one anomalistic period P_A which is the time interval between successive passages of the satellite through perigee. Thus P_A at time t is given by

$$P_A = \dot{M}_t^{-1} = (\dot{M} + (t - t_0).\ddot{M})^{-1}$$

The values of the elements M, e and ω at time t are used to calculate the position of the satellite in its orbit with the equations

$$u = v + \omega \qquad (2)$$

$$\text{where } \tan\left(\frac{v}{2}\right) = \left(\frac{1+e}{1-e}\right)^{\frac{1}{2}}.\tan\left(\frac{E}{2}\right)$$

$$E - e\sin E = M$$

and E is the eccentric anomaly.

V, the true anomaly, and u, the argument of latitude, are angles between the satellite and perigee and the satellite and ascending node respectively; both v and u are measured positively in the direction of motion. For near circular orbits (e small) v and M are related by the equation

$$v = M + 2e\sin M + \text{terms of } O(e^2, e^3 \ldots) \qquad (3)$$

The further formulae (also embodied in the LOCAT computer subroutine) for calculating the latitude and longitude of the satellite from u, i, Ω and the sidereal angle of the earth in space are omitted here. The elements i, ω, v and u are shown in figure 1.

Each set of SDC elements are calculated from radar observations of the direction, range and range-rate of a satellite over a relatively short interval of time; it is not known definitely how long this interval is but it is believed to be only a few hours or 2-3 orbits. As a consequence the Space Defence Centre do warn users that the elements are intended only to represent the motion of the satellite during the period of the observations and that their indiscriminate use for prediction purposes is not recommended.

Now May (ref 1) has suggested that P_A (and ω) are inherently difficult elements to determine anyway, particularly for a near-circular orbit, because of the problem of identifying the position of perigee. This is demonstrated for P_A in figure 2 in which the SDC values of $\dot{M}$ in revs.day^{-1} for NOAA 4 Satellite ($e \simeq 0.0008$) are plotted against orbit number. There is a large scatter (some points lie right outside the area of the diagram) which almost completely conceals a slow decrease of $\dot{M}$ with time. It is suggested that these SDC values of $\dot{M}$ are unreliable but that alternative improved values can be calculated using a method which is now described.

3. Improvement of Values of $\dot{M}$ and $\ddot{M}$

Observations of satellites are made from sites which are fixed with respect to the earth but not with respect to the (rotating) perigee of the orbit. May (ref 1) has therefore suggested that an alternative mean motion $\dot{M}_N$, of the satellite relative to a fixed point on the earth can be determined more accurately than $\dot{M}$, and that $\dot{M}$ should be derived from $\dot{M}_N$.

Now $\dot{M}$ and $\dot{M}_N$ are related by

$$\dot{M} = \dot{M}_N \left\{ 1 - \frac{\Delta\omega}{2\pi} \frac{(1-e^2)^{3/2}}{(1+e\cos(u-\omega))^2} \right\}$$

where $\Delta\omega$ is the secular change of ω in each orbit (ref 2). Apart from orbits with the critical inclination $i = 63^\circ.4$, for which $\omega = 0$, $\dot{M} \neq \dot{M}_N$. In this memorandum we are concerned with the mean motion referred to the ascending node ($u = 0$) so that

$$\dot{M} = \dot{M}_N \left\{ 1 + \frac{\Delta\omega}{2\pi} \frac{(1-e^2)^{3/2}}{(1+e\cos\omega)^2} \right\} \quad \text{------} \quad (4)$$

where M'_N is simply called the nodal mean motion, with its related nodal period $P_N = M_N'^{-1}$.

In reference 1 it was demonstrated that suitable mean values of M'_N could be calculated from values of ascending node crossing time t_n deduced from visual observations of satellites on two orbits n_1 and n_2 using the formula

$$M'_N = P_N^{-1} = \frac{(n_1 - n_2)}{(t_{n_1} - t_{n_2})} \qquad (5)$$

where the value of M'_N is ascribed to the mean epoch $(t_{n1} + t_{n2})/2$. Values of M'_N obtained in the way from the successive values of t_n tabulated in the SDC element sets for NOAA 4 satellite are plotted in figure 3. They lie on a well-defined curve with a marked periodicity associated with the rotation of the perigee around the orbit (values of ω are also shown in this figure). The magnitude of the scatter of the points about the smooth curve, which is much less than the SDC values of M' in figure 2, suggests that the t_n have a relative accuracy of about 0.1 second (or 0.6 km in along-track travel). These values of M'_N have been converted to M' using equation 4 with a numerical value of $\Delta\omega/2\pi = 4.28551 \times 10^{-4}$ (see appendix 1), and are plotted in figure 2 for comparison with the SDC values. The conversion has eliminated the periodicity in ω leaving an approximately linear variation of M' with time through the centre of the more widely-scattered SDC values. The results are sufficiently self-consistent to enable some values of M' to be discarded because they are derived from obviously incorrect values of t_n; these values of t_n usually occur in element sets which are rapidly followed by the next issue of elements.

It can be argued that the values of M' obtained using equations 4 and 5 are no more accurate, relatively, than the SDC values bearing in mind the different time intervals over which they are deduced (a few days compared with a few hours). However this can be disproved; mean values of M' calculated directly from successive SDC values of M_o using the formula

$$M' = \frac{(M_o(t_{n_1}) - M_o(t_{n_2})) + 2\pi(n_1 - n_2)}{(t_{n_1} - t_{n_2})}$$

have a large scatter as the SDC values of M' in figure 2.

The underlying process described in this memorandum to predict the along-track position more accurately is one of forward extrapolation from accurate ascending node crossing times. This essentially involves the nodal period P_N which, as has been demonstrated can be deduced from observations with a greater accuracy than P_A; it is natural therefore, to ask why P_A is used at all in the SDC orbital theory. The orbital energy of a satellite is proportional to the semi-major axis, a, which is constant in the absence of perturbing forces such as air drag, solar radiation pressure and external gravitational influences. If a $\propto P_N^{2/3}$ (Kepler's law) then it would vary with ω as in figure 3. The semi-major axis, which is required to satisfy the distance information in the observations, is assumed by SDC to vary with $P_A^{2/3}$.

The SDC values of M'' for NOAA 4 satellite are plotted in figure 4 for comparison with the mean values indicated by the near-linear variation of M'' in figure 2. In contrast to the random behaviour of the SDC values of M', the values of M'' show a systematic "sawtooth" variation with an improbably large maximum (negative) value of -3.3×10^{-5} revs. day^{-2} on orbit 7140. In comparison the values from the linear variations in figure 2 are -1.9×10^{-7} and -1.2×10^{-7} revs. day^{-2} before and after orbit 6400. Clearly M' is decreasing with time (P_A increasing) so that this change cannot be attributed to air-dray which always reduces P_A. An alternative explanation is that it is caused by solar-radiation pressure on the gravitational effect of the sun and moon. It is noticeable that discontinuities in the SDC values of M'' occur close to the times at which $\omega = 180^\circ$ but otherwise there is no obvious relationship between the two. It is the author's opinion that these values of M'' are fictitious and should be discarded.

In summary it is suggested that the method described here produces a variation of M' with time which can be extrapolated with confidence in order to provide better estimates of M' (and M'') to replace those in later sets of SDC elements. The reductions in along-track error to be gained in this way are described in the next section.

4. Along-track errors resulting from the use of SDC values of $\dot{M}$ and $\ddot{M}$

If it is assumed that the smooth variations of $\dot{M}$ and $\ddot{M}$ in figures 2 and 4 represent the true motion of the satellite when used with equations 1, then it is possible to calculate typical errors in the predicted along-track positions arising from errors in the SDC values of $\dot{M}$ and $\ddot{M}$.

The SDC orbital elements are used to predict positions within an interval from about 7 to 13 days after the epoch. Adopting an average prediction interval of 10 days for simplicity the errors $\Delta \ell$ in along-track position arising from errors $\Delta \dot{M}$ and $\Delta \ddot{M}$ are given approximately by

$$\Delta \ell_1 = \Delta \dot{M} \times 10 \times 40{,}000 \text{ km}$$

$$\Delta \ell_2 = \tfrac{1}{2} \Delta \ddot{M} \times 100 \times 40{,}000 \text{ km}$$

From 64 sets of SDC elements for NOAA 4 satellite $\Delta \ell_1$ has a mean value of 0.4 km (a $-$ve sign indicates that the satellite lags behind the predicted position) and a standard deviation of 6.8 km. The mean value of $\Delta \ell_2$ is -16 km with a maximum value of -66 km (orbit 7140).

These can be compared with corresponding results from the 63 values of $\dot{M}$ deduced from $\dot{M}_N$ in the manner previously described for which $\Delta \ell_1 = +0.1$ km with a standard deviation of 1.5 km. From these figures we can conclude that an immediate benefit can be gained simply by discarding the SDC values of $\ddot{M}$ and replacing them by zero while retaining the values of $\dot{M}$. This will result in a random along-track error which is likely to be less than $\pm$ 10 km after a prediction period of 10 days. However a greater improvement can be obtained by discarding the SDC values of both $\dot{M}$ and $\ddot{M}$ and replacing them by predicted values obtained in the way described here, resulting in an error of less than + 2 km.

A mention should be made of the influence of errors in the initial position of the satellites as given by M_o. In view of the remarks made previously about the difficulty of specifying the position of the satellite with respect to its perigee it may seem dangerous to suggest that errors in the initial position are likely to be negligible. The satellite is on the equator at the epoch (R.H. Gooding, private communication) so that $u=0$ and hence an error in M is related to an error in ω by $\Delta M_o \; (\simeq \Delta \nu_o \;) \simeq \; -\Delta\omega_o$ (equation 2 and 3). Thus if the SDC values of M_o and ω_o (from the same data set) are used to predict the future value of M_t, ω_t and hence u_t then the errors in M_o and ω_o are complementary and cancel out (ref 1).

5. Application to NOAA 5 satellite

NOAA 4 satellite has been used as an example in this work because a long sequence of SDC elements was available for this object and the benefits of the method advocated here could be more clearly seen in the results. NOAA 5 is the present operational polar-orbiting meteorological satellite; figures 5 and 4 show the SDC tabulated values of $\overset{\prime}{M}$ and $\overset{\prime\prime}{M}$ for this satellite, along with those deduced from the P_N using a value of $\Delta\omega/2\pi = 4.14763 \times 10^{-4}$. The remarks made previously concerning the scatter of the SDC values of $\overset{\prime}{M}$ and the anomalous behaviour of $\overset{\prime\prime}{M}$ for NOAA 4 apply equally well to NOAA 5. Again a more well-defined variation of $\overset{\prime}{M}$ with time results from using the method described. For NOAA5 satellite the recommended value of $\overset{\prime}{M}$ for Jan 1.0 1977 is 12.383026 revs. day^{-1} and of $\overset{\prime\prime}{M}$ is -1.5×10^{-7} revs. day^{-2}.

6. An alternative source of orbital data

The Meteorological Office receives regularly the TBUS bulletins which are distributed daily by NESS via the GTS. Part 4 of each bulletin consists of a set of orbital elements (which are at least 30 days old) containing apparently similar information to the SDC elements but without a value for $\overset{\prime\prime}{M}$. It is not known what orbital theory is used but it is not identical to the SDC theory because the values of $\overset{\prime}{M}$ from the two sources differ by about 0.0057 revs. day^{-1} which is equivalent to a difference of about 3 seconds in P_A. Thus the TBUS elements cannot be substituted directly for SDC elements in the LOCAT subroutine. Part 1 of each bulletin consists of the ascending node crossing time and longitude of one specified reference orbit each day, along with the nodal period (to the nearest second) and the longitude difference between successive equatorial crossings; these are predicted from the elements contained in Part 4 of the bulletin. These enable the user to calculate the times and longitudes of equatorial crossings for all other orbits on that day while the path of the satellite between equatorial crossings is summarised in tabular form in Parts 2 and 3 of each bulletin. A comparison of the predicted equatorial crossing times for the TBUS daily reference orbits with those from the SDC elements for NOAA 4 and NOAA 5 satellites reveal differences of up to $\pm$ 2 seconds or 12 km in along-track position.

Thus the information in Parts 1, 2 and 3 of the TBUS bulletins can be used to predict the position of the space craft with an acceptable accuracy but in a cumbersome and inefficient way if large numbers of locations have to be calculated. In comparison the LOCAT subroutine requires only the seven SDC orbital elements to be renewed roughly about every seven days and can attain a comparable accuracy if improved values of M' and M'' are used.

7. References:-

1. The use of directional observations to give accurate orbital decay rates for studies of upper-air density.

   by B.R. May. Plan. Spa. Sci. 19, 685 (1971)

2. The orbital periods of revolution of a satellite.

   by D.E. Smith. Plan. Spa. Sci. 13, 1283 (1965)

Appendix 1

Expression for the secular change $\dot{\omega}$ of the argument of perigee

$$\dot{\omega} = 0.5\,\tau(5\cos^2 i - 1)$$

where $\tau = 1.5\,\dot{M}\,J_2\,(R/a(1-e^2))^2$

$J_2 = 1.08268 \times 10^{-3}$ (constant of the earth's non-spherical gravitational field)

$R = 6378.163$ Km (earth's equatorial radius)

$a = a_0 + a_1$ (a is the semi-major axis of the orbit in km)

$a_0 = (\mu/M^2)^{\frac{1}{3}}$ km (Keplers' Law)

$\mu = 398601.3$ km^3 sec^{-2}

$a_1 = 0.75\,a_0\,J_2\,(R/a_0)^2(1-e^2)^{-3/2}\,(3\cos^2 i - 1)$ km

For both NOAA4 and 5 satellites e undergoes a periodic oscillation between about 0.0008 and 0.001 associated with the motion of perigee around the orbit and caused by the north-south asymmetry of the earth's gravitational field. This produces a negligible oscillation in $\dot{\omega}$ so that constant values of $\Delta\omega/2\pi$ can be used. These values are -4.28551×10^{-4} and -4.14763×10^{-4} for NOAA 4 and 5 respectively.

Appendix 2

The LOCAT subroutine

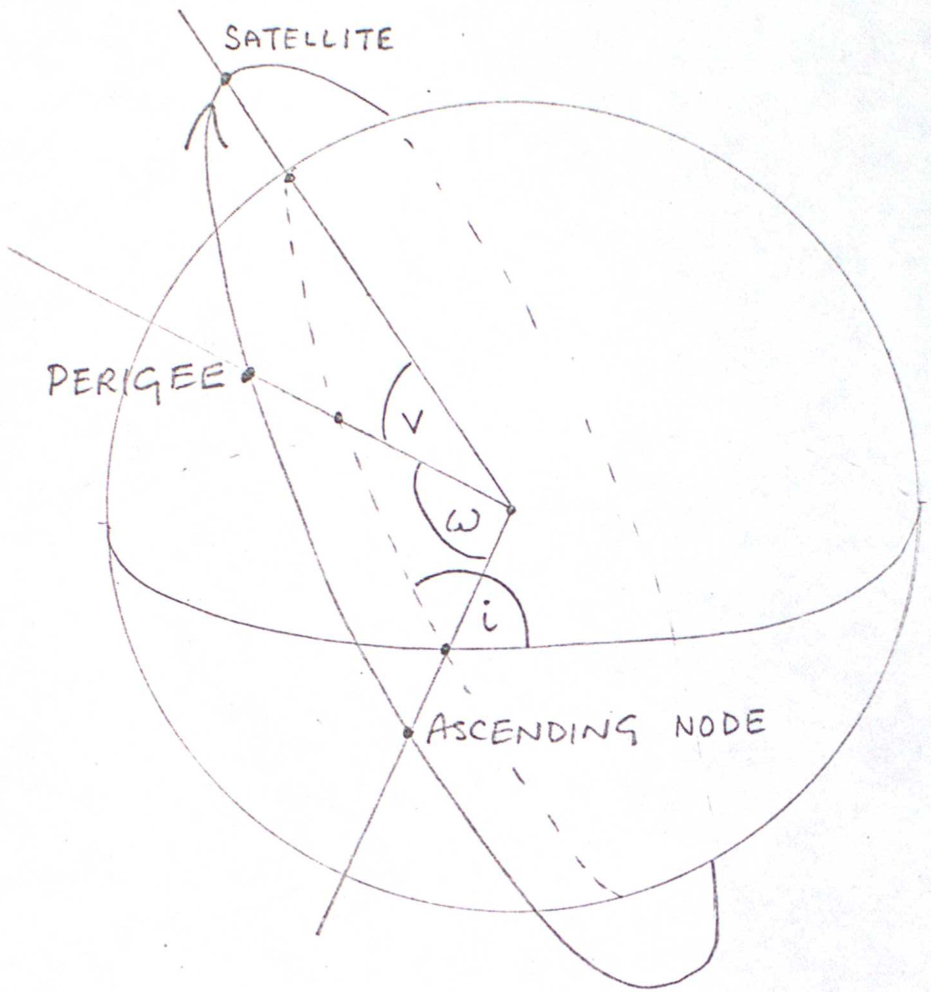
This subroutine was written to locate observations made by a satellite-borne scanning radiometer. The routine calculates the intersection point on the earth's surface of a line from the satellite making an angle θ to the vertical and in a plane perpendicular to the orbit plane passing through the centre of the earth. The position and height of the satellite at the specified time are calculated using SDC orbital elements.

The subroutine may be obtained from the following libraries:-

i) Member JLOCAT on M19.NSRCELIB on METO09 for a source listing.

ii) Member VLOCAT on M19.OBJLIB.NONRES on METO09 for the object module.

Queries concerning this subroutine and requests for SDC elements should be addressed to Met O 19.



ARGUMENT OF LATITUDE, $u = v + \omega$

FIGURE 1 ELEMENTS u, v, ω AND i OF A SATELLITE ORBIT

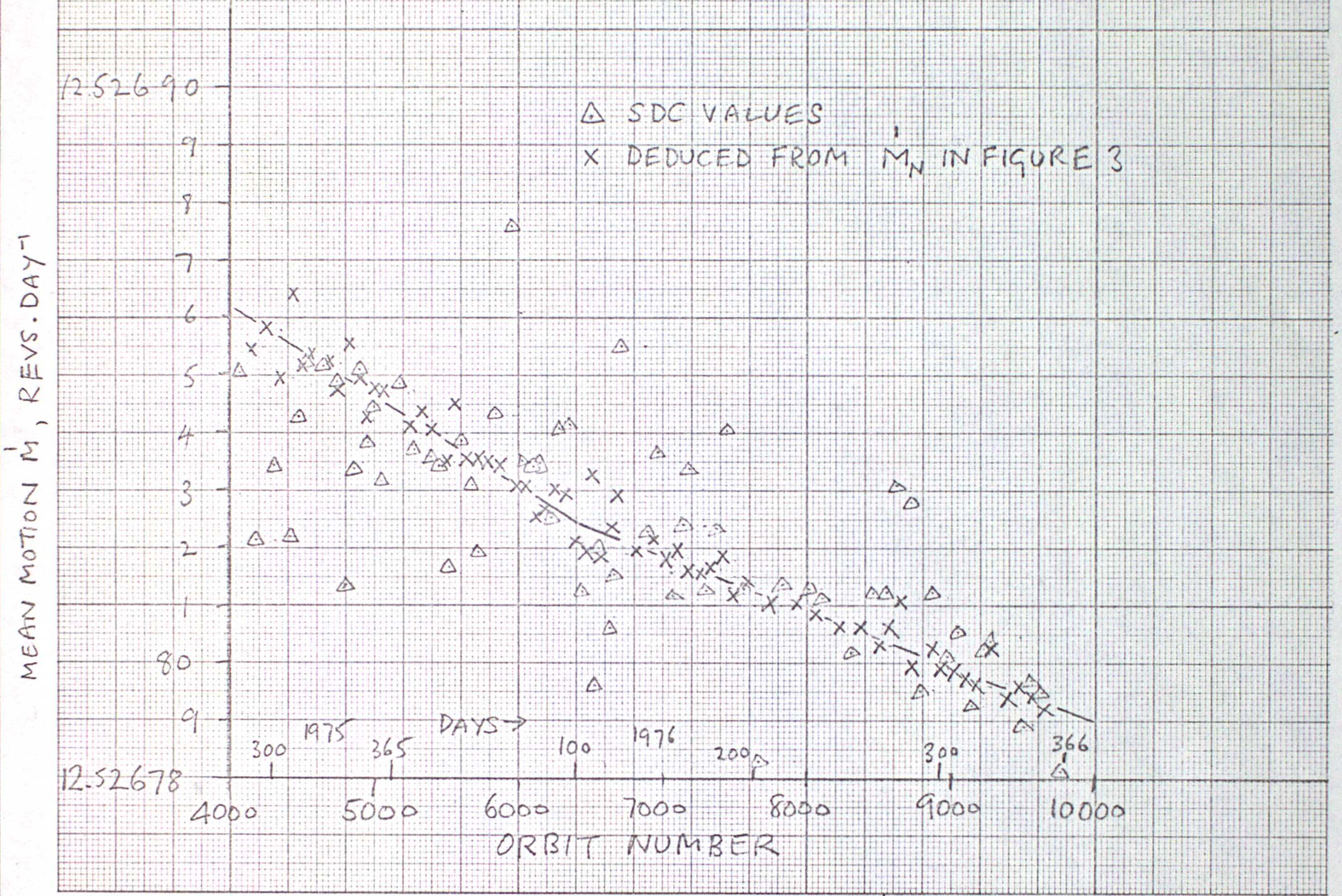


FIGURE 2 VARIATION OF MEAN MOTION $\dot{M}$ FOR NOAA4 SATELLITE.

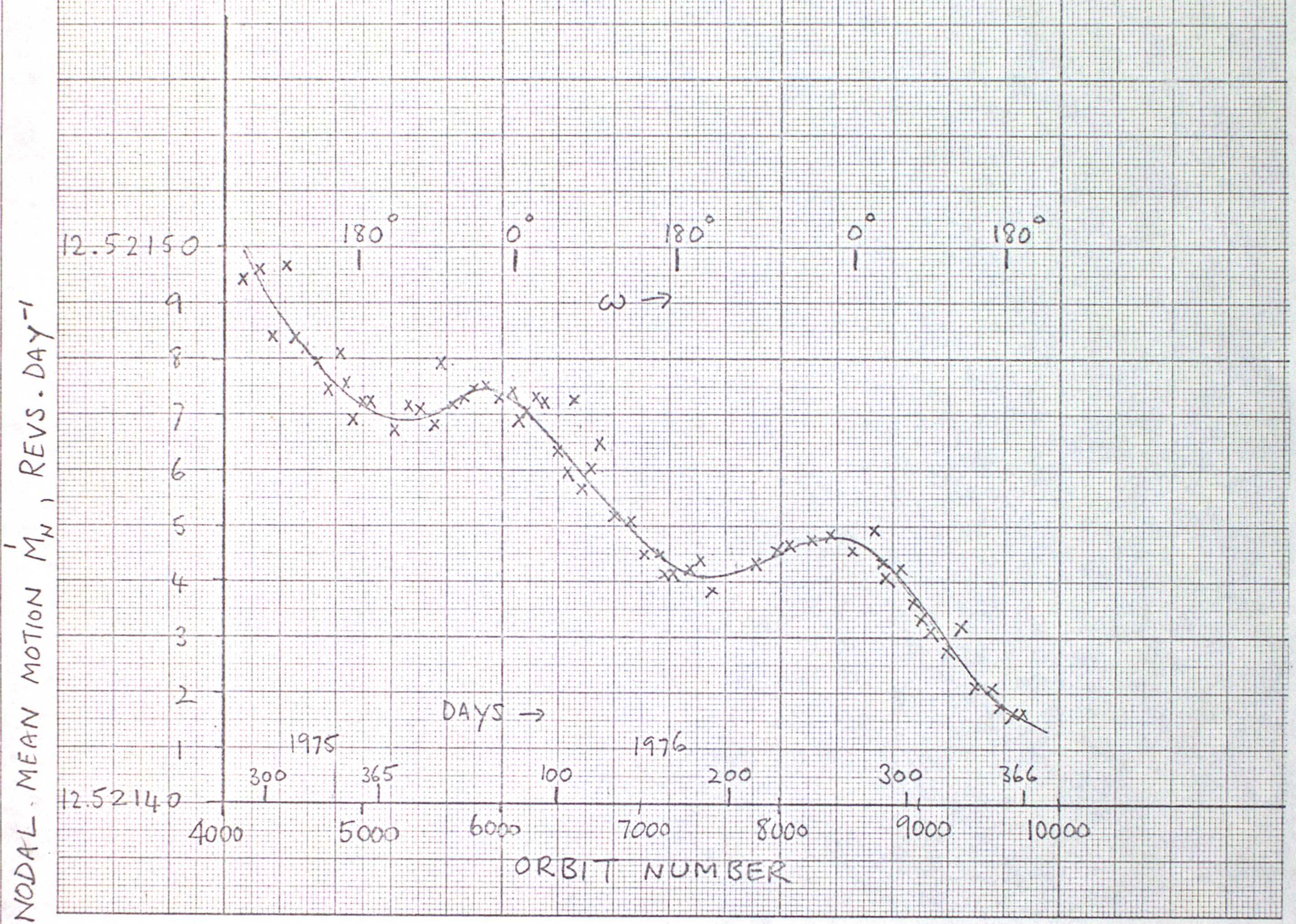


FIGURE 3 VARIATION OF $\dot{M}_N$ FOR NOAA 4 SATELLITE.

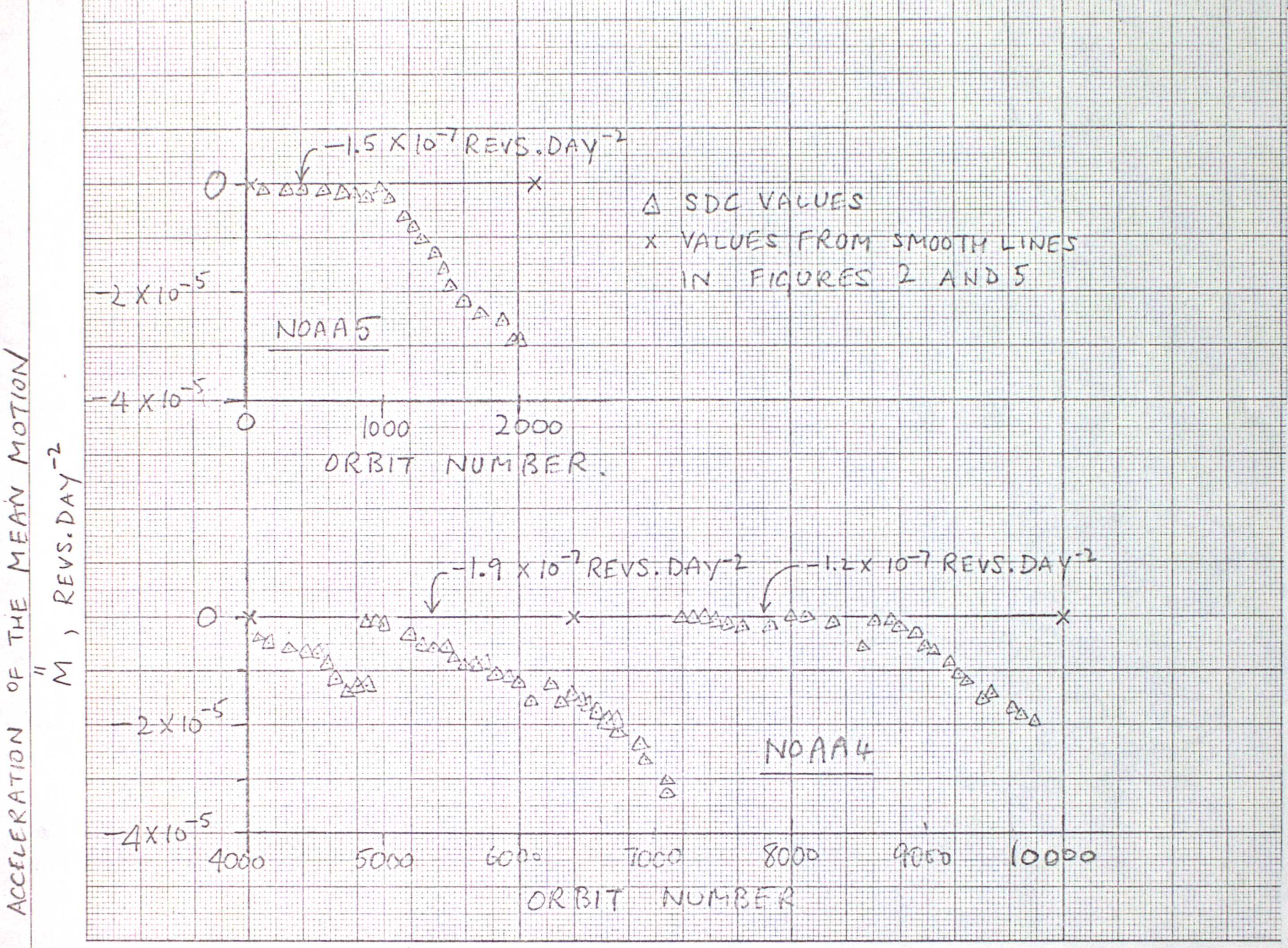


FIGURE 4 VARIATION OF THE ACCELERATION OF THE MEAN MOTION $\ddot{M}$ FOR NOAA 4 AND 5 SATELLITES

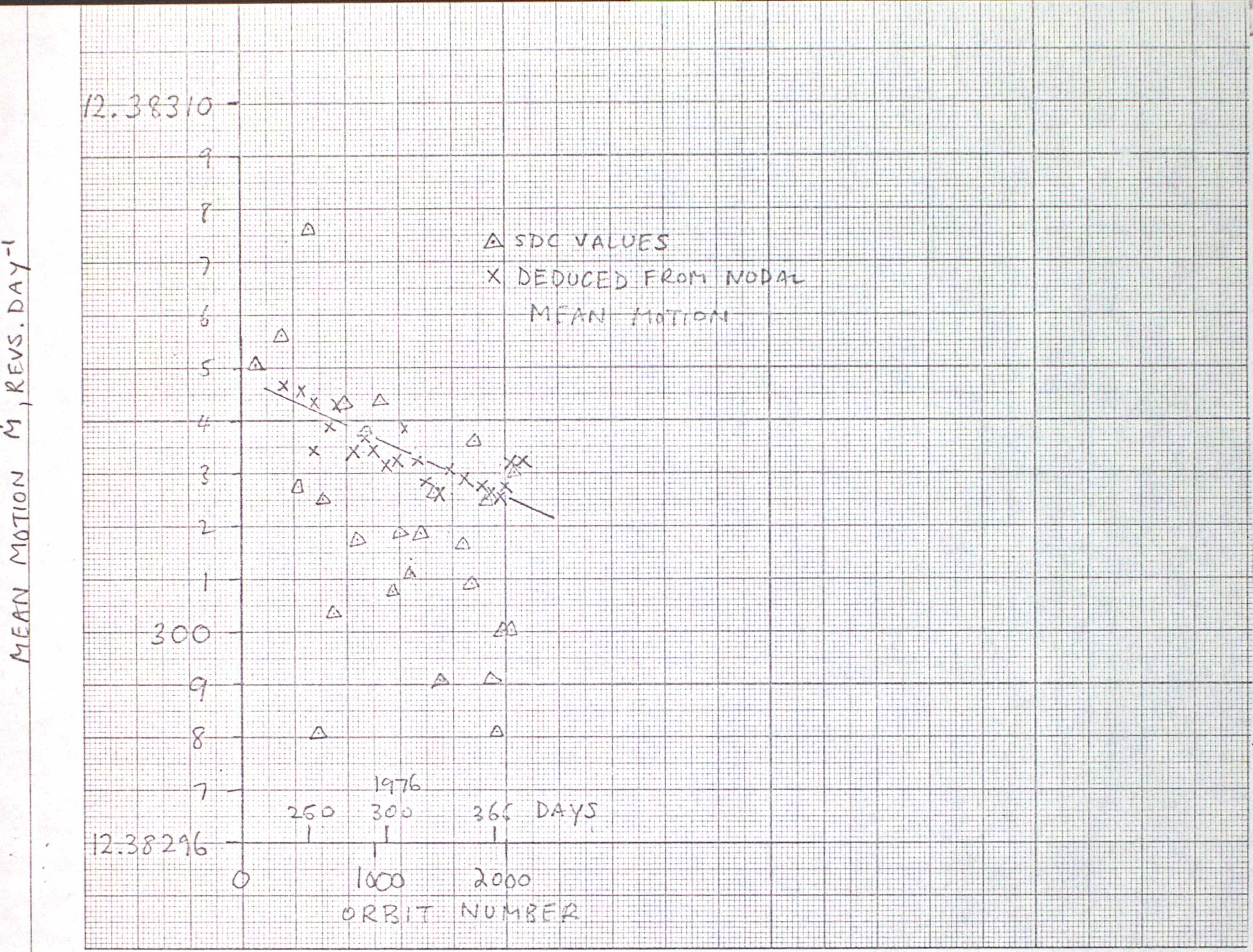


FIGURE 5 VARIATION OF $\dot{M}$ FOR NOAA5 SATELLITE